This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

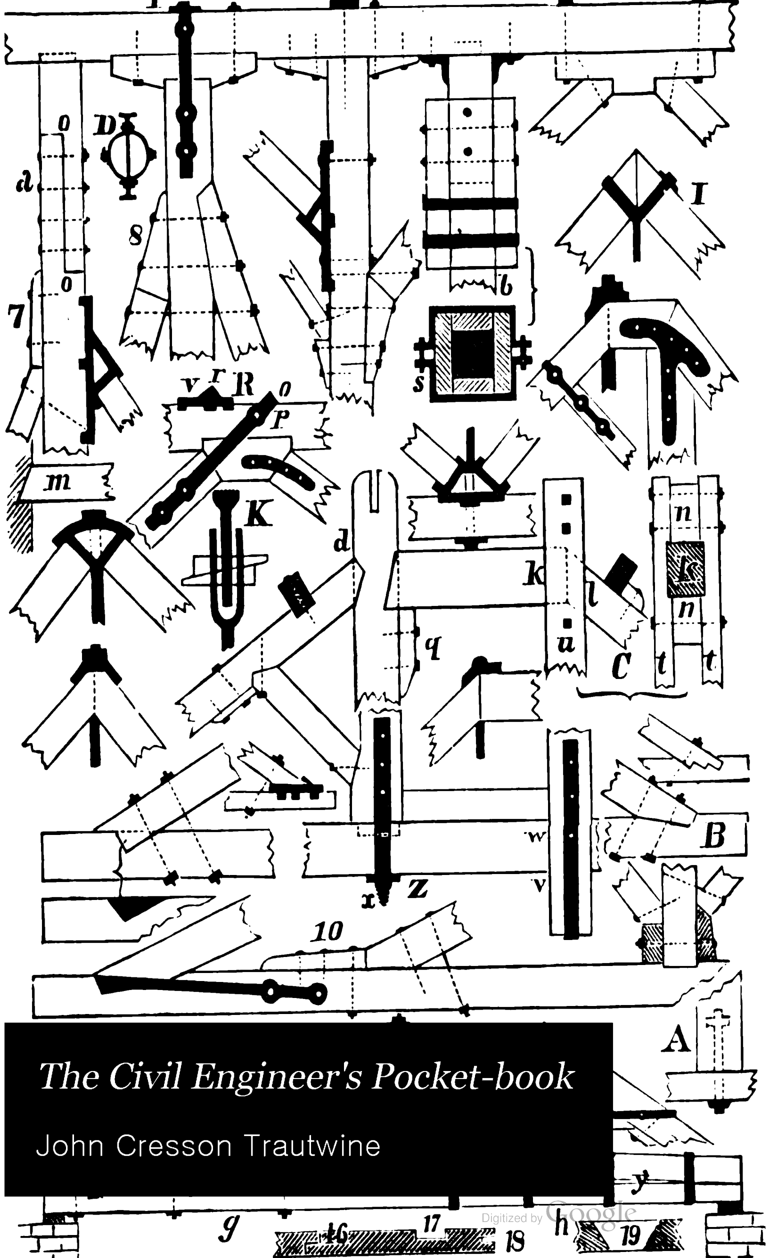
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



The Civil Engineer's Pocket-book

John Cresson Trautwine

ENGINEERING LIBRARY
OF
WILLIAM B. STOREY
A GRADUATE OF
THE COLLEGE OF MECHANICS
CLASS OF 1881
PRESENTED TO THE UNIVERSITY
1922

Gift

GIFT OF

SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
FIAT LVX
MDCCCLXVIII

EX LIBRIS

THE

CIVIL ENGINEER'S POCKET-BOOK

BY

JOHN C. TRAUTWINE

CIVIL ENGINEER

REVISED BY

JOHN C. TRAUTWINE, JR.

AND

JOHN C. TRAUTWINE, 3D.

CIVIL ENGINEERS

EIGHTEENTH EDITION, EIGHTY-FOURTH THOUSAND

NEW YORK
JOHN WILEY & SONS
LONDON: CHAPMAN & HALL, LIMITED
1906

TA151
T7
1906

Entered, according to Act of Congress, in the year 1882, by
JOHN C. TRAUTWINE,
in the Office of the Librarian of Congress at Washington.

Copyright by JOHN C. TRAUTWINE, JR., 1902.

WM. F. FELL COMPANY
ELECTROTYPERS AND PRINTERS
PHILADELPHIA

WM. RUTTER COMPANY
BINDERS
PHILADELPHIA

THE AUTHOR

DEDICATES THIS BOOK

TO THE MEMORY OF HIS FRIEND,

THE LATE

BENJAMIN H. LATROBE, Esq.,

CIVIL ENGINEER.

501816

No pains have been spared to maintain the position of this as the foremost Civil Engineer's Pocket-book, not only in the United States, but in the English language.

JOHN WILEY & SONS,
SCIENTIFIC PUBLISHERS,
43 East Nineteenth Street, New York City.

PREFACE

TO FIRST EDITION, 1872.

SHOULD experts in engineering complain that they do not find anything of interest in this volume, the writer would merely remind them that it was not his intention that they should. The book has been prepared for *young* members of the profession; and one of the leading objects has been to elucidate, in plain English, a few important elementary principles which the savants have enveloped in such a haze of mystery as to render pursuit hopeless to any but a confirmed mathematician.

Comparatively few engineers are good mathematicians; and in the writer's opinion, it is fortunate that such is the case; for nature rarely combines high mathematical talent, with that practical tact, and observation of outward things, so essential to a successful engineer.

There have been, it is true, brilliant exceptions; but they are very rare. But few even of those who have been tolerable mathematicians when young, can, as they advance in years, and become engaged in business, spare the time necessary for retaining such accomplishments.

Nearly all the scientific principles which constitute the foundation of civil engineering are susceptible of complete and satisfactory explanation to any person who *really* possesses only so much elementary knowledge of arithmetic and natural philosophy as is *supposed* to be taught to boys of twelve or fourteen in our public schools.*

---

* Let two little boys weigh each other on a platform scale. Then when they balance each other on their board see-saw, let them see (and measure for themselves) that the lighter one is farther from the fence-rail on which their board is placed, in the same proportion as the heavier boy outweighs the lighter one. They will then have learned the grand principle of the *lever*. Then let them measure and see that the light one see-saws farther than the heavy one, in the same proportion; and they will have acquired the principle of *virtual velocities*. Explain to them that *equality of moments* means nothing more than that when

The little that is beyond this, might safely be intrusted to the savants. Let them work out the results, and give them to the engineer *in intelligible language.* We could afford to take their words for it, because such things are their specialty; and because we know that they are the best qualified to investigate them. On the same principle we intrust our lives to our physician, or to the captain of the vessel at sea. Medicine and seamanship are their respective specialties.

If there is any point in which the writer may hope to meet the approbation of proficients, it is in the accuracy of the tables. The pains taken in this respect have been very great. Most of the tables have been entirely recalculated expressly for this book; and one of the results has been the detection of a great many errors in those in common use. He trusts that none will be found exceeding one, or sometimes two, in the last figure of any table in which great accuracy is required. There are many errors to that amount,

---

they seat themselves at their measured distances on their see-saw, *they balance each other.* Let them see that the weight of the heavy boy, when multiplied by his distance in feet from the fence-rail amounts to just as much as the weight of the light one when multiplied by his distance. Explain to them that each of the amounts is in *foot-pounds.* Tell them that the lightest one, because he see-saws so much faster than the other, will bump against the ground just as hard as the heavy one; and that this means that *their momentums are equal.* The boys may then go in to dinner, and probably puzzle their big lout of a brother who has just passed through college with high honors. They will not forget what they have learned, for they learned it *as play,* without any ear-pulling, spanking, or keeping in. Let their bats and balls, their marbles, their swings, &c, once become their philosophical apparatus, and children may be taught (*really* taught) many of the most important principles of engineering before they can read or write. It is the ignorance of these principles, so easily taught even to children, that constitutes what is popularly called "THE PRACTICAL ENGINEER;" which, in the great majority of cases, means simply an ignoramus, who blunders along without knowing any other reason for what he does, than that he has seen it done so before. And it is this same ignorance that causes employers to prefer this practical man to one who is conversant with principles. They, themselves, were spanked, kept in, &c, when boys, because they could not master leverage, equality of moments, and virtual velocities, enveloped in x's, p's, Greek letters, square-roots, cube-roots, &c, and they naturally set down any man as a fool who could. They turn up their noses at science, not dreaming that the word means simply, *knowing why.* And it must be confessed that they are not altogether without reason; for the savants appear to prepare their books with the express object of preventing purchasers, (they have but few *readers,*) from learning why.

especially where the recalculation was very tedious, and where, consequently, interpolation was resorted to. They are too small to be of practical importance. He knows, however, the almost impossibility of avoiding larger errors *entirely;* and will be glad to be informed of any that may be detected, except the final ones alluded to, that they may be corrected in case another edition should be called for. Tables which are absolutely reliable, possess an intrinsic value that is not to be measured by money alone. With this consideration the volume has been made a trifle larger than would otherwise have been necessary, in order to admit the stereotyped sines and tangents from his book on railroad curves. These have been so thoroughly compared with standards prepared independently of each other, that the writer believes them to be absolutely correct.

In order to reduce the volume to pocket-size, smaller type has been used than would otherwise have been desirable.

Many abbreviations of common words in frequent use have been introduced, such as abut, cen, diag, hor, vert, pres, &c, instead of abutment, center, diagonal, horizontal, vertical, pressure, &c. They can in no case lead to doubt; while they appreciably reduce the thickness of the volume.

Where prices have been added, they are placed in footnotes. They are intended merely to give an approximate or comparative idea of value; for constant fluctuations prevent anything farther.

The addresses of a few manufacturing establishments have also been inserted in notes, in the belief that they might at times be found convenient. They have been given without the knowledge of the proprietors.

The writer is frequently asked to name good elementary books on civil engineering; but regrets to say that there are very few such in our language. "Civil Engineering," by Prof. Mahan of West Point; "Roads and Railroads," by the late Prof. Gillespie; and the "Handbook of Railroad Construction," by Mr. George L. Vose, Civ. Eng. of Boston, are the best. The writer has reason to know that a new edition of the last, now in press, will be far superior to all predecessors; and better adapted to the wants of the young engineer than any book that has appeared.

Many of Weale's series are excellent. Some few of them are behind the times; but it is to be hoped that this may be rectified in future editions. Among pocket-books, Haswell, Hamilton's Useful Information, Henck, Molesworth, Nystrom, Weale, &c, abound in valuable matter.

The writer does not include Rankine, Moseley, and Weisbach, because, although their books are the productions of master-minds, and exhibit a profundity of knowledge beyond the reach of ordinary men, yet their language also is so profound that very few engineers can read them. The writer himself, having long since forgotten the little higher mathematics he once knew, cannot. To him they are but little more than striking instances of how completely the most simple facts may be buried out of sight under heaps of mathematical rubbish.

Where the word "ton" is used in this volume, it always means 2240 ℔s.

There is no table of errata, because no errors are known to exist except two or three of a single letter in spelling; and which will probably escape notice.

JOHN C. TRAUTWINE.

PHILADELPHIA, *November* 13th, 1871.

PREFACE TO NINTH EDITION.

TWENTY-SECOND THOUSAND, 1885.

SINCE the appearance of its last edition (the twentieth thousand) in 1883, the "Pocket-Book" has been thoroughly revised, and many important additions and other alterations have been made. These necessitated considerable change in the places of the former matter, and it was deemed best to turn this necessity to advantage, and to make a thorough re-arrangement, putting all of the articles, as far as possible, in a rational order.

The list of new matter and of revisions and extensions is condensed as follows, 1902:

New articles on the steam-hammer pile driver, machine rock drills, air compressors, high explosives, cost of earthwork by drag and wheel scrapers and by steam excavators, iron trestles, track tanks, artesian well-boring and standard time, and new tables of railroad curves in metric measure, circumferences and areas of circles, thermometric scales, and fractions with their decimal equivalents.

Articles revised and extended, on circular arcs, thermometers, flotation, flow in pipes, waterworks appliances, velocities, &c, of falling bodies, centrifugal force, strength of timber, strength of beams, riveting, riveted girders, trusses, suspension bridges, rail joints, turnouts, turntables, locomotives, cars, railroad statistics and manufactured articles, including columns, beams, channels, angles and tees.

Most of the new matter is in nonpareil, the *larger* of the two types heretofore used. **Boldfaced** type has been freely used; but only for the purpose of guiding the reader rapidly to a desired division of a subject. For *emphasis*, *italics* have been employed.

Illustrations which were lacking in clearness or neatness have been re-touched and re-lettered, or replaced with new and better cuts. The new matter is very freely illustrated.

New rules have been put in the shape of formulæ, and many of the old rules have been re-cast into the same form.

* * * * * * * * * * * *

The addition of new matter, and a number of blank spaces necessarily left in making the re-arrangement, have increased the number of pages about one-fifth.

The new index is in stricter alphabetical order than that of former editions, and contains more than twice as many entries, although much repetition has been avoided by the free use of cross-references, without which this part of the work might have been indefinitely extended.

The selection of articles of manufacture or merchandise for illustration, has been guided by no other consideration than their fitness for the purpose, and the courtesy of the parties representing them, in supplying information.

The writer gratefully acknowledges the kindness of those who have assisted in furnishing and arranging data.

PHILADELPHIA, January, 1885. J. C. T., JR.

PREFACE TO EIGHTEENTH EDITION.

(SEVENTIETH THOUSAND, 1902.)

IN preparation for its eighteenth edition, The Civil Engineer's Pocket Book, the first edition of which appeared thirty years ago, has undergone a far more extensive revision than at any other time. More than 370 pages of new matter have been added; and the new edition is larger, by about 100 pages, than its recent predecessors.

Among the new matter in this edition will be found:

PAGES	
43– 46	Annuities, Depreciation, etc.
70– 72	Logarithms.
73– 77	Logarithmic Chart and Slide Rule.
80– 91	New Table of Logarithms.
228– 253	Conversion Table of Units of Measurement.
300– 301	Isogonic Chart.
532– 535	Venturi Meter.
536	Ferris-Pitot Meter.
546	Miner's Inch.
649	Water Consumption in Cities.
658– 659	Cost of Water Pipe and Laying.
745– 764	Digests of Specifications for Bridges and Buildings.
816	Tie Plates.
870– 873	Digest of Specification for Iron and Steel.
905– 906	Gray Column.
914	Trough Floor Sections.
983– 995	Price List of Manufactured Articles.
996–1007	Business Directory.
1008–1023	Bibliography.

The following articles have been almost or entirely rewritten:

NEW PAGES		OLD PAGES
35– 47	Arithmetic	33– 37
210–211	Specific Gravity	380–381
265–266	Time	395
282–283	Chains and Chaining	176
284–290	Location of the Meridian	177–179
322–325	Rain and Snow	220–221
358–453	Statics	318 f–361, 370–375

NEW PAGES		OLD PAGES
466–494	Strength of Beams	478–520, 528–536
499	Shearing Strength	476
499–500	Torsional Strength	476–477
501–503	Opening Remarks on Hydrostatics	222–224
537–538	Effect of Curves and Bends on Flow in Pipes	255–256
689–744	Trusses	547–614
856–864	Locomotives	805–810
865–866	Cars	811–813
867–869	Railroad Statistics	814–818
892–899	I Beams, Channels, Angles and T Shapes	521–527
930–942	Cement	673–678
943–947	Concrete	678–682
954–956	Timber Preservation	425–425 a

The articles on arithmetic are considerably extended, notably by the addition of new matter relating to interest, annuities, depreciation, etc., including several tables.

The new and greatly enlarged table of five-place logarithms is arranged in a somewhat novel form. In constructing this table, the effort has been to obviate the difficulty, present in all tables where the difference between successive numbers is constant throughout, that the differences between successive logarithms of the lower numbers are relatively very great. In the new table the differences between logarithms are much more nearly constant. For convenience in rough calculations, the old table of five-place logarithms, on two facing pages, is retained.

The Conversion Tables contain the equivalents of both English and metric units, and of each of these in terms of the other; but, owing to the extreme ease with which one metric unit may be converted into others of the same system, it has been unnecessary to burden the table with many of the metric units. The tables have been separately calculated by at least two persons, and their results compared and corrected. One of these results has then been used by the compositor in setting the type, and the proofs have been compared with the other.

The new article on the location of the meridian is much more complete than its predecessors, and a new table of azimuths of Polaris, corresponding to different hour-angles, has been added.

Perhaps the most radical and extensive of all the changes in this edition are those in the articles on Statics, on Beams and on Trusses These have been almost entirely rewritten and completely modernized. Under Trusses, modern methods of calculating the stresses in and the dimensions of the several members, and modern methods of construction, are explained, and several modern roofs and bridges are described and illustrated. One of the most notable features in the new article is the digest of prominent modern specifications for bridges for steam and electric railroads and for highways. The articles on the strength of beams are greatly simplified and brought into harmony with modern methods of dealing with that subject.

In preparing the digests of specifications for iron and steel, use has been made of the specifications recently adopted by the American Section of the International Association for Testing Materials; while those of the American Society of Civil Engineers and of the recent report of a Board of United States Army engineer officers have been similarly used in connection with cement.

The price list of engineering materials and appliances has been prepared merely as a useful guide in roughly estimating the approximate costs of work, and it is not to be supposed that it can, in any important case, take the place of personal inquiry and correspondence with manufacturers or their agents, nearly 700 of whom are named in the accompanying list of names and addresses of manufacturers, etc. From its first appearance, the Pocket Book has undertaken to give prices of certain manufactured articles, and addresses of those from whom they may be obtained; but these, scattered as they were throughout the volume, were necessarily desultory, and limited in their extent and usefulness. It is hoped that the present articles will be found at least an acceptable substitute for them.

As in preceding editions, all new work and all revisions have been the subject of our personal attention, and "scissors-and-paste" methods have been scrupulously avoided. Even in using lists of manufactured articles, etc., although their statements have in general been left unchanged, the matter has in most or all cases been rearranged and classified, to suit the requirements of this work.

For instance, the "digests" of specifications for Cement, for Steel and Iron, for Railroad and Highway Bridges and for Steel Buildings, are by no means mere quotations from the originals; but, as their name implies, the result of careful *digesting* of the contents of the specifications selected for the purpose; their several provisions being carefully studied, in nearly all cases reworded or reduced to figures, and tabulated in form convenient for reference, the whole being arranged in such logical order as to facilitate reference.

As in all cases heretofore, every rule or formula and every description of methods, etc., can be readily understood and applied by any one, engineer or layman, understanding the use of common and decimal fractions, of roots and powers, of logarithms, and of sines, tangents, etc., of angles. On the other hand, one who is not possessed of this very meager stock of mathematical knowledge will hardly approach engineering problems, even as an amateur; and we have therefore followed the precedent, established seventeen years ago, of putting rules in the shape of formulas, which have "the great advantage of showing the whole operation at a glance, of making its *principle* more apparent, and of being much more convenient for reference" (From Preface to ninth edition, 1885).

The new matter is very fully illustrated. As heretofore, all cuts have been engraved expressly for this work.

As in preparing for the ninth edition (1885), all the matter of the book has been rearranged. This has necessitated a new paging; and, in making this, the lettering of pages, introduced from time to time as new editions have appeared in the past, has been eliminated. The rearrangement and the addition of so much new matter have of course necessitated the preparation of a new table of contents and a new index.

In this, as in all previous editions since the eighth (1883), practically all new matter has been set in nonpareil, the larger of the two types hitherto used, and much of the old matter retained has been reset in the larger type.

We take pleasure in acknowledging our indebtedness to many who have kindly assisted us in our work, notably to Messrs. Otis E. Hovey and Wm. M. White, of the American Bridge Co., for painstaking examination of the article on Trusses; to Mr. C. Robert Grimm and Professor E. J. McCaustland for similar assistance in connection with the article on Statics; to Misses Laura Agnes Whyte and Louise C. Hazen for suggestions respecting mathematics and astronomy; and to the following gentlemen for valuable information respecting the subjects named:

Isogonic Chart, Mr. O. H. Tittmann, Sup't, U. S. Coast and Geodetic Survey.

Trusses, Messrs. Wm. A. Pratt, Engineer of Bridges, Pennsylvania Railroad; W. B. Riegner, Engineer of Bridges, Philadelphia and Reading Railway; Paul L. Wolfel, Chief Engineer, American Bridge Co.; J. Sterling Deans, Chief Engineer, and Moritz G. Lippert, Assistant Engineer, Phœnix Bridge Co.; Ralph Modjeski, Northern Pacific Railway; D. J. Whittemore, Chief Engineer, and C. F. Loweth, Engineer and Superintendent of Bridges and Buildings, Chicago, Milwaukee and St. Paul Railway.

Specifications for Bridges and Buildings, Messrs. C. C. Schneider, Vice President, American Bridge Company; J. E. Greiner, Engineer of Bridges and Buildings, Baltimore and Ohio Railroad; Theodore Cooper; W. K. McFarlin, Chief Engineer, Delaware, Lackawanna and Western Railway; Mason B. Strong, Bridge Engineer, Erie Railroad; F. C. Osborn, President, Osborn Engineering Co.; Wm. A. Pratt, Engineer of Bridges, Pennsylvania Railroad; W. B. Riegner, Engineer of Bridges, Philadelphia and Reading Railway; W. J. Wilgus, Chief Engineer, New York Central Railroad.

Locomotives, Baldwin Locomotive Works; Messrs. Wilson Miller, President, Pittsburgh Locomotive and Car Works; Theo. N. Ely, Chief of Motive Power, Pennsylvania Railroad; A. E. Mitchell, C. W. Buchholz and A. Mordecai, of the Erie Railroad; Edwin F. Smith, Wm. Hunter, A. T. Dice and Samuel F. Prince, Jr., of the Philadelphia and Reading Railway; and Thomas Tait, Manager, Canadian Pacific Railway; and Major E. T. D. Myers, of the Richmond, Fredericksburg and Potomac Railroad.

Cars, Allison Manufacturing Co., Harlan & Hollingsworth Co., and Mr. Jos. W. Taylor, Secretary, Master Car Builders' Association.

Railroad Statistics, Mr. Edward A. Moseley, Secretary, Interstate Commerce Commission.

Iron and Steel, Mr. Wm. R. Webster.

Cement, Mr. Richard L. Humphrey.

Concrete Beams, Mr. Howard A. Carson, Chief Engineer, Boston Transit Commission.

Preservation of Timber, Mr. O. Chanute.

Building Material, Mr. John T. Willis.

JOHN C. TRAUTWINE, JR.,
JOHN C. TRAUTWINE, 3D.

PHILADELPHIA, *October, 1902.*

Folios xvi to xxiv inclusive are left blank, to provide for future additions to prefaces.

CONTENTS.

MATHEMATICS. PAGE

- Mathematical Symbols 33
- Greek Alphabet 34

Arithmetic.

- Factors and Multiples 35
- Fractions 35
- Decimals 37
- Ratio and Proportion 38
- Progression 39
- Permutation, Combination, Alligation 40
- Percentage, Interest, Annuities 40
  - Simple Interest 41
  - Equation of Payments 42
  - Compound Interest 42
  - Annuity, Sinking Fund, Depreciation, etc. 43
    - Equations and Tables 44–46
- Duodenal Notation 47
- Reciprocals 48–52
- Roots and Powers.
  - Square and cube.
    - Tables 54
    - Rules 66
  - Fifth Roots and Powers 67
- Logarithms 70
  - Rules 70
  - Logarithmic Chart and Slide Rule 73
  - Two-page Table 78
  - Twelve-page Table 80

Geometry, Mensuration, and Trigonometry.

Lines.

- Definitions 92

Angles.

- Definitions 92
- Construction 93
- Bisection 94
- Inscribed 94
- Complement and Supplement 94
- In a Parallelogram 95
- Minutes and Seconds in Decimals of a Degree, Table of— 95
- Approximate Measurement of Angles 96
- Sine, Tangent, etc. 97
  - Definitions 97
  - Table 98
- Chords. Table of — 143

Surfaces. PAGE

- Polygons.
  - Regular —, Tables, etc., of — 148
- Triangles.
  - Definitions. Properties 148
  - Right-angled — 150
  - Trigonometrical Problems 150
- Parallelogram 157
- Trapezoid. Trapezium 158
- Polygons 159
  - Regular 159
  - Reduction of Figures 159, 160
- Circle 161
  - Radius, Diameter 161
  - Area, Center, to Find — 161
  - Problems 161, 162
  - Tables of —.
    - Diameter in Units, Eighths, etc. 163
    - Diameters in Units and Tenths 166
    - Diameters in Units and Twelfths 172
- Arc. Circular.
  - Chord, Length 179
  - Radius, Rise, and Ordinates 180
  - Of Large Radius, to Draw — 181
  - Tables of — 182–185
- Circular Sector, Ring, Zone, and Lune 186
- Circular Segment.
  - Area of —; to Find 186
  - Area of —; Table 187
- Ellipse.
  - Properties of — 189
  - Ordinates and Circumference of —; to Find — 189
  - Elliptic Arc 189
    - Tables of Lengths of — 190
  - Area of; to Find — 190
  - Construction. Tangents 190
  - Oval or False — 191
- Cyma Recta, Cyma Reversa, Ogee 191
- Parabola.
  - Properties of — 192
  - Parabolic Curve. Length of— 192
  - Area 192
  - Parabolic Zone or Frustum 192
  - Construction 193
- Cycloid 194

Solids.

- Regular Bodies. Tetrahedron, Hexahedron, etc. 194
  - Guldinus Theorem 194
- Parallelopiped, Properties 195

PAGE
- Prism 195
  - Frustum 195
- Cylinder.
  - Volume and Surface of — .. 196
  - Volume. Table of —, in Cu. Ft. and U. S. Gals 197
  - Wells; Contents of — and Masonry in Walls of — ... 198
- Cylindric Ungula 199
- Pyramid and Cone 200
  - Frustums of 201
- Prismoid 202
- Wedge 203
- Sphere.
  - Properties 204
  - Volume, Surface, etc.
    - Formulas for — 204
    - Tables of — 205–207
  - Segment and Zone of — 208
  - Spherical Shell 208
- Spheroid or Ellipsoid 208
- Paraboloid 209
  - Frustum of — 209
- Circular Spindle 209
- Circular Ring 209

Specific Gravity.

- Principles 210
- Table 212–215

Weights and Measures.

- U. S., British and Metric —, Units of — 216
- Coins; Foreign and U. S. — 218
- Gold and Silver 219
- Weights; Troy, Apothecaries' and Avoirdupois — 220
- Long Measure 220
- Degrees of Longitude. Length. 221
- Inches Reduced to Decimals of a Foot. Table 221
- Square or Land Measure 222
- Cubic or Solid Measure 222
- Liquid Measures 223
- Dry Measure 223
- British Imperial Measures 224
- Volumes and Weights of Water 224
- Metric Units 225
- Système Usuel, —Ancien 226
- Russian 227
- Spanish 227
- Conversion Tables 228
  - Introduction and Explanation 228
  - List of Tables 229
  - Fundamental Equivalents .. 230
  - Abbreviations 230
  - Equivalents and Numbers in Common Use 231
  - Metric Prefixes 231
  - Tables 232
- Acres per Mile and per 100 feet. Table 254

PAGE
- Grades, Tables of — 255–257
- Heads and Pressures of Water; Tables of — 258–260
- Discharges in Gals. per Day and Cu. Ft. per Second; Tables 261–265
- Time. Definitions, etc 265
  - Standard Railway — 267
  - Dialing 268
- Board Measure. Table 269

Surveying.

- Tests of Accuracy, Distribution of Error, etc 274
- Chaining 282
- Location of Meridian 284
  - By Circumpolar Stars 284
  - Definitions 284
  - By Means of Polaris 285
  - By Means of Any Star at Equal Altitudes 287
  - Times of Elongation and Culmination of Polaris 288
  - Azimuths of Polaris, Table .. 289
  - Polar Distances and Azimuths of Polaris, Table .. 290
- Engineer's Transit 291
  - Adjustment and Repairs 294
  - Vernier 296
  - Cross-hairs; to Replace 296
  - Bubble Glass; to Replace ... 296
- Theodolite 296
- Pocket Sextant 297
- Compass.
  - Adjustment 298
- Magnetic Declination and Variation.
  - Isogonic Chart of U. S 300
  - Declination 301
  - Variation 301
- Demagnetization 302
- Leveling.
  - Contour Lines 302
  - Y Level 306
    - Adjustment 307
    - Forms for Notes 309
  - Hand Level, Adjustment ... 310
  - Builder's Plumb Level 311
  - Clinometer or Slope Inst 311
  - Leveling by the Barometer or Boiling Point 312
    - Table 315

NATURAL PHENOMENA.

Sound.

- Velocity of 316

Heat.

- Expansion and Melting Points.
  - Table 317
- Thermometer.
  - Conversion of Scales 318
  - Tables 318, 319

Air. Atmosphere.

PAGE
Properties 320
Pressure in Diving Bells, etc... 321
Dew Point 321
Heat and Cold, Records of 321

Wind.

Velocity and Pressure. Table. 321

Rain and Snow.

Precipitation.
  Average 322
  Effect of Climate on — 322
  and Stream-flow 323
  Maximum Rates of — 323
Weight of Snow 323
Rain Gauge 324
Precipitation, Details of — in U. S., Table 325

Water.

Composition, Properties 326
Ice 326
Effects of Water on Metals, etc. 327
Tides 328

Evaporation, Filtration, Leakage 329

MECHANICS, FORCE IN RIGID BODIES.

Definitions 330
Matter; Body 330

Dynamics.

Motion, Velocity 331
Force 332
Action and Reaction 333
Acceleration 334
Mass 336
Impulse 337
Density; Inertia 338
Opposite Forces 339
Work 341
Power 342
Kinetic Energy 343
Momentum 345
Potential Energy 346
Impact 347
Gravity, Falling Bodies 348
Descent on Inclined Planes ... 349
Pendulums 350
Center of Oscillation 351
Center of Percussion 351
Angular Velocity 351
Moment of Inertia 351
Radius of Gyration 352
Centrifugal Force 354

Statics.

PAGE
Forces 358
Line of Action 359
Stress 359
Moments 360
Classification of Forces 361
Composition and Resolution of Forces 362
Force Parallelogram 364
Force Triangle 367
Rectangular Components 369
Inclined Plane 369
Stress Components 371
Applied and Imparted Forces .. 372
Resolution, etc., by means of Co-ordinates 372
Force Polygon 374
Non-concurrent Coplanar Forces 375
Equilibrium of Moments 376
Cord Polygon 377
Concurrent Non-coplanar Forces 380
Non-concurrent Non-coplanar Forces 381
Parallel Forces 382
  Coplanar 382
  Non-coplanar 385
Center of Gravity 386
  Stable, Unstable, and Indifferent Equilibrium 387
  General Rules 387
  Special Rules 391
Line of Pressure. Center of Force or of Pressure 399
  Position of Resultant 399
  Distribution of Pressure 400
  "Middle Third" 402
Couples 404
Friction 407
  Coefficient 408
  Morin's Laws 410
  Table of Coefficients 411
  Other Experiments 412
  Rolling Friction 414
  Lubricated Surfaces 415
  Friction Rollers 417
  Resistance of Trains 417
  Work of Overcoming Friction 418
  Natural Slope 419
  Friction of Revolving Shaft 419
Levers 419
Stability 422
  Work of Overturning 422
  On Inclined Planes 424
The Cord 425
  Funicular Machine 427
  Toggle Joint 427
  Pulley 428
  Loaded Cord or Chain 428
Arches, Dams, etc. Thrust and Resistance Lines 430
  Arches 430
    Graphic Method 430
    Practical Considerations .. 432
  Masonry Dam 433
    Graphic Method 435
    Practical Considerations .. 436
The Screw 436

PAGE
- Forces Acting upon Beams and Trusses 437
  - Conditions of Equilibrium 437
  - End Reactions 439
  - Moments 440
    - In Cantilevers 442
    - In Beams 443
      - Inclined Beams 445
      - Curved Beams 446
  - Shear 446
  - Influence Diagrams 449
    - For Moments 449
    - For Shear 450
  - Relation between Moment and Shear 452

STRENGTH OF MATERIALS.

General Principles. 454

- Stretch, Stress and Strain 455
- Modulus of Elasticity 456
- Limit of Elasticity 458
- Yield Point 459
- Resilience 460
- Suddenly Applied Loads 460
- Elastic Ratio 461
- Strengths of Sections 462
- Fatigue of Materials 465

Transverse Strength

- Conditions of Equilibrium 466
- Neutral Axis 466
- Resisting Moment 467
- Modulus of Rupture 468
- Moment of Inertia 468
  - Table 469
- Section Modulus 473
- Loading. Strength 473
  - Table 474
- Beam of Unit Dimensions 475
  - Coefficients, Table 476
- Weight of Beam as Load 477
- Comparison of Similar Beams 478
- Horizontal Shear 478
- Deflections 480
  - Elastic Limit 482
  - Elastic Curve 482
  - Deflection Coefficient 483
  - Eccentric Loads 484
  - Uniform Loads 485
  - Inclined Beams 485
  - Cylindrical Beams 485
  - Maximum Permissible— 485
  - Suddenly Applied Loads 486
- Uniform Strength 486
  - Cantilevers. Table 487
  - Beams. Table 488
- Continuous Beams 489
  - Table 490
- Cross-shaped Beam 492
- Plates 492
- Transverse and Longitudinal Stresses Combined 493

Strength of Pillars. 495

- Radius of Gyration 496
  - Table 496
- Remarks 498

Shearing Strength . 499

Torsional Strength. 499

HYDROSTATICS.

- Principles 501
  - Center of Pressure 501
  - Air Pressure 502
  - Horizontal and Vertical Components 503
  - Pressure in Vessels 503
  - Opposite Pressures 503
- Rules 504
- Transmission of Pressure 506
- Center of Pressure 506
- Walls to Resist Pressure 508
  - Thickness at Base 509
  - Stability 510
  - Contents 510
  - Liability to Crush 510
- Thickness for Cylinders 511
  - Iron Pipes 512
  - Lead Pipes 513
- Buoyancy 513
  - Flotation. Metacenter 514
  - Draught of Vessels 515

HYDRAULICS.

Flow of Water through Pipes 516

- Head of Water 516
  - Velocity Head 516
  - Entry Head 516
  - Friction Head 516
  - Pressure Head 518
- Piezometers 518
- Hydraulic Grade Line 519
- Siphon 520
- Velocity Formulæ 522
  - Kutter's Formula 523
- Weight of Water in Pipes 525
- Areas and Contents of Pipes 526
- Total Head Required 527
- Table of Velocity and Friction Heads and Discharge 528
- Compound Pipe 531
- Venturi Meter.
  - Theory 532
  - Tube 534
  - Register 535
- Ferris-Pitot Meter 535
- Curves and Bends 537

PAGE

Flow through Orifices

- Theoretical Velocities 539
- With Short Tubes 540
- Through Thin Partition 541
- Discharge from One Reservoir to Another 543
- Rectangular Openings 544
  - Time of Emptying Pond 545
  - Miner's Inch 546

Flow over Weirs

- End Contractions 547
- Measurement of Head 548
- Formulæ 549
  - Francis 550
    - Table of Discharges 551
  - Bazin 552
    - Values of m 553
- Submerged Weirs 554
- Velocity of Approach 556
- Inclined Weirs 558
- Broad-crested Overfall 559
- Triangular Notch 559
- Trapezoidal Notch 559

Flow in Open Channels

- Relations of Velocities 560
- Steam Gauging 560
  - Pitot Tube, etc. 561
  - Wheel Meter 562
- Abrasion of Channel 563
- Theory of Flow 563
- Kutter's Formula 564
  - Coefficient of Roughness 564
  - Coeffs of Roughness. Table 565
  - Coefficient, c, Table 566
  - To Draw Kutter Diagram. 570
  - Flow in Sewers 574
- Flow to Sewers 575
- Flow in Drain-pipes 575
- Constriction of Channel 576
- Scour 577
- Obstructions in Streams 577
- Power of Falling Water 578
- Water Wheels 578
- Hydraulic Ram 578
- Power of Running Stream 578

CONSTRUCTIONS, ETC.

Dredging.

- Cost of Dredging 580
- Horse Dredges 581
- Weight of Material 581

Foundations.

- Foundations 582
- Borings in Common Soils 582
- Unreliable Soils 583
- Resistance of Soils 583

PAGE

- Rip-rap 583
- Protection from Scour 583
- Timber Cribs 584
- Caissons 585
- Coffer-dams 586
- Earth Banks 586
- Crib Coffer-dams 587
- Mooring Caissons or Cribs 589
- Sinking through Soft Soil 589
- Piles 589
  - Sheet Piles 590
  - Grillage 590
  - Pile Drivers 590
  - Resistance of Piles 592
  - Penetrability of Soils 593
  - Driving 593
  - Screw Piles 594
  - Driving by Water Jet 595
- Hollow Iron Cylinders 596
  - Pneumatic Process 596
- Timber Caisson 598
- Masonry Cylinders 599
- Fascines 599
- Sand-Piles 599

Stonework.

- Cost, etc 600

Retaining Walls.

- General Remarks 603
- Theory 606
  - Surcharged Walls 609
- Wharf Walls 611
- Transformation of profile 611
- Sliding, etc. 612

Stone Bridges.

- Definitions 613
- Depth of Keystone 613
- Pressures on Arch-stones 614
- Table of Arches 615
- Abutments 617
- Abutment Piers 619
- Inclination of Courses 620
- Culverts 622
  - Wing Walls 624
  - Foundations 627
- Drains 627
- Drainage of Roadway 628
- Contents of Piers 628
- Brick Arches 629
- Centers 631

Timber Dams.

- Primary Requisites 642
- Examples 642
- Abutments, Sluices, Ground Plan, Cost 645
- Measuring Weirs 646
- Trembling 648
- Thickness of Planking Required 648

WATER SUPPLY.

PAGE
- Consumption, Use and Waste. 649
  - Waste Restriction; Water Meters 649
  - Water for Fire Protection 650
- Reservoirs 650
  - Leakage through —, Mud in — 651
  - Storage Reservoirs 652
  - Valve Towers, etc. 652
  - Compensation 653
  - Distributing Reservoirs 653
- Water Pipes 653
  - Concretions in —, prevention of — 655
  - Weights of Cast Iron Pipes 656
  - Wrought Iron Pipes 656
  - Wooden and Other Pipes 657
  - Costs of Pipes and Laying 658
  - Pipe Joints 660
    - Pipe Jointer 660
    - Flexible Joints 661
  - Special Castings 661
  - Repairs and Connections 662
- Air Valves 662
- Air Vessels, Stand-pipes 663
- Service Pipes 664
  - Tapping Machines 664
- Anti-bursting Device 665
- Valves, Gates 666
- Fire Hydrants 668

TEST AND WELL BORING.

- Test Boring Tools 670
- Artesian Well Drilling 671

ROCK DRILLS.

- Diamond Drills 675
- Percussion Drills 676
- Hand Drills 681
- Channeling 681
- Air Compressors 681

TRACTION, ANIMAL POWER.

- On Roads, Canals, etc. 683

TRUSSES.

Introduction.

- General Principles 689
- Loading, Counterbracing 690
- Cross bracing 691
- Types of Trusses 691
- Camber 696
- Cantilevers 696
- Movable Bridges 696
- Skew Bridges 697
- Roof Trusses 698

Stresses in Truss Members

- General Principles 698
- Method by Sections 700
- Chord Stresses, Moments, Chord Increments 701

PAGE
- Shear 702
  - Influence Diagram 702
- Dead Load Stresses 703
- Live Load Stresses 705
  - Typical Wheel Loads 705
    - Cooper's 706
  - Live Load Web Stresses 706
  - Live Load Chord Stresses 709
- Wind Loads 710
- Impact, etc. 711
- Maximum and Minimum Stresses 712
- Effect of Curves 712
- Counterbracing 713
- Stresses in Roof Trusses 713
  - Weights and Loads 713
  - Wind Pressures 714
    - Graphic Method 715
  - Timber Roof Trusses 716
- Deflections 718
- Redundant Members 720

Bridge Details and Construction

- General Principles 720
  - Floor System and Bearings 720
- Design 721
  - Flexible and Rigid Tension Members 721
  - Compression Members 721
  - Pin and Riveted Connections 721
  - Floor Beam Connections 721
  - Tension Members, Detail 722
  - Compression Members, Detail 722
  - End Post and Portal Bracing 723
  - Joints 724
    - Pin Plates 724
    - Pins 725
  - Expansion Bearings 725
  - Loads, Clearance, etc., for Highway Bridges 726
  - Camber 726
  - Examples 726
- Weights of Steel Railroad Bridges 731
- List of Large Bridges 732
- Timber Trusses 732
  - Joints 733
  - Howe Truss Bridges 736
    - Examples 738
- Metal Roof Trusses 740
  - Broad Street Station, Phila. 740
  - List of Large Arched Roofs 742
- Timber Roof Trusses 742
- Transportation and Erection 743

Digests of Specifications for Bridges and Buildings.

For Steel Railroad and Highway Bridges.

- General Design 745
- Material 751
- Loads 755

PAGE
Stresses and Dimensions...... 759
Protection 763
Erection 763

For Combination Railroad Bridges.

General Design 763
Material 763
Loads 764
Stresses and Dimensions 764
Protection 764

For Roofs, Buildings, etc.

General Design, Material, etc.. 764

SUSPENSION BRIDGES.

Data Required............... 765
Formulas 766
Anchorages 770

RIVETS AND RIVETING.

Rules and Tables............ 772

RAILROADS.

Curves.

Definitions 780
Tables, etc.................. 784

Earthwork.

Table of Level Cuttings....... 790
Shrinkage of Embankment.... 799
Cost of Earthwork........... 800

Tunnels.

Construction 812

Trestles.

Construction 813

Track.

Ballast 815
Ties 815
Tie Plates................... 816
Rails 817
Spikes 818
Rail Joints 819
Turnouts 824

Equipment.

Turntables 845
Water Stations 851
  Track Tanks................ 853
Track Scales, Fences, etc...... 854
Cost of Mile of Track......... 855

Rolling Stock.

PAGE
Locomotives.
  Dimensions, Weights, etc... 856
  Performance 860
  Tonnage Rating............. 862
  Fast Runs.................. 863
  Running Expenses........... 864
Cars 865

Statistics.

Earnings, Expenses, etc........ 867

MATERIALS.

Metals.

Iron and Steel.

Requirements. International Ass'n for Testing Materials. 870
Cast Iron.................... 874
  Weight 875
  Weight of Cast Iron Pipes.. 876
Weight of Wrought Iron and Steel 877
Roofing Iron 880
Corrugated Iron.............. 881
Wrought Iron Pipes and Fittings 882
Screw Threads, Bolts, Nuts and Washers............ 883
  Lock-nut Washers........... 885
Buckle Plates................ 885
Bolts. Weight and Strength, Table 886
Wire Gauges.................. 887
Circular Measure............. 889
Wire, Table.................. 891
Structural Shapes.
  I Beams 892
  Channels 894
  Angles and T Shapes........ 896
  Separators for I Beams..... 900
  Z-Bar Columns.............. 901
  Phœnix Segment Columns.. 904
  Gray Column................ 905
  Strengths of Iron Pillars, Tables 907
  Floor Sections 914
Chains 915

Other Metals.

Tin and Zinc................ 916
Copper, Lead, etc............ 918
Tensile Strengths, Table 920
Compressive Strengths, Table. 921

Stone, etc.

Tensile Strengths, Table...... 922
Compressive Strengths, Table 923
Transverse Strength, Table... 924

Mortar, Bricks, etc.

PAGE
Lime Mortar.................. 925
Bricks 927
Cement 930
  Cement Mortar............. 931
  Sand 935
  Effects on Metals......... 936
  Efflorescence 936
  Silica Cement............. 937
  Recommendations, Am. Soc. C. E.................. 937
    Tests 938
  Report of Board of U. S. A. Engineer Officers...... 940
    Tests 941
    Requirements 942
Concrete 943
  Properties 943
  Handling 946

Explosives.

Nitro-glycerine and Dynamite. 948
Blasting Powders............. 951
  Firing 952
Gunpowder 953

Timber.

Decay and Preservation...... 954
Tensile Strength............. 957
Compressive Strength......... 958
Transverse Strength.......... 959
Strength as Pillars.......... 963

Building Materials and Operations.

PAGE
Plastering 968
Slating 969
Shingles 971
Painting 971
Glass and Glazing 973

Sundry Materials.

Rope 975
Wire Ropes................... 976
Paper 978
Blue Prints, etc............. 979

Price List and Business Directory.

Price List................... 984
Business Directory........... 996

Bibliography.

List of Engineering Books....1008

GLOSSARY 1025

INDEX 1039

MATHEMATICS.

MATHEMATICAL SYMBOLS.

$+$ Plus, positive, add. 1.414+ means 1.414 + other decimals.
$-$ Minus, negative, subtract.
$\pm$ Plus or minus, positive or negative. Thus, $\sqrt{a^2} = \pm a$.
$\mp$ Minus or plus.
$\times$ Multiplied by, times. Thus, $x \times y = x.y = x\,y$; $3 \times 4 = 12$.
$\left.\begin{matrix}\div\\ :\\ /\end{matrix}\right\}$ Divided by. Thus, $a \div b = a : b = a/b = \frac{a}{b}$.
$:$ $::$ Proportion. Thus, $a : b :: c : d$, as a is to b, so is c to d.
$=$ Equals, is equal to.
$>$ Is greater than. Thus, $6 > 5$.
$<$ Is less than. Thus, $5 < 6$.
$\neq$ Is not equal to.
$\gtrless$ Is greater or less than.
$\ngtr$ Is not greater than.
$\nless$ Is not less than.
$\geqq$ Is equal to or greater than.
$\leqq$ Is equal to or less than.
$\propto$ Is proportional to, varies with.
∞ Infinity.
$\perp$ Is perpendicular to.
$\left.\begin{matrix}\angle\\ \measuredangle\end{matrix}\right\}$ Angle.
$\sim$ Is similar to.
$\parallel$ Is parallel to.
$\surd$ $\sqrt{\ }$ Root of. Thus, $\sqrt[2]{a}$ or $\sqrt{a}$ = square root of a, $\sqrt[3]{a}$ = 3d or cube root of a, $\sqrt[n]{a}$ = nth root of a.

( ) Parenthesis.
[ ] Brackets.
{ }
—— Vinculum.
Quantities enclosed or covered by the symbol are to be taken together.

$\because$ Since, because.
$\therefore$ Hence, therefore.
$^\circ$ Degrees.
$'$ Minutes of arc,* feet.
$''$ Seconds of arc,* inches.
$'$ $''$ $'''$ etc. Prime, second, third, etc. Distinguishing accents. Thus, a', a prime; a'', a second, etc.
π $\frac{\text{Circumference}}{\text{Diameter}} = 3.14159265+$, arc of semicircle, or 180°.
E, Modulus of elasticity.
e ϵ, Base of Napierian, natural or hyperbolic logarithms = 2.718281828.
g, Acceleration of gravity = approximately 32.2 feet per second per second = approximately 9.81 meters per second per second.

---

*Minutes and seconds of time, formerly also denoted by $'$ and $''$, are now denoted by m and s, or by min and sec, respectively.

THE GREEK ALPHABET.

This alphabet is inserted for the benefit of those who have occasion to consult scientific works in which Greek letters are used, and who find it inconvenient to memorize the letters.

Greek letters.		Name.	Approximate equivalent.	Commonly used to designate
Capital.	Small.			
Α	α	Alpha	a	Angles, Coefficients.
Β	β	Beta	b	" "
Γ	γ	Gamma	g	" " Specific gravity.
Δ	δ	Delta	d	" " Density, Variation.
Ε	ε	Epsilon	e (short)	Base of hyperbolic logarithms = 2.7182818. Eccentricity in conic sections.
Ζ	ζ	Zeta	z	Co-ordinates, Coefficients.
Η	η	Eta	e (long)	" "
Θ	θ ϑ	Theta	th	Angles.
Ι	ι	Iota	i	
Κ	κ	Kappa	k	
Λ	λ	Lambda	l	Angles, Coefficients, Latitude.
Μ	μ	Mu	m	" "
Ν	ν	Nu	n	"
Ξ	ξ	Xi	x	Co-ordinates.
Ο	ο	Omicron	o (short)	
Π	π	Pi	p	Circumference ÷ radius.*
Ρ	ρ	Rho	r	Radius, Ratio.
Σ	σ ς	Sigma	s	Distance (space).†
Τ	τ	Tau	t	Temperature, Time.
Υ	υ	Upsilon	u or y	
Φ	φ	Phi	ph	Angles, Coefficients.
Χ	χ	Chi	ch	
Ψ	ψ	Psi	ps	Angles.
Ω	ω	Omega	o (long)	Angular velocities.

---

* The small letter π (*pi*) is universally employed to designate the number of times (= 3.14159265 . . .) the diameter of a circle is contained in the circumference, or the radius in the semi-circumference. In the circular measure of angles, an angle is designated by the number of times the radius of any circle is contained in an arc of the same circle subtending that angle. π then stands for an angle of 180° (= two right angles), because, in any circle, π × radius = the semi-circumference.

The capital letter Π (*pi*) is used by some mathematical writers to indicate the product obtained by multiplying together the numbers 1, 2, 3, 4, 5 . . . etc., up to any given point. Thus, $\Pi\, 4 = 1 \times 2 \times 3 \times 4 = 24$.

† The capital letter Σ (*sigma*) is used to designate a *sum*. Thus, in a system of parallel forces, if we call each of the forces (irrespective of their amounts) F, then their resultant, which is equal to the (algebraic) sum of the forces, may be written $R = \Sigma F$.

ARITHMETIC.

FACTORS AND MULTIPLES.

(1) **Factors** of any number, n, are numbers whose product is $= n$. Thus, 17 and 4 are factors of 68; so also are 34 and 2; also 17, 2, and 2.

(2) **A prime number,** or **prime,** is a number which has no factors, except itself and 1; as 2, 3, 5, 19, 233.

(3) **A common factor, common divisor** or **common measure,** of two or more numbers, is a number which exactly divides each of them. Thus, 3 is a common divisor of 6, 12, and 18.

(4) **The highest common factor** or **greatest common divisor,** of two or more numbers, is called their **H. C. F.** or their **G. C. D.** Thus, 6 is the H. C. F. of 6, 12, and 18.

(5) **To find the H. C. F.** of two or more numbers; find the prime factors of each, and multiply together those factors which are common to all, taking each factor only once. Thus, required the H. C. F. of 78, 126, and 234.

$$\begin{aligned} 78 &= 2 \times 3 \times 13 \\ 126 &= 2 \times 3 \times 3 \times 7 \\ 234 &= 2 \times 3 \times 3 \times 13 \end{aligned}$$

and H. C. F. $= 2 \times 3 = 6$.

(6) **To find the H. C. F.** of two large numbers; divide the greater by the less; then the less by the remainder, A; A by the second remainder, B; B by the third remainder, C; and so on until there is no remainder. The last divisor is the H. C. F. Thus, required the H. C. F. of 575 and 782.

```
575)782(1
    575
  A 207)575(2
        414
      B 161)207(1
            161
          C 46)161(3
               138
             D 23)46(2      H. C. F. = D = 23.
                  46
                   0
```

(7) **A common multiple** of two or more numbers is a number which is exactly divisible by each of them.

(8) **The least common multiple** of two or more numbers is called their **L. C. M.**

(9) **To find the L. C. M.** of two or more numbers; find the prime factors of each. Multiply the factors together, taking each as many times as it is contained in that number in which it is oftenest repeated. Thus, required the L. C. M. of 7, 30, and 48.

$$\begin{aligned} 7 &= 7 \\ 30 &= 2 \times 3 \times 5 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 \\ \text{L. C. M.} &= 7 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 1680. \end{aligned}$$

(10) **To find the L. C. M.** of two large numbers; find the H. C. F., as above; and, by means of it, find the other factors. Then find the product of the factors, as before. Thus, required the L. C. M. of 575 and 782. As above, H. C. F. $= 23$; $\frac{575}{23} = 25$; and $\frac{782}{23} = 34$. Hence,

$$\begin{aligned} 575 &= 23 \times 25 \\ 782 &= 23 \times 34 \end{aligned}$$

and L. C. M. $= 23 \times 25 \times 34 = 19{,}550$.

FRACTIONS.

(1) **A common denominator** of two or more fractions is a common multiple of their denominators.

(2) **The least common denominator, or L. C. D.,** of two or more fractions is the L. C. M. of their denominators.

(3) To reduce to a common denominator. Let

N = the new numerator of any fraction
n = its old numerator
d = its old denominator
C = the common denominator

Then

$$N = n\frac{C}{d}$$

Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$. C = L. C. M. of denominators = 24.

$$\frac{3}{4} = \frac{3 \times \frac{24}{4}}{4 \times \frac{24}{4}} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24};\quad \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24};\quad \frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}.$$

If none of the denominators have a common factor, then C = the product of all the denominators, $\frac{C}{d}$ = the product, P, of all the *other* denominators, and N = P n.

Thus, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{7}$. C = 84

$$\frac{2}{3} = \frac{2 \times 4 \times 7}{84} = \frac{56}{84};\quad \frac{1}{4} = \frac{1 \times 3 \times 7}{84} = \frac{21}{84};\quad \frac{5}{7} = \frac{5 \times 3 \times 4}{84} = \frac{60}{84}.$$

(4) Addition and Subtraction. If necessary, reduce the fractions to a common denominator, the lower the better. Add or subtract the numerators. Thus,

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1;\ \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1;\ \frac{3}{4} + \frac{5}{9} = \frac{27}{36} + \frac{20}{36} = \frac{47}{36} = 1\,\frac{11}{36};$$

$$\frac{3}{4} + \frac{7}{8} = \frac{6}{8} + \frac{7}{8} = \frac{13}{8} = 1\,\frac{5}{8}.$$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2};\ \frac{3}{4} - \frac{5}{9} = \frac{27}{36} - \frac{20}{36} = \frac{7}{36};\ \frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}.$$

(5) Multiplication. Multiply together the numerators, also the denominators, cancelling where possible. Thus,

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4};\quad \frac{3}{4} \times \frac{1}{4} = \frac{3}{16};\quad \frac{3}{4} \times \frac{5}{9} \times \frac{2}{3} = \frac{5}{18};$$

$$3\frac{4}{7} \times 1\frac{2}{3} = \frac{25}{7} \times \frac{5}{3} = \frac{125}{21} = 5\frac{20}{21};\quad \frac{2}{3} \times \frac{3}{5} = \frac{2}{5};$$

$$\frac{3}{4} \text{ of } \frac{1}{2} \text{ of } \frac{5}{3} \text{ of } \frac{7}{5} = \frac{3}{4} \times \frac{1}{2} \times \frac{5}{3} \times \frac{7}{5} = \frac{7}{8}.$$

(6) Division. Invert the divisor and multiply. Thus,

$$\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1;\quad \frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = \frac{3}{1} = 3;$$

$$\frac{3}{4} \div \frac{5}{9} = \frac{3}{4} \times \frac{9}{5} = \frac{27}{20} = 1\frac{7}{20};$$

$$3\frac{4}{7} \div 1\frac{2}{3} = \frac{25}{7} \div \frac{5}{3} = \frac{25}{7} \times \frac{3}{5} = \frac{5}{7} \times \frac{3}{1} = \frac{15}{7} = 2\frac{1}{7};$$

$$5 \div \frac{7}{8} = 5 \times \frac{8}{7} = \frac{40}{7} = 5\frac{5}{7}.$$

(7) A fraction is said to be in its **lowest terms,** or to be **simplified,** when its numerator and denominator have no common factor. Thus,

$$\frac{34}{85} \text{ simplified } = \frac{2}{5}.$$

(8) To reduce to lowest terms. Divide numerator and denominator by their H. C. F. Thus, required the lowest terms of $\frac{34}{85}$.

$$\text{H. C. F. of 34 and 85} = 17;\ \text{and } \frac{34}{85} = \frac{34 \div 17}{85 \div 17} = \frac{2}{5}.$$

DECIMALS.

(9) Multiplication. The product has as many decimal places as the factors combined. Thus,

Factors: $100 \times 3 \times 3.5 \times 0.004 \times 465.21 = 1953.882000$
Number of decimal places: $0 + 0 + 1 + 3 + 2 = 6$

(10) Division. The number of decimal places in the quotient = those in the dividend minus those in the divisor. Thus,

$$\frac{5.125}{4.1} = 1.25;\ \frac{5}{4} = \frac{5.00}{4} = 1.25;\ \frac{3}{4} = \frac{3.00}{4} = 0.75;\ \frac{0.42}{0.0021} = \frac{0.4200}{0.0021} = 200;$$

When the divisor is a fraction or a mixed number, we may multiply both divisor and dividend by the least power of 10 which will make the divisor a whole number. Thus,

$$\frac{2.679454}{0.0062} = \frac{26{,}794.54}{62} = 432.17.$$

(11) To reduce a common fraction to decimal form; divide the numerator by the denominator. Thus, $\frac{32}{40} = 0.8$; $1\frac{3}{5} = \frac{8}{5} = 1.6$.

Table 1. Decimal equivalents of common fractions.

8ths	16ths	32ds	64ths		8ths	16ths	32ds	64ths	
			1	.015625				33	.515625
		1	2	.03125			17	34	.53125
			3	.046875				35	.546875
	1	2	4	.0625		9	18	36	.5625
			5	.078125				37	.578125
		3	6	.09375			19	38	.59375
			7	.109375				39	.609375
1	2	4	8	.125	5	10	20	40	.625
			9	.140625				41	.640625
		5	10	.15625			21	42	.65625
			11	.171875				43	.671875
	3	6	12	.1875		11	22	44	.6875
			13	.203125				45	.703125
		7	14	.21875			23	46	.71875
			15	.234375				47	.734375
2	4	8	16	.25	6	12	24	48	.75
			17	.265625				49	.765625
		9	18	.28125			25	50	.78125
			19	.296875				51	.796875
	5	10	20	.3125		13	26	52	.8125
			21	.328125				53	.828125
		11	22	.34375			27	54	.84375
			23	.359375				55	.859375
3	6	12	24	.375	7	14	28	56	.875
			25	.390625				57	.890625
		13	26	.40625			29	58	.90625
			27	.421875				59	.921875
	7	14	28	.4375		15	30	60	.9375
			29	.453125				61	.953125
		15	30	.46875			31	62	.96875
			31	.484375				63	.984375
4	8	16	32	.5	8	16	32	64	1.

(12) To reduce a decimal fraction to common form. Supply the denominator (1), and reduce the resulting fraction to its lowest terms. Thus:

$$0.25 = \frac{0.25}{1.00} = \frac{25}{100} = \frac{1}{4};\ 0.75 = \frac{75}{100} = \frac{3}{4};\ 0.890625 = \frac{890625}{1000000} = \frac{57}{64}.$$

(13) **Recurring, circulating, or repeating decimals** are those in which certain digits, or series of digits, recur indefinitely. Thus, $\frac{1}{3} = 0.3333....$, and so on; $\frac{10}{7} = 1.428571428571.....$, and so on. Recurring decimals may be indicated thus: $0.\bar{3}$, $1.\overline{428571}$; or thus: 0.*3, 1.*428571.

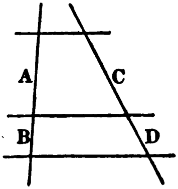
RATIO AND PROPORTION.

(1) **Ratio.** The ratio of two quantities, as A and B, is expressed by their quotient, $\frac{A}{B}$ or $\frac{B}{A}$. Thus, the ratio of 10 to 5 is $= \frac{10}{5} = 2$; the ratio of 5 to 10 is $= \frac{5}{10} = \frac{1}{2} = 0.5$.

(2) **Duplicate ratio** is the ratio of the *squares* of numbers. Thus, $\frac{A^2}{B^2}$ is the duplicate ratio of A and B.

(3) **Proportion** is equality of ratios. Thus, $\frac{10}{5} = \frac{12}{6} = \frac{1266}{633} = 2$. In the figure, which represents segments, A, B, C, and D, between parallel lines; $A : B :: C : D$, or $\frac{A}{B} = \frac{C}{D}$.

(4) The first and fourth terms, A and D, are called the **extremes,** and the second and third, B and C, are called the **means.** The first term, A or C, of each ratio, is called the **antecedent,** and the second term, B or D, is called the **consequent.** D is called the **fourth proportional** of A, B, and C.



(5) In a proportion, $A : B = C : D$, we have:

Product of extremes = product of means. $A\,D = B\,C$.

Alternation. $\frac{A}{B} = \frac{C}{D}$; $\frac{A}{C} = \frac{B}{D}$.

Inversion. $\frac{B}{A} = \frac{D}{C}$; $\frac{B}{D} = \frac{A}{C}$; $\frac{D}{B} = \frac{C}{A}$.

Composition. $\frac{A+B}{A} = \frac{C+D}{C}$; $\frac{A+B}{B} = \frac{C+D}{D}$.

Division. $\frac{A-B}{A} = \frac{C-D}{C}$; $\frac{A-B}{B} = \frac{C-D}{D}$.

Composition and division. $\frac{A+B}{A-B} = \frac{C+D}{C-D}$.

We have, also:

$$\frac{m\,A}{m\,B} = \frac{A}{B} = \frac{C}{D} = \frac{n\,A}{n\,B} = \frac{n\,C}{n\,D};\quad \frac{m\,A}{n\,B} = \frac{m\,C}{n\,D};\quad \frac{A^n}{B^n} = \frac{C^n}{D^n};\quad \frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \frac{\sqrt[n]{C}}{\sqrt[n]{D}}.$$

(6) If, in the proportion, $A : B = C : D$, we have $B = C = m$, then $A : m = m : D$, or $\frac{A}{m} = \frac{m}{D}$ or $m^2 = A\,D$, or $m = \sqrt{A\,D}$.

(7) In such cases, m is called the **mean proportional** between A and D, and D is called the **third proportional** of A and m.

A continued proportion is a series of equal ratios, as

$$A : B = C : D = E : F, \text{ etc.} = R; \quad \text{or} \quad \frac{A}{B} = \frac{C}{D} = \frac{E}{F}, \text{ etc.} = R$$

In continued proportion,

$$\frac{A + C + E + \text{etc.}}{B + D + F + \text{etc.}} = \frac{A}{B} = \frac{C}{D} = \frac{E}{F} \text{ etc.} = R$$

$$\text{If} \quad \frac{A}{B} = \frac{C}{D};\quad \frac{A'}{B'} = \frac{C'}{D'};\quad \frac{A''}{B''} = \frac{C''}{D''};\quad \text{then} \quad \frac{A\,A'\,A''}{B\,B'\,B''} = \frac{C\,C'\,C''}{D\,D'\,D''} \text{ etc.}$$

(8) Let A, B, and C be any three numbers. Then

$$\frac{A}{C} = \frac{A}{B} \cdot \frac{B}{C}, \text{ and } \frac{A}{B} = \frac{A}{C} \cdot \frac{C}{B}.$$

---

* 0.*3, 1.*428571, etc., standing for 0.3333...., 1.428571428571...., etc.

(9) Reciprocal or inverse proportion. Two quantities are said to be *reciprocally* or *inversely* proportional, when the ratio $\frac{A}{B}$ of two values, A and B, of the one, is = the reciprocal, $\frac{B'}{A'}$, of the ratio of the two corresponding values of the other. Thus, let A = a velocity of 2 miles per hour, and B = 3 miles per hour. Then the *hours required per mile* are respectively, $A' = \frac{1}{A} = \frac{1}{2}$, and $B' = \frac{1}{B} = \frac{1}{3}$. Here $A : B = B' : A'$, or $\frac{A}{B} = \frac{B'}{A'}$, or $\frac{2}{3} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{\frac{2}{6}}{\frac{3}{6}} = 1 \div \frac{A'}{B'}$.

(10) If two variable numbers, A and B, are reciprocally proportional, so that $A' : B' = B'' : A''$, the product, $A'\,A''$, of any two values of one of the numbers is equal to the product, $B'\,B''$ of the two corresponding values of the other.

(11) The application of proportion to practical problems is sometimes called the **rule of three.** Thus: **single rule of three:** If 3 men lay 10,000 bricks in a certain time, how many could 6 men lay in the same time?

As 3 men are to 6 men, so are 10,000 bricks to 20,000 bricks; or, 10,000 bricks $\times \frac{6}{3}$ = 20,000 bricks.

If 3 men require 10 hours to lay a certain number of bricks, how many hours would 6 men require to lay the same number?

As 6 men are to 3 men, so are 10 hours to 5 hours; or, 10 hours $\times \frac{3}{6}$ = 5 hours.

(12) Double rule of three.

If 3 men can lay 4,000 bricks in 2 days, how many men can lay 12,000 bricks in 3 days? Here 4,000 bricks require 3 men 2 days, or 6 man-days, and 12,000 bricks will require $6 \times \frac{12{,}000}{4{,}000} = 6 \times 3 = 18$ man-days; and, as the work is to be done in 3 days, $\frac{18}{3} = 6$ men will be required.

PROGRESSION.

(1) Arithmetical Progression. A series of numbers is said to be in arithmetical progression when each number differs from the preceding one by the same amount. Thus, —2, —1, 0, 1, 2, 3, 4, etc., where difference = 1; or 4, 3, 2, 1, 0, —1, —2, etc., where difference = —1; or —4, —2, 0, 2, 4. 6, 8, 10, where difference = 2; or 1½, 1¼, 1, ¾, ½, ¼, 0, —¼, —½, etc., where difference = —¼.

(2) In any such series the numbers are called terms. Let a be the first term, l the last term, d the common difference, n the number of terms, and s the sum of the terms. Then

Required	Given	
l	$a\ d\ n$	$l = a + (n - 1)\,d$
l	$a\ d\ s$	$l = -\frac{1}{2}d \pm \sqrt{2\,d\,s + (a - \frac{1}{2}d)^2}$
s	$a\ d\ n$	$s = \frac{1}{2}n\left[2\,a + (n - 1)\,d\right]$
a	$d\ l\ s$	$a = \frac{1}{2}d \pm \sqrt{(l + \frac{1}{2}d)^2 - 2\,d\,s}$
n	$a\ d\ s$	$n = \frac{d - 2\,a \pm \sqrt{(2\,a - d)^2 + 8\,d\,s}}{2\,d}$
n	$d\ l\ s$	$n = \frac{2\,l + d \pm \sqrt{(2\,l + d)^2 - 8\,d\,s}}{2\,d}$

(3) Geometrical Progression. A series of numbers is said to be in geometrical progression when each number stands to the preceding one in the same ratio. Thus: $\frac{1}{9}$, $\frac{1}{3}$, 1, 3, 9, 27, 81, etc., where ratio = 3; or 48, 24, 12, 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, etc., where ratio = $\frac{1}{2}$; or $\frac{27}{32}$, $1\frac{11}{16}$, $3\frac{3}{8}$, $6\frac{3}{4}$, $13\frac{1}{2}$, 27, etc., where ratio = 2.

(4) Let a be the first term, l the last term, r the constant ratio, n the number of terms, and s the sum of the terms. Then:

Required	Given	
l	$a\ r\ n$	$l = a r^{n-1}$*
l	$a\ r\ s$	$l = \frac{a + (r - 1) s}{r}$
l	$r\ n\ s$	$l = \frac{(r - 1) s r^{n-1}}{r^n - 1}$
s	$a\ n\ l$	$s = \frac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}$
s	$r\ n\ l$	$s = \frac{l r^n - l}{r^n - r^{n-1}}$
r	$a\ n\ l$	$r = \sqrt[n-1]{l \div a}$

PERMUTATION, Etc.

(1) **Permutation** shows in how many positions any number of things can be arranged in a row. To do this, multiply together all the numbers used in counting the things. Thus, in how many positions in a row can 9 things be placed? Here,

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ positions. Ans.

(2) **Combination** shows how many combinations of a few things can be made out of a greater number of things. To do this, first set down that number which indicates the greater number of things; and after it a series of numbers, diminishing by 1, until there are *in all* as many as the number of the few things that are to form each combination. Then beginning under the last one, set down said number of few things; and going backward, set down another series, also diminishing by 1, until arriving under the first of the upper numbers. Multiply together all the upper numbers to form one product; and all the lower ones to form another. Divide the upper product by the lower one.

Ex. How many combinations of 4 figures each, can be made from the 9 figures 1, 2, 3, 4, 5, 6, 7, 8, 9, or from 9 any things?

$$\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = \frac{3024}{24} = 126 \text{ combinations. Ans.}$$

(3) **Alligation** shows the value of a mixture of different ingredients, when the quantity and value of each of these last is known.

Ex. What is the value of a pound of a mixture of 20 lbs of sugar worth 15 cts per lb; with 30 lbs worth 25 cts per lb?

lbs. cts. cts.
$20 \times 15 = 300$
$30 \times 25 = 750$
50 lbs. 1050 cts.

Therefore, $\frac{1050}{50} = 21$ cts. Ans.

PERCENTAGE, INTEREST, ANNUITIES.

Percentage.

(1) Ratio is often expressed by means of the word "per." Thus, we speak of a grade of 105.6 feet per mile, *i. e.*, per 5280 feet. When the two numbers in the ratio refer to quantities of the same kind and denomination, the ratio is often expressed as a per*cent*age (per*hundred*age). Thus, a grade of 105.6 feet per mile,

---

* Equations involving powers and roots are conveniently solved by means of logarithms.

or per 5280 feet, is equivalent to a grade of 0.02 foot per foot,* or 2 feet per 100 feet, or simply (since both dimensions are in feet) 2 per 100, or 2 per "cent."

(2) One-fiftieth, or 1 per 50, is plainly equal to two hundredths, or 2 per *hundred*, or 2 per *cent.* Similarly, $\frac{1}{4}$ = 25 per cent., $\frac{3}{4} = 3 \times 25$ per cent. = 75 per cent., etc. Hence, to reduce a ratio to the form of percentage, divide 100 times* the first term by the second. Thus, in a concrete of 1 part cement to 2 of sand and 5 of broken stone, there are 8 parts in all, and we have, by weight—†

Cement $= \frac{1}{8} = 0.125 =$ 12.5 per cent. of the whole.
Sand $= \frac{2}{8} = 0.250 =$ 25.0 " "
Stone $= \frac{5}{8} = 0.625 =$ 62.5 " "
Concrete $= \frac{8}{8} = 1.000 =$ 100.0 " "

(3) Percentage is of very wide application in money matters, payment for service in such matters being often based upon the amount of money involved. Thus, a purchasing or selling agent may be paid a brokerage or commission which forms a certain percentage of the money value of the goods bought or sold; the premium paid for insurance is a percentage upon the value of the goods insured; etc.

Interest.

(4) Interest is hire or rental paid for the loan of money. The sum loaned is called the **principal,** and the number of cents paid annually for the loan of each dollar, or of dollars per hundred dollars, is called the **rate** of interest. The rate is always stated as a percentage.

(5) If the interest is paid to the lender as it accrues, the money is said to be at **simple interest;** but if the interest is periodically added to the principal, so that it also earns interest, the money is said to be at **compound interest,** and the interest is said to be compounded.

Simple Interest.

(6) At the end of a year, the interest on the principal, P, at the rate, r, is = P.r, and the **amount,** A, or sum of principal and interest, is

$$A = P + P\,r = P\,(1 + r).$$

(7) At the end of a number, n, of years, the interest is = $P\,r\,n$ (see right-hand side of Fig. 1), and

$$A = P + P\,r\,n = P\,(1 + r\,n).$$

Thus, let P = \$865.32, r = 3 per cent., or 0.03, n = 1 year, 3 months and 10 days = 1 year and 100 days = $1\frac{100}{365}$ years = 1.274 years. Then $A = P\,(1 + r\,n)$ $= \$865.32 \times (1 + 0.03 \times 1.274) = \$865.32 \times 1.03822 = \$898.39$.

(8) For the **present worth, principal,** or **capitalization,** P, of the amount, A, we have

$$P = \frac{A}{1 + r\,n}$$

Thus, for the sum, P, which, in 1 year, 3 months, 10 days, at 3 per cent. simple interest, will amount to \$898.39, we have $P = \dfrac{898.39}{1 + 0.03 \times 1.274} = \$865.32$.

(9) In commercial business, interest is commonly **calculated approximately** by taking the year as consisting of 12 months of 30 days each. Then, at 6 per cent., the interest for 2 months, or 60 days, = 1 per cent.; 1 month, or 30 days, = $\frac{1}{2}$ per cent.; 6 days = 0.1 per cent. Thus, required the interest on \$1264.35 for 5 months, 28 days, at 5 per cent.

---

* A *fraction*, as $\frac{1}{8}$, $\frac{5}{16}$, etc., or its decimal equivalent, as 0.125, 0.3125, etc., is compared with *unity* or *one;* but in *percentage* the first term of the ratio is compared with *one hundred* units of the second term. Mistakes often occur through neglect of this distinction. Thus, 0.06 (six per cent. or six per hundred) is sometimes mis-read six one-hundredths of one per cent. or six one-hundredths per cent.

† For proportions by volume, see pp 935 and 943.

Principal	$1264.35
Interest, 2 mos, 1 per cent	12.64
" 2 mos, 1 "	12.64
" 1 mo, ½ "	6.32
" 20 days, ⅓ "	4.21
" 6 days, 0.1 "	1.26
" 2 days, $\frac{1}{30}$ "	0.42
Interest at 6 per cent	$37.49
Deduct one-sixth	6.25
Interest at 5 per cent	$31.24

Equation of Payments.

(10) A owes B $1200; of which $400 are to be paid in 3 months; $500 in 4 months; and $300 in 6 months; all bearing interest until paid; but it has been agreed to pay all at once. Now, at what time must this payment be made so that neither party shall lose any interest?

$$\begin{array}{rcccr} \$ & & \text{months.} & & \\ 400 & \times & 3 & = & 1200 \\ 500 & \times & 4 & = & 2000 \\ \underline{300} & \times & 6 & = & \underline{1800} \\ 1200 & & & & 5000 \end{array}$$

$$\text{Average time} = \frac{5000}{1200} = 4\tfrac{1}{6} \text{ months.} \quad \text{Ans.}$$

Compound Interest.

(11) Interest is usually compounded annually, semi-annually, or quarterly. If it is compounded annually, then (see left side of Fig. 1)

at the end of 1 year $A = P(1 + r)$
" " 2 years $A = P(1 + r)(1 + r) = P(1 + r)^2$
" " n years $A = P(1 + r)^n$; and

$$P = \frac{A}{(1 + r)^n} = A(1 + r)^{-n}$$

$$\frac{A}{P} = (1 + r)^n$$

(12) If the interest is compounded q times per year, we have

$$\frac{A}{P} = \left(1 + \frac{r}{q}\right)^{qn}$$

(13) The principal, P, is sometimes called the **present worth** or **present value** of the amount, A. Thus, in the following table, $1.00 is the present worth of $2.191 due in 20 years at 4 per cent. compound interest, etc, etc.

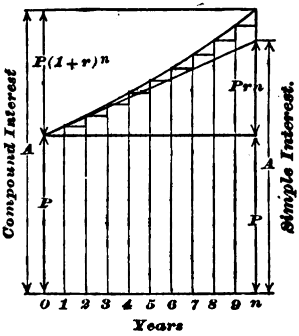


Fig. 1.

Table 2. Compound Interest.

Amount of $1 at Compound Interest.

Years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	6½ per cent.
1	1.030	1.035	1.040	1.045	1.050	1.055	1.060	1.065
2	1.061	1.071	1.082	1.092	1.103	1.113	1.124	1.134
3	1.093	1.109	1.125	1.141	1.158	1.174	1.191	1.208
4	1.126	1.148	1.170	1.193	1.216	1.239	1.262	1.286
5	1.159	1.188	1.217	1.246	1.276	1.307	1.338	1.370
6	1.194	1.229	1.265	1.302	1.340	1.379	1.419	1.459
7	1.230	1.272	1.316	1.361	1.407	1.455	1.504	1.554
8	1.267	1.317	1.369	1.422	1.477	1.535	1.594	1.655
9	1.305	1.363	1.423	1.486	1.551	1.619	1.689	1.763
10	1.344	1.411	1.480	1.553	1.629	1.708	1.791	1.877
11	1.384	1.460	1.539	1.623	1.710	1.802	1.898	1.999
12	1.426	1.511	1.601	1.696	1.796	1.901	2.012	2.129
13	1.469	1.564	1.665	1.772	1.886	2.006	2.133	2.267
14	1.513	1.619	1.732	1.852	1.980	2.116	2.261	2.415
15	1.558	1.675	1.801	1.935	2.079	2.232	2.397	2.572
16	1.605	1.734	1.873	2.022	2.183	2.355	2.540	2.739
17	1.653	1.795	1.948	2.113	2.292	2.485	2.693	2.917
18	1.702	1.858	2.026	2.208	2.407	2.621	2.854	3.107
19	1.754	1.923	2.107	2.308	2.527	2.766	3.026	3.309
20	1.806	1.990	2.191	2.412	2.653	2.918	3.207	3.524
21	1.860	2.059	2.279	2.520	2.786	3.078	3.400	3.753
22	1.916	2.132	2.370	2.634	2.925	3.248	3.604	3.997
23	1.974	2.206	2.465	2.752	3.072	3.426	3.820	4.256
24	2.033	2.283	2.563	2.876	3.225	3.615	4.049	4.533
25	2.094	2.363	2.666	3.005	3.386	3.813	4.292	4.828
26	2.157	2.446	2.772	3.141	3.556	4.023	4.549	5.141
27	2.221	2.532	2.883	3.282	3.733	4.244	4.822	5.476
28	2.288	2.620	2.999	3.430	3.920	4.478	5.112	5.832
29	2.357	2.712	3.119	3.584	4.116	4.724	5.418	6.211
30	2.427	2.807	3.243	3.745	4.322	4.984	5.743	6.614
31	2.500	2.905	3.373	3.914	4.538	5.258	6.088	7.044
32	2.575	3.007	3.508	4.090	4.765	5.547	6.453	7.502
33	2.652	3.112	3.648	4.274	5.003	5.852	6.841	7.990
34	2.732	3.221	3.794	4.466	5.253	6.174	7.251	8.509
35	2.814	3.334	3.946	4.667	5.516	6.514	7.686	9.062
36	2.898	3.450	4.104	4.877	5.792	6.872	8.147	9.651
37	2.985	3.571	4.268	5.097	6.081	7.250	8.636	10.279
38	3.075	3.696	4.439	5.326	6.385	7.649	9.154	10.947
39	3.167	3.825	4.616	5.566	6.705	8.069	9.704	11.658
40	3.262	3.959	4.801	5.816	7.040	8.513	10.286	12.416

Compound interest on M dollars, at any rate *r* for *n* years = M × compound interest on $1 at same rate, *r*, and for *n* years.

Annuity, Sinking Fund, Amortization, Depreciation.

(**14**) Under "Interest" we deal with cases where a certain sum or "principal," P, paid once for all, is allowed to accumulate either simple or compound interest; but in many cases equal periodical payments or appropriations, called **annuities**, are allowed to accumulate, each earning its own interest, usually compound.

(15) Thus, a sum of money is set aside annually to accumulate compound interest and thus form a **sinking fund,** in order to extinguish a debt. In this way, the cost of engineering works is frequently paid virtually in instalments. This process is called **amortization.**

(16) In estimating the operating expenses of engineering works, an allowance is made for **depreciation.** In calculating this allowance, we estimate or assume the life-time, n, of the plant, and find that annuity, p, which, at an assumed rate, r, of compound interest, will, in the time n, amount to the cost of the plant, and thus provide a fund by means of which the plant may be replaced when worn out or superseded.

(17) The **present worth, present value,** or **capitalization,** W, Fig. 2, of an annuity, p, for a given number, n, of years, is that sum which, if now placed at compound interest at the assumed rate, r, will, at the end of that time, reach the same amount, A, as will be reached by that annuity.

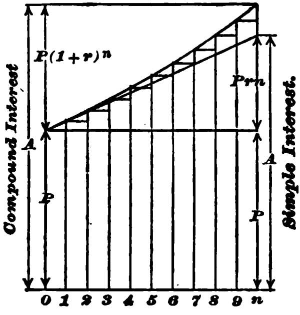


Fig. 1.

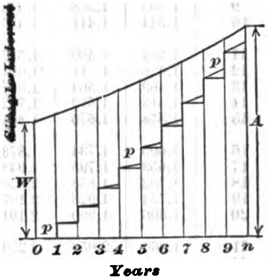


Fig. 2.

(18) Equations for Compound Interest and Annuities. (See Figs. 1 and 2.)

P = principal; r = rate of interest; n = number of years;
A = amount; p = annuity; W = present worth.

The interest is supposed to be compounded, and the annuities to be set aside, at the *end* of each year.

Compound Interest.

(1) **The amount, A,** of \$1, at the end of n years, see **(11)**, is $A = (1 + r)^n$.

(2) Since the present worth of $(1 + r)^n$, due in n years, is \$1, see (1), it follows, by proportion, that **the present worth, W,** of \$1, due in n years, is $W = \frac{1}{(1 + r)^n} = (1 + r)^{-n}$.

Annuities.

(3) In n years, an annuity of \$$r$ will amount to $(1 + r)^n - 1$.* Hence, the **amount, A, of an annuity** of \$1, at the end of n years, is

$$A = \frac{(1 + r)^n - 1}{r}.$$

---

*In the case of compound interest on \$1, the rate, r, may be regarded as an annuity, earning its interest; and, at the end of n years, the amount of the several annuities (each = the annual interest, r, on the \$1 principal) with the interests earned by them, is = the amount, $(1 + r)^n$, of \$1 in n years at rate, r, minus the \$1 principal itself; or, amount of annuity $= (1 + r)^n - 1$.

(4) For **the present worth, W, of an annuity** of $1 for n years, we have, from Equations (1) and (3):

$$(1+r)^n : 1 = \frac{(1+r)^n - 1}{r} : \text{W}. \quad \text{Hence, W} = \frac{(1+r)^n - 1}{(1+r)^n\, r} = \frac{1 - \dfrac{1}{(1+r)^n}}{r}$$

See Table 3.

(5) **The annuity** for n years, which $1 will purchase, is

$$p = \frac{1}{\text{W}}^{*} = \frac{r}{1 - \dfrac{1}{(1+r)^n}}$$

(6) **The annuity** which, in n years, will amount to $1, is

$$p' = p - r = \frac{1}{\text{W}} - r = \frac{r}{1 - \dfrac{1}{(1+r)^n}} - r = \frac{r\,\dagger}{(1+r)^n - 1}$$

See Table 4.

Table 3. Present Value of Annuity of $1000. See Equation (4).

	Rate of Interest (Compound).							
Years.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
5	4,646	4,580	4,515	4,452	4,390	4,329	4,268	4,212
10	8,752	8,530	8,316	8,111	7,913	7,722	7,538	7,360
15	12,381	11,938	11,517	11,118	10,740	10,380	10,037	9,712
20	15,589	14,877	14,212	13,590	13,008	12,462	11,950	11,470
25	18,424	17,413	16,482	15,622	14,828	14,094	13,414	12,783
30	20,930	19,600	18,392	17,292	16,289	15,372	14,534	13,765
35	23,145	21,487	20,000	18,664	17,461	16,374	15,391	14,498
40	25,103	23,115	21,355	19,793	18,401	17,159	16,045	15,046
45	26,833	24,519	22,495	20,720	19,156	17,774	16,548	15,456
50	28,362	25,730	23,456	21,482	19,762	18,256	16,932	15,762
100	36,614	31,599	27,655	24,505	21,950	19,848	18,096	16,618

(19) In comparing the merits of proposed systems of improvement, it is usual to add, to the operating expenses and to the cost of ordinary repairs and maintenance, (1) the interest on the cost, (2) an allowance for depreciation, and sometimes (3) an annuity to form a sinking fund for the extinction of the debt incurred by construction. The capitalization of the total annual expense, thus obtained, is then regarded as the true first cost of the construction. All the elements of cost are thus reduced to a common basis, and the several propositions become properly comparable.

(20) Thus, in estimating, in 1899,‡ the cost of improving the water supply of Philadelphia, the rate, r, of interest was assumed at 3 per cent, and depreciation was assumed as below. Under "Life" is given the assumed life-time of each class of structure or apparatus, and under "Annuity" the sum which must be set aside annually in order to replace, at the expiration of that life, $1,000 of the corresponding value.

$$^{*}\text{Because,}\ \underset{\text{Equation (4)}}{\overset{\text{Present worth}}{\text{W}} : \overset{\text{Annuity}}{\$1.00}} :: \underset{\text{Equation (5)}}{\overset{\text{Present worth}}{\$1.00} : \overset{\text{Annuity}}{p}}.\quad \text{Hence, } p = \frac{1}{\text{W}}.$$

$$^{\dagger}\text{Because,}\ \underset{\text{Equation (3)}}{\overset{\text{Annuity}}{r} : \overset{\text{Amount}}{(1+r)^n - 1}} :: \underset{\text{Equation (6)}}{\overset{\text{Annuity}}{p'} : \overset{\text{Amount}}{\$1.00}}.\quad \text{Hence, } p' = \frac{r}{(1+r)^n - 1}.$$

‡ Report by Rudolph Hering, Samuel M. Gray, and Joseph M. Wilson.

STRUCTURES, APPARATUS, ETC.	LIFE, in years	ANNUITY $
Masonry conduits, filter beds, reservoirs	Indefinite	0.00
Permanent buildings	100	1.65
Cast iron pipe, railroad side-tracks	80	3.11
Steel pipe, valves, blow-offs, and gates	35	16.54
Engines and pumps	30	21.02
Boilers, electric light plants, tramways and equipment, iron fences	20	37.22
Telephone lines, sand-washer, and regulating apparatus	10	87.24

(21) Calculated upon this basis, two projects, each designed to furnish 450 million gallons per day, compared as follows:

UNFILTERED WATER, BY AQUEDUCT.

First Cost.		
Storage reservoirs		$30,900,000
Aqueducts		47,730,000
Distribution		3,555,000
Distributing reservoir		1,000,000
Total		$83,185,000
Annual.		
Interest on $83,185,000		$2,495,550
Depreciation		198,640
Operation and Maintenance.		
Analyses and inspection	$41,620	
Ordinary repairs	49,150	
Pumping and wages	140,770	
		231,540
		$2,925,730

RIVER WATER, TAKEN WITHIN CITY LIMITS AND FILTERED.

First Cost.		
Filter plants		$23,174,680
Mains		10,980,000
Total		$34,154,680
Annual.		
Interest on $34,154,680		$1,024,640
Depreciation		205,540
Operation and Maintenance.		
Pumping	$1,216,021	
Filtration	525,600	
		1,741,621
		$2,971,801

It will be noticed that, although the first cost of the filtration project was much less than half that of the aqueduct project, its large proportion of perishable parts made its charge for depreciation somewhat greater, while its cost for operation and maintenance was more than seven times as great, and its total annual charge a little greater.

Table 4. Annuity required to redeem $1000. See Equation (6).

	Rate of Interest (Compound).							
Years.	1 per cent.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
5	196.04	192.16	190.24	188.36	186.49	184.63	180.98	177.39
10	95.58	91.33	89.25	87.23	85.24	83.29	79.50	75.87
15	62.12	57.83	55.77	53.77	51.82	49.94	46.34	42.96
20	45.42	41.15	39.14	37.22	35.36	33.58	30.24	27.18
25	35.41	31.22	29.27	27.43	25.67	24.01	20.95	18.23
30	28.75	24.65	22.78	21.02	19.37	17.83	15.05	12.65
35	24.00	20.00	18.20	16.54	15.00	13.58	11.07	8.97
40	20.46	16.55	14.84	13.26	11.83	10.52	8.28	6.46
45	17.71	13.91	12.27	10.79	9.45	8.26	6.26	4.70
50	15.51	11.82	10.26	8.87	7.63	6.55	4.78	3.44
60	12.24	8.77	7.35	6.13	5.09	4.20	2.83	1.88
70	9.93	6.67	5.40	4.34	3.46	2.74	1.70	1.03
80	8.22	5.16	4.03	3.11	2.38	1.81	1.03	0.573
90	6.91	4.05	3.04	2.26	1.66	1.21	0.627	0.318
100	5.87	3.20	2.31	1.65	1.16	0.808	0.383	0.177

DUODENAL OR DUODENARY NOTATION.*

(1) In the Arabic system of notation 10 is taken as the base, but in duodenal notation 12, or "a dozen," is the base. While 10 is divisible only by 5, and (once only) by 2, 12 is divisible twice by 2, and once by 3, by 4, and by 6. This accounts for the popularity of the dozen as a basis of enumeration; of weights, as in the Troy pound of 12 ounces; of measures, as in the foot of 12 inches; the year of 12 months, and the half day of 12 hours; and of coinage, as in the British shilling of 12 pence.

(2) The duodenal notation uses the dozen (12), the gross ($12^2 = 144$), and the great gross ($12^3 = 12$ gross $= 1728$), as the decimal system uses the ten (10), the hundred ($10^2 = 100$), and the thousand ($10^3 = 10$ hundred $= 1000$). Two arbitrary single characters, such as T and E, represent ten and eleven respectively; the symbol 10 represents a dozen; 11 represents thirteen, and so on. Thus, the numerals of the two systems compare as follows:

Decimal	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...	20	21	22	23	24	25	36	48	60
Duodenal	1	2	3	4	5	6	7	8	9	T	E	10	11	12	...	18	19	1T	1E	20	21	30	40	50

Decimal	72	84	96	99	100	108	109	110	111	112	113	117	118	119	120	121	122
Duodenal	60	70	80	83	84	90	91	92	93	94	95	99	9T	9E	T0	T1	T2

Decimal	129	130	131	132	133	138	140	141	142	143	144	288	1728	20736	etc.
Duodenal	T9	TT	TE	E0	E1	E6	E8	E9	ET	EE	100	200	1000	10000	etc.

(3) **Duodecimals.** Areas of rectangular figures, the sides of which are expressed in feet and inches, are still sometimes found by a method called "Duodecimals," in which the products are in square feet, in twelfths of a square foot (each equal to 12 square inches) and in square inches; but, by means of our table of "Inches, reduced to decimals of a foot," page 221, the sides may be taken in feet and decimals of a foot, and the multiplication thus more conveniently performed, after which the decimal fraction of a foot in the product may, if desired, be converted into square inches by multiplying by 144.

---

*See Elements of Mechanics, by the late John W. Nystrom.

Table of Reciprocals of Numbers. See p. 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
1	1.000000000	56	.017857143	111	.009009009	166	.006024096
2	0.500000000	57	.017543860	112	.008928571	167	.005988024
3	.333333333	58	.017241379	113	.008849558	168	.005952381
4	.250000000	59	.016949153	114	.008771930	169	.005917160
5	.200000000	60	.016666667	115	.008695652	170	.005882353
6	.166666667	61	.016393443	116	.008620690	171	.005847953
7	.142857143	62	.016129032	117	.008547009	172	.005813953
8	.125000000	63	.015873016	118	.008474576	173	.005780347
9	.111111111	64	.015625000	119	.008403361	174	.005747126
10	.100000000	65	.015384615	120	.008333333	175	.005714286
11	.090909091	66	.015151515	121	.008264463	176	.005681818
12	.083333333	67	.014925373	122	.008196721	177	.005649718
13	.076923077	68	.014705882	123	.008130081	178	.005617978
14	.071428571	69	.014492754	124	.008064516	179	.005586592
15	.066666667	70	.014285714	125	.008000000	180	.005555556
16	.062500000	71	.014084507	126	.007936508	181	.005524862
17	.058823529	72	.013888889	127	.007874016	182	.005494505
18	.055555556	73	.013698630	128	.007812500	183	.005464481
19	.052631579	74	.013513514	129	.007751938	184	.005434783
20	.050000000	75	.013333333	130	.007692308	185	.005405405
21	.047619048	76	.013157895	131	.007633588	186	.005376344
22	.045454545	77	.012987013	132	.007575758	187	.005347594
23	.043478261	78	.012820513	133	.007518797	188	.005319149
24	.041666667	79	.012658228	134	.007462687	189	.005291005
25	.040000000	80	.012500000	135	.007407407	190	.005263158
26	.038461538	81	.012345679	136	.007352941	191	.005235602
27	.037037037	82	.012195122	137	.007299270	192	.005208333
28	.035714286	83	.012048193	138	.007246377	193	.005181347
29	.034482759	84	.011904762	139	.007194245	194	.005154639
30	.033333333	85	.011764706	140	.007142857	195	.005128205
31	.032258065	86	.011627907	141	.007092199	196	.005102041
32	.031250000	87	.011494253	142	.007042254	197	.005076142
33	.030303030	88	.011363636	143	.006993007	198	.005050505
34	.029411765	89	.011235955	144	.006944444	199	.005025126
35	.028571429	90	.011111111	145	.006896552	200	.005000000
36	.027777778	91	.010989011	146	.006849315	201	.004975124
37	.027027027	92	.010869565	147	.006802721	202	.004950495
38	.026315789	93	.010752688	148	.006756757	203	.004926108
39	.025641026	94	.010638298	149	.006711409	204	.004901961
40	.025000000	95	.010526316	150	.006666667	205	.004878049
41	.024390244	96	.010416667	151	.006622517	206	.004854369
42	.023809524	97	.010309278	152	.006578947	207	.004830918
43	.023255814	98	.010204082	153	.006535948	208	.004807692
44	.022727273	99	.010101010	154	.006493506	209	.004784689
45	.022222222	100	.010000000	155	.006451613	210	.004761905
46	.021739130	101	.009900990	156	.006410256	211	.004739336
47	.021276600	102	.009803922	157	.006369427	212	.004716981
48	.020833333	103	.009708738	158	.006329114	213	.004694836
49	.020408163	104	.009615385	159	.006289308	214	.004672897
50	.020000000	105	.009523810	160	.006250000	215	.004651163
51	.019607843	106	.009433962	161	.006211180	216	.004629630
52	.019230769	107	.009345794	162	.006172840	217	.004608295
53	.018867925	108	.009259259	163	.006134969	218	.004587156
54	.018518519	109	.009174312	164	.006097561	219	.004566210
55	.018181818	110	.009090909	165	.006060606	220	.004545455

Table of Reciprocals of Numbers.—(*Continued.*) See p. 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
221	.004524887	276	.003623188	331	.003021148	386	.002590674
222	.004504505	277	.003610108	332	.003012048	387	.002583979
223	.004484305	278	.003597122	333	.003003003	388	.002577320
224	.004464286	279	.003584229	334	.002994012	389	.002570694
225	.004444444	280	.003571429	335	.002985075	390	.002564103
226	.004424779	281	.003558719	336	.002976190	391	.002557545
227	.004405286	282	.003546099	337	.002967359	392	.002551020
228	.004385965	283	.003533569	338	.002958580	393	.002544529
229	.004366812	284	.003521127	339	.002949853	394	.002538071
230	.004347826	285	.003508772	340	.002941176	395	.002531646
231	.004329004	286	.003496503	341	.002932551	396	.002525253
232	.004310345	287	.003484321	342	.002923977	397	.002518892
233	.004291845	288	.003472222	343	.002915452	398	.002512563
234	.004273504	289	.003460208	344	.002906977	399	.002506266
235	.004255319	290	.003448276	345	.002898551	400	.002500000
236	.004237288	291	.003436426	346	.002890173	401	.002493766
237	.004219409	292	.003424658	347	.002881844	402	.002487562
238	.004201681	293	.003412969	348	.002873563	403	.002481390
239	.004184100	294	.003401361	349	.002865330	404	.002475248
240	.004166667	295	.003389831	350	.002857143	405	.002469136
241	.004149378	296	.003378378	351	.002849003	406	.002463054
242	.004132231	297	.003367003	352	.002840909	407	.002457002
243	.004115226	298	.003355705	353	.002832861	408	.002450980
244	.004098361	299	.003344482	354	.002824859	409	.002444988
245	.004081633	300	.003333333	355	.002816901	410	.002439024
246	.004065041	301	.003322259	356	.002808989	411	.002433090
247	.004048583	302	.003311258	357	.002801120	412	.002427184
248	.004032258	303	.003300330	358	.002793296	413	.002421308
249	.004016064	304	.003289474	359	.002785515	414	.002415459
250	.004000000	305	.003278689	360	.002777778	415	.002409639
251	.003984064	306	.003267974	361	.002770083	416	.002403846
252	.003968254	307	.003257329	362	.002762431	417	.002398082
253	.003952569	308	.003246753	363	.002754821	418	.002392344
254	.003937008	309	.003236246	364	.002747253	419	.002386635
255	.003921569	310	.003225806	365	.002739726	420	.002380952
256	.003906250	311	.003215434	366	.002732240	421	.002375297
257	.003891051	312	.003205128	367	.002724796	422	.002369668
258	.003875969	313	.003194888	368	.002717391	423	.002364066
259	.003861004	314	.003184713	369	.002710027	424	.002358491
260	.003846154	315	.003174603	370	.002702703	425	.002352941
261	.003831418	316	.003164557	371	.002695418	426	.002347418
262	.003816794	317	.003154574	372	.002688172	427	.002341920
263	.003802281	318	.003144654	373	.002680965	428	.002336449
264	.003787879	319	.003134796	374	.002673797	429	.002331002
265	.003773585	320	.003125000	375	.002666667	430	.002325581
266	.003759398	321	.003115265	376	.002659574	431	.002320186
267	.003745318	322	.003105590	377	.002652520	432	.002314815
268	.003731343	323	.003095975	378	.002645503	433	.002309469
269	.003717472	324	.003086420	379	.002638522	434	.002304147
270	.003703704	325	.003076923	380	.002631579	435	.002298851
271	.003690037	326	.003067485	381	.002624672	436	.002293578
272	.003676471	327	.003058104	382	.002617801	437	.002288330
273	.003663004	328	.003048780	383	.002610966	438	.002283105
274	.003649635	329	.003039514	384	.002604167	439	.002277904
275	.003636364	330	.003030303	385	.002597403	440	.002272727

Table of Reciprocals of Numbers.—(*Continued.*) See p. 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal
441	.002267574	496	.002016129	551	.001814882	606	.001650165
442	.002262443	497	.002012072	552	.001811594	607	.001647446
443	.002257336	498	.002008032	553	.001808318	608	.001644737
444	.002252252	499	.002004008	554	.001805054	609	.001642036
445	.002247191	500	.002000000	555	.001801802	610	.001639344
446	.002242152	501	.001996008	556	.001798561	611	.001636661
447	.002237136	502	.001992032	557	.001795332	612	.001633987
448	.002232143	503	.001988072	558	.001792115	613	.001631321
449	.002227171	504	.001984127	559	.001788909	614	.001628664
450	.002222222	505	.001980198	560	.001785714	615	.001626016
451	.002217295	506	.001976285	561	.001782531	616	.001623377
452	.002212389	507	.001972387	562	.001779359	617	.001620746
453	.002207506	508	.001968504	563	.001776199	618	.001618123
454	.002202643	509	.001964637	564	.001773050	619	.001615509
455	.002197802	510	.001960784	565	.001769912	620	.001612903
456	.002192982	511	.001956947	566	.001766784	621	.001610306
457	.002188184	512	.001953125	567	.001763668	622	.001607717
458	.002183406	513	.001949318	568	.001760563	623	.001605136
459	.002178649	514	.001945525	569	.001757469	624	.001602564
460	.002173913	515	.001941748	570	.001754386	625	.001600000
461	.002169197	516	.001937984	571	.001751313	626	.001597444
462	.002164502	517	.001934236	572	.001748252	627	.001594896
463	.002159827	518	.001930502	573	.001745201	628	.001592357
464	.002155172	519	.001926782	574	.001742160	629	.001589825
465	.002150538	520	.001923077	575	.001739130	630	.001587302
466	.002145923	521	.001919386	576	.001736111	631	.001584786
467	.002141328	522	.001915709	577	.001733102	632	.001582278
468	.002136752	523	.001912046	578	.001730104	633	.001579779
469	.002132196	524	.001908397	579	.001727116	634	.001577287
470	.002127660	525	.001904762	580	.001724138	635	.001574803
471	.002123142	526	.001901141	581	.001721170	636	.001572327
472	.002118644	527	.001897533	582	.001718213	637	.001569859
473	.002114165	528	.001893939	583	.001715266	638	.001567398
474	.002109705	529	.001890359	584	.001712329	639	.001564945
475	.002105263	530	.001886792	585	.001709402	640	.001562500
476	.002100840	531	.001883239	586	.001706485	641	.001560062
477	.002096436	532	.001879699	587	.001703578	642	.001557632
478	.002092050	533	.001876173	588	.001700680	643	.001555210
479	.002087683	534	.001872659	589	.001697793	644	.001552795
480	.002083333	535	.001869159	590	.001694915	645	.001550388
481	.002079002	536	.001865672	591	.001692047	646	.001547988
482	.002074689	537	.001862197	592	.001689189	647	.001545595
483	.002070393	538	.001858736	593	.001686341	648	.001543210
484	.002066116	539	.001855288	594	.001683502	649	.001540832
485	.002061856	540	.001851852	595	.001680672	650	.001538462
486	.002057613	541	.001848429	596	.001677852	651	.001536098
487	.002053388	542	.001845018	597	.001675042	652	.001533742
488	.002049180	543	.001841621	598	.001672241	653	.001531394
489	.002044990	544	.001838235	599	.001669449	654	.001529052
490	.002040816	545	.001834862	600	.001666667	655	.001526718
491	.002036660	546	.001831502	601	.001663894	656	.001524390
492	.002032520	547	.001828154	602	.001661130	657	.001522070
493	.002028398	548	.001824818	603	.001658375	658	.001519757
494	.002024291	549	.001821494	604	.001655629	659	.001517451
495	.002020202	550	.001818182	605	.001652893	660	.001515152

Table of Reciprocals of Numbers.—(*Continued.*) See p. 52.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
661	.001512859	716	.001396648	771	.001297017	826	.001210654
662	.001510574	717	.001394700	772	.001295337	827	.001209190
663	.001508296	718	.001392758	773	.001293661	828	.001207729
664	.001506024	719	.001390821	774	.001291990	829	.001206273
665	.001503759	720	.001388889	775	.001290323	830	.001204819
666	.001501502	721	.001386963	776	.001288660	831	.001203369
667	.001499250	722	.001385042	777	.001287001	832	.001201923
668	.001497006	723	.001383126	778	.001285347	833	.001200480
669	.001494768	724	.001381215	779	.001283697	834	.001199041
670	.001492537	725	.001379310	780	.001282051	835	.001197605
671	.001490313	726	.001377410	781	.001280410	836	.001196172
672	.001488095	727	.001375516	782	.001278772	837	.001194743
673	.001485884	728	.001373626	783	.001277139	838	.001193317
674	.001483680	729	.001371742	784	.001275510	839	.001191895
675	.001481481	730	.001369863	785	.001273885	840	.001190476
676	.001479290	731	.001367989	786	.001272265	841	.001189061
677	.001477105	732	.001366120	787	.001270648	842	.001187648
678	.001474926	733	.001364256	788	.001269036	843	.001186240
679	.001472754	734	.001362398	789	.001267427	844	.001184834
680	.001470588	735	.001360544	790	.001265823	845	.001183432
681	.001468429	736	.001358696	791	.001264223	846	.001182033
682	.001466276	737	.001356852	792	.001262626	847	.001180638
683	.001464129	738	.001355014	793	.001261034	848	.001179245
684	.001461988	739	.001353180	794	.001259446	849	.001177856
685	.001459854	740	.001351351	795	.001257862	850	.001176471
686	.001457726	741	.001349528	796	.001256281	851	.001175088
687	.001455604	742	.001347709	797	.001254705	852	.001173709
688	.001453488	743	.001345895	798	.001253133	853	.001172333
689	.001451379	744	.001344086	799	.001251564	854	.001170960
690	.001449275	745	.001342282	800	.001250000	855	.001169591
691	.001447178	746	.001340483	801	.001248439	856	.001168224
692	.001445087	747	.001338688	802	.001246883	857	.001166861
693	.001443001	748	.001336898	803	.001245330	858	.001165501
694	.001440922	749	.001335113	804	.001243781	859	.001164144
695	.001438849	750	.001333333	805	.001242236	860	.001162791
696	.001436782	751	.001331558	806	.001240695	861	.001161440
697	.001434720	752	.001329787	807	.001239157	862	.001160093
698	.001432665	753	.001328021	808	.001237624	863	.001158749
699	.001430615	754	.001326260	809	.001236094	864	.001157407
700	.001428571	755	.001324503	810	.001234568	865	.001156069
701	.001426534	756	.001322751	811	.001233046	866	.001154734
702	.001424501	757	.001321004	812	.001231527	867	.001153403
703	.001422475	758	.001319261	813	.001230012	868	.001152074
704	.001420455	759	.001317523	814	.001228501	869	.001150748
705	.001418440	760	.001315789	815	.001226994	870	.001149425
706	.001416431	761	.001314060	816	.001225490	871	.001148106
707	.001414427	762	.001312336	817	.001223990	872	.001146789
708	.001412429	763	.001310616	818	.001222494	873	.001145475
709	.001410437	764	.001308901	819	.001221001	874	.001144165
710	.001408451	765	.001307190	820	.001219512	875	.001142857
711	.001406470	766	.001305483	821	.001218027	876	.001141553
712	.001404494	767	.001303781	822	.001216545	877	.001140251
713	.001402525	768	.001302083	823	.001215067	878	.001138952
714	.001400560	769	.001300390	824	.001213592	879	.001137656
715	.001398601	770	.001298701	825	.001212121	880	.001136364

Table of Reciprocals of Numbers.—(*Continued.*) See below.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
881	.001135074	911	.001097695	941	.001062699	971	.001029866
882	.001133787	912	.001096491	942	.001061571	972	.001028807
883	.001132503	913	.001095290	943	.001060445	973	.001027749
884	.001131222	914	.001094092	944	.001059322	974	.001026694
885	.001129944	915	.001092896	945	.001058201	975	.001025641
886	.001128668	916	.001091703	946	.001057082	976	.001024590
887	.001127396	917	.001090513	947	.001055966	977	.001023541
888	.001126126	918	.001089325	948	.001054852	978	.001022495
889	.001124859	919	.001088139	949	.001053741	979	.001021450
890	.001123596	920	.001086957	950	.001052632	980	.001020408
891	.001122334	921	.001085776	951	.001051525	981	.001019368
892	.001121076	922	.001084599	952	.001050420	982	.001018330
893	.001119821	923	.001083424	953	.001049318	983	.001017294
894	.001118568	924	.001082251	954	.001048218	984	.001016260
895	.001117318	925	.001081081	955	.001047120	985	.001015228
896	.001116071	926	.001079914	956	.001046025	986	.001014199
897	.001114827	927	.001078749	957	.001044932	987	.001013171
898	.001113586	928	.001077586	958	.001043841	988	.001012146
899	.001112347	929	.001076426	959	.001042753	989	.001011122
900	.001111111	930	.001075269	960	.001041667	990	.001010101
901	.001109878	931	.001074114	961	.001040583	991	.001009082
902	.001108647	932	.001072961	962	.001039501	992	.001008065
903	.001107420	933	.001071811	963	.001038422	993	.001007049
904	.001106195	934	.001070664	964	.001037344	994	.001006036
905	.001104972	935	.001069519	965	.001036269	995	.001005025
906	.001103753	936	.001068376	966	.001035197	996	.001004016
907	.001102536	937	.001067236	967	.001034126	997	.001003009
908	.001101322	938	.001066098	968	.001033058	998	.001002004
909	.001100110	939	.001064963	969	.001031992	999	.001001601
910	.001098901	940	.001063830	970	.001030928	1000	.001000000

RECIPROCALS.

(a) **The reciprocal of a number is** the quantity obtained by dividing unity or 1 by that number. In other words, if n be any number, then Recip $n = \frac{1}{n}$. Thus, Recip $40 = \frac{1}{40} = 0.025$; Recip $0.4 = \frac{1}{0.4} = 2.5$, etc., etc.

Hence, Recip $\frac{a}{b} = \frac{b}{a}$, because Recip $\frac{a}{b} = 1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$.

Thus, since 1 yard = 36 inches, 1 inch = $\frac{1}{36}$ yard = .027777778 yard, for Recip 36 = .027777778. Again, 1 foot head of water gives a pressure of .4335 lbs. per square inch. Hence a pressure of 1 lb. per square inch corresponds to a head of $\frac{1}{.4335}$ feet = 2.306805 feet, for Recip .4335 = 2.306805. (See **h**, below.)

(**b**) It follows that if any number in the column headed "No." be taken as the denominator of a common fraction whose numerator is 1, the corresponding reciprocal is the value of that fraction expressed in decimals.* Thus, $\frac{1}{32} = .03125$. Hence, **to reduce a common fraction to decimal form,** multiply the reciprocal of the denominator by the numerator. Thus, $\frac{17}{32} = .53125$, because Recip 32 = .03125, and .03125 × 17 = .53125.

(**c**) Conversely, if the reciprocal of a number n be taken as a number, then the number n itself becomes the reciprocal. In other words, Recip $\frac{1}{n} = n$. Thus, Recip 0.025 = Recip $\frac{1}{40} = 40$; Recip 2.5 = Recip $\frac{1}{0.4} = 0.4$, etc., etc.

---

* The numbers 2 and 5, and their powers and products, are the only ones whose reciprocals can be *exactly* expressed in decimals.

(d) The product of any number by its own reciprocal is equal to unity or 1; or, $n \times \frac{1}{n} = \frac{n}{n} = 1$.

(e) Any number, $a \times$ Recip of a number, $n = a \times \frac{1}{n} = \frac{a}{n}$.

Hence, **to avoid the labor of dividing,** we may *multiply* by the *reciprocal* of the divisor. Thus,
200 ÷ 48750 = 200 × Recip 48750 = 200 × .00002051282 (see **h,** below) = .004102564. . . .

(f) Any number, $a \div$ Recip of a number, $n = a \div \frac{1}{n} = a\,n$.

Hence, $a \div \text{Recip}\ a = a \div \frac{1}{a} = a \times \frac{a}{1} = a^2$.

Thus, $\text{Recip}\ 2 = 0.5$, and $\frac{2}{\text{Recip}\ 2} = \frac{2}{0.5} = 4 = 2^2$.

(g) The numbers in the foregoing table extend from 1 to 1000; but the reciprocals of **multiples of these numbers by 10** may be taken from the table by adding one cipher to the left of the reciprocal (after the decimal point) for each cipher added to the number. Thus,

Recip 390 = .002564103;
Recip 3900 = .0002564103;
Recip 39000 = .00002564103;

and the reciprocals of **numbers containing decimals** may be taken from the table by shifting the decimal point in the tabular reciprocal one place to the right for each decimal place in the number. Thus:

Recip 227 = .004405286;
Recip 22.7 = .04405286;
Recip 2.27 = .4405286;
Recip .227 = 4.405286;
Recip .0227 = 44.05286.

(h) The reciprocal of **a number of more than three figures** may be taken from the table approximately by interpolation. Thus, to find Recip 236.4:

Recip 236 = .004237288
Recip 237 = .004219409
Differences: 1, .000017879, 236.4 — 236 = 0.4.
Then, 0.4 × .000017879 = .000007152,
and, Recip 236 = .004237288
minus .000007152
= Recip 236.4 = .004230136 by interpolation.

The correct reciprocal is .004230118.

(i) The reciprocals of numbers not in the table may be conveniently found **by means of logarithms.** Thus, to find the Recip 236.4 = $\frac{1}{236.4}$:

Log 1 = 0.000000
Subtract Log 236.4 = 2.373647
$\bar{3}$.626353 = Log 0.00423012
Recip 236.4 = 0.00423012.

To find Recip $\frac{8424}{236.4} = \frac{236.4}{8424}$,

Log 236.4 = 2.373647
Subtract Log 8424 = 3.925518
$\bar{2}$.448129 = Log 0.0280627.
Recip $\frac{8424}{236.4}$ = 0.0280627.

(j) **Position of the decimal point.** For the Nos. 10, 100, 1000, etc., the number of the decimal place occupied by the first significant figure in the reciprocal is equal to the number of *ciphers* in the No.; but for all other Nos. it is equal to the number of the *digits* in the integral portion of the No. Thus: Recip 143.7 = .0069.., etc. Here the number of digits in the integral portion (143) of the No. is 3, and the first significant figure (6) of the reciprocal occupies the *third* decimal place.

Square Roots and Cube Roots of Numbers from .1 to 28.

No errors.

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Sq. Rt.	C. Rt.	No.	Sq. Rt.	C. Rt.
.1	.01	.001	.316	.464	.7	2.387	1.786	.4	3.661	2.375
.15	.0225	.0034	.387	.531	.8	2.408	1.797	.6	3.688	2.387
.2	.04	.008	.447	.585	.9	2.429	1.807	.8	3.715	2.399
.25	.0625	.0156	.500	.630	6.	2.449	1.817	14.	3.742	2.410
.3	.09	.027	.548	.669	.1	2.470	1.827	.2	3.768	2.422
.35	.1225	.0429	.592	.705	.2	2.490	1.837	.4	3.795	2.433
.4	.16	.064	.633	.737	.3	2.510	1.847	.6	3.821	2.444
45	.2025	.0911	.671	.766	.4	2.530	1.857	.8	3.847	2.455
.5	.25	.125	.707	.794	.5	2.550	1.866	15.	3.873	2.466
.55	.3025	.1664	.742	.819	.6	2.569	1.876	.2	3.899	2.477
.6	.36	.216	.775	.843	.7	2.588	1.885	.4	3.924	2.488
.65	.4225	.2746	.806	.866	.8	2.608	1.895	.6	3.950	2.499
.7	.49	.343	.837	.888	.9	2.627	1.904	.8	3.975	2.509
.75	.5625	.4219	.866	.909	7.	2.646	1.913	16.	4.	2.520
.8	.64	.512	.894	.928	.1	2.665	1.922	.2	4.025	2.530
.85	.7225	.6141	.922	.947	.2	2.683	1.931	.4	4.050	2.541
.9	.81	.729	.949	.965	.3	2.702	1.940	.6	4.074	2.551
.95	.9025	.8574	.975	.983	.4	2.720	1.949	.8	4.099	2.561
1.	1.000	1.000	1.000	1.000	.5	2.739	1.957	17.	4.123	2.571
.05	1.103	1.158	1.025	1.016	.6	2.757	1.966	.2	4.147	2.581
1.1	1.210	1.331	1.049	1.032	.7	2.775	1.975	.4	4.171	2.591
.15	1.323	1.521	1.072	1.048	.8	2.793	1.983	.6	4.195	2.601
1.2	1.440	1.728	1.095	1.063	.9	2.811	1.992	.8	4.219	2.611
.25	1.563	1.953	1.118	1.077	8.	2.828	2.000	18.	4.243	2.621
1.3	1.690	2.197	1.140	1.091	.1	2.846	2.008	.2	4.266	2.630
.35	1.823	2.460	1.162	1.105	.2	2.864	2.017	.4	4.290	2.640
1.4	1.960	2.744	1.183	1.119	.3	2.881	2.025	.6	4.313	2.650
.45	2.103	3.049	1.204	1.132	.4	2.898	2.033	.8	4.336	2.659
1.5	2.250	3.375	1.225	1.145	.5	2.915	2.041	19.	4.359	2.668
.55	2.403	3.724	1.245	1.157	.6	2.938	2.049	.2	4.382	2.678
1.6	2.560	4.096	1.265	1.170	.7	2.950	2.057	.4	4.405	2.687
.65	2.723	4.492	1.285	1.182	.8	2.966	2.065	.6	4.427	2.696
1.7	2.890	4.913	1.304	1.193	.9	2.983	2.072	.8	4.450	2.705
.75	3.063	5.359	1.323	1.205	9.	3.	2.080	20.	4.472	2.714
1.8	3.240	5.832	1.342	1.216	.1	3.017	2.088	.2	4.494	2.723
.85	3.423	6.332	1.360	1.228	.2	3.033	2.095	.4	4.517	2.732
1.9	3.610	6.859	1.378	1.239	.3	3.050	2.103	.6	4.539	2.741
.95	3.803	7.415	1.396	1.249	.4	3.066	2.110	.8	4.561	2.750
2.	4.000	8.000	1.414	1.260	.5	3.082	2.118	21.	4.583	2.759
.1	4.410	9.261	1.449	1.281	.6	3.098	2.125	.2	4.604	2.768
.2	4.840	10.65	1.483	1.301	.7	3.114	2.133	.4	4.626	2.776
.3	5.290	12.17	1.517	1.320	.8	3.130	2.140	.6	4.648	2.785
.4	5.760	13.82	1.549	1.339	.9	3.146	2.147	.8	4.669	2.794
.5	6.250	15.63	1.581	1.357	10.	3.162	2.154	22.	4.690	2.802
.6	6.760	17.58	1.612	1.375	.1	3.178	2.162	.2	4.712	2.810
.7	7.290	19.68	1.643	1.392	.2	3.194	2.169	.4	4.733	2.819
.8	7.840	21.95	1.673	1.409	.3	3.209	2.176	.6	4.754	2.827
.9	8.410	24.39	1.703	1.426	.4	3.225	2.183	.8	4.775	2.836
3.	9.	27.	1.732	1.442	.5	3.240	2.190	23.	4.796	2.844
.1	9.61	29.79	1.761	1.458	.6	3.256	2.197	.2	4.817	2.852
.2	10.24	32.77	1.789	1.474	.7	3.271	2.204	.4	4.837	2.860
.3	10.89	35.94	1.817	1.489	.8	3.286	2.210	.6	4.858	2.868
.4	11.56	39.30	1.844	1.504	.9	3.302	2.217	.8	4.879	2.876
.5	12.25	42.88	1.871	1.518	11.	3.317	2.224	24.	4.899	2.884
.6	12.96	46.66	1.897	1.533	.1	3.332	2.231	.2	4.919	2.892
.7	13.69	50.65	1.924	1.547	.2	3.347	2.237	.4	4.940	2.900
.8	14.44	54.87	1.949	1.560	.3	3.362	2.244	.6	4.960	2.908
.9	15.21	59.32	1.975	1.574	.4	3.376	2.251	.8	4.980	2.916
4.	16.	64.	2.	1.587	.5	3.391	2.257	25.	5.	2.924
.1	16.81	68.92	2.025	1.601	.6	3.406	2.264	.2	5.020	2.932
.2	17.64	74.09	2.049	1.613	.7	3.421	2.270	.4	5.040	2.940
.3	18.49	79.51	2.074	1.626	.8	3.435	2.277	.6	5.060	2.947
.4	19.36	85.18	2.098	1.639	.9	3.450	2.283	.8	5.079	2.955
.5	20.25	91.13	2.121	1.651	12.	3.464	2.289	26.	5.099	2.962
.6	21.16	97.34	2.145	1.663	.1	3.479	2.296	.2	5.119	2.970
.7	22.09	103.8	2.168	1.675	.2	3.493	2.302	.4	5.138	2.978
.8	23.04	110.6	2.191	1.687	.3	3.507	2.308	.6	5.158	2.985
.9	24.01	117.6	2.214	1.698	.4	3.521	2.315	.8	5.177	2.993
5.	25.	125.	2.236	1.710	.5	3.536	2.321	27.	5.196	3.000
.1	26.01	132.7	2.258	1.721	.6	3.550	2.327	.2	5.215	3.007
.2	27.04	140.6	2.280	1.732	.7	3.564	2.333	.4	5.235	3.015
.3	28.09	148.9	2.302	1.744	.8	3.578	2.339	.6	5.254	3.022
.4	29.16	157.5	2.324	1.754	.9	3.592	2.345	.8	5.273	3.029
.5	30.25	166.4	2.345	1.765	13.	3.606	2.351	28.	5.292	3.037
.6	31.36	175.6	2.366	1.776	.2	3.633	2.363	.2	5.310	3.044

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000.

REMARK ON THE FOLLOWING TABLE. Wherever the effect of a fifth decimal in the roots would be to add 1 to the fourth and final decimal in the table, the addition has been made. No errors.

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
1	1	1	1.0000	1.0000	61	3721	226981	7.8102	3.9365
2	4	8	1.4142	1.2599	62	3844	238328	7.8740	3.9579
3	9	27	1.7321	1.4422	63	3969	250047	7.9373	3.9791
4	16	64	2.0000	1.5874	64	4096	262144	8.0000	4.
5	25	125	2.2361	1.7100	65	4225	274625	8.0623	4.0207
6	36	216	2.4495	1.8171	66	4356	287496	8.1240	4.0412
7	49	343	2.6458	1.9129	67	4489	300763	8.1854	4.0615
8	64	512	2.8284	2.0000	68	4624	314432	8.2462	4.0817
9	81	729	3.0000	2.0801	69	4761	328509	8.3066	4.1016
10	100	1000	3.1623	2.1544	70	4900	343000	8.3666	4.1213
11	121	1331	3.3166	2.2240	71	5041	357911	8.4261	4.1408
12	144	1728	3.4641	2.2894	72	5184	373248	8.4853	4.1602
13	169	2197	3.6056	2.3513	73	5329	389017	8.5440	4.1793
14	196	2744	3.7417	2.4101	74	5476	405224	8.6023	4.1983
15	225	3375	3.8730	2.4662	75	5625	421875	8.6603	4.2172
16	256	4096	4.0000	2.5198	76	5776	438976	8.7178	4.2358
17	289	4913	4.1231	2.5713	77	5929	456533	8.7750	4.2543
18	324	5832	4.2426	2.6207	78	6084	474552	8.8318	4.2727
19	361	6859	4.3589	2.6684	79	6241	493039	8.8882	4.2908
20	400	8000	4.4721	2.7144	80	6400	512000	8.9443	4.3089
21	441	9261	4.5826	2.7589	81	6561	531441	9.	4.3267
22	484	10648	4.6904	2.8020	82	6724	551368	9.0554	4.3445
23	529	12167	4.7958	2.8439	83	6889	571787	9.1104	4.3621
24	576	13824	4.8990	2.8845	84	7056	592704	9.1652	4.3795
25	625	15625	5.0000	2.9240	85	7225	614125	9.2195	4.3968
26	676	17576	5.0990	2.9625	86	7396	636056	9.2736	4.4140
27	729	19683	5.1962	3.0000	87	7569	658503	9.3274	4.4310
28	784	21952	5.2915	3.0366	88	7744	681472	9.3808	4.4480
29	841	24389	5.3852	3.0723	89	7921	704969	9.4340	4.4647
30	900	27000	5.4772	3.1072	90	8100	729000	9.4868	4.4814
31	961	29791	5.5678	3.1414	91	8281	753571	9.5394	4.4979
32	1024	32768	5.6569	3.1748	92	8464	778688	9.5917	4.5144
33	1089	35937	5.7446	3.2075	93	8649	804357	9.6437	4.5307
34	1156	39304	5.8310	3.2396	94	8836	830584	9.6954	4.5468
35	1225	42875	5.9161	3.2711	95	9025	857375	9.7468	4.5629
36	1296	46656	6.0000	3.3019	96	9216	884736	9.7980	4.5789
37	1369	50653	6.0828	3.3322	97	9409	912673	9.8489	4.5947
38	1444	54872	6.1644	3.3620	98	9604	941192	9.8995	4.6104
39	1521	59319	6.2450	3.3912	99	9801	970299	9.9499	4.6261
40	1600	64000	6.3246	3.4200	100	10000	1000000	10.	4.6416
41	1681	68921	6.4031	3.4482	101	10201	1030301	10.0499	4.6570
42	1764	74088	6.4807	3.4760	102	10404	1061208	10.0995	4.6723
43	1849	79507	6.5574	3.5034	103	10609	1092727	10.1489	4.6875
44	1936	85184	6.6332	3.5303	104	10816	1124864	10.1980	4.7027
45	2025	91125	6.7082	3.5569	105	11025	1157625	10.2470	4.7177
46	2116	97336	6.7823	3.5830	106	11236	1191016	10.2956	4.7326
47	2209	103823	6.8557	3.6088	107	11449	1225043	10.3441	4.7475
48	2304	110592	6.9282	3.6342	108	11664	1259712	10.3923	4.7622
49	2401	117649	7.0000	3.6593	109	11881	1295029	10.4403	4.7769
50	2500	125000	7.0711	3.6840	110	12100	1331000	10.4881	4.7914
51	2601	132651	7.1414	3.7084	111	12321	1367631	10.5357	4.8059
52	2704	140608	7.2111	3.7325	112	12544	1404928	10.5830	4.8203
53	2809	148877	7.2801	3.7563	113	12769	1442897	10.6301	4.8346
54	2916	157464	7.3485	3.7798	114	12996	1481544	10.6771	4.8488
55	3025	166375	7.4162	3.8030	115	13225	1520875	10.7238	4.8629
56	3136	175616	7.4833	3.8259	116	13456	1560896	10.7703	4.8770
57	3249	185193	7.5498	3.8485	117	13689	1601613	10.8167	4.8910
58	3364	195112	7.6158	3.8709	118	13924	1643032	10.8628	4.9049
59	3481	205379	7.6811	3.8930	119	14161	1685159	10.9087	4.9187
60	3600	216000	7.7460	3.9149	120	14400	1728000	10.9545	4.9324

TABLE of Squares, Cubes, Square Roots, and Cube Roots of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. R
121	14641	1771561	11.	4.9461	186	34596	6434856	13.6382	5.7083
122	14884	1815848	11.0454	4.9597	187	34969	6539203	13.6748	5.7185
123	15129	1860867	11.0905	4.9732	188	35344	6644672	13.7113	5.7287
124	15376	1906624	11.1355	4.9866	189	35721	6751269	13.7477	5.7388
125	15625	1953125	11.1803	5.	190	36100	6859000	13.7840	5.7489
126	15876	2000376	11.2250	5.0133	191	36481	6967871	13.8203	5.7590
127	16129	2048383	11.2694	5.0265	192	36864	7077888	13.8564	5.7690
128	16384	2097152	11.3137	5.0397	193	37249	7189057	13.8924	5.7790
129	16641	2146689	11.3578	5.0528	194	37636	7301384	13.9284	5.7890
130	16900	2197000	11.4018	5.0658	195	38025	7414875	13.9642	5.7989
131	17161	2248091	11.4455	5.0788	196	38416	7529536	14.	5.8088
132	17424	2299968	11.4891	5.0916	197	38809	7645373	14.0357	5.8186
133	17689	2352637	11.5326	5.1045	198	39204	7762392	14.0712	5.8285
134	17956	2406104	11.5758	5.1172	199	39601	7880599	14.1067	5.8383
135	18225	2460375	11.6190	5.1299	200	40000	8000000	14.1421	5.8480
136	18496	2515456	11.6619	5.1426	201	40401	8120601	14.1774	5.8578
137	18769	2571353	11.7047	5.1551	202	40804	8242408	14.2127	5.8675
138	19044	2628072	11.7473	5.1676	203	41209	8365427	14.2478	5.8771
139	19321	2685619	11.7898	5.1801	204	41616	8489664	14.2829	5.8868
140	19600	2744000	11.8322	5.1925	205	42025	8615125	14.3178	5.8964
141	19881	2803221	11.8743	5.2048	206	42436	8741816	14.3527	5.9059
142	20164	2863288	11.9164	5.2171	207	42849	8869743	14.3875	5.9155
143	20449	2924207	11.9583	5.2293	208	43264	8998912	14.4222	5.9250
144	20736	2985984	12.	5.2415	209	43681	9129329	14.4568	5.9345
145	21025	3048625	12.0416	5.2536	210	44100	9261000	14.4914	5.9439
146	21316	3112136	12.0830	5.2656	211	44521	9393931	14.5258	5.9533
147	21609	3176523	12.1244	5.2776	212	44944	9528128	14.5602	5.9627
148	21904	3241792	12.1655	5.2896	213	45369	9663597	14.5945	5.9721
149	22201	3307949	12.2066	5.3015	214	45796	9800344	14.6287	5.9814
150	22500	3375000	12.2474	5.3133	215	46225	9938375	14.6629	5.9907
151	22801	3442951	12.2882	5.3251	216	46656	10077696	14.6969	6.
152	23104	3511808	12.3288	5.3368	217	47089	10218313	14.7309	6.0092
153	23409	3581577	12.3693	5.3485	218	47524	10360232	14.7648	6.0185
154	23716	3652264	12.4097	5.3601	219	47961	10503459	14.7986	6.0277
155	24025	3723875	12.4499	5.3717	220	48400	10648000	14.8324	6.0368
156	24336	3796416	12.4900	5.3832	221	48841	10793861	14.8661	6.0459
157	24649	3869893	12.5300	5.3947	222	49284	10941048	14.8997	6.0550
158	24964	3944312	12.5698	5.4061	223	49729	11089567	14.9332	6.0641
159	25281	4019679	12.6095	5.4175	224	50176	11239424	14.9666	6.0732
160	25600	4096000	12.6491	5.4288	225	50625	11390625	15.	6.0822
161	25921	4173281	12.6886	5.4401	226	51076	11543176	15.0333	6.0912
162	26244	4251528	12.7279	5.4514	227	51529	11697083	15.0665	6.1002
163	26569	4330747	12.7671	5.4626	228	51984	11852352	15.0997	6.1091
164	26896	4410944	12.8062	5.4737	229	52441	12008989	15.1327	6.1180
165	27225	4492125	12.8452	5.4848	230	52900	12167000	15.1658	6.1269
166	27556	4574296	12.8841	5.4959	231	53361	12326391	15.1987	6.1358
167	27889	4657463	12.9228	5.5069	232	53824	12487168	15.2315	6.1446
168	28224	4741632	12.9615	5.5178	233	54289	12649337	15.2643	6.1534
169	28561	4826809	13.	5.5288	234	54756	12812904	15.2971	6.1622
170	28900	4913000	13.0384	5.5397	235	55225	12977875	15.3297	6.1710
171	29241	5000211	13.0767	5.5505	236	55696	13144256	15.3623	6.1797
172	29584	5088448	13.1149	5.5613	237	56169	13312053	15.3948	6.1885
173	29929	5177717	13.1529	5.5721	238	56644	13481272	15.4272	6.1972
174	30276	5268024	13.1909	5.5828	239	57121	13651919	15.4596	6.2058
175	30625	5359375	13.2288	5.5934	240	57600	13824000	15.4919	6.2145
176	30976	5451776	13.2665	5.6041	241	58081	13997521	15.5242	6.2231
177	31329	5545233	13.3041	5.6147	242	58564	14172488	15.5563	6.2317
178	31684	5639752	13.3417	5.6252	243	59049	14348907	15.5885	6.2403
179	32041	5735339	13.3791	5.6357	244	59536	14526784	15.6205	6.2488
180	32400	5832000	13.4164	5.6462	245	60025	14706125	15.6525	6.2573
181	32761	5929741	13.4536	5.6567	246	60516	14886936	15.6844	6.2658
182	33124	6028568	13.4907	5.6671	247	61009	15069223	15.7162	6.2743
183	33489	6128487	13.5277	5.6774	248	61504	15252992	15.7480	6.2828
184	33856	6229504	13.5647	5.6877	249	62001	15438249	15.7797	6.2912
185	34225	6331625	13.6015	5.6980	250	62500	15625000	15.8114	6.2996

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
251	63001	15813251	15.8430	6.3080	316	99856	31554496	17.7764	6.8113
252	63504	16003008	15.8745	6.3164	317	100489	31855013	17.8045	6.8185
253	64009	16194277	15.9060	6.3247	318	101124	32157432	17.8326	6.8256
254	64516	16387064	15.9374	6.3330	319	101761	32461759	17.8606	6.8328
255	65025	16581375	15.9687	6.3413	320	102400	32768000	17.8885	6.8399
256	65536	16777216	16.	6.3496	321	103041	33076161	17.9165	6.8470
257	66049	16974593	16.0312	6.3579	322	103684	33386248	17.9444	6.8541
258	66564	17173512	16.0624	6.3661	323	104329	33698267	17.9722	6.8612
259	67081	17373979	16.0935	6.3743	324	104976	34012224	18.	6.8683
260	67600	17576000	16.1245	6.3825	325	105625	34328125	18.0278	6.8753
261	68121	17779581	16.1555	6.3907	326	106276	34645976	18.0555	6.8824
262	68644	17984728	16.1864	6.3988	327	106929	34965783	18.0831	6.8894
263	69169	18191447	16.2173	6.4070	328	107584	35287552	18.1108	6.8964
264	69696	18399744	16.2481	6.4151	329	108241	35611289	18.1384	6.9034
265	70225	18609625	16.2788	6.4232	330	108900	35937000	18.1659	6.9104
266	70756	18821096	16.3095	6.4312	331	109561	36264691	18.1934	6.9174
267	71289	19034163	16.3401	6.4393	332	110224	36594368	18.2209	6.9244
268	71824	19248832	16.3707	6.4473	333	110889	36926037	18.2483	6.9313
269	72361	19465109	16.4012	6.4553	334	111556	37259704	18.2757	6.9382
270	72900	19683000	16.4317	6.4633	335	112225	37595375	18.3030	6.9451
271	73441	19902511	16.4621	6.4713	336	112896	37933056	18.3303	6.9521
272	73984	20123648	16.4924	6.4792	337	113569	38272753	18.3576	6.9589
273	74529	20346417	16.5227	6.4872	338	114244	38614472	18.3848	6.9658
274	75076	20570824	16.5529	6.4951	339	114921	38958219	18.4120	6.9727
275	75625	20796875	16.5831	6.5030	340	115600	39304000	18.4391	6.9795
276	76176	21024576	16.6132	6.5108	341	116281	39651821	18.4662	6.9864
277	76729	21253933	16.6433	6.5187	342	116964	40001688	18.4932	6.9932
278	77284	21484952	16.6733	6.5265	343	117649	40353607	18.5203	7.
279	77841	21717639	16.7033	6.5343	344	118336	40707584	18.5472	7.0068
280	78400	21952000	16.7332	6.5421	345	119025	41063625	18.5742	7.0136
281	78961	22188041	16.7631	6.5499	346	119716	41421736	18.6011	7.0203
282	79524	22425768	16.7929	6.5577	347	120409	41781923	18.6279	7.0271
283	80089	22665187	16.8226	6.5654	348	121104	42144192	18.6548	7.0338
284	80656	22906304	16.8523	6.5731	349	121801	42508549	18.6815	7.0406
285	81225	23149125	16.8819	6.5808	350	122500	42875000	18.7083	7.0473
286	81796	23393656	16.9115	6.5885	351	123201	43243551	18.7350	7.0540
287	82369	23639903	16.9411	6.5962	352	123904	43614208	18.7617	7.0607
288	82944	23887872	16.9706	6.6039	353	124609	43986977	18.7883	7.0674
289	83521	24137569	17.	6.6115	354	125316	44361864	18.8149	7.0740
290	84100	24389000	17.0294	6.6191	355	126025	44738875	18.8414	7.0807
291	84681	24642171	17.0587	6.6267	356	126736	45118016	18.8680	7.0873
292	85264	24897088	17.0880	6.6343	357	127449	45499293	18.8944	7.0940
293	85849	25153757	17.1172	6.6419	358	128164	45882712	18.9209	7.1006
294	86436	25412184	17.1464	6.6494	359	128881	46268279	18.9473	7.1072
295	87025	25672375	17.1756	6.6569	360	129600	46656000	18.9737	7.1138
296	87616	25934336	17.2047	6.6644	361	130321	47045881	19.	7.1204
297	88209	26198073	17.2337	6.6719	362	131044	47437928	19.0263	7.1269
298	88804	26463592	17.2627	6.6794	363	131769	47832147	19.0526	7.1335
299	89401	26730899	17.2916	6.6869	364	132496	48228544	19.0788	7.1400
300	90000	27000000	17.3205	6.6943	365	133225	48627125	19.1050	7.1466
301	90601	27270901	17.3494	6.7018	366	133956	49027896	19.1311	7.1531
302	91204	27543608	17.3781	6.7092	367	134689	49430863	19.1572	7.1596
303	91809	27818127	17.4069	6.7166	368	135424	49836032	19.1833	7.1661
304	92416	28094464	17.4356	6.7240	369	136161	50243409	19.2094	7.1726
305	93025	28372625	17.4642	6.7313	370	136900	50653000	19.2354	7.1791
306	93636	28652616	17.4929	6.7387	371	137641	51064811	19.2614	7.1855
307	94249	28934443	17.5214	6.7460	372	138384	51478848	19.2873	7.1920
308	94864	29218112	17.5499	6.7533	373	139129	51895117	19.3132	7.1984
309	95481	29503629	17.5784	6.7606	374	139876	52313624	19.3391	7.2048
310	96100	29791000	17.6068	6.7679	375	140625	52734375	19.3649	7.2112
311	96721	30080231	17.6352	6.7752	376	141376	53157376	19.3907	7.2177
312	97344	30371328	17.6635	6.7824	377	142129	53582633	19.4165	7.2240
313	97969	30664297	17.6918	6.7897	378	142884	54010152	19.4422	7.2304
314	98596	30959144	17.7200	6.7969	379	143641	54439939	19.4679	7.2368
315	99225	31255875	17.7482	6.8041	380	144400	54872000	19.4936	7.2432

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
381	145161	55306341	19.5192	7.2495	446	198916	88716536	21.1187	7.6403
382	145924	55742968	19.5448	7.2558	447	199809	89314623	21.1424	7.6460
383	146689	56181887	19.5704	7.2622	448	200704	89915392	21.1660	7.6517
384	147456	56623104	19.5959	7.2685	449	201601	90518849	21.1896	7.6574
385	148225	57066625	19.6214	7.2748	450	202500	91125000	21.2132	7.6631
386	148996	57512456	19.6469	7.2811	451	203401	91733851	21.2368	7.6688
387	149769	57960603	19.6723	7.2874	452	204304	92345408	21.2603	7.6744
388	150544	58411072	19.6977	7.2936	453	205209	92959677	21.2838	7.6801
389	151321	58863869	19.7231	7.2999	454	206116	93576664	21.3073	7.6857
390	152100	59319000	19.7484	7.3061	455	207025	94196375	21.3307	7.6914
391	152881	59776471	19.7737	7.3124	456	207936	94818816	21.3542	7.6970
392	153664	60236288	19.7990	7.3186	457	208849	95443993	21.3776	7.7026
393	154449	60698457	19.8242	7.3248	458	209764	96071912	21.4009	7.7082
394	155236	61162984	19.8494	7.3310	459	210681	96702579	21.4243	7.7138
395	156025	61629875	19.8746	7.3372	460	211600	97336000	21.4476	7.7194
396	156816	62099136	19.8997	7.3434	461	212521	97972181	21.4709	7.7250
397	157609	62570773	19.9249	7.3496	462	213444	98611128	21.4942	7.7306
398	158404	63044792	19.9499	7.3558	463	214369	99252847	21.5174	7.7362
399	159201	63521199	19.9750	7.3619	464	215296	99897344	21.5407	7.7418
400	160000	64000000	20.	7.3681	465	216225	100544625	21.5639	7.7473
401	160801	64481201	20.0250	7.3742	466	217156	101194696	21.5870	7.7529
402	161604	64964808	20.0499	7.3803	467	218089	101847563	21.6102	7.7584
403	162409	65450827	20.0749	7.3864	468	219024	102503232	21.6333	7.7639
404	163216	65939264	20.0998	7.3925	469	219961	103161709	21.6564	7.7695
405	164025	66430125	20.1246	7.3986	470	220900	103823000	21.6795	7.7750
406	164836	66923416	20.1494	7.4047	471	221841	104487111	21.7025	7.7805
407	165649	67419143	20.1742	7.4108	472	222784	105154048	21.7256	7.7860
408	166464	67917312	20.1990	7.4169	473	223729	105823817	21.7486	7.7915
409	167281	68417929	20.2237	7.4229	474	224676	106496424	21.7715	7.7970
410	168100	68921000	20.2485	7.4290	475	225625	107171875	21.7945	7.8025
411	168921	69426531	20.2731	7.4350	476	226576	107850176	21.8174	7.8079
412	169744	69934528	20.2978	7.4410	477	227529	108531333	21.8403	7.8134
413	170569	70444997	20.3224	7.4470	478	228484	109215352	21.8632	7.8188
414	171396	70957944	20.3470	7.4530	479	229441	109902239	21.8861	7.8243
415	172225	71473375	20.3715	7.4590	480	230400	110592000	21.9089	7.8297
416	173056	71991296	20.3961	7.4650	481	231361	111284641	21.9317	7.8352
417	173889	72511713	20.4206	7.4710	482	232324	111980168	21.9545	7.8406
418	174724	73034632	20.4450	7.4770	483	233289	112678587	21.9773	7.8460
419	175561	73560059	20.4695	7.4829	484	234256	113379904	22.	7.8514
420	176400	74088000	20.4939	7.4889	485	235225	114084125	22.0227	7.8568
421	177241	74618461	20.5183	7.4948	486	236196	114791256	22.0454	7.8622
422	178084	75151448	20.5426	7.5007	487	237169	115501303	22.0681	7.8676
423	178929	75686967	20.5670	7.5067	488	238144	116214272	22.0907	7.8730
424	179776	76225024	20.5913	7.5126	489	239121	116930169	22.1133	7.8784
425	180625	76765625	20.6155	7.5185	490	240100	117649000	22.1359	7.8837
426	181476	77308776	20.6398	7.5244	491	241081	118370771	22.1585	7.8891
427	182329	77854483	20.6640	7.5302	492	242064	119095488	22.1811	7.8944
428	183184	78402752	20.6882	7.5361	493	243049	119823157	22.2036	7.8998
429	184041	78953589	20.7123	7.5420	494	244036	120553784	22.2261	7.9051
430	184900	79507000	20.7364	7.5478	495	245025	121287375	22.2486	7.9105
431	185761	80062991	20.7605	7.5537	496	246016	122023936	22.2711	7.9158
432	186624	80621568	20.7846	7.5595	497	247009	122763473	22.2935	7.9211
433	187489	81182737	20.8087	7.5654	498	248004	123505992	22.3159	7.9264
434	188356	81746504	20.8327	7.5712	499	249001	124251499	22.3383	7.9317
435	189225	82312875	20.8567	7.5770	500	250000	125000000	22.3607	7.9370
436	190096	82881856	20.8806	7.5828	501	251001	125751501	22.3830	7.9423
437	190969	83453453	20.9045	7.5886	502	252004	126506008	22.4054	7.9476
438	191844	84027672	20.9284	7.5944	503	253009	127263527	22.4277	7.9528
439	192721	84604519	20.9523	7.6001	504	254016	128024064	22.4499	7.9581
440	193600	85184000	20.9762	7.6059	505	255025	128787625	22.4722	7.9634
441	194481	85766121	21.	7.6117	506	256036	129554216	22.4944	7.9686
442	195364	86350888	21.0238	7.6174	507	257049	130323843	22.5167	7.9739
443	196249	86938307	21.0476	7.6232	508	258064	131096512	22.5389	7.9791
444	197136	87528384	21.0713	7.6289	509	259081	131872229	22.5610	7.9843
445	198025	88121125	21.0950	7.6346	510	260100	132651000	22.5832	7.9896

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000.—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
511	261121	133432831	22.6053	7.9948	576	331776	191102976	24.	8.3203
512	262144	134217728	22.6274	8.	577	332929	192100033	24.0208	8.3251
513	263169	135005697	22.6495	8.0052	578	334084	193100552	24.0416	8.3300
514	264196	135796744	22.6716	8.0104	579	335241	194104539	24.0624	8.3348
515	265225	136590875	22.6936	8.0156	580	336400	195112000	24.0832	8.3396
516	266256	137388096	22.7156	8.0208	581	337561	196122941	24.1039	8.3443
517	267289	138188413	22.7376	8.0260	582	338724	197137368	24.1247	8.3491
518	268324	138991832	22.7596	8.0311	583	339889	198155287	24.1454	8.3539
519	269361	139798359	22.7816	8.0363	584	341056	199176704	24.1661	8.3587
520	270400	140608000	22.8035	8.0415	585	342225	200201625	24.1868	8.3634
521	271441	141420761	22.8254	8.0466	586	343396	201230056	24.2074	8.3682
522	272484	142236648	22.8473	8.0517	587	344569	202262003	24.2281	8.3730
523	273529	143055667	22.8692	8.0569	588	345744	203297472	24.2487	8.3777
524	274576	143877824	22.8910	8.0620	589	346921	204336469	24.2693	8.3825
525	275625	144703125	22.9129	8.0671	590	348100	205379000	24.2899	8.3872
526	276676	145531576	22.9347	8.0723	591	349281	206425071	24.3105	8.3920
527	277729	146363183	22.9565	8.0774	592	350464	207474688	24.3311	8.3967
528	278784	147197952	22.9783	8.0825	593	351649	208527857	24.3516	8.4014
529	279841	148035889	23.	8.0876	594	352836	209584584	24.3721	8.4061
530	280900	148877000	23.0217	8.0927	595	354025	210644875	24.3926	8.4108
531	281961	149721291	23.0434	8.0978	596	355216	211708736	24.4131	8.4155
532	283024	150568768	23.0651	8.1028	597	356409	212776173	24.4336	8.4202
533	284089	151419437	23.0868	8.1079	598	357604	213847192	24.4540	8.4249
534	285156	152273304	23.1084	8.1130	599	358801	214921799	24.4745	8.4296
535	286225	153130375	23.1301	8.1180	600	360000	216000000	24.4949	8.4343
536	287296	153990656	23.1517	8.1231	601	361201	217081801	24.5153	8.4390
537	288369	154854153	23.1733	8.1281	602	362404	218167208	24.5357	8.4437
538	289444	155720872	23.1948	8.1332	603	363609	219256227	24.5561	8.4484
539	290521	156590819	23.2164	8.1382	604	364816	220348864	24.5764	8.4530
540	291600	157464000	23.2379	8.1433	605	366025	221445125	24.5967	8.4577
541	292681	158340421	23.2594	8.1483	606	367236	222545016	24.6171	8.4623
542	293764	159220088	23.2809	8.1533	607	368449	223648543	24.6374	8.4670
543	294849	160103007	23.3024	8.1583	608	369664	224755712	24.6577	8.4716
544	295936	160989184	23.3238	8.1633	609	370881	225866529	24.6779	8.4763
545	297025	161878625	23.3452	8.1683	610	372100	226981000	24.6982	8.4809
546	298116	162771336	23.3666	8.1733	611	373321	228099131	24.7184	8.4856
547	299209	163667323	23.3880	8.1783	612	374544	229220928	24.7386	8.4902
548	300304	164566592	23.4094	8.1833	613	375769	230346397	24.7588	8.4948
549	301401	165469149	23.4307	8.1882	614	376996	231475544	24.7790	8.4994
550	302500	166375000	23.4521	8.1932	615	378225	232608375	24.7992	8.5040
551	303601	167284151	23.4734	8.1982	616	379456	233744896	24.8193	8.5086
552	304704	168196608	23.4947	8.2031	617	380689	234885113	24.8395	8.5132
553	305809	169112377	23.5160	8.2081	618	381924	236029032	24.8596	8.5178
554	306916	170031464	23.5372	8.2130	619	383161	237176659	24.8797	8.5224
555	308025	170953875	23.5584	8.2180	620	384400	238328000	24.8998	8.5270
556	309136	171879616	23.5797	8.2229	621	385641	239483061	24.9199	8.5316
557	310249	172808693	23.6008	8.2278	622	386884	240641848	24.9399	8.5362
558	311364	173741112	23.6220	8.2327	623	388129	241804367	24.9600	8.5408
559	312481	174676879	23.6432	8.2377	624	389376	242970624	24.9800	8.5453
560	313600	175616000	23.6643	8.2426	625	390625	244140625	25.	8.5499
561	314721	176558481	23.6854	8.2475	626	391876	245314376	25.0200	8.5544
562	315844	177504328	23.7065	8.2524	627	393129	246491883	25.0400	8.5590
563	316969	178453547	23.7276	8.2573	628	394384	247673152	25.0599	8.5635
564	318096	179406144	23.7487	8.2621	629	395641	248858189	25.0799	8.5681
565	319225	180362125	23.7697	8.2670	630	396900	250047000	25.0998	8.5726
566	320356	181321496	23.7908	8.2719	631	398161	251239591	25.1197	8.5772
567	321489	182284263	23.8118	8.2768	632	399424	252435968	25.1396	8.5817
568	322624	183250432	23.8328	8.2816	633	400689	253636137	25.1595	8.5862
569	323761	184220009	23.8537	8.2865	634	401956	254840104	25.1794	8.5907
570	324900	185193000	23.8747	8.2913	635	403225	256047875	25.1992	8.5952
571	326041	186169411	23.8956	8.2962	636	404496	257259456	25.2190	8.5997
572	327184	187149248	23.9165	8.3010	637	405769	258474853	25.2389	8.6043
573	328329	188132517	23.9374	8.3059	638	407044	259694072	25.2587	8.6088
574	329476	189119224	23.9583	8.3107	639	408321	260917119	25.2784	8.6132
575	330625	190109375	23.9792	8.3155	640	409600	262144000	25.2982	8.6177

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
641	410881	263374721	25.3180	8.6222	706	498436	351895816	26.5707	8.9043
642	412164	264609288	25.3377	8.6267	707	499849	353393243	26.5895	8.9085
643	413449	265847707	25.3574	8.6312	708	501264	354894912	26.6083	8.9127
644	414736	267089984	25.3772	8.6357	709	502681	356400829	26.6271	8.9169
645	416025	268336125	25.3969	8.6401	710	504100	357911000	26.6458	8.9211
646	417316	269586136	25.4165	8.6446	711	505521	359425431	26.6646	8.9253
647	418609	270840023	25.4362	8.6490	712	506944	360944128	26.6833	8.9295
648	419904	272097792	25.4558	8.6535	713	508369	362467097	26.7021	8.9337
649	421201	273359449	25.4755	8.6579	714	509796	363994344	26.7208	8.9378
650	422500	274625000	25.4951	8.6624	715	511225	365525875	26.7395	8.9420
651	423801	275894451	25.5147	8.6668	716	512656	367061696	26.7582	8.9462
652	425104	277167808	25.5343	8.6713	717	514089	368601813	26.7769	8.9503
653	426409	278445077	25.5539	8.6757	718	515524	370146232	26.7955	8.9545
654	427716	279726264	25.5734	8.6801	719	516961	371694959	26.8142	8.9587
655	429025	281011375	25.5930	8.6845	720	518400	373248000	26.8328	8.9628
656	430336	282300416	25.6125	8.6890	721	519841	374805361	26.8514	8.9670
657	431649	283593393	25.6320	8.6934	722	521284	376367048	26.8701	8.9711
658	432964	284890312	25.6515	8.6978	723	522729	377933067	26.8887	8.9752
659	434281	286191179	25.6710	8.7022	724	524176	379503424	26.9072	8.9794
660	435600	287496000	25.6905	8.7066	725	525625	381078125	26.9258	8.9835
661	436921	288804781	25.7099	8.7110	726	527076	382657176	26.9444	8.9876
662	438244	290117528	25.7294	8.7154	727	528529	384240583	26.9629	8.9918
663	439569	291434247	25.7488	8.7198	728	529984	385828352	26.9815	8.9959
664	440896	292754944	25.7682	8.7241	729	531441	387420489	27.	9.
665	442225	294079625	25.7876	8.7285	730	532900	389017000	27.0185	9.0041
666	443556	295408296	25.8070	8.7329	731	534361	390617891	27.0370	9.0082
667	444889	296740963	25.8263	8.7373	732	535824	392223168	27.0555	9.0123
668	446224	298077632	25.8457	8.7416	733	537289	393832837	27.0740	9.0164
669	447561	299418309	25.8650	8.7460	734	538756	395446904	27.0924	9.0205
670	448900	300763000	25.8844	8.7503	735	540225	397065375	27.1109	9.0246
671	450241	302111711	25.9037	8.7547	736	541696	398688256	27.1293	9.0287
672	451584	303464448	25.9230	8.7590	737	543169	400315553	27.1477	9.0328
673	452929	304821217	25.9422	8.7634	738	544644	401947272	27.1662	9.0369
674	454276	306182024	25.9615	8.7677	739	546121	403583419	27.1846	9.0410
675	455625	307546875	25.9808	8.7721	740	547600	405224000	27.2029	9.0450
676	456976	308915776	26.	8.7764	741	549081	406869021	27.2213	9.0491
677	458329	310288733	26.0192	8.7807	742	550564	408518488	27.2397	9.0532
678	459684	311665752	26.0384	8.7850	743	552049	410172407	27.2580	9.0572
679	461041	313046839	26.0576	8.7893	744	553536	411830784	27.2764	9.0613
680	462400	314432000	26.0768	8.7937	745	555025	413493625	27.2947	9.0654
681	463761	315821241	26.0960	8.7980	746	556516	415160936	27.3130	9.0694
682	465124	317214568	26.1151	8.8023	747	558009	416832723	27.3313	9.0735
683	466489	318611987	26.1343	8.8066	748	559504	418508992	27.3496	9.0775
684	467856	320013504	26.1534	8.8109	749	561001	420189749	27.3679	9.0816
685	469225	321419125	26.1725	8.8152	750	562500	421875000	27.3861	9.0856
686	470596	322828856	26.1916	8.8194	751	564001	423564751	27.4044	9.0896
687	471969	324242703	26.2107	8.8237	752	565504	425259008	27.4226	9.0937
688	473344	325660672	26.2298	8.8280	753	567009	426957777	27.4408	9.0977
689	474721	327082769	26.2488	8.8323	754	568516	428661064	27.4591	9.1017
690	476100	328509000	26.2679	8.8366	755	570025	430368875	27.4773	9.1057
691	477481	329939371	26.2869	8.8408	756	571536	432081216	27.4955	9.1098
692	478864	331373888	26.3059	8.8451	757	573049	433798093	27.5136	9.1138
693	480249	332812557	26.3249	8.8493	758	574564	435519512	27.5318	9.1178
694	481696	334255384	26.3439	8.8536	759	576081	437245479	27.5500	9.1218
695	483025	335702375	26.3629	8.8578	760	577600	438976000	27.5681	9.1258
696	484416	337153536	26.3818	8.8621	761	579121	440711081	27.5862	9.1298
697	485809	338608873	26.4008	8.8663	762	580644	442450728	27.6043	9.1338
698	487204	340068392	26.4197	8.8706	763	582169	444194947	27.6225	9.1378
699	488601	341532099	26.4386	8.8748	764	583696	445943744	27.6405	9.1418
700	490000	343000000	26.4575	8.8790	765	585225	447697125	27.6586	9.1458
701	491401	344472101	26.4764	8.8833	766	586756	449455096	27.6767	9.1498
702	492804	345948408	26.4953	8.8875	767	588289	451217663	27.6948	9.1537
703	494209	347428927	26.5141	8.8917	768	589824	452984832	27.7128	9.1577
704	495616	348913664	26.5330	8.8959	769	591361	454756609	27.7308	9.1617
705	497025	350402625	26.5518	8.9001	770	592900	456533000	27.7489	9.1657

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
771	594441	458314011	27.7669	9.1696	836	698896	584277056	28.9137	9.4204
772	595984	460099648	27.7849	9.1736	837	700569	586376253	28.9310	9.4241
773	597529	461889917	27.8029	9.1775	838	702244	588480472	28.9482	9.4279
774	599076	463684824	27.8209	9.1815	839	703921	590589719	28.9655	9.4316
775	600625	465484375	27.8388	9.1855	840	705600	592704000	28.9828	9.4354
776	602176	467288576	27.8568	9.1894	841	707281	594823321	29.	9.4391
777	603729	469097433	27.8747	9.1933	842	708964	596947688	29.0172	9.4429
778	605284	470910952	27.8927	9.1973	843	710649	599077107	29.0345	9.4466
779	606841	472729139	27.9106	9.2012	844	712336	601211584	29.0517	9.4503
780	608400	474552000	27.9285	9.2052	845	714025	603351125	29.0689	9.4541
781	609961	476379541	27.9464	9.2091	846	715716	605495736	29.0861	9.4578
782	611524	478211768	27.9643	9.2130	847	717409	607645423	29.1033	9.4615
783	613089	480048687	27.9821	9.2170	848	719104	609800192	29.1204	9.4652
784	614656	481890304	28.	9.2209	849	720801	611960049	29.1376	9.4690
785	616225	483736625	28.0179	9.2248	850	722500	614125000	29.1548	9.4727
786	617796	485587656	28.0357	9.2287	851	724201	616295051	29.1719	9.4764
787	619369	487443403	28.0535	9.2326	852	725904	618470208	29.1890	9.4801
788	620944	489303872	28.0713	9.2365	853	727609	620650477	29.2062	9.4838
789	622521	491169069	28.0891	9.2404	854	729316	622835864	29.2233	9.4875
790	624100	493039000	28.1069	9.2443	855	731025	625026375	29.2404	9.4912
791	625681	494913671	28.1247	9.2482	856	732736	627222016	29.2575	9.4949
792	627264	496793088	28.1425	9.2521	857	734449	629422793	29.2746	9.4986
793	628849	498677257	28.1603	9.2560	858	736164	631628712	29.2916	9.5023
794	630436	500566184	28.1780	9.2599	859	737881	633839779	29.3087	9.5060
795	632025	502459875	28.1957	9.2638	860	739600	636056000	29.3258	9.5097
796	633616	504358336	28.2135	9.2677	861	741321	638277381	29.3428	9.5134
797	635209	506261573	28.2312	9.2716	862	743044	640503928	29.3598	9.5171
798	636804	508169592	28.2489	9.2754	863	744769	642735647	29.3769	9.5207
799	638401	510082399	28.2666	9.2793	864	746496	644972544	29.3939	9.5244
800	640000	512000000	28.2843	9.2832	865	748225	647214625	29.4109	9.5281
801	641601	513922401	28.3019	9.2870	866	749956	649461896	29.4279	9.5317
802	643204	515849608	28.3196	9.2909	867	751689	651714363	29.4449	9.5354
803	644809	517781627	28.3373	9.2948	868	753424	653972032	29.4618	9.5391
804	646416	519718464	28.3549	9.2986	869	755161	656234909	29.4788	9.5427
805	648025	521660125	28.3725	9.3025	870	756900	658503000	29.4958	9.5464
806	649636	523606616	28.3901	9.3063	871	758641	660776311	29.5127	9.5501
807	651249	525557943	28.4077	9.3102	872	760384	663054848	29.5296	9.5537
808	652864	527514112	28.4253	9.3140	873	762129	665338617	29.5466	9.5574
809	654481	529475129	28.4429	9.3179	874	763876	667627624	29.5635	9.5610
810	656100	531441000	28.4605	9.3217	875	765625	669921875	29.5804	9.5647
811	657721	533411731	28.4781	9.3255	876	767376	672221376	29.5973	9.5683
812	659344	535387328	28.4956	9.3294	877	769129	674526133	29.6142	9.5719
813	660969	537367797	28.5132	9.3332	878	770884	676836152	29.6311	9.5756
814	662596	539353144	28.5307	9.3370	879	772641	679151439	29.6479	9.5792
815	664225	541343375	28.5482	9.3408	880	774400	681472000	29.6648	9.5828
816	665856	543338496	28.5657	9.3447	881	776161	683797841	29.6816	9.5865
817	667489	545338513	28.5832	9.3485	882	777924	686128968	29.6985	9.5901
818	669124	547343432	28.6007	9.3523	883	779689	688465387	29.7153	9.5937
819	670761	549353259	28.6182	9.3561	884	781456	690807104	29.7321	9.5973
820	672400	551368000	28.6356	9.3599	885	783225	693154125	29.7489	9.6010
821	674041	553387661	28.6531	9.3637	886	784996	695506456	29.7658	9.6046
822	675684	555412248	28.6705	9.3675	887	786769	697864103	29.7825	9.6082
823	677329	557441767	28.6880	9.3713	888	788544	700227072	29.7993	9.6118
824	678976	559476224	28.7054	9.3751	889	790321	702595369	29.8161	9.6154
825	680625	561515625	28.7228	9.3789	890	792100	704969000	29.8329	9.6190
826	682276	563559976	28.7402	9.3827	891	793881	707347971	29.8496	9.6226
827	683929	565609283	28.7576	9.3865	892	795664	709732288	29.8664	9.6262
828	685584	567663552	28.7750	9.3902	893	797449	712121957	29.8831	9.6298
829	687241	569722789	28.7924	9.3940	894	799236	714516984	29.8998	9.6334
830	688900	571787000	28.8097	9.3978	895	801025	716917375	29.9166	9.6370
831	690561	573856191	28.8271	9.4016	896	802816	719323136	29.9333	9.6406
832	692224	575930368	28.8444	9.4053	897	804609	721734273	29.9500	9.6442
833	693889	578009537	28.8617	9.4091	898	806404	724150792	29.9666	9.6477
834	695556	580093704	28.8791	9.4129	899	808201	726572699	29.9833	9.6513
835	697225	582182875	28.8964	9.4166	900	810000	729000000	30.	9.6549

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(Continued.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
901	811801	731432701	30.0167	9.6585	951	904401	860085351	30.8383	9.8339
902	813604	733870808	30.0333	9.6620	952	906304	862801408	30.8545	9.8374
903	815409	736314327	30.0500	9.6656	953	908209	865523177	30.8707	9.8408
904	817216	738763264	30.0666	9.6692	954	910116	868250664	30.8869	9.8443
905	819025	741217625	30.0832	9.6727	955	912025	870983875	30.9031	9.8477
906	820836	743677416	30.0998	9.6763	956	913936	873722816	30.9192	9.8511
907	822649	746142643	30.1164	9.6799	957	915849	876467493	30.9354	9.8546
908	824464	748613312	30.1330	9.6834	958	917764	879217912	30.9516	9.8580
909	826281	751089429	30.1496	9.6870	959	919681	881974079	30.9677	9.8614
910	828100	753571000	30.1662	9.6905	960	921600	884736000	30.9839	9.8648
911	829921	756058031	30.1828	9.6941	961	923521	887503681	31.	9.8683
912	831744	758550528	30.1993	9.6976	962	925444	890277128	31.0161	9.8717
913	833569	761048497	30.2159	9.7012	963	927369	893056347	31.0322	9.8751
914	835396	763551944	30.2324	9.7047	964	929296	895841344	31.0483	9.8785
915	837225	766060875	30.2490	9.7082	965	931225	898632125	31.0644	9.8819
916	839056	768575296	30.2655	9.7118	966	933156	901428696	31.0805	9.8854
917	840889	771095213	30.2820	9.7153	967	935089	904231063	31.0966	9.8888
918	842724	773620632	30.2985	9.7188	968	937024	907039232	31.1127	9.8922
919	844561	776151559	30.3150	9.7224	969	938961	909853209	31.1288	9.8956
920	846400	778688000	30.3315	9.7259	970	940900	912673000	31.1448	9.8990
921	848241	781229961	30.3480	9.7294	971	942841	915498611	31.1609	9.9024
922	850084	783777448	30.3645	9.7329	972	944784	918330048	31.1769	9.9058
923	851929	786330467	30.3809	9.7364	973	946729	921167317	31.1929	9.9092
924	853776	788889024	30.3974	9.7400	974	948676	924010424	31.2090	9.9126
925	855625	791453125	30.4138	9.7435	975	950625	926859375	31.2250	9.9160
926	857476	794022776	30.4302	9.7470	976	952576	929714176	31.2410	9.9194
927	859329	796597983	30.4467	9.7505	977	954529	932574833	31.2570	9.9227
928	861184	799178752	30.4631	9.7540	978	956484	935441352	31.2730	9.9261
929	863041	801765089	30.4795	9.7575	979	958441	938313739	31.2890	9.9295
930	864900	804357000	30.4959	9.7610	980	960400	941192000	31.3050	9.9329
931	866761	806954491	30.5123	9.7645	981	962361	944076141	31.3209	9.9363
932	868624	809557568	30.5287	9.7680	982	964324	946966168	31.3369	9.9396
933	870489	812166237	30.5450	9.7715	983	966289	949862087	31.3528	9.9430
934	872356	814780504	30.5614	9.7750	984	968256	952763904	31.3688	9.9464
935	874225	817400375	30.5778	9.7785	985	970225	955671625	31.3847	9.9497
936	876096	820025856	30.5941	9.7819	986	972196	958585256	31.4006	9.9531
937	877969	822656953	30.6105	9.7854	987	974169	961504803	31.4166	9.9565
938	879844	825293672	30.6268	9.7889	988	976144	964430272	31.4325	9.9598
939	881721	827936019	30.6431	9.7924	989	978121	967361669	31.4484	9.9632
940	883600	830584000	30.6594	9.7959	990	980100	970299000	31.4643	9.9666
941	885481	833237621	30.6757	9.7993	991	982081	973242271	31.4802	9.9699
942	887364	835896888	30.6920	9.8028	992	984064	976191488	31.4960	9.9733
943	889249	838561807	30.7083	9.8063	993	986049	979146657	31.5119	9.9766
944	891136	841232384	30.7246	9.8097	994	988036	982107784	31.5278	9.9800
945	893025	843908625	30.7409	9.8132	995	990025	985074875	31.5436	9.9833
946	894916	846590536	30.7571	9.8167	996	992016	988047936	31.5595	9.9866
947	896809	849278123	30.7734	9.8201	997	994009	991026973	31.5753	9.9900
948	898704	851971392	30.7896	9.8236	998	996004	994011992	31.5911	9.9933
949	900601	854670349	30.8058	9.8270	999	998001	997002999	31.6070	9.9967
950	902500	857375000	30.8221	9.8305	1000	1000000	1000000000	31.6228	10.

To find the square or cube of any whole number ending with ciphers. First, omit all the final ciphers. Take from the table the square or cube (as the case may be) of the rest of the number. To this *square* add *twice* as many ciphers as there were final ciphers in the original number. To the *cube* add *three* times as many as in the original number. Thus, for 90500^2; $905^2 = 819025$. Add twice 2 ciphers, obtaining 8190250000. For 90500^3, $905^3 = 741217625$. Add 3 times 2 ciphers, obtaining 741217625000000.

Square Roots and Cube Roots of Numbers from 1000 to 10000.

No errors.

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
1005	31.70	10.02	1405	37.48	11.20	1805	42.49	12.18	2205	46.96	13.02
1010	31.78	10.03	1410	37.55	11.21	1810	42.54	12.19	2210	47.01	13.03
1015	31.86	10.05	1415	37.62	11.23	1815	42.60	12.20	2215	47.06	13.04
1020	31.94	10.07	1420	37.68	11.24	1820	42.66	12.21	2220	47.12	13.05
1025	32.02	10.08	1425	37.75	11.25	1825	42.72	12.22	2225	47.17	13.05
1030	32.09	10.10	1430	37.82	11.27	1830	42.78	12.23	2230	47.22	13.06
1035	32.17	10.12	1435	37.88	11.28	1835	42.84	12.24	2235	47.28	13.07
1040	32.25	10.13	1440	37.95	11.29	1840	42.90	12.25	2240	47.33	13.08
1045	32.33	10.15	1445	38.01	11.31	1845	42.95	12.26	2245	47.38	13.09
1050	32.40	10.16	1450	38.08	11.32	1850	43.01	12.28	2250	47.43	13.10
1055	32.48	10.18	1455	38.14	11.33	1855	43.07	12.29	2255	47.49	13.11
1060	32.56	10.20	1460	38.21	11.34	1860	43.13	12.30	2260	47.54	13.12
1065	32.63	10.21	1465	38.28	11.36	1865	43.19	12.31	2265	47.59	13.13
1070	32.71	10.23	1470	38.34	11.37	1870	43.24	12.32	2270	47.64	13.14
1075	32.79	10.24	1475	38.41	11.38	1875	43.30	12.33	2275	47.70	13.15
1080	32.86	10.26	1480	38.47	11.40	1880	43.36	12.34	2280	47.75	13.16
1085	32.94	10.28	1485	38.54	11.41	1885	43.42	12.35	2285	47.80	13.17
1090	33.02	10.29	1490	38.60	11.42	1890	43.47	12.36	2290	47.85	13.18
1095	33.09	10.31	1495	38.67	11.43	1895	43.53	12.37	2295	47.91	13.19
1100	33.17	10.32	1500	38.73	11.45	1900	43.59	12.39	2300	47.96	13.20
1105	33.24	10.34	1505	38.79	11.46	1905	43.65	12.40	2305	48.01	13.21
1110	33.32	10.35	1510	38.86	11.47	1910	43.70	12.41	2310	48.06	13.22
1115	33.39	10.37	1515	38.92	11.49	1915	43.76	12.42	2315	48.11	13.23
1120	33.47	10.38	1520	38.99	11.50	1920	43.82	12.43	2320	48.17	13.24
1125	33.54	10.40	1525	39.05	11.51	1925	43.87	12.44	2325	48.22	13.25
1130	33.62	10.42	1530	39.12	11.52	1930	43.93	12.45	2330	48.27	13.26
1135	33.69	10.43	1535	39.18	11.54	1935	43.99	12.46	2335	48.32	13.27
1140	33.76	10.45	1540	39.24	11.55	1940	44.05	12.47	2340	48.37	13.28
1145	33.84	10.46	1545	39.31	11.56	1945	44.10	12.48	2345	48.43	13.29
1150	33.91	10.48	1550	39.37	11.57	1950	44.16	12.49	2350	48.48	13.30
1155	33.99	10.49	1555	39.43	11.59	1955	44.22	12.50	2355	48.53	13.30
1160	34.06	10.51	1560	39.50	11.60	1960	44.27	12.51	2360	48.58	13.31
1165	34.13	10.52	1565	39.56	11.61	1965	44.33	12.53	2365	48.63	13.32
1170	34.21	10.54	1570	39.62	11.62	1970	44.38	12.54	2370	48.68	13.33
1175	34.28	10.55	1575	39.69	11.63	1975	44.44	12.55	2375	48.73	13.34
1180	34.35	10.57	1580	39.75	11.65	1980	44.50	12.56	2380	48.79	13.35
1185	34.42	10.58	1585	39.81	11.66	1985	44.55	12.57	2385	48.84	13.36
1190	34.50	10.60	1590	39.87	11.67	1990	44.61	12.58	2390	48.89	13.37
1195	34.57	10.61	1595	39.94	11.68	1995	44.67	12.59	2395	48.94	13.38
1200	34.64	10.63	1600	40.00	11.70	2000	44.72	12.60	2400	48.99	13.39
1205	34.71	10.64	1605	40.06	11.71	2005	44.78	12.61	2405	49.04	13.40
1210	34.79	10.66	1610	40.12	11.72	2010	44.83	12.62	2410	49.09	13.41
1215	34.86	10.67	1615	40.19	11.73	2015	44.89	12.63	2415	49.14	13.42
1220	34.93	10.69	1620	40.25	11.74	2020	44.94	12.64	2420	49.19	13.43
1225	35.00	10.70	1625	40.31	11.76	2025	45.00	12.65	2425	49.24	13.43
1230	35.07	10.71	1630	40.37	11.77	2030	45.06	12.66	2430	49.30	13.44
1235	35.14	10.73	1635	40.44	11.78	2035	45.11	12.67	2435	49.35	13.45
1240	35.21	10.74	1640	40.50	11.79	2040	45.17	12.68	2440	49.40	13.46
1245	35.28	10.76	1645	40.56	11.80	2045	45.22	12.69	2445	49.45	13.47
1250	35.36	10.77	1650	40.62	11.82	2050	45.28	12.70	2450	49.50	13.48
1255	35.43	10.79	1655	40.68	11.83	2055	45.33	12.71	2460	49.60	13.50
1260	35.50	10.80	1660	40.74	11.84	2060	45.39	12.72	2470	49.70	13.52
1265	35.57	10.82	1665	40.80	11.85	2065	45.44	12.73	2480	49.80	13.54
1270	35.64	10.83	1670	40.87	11.86	2070	45.50	12.74	2490	49.90	13.55
1275	35.71	10.84	1675	40.93	11.88	2075	45.55	12.75	2500	50.00	13.57
1280	35.78	10.86	1680	40.99	11.89	2080	45.61	12.77	2510	50.10	13.59
1285	35.85	10.87	1685	41.05	11.90	2085	45.66	12.78	2520	50.20	13.61
1290	35.92	10.89	1690	41.11	11.91	2090	45.72	12.79	2530	50.30	13.63
1295	35.99	10.90	1695	41.17	11.92	2095	45.77	12.80	2540	50.40	13.64
1300	36.06	10.91	1700	41.23	11.93	2100	45.83	12.81	2550	50.50	13.66
1305	36.12	10.93	1705	41.29	11.95	2105	45.88	12.82	2560	50.60	13.68
1310	36.19	10.94	1710	41.35	11.96	2110	45.93	12.83	2570	50.70	13.70
1315	36.26	10.96	1715	41.41	11.97	2115	45.99	12.84	2580	50.79	13.72
1320	36.33	10.97	1720	41.47	11.98	2120	46.04	12.85	2590	50.89	13.73
1325	36.40	10.98	1725	41.53	11.99	2125	46.10	12.86	2600	50.99	13.75
1330	36.47	11.00	1730	41.59	12.00	2130	46.15	12.87	2610	51.09	13.77
1335	36.54	11.01	1735	41.65	12.02	2135	46.21	12.88	2620	51.19	13.79
1340	36.61	11.02	1740	41.71	12.03	2140	46.26	12.89	2630	51.28	13.80
1345	36.67	11.04	1745	41.77	12.04	2145	46.31	12.90	2640	51.38	13.82
1350	36.74	11.05	1750	41.83	12.05	2150	46.37	12.91	2650	51.48	13.84
1355	36.81	11.07	1755	41.89	12.06	2155	46.42	12.92	2660	51.58	13.86
1360	36.88	11.08	1760	41.95	12.07	2160	46.48	12.93	2670	51.67	13.87
1365	36.95	11.09	1765	42.01	12.09	2165	46.53	12.94	2680	51.77	13.89
1370	37.01	11.11	1770	42.07	12.10	2170	46.58	12.95	2690	51.87	13.91
1375	37.08	11.12	1775	42.13	12.11	2175	46.64	12.96	2700	51.96	13.92
1380	37.15	11.13	1780	42.19	12.12	2180	46.69	12.97	2710	52.06	13.94
1385	37.22	11.15	1785	42.25	12.13	2185	46.74	12.98	2720	52.15	13.96
1390	37.28	11.16	1790	42.31	12.14	2190	46.80	12.99	2730	52.25	13.98
1395	37.35	11.17	1795	42.37	12.15	2195	46.85	13.00	2740	52.35	13.99
1400	37.42	11.19	1800	42.43	12.16	2200	46.90	13.01	2750	52.44	14.01

Square Roots and Cube Roots of Numbers from 1000 to 10000 —(Continued.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
2760	52.54	14.03	3550	59.58	15.25	4340	65.88	16.31	5130	71.62	17.25
2770	52.63	14.04	3560	59.67	15.27	4350	65.95	16.32	5140	71.69	17.26
2780	52.73	14.06	3570	59.75	15.28	4360	66.03	16.34	5150	71.76	17.27
2790	52.82	14.08	3580	59.83	15.30	4370	66.11	16.35	5160	71.83	17.28
2800	52.92	14.09	3590	59.92	15.31	4380	66.18	16.36	5170	71.90	17.29
2810	53.01	14.11	3600	60.00	15.33	4390	66.26	16.37	5180	71.97	17.30
2820	53.10	14.13	3610	60.08	15.34	4400	66.33	16.39	5190	72.04	17.31
2830	53.20	14.14	3620	60.17	15.35	4410	66.41	16.40	5200	72.11	17.32
2840	53.29	14.16	3630	60.25	15.37	4420	66.48	16.41	5210	72.18	17.34
2850	53.39	14.18	3640	60.33	15.38	4430	66.56	16.42	5220	72.25	17.35
2860	53.48	14.19	3650	60.42	15.40	4440	66.63	16.44	5230	72.32	17.36
2870	53.57	14.21	3660	60.50	15.41	4450	66.71	16.45	5240	72.39	17.37
2880	53.67	14.23	3670	60.58	15.42	4460	66.78	16.46	5250	72.46	17.38
2890	53.76	14.24	3680	60.66	15.44	4470	66.86	16.47	5260	72.53	17.39
2900	53.85	14.26	3690	60.75	15.45	4480	66.93	16.49	5270	72.59	17.40
2910	53.94	14.28	3700	60.83	15.47	4490	67.01	16.50	5280	72.66	17.41
2920	54.04	14.29	3710	60.91	15.48	4500	67.08	16.51	5290	72.73	17.42
2930	54.13	14.31	3720	60.99	15.49	4510	67.16	16.52	5300	72.80	17.44
2940	54.22	14.33	3730	61.07	15.51	4520	67.23	16.53	5310	72.87	17.45
2950	54.31	14.34	3740	61.16	15.52	4530	67.31	16.55	5320	72.94	17.46
2960	54.41	14.36	3750	61.24	15.54	4540	67.38	16.56	5330	73.01	17.47
2970	54.50	14.37	3760	61.32	15.55	4550	67.45	16.57	5340	73.08	17.48
2980	54.59	14.39	3770	61.40	15.56	4560	67.53	16.58	5350	73.14	17.49
2990	54.68	14.41	3780	61.48	15.58	4570	67.60	16.59	5360	73.21	17.50
3000	54.77	14.42	3790	61.56	15.59	4580	67.68	16.61	5370	73.28	17.51
3010	54.86	14.44	3800	61.64	15.60	4590	67.75	16.62	5380	73.35	17.52
3020	54.95	14.45	3810	61.73	15.62	4600	67.82	16.63	5390	73.42	17.53
3030	55.05	14.47	3820	61.81	15.63	4610	67.90	16.64	5400	73.48	17.54
3040	55.14	14.49	3830	61.89	15.65	4620	67.97	16.66	5410	73.55	17.55
3050	55.23	14.50	3840	61.97	15.66	4630	68.04	16.67	5420	73.62	17.57
3060	55.32	14.52	3850	62.05	15.67	4640	68.12	16.68	5430	73.69	17.58
3070	55.41	14.53	3860	62.13	15.69	4650	68.19	16.69	5440	73.76	17.59
3080	55.50	14.55	3870	62.21	15.70	4660	68.26	16.70	5450	73.82	17.60
3090	55.59	14.57	3880	62.29	15.71	4670	68.34	16.71	5460	73.89	17.61
3100	55.68	14.58	3890	62.37	15.73	4680	68.41	16.73	5470	73.96	17.62
3110	55.77	14.60	3900	62.45	15.74	4690	68.48	16.74	5480	74.03	17.63
3120	55.86	14.61	3910	62.53	15.75	4700	68.56	16.75	5490	74.09	17.64
3130	55.95	14.63	3920	62.61	15.77	4710	68.63	16.76	5500	74.16	17.65
3140	56.04	14.64	3930	62.69	15.78	4720	68.70	16.77	5510	74.23	17.66
3150	56.12	14.66	3940	62.77	15.79	4730	68.77	16.79	5520	74.30	17.67
3160	56.21	14.67	3950	62.85	15.81	4740	68.85	16.80	5530	74.36	17.68
3170	56.30	14.69	3960	62.93	15.82	4750	68.92	16.81	5540	74.43	17.69
3180	56.39	14.71	3970	63.01	15.83	4760	68.99	16.82	5550	74.50	17.71
3190	56.48	14.72	3980	63.09	15.85	4770	69.07	16.83	5560	74.57	17.72
3200	56.57	14.74	3990	63.17	15.86	4780	69.14	16.85	5570	74.63	17.73
3210	56.66	14.75	4000	63.25	15.87	4790	69.21	16.86	5580	74.70	17.74
3220	56.75	14.77	4010	63.32	15.89	4800	69.28	16.87	5590	74.77	17.75
3230	56.83	14.78	4020	63.40	15.90	4810	69.35	16.88	5600	74.83	17.76
3240	56.92	14.80	4030	63.48	15.91	4820	69.43	16 89	5610	74.90	17.77
3250	57.01	14.81	4040	63.56	15.93	4830	69.50	16.90	5620	74.97	17.78
3260	57.10	14.83	4050	63.64	15.94	4840	69.57	16.92	5630	75.03	17.79
3270	57.18	14.84	4060	63.72	15.95	4850	69.64	16.93	5640	75.10	17.80
3280	57.27	14.86	4070	63.80	15.97	4860	69.71	16.94	5650	75.17	17.81
3290	57.36	14.87	4080	63.87	15.98	4870	69.79	16.95	5660	75.23	17.82
3300	57.45	14.89	4090	63.95	15.99	4880	69.86	16.96	5670	75.30	17.83
3310	57.53	14.90	4100	64.03	16.01	4890	69.93	16.97	5680	75.37	17.84
3320	57.62	14.92	4110	64.11	16.02	4900	70.00	16.98	5690	75.43	17.85
3330	57.71	14.93	4120	64.19	16.03	4910	70.07	17.00	5700	75.50	17.86
3340	57.79	14.95	4130	64.27	16.04	4920	70.14	17.01	5710	75.56	17.87
3350	57.88	14.96	4140	64.34	16.06	4930	70.21	17.02	5720	75.63	17.88
3360	57.97	14.98	4150	64.42	16.07	4940	70.29	17.03	5730	75.70	17.89
3370	58.05	14.99	4160	64.50	16.08	4950	70.36	17.04	5740	75.76	17.90
3380	58.14	15,01	4170	64.58	16.10	4960	70.43	17.05	5750	75.83	17.92
3390	58.22	15.02	4180	64.65	16.11	4970	70.50	17.07	5760	75.89	17.93
3400	58.31	15.04	4190	64.73	16.12	4980	70.57	17.08	5770	75.96	17.94
3410	58.40	15.05	4200	64.81	16.13	4990	70.64	17.09	5780	76.03	17.95
3420	58.48	15.07	4210	64.88	16.15	5000	70.71	17.10	5790	76.09	17.96
3430	58.57	15.08	4220	64.96	16.16	5010	70.78	17.11	5800	76.16	17.97
3440	58.65	15.10	4230	65.04	16.17	5020	70.85	17.12	5810	76.22	17.98
3450	58.74	15.11	4240	65.12	16.19	5030	70.92	17.13	5820	76.29	17.99
3460	58.82	15.12	4250	65.19	16.20	5040	70.99	17.15	5830	76.35	18.00
3470	58.91	15.14	4260	65.27	16.21	5050	71.06	17.16	5840	76.42	18.01
3480	58.99	15.15	4270	65.35	16.22	5060	71.13	17.17	5850	76.49	18.02
3490	59.08	15.17	4280	65.42	16.24	5070	71.20	17.18	5860	76.55	18.03
3500	59.16	15.18	4290	65.50	16.25	5080	71.27	17.19	5870	76.62	18.04
3510	59.25	15.20	4300	65.57	16.26	5090	71.34	17.20	5880	76.68	18.05
3520	59.33	15.21	4310	65.65	16.27	5100	71.41	17.21	5890	76.75	18.06
3530	59.41	15.23	4320	65.73	16.29	5110	71.48	17.22	5900	76.81	18.07
3540	59.50	15.24	4330	65.80	16.30	5120	71.55	17.24	5910	76.88	18.08

Square Roots and Cube Roots of Numbers from 1000 to 10000 —(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. R
5920	76.94	18.09	6710	81.91	18.86	7500	86.60	19.57	8290	91.05	20.24
5930	77.01	18.10	6720	81.98	18.87	7510	86.66	19.58	8300	91.10	20.25
5940	77.07	18.11	6730	82.04	18.88	7520	86.72	19.59	8310	91.16	20.26
5950	77.14	18.12	6740	82.10	18.89	7530	86.78	19.60	8320	91.21	20.26
5960	77.20	18.13	6750	82.16	18.90	7540	86.83	19.61	8330	91.27	20.27
5970	77.27	18.14	6760	82.22	18.91	7550	86.89	19.62	8340	91.32	20.28
5980	77.33	18.15	6770	82.28	18.92	7560	86.95	19.63	8350	91.38	20.29
5990	77.40	18.16	6780	82.34	18.93	7570	87.01	19.64	8360	91.43	20.30
6000	77.46	18.17	6790	82.40	18.94	7580	87.06	19.64	8370	91.49	20.30
6010	77.52	18.18	6800	82.46	18.95	7590	87.12	19.65	8380	91.54	20.31
3020	77.59	18.19	6810	82.52	18.95	7600	87.18	19.66	8390	91.60	20.32
6030	77.65	18.20	6820	82.58	18.96	7610	87.24	19.67	8400	91.65	20.33
6040	77.72	18.21	6830	82.64	18.97	7620	87.29	19.68	8410	91.71	20.34
6050	77.78	18.22	6840	82.70	18.98	7630	87.35	19.69	8420	91.76	20.34
6060	77.85	18.23	6850	82.76	18.99	7640	87.41	19.70	8430	91.82	20.35
6070	77.91	18.24	6860	82.83	19.00	7650	87.46	19.70	8440	91.87	20.36
6080	77.97	18.25	6870	82.89	19.01	7660	87.52	19.71	8450	91.92	20.37
6090	78.04	18.26	6880	82.95	19.02	7670	87.58	19.72	8460	91.98	20.38
6100	78.10	18.27	6890	83.01	19.03	7680	87.64	19.73	8470	92.03	20.38
6110	78.17	18.28	6900	83.07	19.04	7690	87.69	19.74	8480	92.09	20.39
6120	78.23	18.29	6910	83.13	19.05	7700	87.75	19.75	8490	92.14	20.40
6130	78.29	18.30	6920	83.19	19.06	7710	87.81	19.76	8500	92.20	20.41
6140	78.36	18.31	6930	83.25	19.07	7720	87.86	19.76	8510	92.25	20.42
6150	78.42	18.32	6940	83.31	19.07	7730	87.92	19.77	8520	92.30	20.42
6160	78.49	18.33	6950	83.37	19.08	7740	87.98	19.78	8530	92.36	20.43
6170	78.55	18.34	6960	83.43	19.09	7750	88.03	19.79	8540	92.41	20.44
6180	78.61	18.35	6970	83.49	19.10	7760	88.09	19.80	8550	92.47	20.45
6190	78.68	18.36	6980	83.55	19.11	7770	88.15	19.81	8560	92.52	20.46
6200	78.74	18.37	6990	83.61	19.12	7780	88.20	19.81	8570	92.57	20.46
6210	78.80	18.38	7000	83.67	19.13	7790	88.26	19.82	8580	92.63	20.47
6220	78.87	18.39	7010	83.73	19.14	7800	88.32	19.83	8590	92.68	20.48
6230	78.93	18.40	7020	83.79	19.15	7810	88.37	19.84	8600	92.74	20.49
6240	78.99	18.41	7030	83.85	19.16	7820	88.43	19.85	8610	92.79	20.50
6250	79.06	18.42	7040	83.90	19.17	7830	88.49	19.86	8620	92.84	20.50
6260	79.12	18.43	7050	83.96	19.17	7840	88.54	19.87	8630	92.90	20.51
6270	79.18	18.44	7060	84.02	19.18	7850	88.60	19.87	8640	92.95	20.52
6280	79.25	18.45	7070	84.08	19.19	7860	88.66	19.88	8650	93.01	20.53
6290	79.31	18.46	7080	84.14	19.20	7870	88.71	19.89	8660	93.06	20.54
6300	79.37	18.47	7090	84.20	19.21	7880	88.77	19.90	8670	93.11	20.54
6310	79.44	18.48	7100	84.26	19.22	7890	88.83	19.91	8680	93.17	20.55
6320	79.50	18.49	7110	84.32	19.23	7900	88.88	19.92	8690	93.22	20.56
6330	79.56	18.50	7120	84.38	19.24	7910	88.94	19.92	8700	93.27	20.57
6340	79.62	18.51	7130	84.44	19.25	7920	88.99	19.93	8710	93.33	20.57
6350	79.69	18.52	7140	84.50	19.26	7930	89.05	19.94	8720	93.38	20.58
6360	79.75	18.53	7150	84.56	19.26	7940	89.11	19.95	8730	93.43	20.59
6370	79.81	18.54	7160	84.62	19.27	7950	89.16	19.96	8740	93.49	20.60
6380	79.87	18.55	7170	84.68	19.28	7960	89.22	19.97	8750	93.54	20.61
6390	79.94	18.56	7180	84.73	19.29	7970	89.27	19.97	8760	93.59	20.61
6400	80.00	18.57	7190	84.79	19.30	7980	89.33	19.98	8770	93.65	20.62
6410	80.06	18.58	7200	84.85	19.31	7990	89.39	19.99	8780	93.70	20.63
6420	80.12	18.59	7210	84.91	19.32	8000	89.44	20.00	8790	93.75	20.64
6430	80.19	18.60	7220	84.97	19.33	8010	89.50	20.01	8800	93.81	20.65
6440	80.25	18.60	7230	85.03	19.34	8020	89.55	20.02	8810	93.86	20.65
6450	80.31	18.61	7240	85.09	19.35	8030	89.61	20.02	8820	93.91	20.66
6460	80.37	18.62	7250	85.15	19.35	8040	89.67	20.03	8830	93.97	20.67
6470	80.44	18.63	7260	85.21	19.36	8050	89.72	20.04	8840	94.02	20.68
6480	80.50	18.64	7270	85.26	19.37	8060	89.78	20.05	8850	94.07	20.68
6490	80.56	18.65	7280	85.32	19.38	8070	89.83	20.06	8860	94.13	20.69
6500	80.62	18.66	7290	85.38	19.39	8080	89.89	20.07	8870	94.18	20.70
6510	80.68	18.67	7300	85.44	19.40	8090	89.94	20.07	8880	94.23	20.71
6520	80.75	18.68	7310	85.50	19.41	8100	90.00	20.08	8890	94.29	20.72
6530	80.81	18.69	7320	85.56	19.42	8110	90.06	20.09	8900	94.34	20.72
6540	80.87	18.70	7330	85.62	19.43	8120	90.11	20.10	8910	94.39	20.73
6550	80.93	18.71	7340	85.67	19.43	8130	90.17	20.11	8920	94.45	20.74
6560	80.99	18.72	7350	85.73	19.44	8140	90.22	20.12	8930	94.50	20.75
6570	81.06	18.73	7360	85.79	19.45	8150	90.28	20.12	8940	94.55	20.75
6580	81.12	18.74	7370	85.85	19.46	8160	90.33	20.13	8950	94.60	20.76
6590	81.18	18.75	7380	85.91	19.47	8170	90.39	20.14	8960	94.66	20.77
6600	81.24	18.76	7390	85.97	19.48	8180	90.44	20.15	8970	94.71	20.78
6610	81.30	18.77	7400	86.02	19.49	8190	90.50	20.16	8980	94.76	20.79
6620	81.36	18.78	7410	86.08	19.50	8200	90.55	20.17	8990	94.82	20.79
6630	81.42	18.79	7420	86.14	19.50	8210	90.61	20.17	9000	94.87	20.80
6640	81.49	18.80	7430	86.20	19.51	8220	90.66	20.18	9010	94.92	20.81
6650	81.55	18.81	7440	86.26	19.52	8230	90.72	20.19	9020	94.97	20.82
6660	81.61	18.81	7450	86.31	19.53	8240	90.77	20.20	9030	95.03	20.82
6670	81.67	18.82	7460	86.37	19.54	8250	90.83	20.21	9040	95.08	20.83
6680	81.73	18.83	7470	86.43	19.55	8260	90.88	20.21	9050	95.13	20.84
6690	81.79	18.84	7480	86.49	19.56	8270	90.94	20.22	9060	95.18	20.85
6700	81.85	18.85	7490	86.54	19.57	8280	90.99	20.23	9070	95.24	20.85

Square Roots and Cube Roots of Numbers from 1000 to 10000 —(Continued.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
9080	95.29	20.86	9320	96.54	21.04	9550	97.72	21.22	9780	98.89	21.39
9090	95.34	20.87	9330	96.59	21.05	9560	97.78	21.22	9790	98.94	21.39
9100	95.39	20.88	9340	96.64	21.06	9570	97.83	21.23	9800	98.99	21.40
9110	95.45	20.89	9350	96.70	21.07	9580	97.88	21.24	9810	99.05	21.41
9120	95.50	20.89	9360	96.75	21.07	9590	97.93	21.25	9820	99.10	21.41
9130	95.55	20.90	9370	96.80	21.08	9600	97.98	21.25	9830	99.15	21.42
9140	95.60	20.91	9380	96.85	21.09	9610	98.03	21.26	9840	99.20	21.43
9150	95.66	20.92	9390	96.90	21.10	9620	98.08	21.27	9850	99.25	21.44
9160	95.71	20.92	9400	96.95	21.10	9630	98.13	21.28	9860	99.30	21.44
9170	95.76	20.93	9410	97.01	21.11	9640	98.18	21.28	9870	99.35	21.45
9180	95.81	20.94	9420	97.06	21.12	9650	98.23	21.29	9880	99.40	21.46
9190	95.86	20.95	9430	97.11	21.13	9660	98.29	21.30	9890	99.45	21.47
9200	95.92	20.95	9440	97.16	21.13	9670	98.34	21.30	9900	99.50	21.47
9210	95.97	20.96	9450	97.21	21.14	9680	98.39	21.31	9910	99.55	21.48
9220	96.02	20.97	9460	97.26	21.15	9690	98.44	21.32	9920	99.60	21.49
9230	96.07	20.98	9470	97.31	21.16	9700	98.49	21.33	9930	99.65	21.49
9240	96.12	20.98	9480	97.37	21.16	9710	98.54	21.33	9940	99.70	21.50
9250	96.18	20.99	9490	97.42	21.17	9720	98.59	21.34	9950	99.75	21.51
9260	96.23	21.00	9500	97.47	21.18	9730	98.64	21.35	9960	99.80	21.52
9270	96.28	21.01	9510	97.52	21.19	9740	98.69	21.36	9970	99.85	21.52
9280	96.33	21.01	9520	97.57	21.19	9750	98.74	21.36	9980	99.90	21.53
9290	96.38	21.02	9530	97.62	21.20	9760	98.79	21.37	9990	99.95	21.54
9300	96.44	21.03	9540	97.67	21.21	9770	98.84	21.38	10000	100.00	21.54
9310	96.49	21.04									

To find Square or Cube Roots of large numbers not contained in the column of numbers of the table.

Such roots may sometimes be taken at once from the table, by merely regarding the columns of powers as being columns of numbers; and those of numbers as being those of roots. Thus, if the *sq rt* of 25281 is reqd, first find that number in the column of *squares;* and opposite to it, in the column of numbers, is its sq rt 159. For the *cube rt* of 857375, find that number in the column of *cubes;* and opposite to it, in the col of numbers, is its cube rt 95. When the exact number is not contained in the column of squares, or cubes, as the case may be, we may use instead the number nearest to it, if no great accuracy is reqd. But when a considerable degree of accuracy is necessary, the following very correct methods may be used.

For the square root.

This rule applies both to whole numbers, and to those which are *partly* (not wholly) decimal. First, in the foregoing manner, take out the tabular number, which is nearest to the given one; and also its tabular sq rt. Mult this tabular number by 3; to the prod add the given number. Call the sum A. Then mult the given number by 3; to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 946.53. Here we find the nearest tabular number to be 947; and its tabular sq rt 30.7734. Hence,

$$\left.\begin{array}{rl} 947 & = \text{tab num} \\ 3 & \\ \hline 2841 & \\ 946.53 & = \text{given num.} \\ \hline 3787.53 & = \text{A.} \end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{rl} 946.53 & = \text{given num.} \\ 3 & \\ \hline 2839.59 & \\ 947 & = \text{tab num.} \\ \hline 3786.59 & = \text{B.} \end{array}\right.$$

Then A. 3787.53 : B. 3786.59 : : Tab root. 30.7734 : Reqd root. 30.7657 +.

The root as found by actual mathematical process is also 30.7657 +.

For the cube root.

This rule applies both to whole numbers, and to those which are *partly* decimal. First take out the tabular number which is nearest to the given one; and also its tabular cube rt. Mult this tabular number by 2; and to the prod add the given number. Call the sum A. Then mult the given number by 2; and to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 7368. Here we find the nearest tabular number (in the column of cubes) to be 6859; and its tabular cube rt 19. Hence,

$$\left.\begin{array}{rl} 6859 & = \text{tab num.} \\ 2 & \\ \hline 13718 & \\ 7368 & = \text{given num.} \\ \hline 21086 & = \text{A.} \end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{rl} 7368 & = \text{given num.} \\ 2 & \\ \hline 14736 & \\ 6859 & = \text{tab num.} \\ \hline 21595 & = \text{B.} \end{array}\right.$$

Then, as A. 21086 : B. 21595 : : Tab Root. 19 : Reqd Rt. 19.4585

The root as found by correct mathematical process is 19.4586. The engineer rarely requires even

this degree of accuracy; for his purposes, therefore, this process is greatly preferable to the ordinary laborious one.

To find the square root of a number which is wholly decimal.

Very simple, and correct to the third numeral figure inclusive. If the number does not contain at least five figures, *counting from the first numeral, and including it*, add one or more ciphers to make five. If, after that, the *whole number* is not separable into twos, add another cipher to make it so. Then beginning at the first numeral figure, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one; take out its tabular sq rt; move the decimal point of this tabular root to the left, *half* as many places as the finally modified *decimal* number has figures.

Ex. What is the sq rt of the decimal .002? Here, in order to have at least five decimal figures, counting from the first numeral (2), and including it, add ciphers thus, .00.20,00,0. But, as it is not now separable into twos, add another cipher, thus, .00,20,00,00. Then beginning at the first numeral (2), assume this decimal to be the whole number 200000. The nearest to this in the table is 199809; and the sq rt of this is 447. Now, the decimal number as finally modified, namely, .00.20,00,00, has eight figures; one-half of which is 4; therefore, move the decimal point of the root 447, four places to the left; making it .0447. This is the reqd sq rt of .002, correct to the third numeral 7 included.

To find the cube root of a number which is wholly decimal.

Very simple, and correct to the third numeral inclusive.

If the number does not contain at least five figures, counting from the first numeral, and including it, add one or more ciphers to make five. If, after that, the number is not separable into threes, add one or more ciphers to make it so. Then beginning at the first numeral, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one, and take out its tabular cub rt. Move the decimal point of this rt to the left, one-third as many places as the finally modified *decimal* number has figures.

Ex. What is the cube rt of the decimal .002? Here, in order to have at least five figures, counting from the first numeral (2), and including it, add ciphers thus, .002,000,0. But as it is not now separable into threes, add two more ciphers to make it so; thus, .002,000,000. Then beginning with the first numeral (2), assume the decimal to be the whole number 2000000. The nearest cube to this in the table in the column of cubes, is 2000376; and its tabular cube rt as found in the col of numbers, is 126. Now, the decimal number as finally modified, namely, .002 000 000, has nine figures; one-third of which is 3; therefore, move the decimal point of the root 126, three places to the left, making it .126. This is the reqd cube rt of the decimal .002, correct to the third numeral 6 included.

Fifth roots and fifth powers.

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.
.0000100	.1	.000142	.170	.004219	.335	.077760	.60	.695688	.93	8.11368	1.52
		.000164	.175	.004544	.340	.084460	.61	.733904	.94	8.66171	1.54
.0000110	.102	.000189	.180	.004888	.345	.091613	.62	.773781	.95	9.23896	1.56
		.000217	.185	.005252	.350	.099244	.63	.815373	.96	9.84658	1.58
.0000122	.104	.000248	.190	.005638	.355	.107374	.64	.858734	.97	10.4858	1.60
		.000282	.195	.006047	.360	.116029	.65	.903921	.98	11.1577	1.62
.0000134	.106	.000320	.200	.006478	.365	.125233	.66	.950990	.99	11.8637	1.64
		.000362	.205	.006934	.370	.135012	.67	1.	1.	12.6049	1.66
.0000147	.108	.000408	.210	.007416	.375	.145393	.68	1.10408	1.02	13.3828	1.68
.0000161	.110	.000459	.215	.007924	.380	,156403	.69	1.21665	1.04	14.1986	1.70
.0000176	.112	.000515	.220	.008459	.385	.168070	.70	1.33823	1.06	15.0537	1.72
.0000193	.114	.000577	.225	.009022	.390	.180423	.71	1.46933	1.08	15.9495	1.74
.0000210	.116	.000644	.230	.009616	.395	.193492	.72	1.61051	1.10	16.8874	1.76
.0000229	.118	.000717	.235	.010240	.400	.207307	.73	1.76234	1.12	17.8690	1.78
.0000249	.120	.000796	.240	.011586	.41	.221901	.74	1.92541	1.14	18.8957	1.80
.0000270	.122	.000883	.245	.013069	.42	.237305	.75	2.10034	1.16	19.9690	1.82
.0000293	.124	.000977	.250	.014701	.43	.253553	.76	2.28775	1.18	21.0906	1.84
.0000318	.126	.001078	.255	.016492	.44	.270678	.77	2.48832	1.20	22.2620	1.86
.0000344	.128	.001188	.260	.018453	.45	.288717	.78	2.70271	1.22	23.4849	1.88
.0000371	.130	.001307	.265	.020596	.46	.307706	.79	2.93163	1.24	24.7610	1.90
.0000401	.132	.001435	.270	.022935	.47	.327680	.80	3.17580	1.26	26.0919	1.92
.0000432	.134	.001573	.275	·025480	.48	.348678	.81	3.43597	1.28	27.4795	1.94
.0000465	.136	.001721	.280	.028248	.49	.370740	.82	3.71293	1.30	28.9255	1.96
.0000500	.138	.001880	.285	.031250	.50	.393904	.83	4.00746	1.32	30.4317	1.98
.0000538	.140	.002051	.290	.034503	.51	.418212	.84	4.32040	1.34	32.0000	2.00
.0000577	.142	.002234	.295	.038020	.52	.443705	.85	4.65259	1.36	36.2051	2.05
.0000619	.144	.002430	.300	.041820	.53	.470427	.86	5.00490	1.38	40.8410	2.10
.0000663	.146	.002639	.305	.045917	.54	.498421	.87	5.37824	1.40	45.9401	2.15
.0000710	.148	.002863	.310	.050328	.55	.527732	.88	5.77353	1.42	51.5363	2.20
.0000754	.150	.003101	.315	.055073	.56	.558406	.89	6.19174	1.44	57.6650	2.25
.0000895	.155	.003355	.320	.060169	.57	.590490	.90	6.63383	1.46	64.3634	2.30
.000105	.160	.003626	.325	.065636	.58	.624032	.91	7.10082	1.48	71.6703	2.35
.000122	.165	.003914	.330	.071492	.59	.659082	.92	7.59375	1.50	79.6262	2.40

Fifth roots and fifth powers—(Continued).

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.
88.2735	2.45	2824.75	4.90	85873	9.70	2609193	19.2	20511149	29.0	459165024	54.
97.6562	2.50	2971.84	4.95	90392	9.80	2747949	19.4	21228253	29.2	503284375	55.
107.820	2.55	3125.00	5.00	95099	9.90	2892547	19.6	21965275	29.4	550731776	56.
118.814	2.60	3450.25	5.10	100000	10.0	3043168	19.8	22722628	29.6	601692057	57.
130.686	2.65	3802.04	5.20	110408	10.2	3200000	20.0	23500728	29.8	656356768	58.
143.489	2.70	4181.95	5.30	121665	10.4	3363232	20.2	24300000	30.0	714924299	59.
157.276	2.75	4591.65	5.40	133823	10.6	3533059	20.4	26393634	30.5	777600000	60.
172.104	2.80	5032.84	5.50	146933	10.8	3709677	20.6	28629151	31.0	844596301	61.
188.029	2.85	5507.32	5.60	161051	11.0	3893289	20.8	31013642	31.5	916132832	62.
205.111	2.90	6016.92	5.70	176234	11.2	4084101	21.0	33554432	32.0	992436543	63.
223.414	2.95	6563.57	5.80	192541	11.4	4282322	21.2	36259082	32.5	1073741824	64.
243.000	3.00	7149.24	5.90	210034	11.6	4488166	21.4	39135393	33.0	1160290625	65.
263.936	3.05	7776.00	6.00	228776	11.8	4701850	21.6	42191410	33.5	1252332576	66.
286.292	3.10	8445.96	6.10	248832	12.0	4923597	21.8	45435424	34.0	1350125107	67.
310.136	3.15	9161.33	6.20	270271	12.2	5153632	22.0	48875980	34.5	1453933568	68.
335.544	3.20	9924.37	6.30	293163	12.4	5392186	22.2	52521875	35.0	1564031349	69.
362.591	3.25	10737	6.40	317580	12.6	5639493	22.4	56382167	35.5	1680700000	70.
391.354	3.30	11603	6.50	343597	12.8	5895793	22.6	60466176	36 0	1804229351	71.
421.419	3.35	12523	6.60	371293	13.0	6161327	22.8	64783487	36 5	1934917632	72.
454.354	3.40	13501	6.70	400746	13.2	6436343	23.0	69343957	37.0	2073071593	73.
488.760	3.45	14539	6.80	432040	13.4	6721093	23.2	74157715	37.5	2219006624	74.
525.219	3.50	15640	6.90	465259	13.6	7015834	23.4	79235168	38.0	2373046875	75.
563.822	3.55	16807	7.00	500490	13.8	7320825	23.6	84587005	38.5	2535525376	76
604.662	3.60	18042	7.10	537824	14.0	7636332	23.8	90224199	39.0	2706784157	77.
647.835	3.65	19349	7.20	577353	14.2	7962624	24.0	96158012	39.5	2887174368	78
693.440	3.70	20731	7.30	619174	14.4	8299976	24.2	102400000	40.0	3077056399	79.
741.577	3.75	22190	7.40	663383	14.6	8648666	24.4	108962013	40.5	3276800000	80
792.352	3.80	23730	7.50	710082	14.8	9008978	24.6	115856201	41.0	3486784401	81.
845.870	3.85	25355	7.60	759375	15.0	9381200	24.8	123095020	41.5	3707398432	82.
902.242	3.90	27068	7.70	811368	15.2	9765625	25.0	130691232	42.0	3939040643	83.
961.580	3.95	28872	7.80	866171	15.4	10162550	25.2	138657910	42.5	4182119424	84.
1024.00	4.00	30771	7.90	923896	15.6	10572278	25.4	147008443	43.0	4437053125	85.
1089.62	4.05	32768	8.00	984658	15.8	10995116	25.6	155756538	43.5	4704270176	86.
1158.56	4.10	34868	8.10	1048576	16.0	11431377	25.8	164916224	44 0	4984209207	87.
1230.95	4.15	37074	8.20	1115771	16.2	11881376	26.0	174501858	44.5	5277319168	88.
1306.91	4.20	39390	8.30	1186367	16.4	12345437	26.2	184528125	45.0	5584059449	89.
1386.58	4.25	41821	8.40	1260493	16.6	12823886	26.4	195010045	45.5	5904900000	90.
1470.08	4.30	44371	8.50	1338278	16.8	13317055	26.6	205962976	46.0	6240321451	91.
1557.57	4.35	47043	8.60	1419857	17.0	13825281	26.8	217402615	46.5	6590815232	92.
1649.16	4.40	49842	8.70	1505366	17.2	14348907	27.0	229345007	47 0	6956883693	93.
1745.02	4.45	52773	8.80	1594947	17.4	14888280	27.2	241806543	47.5	7339040224	94.
1845.28	4.50	55841	8.90	1688742	17.6	15443752	27.4	254803968	48.0	7737809375	95.
1950.10	4.55	59049	9.00	1786899	17.8	16015681	27.6	268354383	48.5	8153726976	96.
2059.63	4.60	62403	9.10	1889568	18.0	16604430	27.8	282475249	49.0	8587340257	97.
2174.03	4.65	65908	9.20	1996903	18.2	17210368	28.0	297184391	49.5	9039207968	98.
2293.45	4.70	69569	9.30	2109061	18.4	17833868	28.2	312500000	50.0	9509900499	99.
2418.07	4.75	73390	9.40	2226203	18.6	18475309	28.4	345025251	51.		
2548.04	4.80	77378	9.50	2348493	18.8	19135075	28.6	380204032	52.		
2683.54	4.85	81537	9.60	2476099	19.0	19813557	28.8	418195493	53.		

The column headed "12 n" facilitates the use of the table in cases where, for instance, the quantity is given in *inches*, and where it is desired to obtain the $\frac{5}{2}$ power of the same quantity in *feet*. Thus, suppose we have a ½ inch pipe, and we require the $\frac{5}{2}$ power of the diameter in feet. Find ½ (the diameter, in inches) in the column headed "12 n," opposite which, in the column headed "n," is 0.041666 (the diameter, in feet), and, in column headed "$n^{5/2}$," 0.00035 (the $\frac{5}{2}$ power of the diameter, 0.041666, in feet).

Values of n, ending in 0 or in 5, are *exact* values. All others end in repeating decimals. Thus: n = 0.052083 signifies n = 0.052083333 . . . . .

Square roots of fifth powers of numbers. See page 68.

12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$	12 n	n	$n^{\frac{5}{2}}$
¼	0.020833	0.000063	22	1.8333	4.5510	84	7.000	129.64	468	39	9499
⅜	0.031250	0.000173	23	1.9166	5.0859	85	7.083	133.53	480	40	10119
½	0.041666	0.000354	24	2.0000	5.6569	86	7.166	137.50	492	41	10764
⅝	0.052083	0.000619	25	2.0833	6.2647	87	7.250	141.53	504	42	11432
¾	0.062500	0.000977	26	2.1666	6.9100	88	7.333	145.63	516	43	12125
⅞	0.072916	0.001436	27	2.2500	7.5938	89	7.416	149.80	528	44	12842
1	0.083333	0.002005	28	2.3333	8.3165	90	7.500	154.05	540	45	13584
⅛	0.093750	0.002691	29	2.4166	9.0791	91	7.583	158.56	552	46	14351
¼	0.104166	0.003502	30	2.5000	9.8821	92	7.666	162.75	564	47	15144
⅜	0.114583	0.004444	31	2.5833	10.726	93	7.750	167.21	576	48	15964
½	0.125000	0.005524	32	2.6666	11.612	94	7.833	171.74	588	49	16807
⅝	0.135416	0.006748	33	2.7500	12.541	95	7.916	176.34	600	50	17678
¾	0.145833	0.008122	34	2.8333	13.513	96	8.000	181.02	612	51	18575
⅞	0.156250	0.009651	35	2.9166	14.528	97	8.083	185.77	624	52	19499
2	0.166666	0.011340	36	3.0000	15.588	98	8.166	190.60	636	53	20450
¼	0.187500	0.015223	37	3.0833	16.694	99	8.250	195.49	648	54	21428
½	0.208333	0.019811	38	3.1666	17.845	100	8.333	200.47	660	55	22434
¾	0.229166	0.025141	39	3.2500	19.042	102	8.50	210.64	672	56	23468
3	0.250000	0.031250	40	3.3333	20.286	105	8.75	226.47	684	57	24529
¼	0.270833	0.038173	41	3.4166	21.578	108	9.00	243.00	696	58	25619
½	0.291666	0.045943	42	3.5000	22.918	111	9.25	260.23	708	59	26738
¾	0.312500	0.054592	43	3.5833	24.306	114	9.50	278.17	720	60	27885
4	0.333333	0.064150	44	3.6666	25.744	117	9.75	296.83	732	61	29062
¼	0.354166	0.074648	45	3.7500	27.232	120	10.0	316.23	744	62	30268
½	0.375000	0.086115	46	3.8333	28.770	126	10.5	357.25	756	63	31503
¾	0.395833	0.098578	47	3.9166	30.359	132	11.0	401.31	768	64	32768
5	0.416666	0.11207	48	4.0000	32.000	138	11.5	448.48	780	65	34063
¼	0.437500	0.12660	49	4.0833	33.693	144	12.0	498.83	792	66	35388
½	0.458333	0.14222	50	4.1666	35.438	150	12.5	552.43	804	67	36744
¾	0.479166	0.15893	51	4.2500	37.237	156	13.0	609.34	816	68	38130
6	0.500000	0.17678	52	4.3333	39.089	162	13.5	669.63	828	69	39548
½	0.541666	0.21594	53	4.4166	40.996	168	14.0	733.37	840	70	40996
7	0.583333	0.25989	54	4.5000	42.957	174	14.5	800.61	852	71	42476
½	0.625000	0.30882	55	4.5833	44.973	180	15.0	871.42	864	72	43988
8	0.666666	0.36289	56	4.6666	47.045	186	15.5	945.87	876	73	45531
½	0.708333	0.42227	57	4.7500	49.174	192	16.0	1024.0	888	74	47106
9	0.750000	0.48714	58	4.8333	51.359	198	16.5	1105.9	900	75	48714
½	0.791666	0.55764	59	4.9166	53.602	204	17.0	1191.6	912	76	50354
10	0.833333	0.63394	60	5.0000	55.902	210	17.5	1281.1	924	77	52027
½	0.875000	0.71618	61	5.0833	58.260	216	18.0	1374.6	936	78	53732
11	0.916666	0.80451	62	5.1666	60.677	222	18.5	1472.1	948	79	55471
½	0.958333	0.89907	63	5.2500	63.154	228	19.0	1573.6	960	80	57243
12	1.000000	1.00000	64	5.3333	65.690	234	19.5	1679.1	972	81	59049
½	1.041666	1.1074	65	5.4166	68.286	240	20	1788.9	984	82	60888
13	1.083333	1.2215	66	5.5000	70.943	252	21	2020.9	996	83	62762
½	1.125000	1.3424	67	5.5833	73.660	264	22	2270.2	1008	84	64669
14	1.166666	1.4702	68	5.6666	76.440	276	23	2537.0	1020	85	66611
½	1.208333	1.6050	69	5.7500	79.281	288	24	2821.8	1032	86	68588
15	1.250000	1.7469	70	5.8333	82.185	300	25	3125.0	1044	87	70599
½	1.291666	1.8962	71	5.9166	85.152	312	26	3446.9	1056	88	72645
16	1.333333	2.0528	72	6.0000	88.182	324	27	3788.0	1068	89	74727
½	1.375000	2.2170	73	6.0833	91.276	336	28	4148.5	1080	90	76843
17	1.416666	2.3887	74	6.1666	94.434	348	29	4528.9	1092	91	78996
½	1.458333	2.5683	75	6.2500	97.656	360	30	4929.5	1104	92	81184
18	1.500000	2.7557	76	6.3333	100.94	372	31	5350.6	1116	93	83408
½	1.541666	2.9510	77	6.4166	104.30	384	32	5792.6	1128	94	85668
19	1.583333	3.1545	78	6.5000	107.72	396	33	6255.8	1140	95	87965
½	1.625000	3.3662	79	6.5833	111.20	408	34	6740.6	1152	96	90298
20	1.666666	3.5861	80	6.6666	114.76	420	35	7247.2	1164	97	92668
½	1.708333	3.8144	81	6.7500	118.37	432	36	7776.0	1176	98	95075
21	1.750000	4.0513	82	6.8333	122.06	444	37	8327.3	1188	99	97519
½	1.791666	4.2968	83	6.9166	125.82	456	38	8901.4	1200	100	100000

Square roots of the fifth powers of numbers.

No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.
.25	.031	7.	129.64	17.5	1281.1	31.	5351	49	16807	76	50354
.5	.177	7.25	141.53	18.	1374.6	31.5	5569	50	17678	77	52027
.75	.485	7.5	154.05	18.5	1472.1	32.	5793	51	18575	78	53732
1.	1.	7.75	167.21	19.	1573.6	32.5	6022	52	19499	79	55471
1.25	1.747	8.	181.02	19.5	1679.1	33.	6256	53	20450	80	57243
1.5	2.756	8.25	195.50	20.	1788.9	33.5	6496	54	21428	81	59049
1.75	4.051	8.5	210.64	20.5	1902.8	34.	6741	55	22434	82	60888
2.	5.657	8.75	226.48	21.	2020.9	34.5	6991	56	23468	83	62762
2.25	7.594	9.	243.	21.5	2143.4	35.	7247	57	24529	84	64669
2.5	9.882	9.25	260.23	22.	2270.2	35.5	7509	58	25620	85	66611
2.75	12.541	9.5	278.17	22.5	2401.4	36.	7776	59	26738	86	68588
3.	15.588	9.75	296.83	23.	2537.	36.5	8049	60	27886	87	70599
3.25	19.042	10.	316.23	23.5	2677.1	37.	8327	61	29062	88	72646
3.5	22.918	10.5	357.2	24.	2821.8	37.5	8611	62	30268	89	74727
3.75	27.232	11.	401.3	24.5	2971.1	38.	8901	63	31503	90	76843
4.	32.	11.5	448.5	25.	3125.	38.5	9197	64	32768	91	78996
4.25	37.24	12.	498.8	25.5	3283.6	39.	9498	65	34063	92	81184
4.5	42.96	12.5	552.4	26.	3446.9	39.5	9806	66	35388	93	83408
4.75	49.17	13.	609.3	26.5	3615.1	40.	10119	67	36744	94	85668
5.	55.90	13.5	669.6	27.	3788.	41.	10764	68	38131	95	87965
5.25	63.15	14.	733.4	27.5	3965.8	42.	11432	69	39548	96	90298
5.5	70.94	14.5	800.6	28.	4148.5	43.	12125	70	40996	97	92668
5.75	79.28	15.	871.4	28.5	4336.2	44.	12842	71	42476	98	95075
6.	88.18	15.5	945.9	29.	4528.9	45.	13584	72	43988	99	97519
6.25	97.66	16.	1024.	29.5	4726.7	46.	14351	73	45531	100	100000
6.5	107.72	16.5	1105.9	30.	4929.5	47.	15144	74	47106		
6.75	118.38	17.	1191.6	30.5	5138.	48.	15963	75	48714		

Numbers, in inches. Square roots of fifth powers, in feet.

	Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.
Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.
¼	.00006	3¾	.0547	12.	1.000	22½	4.813	42	22.92
⅜	.00017	4.	.0641	½	1.108	23	5.086	43	24.31
½	.00035	¼	.0731	13.	1.221	½	5.365	44	25.74
⅝	.00062	½	.0827	½	1.342	24	5.657	45	27.23
¾	.00096	¾	.0971	14.	1.470	25	6.264	46	28.77
⅞	.00144	5.	.1120	½	1.605	26	6.909	47	30.36
1.	.0020	¼	.1271	15.	1.747	27	7.593	48	32.00
⅛	.0027	½	.1428	½	1.896	28	8.316	49	33.69
¼	.0035	¾	.1590	16.	2.053	29	9.079	50	35.44
⅜	.0044	6	.1768	½	2.217	30	9.882	51	37.25
½	.0055	½	.2160	17.	2.389	31	10.73	52	39.13
⅝	.0067	7.	.2599	½	2.567	32	11.61	53	41.02
¾	.0081	½	.3088	18.	2.756	33	12.54	54	42.96
⅞	.0096	8.	.3628	½	2.950	34	13.51	55	44.97
2.	.0113	½	.4228	19.	3.155	35	14.58	56	47.05
¼	.0152	9.	.4871	½	3.365	36	15.59	57	49.17
½	.0198	½	.5577	20.	3.586	37	16.69	58	51.35
¾	.0252	10.	.6339	½	3.813	38	17.84	59	53.60
3.	.0312	½	.7162	21.	4.051	39	19.04	60	55.90
¼	.0383	11.	.8043	½	4.297	40	20.29	61	58.27
½	.0459	½	.8990	22.	4.551	41	21.58		

Logarithms.

(1) Tables of logarithms greatly facilitate multiplication and division and the finding of powers and roots of numbers.*

(2) The tables pp. 78 to 91 contain the **common, decimal** or **Briggs logarithms** of numbers. The *common* logarithm of a number is the exponent or index of that number as a power of 10. See (18). Thus: $1000 = 10^3$, and log 1000 (logarithm of 1000) = 3.00 000. Similarly, $28.7 = 10^{1.45\,788}$, and log 28.7 = 1.45 788.

(3) In general, let A and B be any two numbers, and k any exponent. Then, from the principle of powers and roots, we have

$$(1)\ \log AB = \log A + \log B; \qquad (2)\ \log \frac{A}{B} = \log A - \log B;$$

$$(3)\ \log A^k = k\,(\log A); \qquad (4)\ \log \sqrt[k]{A} = \frac{\log A}{k}$$

In other words:—
(1) Log of product = sum of logs of factors.
(2) Log of quotient = log of dividend — log of divisor.
Or log of fraction = log of numerator — log of denominator.
(3) Log of power = log of number, multiplied by exponent.
(4) Log of root = log of number, divided by exponent.

(4) From what has been said, it follows that

$\text{Log } 100 = \log 10^2 = 2.00\,000$ | $\text{Log } 0.1 = \log 10^{-1} = \bar{1}.00\,000$†
$\text{Log } 10 = \log 10^1 = 1.00\,000$ | $\text{Log } 0.01 = \log 10^{-2} = \bar{2}.00\,000$
$\text{Log } 1 = \log 10^0 = 0.00\,000$† | $\text{Log } 0.001 = \log 10^{-3} = \bar{3}.00\,000$

(5) Each common logarithm is a mixed number, consisting of an *integral* portion, called the **characteristic** or **index** (*preceding* the decimal point), and a *fractional* portion, called the **mantissa** (*following* the decimal point). The table gives only the *mantissa* of each log, the characteristic being found as explained below. The *mantissa* is always *positive*. The *characteristic*, in the log of any *whole* or *mixed* number, is *positive*, and is equal to the number of digits in the whole number, minus 1; while the characteristic of a *wholly decimal* number is *negative*, and is numerically equal to 1 plus the number of ciphers immediately following the decimal point. Thus:

log 2870 = 3.45 788 | log 0.287 = $\bar{1}$.45 788‡
" 287 = 2.45 788 | " 0.0287 = $\bar{2}$.45 788
" 28.7 = 1.45 788 | " 0.00287 = $\bar{3}$.45 788
" 2.87 = 0.45 788 | " 0.000287 = $\bar{4}$.45 788

It will be noticed that the mantissa remains constant for any given combination of significant figures in a number, wherever the decimal point in the number be placed; while the characteristic depends solely upon the position of the decimal point in the number.

(6) Let the number be resolved into two factors, one of which is an integer power of 10, while the other is greater than 1 and less than 10. Then the index of the power of 10 is the characteristic of the logarithm, and the logarithm of the other factor is the mantissa. Thus, $2870 = 1000 \times 2.87 = 10^3 \times 2.87$, and the logarithm of 2870 (3.45 788) is the sum of the exponent 3 (or 3.00 000) and the log (0.45 788) of 2.87.‡

---

* Logarithms not being exact quantities, operations performed with them are subject to some inaccuracy, especially where a logarithm is multiplied by a large number, the existing error being thus magnified. Logarithms of only five places in the mantissa usually suffice for calculations with numbers of four or five places. Greater accuracy is obtained by the use of tables of logarithms carried out to seven places.

† $\text{Log } 1 = \log \frac{10}{10} = \log 10 - \log 10 = 1 - 1 = 0$; or $1 = 10^0$.
$\text{Log } 0.1 = \log \frac{1}{10} = \log 1 - \log 10 = 0 - 1 = \bar{1}.0$; or $0.1 = 10^{-1}$.

‡ $0.287 = 2.87 \div 10$. Hence, log 0.287 = log 2.87 − log 10 = 0.45 788 − 1, which, for convenience, is written $\bar{1}$.45 788. See (16). Similarly, log 0.0287 = log 2.87 — log 100 = 0.45 788 — 2 = $\bar{2}$.45 788.

(7) **To find the logarithm** of a number. The short table on pages 78; 79 gives logs of numbers up to 1000. The longer table, pages 80 to 91, gives

(1)	The mantissa for *each* number from	1000	to	1750
(2)	The mantissa for each *even* number from	1750	to	3750
(3)	The mantissa for each *fifth* number from	3750	to	10000

(8) Logs of **numbers intermediate** of those given in the tables are found by simple proportion. The procedure necessary in these cases is explained in the examples given in connection with the tables, but it will often be found sufficiently accurate to use the log of the nearest number given in the table, neglecting interpolation.

The antilogarithm or **num log** (*numerus logarithmi*) is the **number corresponding** to a given logarithm. Thus, log. 2 = 0.30 103, and antilog 0.30 103 = 2.

(9) **Multiplication.** To multiply together two or more numbers, add together their logs and find the antilog of their sum. See Proportion (11) below.

(10) **Division.** Subtract the log of the divisor from that of the dividend, and find the antilog of the remainder. See Proportion (11) below.

The **reciprocal** of any number, $n, = \frac{1}{n}$. See page 52. Thus, recip $2 = \frac{1}{2} = 0.5$. Hence, log recip $n = \log \frac{1}{n} = \log 1 - \log n = 0 - \log n$.

Similarly, $\log \text{ recip } \frac{m}{n} = \log \frac{1}{m + n} = 0 - \log \frac{m}{n}$.

Thus, $\log \frac{6.3023}{23057} = 0 - \log \frac{23057}{6.3023} = 0 - 3.56\ 331 = \bar{4}.43\ 669$.

Since $n^{2-1} = n^1 = \frac{n^2}{n}$, $n^{1-1} = n^0 = \frac{n}{n} = 1$, $n^{0-1} = n^{-1} = \frac{1}{n}$, and $n^{0-2} = n^{-2} = \frac{1}{n^2}$ it follows that $\log n^{-1} = \log \frac{1}{n} = \log \text{ recip } n$; $\log n^{-2} = \log \frac{1}{n^2} = \log$ recip n^2, etc.

(11) **Proportion.** Example. 6.3023 : 290.19 = 1260.7 : ?

Multiply Nos, Add Logs.	Log 290.19	= 2.46 269
	" 1260.7	= 3.10 062
	Log 290.19 × 1260.7	= 5.56 331
Divide Nos, Subtract Log.	Log 6.3023	= 0.79 950
	Log 58051	= 4.76 881

The true value is 58049.05 +

(12) Instead of subtracting the log of the divisor, we may add its **cologarithm** or **arithmetical complement,** which is log of reciprocal of divisor, = 0 — log divisor = 10 — log divisor — 10. Thus:

$$\frac{1523}{3.332 \times 8.655} = ?$$

$$\begin{array}{lr}
\text{Log } 1523 & = 3.18\ 270 \\
\text{Colog } 8.332 = 10 - \log 8.332 - 10 = 10 - 0.52\ 270 - 10 = & 9.47\ 730 - 10 \\
\text{Colog } 8.655 = 10 - \log 8.655 - 10 = 10 - 0.93\ 727 - 10 = & 9.06\ 273 - 10 \\
\text{Sum of logs and cologs} & = 21.72\ 273 - 20 \\
= \text{Log } 52.813 = & 1.72\ 273
\end{array}$$

The true value is 52.8114 +

(13) **Involution,** or **finding powers** of numbers. Multiply log of given number by the exponent of the required power, and find the antilog of the product. Thus: $36^3 = ?$
Log .36 = 1.55 630. 1.55 630 × 3 = 4.66 890. Antilog 4.66 890 = 46656.

(14) **Evolution,** or **finding roots** of numbers. Divide log of given number by exponent of required root, and find antilog of quotient. Thus: $\sqrt[3]{46656} = ?$ Log 46656 = 4.66 890. 4.66 890 ÷ 3 = 1.55 630. Antilog 1.55 630 = 36.

(15) In finding roots of numbers, if the given number is a whole or mixed

number, the division of the log is performed in the usual way, as in the preceding example, even where, as in that example, the characteristic is not exactly divisible by the exponent of the required root. But **if the number is a fraction,** and the characteristic of its log therefore negative, and if the characteristic is not exactly divisible by the exponent, division in the usual way would give erroneous results. In such cases we may add a suitable number to the mantissa and deduct the same number from the characteristic, thus: to find $\sqrt[3]{0.00048}$. Log $0.00048 = \bar{4}.68\ 124 = 0.68\ 124 - 4 = 2.68\ 124 - 6 = \bar{6} + 2.68\ 124$, which, divided by 3, $= \bar{2} + 0.89\ 375 = \bar{2}.89\ 375 = \log 0.0783$. Or, see (16) and (17).

(16) To avoid inconvenience from the use of **negative characteristics,** it is customary to **modify** them by adding 10 to them, **afterward** deducting each such 10 from the sum, etc., of the logarithms. **Thus:** in multiplying or dividing 7425 by 0.25, we have

$$\begin{array}{llrl}
 & & \text{Multiplying.} & \text{Dividing.}\\
\text{either} & \log 7425 \ = & 3.87\ 070 & = 3.87\ 070\\
 & \log\quad 0.25 = & \underline{\bar{1}.39\ 794} & = \underline{\bar{1}.39\ 794}\\
 & & \underline{3.26\ 864} & \underline{4.47\ 276}\\
\text{or} & \log 7425 \ = & 3.87\ 070 & = 3.87\ 070\\
 & \text{modified } \log\quad 0.25 = & \underline{9.39\ 794} - 10 & = \underline{9.39\ 794} - 10\\
 & & 13.26\ 864 - 10 & \bar{6}.47\ 276 + 10\\
 & & = 3.26\ 864 & = 4.47\ 276
\end{array}$$

In most cases the actual process of deducting the added tens may be neglected, the nature of the work usually being such that an error so great as that arising from such neglect could hardly pass unnoticed.

(17) **To divide a modified logarithm,** add to it such a multiple of 10 as will make the sum exceed the true log by 10 times the divisor. **Thus:** to divide log 0.00048 by 3. Log $0.00048 = \bar{4}.68\ 124$, which, divided by 3, $= \bar{2}.89\ 375$. See (15).

$$\begin{array}{lrl}
 & \text{Log } 0.00048 = & \underline{\bar{4}.68\ 124}\\
\text{Modified} & \log 0.00048 = & 6.68\ 124 - 10\\
 & \text{Add } 2 \times 10 & \underline{20 \qquad\quad} - 20\\
\text{Dividing by} & 3) & \underline{26.68\ 124} - 30\\
\text{we obtain} & & 8.89\ 375 - 10, \text{ which is } \bar{2}.89\ 375 \text{ modified.}
\end{array}$$

(18) Except 1, any number can (like 10) be made the base of a system of logarithms. The base of the **hyperbolic, Napierian,** or **natural logarithms,** much used in steam engineering, is

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \ldots\ldots = 2.71\ 828 +$$

and is called ϵ (epsilon) or e.

$M = \log_{10} e$ (common log e) $= 0.43\ 429$; $\frac{1}{M} = \log_{\epsilon} 10$ (hyperbolic log 10) $= 2.30\ 259$.

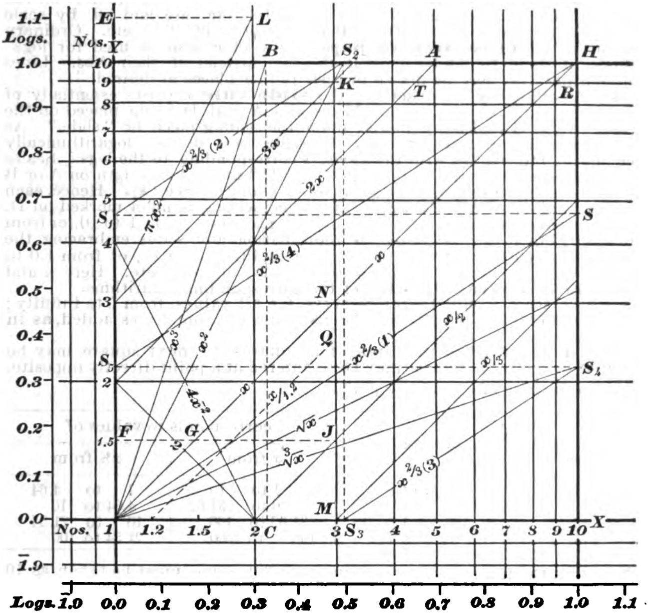
For any number, n,

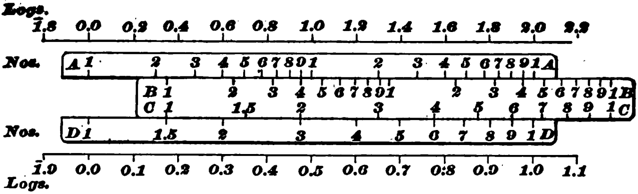
$$\log_{\epsilon} n = \frac{\log_{10} n}{M} = 2.30259 \log_{10} n;\ \log_{10} n = M \log_{\epsilon} n = 0.43429 \log_{\epsilon} n$$

(19) Whatever may be the base chosen for a system of logs, **the mantissas** of the logs of any given numbers bear a **constant ratio** to each other. Thus, in any system of logs, log 4 is always $= 2 \times \log 2$, and $= \frac{2}{3} \times \log 8$, etc., etc.

(20) **Logarithmic sines, tangents,** etc. of angles are the logs of the sines, tangents, etc. of those angles. Thus, sin 30° = 0.5000000, and log sin 30° = log. 0.5 = $\bar{1}.69\ 897$, usually written 9.69 897 — 10, or simply 9.69 897.

(21) Since no power of a positive number can be negative, negative numbers properly have no logs; but **operations with negative numbers** can nevertheless be performed by means of logs, by treating all the numbers as positive and taking care to use the proper sign, + or —, in the result.




Logs.
$\bar{1}.8$ 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2
Nos.
A 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 A
B 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 B
C 1 1.5 2 3 4 5 6 7 8 9 1 C
Nos.
D 1 1.5 2 3 4 5 6 7 8 9 1 D
$\bar{1}.9$ 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
Logs.

The Logarithmic Chart and the Slide Rule.

(1) By means of a logarithmic chart or diagram (often miscalled logarithmic cross-section paper) logarithmic operations are performed graphically, and by means of the slide rule mechanically, without reference to the logarithms themselves *. But see †, p 76. Their use greatly facilitates many hydraulic and other engineering computations.

(*) The ratio between the mantissas of the logs of any given numbers being constant for all systems of logs, the ratio between the distances laid off on the chart or slide rule is the same for all systems, and the use of th chart or rule is independent of the system of logs used.

2 The logarithmic chart consists primarily of a square,* on the sides of which the distances marked 1–2, 1–3, etc., are laid off by scale (according to the logs 0.30 103, 0.47 712, etc.) of 2, 3, etc. Ordinary squared or **cross section paper** may of course be used for logarithmic plotting, by plotting on it the *logs* instead of their *Nos.* Lines representing Nos. may be drawn in their proper places as desired.

3 As ordinarily constructed,‡ **the slide rule** consists essentially of four scales, A, B, C, and D, see (17), scales A and D being placed on the rule, while B and C are placed upon the sliding piece, or "slide." As in the logarithmic chart, see (2), the scales are divided logarithmically (see figure), but *marked with the numbers* corresponding to the logs. Scales A and B are equal, as are also scales C and D, but a given length on A or B represents a logarithm twice as great as on C or D. See (4). Hence, each number marked on A is the *square* of the coinciding number marked on D.

4 A single logarithmic scale is usually numbered from 1 to 10, or from 10 to 100; but it may be taken as representing any series embracing the numbers from 10^n to 10^{n+1}; as from 0.1 to 1.0 ($n = -1$); or from 1.0 to 10.0 ($n = 0$); or from 10.0 to 100.0 ($n = 1$); or—etc., etc. Here n and $n + 1$ are the **characteristics** of the corresponding logarithms.

A single scale would therefore serve for all values, from 0 to infinity; but for convenience several contiguous scales are sometimes added, as in the log chart*.

When a line reaches the limit of a square, the next square may be entered* or the same square may be re-entered at a point directly opposite. Thus, in the case of line $x^{2/3}$ ($= \sqrt[3]{x^2}$).

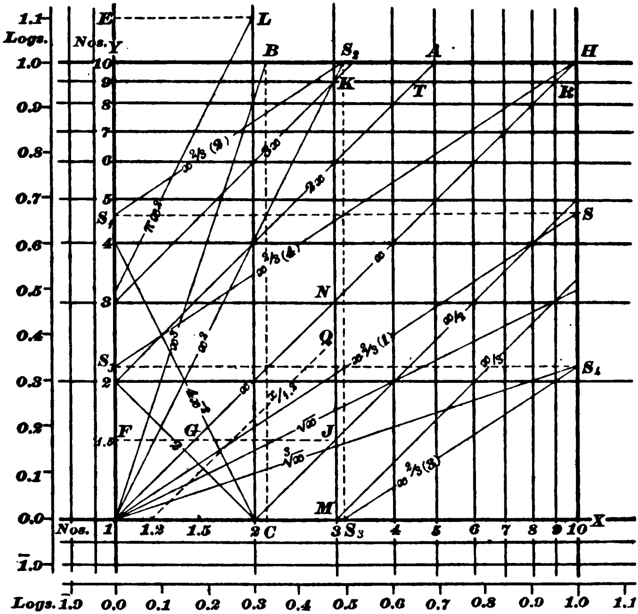
Line marked $x^{2/3}$	between	corresponds to values of	
		x from	$x^{2/3}$ from
(1)	1 and S	1 to 10	1 to 4.64
(2)	S_1 and S_2	10 to 31.62	4.64 to 10
(3)	S_3 and S_4	31.62 to 100	10 to 21.54
(4)	S_5 and H	100 to 1000	21.54 to 100

Note that the numbers, marked on any given scale, must be taken as 10 times the corresponding numbers marked in the next scale preceding, and the characteristics therefore as being greater by 1, and *vice versa.* Thus, in our figure, log 1.5 + log 2 = 1-1.5 + 1-2 = log 3 = distance 1-M. But log 15 + log 20 = (1-1.5 + 1-10) + (1-2 + 1-10), so that the characteristic of the resulting log is greater by 2, and the 3 representing the product of 15 and 20 is really in the second square to the right of that shown. In finding *powers and roots,* remember that multiplying or dividing the *number* by 0.1, 10, 100, etc. (*i. e.,* changing the *characteristic* of its log), changes also the *mantissa* of the log of its *power* or *root.* Thus, $\sqrt[3]{2.7} = 1.39\ldots$, (log = 0.14 379); but $\sqrt[3]{27} = 3$, (log = 0.47 712) and $\sqrt[3]{270} = 6.46\ldots$, (log = 0.81 023). The chart or rule gives *all* such possible roots, and care must be taken to select the proper one. Most operations exceed the limits of one scale, and facility in using either instrument depends largely upon the ability to pass readily and correctly from one scale to another. This ability is best gained by practice, aided by a thorough grasp of the principles involved. Where several successive operations are to be performed, a sliding runner or marker (furnished with each slide rule) is used, in order to avoid error in shifting the slide. Detailed instructions are usually furnished with the slide rule.

(*) A common form of chart has four or more similar squares joined together. See (4). Our figure represents one complete square, with portions of adjoining squares. For actual use, both charts and slide rules are, of course, much more finely subdivided than in our figures, which are given merely to illustrate the principles. Carefully engraved charts are published by Mr. John R. Freeman, Providence, R. I.

(‡) Other forms embodying the same principle are: The "Reaction Scale and General Slide Rule," by W. H. Breithaupt, M. Am. Soc. C. E.; Sexton's Omnimeter or Circular Slide Rule, by Thaddeus Norris; The Goodchild Computing Chart; The Thacher Calculating Machine or Cylindrical Slide
ox Computers, designed for special formulas; and the Pocket
ued by "The Mechanical Engineer," London.

(5) **Multiplication and division.** For example, 2 × 1.5 = ? On 1–X, in the chart, or on C or D, in the slide rule, the distance 1–1.5 represents by scale the logarithm (0.17 609) of 1.5, and 1–2 represents the logarithm (0.30 103) of 2. If now we add these two distances together, by laying off 1-2 from 1.5 on 1-X of the chart, or by placing the slide as in the figure, we obtain the distance 1-3 = .47 712 = the mantissa of log 3 or of log (2 × 1.5).* Conversely, to divide 3 by 2, we graphically or mechanically subtract 1-2 from 1-3.



(6) **In the logarithmic chart,** the scales of both axes, 1-X and 1-Y, being equal, a line 1-H, marked x, bisecting the square and forming an angle of 45° with each axis (tan 45° = 1),† will bisect also the intersections of all *equal* co-ordinates. Thus, points in the line x, immediately over 2, 3, 4, etc., in 1-X, are also opposite 2, 3, 4, etc., respectively, on 1-Y. See (4).

(7) If lines 2-A, 3-K, etc. (marked $2x$, $3x$, etc.), parallel to *and above* 1-H, be drawn through 2, 3, etc., on 1-Y, then points in such lines, immediately over any number, x, in 1-X, will be respectively opposite the

---

(*) In the slide rule, with the slide as shown, *each* number on D is = 1.5 × the coinciding number on C.

(†) In discussing tangents of angles on log chart, we refer to the actual measured distances, as shown on the equally divided scales of *logs* in our figures, and not to the *numbers*, which, for mere convenience, are *marked* on the chart. Thus, in line 1-B, tan C 1 B = $\frac{\text{C B}}{\text{1 C}} = \frac{1.0}{0.33}$, not $\frac{10}{2.15}$.

numbers giving the products $2x$, $3x$, etc., on 1–Y; while similar lines, drawn *below* 1–H and through 2, 3, etc., on 1–X, give values of $\frac{x}{2}$, $\frac{x}{3}$, etc., respectively. If these lines $\frac{x}{2}$, $\frac{x}{3}$, etc., be produced downward, they will cut 1–Y (produced) at 0.5 (= ½), 0.33 . . (= ⅓), etc., respectively.* See (4).

(8) **Powers and roots.** If a line x^2 be drawn through 1, at an angle S_2 1–X, whose tangent, $\frac{S_3-S_2}{1-S_3}$ is 2, it will give values of x^2. Thus, the vertical through 3, on 1–X, cuts the line x^2 opposite 9 (= 3^2) on 1–Y. Similarly, line x^3 (tangent = 3) gives values of x^3; and line $\sqrt[3]{x}$ (tangent = ⅓) gives values of $x^{\frac{1}{3}}$ or $\sqrt[3]{x}$. See (4).

(9) Any equation of the form $y = C x^n$ in which $\log y = \log C + n \log x$, (such as: area of circle = π radius^2), is represented, on a logarithmic chart, by a straight line so drawn that the tangent T of its angle with 1–X is = n, and intersecting 1–Y at that point which represents the value C. Thus, the line marked πx^2, (tangent = 2) is a line of squares, and, being drawn through π (= 3.14. .) on 1–Y, it gives values of πx^2. Thus, for a circle of radius 2, we find, in the line πx^2 over 2, a point L opposite E, or 12.57. . . , the area of such circle.† Conversely, having area = 12.57. . . , we obtain, from the diagram, radius = 2.

(10) If a chart is to be used for solving many equations of a single kind, such as $y = C x^n$, where C is a variable coefficient, and n a constant exponent, parallel lines, forming the proper angle with 1–X, should be permanently ruled across the sheet at short intervals.

(11) For any log, as 1–3 (= log 3), we may substitute its equal, M–N or 3–N, extending to the central diagonal line 1–H, marked x; and then, since, for instance, 1–1.2 = N–Q, 1–3 = N–K, etc., we may *add* any log (as 1–3) by moving *upward* from line x (as from N to K) or to the *right*, and *subtract* any log (as 1–1.2) by moving *downward* (as from N to Q) or to the *left*. This facilitates the performance of a series of operations.

Thus:

To multiply 1.5 by 2 (= 3), by 3 (= 9), and divide by 2 (= 4.5).

F–G = 1–F = log 1.5. Add G–J = 1–2 = log 2; sum = F–J = log 3 = 1–3 = M–N. Add N–K = 1–3 = log 3; sum = M–K = log 9 = 1–9 = 9–R. Subtract R–T = 1–2 = log 2; remainder = 9–T = log 4.5.

For an example of the application of this principle to engineering problems, see "Diagrams for proportioning wooden beams and posts," by Carl S. Fogh, "Engineering News", Sept. 27, 1894.

(12) **Negative exponents.** If x is in the *divisor*, the line will slope in the opposite direction, or downward from left to right. Thus, line 4–2 leaving 1–Y, at 4, and forming, with 1–X, the angle X, 2, 4, with tangent $= \frac{602\ldots}{-301\ldots}$, $= -2$, represents the equation: $y = \frac{4}{x^2} = 4x^{-2}$.

(13) If the lines of products, powers, and roots, Cx, x^n, and $\sqrt[n]{x}$, etc., be drawn at angles whose tangents are less by 1 than those of the angles formed by the corresponding lines in our figure, the results may be read directly from oblique lines drawn parallel to 2–2. Lines (Cx) giving multiples and sub-multiples of the *first* power of x then become *horizontal* lines (tan = 0).‡

(14) **Powers and roots by the slide rule.** Scales C and D being twice as large as scales A and B, these scales, with their ends coinciding, form a table of squares and of square roots. See (3). By moving the slide we solve equations of the forms $y = (Cx)^2$ and $y = Cx^2$. Thus, with the

---

(*) In each of these lines, the product of the two numbers at its ends is = 10. Thus, in line 2–A, 2 × 5 = 10; in 3–K, 3 × 3.33 . . . = 10, etc. The chart thus furnishes a table of **reciprocals.**

(†) Even with full-size charts and slide rules for actual use, accuracy is not to be expected beyond the third or fourth significant figure.

(‡) A chart of this kind, prepared by Major Wm. H. Bixby, U. S. A., after the method of Léon Lalanne, Corps de Ponts et Chaussées, France, is published by Messrs. John Wiley & Sons, New York. Price, 25 cents.

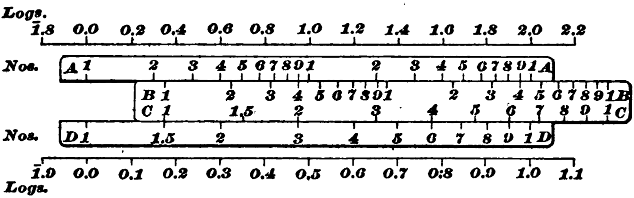
slide as shown, each number on A is = the square of (1.5 × the coinciding number on C); while, with 1 on B opposite 1.5 on A, each number on A is = 1.5 × the square of the coinciding number on C.

(15) Since $x^3 = x^2 \times x$, we find cubes or third powers by placing the slide with 1 on B opposite x^2 on A (*i. e.*, opposite x on D), see **(3)**, and reading x^3 from A opposite x on B. Thus, $1.5^3 = ?$. Place 1 on B opposite 1.5 on D; *i, e.*, opposite 1.5^2 (= 2.25) on A. Then, on A, opposite 1.5 on B, find $3.375 = 1.5^3$. Or, turn the slide end for end. Place 1.5 on B opposite 1.5 on D, *i. e.*, opposite $1.5^2 = 2.25$ on A. Then, adding log 1.5 (on B) to log 2.25 on A, we find 3.375 (= 1.5^3) on A opposite 1 on B.

(16) Conversely, to find $\sqrt[3]{x}$, we shift the slide (in its normal position) until we find, on B, opposite x on A, the same number as we have on D opposite 1 on C, and this number will be = $\sqrt[3]{x}$. Or, turn the slide end for end,* place 1 on C opposite x on A, and find, on B, a number which coincides with its equal on D. This number is = $\sqrt[3]{x}$. See also **(17)**, **(18)**.

(17) On the back of the slide is usually placed a scale of logs (see scale shown below the rule in figure) and two scales of angles, marked "S" and "T" respectively, for finding sines of angles greater than 0° 34′ . . . ″, and tangents of angles between 5° 42′ . . . ″ and 45°.

(18) Placing 1 on C opposite any number x on D (with slide in its normal position), log x is read from the scale of logs by means of an index on the back of the rule. The logs may be used in finding powers and roots.


Logs.
1̄.8 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2
Nos.
A 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 A
B 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 B
C 1 1.5 2 3 4 5 6 7 8 9 1 C
Nos.
D 1 1.5 2 3 4 5 6 7 8 9 1 D
1̄.9 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
Logs.

(19) To find the **sine or tangent** of an angle a; bring a, on scale S or T, as the case may be, opposite the index on back, and read the natural (not logarithmic) sine or tangent opposite 10 at the end of A or D: sines on B, and tangents on C. Or, invert the slide, placing S under A, and T over D, with the ends of the scales coinciding. Then the numbers on A and D are the sines and tangents, respectively, of the angles on S and T.

Caution. Sines of angles less than 5° 45′ . . . ″ are less than 0.1.
" " " " " 90° " " " 1.0.
Tangents " " betw. 5° 42′ . . . ″ and 45° are betw. 0.1 and 1.0.

(20) On the back of the rule is usually printed a table of ratios of numbers in common use, for convenience in operating with the slide rule. Thus:

$$\frac{\text{diameter}}{\text{circumference}} = \frac{113}{355}; \quad \frac{\text{U. S. gallons}}{\text{pounds}} = \frac{3}{25} \text{ (in a given quantity of water).}$$

(21) Soaping the edges of the slide and the groove in which it runs, will often cure sticking, which is apt to be very annoying. If the slide is too loose, the groove may be deepened, and small springs, cut from narrow steel tape, inserted between it and the edge of the slide.

(*) With the slide thus reversed, and with the ends of the scales coinciding, the numbers on A and B are **reciprocals** (page 52), as are also those on C and D.

Common or Briggs Logarithms, Base = 10.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
		00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14613	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282	126
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62118	62221	104
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741	78
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67

Common or Briggs Logarithms. Base = 10.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331	48
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	46
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181	46
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	45
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	45
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	44
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	44

For extended table of logarithms see pages 80–91. The table above, being given on two opposite pages, avoids the necessity of turning leaves. It contains no error as great as 1 in the final figure. The proportional parts, in the last column, give merely the *average* difference for each line. Hence, when dealing with small numbers, and using 5-place logs, it is better to find differences by subtraction; but where a two-page table is used, interpolation is often unnecessary. Indeed, the first four, or even the first three, places of the mantissas here given will often be found sufficient. If the first number dropped is 5 or more, increase by 1 the last figure retained. Thus, for log 660, mantissa = 81954, or 8195, or 820.

Multiplication. $\text{Log } a\,b = \log a + \log b.$

Division. $\text{Log } \frac{a}{b} = \log a - \log b.$

Involution (Powers). $\text{Log } a^n = n. \log a.$

Evolution (Roots). $\text{Log} \sqrt[n]{a} = \frac{\log a}{n}.$

Characteristics.

Log 2870 = 3.45788
" 287 = 2.45788
" 28.7 = 1.45788
" 2.87 = 0.45788

Log 0.287 = 0.45788 — 1 = $\bar{1}.45788$
" 0.0287 = 0.45788 — 2 = $\bar{2}.45788$
" 0.00287 = 0.45788 — 3 = $\bar{3}.45788$
" 0.000287 = 0.45788 — 4 = $\bar{4}.45788$

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
1000	00000	43	**1050**	02119	41	**1100**	04139	40	**1150**	06070	38	**1200**	07918	36
01	043	44	51	160	42	01	—179	39	51	—108	37	01	954	36
02	—087	43	52	—202	41	02	218	40	52	145	38	02	990	37
03	130	43	53	—243	41	03	—258	39	53	—183	38	03	08027	36
04	173	44	54	284	41	04	—297	39	54	—221	37	04	—063	36
05	—217	43	55	325	41	05	336	40	55	258	38	05	—099	36
06	—260	43	56	366	41	06	—376	39	56	—296	37	06	—135	36
07	—303	43	57	407	42	07	—415	39	57	333	38	07	—171	36
08	346	43	58	—449	41	08	—454	39	58	—371	37	08	—207	36
09	389	43	59	—490	41	09	493	39	59	408	38	09	—243	36
1010	432	43	**1060**	—531	41	**1110**	532	39	**1160**	—446	37	**1210**	—279	35
11	475	43	61	—572	40	11	571	39	61	483	38	11	314	36
12	518	43	62	612	41	12	610	40	62	—521	37	12	350	36
13	—561	43	63	653	41	13	—650	39	63	—558	37	13	386	36
14	—604	43	64	694	41	14	—689	38	64	595	38	14	—422	36
15	—647	42	65	—735	41	15	727	39	65	—633	37	15	—458	35
16	689	43	66	—776	40	16	766	39	66	—670	37	16	493	36
17	732	43	67	816	41	17	805	39	67	707	37	17	529	36
18	—775	42	68	857	41	18	844	39	68	744	37	18	—565	35
19	817	43	69	—898	41	19	883	39	69	781	38	19	600	36
1020	860	43	**1070**	938	41	**1120**	—922	39	**1170**	—819	37	**1220**	—636	36
21	—903	42	71	—979	40	21	—961	38	71	—856	37	21	—672	35
22	945	43	72	03019	41	22	999	39	72	—893	37	22	707	36
23	—988	42	73	—060	40	23	05038	39	73	—930	37	23	—743	35
24	01030	42	74	100	41	24	—077	38	74	—967	37	24	778	36
25	072	43	75	—141	40	25	115	39	75	07004	37	25	—814	35
26	—115	42	76	181	41	26	—154	38	76	—041	37	26	849	35
27	157	42	77	—222	40	27	192	39	77	—078	37	27	884	36
28	199	43	78	—262	40	28	—231	38	78	—115	36	28	—920	35
29	—242	42	79	302	40	29	269	39	79	151	37	29	955	36
1030	—284	42	**1080**	342	41	**1130**	—308	38	**1180**	188	37	**1230**	—991	35
31	—326	42	81	—383	40	31	346	39	81	—225	37	31	09026	35
32	—368	42	82	—423	40	32	—385	38	82	—262	36	32	061	35
33	410	42	83	—463	40	33	—423	38	83	298	37	33	096	36
34	452	42	84	—503	40	34	461	39	84	335	37	34	—182	35
35	494	42	85	—543	40	35	—500	38	85	—372	36	35	—167	35
36	—536	42	86	—583	40	36	—538	38	86	408	37	36	—202	35
37	—578	42	87	—623	40	37	576	38	87	445	37	37	—237	35
38	—620	42	88	—663	40	38	614	38	88	—482	36	38	272	35
39	—662	41	89	—703	40	39	652	38	89	518	37	39	307	35
1040	703	42	**1090**	—743	39	**1140**	690	39	**1190**	—555	36	**1240**	342	35
41	745	42	91	782	40	41	—729	38	91	591	37	41	377	35
42	—787	41	92	822	40	42	—767	38	92	—628	36	42	412	35
43	828	42	93	862	40	43	—805	38	93	664	36	43	447	35
44	870	42	94	—902	39	44	—843	38	94	700	37	44	482	35
45	—912	41	95	941	40	45	—881	37	95	—737	36	45	—517	35
46	953	42	96	981	40	46	918	38	96	773	36	46	—552	35
47	—995	41	97	04021	39	47	956	38	97	809	37	47	—587	34
48	02036	42	98	060	40	48	994	38	98	—846	36	48	621	35
49	—078	41	99	—100	39	49	06032	38	99	—882	36	49	656	35

Example:
To find Log. 11826:
Log. 11830 = 07298
Dif. = 10 **36**
Log. 11820 = 07262
11826 — 11820 = **6**
Dif. for **6** under **36**
= 22
Log. 11826 =
07262 + 22 = 07284

	44	43	42	41	40	39	38	37	36	35	34	
1	4	4	4	4	4	4	4	4	4	4	3	**1**
2	9	9	8	8	8	8	8	7	7	7	7	**2**
3	13	13	13	12	12	12	11	11	11	11	10	**3**
4	18	17	17	16	16	16	15	15	14	14	14	**4**
5	22	22	21	21	20	20	19	19	18	18	17	**5**
6	26	26	25	25	24	23	23	22	22	21	20	**6**
7	31	30	29	29	28	27	27	26	25	25	24	**7**
8	35	34	34	33	32	31	30	30	29	28	27	**8**
9	40	39	38	37	36	35	34	33	32	32	31	**9**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
1250	09691	35	**1300**	11394	34	**1350**	13033	33	**1400**	14613	31	**1450**	16137	30
51	—726	34	01	—428	33	51	—066	32	01	—644	31	51	—167	30
52	760	35	02	461	33	52	—098	32	02	—675	31	52	—197	30
53	795	35	03	494	34	53	—130	32	03	—706	31	53	—227	29
54	—830	34	04	—528	33	54	—162	32	04	—737	31	54	256	30
55	864	35	05	561	33	55	—194	32	05	—768	31	55	286	30
56	—899	35	06	594	34	56	—226	32	06	—799	30	56	316	30
57	—934	34	07	—628	33	57	—258	32	07	829	31	57	—346	30
58	968	35	08	—661	33	58	—290	32	08	860	31	58	—376	30
59	10003	34	09	—694	33	59	—322	32	09	891	31	59	—406	29
1260	037	35	**1310**	727	33	**1360**	—354	32	**1410**	—922	31	**1460**	435	30
61	—072	34	11	760	33	61	—386	32	11	—953	30	61	465	30
62	—106	34	12	793	33	62	—418	32	12	983	31	62	—495	29
63	140	35	13	826	34	63	—450	31	13	15014	31	63	524	30
64	—175	34	14	—860	33	64	481	32	14	—045	31	64	554	30
65	209	34	15	—893	33	65	513	32	15	—076	30	65	—584	29
66	243	35	16	—926	33	66	545	32	16	106	31	66	613	30
67	—278	34	17	—959	33	67	—577	32	17	—137	31	67	643	30
68	—312	34	18	—992	32	68	—609	31	18	—168	30	68	—673	29
69	346	34	19	12024	33	69	640	32	19	198	31	69	702	30
1270	380	35	**1320**	057	33	**1370**	672	32	**1420**	—229	30	**1470**	—732	29
71	—415	34	21	090	33	71	—704	31	21	259	31	71	761	30
72	—449	34	22	123	33	72	735	32	22	—290	30	72	—791	29
73	—483	34	23	—156	33	73	767	32	23	320	31	73	820	30
74	—517	34	24	—189	33	74	—799	31	24	—351	30	74	—850	29
75	551	34	25	—222	32	75	830	32	25	381	31	75	879	30
76	585	34	26	254	33	76	—862	31	26	—412	30	76	—909	29
77	619	34	27	287	33	77	893	32	27	442	31	77	938	29
78	653	34	28	—320	32	78	—925	31	28	—473	30	78	967	30
79	687	34	29	352	33	79	956	32	29	503	31	79	—997	29
1280	—721	34	**1330**	385	33	**1380**	—988	31	**1430**	—534	30	**1480**	17026	30
81	—755	34	31	—418	32	81	14019	32	31	—564	30	81	—056	29
82	—789	34	32	450	33	82	—051	31	32	594	31	82	—085	29
83	—823	34	33	483	33	83	082	32	33	—625	30	83	114	29
84	—857	33	34	—516	32	84	—114	31	34	—655	30	84	143	30
85	890	34	35	548	33	85	—145	31	35	685	30	85	—173	29
86	924	34	36	—581	32	86	176	32	36	715	31	86	—202	29
87	—958	34	37	613	33	87	—208	31	37	—746	30	87	231	29
88	—992	33	38	—646	32	88	—239	31	38	—776	30	88	260	29
89	11025	34	39	678	32	89	270	31	39	806	30	89	289	30
1290	—059	34	**1340**	710	33	**1390**	301	32	**1440**	836	30	**1490**	—319	29
91	—093	33	41	—743	32	91	—333	31	41	866	31	91	—348	29
92	126	34	42	775	33	92	—364	31	42	—897	30	92	—377	29
93	—160	33	43	—808	32	93	395	31	43	—927	30	93	—406	29
94	193	34	44	—840	32	94	426	31	44	—957	30	94	435	29
95	—227	34	45	872	33	95	457	32	45	—987	30	95	464	29
96	—261	33	46	—905	32	96	—489	31	46	16017	30	96	493	29
97	—294	33	47	—937	32	97	—520	31	47	—047	30	97	522	29
98	327	34	48	—969	32	98	—551	31	48	—077	30	98	551	29
99	—361	33	49	13001	32	99	—582	31	49	—107	30	99	580	29

Example:
To find Log. 12605:
Log. 12610 = 10072
Dif. = 10 **35**
Log. 12600 = 10037
12605 — 12600 = **5**
Dif. for **5** under **35**
= 18
Log. 12605 =
10037 + 18 = 10055

	35	34	33	32	31	30	29	
1	4	3	3	3	3	3	3	**1**
2	7	7	7	6	6	6	6	**2**
3	11	10	10	10	9	9	9	**3**
4	14	14	13	13	12	12	12	**4**
5	18	17	17	16	16	15	15	**5**
6	21	20	20	19	19	18	17	**6**
7	25	24	23	22	22	21	20	**7**
8	28	27	26	26	25	24	23	**8**
9	32	31	30	29	28	27	26	**9**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 1366=1354507
" 1367=1357685

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
1500	17609	29	**1550**	19033	28	**1600**	20412	— 27	**1650**	21748	27	**1700**	23045	— 25
01	638	29	51	061	28	01	439	27	51	—775	26	01	070	26
02	—667	29	52	089	28	02	466	27	52	801	26	02	—096	25
03	—696	29	53	117	28	03	493	27	53	827	27	03	121	26
04	—725	29	54	145	28	04	520	28	54	—854	26	04	—147	25
05	—754	28	55	173	28	05	—548	27	55	—880	26	05	172	26
06	782	29	56	—201	28	06	—575	27	56	906	26	06	—198	25
07	811	29	57	—229	28	07	—602	27	57	932	26	07	223	26
08	840	29	58	—257	28	08	—629	27	58	958	27	08	—249	25
09	—869	29	59	—285	27	09	—656	27	59	—985	26	09	274	26
1510	—898	28	**1560**	312	28	**1610**	—683	27	**1660**	22011	— 26	**1710**	—300	25
11	926	29	61	340	28	11	—710	27	61	—037	26	11	325	25
12	955	29	62	368	28	12	—737	26	62	063	26	12	350	26
13	—984	29 —	63	—396	28	13	763	27	63	089	26	13	—376	25
14	18013	28	64	—424	27	14	790	27	64	115	26	14	401	25
15	041	29	65	451	28	15	817	27	65	141	26	15	426	26
16	—070	29	66	479	28	16	844	27	66	167	27	16	—452	25
17	—099	28	67	—507	28	17	871	27	67	—194	26	17	477	25
18	127	29	68	—535	27	18	—898	27	68	—220	26	18	502	26
19	—156	28	69	562	28	19	—925	27	69	—246	26	19	—528	25
1520	184	29	**1570**	—590	28	**1620**	—952	26	**1670**	—272	26	**1720**	—553	25
21	—213	28	71	—618	27	21	978	27	71	—298	26	21	578	25
22	241	29	72	645	28	22	21005	27	72	—324	26	22	603	26
23	—270	28	73	—673	27	23	—032	27	73	—350	26	23	—629	25
24	298	29	74	700	28	24	—059	26	74	—376	25	24	—654	25
25	—327	28	75	728	28	25	085	27	75	401	26	25	—679	25
26	355	29	76	—756	27	26	112	27	76	427	26	26	704	25
27	—384	28	77	783	28	27	—139	26	77	453	26	27	729	25
28	412	29	78	—811	27	28	165	27	78	479	26	28	754	25
29	—441	28	79	838	28	29	192	27	79	505	26	29	779	26
1530	469	29	**1580**	—866	27	**1630**	—219	26	**1680**	—531	26	**1730**	—805	25
31	—498	28	81	893	28	31	245	27	81	—557	26	31	—830	25
32	—526	28	82	—921	27	32	272	27	82	—583	25	32	—855	25
33	554	29	83	948	28	33	—299	26	83	608	26	33	—880	25
34	—583	28	84	—976	27 —	34	325	27	84	634	26	34	—905	25
35	—611	28	85	20003	27	35	—352	26	85	—660	26	35	—930	25
36	639	28	86	030	28	36	378	27	86	—686	26	36	—955	25
37	667	29	87	—058	27	37	—405	26	87	—712	25	37	—980	25 —
38	—696	28	88	085	27	38	431	27	88	737	26	38	24005	25
39	—724	28	89	112	28	39	—458	26	89	—763	26	39	—030	25
1540	752	28	**1590**	—140	27	**1640**	484	27	**1690**	—789	25	**1740**	—055	25
41	780	28	91	167	27	41	—511	26	91	814	26	41	—080	25
42	808	29	92	194	28	42	537	27	92	840	26	42	—105	25
43	—837	28	93	—222	27	43	—564	26	93	—866	25	43	—130	25
44	—865	28	94	—249	27	44	590	27	94	891	26	44	—155	25
45	—893	28	95	276	27	45	—617	26	95	—917	26	45	—180	24
46	—921	28	96	303	27	46	—643	26	96	—943	25	46	204	25
47	949	28	97	330	28	47	669	27	97	968	26	47	229	25
48	977	28	98	—358	27	48	—696	26	98	—994	25	48	254	25
49	19005	28	99	—385	27	49	722	26	99	23019	26	49	—279	25

Example:

To find Log. 15414:
Log. 15420 = 18808
Dif. = 10 **28**
Log. 15410 = 18780
15414 — 15410 = **4**
Dif. for **4** under **28** = 11
Log. 15414 =
18780 + 11 = 18791

	29	28	27	26	25	24	
1	3	3	3	3	3	2	**1**
2	6	6	5	5	5	5	**2**
3	9	8	8	8	8	7	**3**
4	12	11	11	10	10	10	**4**
5	15	14	14	13	13	12	**5**
6	17	17	16	16	15	14	**6**
7	20	20	19	18	18	17	**7**
8	23	22	22	21	20	19	**8**
9	26	25	24	23	23	22	**9**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 1562=1936810
" 1563=1939590

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
1750	24304	49	**1850**	26717	47	**1950**	29003	45	**2050**	31175	43	**2150**	33244	40
52	353	50	52	764	47	52	—048	44	52	—218	42	52	284	41
54	—403	49	54	—811	47	54	092	45	54	260	42	54	—325	40
56	452	50	56	—858	47	56	—137	44	56	302	43	56	—365	40
58	—502	49	58	—905	46	58	181	45	58	—345	42	58	405	40
1760	551	50	**1860**	951	47	**1960**	—226	44	**2060**	—387	42	**2160**	445	41
62	—601	49	62	—998	47	62	—270	44	62	—429	42	62	—486	40
64	—650	49	64	27045	46	64	314	44	64	—471	42	64	—526	40
66	699	49	66	091	47	66	358	45	66	513	42	66	—566	40
68	748	49	68	—138	46	68	—403	44	68	555	42	68	—606	40
1770	797	49	**1870**	184	47	**1970**	—447	44	**2070**	597	42	**2170**	—646	40
72	846	49	72	—231	46	72	—491	44	72	—639	42	72	—686	40
74	895	49	74	—277	46	74	—535	44	74	—681	42	74	—726	40
76	944	49	76	323	47	76	—579	44	76	—723	42	76	—766	40
78	993	49	78	—370	46	78	—623	44	78	—765	41	78	—806	40
1780	25042	49	**1880**	—416	46	**1980**	—667	43	**2080**	806	42	**2180**	—846	39
82	—091	48	82	—462	46	82	710	44	82	848	42	82	885	40
84	139	49	84	508	46	84	754	44	84	—890	41	84	925	40
86	188	49	86	554	46	86	—798	44	86	931	42	86	965	40
88	—237	48	88	600	46	88	—842	43	88	973	42	88	34005	39
1790	285	49	**1890**	646	46	**1990**	885	44	**2090**	32015	41	**2190**	044	40
92	—334	48	92	692	46	92	—929	44	92	056	42	92	084	40
94	382	49	94	—738	46	94	—973	43	94	—098	41	94	—124	39
96	—431	48	96	—784	46	96	30016	44	96	139	42	96	163	40
98	—479	48	98	—830	45	98	—060	43	98	—181	41	98	—203	39
1800	527	48	**1900**	875	46	**2000**	—103	43	**2100**	—222	41	**2200**	242	40
02	575	49	02	921	46	02	146	44	02	263	42	02	—282	39
04	—624	48	04	—967	45	04	—190	43	04	—305	41	04	321	40
06	—672	48	06	28012	46	06	233	43	06	—346	41	06	—361	39
08	—720	48	08	—058	45	08	276	44	08	387	41	08	—400	39
1810	—768	48	**1910**	103	46	**2010**	—320	43	**2110**	428	41	**2210**	439	40
12	—816	48	12	—149	45	12	—363	43	12	469	41	12	—479	39
14	—864	48	14	194	46	14	—406	43	14	510	42	14	—518	39
16	—912	47	16	—240	45	16	449	43	16	—552	41	16	—557	39
18	959	48	18	—285	45	18	492	43	18	—593	41	18	596	39
1820	26007	48	**1920**	330	45	**2020**	535	43	**2120**	—634	41	**2220**	635	39
22	—055	47	22	375	46	22	578	43	22	—675	40	22	674	39
24	102	48	24	—421	45	24	621	43	24	715	41	24	713	40
26	150	48	26	—466	45	26	—664	43	26	756	41	26	—753	39
28	—198	47	28	—511	45	28	—707	43	28	797	41	28	—792	38
1830	245	48	**1930**	—556	45	**2030**	—750	42	**2130**	—838	41	**2230**	830	39
32	—293	47	32	—601	45	32	792	43	32	—879	40	32	869	39
34	—340	47	34	—646	45	34	835	43	34	919	41	34	908	39
36	387	48	36	—691	44	36	—878	42	36	960	41	36	947	39
38	—435	47	38	735	45	38	920	43	38	33001	40	38	986	39
1840	—482	47	**1940**	780	45	**2040**	963	43	**2140**	041	41	**2240**	35025	39
42	—529	47	42	—825	45	42	31006	42	42	—082	40	42	—064	38
44	576	47	44	—870	44	44	048	43	44	122	41	44	102	39
46	623	47	46	914	45	46	—091	42	46	—163	40	46	—141	39
48	670	47	48	—959	44	48	—133	42	48	203	41	48	—180	38

To find Log. 18117:
Log. 18120 = 25816
Dif. 20 **48**
Log. 18100 = 25768
18117 — 18100 = **17**
Under **48**
Dif. for 10 = 24
" " 7 = 17
" " **17** = 41
Log. 18117 =
25768 + 41 = 25809.

	50	**49**	**48**	**47**	**46**	**45**	**44**	**43**	**42**	**41**	**40**	**39**	**38**	
1	3	2	2	2	2	2	2	2	2	2	2	2	2	**1**
2	5	5	5	5	5	5	4	4	4	4	4	4	4	**2**
3	8	7	7	7	7	7	7	6	6	6	6	6	6	**3**
4	10	10	10	9	9	9	9	9	8	8	8	8	8	**4**
5	13	12	12	12	12	11	11	11	11	10	10	10	10	**5**
6	15	15	14	14	14	14	13	13	13	12	12	12	11	**6**
7	18	17	17	16	16	16	15	15	15	14	14	14	13	**7**
8	20	20	19	19	18	18	18	17	17	16	16	16	15	**8**
9	23	22	22	21	21	20	20	19	19	18	18	18	17	**9**
10	25	25	24	24	23	23	22	22	21	21	20	20	19	**10**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
2250	35218	39	**2350**	37107	37	**2450**	38917	35	**2550**	40654	34	**2650**	42325	32
52	—257	38	52	—144	37	52	952	35	52	688	34	52	357	33
54	295	39	54	—181	37	54	987	36	54	722	34	54	390	33
56	—334	38	56	—218	36	56	39023	35	56	756	34	56	—423	32
58	372	39	58	254	37	58	058	36	58	790	34	58	455	33
2260	—411	38	**2360**	291	37	**2460**	—094	35	**2560**	—824	34	**2660**	488	33
62	449	39	62	—328	37	62	—129	35	62	—858	34	62	—521	32
64	—488	38	64	—365	36	64	164	35	64	—892	34	64	553	33
66	—526	38	66	401	37	66	199	36	66	—926	34	66	586	33
68	564	39	68	438	37	68	—235	35	68	—960	33	68	—619	32
2270	—603	38	**2370**	—475	36	**2470**	—270	35	**2570**	993	34	**2670**	651	33
72	—641	38	72	511	37	72	—305	35	72	41027	34	72	—684	32
74	679	38	74	548	37	74	—340	35	74	—061	34	74	716	33
76	717	38	76	—585	36	76	375	35	76	—095	33	76	—749	32
78	755	38	78	621	37	78	410	35	78	128	34	78	781	32
2280	793	39	**2380**	—658	36	**2480**	445	35	**2580**	—162	34	**2680**	813	33
82	—832	38	82	694	37	82	480	35	82	—196	33	82	—846	32
84	—870	38	84	—731	36	84	515	35	84	229	34	84	878	33
86	—908	38	86	767	36	86	550	35	86	—263	33	86	—911	32
88	—946	38	88	803	37	88	585	35	88	296	34	88	—943	32
2290	—984	37	**2390**	—840	36	**2490**	—620	35	**2590**	—330	33	**2690**	975	33
92	36021	38	92	876	36	92	—655	35	92	363	34	92	43008	32
94	059	38	94	912	37	94	—690	34	94	—397	33	94	—040	32
96	097	38	96	—949	36	96	724	35	96	430	34	96	—072	32
98	135	38	98	—985	36	98	759	35	98	—464	33	98	104	32
2300	—173	38	**2400**	38021	36	**2500**	794	35	**2600**	497	34	**2700**	136	33
02	—211	37	02	057	36	02	—829	34	02	—531	33	02	—169	32
04	248	38	04	093	37	04	863	35	04	564	33	04	—201	32
06	—286	38	06	—130	36	06	898	35	06	597	34	06	—233	32
08	—324	37	08	—166	36	08	—933	34	08	—631	33	08	—265	32
2310	361	38	**2410**	—202	36	**2510**	967	35	**2610**	664	33	**2710**	—297	32
12	—399	37	12	—238	36	12	40002	35	12	697	34	12	—329	32
14	436	38	14	—274	36	14	—037	34	14	—731	33	14	—361	32
16	—474	37	16	—310	36	16	071	35	16	—764	33	16	—393	32
18	511	38	18	—346	36	18	—106	34	18	—797	33	18	—425	32
2320	—549	37	**2420**	—382	35	**2520**	140	35	**2620**	830	33	**2720**	—457	32
22	586	38	22	417	36	22	—175	34	22	863	33	22	—489	32
24	—624	37	24	453	36	24	—209	34	24	896	33	24	—521	32
26	—661	37	26	489	36	26	243	35	26	929	34	26	—553	31
28	698	38	28	—525	36	28	—278	34	28	—963	33	28	584	32
2330	—736	37	**2430**	—561	35	**2530**	312	34	**2630**	—996	33	**2730**	616	32
32	—773	37	32	596	36	32	346	35	32	42029	33	32	648	32
34	810	37	34	632	36	34	—381	34	34	—062	33	34	—680	32
36	847	37	36	—668	35	36	—415	34	36	—095	32	36	—712	31
38	884	38	38	703	36	38	449	34	38	127	33	38	743	32
2340	—922	37	**2440**	—739	36	**2540**	483	35	**2640**	160	33	**2740**	775	32
42	—959	37	42	—775	35	42	—518	34	42	193	33	42	—807	31
44	—996	37	44	810	36	44	—552	34	44	226	33	44	838	32
46	37033	37	46	—846	35	46	—586	34	46	—259	33	46	870	32
48	—070	37	48	881	36	48	—620	34	48	—292	33	48	—902	31

To find Log. 23335:
Log. 23340 = 36810
Dif. 20 **37**
Log. 23320 = 36773
23335 — 23320 = **15**
Under **37**
Dif. for 10 = 19
“ “ 5 = 9
“ “ **15** = 28
og. 23335 =
36773 + 28 = 36801.

	39	38	37	36	35	34	33	32	31	
1	2	2	2	2	2	2	2	2	2	**1**
2	4	4	4	4	4	3	3	3	3	**2**
3	6	6	6	5	5	5	5	5	5	**3**
4	8	8	7	7	7	7	7	6	6	**4**
5	10	10	9	9	9	9	8	8	8	**5**
6	12	11	11	11	11	10	10	10	9	**6**
7	14	13	13	13	12	12	12	11	11	**7**
8	16	15	15	14	14	14	13	13	12	**8**
9	18	17	17	16	16	15	15	14	14	**9**
10	20	19	19	18	18	17	17	16	16	**10**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
2750	43933	32	**2850**	45484	31	**2950**	46982	30	**3050**	48430	— 28	**3150**	49831	28
52	—965	31	52	—515	30	52	47012	— 29	52	458	29	52	—859	27
54	996	32	54	545	31	54	041	29	54	—487	28	54	886	28
56	44028	— 31	56	—576	30	56	070	30	56	515	29	56	—914	27
58	059	32	58	606	31	58	—100	29	58	—544	28	58	941	28
2760	—091	31	**2860**	—637	30	**2960**	129	30	**3060**	572	29	**3160**	—969	27
62	122	32	62	—667	30	62	—159	29	62	—601	28	62	996	28
64	—154	31	64	697	31	64	—188	29	64	—629	28	64	50024	— 27
66	185	32	66	—728	30	66	217	29	66	657	29	66	051	28
68	—217	31	68	—758	30	68	246	30	68	—686	28	68	—079	27
2770	—248	31	**2870**	788	30	**2970**	—276	29	**3070**	—714	28	**3170**	—106	27
72	279	32	72	818	31	72	--305	29	72	742	28	72	133	28
74	—311	31	74	—849	30	74	334	29	74	770	29	74	—161	27
76	—342	31	76	—879	30	76	363	29	76	—799	28	76	188	27
78	373	31	78	909	30	78	392	30	78	—827	28	78	215	28
2780	404	32	**2880**	939	30	**2980**	—422	29	**3080**	855	28	**3180**	—243	27
82	—436	31	82	969	31	82	—451	29	82	883	28	82	270	27
84	—467	31	84	46000	— 30	84	—480	29	84	911	29	84	297	28
86	498	31	86	—030	30	86	—509	29	86	—940	28	86	—325	27
88	529	31	88	—060	30	88	538	29	88	—968	28	88	—352	27
2790	560	32	**2890**	—090	30	**2990**	567	29	**3090**	—996	28	**3190**	379	27
92	—592	31	92	—120	30	92	596	29	92	49024	— 28	92	406	27
94	—623	31	94	—150	30	94	625	29	94	052	28	94	433	28
96	—654	31	96	—180	30	96	654	29	96	080	28	96	—461	27
98	— 685	31	98	—210	30	98	683	29	98	108	28	98	—488	27
2800	—716	31	**2900**	—240	30	**3000**	712	29	**3100**	136	28	**3200**	—515	27
02	—747	31	02	—270	30	02	741	29	02	164	28	02	542	27
04	—778	31	04	—300	30	04	—770	29	04	192	28	04	569	27
06	—809	31	06	—330	29	06	—799	29	06	220	28	06	596	27
08	—840	31	08	359	30	08	—828	29	08	248	28	08	623	28
2810	—871	31	**2910**	389	30	**3010**	—857	28	**3110**	276	28	**3210**	—651	27
12	—902	30	12	419	30	12	885	29	12	—304	28	12	—678	27
14	932	31	14	—449	30	14	914	29	14	—332	28	14	—705	27
16	963	31	16	—479	30	16	943	29	16	—360	28	16	—732	27
18	994	31	18	—509	29	18	—972	29	18	—388	27	18	—759	27
2820	45025	— 31	**2920**	538	30	**3020**	48001	— 28	**3120**	415	28	**3220**	—786	27
22	—056	30	22	568	30	22	029	29	22	443	28	22	—813	27
24	086	31	24	—598	29	24	058	29	24	471	28	24	—840	26
26	117	31	26	627	30	26	—087	29	26	—499	28	26	866	27
28	—148	31	28	657	30	28	—116	28	28	—527	27	28	893	27
2830	—179	30	**2930**	—687	29	**3030**	144	29	**3130**	554	28	**3230**	920	27
32	209	31	32	716	30	32	—173	29	32	582	28	32	947	27
34	—240	31	34	746	30	34	—202	28	34	—610	28	34	974	27
36	—271	30	36	—776	29	36	230	29	36	—638	27	36	51001	— 27
38	301	31	38	805	30	38	—259	28	38	665	28	38	—028	27
2840	—332	30	**2940**	—835	29	**3040**	287	29	**3140**	—693	28	**3240**	—055	26
42	362	31	42	864	30	42	— 316	28	42	—721	27	42	081	27
44	—393	30	44	—894	29	44	344	29	44	748	28	44	108	27
46	423	31	46	923	30	46	—373	28	46	—776	27	46	—135	27
48	—454	30	48	—953	29	48	401	29	48	803	28	48	—162	26

To find Log. 29019:
Log. 29020 = 46270
Dif. 20 **30**
Log. 29000 = 46240
29019—29000 = **19**
Under **30**
Dif. for 10 = 15
" " 9 = 14
" " **19** = 29
Log. 29019 =
46240 + 29 = 46269.

	32	**31**	**30**	**29**	**28**	**27**	**26**	
1	2	2	2	1	1	1	1	**1**
2	3	3	3	3	3	3	3	**2**
3	5	5	5	4	4	4	4	**3**
4	6	6	6	6	6	5	5	**4**
5	8	8	8	7	7	7	7	**5**
6	10	9	9	9	8	8	8	**6**
7	11	11	11	10	10	9	9	**7**
8	13	12	12	12	11	11	10	**8**
9	14	14	14	13	13	12	12	**9**
10	16	16	15	15	14	14	13	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 3128 = 4952667
" 3130 = 4955443

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
3250	51188	27	**3350**	52504	26	**3450**	53782	25	**3550**	55023	24	**3650**	56229	24
52	215	27	52	530	26	52	807	25	52	047	25	52	253	24
54	—242	26	54	556	26	54	832	25	54	—072	24	54	—277	24
56	268	27	56	582	26	56	857	25	56	096	25	56	—301	23
58	295	27	58	608	26	58	882	26	58	—121	24	58	324	24
3260	—322	26	**3360**	—634	26	**3460**	—908	25	**3560**	—145	24	**3660**	348	24
62	348	27	62	—660	26	62	—933	25	62	169	25	62	—372	24
64	375	27	64	—686	25	64	—958	25	64	—194	24	64	—396	23
66	—402	26	66	711	26	66	—983	25	66	218	24	66	419	24
68	428	27	68	737	26	68	54008	25	68	242	25	68	—443	24
3270	—455	26	**3370**	—763	26	**3470**	—033	25	**3570**	—267	24	**3670**	—467	23
72	481	27	72	—789	26	72	—058	25	72	291	24	72	490	24
74	—508	26	74	—815	25	74	—083	25	74	315	25	74	—514	24
76	534	27	76	840	26	76	—108	25	76	—340	24	76	—538	23
78	—561	26	78	—866	26	78	—133	25	78	364	24	78	561	24
3280	587	27	**3380**	—892	25	**3480**	—158	25	**3580**	388	25	**3680**	—585	23
82	—614	26	82	917	26	82	—183	25	82	—413	24	82	608	24
84	640	27	84	943	26	84	—208	25	84	—437	24	84	—632	24
86	—667	26	86	—969	25	86	—233	25	86	461	24	86	—656	23
88	693	27	88	994	26	88	—258	25	88	485	24	88	679	24
3290	—720	26	**3390**	53020	26	**3490**	—283	24	**3590**	509	25	**3690**	—703	23
92	—746	26	92	—046	25	92	307	25	92	—534	24	92	726	24
94	772	27	94	071	26	94	332	25	94	—558	24	94	—750	23
96	—799	26	96	—097	25	96	357	25	96	—582	24	96	773	24
98	825	26	98	122	26	98	—382	25	98	606	24	98	—797	23
3300	851	27	**3400**	—148	25	**3500**	—407	25	**3600**	630	24	**3700**	820	24
02	—878	26	02	173	26	02	—432	24	02	654	24	02	—844	23
04	904	26	04	—199	25	04	456	25	04	678	25	04	867	24
06	930	27	06	224	26	06	481	25	06	—703	24	06	—891	23
08	—957	26	08	—250	25	08	—506	25	08	—727	24	08	—914	23
3310	—983	26	**3410**	275	26	**3510**	—531	24	**3610**	—751	24	**3710**	937	24
12	52009	26	12	—301	25	12	555	25	12	—775	24	12	—961	23
14	035	26	14	326	26	14	580	25	14	—799	24	14	984	24
16	061	27	16	—352	25	16	—605	25	16	—823	24	16	57008	23
18	—088	26	18	377	26	18	—630	24	18	—847	24	18	—031	23
3320	—114	26	**3420**	—403	25	**3520**	654	25	**3620**	—871	24	**3720**	054	24
22	—140	26	22	428	25	22	—679	25	22	—895	24	22	—078	23
24	166	26	24	453	26	24	—704	24	24	—919	24	24	—101	23
26	192	26	26	—479	25	26	728	25	26	—943	24	26	124	24
28	218	26	28	504	25	28	—753	24	28	—967	24	28	—148	23
3330	244	26	**3430**	529	26	**3530**	777	25	**3630**	—991	24	**3730**	—171	23
32	270	27	32	—555	25	32	802	25	32	56015	23	32	194	23
34	—297	26	34	580	25	34	—827	24	34	038	24	34	217	24
36	—323	26	36	605	26	36	851	25	36	062	24	36	—241	23
38	—349	26	38	—631	25	38	—876	24	38	086	24	38	—264	23
3340	—375	26	**3440**	—656	25	**3540**	900	25	**3640**	110	24	**3740**	287	23
42	—401	26	42	681	25	42	—925	24	42	—134	24	42	310	24
44	—427	26	44	706	26	44	949	25	44	—158	24	44	—334	23
46	—453	26	46	—732	25	46	—974	24	46	—182	23	46	—357	23
48	—479	25	48	—757	25	48	998	25	48	205	24	48	—380	23

To find Log. 36114:
Log. 36120 = 55775
Log. 36100 = 55751
Dif. 20 **24**
36114 — 36100 = **14**
Under **24**
Dif. for 10 = 12
" " 4 = 5
" " **14** = 17
Log. 36114 =
55751 + 17 = 55768.

	27	**26**	**25**	**24**	**23**	
1	1	1	1	1	1	**1**
2	3	3	3	2	2	**2**
3	4	4	4	4	3	**3**
4	5	5	5	5	5	**4**
5	7	7	6	6	6	**5**
6	8	8	8	7	7	**6**
7	10	9	9	8	8	**7**
8	11	10	10	10	9	**8**
9	12	12	11	11	10	**9**
10	14	13	13	12	12	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 3490 = 5428254
" 3492 = 5430742

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
3750	57403	58	**4000**	60206	54	**4250**	62839	51	**4500**	65321	48	**4750**	67669	46
55	—461	58	05	260	54	55	—890	51	05	369	49	55	715	46
60	—519	57	10	314	55	60	—941	51	10	—418	48	60	—761	45
65	576	58	15	—369	54	65	—992	51	15	—466	48	65	806	45
70	634	58	20	—423	54	70	63043	51	20	—514	48	70	—852	45
75	—692	57	25	—477	54	75	—094	50	25	—562	48	75	897	46
80	749	58	30	—531	53	80	144	51	30	—610	48	80	—943	45
85	—807	57	35	584	54	85	195	51	35	—658	48	85	988	46
90	—864	57	40	638	54	90	—246	50	40	—706	47	90	68034	45
95	921	57	45	—692	54	95	296	51	45	753	48	95	—079	45
3800	978	57	**4050**	—746	53	**4300**	—347	50	**4550**	801	48	**4800**	124	45
05	58035	57	55	799	54	05	397	51	55	—849	47	05	169	46
10	092	57	60	—853	53	10	—448	50	60	896	48	10	—215	45
15	149	57	65	906	53	15	498	50	65	944	48	15	—260	45
20	206	57	70	959	54	20	548	51	70	—992	47	20	—305	45
25	263	57	75	61013	53	25	—599	50	75	66039	48	25	—350	45
30	—320	57	80	066	53	30	—649	50	80	—087	47	30	—395	45
35	—377	56	85	119	53	35	—699	50	85	—134	47	35	—440	45
40	433	57	90	172	53	40	—749	50	90	181	48	40	—485	44
45	—490	56	95	225	53	45	—799	50	95	—229	47	45	529	45
3850	546	56	**4100**	278	53	**4350**	—849	50	**4600**	—276	47	**4850**	574	45
55	602	57	05	331	53	55	—899	50	05	—323	47	55	—619	45
60	—659	56	10	384	53	60	—949	49	10	370	47	60	—664	44
65	—715	56	15	—437	53	65	998	50	15	417	47	65	708	45
70	771	56	20	—490	52	70	64048	50	20	464	47	70	—753	44
75	827	56	25	542	53	75	—098	49	25	511	47	75	797	45
80	883	56	30	595	53	80	147	50	30	558	47	80	—842	44
85	939	56	35	—648	52	85	—197	49	35	—605	47	85	886	45
90	—995	56	40	700	52	90	246	50	40	—652	47	90	—931	44
95	59051	55	45	752	53	95	—296	49	45	—699	46	95	975	45
3900	106	56	**4150**	—805	52	**4400**	345	50	**4650**	745	47	**4900**	69020	44
05	162	56	55	857	52	05	—395	49	55	—792	47	05	—064	44
10	—218	55	60	909	53	10	—444	49	60	—839	46	10	108	44
15	273	56	65	—962	52	15	493	49	65	885	47	15	152	45
20	—329	55	70	62014	52	20	542	49	70	—932	46	20	—197	44
25	—384	55	75	—066	52	25	591	49	75	978	47	25	—241	44
30	439	55	80	—118	52	30	640	49	80	67025	46	30	—285	44
35	494	56	85	—170	51	35	689	49	85	—071	46	35	—329	44
40	—550	55	90	221	52	40	738	49	90	117	47	40	—373	44
45	—605	55	95	273	52	45	787	49	95	—164	46	45	—417	44
3950	—660	55	**4200**	—325	52	**4450**	836	49	**4700**	—210	46	**4950**	—461	43
55	—715	55	05	—377	51	55	—885	48	05	—256	46	55	504	44
60	—770	54	10	428	52	60	933	49	10	302	46	60	548	44
65	824	55	15	—480	51	65	982	49	15	348	46	65	—592	44
70	879	55	20	531	52	70	65031	48	20	394	46	70	—636	43
75	—934	54	25	—583	51	75	079	49	25	440	46	75	679	44
80	988	55	30	634	51	80	—128	48	30	486	46	80	—723	44
85	60043	54	35	685	52	85	176	49	35	—532	46	85	—767	43
90	097	55	40	—737	51	90	—225	48	40	—578	46	90	810	44
95	—152	54	45	—788	51	95	—273	48	45	—624	45	95	—854	43

	58	**57**	**56**	**55**	**54**	**53**	**52**	**51**	**50**	**49**	**48**	**47**	**46**	**45**	**44**	**43**	
1	1.2	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.9	**1**
2	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	**2**
3	3.5	3.4	3.4	3.3	3.2	3.2	3.1	3.1	3.0	2.9	2.9	2.8	2.8	2.7	2.6	2.6	**3**
4	4.6	4.6	4.5	4.4	4.3	4.2	4.2	4.1	4.0	3.9	3.8	3.8	3.7	3.6	3.5	3.4	**4**
5	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	**5**
6	7.0	6.8	6.7	6.6	6.5	6.4	6.2	6.1	6.0	5.9	5.8	5.6	5.5	5.4	5.3	5.2	**6**
7	8.1	8.0	7.8	7.7	7.6	7.4	7.3	7.1	7.0	6.9	6.7	6.6	6.4	6.3	6.2	6.0	**7**
8	9.3	9.1	9.0	8.8	8.6	8.5	8.3	8.2	8.0	7.8	7.7	7.5	7.4	7.2	7.0	6.9	**8**
9	10.4	10.3	10.1	9.9	9.7	9.5	9.4	9.2	9.0	8.8	8.6	8.5	8.3	8.1	7.9	7.7	**9**
10	11.6	11.4	11.2	11.0	10.8	10.6	10.4	10.2	10.0	9.8	9.6	9.4	9.2	9.0	8.8	8.6	**10**

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
5000	69897	43	**5250**	72016	41	**5500**	74036	40	**5750**	75967	38	**6000**	77815	36
05	940	44	55	057	42	05	—076	39	55	76005	37	05	851	36
10	—984	43	60	—099	41	10	115	40	60	042	38	10	887	37
15	70027	43	65	—140	41	15	—155	39	65	—080	38	15	—924	36
20	070	44	70	181	41	20	—194	39	70	—118	37	20	—960	36
25	—114	43	75	222	41	25	233	40	75	155	38	25	—996	36
30	—157	43	80	263	41	30	—273	39	80	—193	37	30	78032	36
35	—200	43	85	304	42	35	—312	39	85	230	38	35	—068	36
40	243	43	90	—346	41	40	—351	39	90	—268	37	40	—104	36
45	286	43	95	—387	41	45	390	39	95	305	38	45	—140	36
5050	329	43	**5300**	—428	41	**5550**	429	39	**5800**	—343	37	**6050**	—176	35
55	372	43	05	—469	40	55	468	39	05	380	38	55	211	36
60	415	43	10	509	41	60	507	40	10	—418	37	60	247	36
65	—458	43	15	550	41	65	—547	39	15	—455	37	65	283	36
70	—501	43	20	591	41	70	—586	38	20	492	38	70	—319	36
75	—544	42	25	—632	41	75	624	39	25	—530	37	75	—355	35
80	586	43	30	—673	40	80	663	39	30	—567	37	80	390	36
85	629	43	35	713	41	85	702	39	35	604	37	85	426	36
90	—672	42	40	754	41	90	741	39	40	641	37	90	—462	35
95	714	43	45	—795	40	95	780	39	45	678	38	95	497	36
5100	757	43	**5350**	835	41	**5600**	—819	39	**5850**	—716	37	**6100**	—533	36
05	—800	42	55	—876	40	05	—858	38	55	—753	37	05	—569	35
10	842	43	60	916	41	10	896	39	60	—790	37	10	604	36
15	—885	42	65	—957	40	15	—935	39	65	—827	37	15	—640	35
20	—927	42	70	997	41	20	—974	38	70	—864	37	20	675	36
25	969	43	75	73038	40	25	75012	39	75	—901	37	25	—711	35
30	71012	42	80	078	41	30	—051	38	80	—938	37	30	746	35
35	054	42	85	—119	40	35	089	39	85	—975	37	35	781	36
40	096	43	90	—159	40	40	—128	38	90	77012	36	40	—817	35
45	—139	42	95	199	40	45	166	39	95	048	37	45	852	36
5150	—181	42	**5400**	239	41	**5650**	—205	38	**5900**	085	37	**6150**	—888	35
55	—223	42	05	—280	40	55	243	39	05	—122	37	55	—923	35
60	—265	42	10	—320	40	60	—282	38	10	—159	36	60	958	35
65	307	42	15	—360	40	65	—320	38	15	195	37	65	993	36
70	349	42	20	—400	40	70	358	39	20	232	37	70	79029	35
75	391	42	25	—440	40	75	—397	38	25	—269	36	75	—064	35
80	—433	42	30	—480	40	80	—435	38	30	305	37	80	—099	35
85	—475	42	35	—520	40	85	473	38	35	342	37	85	—134	35
90	—517	42	40	—560	40	90	511	38	40	—379	36	90	169	35
95	—559	41	45	—600	40	95	549	38	45	415	37	95	204	35
5200	600	42	**5450**	—640	39	**5700**	587	39	**5950**	—452	36	**6200**	239	35
05	642	42	55	679	40	05	—626	38	55	488	37	05	274	35
10	—684	41	60	719	40	10	—664	38	60	—525	36	10	309	35
15	725	42	65	759	40	15	—702	38	65	561	36	15	344	35
20	767	42	70	—799	39	20	—740	38	70	597	37	20	379	35
25	—809	41	75	838	40	25	—778	37	75	—634	36	25	—414	35
30	850	42	80	878	40	30	815	38	80	670	36	30	—449	35
35	—892	41	85	—918	39	35	853	38	85	706	37	35	—484	34
40	933	42	90	957	40	40	891	38	90	—743	36	40	518	35
45	—975	41	95	—997	39	45	929	38	95	—779	36	45	553	35

	44	**43**	**42**	**41**	**40**	**39**	**38**	**37**	**36**	**35**	**34**	
1	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	**1**
2	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	**2**
3	2.6	2.6	2.5	2.5	2.4	2.3	2.3	2.2	2.2	2.1	2.0	**3**
4	3.5	3.4	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.7	**4**
5	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5	3.4	**5**
6	5.3	5.2	5.0	4.9	4.8	4.7	4.6	4.4	4.3	4.2	4.1	**6**
7	6.2	6.0	5.9	5.7	5.6	5.5	5.3	5.2	5.0	4.9	4.8	**7**
8	7.0	6.9	6.7	6.6	6.4	6.2	6.1	5.9	5.8	5.6	5.4	**8**
9	7.9	7.7	7.6	7.4	7.2	7.0	6.8	6.7	6.5	6.3	6.1	**9**
10	8.8	8.6	8.4	8.2	8.0	7.8	7.6	7.4	7.2	7.0	6.8	**10**

To find Log. 58636:
Log. 58650 = 76827
Log. 58600 = 76790
Dif. 50 **37**
58636 — 58600 = **36**
Under **37**
Dif. for 10 × 3 = 22.0
" " 6 = 4.4
" " **36** = 26.4
Log. 58636 = 76790 + 26 = 76816

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
6250	79588	35	**6500**	81291	34	**6750**	82930	33	**7000**	84510	31	**7250**	86034	30
55	—623	34	05	—325	33	55	—963	32	05	—541	31	55	—064	30
60	657	35	10	358	33	60	—995	32	10	—572	31	60	—094	30
65	692	35	15	391	34	65	83027	32	15	—603	31	65	—124	29
70	—727	34	20	—425	33	70	—059	32	20	—634	31	70	153	30
75	761	35	25	458	33	75	—091	32	25	—665	31	75	183	30
80	—796	35	30	491	34	80	—123	32	30	—696	30	80	213	30
85	—831	34	35	—525	33	85	—155	32	35	726	31	85	—243	30
90	865	35	40	—558	33	90	—187	32	40	757	31	90	—273	30
95	—900	34	45	—591	33	95	—219	32	45	788	31	95	—303	29
6300	934	35	**6550**	624	33	**6800**	—251	32	**7050**	—819	31	**7300**	332	30
05	—969	34	55	657	33	05	—283	32	55	—850	30	05	362	30
10	80003	34	60	690	33	10	—315	32	60	880	31	10	—392	29
15	037	35	65	723	34	15	—347	31	65	911	31	15	421	30
20	—072	34	70	—757	33	20	378	32	70	—942	31	20	451	30
25	106	34	75	—790	33	25	410	32	75	—973	30	25	—481	29
30	140	35	80	—823	33	30	442	32	80	85003	31	30	510	30
35	—175	34	85	—856	33	35	—474	32	85	—034	31	35	540	30
40	—209	34	90	—889	32	40	—506	31	90	—065	30	40	—570	29
45	243	34	95	921	33	45	537	32	95	095	31	45	599	30
6350	277	35	**6600**	954	33	**6850**	569	32	**7100**	—126	30	**7350**	—629	29
55	—312	34	05	987	33	55	—601	31	05	156	31	55	658	30
60	—346	34	10	82020	33	60	632	32	10	—187	30	60	—688	29
65	—380	34	15	—053	33	65	664	32	15	217	31	65	717	30
70	—414	34	20	—086	33	70	—696	31	20	—248	30	70	—747	29
75	448	34	25	—119	32	75	727	32	25	278	31	75	776	30
80	482	34	30	151	33	80	—759	31	30	—309	30	80	—806	29
85	516	34	35	184	33	85	790	32	35	339	31	85	835	29
90	550	34	40	—217	32	90	—822	31	40	—370	30	90	864	30
95	584	31	45	249	33	95	853	32	45	400	31	95	—894	29
6400	—618	34	**6650**	282	33	**6900**	—885	31	**7150**	—431	30	**7400**	923	30
05	—652	34	55	—315	32	05	916	32	55	—461	30	05	—953	29
10	—686	34	60	347	33	10	—948	31	60	491	31	10	—982	29
15	—720	34	65	380	33	15	979	32	65	—522	30	15	87011	29
20	—754	33	70	—413	32	20	84011	31	70	—552	30	20	040	30
25	787	34	75	445	33	25	—042	31	75	582	30	25	—070	29
30	821	34	80	—478	32	30	073	32	80	612	31	30	—099	29
35	—855	34	85	510	33	35	—105	31	85	—643	30	35	128	29
40	—889	33	90	—543	32	40	—136	31	90	—673	30	40	157	29
45	922	34	95	575	32	45	167	31	95	703	30	45	186	30
6450	—956	34	**6700**	607	33	**6950**	198	32	**7200**	733	30	**7450**	—216	29
55	—990	33	05	—640	32	55	—230	31	05	763	31	55	—245	29
60	81023	34	10	672	33	60	—261	31	10	—794	30	60	—274	29
65	—057	33	15	—705	32	65	292	31	15	—824	30	65	—303	29
70	090	34	20	—737	32	70	323	31	20	—854	30	70	332	29
75	—124	34	25	769	33	75	354	32	25	—884	30	75	361	29
80	—158	33	30	—802	32	80	—386	31	30	—914	30	80	390	29
85	—191	33	35	—834	32	85	—417	31	35	—944	30	85	419	29
90	224	34	40	—866	32	90	—448	31	40	—974	30	90	448	29
95	—258	33	45	898	32	95	—479	31	45	86004	30	95	477	29

To find Log. 63023:
Log. 63050 = 79969
Log. 63000 = 79934
Dif. 50 **35**
63023 — 63000 = **23**
Under **35**
Dif. for 10 × 2 = 14.0
" " 3 = 2.1
" " **23** = 16.1
Log. 63023 =
9934 + 16 = 79950.

	35	34	33	32	31	30	29	
1	0.7	0.7	0.7	0.6	0.6	0.6	0.6	**1**
2	1.4	1.4	1.3	1.3	1.2	1.2	1.2	**2**
3	2.1	2.0	2.0	1.9	1.9	1.8	1.7	**3**
4	2.8	2.7	2.6	2.6	2.5	2.4	2.3	**4**
5	3.5	3.4	3.3	3.2	3.1	3.0	2.9	**5**
6	4.2	4.1	4.0	3.8	3.7	3.6	3.5	**6**
7	4.9	4.8	4.6	4.5	4.3	4.2	4.1	**7**
8	5.6	5.4	5.3	5.1	5.0	4.8	4.6	**8**
9	6.3	6.1	5.9	5.8	5.6	5.4	5.2	**9**
10	7.0	6.8	6.6	6.4	6.2	6.0	5.8	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 7400 = 8692317
" 7405 = 8695251

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
7500	87506	29	**7750**	88930	28	**8000**	90309	27	**8250**	91645	27	**8500**	92942—	25
05	535	29	55	958	28	05	336	27	55	—672	26	05	967	26
10	—564	29	60	986	28	10	363	27	60	698	26	10	—993	25
15	—593	29	65	89014	28	15	390	27	65	724	27	15	93018	26
20	—622	29	70	042	28	20	417	28	70	—751	26	20	—044	25
25	—651	28	75	070	28	25	—445	27	75	—777	26	25	069	26
30	679	29	80	—098	28	30	—472	27	80	803	26	30	—095	25
35	708	29	85	—126	28	35	—499	27	85	829	26	35	120	26
40	737	29	90	—154	28	40	—526	27	90	855	27	40	—146	25
45	—766	29	95	—182	27	45	—553	27	95	—882	26	45	171	26
7550	—795	28	**7800**	209	28	**8050**	—580	27	**8300**	—908	26	**8550**	—197	25
55	823	29	05	237	28	55	—607	27	05	—934	26	55	222	25
60	852	29	10	265	28	60	—634	26	10	960	26	60	247	26
65	—881	29	15	—293	28	65	660	27	15	986	26	65	—273	25
70	—910	28	20	—321	27	70	687	27	20	92012	26	70	298	25
75	938	29	25	348	28	75	714	27	25	038	27	75	323	26
80	—967	29	30	376	28	80	741	27	30	—065	26	80	—349	25
85	—996	28	35	—404	28	85	768	27	35	—091	26	85	374	25
90	88024	29	40	—432	27	90	—795	27	40	—117	26	90	399	26
95	—053	28	45	459	28	95	—822	27	45	—143	26	95	—425	25
7600	081	29	**7850**	—487	28	**8100**	—849	26	**8350**	—169	26	**8600**	—450	25
05	—110	28	55	—515	27	05	875	27	55	—195	26	05	475	25
10	138	29	60	542	28	10	902	27	60	—221	26	10	500	26
15	—167	28	65	—570	27	15	—929	27	65	—247	26	15	—526	25
20	195	29	70	597	28	20	—956	26	70	—273	25	20	—551	25
25	—224	28	75	625	28	25	982	27	75	298	26	25	—576	25
30	252	29	80	—653	27	30	91009	27	80	324	26	30	601	25
35	—281	28	85	680	28	35	—036	26	85	350	26	35	626	25
40	309	29	90	—708	27	40	062	27	90	376	26	40	651	25
45	—338	28	95	735	28	45	089	27	95	402	26	45	676	26
7650	366	29	**7900**	—763	27	**8150**	—116	26	**8400**	—428	26	**8650**	—702	25
55	—395	28	05	790	28	55	142	27	05	—454	26	55	—727	25
60	—423	28	10	—818	27	60	169	27	10	—480	25	60	—752	25
65	451	29	15	845	28	65	—196	26	15	505	26	65	—777	25
70	—480	28	20	—873	27	70	222	27	20	531	26	70	—802	25
75	—508	28	25	—900	27	75	—249	26	25	—557	26	75	—827	25
80	536	28	30	927	28	80	275	27	30	—583	26	80	—852	25
85	564	29	35	—955	27	85	—302	26	35	—609	25	85	—877	25
90	—593	28	40	982	27	90	328	27	40	634	26	90	—902	25
95	—621	28	45	90009	28	95	—355	26	45	—660	26	95	—927	25
7700	649	28	**7950**	—037	27	**8200**	381	27	**8450**	—686	25	**8700**	—952	25
05	677	28	55	064	27	05	—408	26	55	711	26	05	—977	25
10	705	29	60	091	28	10	434	27	60	737	26	10	94002—	25
15	—734	28	65	—119	27	15	—461	26	65	—763	25	15	—027	25
20	—762	28	70	—146	27	20	487	27	70	788	26	20	—052	25
25	—790	28	75	173	27	25	—514	26	75	—814	26	25	—077	24
30	—818	28	80	200	27	30	—540	26	80	—840	25	30	101	25
35	846	28	85	227	28	35	566	27	85	865	26	35	126	25
40	874	28	90	—255	27	40	—593	26	90	—891	25	40	151	25
45	902	28	95	—282	27	45	619	26	95	916	26	45	—176	25

To find Log. 83678:
Log. 83700 = 92273
Log. 83650 = 92247
Dif. 50 **26**
83678 — 83650 = **28**
Under **26**
Dif. for 10 × 2 = 10.0
" " 8 = 4.2
" " **28** = 14.2
Log. 83678 =
92247 + 14 = 92261.

	29	**28**	**27**	**26**	**25**	**24**	
1	0.6	0.6	0.5	0.5	0.5	0.5	**1**
2	1.2	1.1	1.1	1.0	1.0	1.0	**2**
3	1.7	1.7	1.6	1.6	1.5	1.4	**3**
4	2.3	2.2	2.2	2.1	2.0	1.9	**4**
5	2.9	2.8	2.7	2.6	2.5	2.4	**5**
6	3.5	3.4	3.2	3.1	3.0	2.9	**6**
7	4.1	3.9	3.8	3.6	3.5	3.4	**7**
8	4.6	4.5	4.3	4.2	4.0	3.8	**8**
9	5.2	5.0	4.9	4.7	4.5	4.3	**9**
10	5.8	5.6	5.4	5.2	5.0	4.8	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 8325 = 9203842
" 8330 = 9206450

Common or Briggs Logarithms. Base = 10.

No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.	No.	Log.	Dif.
8750	94201	25	**9000**	95424	24	**9250**	96614	24	**9500**	97772	23	**9750**	98900	23
55	—226	24	05	448	24	55	—638	23	05	795	23	55	—923	22
60	250	25	10	472	25	60	661	24	10	818	23	60	—945	22
65	275	25	15	—497	24	65	—685	23	15	—841	23	65	967	22
70	—300	25	20	—521	24	70	—708	23	20	—864	22	70	989	23
75	—325	24	25	—545	24	75	731	24	25	886	23	75	99012	22
80	349	25	30	—569	24	80	—755	23	30	909	23	80	—034	22
85	374	25	35	—593	24	85	778	24	35	932	23	85	056	22
90	—399	25	40	—617	24	90	—802	23	40	—955	23	90	078	22
95	—424	24	45	—641	24	95	—825	23	45	—978	22	95	100	23
8800	448	25	**9050**	—665	24	**9300**	848	24	**9550**	98000	23	**9800**	—123	22
05	—473	25	55	—689	24	05	—872	23	55	023	23	05	—145	22
10	—498	24	60	—713	24	10	—895	23	60	—046	22	10	—167	22
15	522	25	65	—737	24	15	918	24	65	068	23	15	189	22
20	—547	24	70	—761	24	20	—942	23	70	091	23	20	211	22
25	571	25	75	—785	24	25	—965	23	75	—114	23	25	233	22
30	596	25	80	—809	23	30	988	23	80	—137	22	30	255	22
35	—621	24	85	832	24	35	97011	24	85	159	23	35	277	23
40	645	25	90	856	24	40	—035	23	90	—182	22	40	—300	22
45	—670	24	95	880	24	45	—058	23	95	204	23	45	—322	22
8850	694	25	**9100**	904	24	**9350**	081	23	**9600**	227	23	**9850**	—344	22
55	—719	24	05	—928	24	55	104	24	05	—250	22	55	—366	22
60	743	25	10	—952	24	60	—128	23	10	272	23	60	—388	22
65	—768	24	15	—976	23	65	—151	23	15	—295	23	65	—410	22
70	792	25	20	999	24	70	—174	23	20	—318	22	70	—432	22
75	—817	24	25	96023	24	75	197	23	25	340	23	75	—454	22
80	841	25	30	047	24	80	220	23	30	—363	22	80	—476	22
85	—866	24	35	—071	24	85	243	24	35	385	23	85	—498	22
90	890	25	40	—095	23	90	—267	23	40	—408	22	90	—520	22
95	—915	24	45	118	24	95	—290	23	45	430	23	95	—542	22
8900	939	24	**9150**	142	24	**9400**	—313	23	**9650**	—453	22	**9900**	—564	21
05	963	25	55	—166	24	05	—336	23	55	475	23	05	585	22
10	—988	24	60	—190	23	10	—359	23	60	—498	22	10	607	22
15	95012	24	65	213	24	15	382	23	65	520	23	15	629	22
20	036	25	70	—237	24	20	405	23	70	—543	22	20	651	22
25	—061	24	75	—261	23	25	428	23	75	565	23	25	673	22
30	085	24	80	284	24	30	451	23	80	—588	22	30	—695	22
35	109	25	85	—308	24	35	474	23	85	—610	22	35	—717	22
40	—134	24	90	—332	23	40	497	23	90	632	23	40	—739	21
45	158	24	95	355	24	45	520	23	95	—655	22	45	760	22
8950	182	25	**9200**	—379	23	**9450**	543	23	**9700**	677	23	**9950**	782	22
55	—207	24	05	402	24	55	566	23	05	—700	22	55	804	22
60	—231	24	10	—426	24	60	589	23	10	—722	22	60	—826	22
65	255	24	15	—450	23	65	612	23	15	744	23	65	—848	22
70	279	24	20	473	24	70	—635	23	20	—767	22	70	—870	21
75	303	25	25	—497	23	75	—658	23	25	—789	22	75	891	22
80	—328	24	30	520	24	80	—681	23	30	811	23	80	913	22
85	—352	24	35	—544	23	85	—704	23	35	—834	22	85	—935	22
90	—376	24	40	567	24	90	—727	22	40	—856	22	90	—957	22
95	400	24	45	—591	23	95	749	23	45	878	22	95	978	22

To find Log. 95544:
Log. 95550 = 98023
Log. 95500 = 98000
Dif. 50 **23**
95544 — 95500 = **44**
Under **23**
Dif. for 10 × 4 = 18.0
" " 4 = 1.8
" " **44** = 19.8
Log. 95544 =
98000 + 20 = 98020.

	25	**24**	**23**	**22**	**21**	
1	0.5	0.5	0.5	0.4	0.4	**1**
2	1.0	1.0	0.9	0.9	0.8	**2**
3	1.5	1.4	1.4	1.3	1.3	**3**
4	2.0	1.9	1.8	1.8	1.7	**4**
5	2.5	2.4	2.3	2.2	2.1	**5**
6	3.0	2.9	2.8	2.6	2.5	**6**
7	3.5	3.4	3.2	3.1	2.9	**7**
8	4.0	3.8	3.7	3.5	3.4	**8**
9	4.5	4.3	4.1	4.0	3.8	**9**
10	5.0	4.8	4.6	4.4	4.2	**10**

A dash before or after a log. denotes that its true value is *less* than the tabular value by less than half a unit in the last place. Thus:
Log. 9600 = 9822712
" 9605 = 9824974

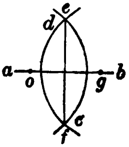
GEOMETRY.

Lines, Figures, Solids. defined. Strictly speaking a geometrical **line** is simply length, or distance. The lines we draw on paper have not only length, but breadth and thickness; still they are the most convenient symbol we can employ for denoting a geometrical line. **Straight** lines are also called **right** lines. A **vertical** line is one that points toward the center of the earth; and a **horizontal** one is at right angles to a vert one. **A plane figure** is merely any flat surface or area entirely enclosed by lines either straight or curved; which are called its outline, boundary, circumf, or periphery. We often confound the outline with the fig itself as when we speak of drawing circles, squares, &c; for we actually draw only their outlines. Geometrically speaking, a fig has length and breadth only; no thickness. **A solid** is any body; it has length, breadth, and thickness.

Geometrically similar figs or solids, are not necessarily of the same **size;** but only of precisely the same **shape.** Thus, any two squares are, scientifically speaking, similar to each other; so also any two circles, cubes, &c, no matter how different they may be in size. When they are not only of the same shape, but of the same size, they are said to be **similar,** and **equal.**

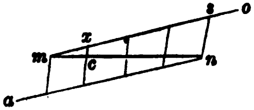
The **quantities of lines** are to each other simply as their **lengths;** but the quantities, or areas, or surfaces of similar **figures,** are as, or in proportion to, the **squares** of any one of the corresponding lines or sides which enclose the figures, or which may be drawn upon them; and the quantities, or solidities of similar **solids,** are as the **cubes** of any of the corresponding lines which form their edges, or the figures by which they are enclosed.

Rem.—Simple as the following operations appear, it is only by care, and good instruments, that they are made to give accurate results. Several of them can be much better performed by means of a metallic triangle having one perfectly accurate right angle. In the field, the tape-line, chain, or a measuring-rod will take the place of the dividers and ruler used indoors.



To divide a given line, *a b*, into two equal parts.

From its ends *a* and *b* as centers, and with any rad greater than one-half of *a b*, describe the arcs *c* and *d*, and join *e f*. If the line *a b* is very long, first lay off equal dists *a o* and *b g*, each way from the ends, so as to approach conveniently near to each other; and then proceed as if *o g* were the line to be divided. Or measure *a b* by a scale, and thus ascertain its center.



To divide a given line, *m n*, into any given number of equal parts.

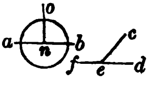
From *m* and *n* draw any two parallel lines *m o* and *n a*, to an indefinite dist; and on them, from *m* and *n* step off the reqd number of equal parts of any convenient length: finally, join the corresponding points thus stepped off. Or only one line, as *m o*, may be drawn and stepped off, as to *s*; then join *s n*; and draw the other short lines parallel to it.

To divide a given line, *m n*, into two parts which shall have a given proportion to each other.

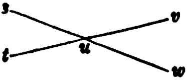
This is done on the same principle as the last; thus, let the proportion be as 1 to 3. First draw any line *m o*; and with any convenient opening of the dividers, make *m x* equal to one step; and *m s* equal to three steps. Join *s n*; and parallel to it draw *x c*. Then *m c* is to *c n* as 1 is to 3.

ANGLES.

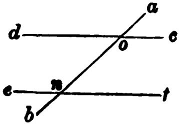
Angles. When two straight, or right lines meet each other at any inclination, the inclination is called an **angle;** and is measured by the degrees contained in the arc of a circle described from the point of meeting as a center. Since all circles, whether large or small, are supposed to be divided into 360 degrees, it follows that any number of degrees of a small circle will measure the same degree of inclination as will the same number of a large one.



When two straight lines, as *o n* and *a b*, meet in such a manner that the inclination *o n a* is equal to the inclination *o n b*, then the two lines are said to be **perpendicular** to each other; and the angles *o n a* and *o n b*, are called **right angles;** and are each meas'd by, or are equal to, 90°, or one-fourth part of the circumf of a circle. Any angle, as *c e d*, smaller than a right angle, is called **acute** or sharp; and one *c e f*, larger than a right angle, is called **obtuse,** or blunt. When one line meets another, as in the first Fig on opposite page, the two angles on the same side of either line are called **contiguous,** or **adjacent.** Thus, *v u s* and *v u w* are adjacent; also *t u s* and *t u w*; *s u t* and *s u v*; *w u t* and *w u v*. The sum of two adjacent angles is always equal to two right angles; or to 180°. Therefore, if we know the number of degrees contained in one of them, and subtract it from 180°, we obtain the other.

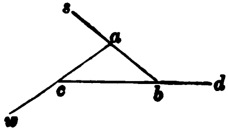


When two straight lines *cross* each other, forming four angles, either pair of those angles which point in exactly opposite directions are called **opposite, or vertical angles**; thus, the pair *s u t* and *v u w* are opposite angles; also the pair *s u v* and *t u w*. The opposite angles of any pair are always equal to each other.



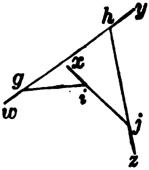
When a straight line *a b* crosses two parallel lines *c d*, *e f*, the **alternate** angles which form a kind of Z are equal to each other. Thus, the angles *d o n* and *o n f* are equal: as are also *c o n* and *o n e*. Also the sum of the two internal angles on the same side of *a b*, is equal to two right angles, or 180°; thus, $c\,o\,n + o\,n\,f = 180^\circ$; also $d\,o\,n + o\,n\,e = 180^\circ$.

An interior angle,



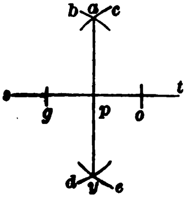
In any fig, is any angle formed *inside* of that fig, by the meeting of two of its sides, as the angles *c a b*, *a b c*, *b c a*, of this triangle. All the interior angles of any straight-lined figure of any number of sides whatever, are together equal to twice as many right angles minus four, as the figure has sides. Thus, a triangle has 3 sides; twice that number is 6; and 6 right angles, or $6 \times 90^\circ = 540^\circ$; from which take 4 right angles, or 360°; and there remain 180°, which is the number of degrees in every plane, or straight-lined triangle. This principle furnishes an easy means of testing our measurements of the angles of any fig; for if the sum of all our measurements does not agree with the sum given by the rule, it is a proof that we have committed some error.

An exterior angle



Of any straight-lined figure, is any angle, as *a b d*, formed by the meeting of any side, as *a b*, with the prolongation of an adjacent side, as *c b*; so likewise the angles *c a s* and *b c w*. All the exterior angles of any straight-lined fig, no matter how many sides it may have, amount to 360°; but, in the case of a *re-entering* angle, as *g i j*, the interior angle, *g i j*, exceeds 180°, and the "exterior" angle, *g i x*, being = 180° — interior angle, is *negative*. Thus $a\,b\,d + b\,c\,w + c\,a\,s = 360^\circ$; and $y\,h\,j + z\,j\,i - g\,i\,x + i\,g\,w = 360^\circ$. Angles, as *a*, *b*, *c*, *g*, *h*, and *j*, which point *outward*, are called **salient.**

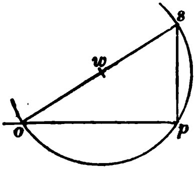
From any given point, *p*, on a line *s t*, to draw a perp, *p a*.



From *p*, with any convenient opening of the dividers, step off the equals *p o*, *p g*. From *o* and *g* as centers, with any opening greater than half *o g*, describe the two short arcs *b* and *c*; and join *a p*. Or still better, describe four arcs, and join *a y*.

Or from *p* with any convenient scale describe two short arcs *g* and *c* either one of them with a radius 3, and the other with a rad 4. Then from *g* with rad 5 describe the arc *b*. Join *p a*.

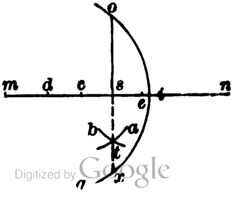
If the point *p* is at one end of the line, or very near it,



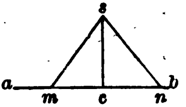
Extend the line, if possible, and proceed as above. But if this cannot be done, then from any convenient point, *w*, open the dividers to *p*, and describe the semicircle, *s p o*; through *o w* draw *o w s*; join *p s*.

Or use the last foregoing process with rads 3, 4, and 5.

From a given point, *o*, to let fall a perp *o s*, to a given line, *m n*.

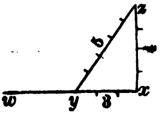


From *o*, measure to the line *m n*, any two equal dists, *o c*, *o e*; and from *c* and *e* as centers, with any opening greater than half of *c e*, describe the two arcs *a* and *b*; join *o t*. Or from any point, as *d* on the line, open the dividers to *o*, and describe the arc *o g*; make *t x* equal to *t o*; and join *o x*.



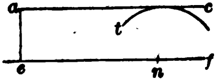
If the line, $a b$, is on the ground,

And a perp is reqd to be drawn from c, first measure off any two equal dists, $c m$, $c n$. At m and n, hold the ends of a piece of string, tape-line, or chain, $m s n$; then tighten out the string, &c, as shown by $m s n$; s being its center. Then will $s c$ be the reqd perp. Or if the perp $x z$ is to be drawn from the *end* of the line $w x$, first measure $x y$ upon the line, and equal to three feet; then holding the end of a tape-line at z, and its nine feet mark at y, hold the four feet mark at x, keeping $z x$ and $z y$ equally stretched. Then $z x$ will be the reqd perp, because 3, 4, and 5, make the sides of a right-angled triangle. Instead of 3, 4, and 5, any multiples of those numbers may be used, such as 6, 8, and 10; or 9, 12, 15, &c: also instead of feet, we may use yards, chains, &c.



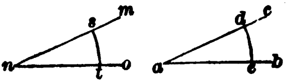
Through a given point, a, to draw a line, $a c$, parallel to another line, $e f$.

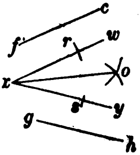
With the perp dist, $a e$, from any point, n, in $e f$, describe an arc, t; draw $a c$ just touching the arc.



At any point, a, in a line $a b$, to make an angle $c a b$, equal to a given angle, $m n o$.

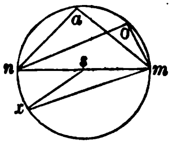
From n and a, with any convenient rad, describe the arcs $s t$, $d e$; measure $s t$, and make $e d$ equal to it; through $a d$ draw $a c$.



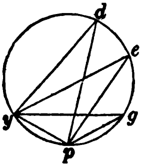


To bisect, or divide any angle, $w x y$, into two equal parts.

From x set off any two equal dists, $x r$, $x s$. From r and s with any rad describe two arcs intersecting, as at o; and join $o x$. If the two sides of the angle do not meet, as $c f$ and $g h$, either first extend them until they do meet; or else draw lines $x w$, and $x y$, parallel to them, and at equal dists from them, so as to meet; then proceed as before.



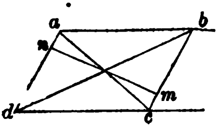
All angles, as $n a m$, $n o m$, at the circumf of a semicircle, and standing on its diam $n m$, are right angles; or, as it is usually expressed, **all angles in a semicircle are right angles.** An angle $n s x$ at the centre of a circle, is twice as great as an angle $n m x$ at the circumf, when both stand upon the same arc $n x$.



All angles, as $y d p$, $y e p$, $y g p$, at the circumf of a circle, and standing upon the same arc, as $y p$, are equal to each other; or, as usually expressed, **all angles in the same segment of a circle are equal.**

The **complement** of an angle is what it lacks of 90°. Thus, the complement of 80° is $90^\circ - 80^\circ = 10^\circ$; and that of 210° is $90^\circ - 210^\circ = -120^\circ$. The **supplement** of an angle is what it lacks of 180°. Thus, the supplement of 80° is $180^\circ - 80^\circ = 100^\circ$; and that of 210° is $180^\circ - 210^\circ = -30^\circ$. But ordinarily we may neglect the signs + and —, before complements and supplements, and call the complement of an angle its *diff* from 90°· and the supplement its *diff* from 180°.

Angles in a Parallelogram.



A parallelogram is any four-sided straight-lined figure whose opposite sides are equal, as *a b c d*; or a square, &c. Any line drawn across a parallelogram between 2 opposite angles, is called a *diagonal*, as *a c*, or *b d*. A diag divides a parallelogram into two equal parts; as does also any line *m n* drawn through the center of either diag; and moreover, the line *m n* itself is div into two equal parts by the diag. Two diags bisect each other; they also divide the parallelogram into four triangles of equal areas. The sum of the two angles at the ends of any one side is = 180°; thus, *d a b* + *a b c* = *a b c* + *b c d* = 180°; and the sum of the four angles, *d a b*, *a b c*, *b c d*, *c d a* = 360°.

The sum of the squares of the four sides, is equal to the sum of the squares of the two diags.

To reduce Minutes and Seconds to Degrees and decimals of a Degree, etc.

In any given angle—

Number of degrees = Number of minutes ÷ 60.
= Number of seconds ÷ 3600.

Number of minutes = Number of degrees × 60.
= Number of seconds ÷ 60.

Number of seconds = Number of degrees × 3600.
= Number of minutes × 60.

Table of Minutes and Seconds in Decimals of a Degree, and of Seconds in Decimals of a Minute.

(The columns of Mins and Degs answer equally for Secs and Mins.)

Mins.	Deg.	Mins.	Deg.	Mins.	Deg.	Secs.	Deg.	Secs.	Deg.	Secs.	Deg.
In each equivalent, **the last digit repeats indefinitely.** See * below:											
1	0.016	21	0.350	41	0.683	1	0.00027	21	0.00583	41	0.01138
2	0.033	22	0.366	42	0.700	2	0.00055	22	0.00611	42	0.01166
3	0.050	23	0.383	43	0.716	3	0.00083	23	0.00638	43	0.01194
4	0.066	24	0.400	44	0.733	4	0.00111	24	0.00666	44	0.01222
5	0.083	25	0.416	45	0.750	5	0.00138	25	0.00694	45	0.01250
6	0.100	26	0.433	46	0.766	6	0.00166	26	0.00722	46	0.01277
7	0.116	27	0.450	47	0.783	7	0.00194	27	0.00750	47	0.01305
8	0.133	28	0.466	48	0.800	8	0.00222	28	0.00777	48	0.01333
9	0.150	29	0.483	49	0.816	9	0.00250	29	0.00805	49	0.01361
10	0.166	30	0.500	50	0.833	10	0.00277	30	0.00833	50	0.01388
11	0.183	31	0.516	51	0.850	11	0.00305	31	0.00861	51	0.01416
12	0.200	32	0.533	52	0.866	12	0.00333	32	0.00888	52	0.01444
13	0.216	33	0.550	53	0.883	13	0.00361	33	0.00916	53	0.01472
14	0.233	34	0.566	54	0.900	14	0.00388	34	0.00944	54	0.01500
15	0.250	35	0.583	55	0.916	15	0.00416	35	0.00972	55	0.01527
16	0.266	36	0.600	56	0.933	16	0.00444	36	0.01000	56	0.01555
17	0.283	37	0.616	57	0.950	17	0.00472	37	0.01027	57	0.01583
18	0.300	38	0.633	58	0.966	18	0.00500	38	0.01055	58	0.01611
19	0.316	39	0.650	59	0.983	19	0.00527	39	0.01083	59	0.01638
20	0.333	40	0.666	60	1.000	20	0.00555	40	0.01111	60	0.01666
Secs.	Min.	Secs.	Min.	Secs.	Min.	Secs.	Deg.	Secs.	Deg.	Secs.	Deg.

* Each equivalent is a **repeating decimal,** thus:

2 minutes = 0.0333333 . . . . degree	12 seconds = 0.2000000 . . . . minute
7 " = 0.1166666 . . . . "	1 second = 0.0002777 . . . . degree
12 " = 0.2000000 . . . . "	50 seconds = 0.0138888 . . . . "

Approximate Measurement of Angles.

(1) **The four fingers of the hand,** held at right angles to the arm and at arm's length from the eye, cover about 7 degrees. And an angle of 7° corresponds to about 12.2 feet in 100 feet; or to 36.6 feet in 100 yards; or to 645 feet in a mile.

(2) **By means of a two-foot rule,** either on a drawing or between distant objects in the field. If the inner edges of a common two-foot rule be opened to the extent shown in the column of inches, they will be inclined to each other at the angles shown in the column of angles. Since an opening of ⅛ inch (up to 19 inches or about 105°) corresponds to from about ½° to 1°, no great accuracy is to be expected, and beyond 105° still less; for the liability to error then increases very rapidly as the opening becomes greater. Thus, the last ⅛ inch corresponds to about 12°.

Angles for openings intermediate of those given may be calculated to the nearest minute or two, by simple proportion, up to 23 inches of opening, or about 147°.

Table of Angles corresponding to openings of a 2-foot rule.

(Original).

Correct.

Ins.	Deg.	min.	Ins.	Deg.	min.	Ins.	Deg.	min.	Ins.	Deg.	min.	Ins.	Deg.	min.	Ins.	Deg.	min.
¼	1	12	4¼	20	24	8¼	40	13	12¼	61	23	16¼	85	14	20¼	115	5
	1	48		21			40	51		62	5		86	3		116	12
½	2	24	½	21	37	½	41	29	½	62	47	½	86	52	½	117	20
	3	00		22	13		42	7		63	28		87	41		118	30
¾	3	36	¾	22	50	¾	42	46	¾	64	11	¾	88	31	¾	119	40
	4	11		23	27		43	24		64	53		89	21		120	52
1	4	47	5	24	3	9	44	3	13	65	35	17	90	12	21	122	6
	5	23		24	39		44	42		66	18		91	3		123	20
¼	5	58	¼	25	16	¼	45	21	¼	67	1	¼	91	54	¼	124	36
	6	34		25	53		45	59		67	44		92	46		125	54
½	7	10	½	26	30	½	46	38	½	68	28	½	93	38	½	127	14
	7	46		27	7		47	17		69	12		94	31		128	35
¾	8	22	¾	27	44	¾	47	56	¾	69	55	¾	95	24	¾	129	59
	8	58		28	21		48	35		70	38		96	17		131	25
2	9	34	6	28	58	10	49	15	14	71	22	18	97	11	22	132	53
	10	10		29	35		49	54		72	6		98	5		134	24
¼	10	46	¼	30	11	¼	50	34	¼	72	51	¼	99	00	¼	135	58
	11	22		30	49		51	13		73	36		99	55		137	35
½	11	58	½	31	26	½	51	53	½	74	21	½	100	51	½	139	16
	12	34		32	3		52	33		75	6		101	48		141	1
¾	13	10	¾	32	40	¾	53	13	¾	75	51	¾	102	45	¾	142	51
	13	46		33	17		53	53		76	36		103	43		144	46
3	14	22	7	33	54	11	54	34	15	77	22	19	104	41	23	146	48
	14	58		34	33		55	14		78	8		105	40		148	58
¼	15	34	¼	35	10	¼	55	55	¼	78	54	¼	106	39	¼	151	17
	16	10		35	47		56	35		79	40		107	40		153	48
½	16	46	½	36	25	½	57	16	½	80	27	½	108	41	½	156	34
	17	22		37	3		57	57		81	14		109	43		159	48
¾	17	59	¾	37	41	¾	58	38	¾	82	2	¾	110	46	¾	163	27
	18	35		38	19		59	19		82	49		111	49		168	18
4	19	12	8	38	57	12	60	00	16	83	37	20	112	53	24	180	00
	19	48		39	35		60	41		84	26		113	58			

(3) **With the same table, using feet instead of inches.** From the given point measure 12 *feet* toward * each object, and place marks. Measure the distance in *feet* between these marks. Suppose the first column in the table to be *feet* instead of inches. Then opposite the distance in *feet* will be the angle.

⅛ foot = 1.5 inches.

1 in. = .083 ft.	4 ins. = .333 ft.	7 ins. = .583 ft.	10 ins. = .833 ft.
2 ins. = .167 ft.	5 ins. = .416 ft.	8 ins. = .667 ft.	11 ins. = .917 ft.
3 ins. = .25 ft.	6 ins. = .5 ft.	9 ins. = .75 ft.	12 ins. = 1.0 ft.

(4) **Or, measure toward * each object** 100 or any other number of feet, and place marks. Measure the distance in feet between the marks. Then

$$\text{Sine of } \textit{half} \text{ the angle} = \frac{\textit{half} \text{ the distance between the marks}}{\text{the distance measured toward one of the objects}}.$$

Find this sine in the table pp. 98, etc.; take out the corresponding angle and multiply it by 2

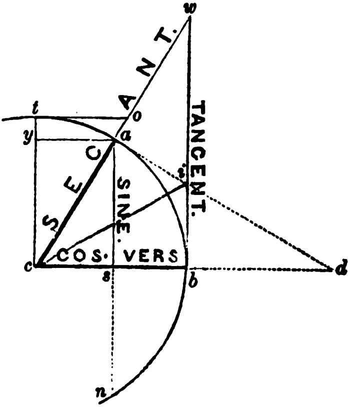
(5) **See** last paragraph of foot-note, pp. 152 and 153.

---

* If it is inconvenient to measure *toward* the objects, measure directly *from* them.

Sines, Tangents, &c.

Sine, *a s*, of any angle, *a c b*, or which is the same thing, the sine of any circular arc, *a b*, which subtends or measures the angle, is a straight line drawn from one end, as *a*, of the arc, at right angles to, and terminating at, the rad *c b*, drawn to the other end *b* of the arc. It is, therefore, equal to half the chord *a n*, of the arc *a b n*, which is equal to twice the arc *a b*; or, the sine of an angle is always equal to half the chord of twice that angle; and vice versa, the chord of an angle is always equal to twice the sine of half the angle.



The sine *t c* of an angle *t c b*, or of an arc *t a b*, of 90°, is equal to the rad of the arc or of the circle; and this sine of 90° is greater than that of any other angle.

Cosine *c s* of an angle *a c b*, is that part of the rad which lies between the sine and the center of the circle. It is always equal to the sine *y a* of the complement *t c a* of *a c b*; or of what *a c b* wants of being 90°. The prefix *co* before sines, &c, means complement; thus, cosine means sine of the complement.

Versed sine *s b* of any angle *a c b*, is that part of the diam which lies between the sine, and the outer end *b*. It is very common, but erroneous, when speaking of bridges, &c, to call the rise or height *s b* of a circular arch *a b n*, its versed sine; while it is actually the versed sine of only half the arch. This absurdity should cease; for the word rise or height is not only more expressive, but is correct.

Tangent *b w* or *a d*, of any angle *a c b*, is a line drawn from, and at right angles to, the end *b* or *a* of either rad *c b*, or *c a*, which forms one of the legs of the angle; and terminating as at *w*, or *d*, in the prolongation of the rad which forms the other leg. This last rad thus prolonged, that is, *c w*, or *c d*, as the case may be, is the **secant** of the angle *a c b*. The angle *t c b* being supposed to be equal to 90°, the angle *t c a* becomes the complement of the angle *a c b*, or what *a c b* wants of being 90°; and the sine *y a* of this complement; its versed sine *t y*; its tangent *t o*; and its secant *c o*, are respectively the co-sine, co-versed sine; co-tangent; and co-secant, of the angle *a c b*. Or, vice versa, the sine, &c, of *a c b*, are the cosine, &c, of *t c a*; because the angle *a c b* is the complement of the angle *t c a*. When the rad *c b*, *c a*, or *c t*, is assumed to be equal to unity, or 1, the corresponding sines, tangents, &c, are called *natural* ones; and their several lengths for diff angles, for said rad of unity, have been calculated; constituting the well-known tables of nat sines, &c. In any circle whose rad is either larger or smaller than 1, the sines, &c, of the angles will be in the same proportion larger or smaller than those in the tables, and are consequently found by mult the sine, &c, of the table, by said larger or smaller rad.

The following table of natural sines, &c. does not contain nat versed sines, co-versed sines, secants, nor cosecants, but these may be found thus; for any angle not exceeding 90 degrees.

Versed Sine. From 1 take the nat cosine.
Co-versed Sine. From 1 take the nat sine.
Secant. Divide 1 by the nat cosine.
Cosecant. Divide 1 by the nat sine.

For angles exceeding 90°; to find the sine, cosine, tangent, cotang, secant, or cosec, (but not the versed sine or co-versed sine), take the angle from 180°; if between 180° and 270° take 180° from the angle; if bet 270° and 360°, take the angle from 360°. Then in each case take from the table the sine, cosine, tang, or cotang of the remainder. Find its secant or cosec as directed above. *For the versed sine;* if between 90° and 270°, add cosine to 1; if bet 270° and 360°, take cosine from 1. (The engineer seldom needs sines, &c, exceeding 180°.

To find the nat sine, cosine, tang, secant, versed sine, &c, of an angle containing seconds. First find that due to the given deg and min; then the next greater one. Take their diff. Then as 60 sec are to this diff, so are the sec only of the given angle to a dec quantity to be **added** to the one first taken out if it is a sine, tang, secant, &c; or to be **subtracted** from it if it is a cosine, cotang, cosecant, &c.

The tangents in the table are strict **trigonometrical** ones; that is, tangents to given *angles;* and which must extend to meet the secants of the angles to which they belong. **Ordinary,** or **geometrical tangents,** as those on p 162, may extend as far as we please. **In the field practice of railroad curves,** two trigonometrical tangents terminate where they meet each other. Each of these tangs is the tang of *half* the curve. It is usually, but improperly, called "the tang of the *curve.*" "Apex dist of the curve," as suggested by Mr Shunk, would be better.

TABLE OF SINES, TANGENTS, COTANGENTS AND COSINES.

0 Deg.						0 Deg.						0 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0000000	·000000	Infinite.	1·000000	60	21	·0061086	·006108	163·7001	·9999813	39	41	·0119261	·011927	83·84350	·9999289	19
1	·0002909	·000291	3437·746	1·000000	59	22	·0063995	·006399	156·2590	·9999795	38	42	·0122170	·012217	81·84704	·9999254	18
2	·0005818	·000582	1718·873	·9999998	58	23	·0066904	·006690	149·4650	·9999776	37	43	·0125079	·012508	79·94343	·9999218	17
3	·0008727	·000872	1145·915	·9999996	57	24	·0069813	·006981	143·2371	·9999756	36	44	·0127987	·012799	78·12634	·9999181	16
4	·0011636	·001163	859·4363	·9999993	56	25	·0072721	·007272	137·5075	·9999736	35	45	·0130896	·013090	76·39000	·9999143	15
5	·0014544	·001454	687·5488	·9999989	55	26	·0075630	·007563	132·2185	·9999714	34	46	·0133805	·013381	74·72916	·9999105	14
6	·0017453	·001745	572·9572	·9999985	54	27	·0078539	·007854	127·3213	·9999692	33	47	·0136713	·013672	73·13899	·9999065	13
7	·0020362	·002036	491·1060	·9999979	53	28	·0081448	·008145	122·7739	·9999668	32	48	·0139622	·013963	71·61507	·9999025	12
8	·0023271	·002327	429·7175	·9999973	52	29	·0084357	·008436	118·5401	·9999644	31	49	·0142530	·014254	70·15334	·9998984	11
9	·0026180	·002618	381·9709	·9999966	51	30	·0087265	·008726	114·5886	·9999619	30	50	·0145439	·014545	68·75008	·9998942	10
10	·0029089	·002908	343·7737	·9999958	50	31	·0090174	·009017	110·8920	·9999593	29	51	·0148348	·014836	67·40185	·9998900	9
11	·0031998	·003199	312·5213	·9999949	49	32	·0093083	·009308	107·4264	·9999567	28	52	·0151256	·015127	66·10547	·9998856	8
12	·0034907	·003490	286·4777	·9999939	48	33	·0095992	·009599	104·1709	·9999539	27	53	·0154165	·015418	64·85800	·9998812	7
13	·0037815	·003781	264·4408	·9999928	47	34	·0098900	·009890	101·1069	·9999511	26	54	·0157073	·015709	63·65674	·9998766	6
14	·0040724	·004072	245·5519	·9999917	46	35	·0101809	·010181	98·21794	·9999482	25	55	·0159982	·016000	62·49915	·9998720	5
15	·0043633	·004363	229·1816	·9999905	45	36	·0104718	·010472	95·48947	·9999452	24	56	·0162890	·016291	61·38290	·9998673	4
16	·0046542	·004654	214·8576	·9999892	44	37	·0107627	·010763	92·90848	·9999421	23	57	·0165799	·016582	60·30582	·9998625	3
17	·0049451	·004945	202·2187	·9999878	43	38	·0110535	·011054	90·46333	·9999389	22	58	·0168707	·016873	59·26587	·9998577	2
18	·0052360	·005236	190·9841	·9999863	42	39	·0113444	·011345	88·14357	·9999357	21	59	·0171616	·017164	58·26117	·9998527	1
19	·0055268	·005526	180·9322	·9999847	41	40	·0116353	·011636	85·93979	·9999323	20	60	·0174524	·017455	57·28996	·9998477	0
20	·0058177	·005817	171·8854	·9999831	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
				Deg. 89.						Deg. 89.						Deg. 89.	

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0174524	·017455	57·28996	·9998477	60
1	·0177432	·017746	56·35059	·9998426	59
2	·0180341	·018037	55·44151	·9998374	58
3	·0183249	·018328	54·56130	·9998321	57
4	·0186158	·018619	53·70858	·9998267	56
5	·0189066	·018910	52·88211	·9998213	55
6	·0191974	·019201	52·08067	·9998157	54
7	·0194883	·019492	51·30315	·9998101	53
8	·0197791	·019783	50·54850	·9998044	52
9	·0200699	·020074	49·81572	·9997986	51
10	·0203608	·020365	49·10388	·9997927	50
11	·0206516	·020656	48·41208	·9997867	49
12	·0209424	·020947	47·73950	·9997807	48
13	·0212332	·021238	47·08534	·9997745	47
14	·0215241	·021529	46·44886	·9997683	46
15	·0218149	·021820	45·82935	·9997620	45
16	·0221057	·022111	45·22614	·9997556	44
17	·0223965	·022402	44·63859	·9997492	43
18	·0226873	·022693	44·06611	·9997426	42
19	·0229781	·022984	43·50812	·9997360	41
20	·0232690	·023275	42·96407	·9997292	40
′	Cosine.	Cotan.	Tang.	Sine.	′

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0235598	·023566	42·43346	·9997224	39
22	·0238506	·023857	41·91579	·9997156	38
23	·0241414	·024148	41·41058	·9997086	37
24	·0244322	·024439	40·91741	·9997015	36
25	·0247230	·024730	40·43583	·9996943	35
26	·0250138	·025021	39·96546	·9996871	34
27	·0253046	·025312	39·50589	·9996798	33
28	·0255954	·025603	39·05677	·9996724	32
29	·0258862	·025894	38·61773	·9996649	31
30	·0261769	·026185	38·18845	·9996573	30
31	·0264677	·026477	37·76861	·9996497	29
32	·0267585	·026768	37·35789	·9996419	28
33	·0270493	·027059	36·95600	·9996341	27
34	·0273401	·027350	36·56265	·9996262	26
35	·0276309	·027641	36·17759	·9996182	25
36	·0279216	·027932	35·80055	·9996101	24
37	·0282124	·028223	35·43128	·9996020	23
38	·0285032	·028514	35·06954	·9995937	22
39	·0287940	·028805	34·71511	·9995854	21
40	·0290847	·029097	34·36777	·9995770	20
′	Cosine.	Cotan.	Tang.	Sine.	′

1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0293755	·029388	34·02730	·9995684	19
42	·0296662	·029679	33·69350	·9995599	18
43	·0299570	·029970	33·36619	·9995512	17
44	·0302478	·030261	33·04517	·9995424	16
45	·0305385	·030552	32·73026	·9995336	15
46	·0308293	·030843	32·42129	·9995247	14
47	·0311200	·031135	32·11809	·9995157	13
48	·0314108	·031426	31·82051	·9995066	12
49	·0317015	·031717	31·52839	·9994974	11
50	·0319922	·032008	31·24157	·9994881	10
51	·0322830	·032299	30·95992	·9994788	9
52	·0325737	·032591	30·68330	·9994693	8
53	·0328644	·032882	30·41158	·9994598	7
54	·0331552	·033173	30·14461	·9994502	6
55	·0334459	·033464	29·88229	·9994405	5
56	·0337366	·033755	29·62449	·9994308	4
57	·0340274	·034047	29·37110	·9994209	3
58	·0343181	·034338	29·12200	·9994110	2
59	·0346088	·034629	28·87708	·9994009	1
60	·0348995	·034920	28·63625	·9993908	0
′	Cosine.	Cotan.	Tang.	Sine.	′

2 Deg. 2 Deg. 2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0348995	·034920	28·63625	·9993908	60	21	·0410037	·041038	24·36750	·9991590	39	41	·0468159	·046867	21.33685	·9989035	19
1	·0351902	·035212	28·39939	·9993806	59	22	·0412944	·041329	24·19571	·9991470	38	42	·0471065	·047158	21·20494	·9988899	18
2	·0354809	·035503	28·16642	·9993704	58	23	·0415850	·041621	24·02632	·9991350	37	43	·0473970	·047450	21·07466	·9988761	17
3	·0357716	·035794	27·93723	·9993600	57	24	·0418757	·041912	23·85927	·9991228	36	44	·0476876	·047741	20·94596	·9988623	16
4	·0360623	·036085	27·71174	·9993495	56	25	·0421663	·042203	23·69453	·9991106	35	45	·0479781	·048033	20·81882	·9988484	15
5	·0363530	·036377	27·48985	·9993390	55	26	·0424569	·042495	23·53205	·9990983	34	46	·0482687	·048325	20·69322	·9988344	14
6	·0366437	·036668	27·27148	·9993284	54	27	·0427475	·042786	23·37177	·9990859	33	47	·0485592	·048616	20·56911	·9988203	13
7	·0369344	·036959	27·05655	·9993177	53	28	·0430382	·043078	23·21366	·9990734	32	48	·0488498	·048908	20·44648	·9988061	12
8	·0372251	·037250	26·84498	·9993069	52	29	·0433288	·043369	23·05767	·9990609	31	49	·0491403	·049199	20·32530	·9987919	11
9	·0375158	·037542	26·63669	·9992960	51	30	·0436194	·043660	22·90376	·9990482	30	50	·0494308	·049491	20·20555	·9987775	10
10	·0378065	·037833	26·43160	·9992851	50	31	·0439100	·043952	22·75189	·9990355	29	51	·0497214	·049782	20·08719	·9987631	9
11	·0380971	·038124	26·22963	·9992740	49	32	·0442006	·044243	22·60201	·9990227	28	52	·0500119	·050074	19·97021	·9987486	8
12	·0383878	·038416	26·03073	·9992629	48	33	·0444912	·044535	22·45409	·9990098	27	53	·0503024	·050366	19·85459	·9987340	7
13	·0386785	·038707	25·83482	·9992517	47	34	·0447818	·044826	22·30809	·9989968	26	54	·0505929	·050657	19·74029	·9987194	6
14	·0389692	·038998	25·64183	·9992404	46	35	·0450724	·045118	22·16398	·9989837	25	55	·0508835	·050949	19·62729	·9987046	5
15	·0392598	·039290	25·45170	·9992290	45	36	·0453630	·045409	22·02171	·9989706	24	56	·0511740	·051241	19·51558	·9986898	4
16	·0395505	·039581	25·26436	·9992176	44	37	·0456536	·045701	21·88125	·9989573	23	57	·0514645	·051532	19·40513	·9986748	3
17	·0398411	·039872	25·07975	·9992060	43	38	·0459442	·045992	21·74256	·9989440	22	58	·0517550	·051824	19·29592	·9986598	2
18	·0401318	·040164	24·89782	·9991944	42	39	·0462347	·046284	21·60563	·9989306	21	59	·0520455	·052116	19·18793	·9986447	1
19	·0404224	·040455	24·71851	·9991827	41	40	·0465253	·046575	21·47040	·9989171	20	60	·0523360	·052407	19·08113	·9986295	0
20	·0407131	·040746	24·54175	·9991709	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87. Deg. 87. Deg. 87.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526264	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	.053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

4 Deg. 4 Deg. 4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0697565	·069926	14·30066	·9975641	60	21	·0758489	·076068	13·14612	·9971193	39	41	·0816486	·081922	12·20671	·9966612	19
1	·0700467	·070219	14·24113	·9975437	59	22	·0761390	·076360	13·09575	·9970972	38	42	·0819385	·082215	12·16323	·9966374	18
2	·0703368	·070511	14·18209	·9975233	58	23	·0764290	·076653	13.04576	·9970750	37	43	·0822284	·082507	12·12006	·9966135	17
3	·0706270	·070803	14·12353	·9975028	57	24	·0767190	·076945	12·99616	·9970528	36	44	·0825183	·082800	12·07719	·9965895	16
4	·0709171	·071096	14·06545	·9974822	56	25	·0770091	·077238	12·94692	·9970304	35	45	·0828082	·083093	12·03462	·9965655	15
5	·0712073	·071388	14·00785	·9974615	55	26	·0772991	·077531	12·89805	·9970080	34	46	·0830981	·083386	11·99234	·9965414	14
6	·0714974	·071680	13·95071	·9974408	54	27	·0775891	·077823	12·84955	·9969854	33	47	·0833880	·083679	11·95037	·9965172	13
7	·0717876	·071973	13·89404	·9974199	53	28	·0778791	·078116	12·80141	·9969628	32	48	·0836778	·083972	11·90868	·9964929	12
8	·0720777	·072265	13·83782	·9973990	52	29	·0781691	·078409	12·75363	·9969401	31	49	·0839677	·084265	11·86728	·9964685	11
9	·0723678	·072558	13·78206	·9973780	51	30	·0784591	·078701	12·70620	·9969173	30	50	·0842576	·084558	11·82616	·9964440	10
10	·0726580	·072850	13·72673	·9973569	50	31	·0787491	·078994	12·65912	·9968945	29	51	·0845474	·084851	11·78533	·9964195	9
11	·0729481	·073143	13·67185	·9973357	49	32	·0790391	·079287	12·61239	·9968715	28	52	·0848373	·085144	11·74477	·9963948	8
12	·0732382	·073435	13·61740	·9973145	48	33	·0793290	·079579	12·56599	·9968485	27	53	·0851271	·085437	11·70450	·9963701	7
13	·0735283	·073727	13·56339	·9972931	47	34	·0796190	·079872	12·51994	·9968254	26	54	·0854169	·085730	11·66449	·9963453	6
14	·0738184	·074020	13·50979	·9972717	46	35	·0799090	.080165	12·47422	·9968022	25	55	·0857067	·086023	11·62476	·9963204	5
15	·0741085	·074312	13·45662	·9972502	45	36	·0801989	·080458	12·42883	·9967789	24	56	·0859966	·086316	11·58529	·9962954	4
16	·0743986	·074605	13·40386	·9972286	44	37	·0804889	·080750	12·38376	·9967555	23	57	·0862864	·086609	11·54609	·9962704	3
17	·0746887	·074897	13·35151	·9972069	43	38	·0807788	·081043	12·33902	·9967321	22	58	·0865762	·086902	11·50715	·9962452	2
18	·0749787	·075190	13·29957	·9971851	42	39	·0810687	·081336	12·29460	·9967085	21	59	·0868660	·087195	11·46847	·9962200	1
19	·0752688	·075482	13·24803	·9971633	41	40	·0813587	·081629	12·25050	·9966849	20	60	·0871557	·087488	11·43005	·9961947	0
20	·0755589	·075775	13·19688	·9971413	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85. Deg. 85. Deg. 85.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0871557	·087488	11·43005	·9961947	60
1	·0874455	·087781	11·39188	·9961693	59
2	·0877353	·088074	11·35397	·9961438	58
3	·0880251	·088368	11·31630	·9961183	57
4	·0883148	·088661	11·27888	·9960926	56
5	·0886046	·088954	11·24171	·9960669	55
6	·0888943	·089247	11·20478	·9960411	54
7	·0891840	·089540	11·16808	·9960152	53
8	·0894738	·089834	11·13163	·9959892	52
9	·0897635	·090127	11·09541	·9959631	51
10	·0900532	·090420	11·05943	·9959370	50
11	·0903429	·090713	11·02367	·9959107	49
12	·0906326	·091007	10·98815	·9958844	48
13	·0909223	·091300	10·95285	·9958580	47
14	·0912119	·091593	10·91777	·9958315	46
15	·0915016	·091887	10·88292	·9958049	45
16	·0917913	·092180	10·84828	·9957783	44
17	·0920809	·092473	10·81387	·9957515	43
18	·0923706	·092767	10·77967	·9957247	42
19	·0926602	·093060	10·74568	·9956978	41
20	·0929499	·093354	10·71191	·9956708	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0932395	·093647	10·67834	·9956437	39
22	·0935291	·093940	10·64499	·9956165	38
23	·0938187	·094234	10·61184	·9955892	37
24	·0941083	·094527	10·57889	·9955620	36
25	·0943979	·094821	10·54615	·9955345	35
26	·0946875	·095114	10·51360	·9955070	34
27	·0949771	·095408	10·48126	·9954794	33
28	·0952666	·095701	10·44911	·9954517	32
29	·0955562	·095995	10·41715	·9954240	31
30	·0958458	·096289	10·38539	·9953962	30
31	·0961353	·096582	10·35382	·9953683	29
32	·0964248	·096876	10·32244	·9953403	28
33	·0967144	·097169	10·29125	·9953122	27
34	·0970039	·097463	10·26024	·9952840	26
35	·0972934	·097757	10·22942	·9952557	25
36	·0975829	·098050	10·19878	·9952274	24
37	·0978724	·098344	10·16833	·9951990	23
38	·0981619	·098638	10·13805	·9951705	22
39	·0984514	·098932	10·10795	·9951419	21
40	·0987408	·099225	10·07803	·9951132	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0990303	·099519	10·04828	·9950844	19
42	·0993197	·099813	10·01871	·9950556	18
43	·0996092	·100107	9·989305	·9950266	17
44	·0998986	·100400	9·960072	·9949976	16
45	·1001881	·100694	9·931008	·9949685	15
46	·1004775	·100988	9·902112	·9949393	14
47	·1007669	·101282	9·873382	·9949101	13
48	·1010563	·101576	9·844816	·9948807	12
49	·1013457	·101870	9·816414	·9948513	11
50	·1016351	·102164	9·788173	·9948217	10
51	·1019245	·102458	9·760092	·9947921	9
52	·1022138	·102752	9·732171	·9947625	8
53	·1025032	·103046	9·704407	·9947327	7
54	·1027925	·103339	9·676800	·9947028	6
55	·1030819	·103634	9·649347	·9946729	5
56	·1033712	·103928	9·622048	·9946428	4
57	·1036605	·104222	9·594902	·9946127	3
58	·1039499	·104516	9·567906	·9945825	2
59	·1042392	·104810	9·541061	·9945523	1
60	·1045285	·105104	9·514364	·9945219	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60
1	·1048178	·105398	9·487814	·9944914	59
2	·1051070	·105692	9·461411	·9944609	58
3	·1053963	·105986	9·435153	·9944303	57
4	·1056856	·106280	9·409038	·9943996	56
5	·1059748	·106575	9·383066	·9943688	55
6	·1062641	·106869	9·357235	·9943379	54
7	·1065533	·107163	9·331545	·9943070	53
8	·1068425	·107457	9·305993	·9942760	52
9	·1071318	·107751	9·280580	·9942448	51
10	·1074210	·108046	9·255303	·9942136	50
11	·1077102	·108340	9·230162	·9941823	49
12	·1079994	·108634	9·205156	·9941510	48
13	·1082885	·108929	9·180283	·9941195	47
14	·1085777	·109223	9·155543	·9940880	46
15	·1088669	·109517	9·130934	·9940563	45
16	·1091560	·109812	9·106456	·9940246	44
17	·1094452	·110106	9·082107	·9939928	43
18	·1097343	·110401	9·057886	·9939610	42
19	·1100234	·110695	9·033793	·9939290	41
20	·1103126	·110989	9·009826	·9938969	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1106017	·111284	8·985984	·9938648	39
22	·1108908	·111578	8·962266	·9938326	38
23	·1111799	·111873	8·938672	·9938003	37
24	·1114689	·112168	8·915200	·9937679	36
25	·1117580	·112462	8·891850	·9937355	35
26	·1120471	·112757	8·868620	·9937029	34
27	·1123361	·113051	8·845510	·9936703	33
28	·1126252	·113346	8·822518	·9936375	32
29	·1129142	·113641	8·799644	·9936047	31
30	·1132032	·113935	8·776887	·9935719	30
31	·1134922	·114230	8·754246	·9935389	29
32	·1137812	·114525	8·731719	·9935058	28
33	·1140702	·114819	8·709307	·9934727	27
34	·1143592	·115114	8·687008	·9934395	26
35	·1146482	·115409	8·664822	·9934062	25
36	·1149372	·115703	8·642747	·9933728	24
37	·1152261	·115998	8·620783	·9933393	23
38	·1155151	·116293	8·598929	·9933057	22
39	·1158040	·116588	8·577183	·9932721	21
40	·1160929	·116883	8·555546	·9932384	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1163818	·117178	8·534017	·9932045	19
42	·1166707	·117473	8·512594	·9931706	18
43	·1169596	·117767	8·491277	·9931367	17
44	·1172485	·118062	8·470065	·9931026	16
45	·1175374	·118357	8·448957	·9930685	15
46	·1178263	·118652	8·427953	·9930342	14
47	·1181151	·118947	8·407051	·9929999	13
48	·1184040	·119242	8·386251	·9929655	12
49	·1186928	·119537	8·365553	·9929310	11
50	·1189816	·119832	8·344955	·9928965	10
51	·1192704	·120127	8·324457	·9928618	9
52	·1195593	·120423	8·304058	·9928271	8
53	·1198481	·120718	8·283757	·9927922	7
54	·1201368	·121013	8·263554	·9927573	6
55	·1204256	·121308	8·243448	·9927224	5
56	·1207144	·121603	8·223488	·9926873	4
57	·1210031	·121898	8·203523	·9926521	3
58	·1212919	·122194	8·183704	·9926169	2
59	·1215806	·122489	8·163978	·9925816	1
60	·1218693	·122784	8·144346	·9925162	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60
1	·1221581	·123079	8·124807	·9925107	59
2	·1224468	·123375	8·105359	·9924751	58
3	·1227355	·123670	8·086004	·9924394	57
4	·1230241	·123965	8·066739	·9924037	56
5	·1233128	·124261	8·047564	·9923679	55
6	·1236015	·124556	8·028479	·9923319	54
7	·1238901	·124852	8·009483	·9922959	53
8	·1241788	·125147	7·990575	·9922599	52
9	·1244674	·125442	7·971755	·9922237	51
10	·1247560	·125738	7·953022	·9921874	50
11	·1250446	·126033	7·934375	·9921511	49
12	·1253332	·126329	7·915815	·9921147	48
13	·1256218	·126624	7·897339	·9920782	47
14	·1259104	·126920	7·878948	·9920416	46
15	·1261990	·127216	7·860642	·9920049	45
16	·1264875	·127511	7·842419	·9919682	44
17	·1267761	·127807	7·824279	·9919314	43
18	·1270646	·128103	7·806221	·9918944	42
19	·1273531	·128398	7·788245	·9918574	41
20	·1276416	·128694	7·770350	·9918204	40
′	Cosine.	Cotan.	Tang.	Sine.	′

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1279302	·128990	7·752536	·9917832	39
22	·1282186	·129285	7·734802	·9917459	38
23	·1285071	·129581	7·717148	·9917086	37
24	·1287956	·129877	7·699573	·9916712	36
25	·1290841	·130173	7·682076	·9916337	35
26	·1293725	·130469	7·664658	·9915961	34
27	·1296609	·130764	7·647317	·9915584	33
28	·1299494	·131060	7·630053	·9915206	32
29	·1302378	·131356	7·612865	·9914828	31
30	·1305262	·131652	7·595754	·9914449	30
31	·1308146	·131948	7·578717	·9914069	29
32	·1311030	·132244	7·561756	·9913688	28
33	·1313913	·132540	7·544869	·9913306	27
34	·1316797	·132836	7·528057	·9912923	26
35	·1319681	·133132	7·511317	·9912540	25
36	·1322564	·133428	7·494651	·9912155	24
37	·1325447	·133724	7·478057	·9911770	23
38	·1328330	·134020	7·461535	·9911384	22
39	·1331213	·134316	7·445085	·9910997	21
40	·1334096	·134612	7·428706	·9910610	20
′	Cosine.	Cotan.	Tang.	Sine.	′

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1336979	·134909	7·412397	·9910221	19
42	·1339862	·135205	7·396159	·9909832	18
43	·1342744	·135501	7·379990	·9909442	17
44	·1345627	·135797	7·363891	·9909051	16
45	·1348509	·136094	7·347861	·9908659	15
46	·1351392	·136390	7·331898	·9908266	14
47	·1354274	·136686	7·316004	·9907873	13
48	·1357156	·136983	7·300178	·9907478	12
49	·1360038	·137279	7·284418	·9907083	11
50	·1362919	·137575	7·268725	·9906687	10
51	·1365801	·137872	7·253098	·9906290	9
52	·1368683	·138168	7·237537	·9905893	8
53	·1371564	·138465	7·222042	·9905494	7
54	·1374445	·138761	7·206611	·9905095	6
55	·1377327	·139058	7·191245	·9904694	5
56	·1380208	·139354	7·175943	·9904293	4
57	·1383089	·139651	7·160705	·9903891	3
58	·1385970	·139947	7·145530	·9903489	2
59	·1388850	·140244	7·130419	·9903085	1
60	·1391731	·140540	7·115369	·9902681	0
′	Cosine.	Cotan.	Tang.	Sine.	′

8 Deg.

8 Deg.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1391731	·140540	7·115369	·9902681	60	21	·1452197	·146775	6·813122	·9893994	39	41	·1509733	·152723	6·547767	·9885378	19
1	·1394612	·140837	7·100382	·9902275	59	22	·1455075	·147072	6·799356	·9893572	38	42	·1512608	·153021	6·535029	·9884939	18
2	·1397492	·141134	7·085457	·9901869	58	23	·1457953	·147369	6·785644	·9893148	37	43	·1515484	·153319	6·522339	·9884498	17
3	·1400372	·141430	7·070593	·9901462	57	24	·1460830	·147667	6·771986	·9892723	36	44	·1518359	·153617	6·509698	·9884057	16
4	·1403252	·141727	7·055790	·9901055	56	25	·1463708	·147964	6·758382	·9892298	35	45	·1521234	·153914	6·497104	·9883615	15
5	·1406132	·142024	7·041048	·9900646	55	26	·1466585	·148261	6·744831	·9891872	34	46	·1524109	·154212	6·484558	·9883172	14
6	·1409012	·142321	7·026366	·9900237	54	27	·1469463	·148559	6·731334	·9891445	33	47	·1526984	·154510	6·472059	·9882728	13
7	·1411892	·142617	7·011744	·9899826	53	28	·1472340	·148856	6·717889	·9891017	32	48	·1529858	·154808	6·459607	·9882284	12
8	·1414772	·142914	6·997180	·9899415	52	29	·1475217	·149153	6·704496	·9890588	31	49	·1532733	·155106	6·447201	·9881838	11
9	·1417651	·143211	6·982678	·9899003	51	30	·1478094	·149451	6·691156	·9890159	30	50	·1535607	·155404	6·434842	·9881392	10
10	·1420531	·143508	6·968233	·9898590	50	31	·1480971	·149748	6·677867	·9889728	29	51	·1538482	·155701	6·422530	·9880945	9
11	·1423410	·143805	6·953847	·9898177	49	32	·1483848	·150045	6·664630	·9889297	28	52	·1541356	·155999	6·410263	·9880497	8
12	·1426289	·144102	6·939519	·9897762	48	33	·1486724	·150343	6·651444	·9888865	27	53	·1544230	·156297	6·398042	·9880048	7
13	·1429168	·144399	6·925248	·9897347	47	34	·1489601	·150640	6·638310	·9888432	26	54	·1547104	·156595	6·385866	·9879599	6
14	·1432047	·144696	6·911035	·9896931	46	35	·1492477	·150938	6·625225	·9887998	25	55	·1549978	·156893	6·373735	·9879148	5
15	·1434926	·144993	6·896879	·9896514	45	36	·1495353	·151235	6·612191	·9887564	24	56	·1552851	·157191	6·361650	·9878697	4
16	·1437805	·145290	6·882780	·9896096	44	37	·1498230	·151533	6·599208	·9887128	23	57	·1555725	·157490	6·349609	·9878245	3
17	·1440684	·145587	6·868737	·9895677	43	38	·1501106	·151830	6·586273	·9886692	22	58	·1558598	·157788	6·337612	·9877792	2
18	·1443562	·145884	6·854750	·9895258	42	39	·1503981	·152128	6·573389	·9886255	21	59	·1561472	·158086	6·325660	·9877338	1
19	·1446440	·146181	6·840819	·9894838	41	40	·1506857	·152426	6·560553	·9885817	20	60	·1564345	·158384	6·313751	·9876883	0
20	·1449319	·146478	6·826943	·9894416	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81.

Deg. 81.

Deg. 81.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1564345	·158384	6·313751	·9876883	60
1	·1567218	·158682	6·301886	·9876428	59
2	·1570091	·158980	6·290065	·9875972	58
3	·1572963	·159279	6·278286	·9875514	57
4	·1575836	·159577	6·266551	·9875057	56
5	·1578708	·159875	6·254858	·9874598	55
6	·1581581	·160174	6·243208	·9874138	54
7	·1584453	·160472	6·231600	·9873678	53
8	·1587325	·160770	6·220034	·9873216	52
9	·1590197	·161069	6·208510	·9872754	51
10	·1593069	·161367	6·197027	·9872291	50
11	·1595940	·161666	6·185586	·9871827	49
12	·1598812	·161964	6·174186	·9871363	48
13	·1601683	·162263	6·162827	·9870897	47
14	·1604555	·162561	6·151508	·9870431	46
15	·1607426	·162860	6·140230	·9869964	45
16	·1610297	·163159	6·128992	·9869496	44
17	·1613167	·163457	6·117794	·9869027	43
18	·1616038	·163756	6·106636	·9868557	42
19	·1618909	·164055	6·095517	·9868087	41
20	·1621779	·164353	6·084438	·9867615	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1624650	·164652	6·073397	·9867143	39
22	·1627520	·164951	6·062396	·9866670	38
23	·1630390	·165250	6·051434	·9866196	37
24	·1633260	·165548	6·040510	·9865722	36
25	·1636129	·165847	6·029624	·9865246	35
26	·1638999	·166146	6·018777	·9864770	34
27	·1641868	·166445	6·007967	·9864293	33
28	·1644738	·166744	5·997195	·9863815	32
29	·1647607	·167043	5·986461	9863336	31
30	·1650476	·167342	5·975764	·9862856	30
31	·1653345	·167641	5·965104	·9862375	29
32	·1656214	·167940	5·954481	·9861894	28
33	·1659082	·168239	5·943895	·9861412	27
34	·1661951	·168539	5·933345	·9860929	26
35	·1664819	·168838	5·922832	·9860445	25
36	·1667687	·169137	5·912355	·9859960	24
37	·1670556	·169436	5·901913	·9859475	23
38	·1673423	·169735	5·891508	·9858988	22
39	·1676291	·170035	5·881138	·9858501	21
40	·1679159	·170334	5·870804	·9858013	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1682026	·170633	5·860505	·9857524	19
42	·1684894	·170933	5·850241	·9857035	18
43	·1687761	·171232	5·840011	·9856544	17
44	·1690628	·171532	5·829817	·9856053	16
45	·1693495	·171831	5·819657	·9855561	15
46	·1696362	·172130	5·809531	·9855068	14
47	·1699228	·172430	5·799440	·9854574	13
48	·1702095	·172730	5·789382	·9854079	12
49	·1704961	·173029	5·779358	·9853583	11
50	·1707828	·173329	5·769368	·9853087	10
51	·1710694	·173628	5·759412	·9852590	9
52	·1713560	·173928	5·749488	·9852092	8
53	·1716425	·174228	5·739598	·9851593	7
54	·1719291	·174527	5·729741	·9851093	6
55	·1722156	·174827	5·719917	·9850593	5
56	·1725022	·175127	5·710125	·9850091	4
57	·1727887	·175427	5·700366	·9849589	3
58	·1730752	·175727	5·690639	·9849086	2
59	·1733617	·176027	5·680944	·9848582	1
60	·1736482	·176327	5·671281	·9848078	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1736482	·176327	5·671281	·9848078	60
1	·1739346	·176626	5·661650	·9847572	59
2	·1742211	·176926	5·652051	·9847066	58
3	·1745075	·177226	5·642483	·9846558	57
4	·1747939	·177527	5·632947	·9846050	56
5	·1750803	·177827	5·623442	·9845542	55
6	·1753667	·178127	5·613968	·9845032	54
7	·1756531	·178427	5·604524	·9844521	53
8	·1759395	·178727	5·595112	·9844010	52
9	·1762258	·179027	5·585730	·9843498	51
10	·1765121	·179327	5·576378	·9842985	50
11	·1767984	·179628	5·567057	·9842471	49
12	·1770847	·179928	5·557766	·9841956	48
13	·1773710	·180228	5·548505	·9841441	47
14	·1776573	·180529	5·539274	·9840924	46
15	·1779435	·180829	5·530072	·9840407	45
16	·1782298	·181129	5·520900	·9839889	44
17	·1785160	·181430	5·511757	·9839370	43
18	·1788022	·181730	5·502644	·9838850	42
19	·1790884	·182031	5·493560	·9838330	41
20	·1793746	·182331	5·484505	·9837808	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1796607	·182632	5·475478	·9837286	39
22	·1799469	·182933	5·466481	·9836763	38
23	·1802330	·183233	5·457512	·9836239	37
24	·1805191	·183534	5·448571	·9835715	36
25	·1808052	·183835	5·439659	·9835189	35
26	·1810913	·184135	5·430775	·9834663	34
27	·1813774	·184436	5·421918	·9834136	33
28	·1816635	·184737	5·413090	·9833608	32
29	·1819495	·185038	5·404290	·9833079	31
30	·1822355	·185339	5·395517	·9832549	30
31	·1825215	·185639	5·386771	·9832019	29
32	·1828075	·185940	5·378053	·9831487	28
33	·1830935	·186241	5·369363	·9830955	27
34	·1833795	·186542	5·360699	·9830422	26
35	·1836654	·186843	5·352062	·9829888	25
36	·1839514	·187144	5·343452	·9829353	24
37	·1842373	·187446	5·334869	·9828818	23
38	·1845232	·187747	5·326313	·9828282	22
39	·1848091	·188048	5·317783	·9827744	21
40	·1850949	·188349	5·309279	·9827206	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1853808	·188650	5·300801	·9826668	19
42	·1856666	·188952	5·292350	·9826128	18
43	·1859524	·189253	5·283925	·9825587	17
44	·1862382	·189554	5·275525	·9825046	16
45	·1865240	·189855	5·267151	·9824504	15
46	·1868098	·190157	5·258803	·9823961	14
47	·1870956	·190458	5·250480	·9823417	13
48	·1873813	·190760	5·242183	·9822873	12
49	·1876670	·191061	5·233911	·9822327	11
50	·1879528	·191363	5·225664	·9821781	10
51	·1882385	·191664	5·217442	·9821234	9
52	·1885241	·191966	5·209245	·9820686	8
53	·1888098	·192268	5·201073	·9820137	7
54	·1890954	·192569	5·192926	·9819587	6
55	·1893811	·192871	5·184803	·9819037	5
56	·1896667	·193173	5·176705	·9818485	4
57	·1899523	·193474	5·168631	·9817933	3
58	·1902379	·193776	5·160581	·9817380	2
59	·1905234	·194078	5·152555	·9816826	1
60	·1908090	·194380	5·144554	·9816272	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

11 Deg. 11 Deg. 11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1908090	·194380	5·144554	·9816272	60	21	·1968018	·200727	4·981881	·9804433	39	41	·2025024	·206786	4·835901	·9792818	19
1	·1910945	·194682	5·136576	·9815716	59	22	·1970870	·201030	4·974381	·9803860	38	42	·2027873	·207090	4·828817	·9792228	18
2	·1913801	·194984	5·128622	·9815160	58	23	·1973722	·201332	4·966903	·9803286	37	43	·2030721	·207393	4·821753	·9791638	17
3	·1916656	·195286	5·120692	·9814603	57	24	·1976573	·201635	4·959447	·9802712	36	44	·2033569	·207696	4·814709	·9791047	16
4	·1919510	·195588	5·112785	·9814045	56	25	·1979425	·201938	4·952012	·9802136	35	45	·2036418	·208000	4·807685	·9790455	15
5	·1922365	·195890	5·104902	·9813486	55	26	·1982276	·202240	4·944599	·9801560	34	46	·2039265	·208303	4·800680	·9789862	14
6	·1925220	·196192	5·097042	·9812927	54	27	·1985127	·202543	4·937206	·9800983	33	47	·2042113	·208607	4·793695	·9789268	13
7	·1928074	·196494	5·089206	·9812366	53	28	·1987978	·202846	4·929835	·9800405	32	48	·2044961	·208910	4·786730	·9788674	12
8	·1930928	·196796	5·081392	·9811805	52	29	·1990829	·203149	4·922485	·9799827	31	49	·2047808	·209214	4·779783	·9788079	11
9	·1933782	·197098	5·073602	·9811243	51	30	·1993679	·203452	4·915157	·9799247	30	50	·2050655	·209518	4·772856	·9787483	10
10	·1936636	·197400	5·065835	·9810680	50	31	·1996530	·203755	4·907849	·9798667	29	51	·2053502	·209821	4·765949	·9786886	9
11	·1939490	·197703	5·058090	·9810116	49	32	·1999380	·204058	4·900562	·9798086	28	52	·2056349	·210125	4·759060	·9786288	8
12	·1942344	·198005	5·050369	·9809552	48	33	·2002230	·204361	4·893295	·9797504	27	53	·2059195	·210429	4·752190	·9785689	7
13	·1945197	·198307	5·042670	·9808986	47	34	·2005080	·204664	4·886049	·9796921	26	54	·2062042	·210733	4·745340	·9785090	6
14	·1948050	·198610	5·034993	·9808420	46	35	·2007930	·204967	4·878824	·9796337	25	55	·2064888	·211036	4·738508	·9784490	5
15	·1950903	·198912	5·027339	·9807853	45	36	·2010779	·205270	4·871620	·9795752	24	56	·2067734	·211340	4·731695	·9783889	4
16	·1953756	·199214	5·019707	·9807285	44	37	·2013629	·205573	4·864435	·9795167	23	57	·2070580	·211644	4·724901	·9783287	3
17	·1956609	·199517	5·012098	·9806716	43	38	·2016478	·205876	4·857271	·9794581	22	58	·2073426	·211948	4·718125	·9782684	2
18	·1959461	·199819	5·004511	·9806147	42	39	·2019327	·206180	4·850128	·9793994	21	59	·2076272	·212252	4·711368	·9782080	1
19	·1962314	·200122	4·996945	·9805576	41	40	·2022176	·206483	4·843004	·9793406	20	60	·2079117	·212556	4·704630	·9781476	0
20	·1965166	·200424	4·989402	·9805005	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78. Deg. 78. Deg. 78.

12 Deg. 12 Deg. 12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2079117	·212556	4·704630	·9781476	60	21	·2138829	·218949	4·567261	·9768593	39	41	·2195624	·225054	4·443376	·9755985	19
1	·2081962	·212860	4·697910	·9780871	59	22	·2141671	·219254	4·560911	·9767970	38	42	·2198462	·225359	4·437350	·9755345	18
2	·2084807	·213164	4·691208	·9780265	58	23	·2144512	·219559	4·554577	·9767347	37	43	·2201300	·225665	4·431339	·9754706	17
3	·2087652	·213468	4·684524	·9779658	57	24	·2147353	·219864	4·548260	·9766723	36	44	·2204137	·225971	4·425343	·9754065	16
4	·2090497	·213773	4·677859	·9779050	56	25	·2150194	·220169	4·541960	·9766098	35	45	·2206974	·226276	4·419364	·9753423	15
5	·2093341	·214077	4·671212	·9778441	55	26	·2153035	·220474	4·535677	·9765472	34	46	·2209811	·226582	4·413399	·9752781	14
6	·2096186	·214381	4·664583	·9777832	54	27	·2155876	·220779	4·529410	·9764845	33	47	·2212648	·226888	4·407450	·9752138	13
7	·2099030	·214685	4·657972	·9777222	53	28	·2158716	·221084	4·523160	·9764217	32	48	·2215485	·227194	4·401516	·9751494	12
8	·2101874	·214990	4·651378	·9776611	52	29	·2161556	·221389	4·516926	·9763589	31	49	·2218321	·227500	4·395597	·9750849	11
9	·2104718	·215294	4·644803	·9775999	51	30	·2164396	·221694	4·510708	·9762960	30	50	·2221158	·227806	4·389694	·9750203	10
10	·2107561	·215598	4·638245	·9775386	50	31	·2167236	·221999	4·504507	·9762330	29	51	·2223994	·228112	4·383805	·9749556	9
11	·2110405	·215903	4·631705	·9774773	49	32	·2170076	·222305	4·498322	·9761699	28	52	·2226830	·228418	4·377931	·9748909	8
12	·2113248	·216207	4·625183	·9774159	48	33	·2172915	·222610	4·492153	·9761067	27	53	·2229666	·228724	4·372073	·9748261	7
13	·2116091	·216512	4·618678	·9773544	47	34	·2175754	·222915	4·486000	·9760435	26	54	·2232501	·229030	4·366229	·9747612	6
14	·2118934	·216816	4·612190	·9772928	46	35	·2178593	·223221	4·479863	·9759802	25	55	·2235337	·229336	4·360400	·9746962	5
15	·2121777	·217121	4·605720	·9772311	45	36	·2181432	·223526	4·473742	·9759168	24	56	·2238172	·229642	4·354586	·9746311	4
16	·2124619	·217425	4·599268	·9771693	44	37	·2184271	·223831	4·467637	·9758533	23	57	·2241007	·229949	4·348786	·9745660	3
17	·2127462	·217730	4·592832	·9771075	43	38	·2187110	·224137	4·461548	·9757897	22	58	·2243842	·230255	4·343001	·9745008	2
18	·2130304	·218035	4·586414	·9770456	42	39	·2189948	·224442	4·455475	·9757260	21	59	·2246676	·230561	4·337231	·9744355	1
19	·2133146	·218340	4·580012	·9769836	41	40	·2192786	·224748	4·449418	·9756623	20	60	·2249511	·230868	4·331475	·9743701	0
20	·2135988	·218644	4·573628	·9769215	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77. Deg. 77. Deg 77.

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2249511	·230868	4·331475	·9743701	60
1	·2252345	·231174	4·325734	·9743046	59
2	·2255179	·231481	4·320007	·9742390	58
3	·2258013	·231787	4·314295	·9741734	57
4	·2260846	·232094	4·308597	·9741077	56
5	·2263680	·232400	4·302913	·9740419	55
6	·2266513	·232707	4·297244	·9739760	54
7	·2269346	·233014	4·291588	·9739100	53
8	·2272179	·233320	4·285947	·9738439	52
9	·2275012	·233627	4·280319	·9737778	51
10	·2277844	·233934	4·274706	·9737116	50
11	·2280677	·234241	4·269107	·9736453	49
12	·2283509	·234547	4·263521	·9735789	48
13	·2286341	·234854	4·257950	·9735124	47
14	·2289172	·235161	4·252392	·9734458	46
15	·2292004	·235468	4·246848	·9733792	45
16	·2294835	·235775	4·241317	·9733125	44
17	·2297666	·236082	4·235800	·9732457	43
18	·2300497	·236390	4·230297	·9731789	42
19	·2303328	·236697	4·224808	·9731119	41
20	·2306159	·237004	4·219331	·9730449	40
′	Cosine.	Cotang.	Tang.	Sine.	′

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·2308989	·237311	4·213869	·9729777	39
22	·2311819	·237618	4·208419	·9729105	38
23	·2314649	·237926	4·202983	·9728432	37
24	·2317479	·238233	4·197560	·9727759	36
25	·2320309	·238541	4·192151	·9727084	35
26	·2323138	·238848	4·186754	·9726409	34
27	·2325967	·239156	4·181371	·9725733	33
28	·2328796	·239463	4·176001	·9725056	32
29	·2331625	·239771	4·170644	·9724378	31
30	·2334454	·240078	4·165299	·9723699	30
31	·2337282	·240386	4·159968	·9723020	29
32	·2340110	·240694	4·154650	·9722339	28
33	·2342938	·241001	4·149344	·9721658	27
34	·2345766	·241309	4·144051	·9720976	26
35	·2348594	·241617	4·138771	·9720294	25
36	·2351421	·241925	4·133504	·9719610	24
37	·2354248	·242233	4·128249	·9718926	23
38	·2357075	·242541	4·123007	·9718240	22
39	·2359902	·242849	4·117778	·9717554	21
40	·2362729	·243157	4·112561	·9716867	20
′	Cosine.	Cotang.	Tang.	Sine.	′

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2365555	·243465	4·107356	·9716180	19
42	·2368381	·243773	4·102164	·9715491	18
43	·2371207	·244081	4·096985	·9714802	17
44	·2374033	·244390	4·091817	·9714112	16
45	·2376859	·244698	4·086662	·9713421	15
46	·2379684	·245006	4·081519	·9712729	14
47	·2382510	·245315	4·076389	·9712036	13
48	·2385335	·245623	4·071270	·9711343	12
49	·2388159	·245932	4·066164	·9710649	11
50	·2390984	·246240	4·061070	·9709953	10
51	·2393808	·246519	4·055987	·9709258	9
52	·2396633	·246857	4·050917	·9708561	8
53	·2399457	·247166	4·045859	·9707863	7
54	·2402280	·247475	4·040812	·9707165	6
55	·2405104	·247783	4·035777	·9706466	5
56	·2407927	·248092	4·030755	·9705766	4
57	·2410751	·248401	4·025744	·9705065	3
58	·2413574	·248710	4·020744	·9704363	2
59	·2416396	·249019	4·015757	·9703660	1
60	·2419219	·249328	4·010780	·9702957	0
′	Cosine.	Cotang.	Tang.	Sine.	′

14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2419219	·249328	4.010780	·9702957	60
1	·2422041	·249637	4·005816	·9702253	59
2	·2424863	·249946	4·000863	·9701548	58
3	·2427685	·250255	3·995922	·9700842	57
4	·2430507	·250564	3·990992	·9700135	56
5	·2433329	·250873	3·986073	·9699428	55
6	·2436150	·251182	3·981166	·9698720	54
7	·2438971	·251491	3·976271	·9698011	53
8	·2441792	·251801	3·971386	·9697301	52
9	·2444613	·252110	3·966513	·9696591	51
10	·2447433	·252420	3·961651	·9695879	50
11	·2450254	·252729	3·956801	·9695167	49
12	·2453074	·253038	3·951961	·9694453	48
13	·2455894	·253348	3·947133	·9693740	47
14	·2458713	·253658	3·942315	·9693025	46
15	·2461533	·253967	3·937509	·9692309	45
16	·2464352	·254277	3·932714	·9691593	44
17	·2467171	·254587	3·927929	·9690875	43
18	·2469990	·254896	3·923156	·9690157	42
19	·2472809	·255206	3·918393	·9689438	41
20	·2475627	·255516	3·913642	·9688719	40
21	·2478445	·255826	3·908901	·9687998	39
22	·2481263	·256136	3·904171	·9687277	38
23	·2484081	·256446	3·899451	·9686555	37
24	·2486899	·256756	3·894742	·9685832	36
25	·2489716	·257066	3·890044	·9685108	35
26	·2492533	·257376	3·885357	·9684383	34
27	·2495350	·257686	3·880680	·9683658	33
28	·2498167	·257997	3·876014	·9682931	32
29	·2500984	·258307	3·871358	·9682204	31
30	·2503800	·258617	3·866713	·9681476	30
31	·2506616	·258928	3·862078	·9680748	29
32	·2509432	·259238	3·857453	·9680018	28
33	·2512248	·259548	3·852839	·9679288	27
34	·2515063	·259859	3·848235	·9678557	26
35	·2517879	·260169	3·843642	·9677825	25
36	·2520694	·260480	3·839059	·9677092	24
37	·2523508	·260791	3·834486	·9676358	23
38	·2526323	·261101	3·829923	·9675624	22
39	·2529137	·261412	3·825370	·9674888	21
40	·2531952	·261723	3·820828	·9674152	20
41	·2534766	·262034	3·816295	·9673415	19
42	·2537579	·262345	3·811773	·9672678	18
43	·2540393	·262656	3·807260	·9671939	17
44	·2543206	·262967	3·802758	·9671200	16
45	·2546019	·263278	3·798266	·9670459	15
46	·2548832	·263589	3·793783	·9669718	14
47	·2551645	·263900	3·789310	·9668977	13
48	·2554458	·264211	3·784848	.9668234	12
49	·2557270	·264522	3·780395	·9667490	11
50	·2560082	·264833	3·775951	·9666746	10
51	·2562894	·265145	3·771518	·9666001	9
52	·2565705	·265456	3·767094	·9665255	8
53	·2568517	·265768	3·762680	·9664508	7
54	·2571328	·266079	3·758276	·9663761	6
55	·2574139	·266390	3·753881	·9663012	5
56	·2576950	·266702	3·749496	·9662263	4
57	·2579760	·267014	3·745120	·9661513	3
58	·2582570	·267325	3·740754	·9660762	2
59	·2585381	·267637	3·736398	·9660011	1
60	·2588190	·267949	3·732050	·9659258	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 75.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2588190	·267949	3·732050	·9659258	60	21	·2647147	·274507	3·642891	·9643268	39	41	·2703204	·280773	3·561590	·9627704	19
1	·2591000	·268261	3·727713	·9658505	59	22	·2649952	·274820	3·638744	·9642497	38	42	·2706004	·281087	3·557613	·9626917	18
2	·2593810	·268572	3·723384	·9657751	58	23	·2652757	·275133	3·634606	·9641726	37	43	·2708805	·281401	3·553644	·9626130	17
3	·2596619	·268884	3·719065	·9656996	57	24	·2655561	·275445	3·630477	·9640954	36	44	·2711605	·281715	3·549684	·9625342	16
4	·2599428	·269196	3·714756	·9656240	56	25	·2658366	·275758	3·626356	·9640181	35	45	·2714404	·282029	3·545732	·9624552	15
5	2602237	·269508	3·710455	·9655484	55	26	·2661170	·276071	3·622244	·9639407	34	46	·2717204	·282343	3·541788	·9623762	14
6	·2605045	·269820	3·706164	·9654726	54	27	·2663973	·276385	3·618141	·9638633	33	47	·2720003	·282657	3·537852	·9622972	13
7	·2607853	·270132	3·701883	·9653968	53	28	·2666777	·276698	3·614046	·9637858	32	48	·2722802	·282971	3·533925	·9622180	12
8	·2610662	·270444	3·697610	·9653209	52	29	·2669581	·277011	3·609960	·9637081	31	49	·2725601	·283285	3·530005	·9621387	11
9	·2613469	·270757	3·693346	·9652449	51	30	·2672384	·277324	3·605883	·9636305	30	50	·2728400	·283599	3·526093	·9620594	10
10	·2616277	·271069	3·689092	·9651689	50	31	·2675187	·277637	3·601814	·9635527	29	51	·2731198	·283914	3·522190	·9619800	9
11	·2619085	·271381	3·684847	·9650927	49	32	·2677989	·277951	3·597754	·9634748	28	52	·2733997	·284228	3·518294	·9619005	8
12	·2621892	·271694	3·680611	·9650165	48	33	·2680792	·278264	3·593702	·9633969	27	53	·2736794	·284543	3·514407	·9618210	7
13	·2624699	·272006	3·676384	·9649402	47	34	·2683594	·278578	3·589659	·9633189	26	54	·2739592	·284857	3·510527	·9617413	6
14	·2627506	·272318	3·672166	·9648638	46	35	·2686396	·278891	3·585624	·9632408	25	55	·2742390	·285172	3·506655	·9616616	5
15	·2630312	·272631	3·667957	·9647873	45	36	·2689198	·279205	3·581597	·9631626	24	56	·2745187	·285486	3·502791	·9615818	4
16	·2633118	·272943	3·663757	·9647108	44	37	·2692000	·279518	3·577579	·9630843	23	57	·2747984	·285801	3·498935	·9615019	3
17	·2635925	·273256	3·659566	·9646341	43	38	·2694801	·279832	3·573569	·9630060	22	58	·2750781	·286115	3·495087	·9614219	2
18	·2638730	·273569	3·655384	·9645574	42	39	·2697602	·280145	3·569568	·9629275	21	59	·2753577	·286430	3·491247	·9613418	1
19	·2641536	·273881	3·651211	·9644806	41	40	·2700403	·280459	3·565574	·9628490	20	60	·2756374	·286745	3·487414	·9612617	0
20	·2644342	·274194	3·647046	·9644037	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2756374	·286745	3·487414	·9612617	60
1	·2759170	·287060	3·483589	·9611815	59
2	·2761965	·287375	3·479772	·9611012	58
3	·2764761	·287690	3·475963	·9610208	57
4	·2767556	·288005	3·472161	·9609403	56
5	·2770352	·288320	3·468367	·9608598	55
6	·2773147	·288635	3·464581	·9607792	54
7	·2775941	·288950	3·460802	·9606984	53
8	·2778736	·289265	3·457031	·9606177	52
9	·2781530	·289580	3·453267	·9605368	51
10	·2784324	·289896	3·449512	·9604558	50
11	·2787118	·290211	3·445763	·9603748	49
12	·2789911	·290526	3·442022	·9602937	48
13	·2792704	·290842	3·438289	·9602125	47
14	·2795497	·291157	3·434563	·9601312	46
15	·2798290	·291473	3·430844	·9600499	45
16	·2801083	·291789	3·427133	·9599684	44
17	·2803875	·292104	3·423429	·9598869	43
18	·2806667	·292420	3·419733	·9598053	42
19	·2809459	·292736	3·416044	·9597236	41
20	·2812251	·293052	3·412362	·9596418	40
21	·2815042	·293368	3·408688	·9595600	39
22	·2817833	·293683	3·405021	·9594781	38
23	·2820624	·293999	3·401361	·9593961	37
24	·2823415	·294316	3·397708	·9593140	36
25	·2826205	·294632	3·394063	·9592318	35
26	·2828995	·294948	3·390424	·9591496	34
27	·2831785	·295264	3·386793	·9590672	33
28	·2834575	·295580	3·383169	·9589848	32
29	·2837364	·295897	3·379553	·9589023	31
30	·2840153	·296213	3·375943	·9588197	30
31	·2842942	·296529	3·372340	·9587371	29
32	·2845731	·296846	3·368745	·9586543	28
33	·2848520	·297163	3·365156	·9585715	27
34	·2851308	·297479	3·361575	·9584886	26
35	·2854096	·297796	3·358000	·9584056	25
36	·2856884	·298112	3·354433	·9583226	24
37	·2859671	·298429	3·350872	·9582394	23
38	·2862458	·298746	3·347319	·9581562	22
39	·2865246	·299063	3·343772	·9580729	21
40	·2868032	·299380	3·340232	·9579895	20
41	·2870819	·299697	3·336699	·9579060	19
42	·2873605	·300014	3·333173	·9578225	18
43	·2876391	·300331	3·329654	·9577389	17
44	·2879177	·300648	3·326141	·9576552	16
45	·2881963	·300965	3·322636	·9575714	15
46	·2884748	·301283	3·319137	·9574875	14
47	·2887533	·301600	3·315645	·9574035	13
48	·2890318	·301917	3·312159	·9573195	12
49	·2893103	·302235	3·308681	·9572354	11
50	·2895887	·302552	3·305209	·9571512	10
51	·2898671	·302870	3·301743	·9570669	9
52	·2901455	·303187	3·298285	·9569825	8
53	·2904239	·303505	3·294833	·9568981	7
54	·2907022	·303823	3·291387	·9568136	6
55	·2909805	·304141	3·287948	·9567290	5
56	·2912588	·304458	3·284516	·9566443	4
57	·2915371	·304776	3·281090	·9565595	3
58	·2918153	·305094	3·277671	·9564747	2
59	·2920935	·305412	3·274258	·9563898	1
60	·2923717	·305730	3·270852	·9563048	0
	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73.

17 Deg. 17 Deg. 17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2923717	·305730	3·270852	·9563048	60	21	·2982079	·312422	3·200789	·9545009	39	41	·3037559	·318820	3·136563	·9527499	19
1	·2926499	·306048	3·267452	·9562197	59	22	·2984856	·312742	3·197521	·9544141	38	42	·3040331	·319140	3·133414	·9526615	18
2	·2929280	·306367	3·264059	·9561345	58	23	·2987632	·313061	3·194259	·9543273	37	43	·3043102	·319461	3·130270	·9525730	17
3	·2932061	·306685	3·260672	·9560492	57	24	·2990408	·313381	3·191003	·9542403	36	44	·3045872	·319781	3·127131	·9524844	16
4	·2934842	·307003	3·257292	·9559639	56	25	·2993184	·313700	3·187754	·9541533	35	45	·3048643	·320102	3·123999	·9523958	15
5	·2937623	·307321	3·253918	·9558785	55	26	·2995959	·314020	3·184510	·9540662	34	46	·3051413	·320423	3·120872	·9523071	14
6	·2940403	·307640	3·250550	·9557930	54	27	·2998734	·314339	3·181272	·9539790	33	47	·3054183	·320744	3·117750	·9522183	13
7	·2943183	·307958	3·247189	·9557074	53	28	·3001509	·314659	3·178040	·9538917	32	48	·3056953	·321064	3·114635	·9521294	12
8	·2945963	·308277	3·243834	·9556218	52	29	·3004284	·314979	3·174814	·9538044	31	49	·3059723	·321385	3·111525	·9520404	11
9	·2948743	·308595	3·240486	·9555361	51	30	·3007058	·315298	3·171594	·9537170	30	50	·3062492	·321706	3·108421	·9519514	10
10	·2951522	·308914	3·237143	·9554502	50	31	·3009832	·315618	3·168380	·9536294	29	51	·3065261	·322027	3·105322	·9518623	9
11	·2954302	·309233	3·233807	·9553643	49	32	·3012606	·315938	3·165172	·9535418	28	52	·3068030	·322348	3·102229	·9517731	8
12	·2957081	·309551	3·230478	·9552784	48	33	·3015380	·316258	3·161970	·9534542	27	53	·3070798	·322670	3·099141	·9516838	7
13	·2959859	·309870	3·227154	·9551923	47	34	·3018153	·316578	3·158774	·9533664	26	54	·3073566	·322991	3·096059	·9515944	6
14	·2962638	·310189	3·223837	·9551062	46	35	·3020926	·316898	3·155584	·9532786	25	55	·3076334	·323312	3·092983	·9515050	5
15	·2965416	·310508	3·220526	·9550199	45	36	·3023699	·317218	3·152399	·9531907	24	56	·3079102	·323633	3·089912	·9514154	4
16	·2968194	·310827	3·217221	·9549336	44	37	·3026471	·317538	3·149220	·9531027	23	57	·3081869	·323955	3·086846	·9513258	3
17	·2970971	·311146	3·213922	·9548473	43	38	·3029244	·317859	3·146047	·9530146	22	58	·3084636	·324276	3·083786	·9512361	2
18	·2973749	·311465	3·210630	·9547608	42	39	·3032016	·318179	3·142880	·9529264	21	59	·3087403	·324598	3·080732	·9511464	1
19	·2976526	·311784	3·207344	·9546743	41	40	·3034788	·318499	3·139719	·9528382	20	60	·3090170	·324919	3·077683	·9510565	0
20	·2979303	·312103	3·204063	·9545876	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72. Deg. 72. Deg 72.

8 Deg. 18 Deg. 18 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3090170	·324919	3·077683	·9510565	60	21	·3148209	·331686	3·014892	·9491511	39	41	·3203374	·338157	2·957205	·9473035	19
1	·3092936	·325241	3·074640	·9509666	59	22	·3150969	·332009	3·011960	·9490595	38	42	·3206130	·338481	2·954372	·9472103	18
2	·3095702	·325563	3·071602	·9508766	58	23	·3153730	·332332	3·009033	·9489678	37	43	·3208885	·338805	2·951545	·9471170	17
3	·3098468	·325884	3·068569	·9507865	57	24	·3156490	·332655	3·006110	·9488760	36	44	·3211640	·339129	2·948722	·9470236	16
4	·3101234	·326206	3·065542	·9506963	56	25	·3159250	·332978	3·003193	·9487842	35	45	·3214395	·339454	2·945905	·9469301	15
5	·3103999	·326528	3·062520	·9506061	55	26	·3162010	·333302	3·000282	·9486922	34	46	·3217149	·339778	2·943092	·9468366	14
6	·3106764	·326850	3·059503	·9505157	54	27	·3164770	·333625	2·997375	·9486002	33	47	·3219903	·340103	2·940284	·9467430	13
7	·3109529	·327172	3·056492	·9504253	53	28	·3167529	·333948	2·994473	·9485081	32	48	·3222657	·340427	2·937480	·9466493	12
8	·3112294	·327494	3·053487	·9503348	52	29	·3170288	·334271	2·991576	·9484159	31	49	·3225411	·340752	2·934682	·9465555	11
9	·3115058	·327816	3·050486	·9502443	51	30	·3173047	·334595	2·988685	·9483237	30	50	·3228164	·341077	2·931888	·9464616	10
10	·3117822	·328138	3·047491	·9501536	50	31	·3175805	·334918	2·985798	·9482313	29	51	·3230917	·341401	2·929099	·9463677	9
11	·3120586	·328461	3·044501	·9500629	49	32	·3178563	·335242	2·982916	·9481389	28	52	·3233670	·341726	2·926315	·9462736	8
12	·3123349	·328783	3·041517	·9499721	48	33	·3181321	·335566	2·980040	·9480464	27	53	·3236422	·342051	2·923535	·9461795	7
13	·3126112	·329105	3·038538	·9498812	47	34	·3184079	·335889	2·977168	·9479538	26	54	·3239174	·342376	2·920761	·9460854	6
14	·3128875	·329428	3·035564	·9497902	46	35	·3186836	·336213	2·974301	·9478612	25	55	·3241926	·342701	2·917990	·9459911	5
15	·3131638	·329750	3·032595	·9496991	45	36	·3189593	·336537	2·971439	·9477684	24	56	·3244678	·343026	2·915225	·9458968	4
16	·3134400	·330073	3·029632	·9496080	44	37	·3192350	·336861	2·968583	·9476756	23	57	·3247429	·343351	2·912464	·9458023	3
17	·3137163	·330395	3·026673	·9495168	43	38	·3195106	·337185	2·965731	·9475827	22	58	·3250180	·343677	2·909708	·9457078	2
18	·3139925	·330718	3·023720	·9494255	42	39	·3197863	·337509	2·962884	·9474897	21	59	·3252931	·344002	2·906957	·9456132	1
19	·3142686	·331041	3·020772	·9493341	41	40	·3200619	·337833	2·960042	·9473966	20	60	·3255682	·344327	2·904210	·9455186	0
20	·3145448	·331363	3·017830	·9492426	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 71. Deg. 71. Deg. 71.

19 Deg. **19 Deg.** **19 Deg.**

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3255682	·344327	2·904210	·9455186	60	21	·3313379	·351175	2·847583	·9435122	39	41	·3368214	·357723	2·795453	·9415686	19
1	·3258432	·344653	2·901468	·9454238	59	22	·3316123	·351501	2·844935	·9434157	38	42	·3370953	·358051	2·792891	·9414705	18
2	·3261182	·344978	2·898731	·9453290	58	23	·3318867	·351828	2·842292	·9433192	37	43	·3373691	·358380	2·790333	·9413724	17
3	·3263932	·345304	2·895998	·9452341	57	24	·3321611	·352155	2·839653	·9432227	36	44	·3376429	·358708	2·787780	·9412743	16
4	·3266681	·345629	2·893270	·9451391	56	25	·3324355	·352482	2·837019	·9431260	35	45	·3379167	·359036	2·785230	·9411760	15
5	·3269430	·345955	2·890546	·9450441	55	26	·3327098	·352809	2·834389	·9430293	34	46	·3381905	·359365	2·782685	·9410777	14
6	3272179	·346281	2·887827	·9449489	54	27	·3329841	·353136	2·831763	·9429324	33	47	·3384642	·359693	2·780144	·9409793	13
7	·3274928	·346606	2·885113	·9448537	53	28	·3332584	·353464	2·829142	·9428355	32	48	·3387379	·360022	2·777606	·9408808	12
8	·3277676	·346932	2·882403	·9447584	52	29	·3335326	·353791	2·826525	·9427386	31	49	·3390116	·360350	2·775073	·9407822	11
9	·3280424	·347258	2·879697	·9446630	51	30	·3338069	·354118	2·823912	·9426415	30	50	·3392852	·360679	2·772544	·9406835	10
10	·3283172	·347584	2·876997	·9445675	50	31	·3340810	·354446	2·821304	·9425444	29	51	·3395589	·361008	2·770019	·9405848	9
11	·3285919	·347910	2·874300	·9444720	49	32	·3343552	·354773	2·818700	·9424471	28	52	·3398325	·361337	2·767499	·9404860	8
12	·3288666	·348236	2·871608	·9443764	48	33	·3346293	·355101	2·816100	·9423498	27	53	·3401060	·361666	2·764982	·9403871	7
13	·3291413	·348563	2·868921	·9442807	47	34	·3349034	·355428	2·813504	·9422525	26	54	·3403796	·361994	2·762469	·9402881	6
14	·3294160	·348889	2·866238	·9441849	46	35	·3351775	·355756	2·810913	·9421550	25	55	·3406531	·362324	2·759960	·9401891	5
15	·3296906	·349215	2·863560	·9440890	45	36	·3354516	·356084	2·808326	·9420575	24	56	·3409265	·362653	2·757456	·9400899	4
16	·3299653	·349542	2·860886	·9439931	44	37	·3357256	·356411	2·805743	·9419598	23	57	·3412000	·362982	2·754955	·9399907	3
17	·3302398	·349868	2·858216	·9438971	43	38	·3359996	·356739	2·803164	·9418621	22	58	·3414734	·363311	2·752458	·9398914	2
18	·3305144	·350195	2·855551	·9438010	42	39	·3362735	·357067	2·800590	·9417644	21	59	·3417468	·363640	2·749966	·9397921	1
19	·3307889	·350521	2·852891	·9437048	41	40	·3365475	·357395	2·798019	·9416665	20	60	·3420201	·363970	2·747477	·9396926	0
20	·3310634	·350848	2·850234	·9436085	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70. **Deg. 70.** **Deg. 70.**

20 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3420201	·363970	2·747477	·9396926	60
1	·3422935	·364299	2·744992	·9395931	59
2	·3425668	·364629	2·742512	·9394935	58
3	·3428400	·364958	2·740035	·9393938	57
4	·3431133	·365288	2·737562	·9392940	56
5	·3433865	·365618	2·735093	·9391942	55
6	·3436597	·365948	2·732628	·9390943	54
7	·3439329	·366277	2·730167	·9389943	53
8	·3442060	·366607	2 727710	·9388942	52
9	·3444791	·366937	2·725256	·9387940	51
10	·3447521	·367268	2·722807	·9386938	50
11	·3450252	·367598	2·720362	·9385934	49
12	·3452982	·367928	2·717920	·9384930	48
13	·3455712	·368258	2·715482	·9383925	47
14	·3458441	·368589	2·713048	·9382920	46
15	·3461171	·368919	2·710618	·9381913	45
16	·3463900	·369250	2·708192	·9380906	44
17	·3466628	·369580	2·705769	·9379898	43
18	·3469357	·369911	2·703351	·9378889	42
19	·3472085	·370242	2·700936	·9377880	41
20	·3474812	·370572	2·698525	·9376869	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 69.

20 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3477540	·370903	2·696118	·9375858	39
22	·3480267	·371234	2·693714	·9374846	38
23	·3482994	·371565	2·691314	·9373833	37
24	·3485720	·371896	2·688919	·9372820	36
25	·3488447	·372227	2·686526	·9371806	35
26	·3491173	·372559	2·684138	·9370790	34
27	·3493898	·372890	2·681753	·9369774	33
28	·3496624	·373221	2·679372	·9368758	32
29	·3499349	·373553	2·676995	·9367740	31
30	·3502074	·373884	2·674621	·9366722	30
31	·3504798	·374216	2·672251	·9365703	29
32	·3507523	·374547	2·669885	·9364683	28
33	.3510246	·374879	2·667522	·9363662	27
34	·3512970	·375211	2·665163	·9362641	26
35	·3515693	·375543	2·662808	·9361618	25
36	·3518416	·375875	2·660456	·9360595	24
37	·3521139	·376207	2·658108	·9359571	23
38	·3523862	·376539	2·655764	·9358547	22
39	·3526584	·376871	2·653423	·9357521	21
40	·3529306	·377203	2·651086	·9356495	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 69.

20 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3532027	·377536	2·648753	·9355468	19
42	·3534748	·377868	2·646423	·9354440	18
43	·3537469	·378201	2·644096	·9353412	17
44	·3540190	·378533	2·641774	·9352382	16
45	·3542910	·378866	2·639454	·9351352	15
46	·3545630	·379198	2·637139	·9350321	14
47	·3548350	·379531	2·634827	·9349289	13
48	·3551070	·379864	2·632518	.9348257	12
49	·3553789	·380197	2·630213	·9347223	11
50	·3556508	·380530	2·627912	·9346189	10
51	·3559226	·380863	2·625614	·9345154	9
52	·3561944	·381196	2·623319	·9344119	8
53	·3564662	·381529	2·621028	·9343082	7
54	·3567380	·381862	2·618741	·9342045	6
55	·3570097	·382196	2·616457	·9341007	5
56	·3572814	·382529	2·614176	·9339968	4
57	·3575531	·382863	2·611899	·9338928	3
58	·3578248	·383196	2·609625	·9337888	2
59	·3580964	·383530	2·607355	·9336846	1
60	·3583679	·383864	2·605089	·9335804	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 69.

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3583679	·383864	2·605089	·9335804	60
1	·3586395	·384197	2·602825	·9334761	59
2	·3589110	·384531	2·600565	9333718	58
3	·3591825	·384865	2·598309	·9332673	57
4	·3594540	·385199	2·596056	·9331628	56
5	·3597254	·385533	2·593806	·9330582	55
6	·3599968	·385867	2·591560	·9329535	54
7	·3602682	·386202	2·589317	·9328488	53
8	·3605395	·386536	2·587078	·9327439	52
9	·3608108	·386870	2·584842	·9326390	51
10	·3610821	·387205	2·582609	·9325340	50
11	·3613534	·387539	2·580380	·9324290	49
12	·3616246	·387874	2·578153	·9323238	48
13	·3618958	·388209	2·575931	·9322186	47
14	·3621669	·388543	2·573711	·9321133	46
15	·3624380	·388878	2·571495	·9320079	45
16	·3627091	·389213	2·569283	·9319024	44
17	·3629802	·389548	2·567073	·9317969	43
18	·3632512	·389883	2·564867	·9316912	42
19	·3635222	·390218	2·562664	·9315855	41
20	·3637932	·390554	2·560464	·9314797	40
′	Cosine.	Cotang.	Tang.	Sine.	′

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·3640641	·390889	2·558268	·9313739	39
22	·3643351	·391224	2·556075	·9312679	38
23	·3646059	·391560	2·553885	·9311619	37
24	·3648768	·391895	2·551699	·9310558	36
25	·3651476	·392231	2·549516	·9309496	35
26	·3654184	·392567	2·547335	·9308434	34
27	·3656891	·392902	2·545159	·9307370	33
28	·3659599	·393238	2·542985	·9306306	32
29	·3662306	·393574	2·540815	·9305241	31
30	·3665012	·393910	2·538647	·9304176	30
31	·3667719	·394246	2·536483	·9303109	29
32	·3670425	·394582	2·534323	·9302042	28
33	·3673130	·394918	2·532165	·9300974	27
34	3675836	·395255	2·530011	·9299905	26
35	·3678541	·395591	2·527859	·9298835	25
36	·3681246	·395928	2·525711	·9297765	24
37	·3683950	·396264	2·523566	·9296694	23
38	·3686654	·396601	2·521424	·9295622	22
39	·3689358	·396937	2·519286	9294549	21
40	·3692061	·397274	2·517150	·9293475	20
′	Cosine.	Cotang.	Tang.	Sine.	′

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3694765	·397611	2 515018	·9292401	19
42	·3697468	·397948	2·512889	·9291326	18
43	·3700170	·398285	2·510762	·9290250	17
44	·3702872	·398622	2·508639	·9289173	16
45	·3705574	·398959	2·506519	·9288096	15
46	·3708276	·399296	2·504403	·9287017	14
47	·3710977	·399634	2·502289	·9285938	13
48	·3713678	·399971	2·500178	·9284858	12
49	·3716379	·400308	2·498070	·9283778	11
50	·3719079	·400646	2·495966	·9282696	10
51	·3721780	·400984	2·493864	·9281614	9
52	·3724479	·401321	2·491766	·9280531	8
53	·3727179	·401659	2·489670	·9279447	7
54	·3729878	·401997	2·487578	·9278363	6
55	·3732577	·402335	2·485488	·9277277	5
56	·3735275	·402673	2·483402	·9276191	4
57	·3737973	·403011	2·481319	·9275104	3
58	·3740671	·403349	2·479238	·9274016	2
59	·3743369	·403687	2·477161	·9272928	1
60	·3746066	·404026	2·475086	·9271839	0
′	Cosine.	Cotang.	Tang.	Sine.	′

22 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3746066	·404026	2·475086	·9271839	60
1	·3748763	·404364	2·473015	·9270748	59
2	·3751459	·404703	2·470947	·9269658	58
3	·3754156	·405041	2·468881	·9268566	57
4	·3756852	·405380	2·466819	·9267474	56
5	·3759547	·405719	2·464759	·9266380	55
6	·3762243	·406057	2·462703	·9265286	54
7	·3764938	·406396	2·460649	·9264192	53
8	·3767632	·406735	2·458598	·9263096	52
9	·3770327	·407074	2·456551	·9262000	51
10	·3773021	·407413	2·454506	·9260902	50
11	·3775714	·407753	2·452464	·9259805	49
12	·3778408	·408092	2·450425	·9258706	48
13	·3781101	·408431	2·448389	·9257606	47
14	·3783794	·408771	2·446355	·9256506	46
15	·3786486	·409110	2·444325	·9255405	45
16	·3789178	·409450	2·442298	·9254303	44
17	·3791870	·409790	2·440273	·9253201	43
18	·3794562	·410129	2·438251	·9252097	42
19	·3797253	·410469	2·436233	·9250993	41
20	·3799944	·410809	2·434217	·9249888	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 67.

22 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3802634	·411149	2·432204	·9248782	39
22	·3805324	·411489	2·430193	·9247676	38
23	·3808014	·411830	2·428186	·9246568	37
24	·3810704	·412170	2·426181	·9245460	36
25	·3813393	·412510	2·424180	·9244351	35
26	·3816082	·412851	2·422181	·9243242	34
27	·3818770	·413191	2·420185	·9242131	33
28	·3821459	·413532	2·418191	·9241020	32
29	·3824147	·413872	2·416201	·9239908	31
30	·3826834	·414213	2·414213	·9238795	30
31	·3829522	·414554	2·412228	·9237682	29
32	·3832209	·414895	2·410246	·9236567	28
33	·3834895	·415236	2·408267	·9235452	27
34	·3837582	·415577	2·406290	·9234336	26
35	·3840268	·415918	2·404316	·9233220	25
36	·3842953	·416259	2·402345	·9232102	24
37	·3845639	·416601	2·400377	·9230984	23
38	·3848324	·416942	2·398411	·9229865	22
39	·3851008	·417284	2·396449	·9228745	21
40	·3853693	·417625	2·394488	·9227624	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 67.

22 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3856377	·417967	2·392531	·9226503	19
42	·3859060	·418309	2·390576	·9225381	18
43	·3861744	·418650	2·388625	·9224258	17
44	·3864427	·418992	2·386675	·9223134	16
45	·3867110	·419334	2·384729	·9222010	15
46	·3869792	·419676	2·382785	·9220884	14
47	·3872474	·420019	2·380844	·9219758	13
48	·3875156	·420361	2·378906	·9218632	12
49	·3877837	·420703	2·376970	·9217504	11
50	·3880518	·421046	2·375037	·9216375	10
51	·3883199	·421388	2·373106	·9215246	9
52	·3885880	·421731	2·371179	·9214116	8
53	·3888560	·422073	2·369254	·9212986	7
54	·3891240	·422416	2·367331	·9211854	6
55	·3893919	·422759	2·365411	·9210722	5
56	·3896598	·423102	2·363494	·9209589	4
57	·3899277	·423445	2·361580	·9208455	3
58	·3901955	·423788	2·359668	·9207320	2
59	·3904633	·424131	2·357759	·9206185	1
60	·3907311	·424474	2·355852	·9205049	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 67.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3907311	·424474	2·355852	·9205049	60
1	·3909989	·424818	2·353948	·9203912	59
2	·3912666	·425161	2·352046	·9202774	58
3	·3915343	·425505	2·350148	·9201635	57
4	·3918019	·425848	2·348251	·9200496	56
5	3920695	·426192	2·346358	·9199356	55
6	·3923371	·426536	2·344467	·9198215	54
7	·3926047	·426880	2·342578	·9197073	53
8	·3928722	·427223	2·340692	·9195931	52
9	·3931397	·427568	2·338809	·9194788	51
10	3934071	·427912	2·336928	·9193644	50
11	·3936745	·428256	2·335050	·9192499	49
12	·3939419	·428600	2·333174	·9191353	48
13	·3942093	·428944	2·331301	·9190207	47
14	·3944766	·429289	2·329431	·9189060	46
15	·3947439	·429633	2·327563	·9187912	45
16	·3950111	·429978	2·325697	·9186763	44
17	·3952783	·430323	2·323834	·9185614	43
18	·3955455	·430668	2·321974	·9184464	42
19	·3958127	·431012	2·320116	·9183313	41
20	·3960798	·431357	2·318260	·9182161	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3963468	·431703	2·316407	·9181009	39
22	·3966139	·432048	2·314557	·9179855	38
23	·3968809	·432393	2·312709	·9178701	37
24	·3971479	·432738	2·310863	·9177546	36
25	·3974148	·433084	2·309020	·9176391	35
26	·3976818	·433429	2·307180	·9175234	34
27	·3979486	·433775	2·305342	·9174077	33
28	·3982155	·434120	2·303506	·9172919	32
29	·3984823	·434466	2·301673	·9171760	31
30	·3987491	·434812	2·299842	·9170601	30
31	·3990158	·435158	2·298014	·9169440	29
32	·3992825	·435504	2·296188	·9168279	28
33	·3995492	·435850	2·294365	·9167118	27
34	·3998158	·436196	2·292544	·9165955	26
35	·4000825	·436542	2·290725	·9164791	25
36	·4003490	·436889	2·288909	·9163627	24
37	·4006156	·437235	2·287095	·9162462	23
38	·4008821	·437582	2·285284	·9161297	22
39	·4011486	·437928	2·283475	·9160130	21
40	·4014150	·438275	2·281669	·9158963	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4016814	·438622	2·279865	·9157795	19
42	·4019478	·438969	2·278063	·9156626	18
43	·4022141	·439316	2·276264	·9155456	17
44	·4024804	·439663	2·274467	·9154286	16
45	·4027467	·440010	2·272672	·9153115	15
46	·4030129	·440357	2·270880	·9151943	14
47	·4032791	·440705	2·269090	·9150770	13
48	·4035453	·441052	2·267303	·9149597	12
49	·4038114	·441400	2·265518	·9148422	11
50	·4040775	·441747	2·263735	·9147247	10
51	·4043436	·442095	2·261955	·9146072	9
52	·4046096	·442443	2·260177	·9144895	8
53	·4048756	·442791	2·258401	·9143718	7
54	·4051416	·443139	2·256628	·9142540	6
55	·4054075	·443487	2·254857	·9141361	5
56	·4056734	·443835	2·253088	·9140181	4
57	·4059393	·444183	2·251322	·9139001	3
58	·4062051	·444531	2·249558	·9137819	2
59	·4064709	·444880	2·247796	·9136637	1
60	·4067366	·445228	2·246036	·9135455	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

24 Deg. **24 Deg.** **24 Deg.**

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4067366	·445228	2·246036	·9135455	60	21	·4123096	·452568	2·209611	·9110438	39	41	·4176028	·459596	2·175822	·9086297	19
1	·4070024	·445577	2·244279	·9134271	59	22	·4125745	·452918	2·207901	·9109238	38	42	·4178671	·459948	2·174155	·9085082	18
2	·4072681	·445926	2·242524	·9133087	58	23	·4128395	·453269	2·206193	·9108038	37	43	·4181313	·460301	2·172491	·9083866	17
3	·4075337	·446274	2·240772	·9131902	57	24	·4131044	·453620	2·204487	·9106837	36	44	·4183956	·460653	2·170828	·9082649	16
4	·4077993	·446623	2·239021	·9130716	56	25	·4133693	·453970	2·202784	·9105635	35	45	·4186597	·461006	2·169167	·9081432	15
5	·4080649	·446972	2·237273	·9129529	55	26	·4136342	·454321	2·201083	·9104432	34	46	·4189239	·461359	2·167509	·9080214	14
6	·4083305	·447321	2·235528	·9128342	54	27	·4138990	·454672	2·199384	·9103228	33	47	·4191880	·461711	2·165852	·9078995	13
7	·4085960	·447670	2·233784	·9127154	53	28	·4141638	·455023	2·197687	·9102024	32	48	·4194521	·462064	2·164198	·9077775	12
8	·4088615	·448020	2·232043	·9125965	52	29	·4144285	·455375	2·195992	·9100819	31	49	·4197161	·462417	2·162546	·9076554	11
9	·4091269	·448369	2·230304	·9124775	51	30	·4146932	·455726	2·194299	·9099613	30	50	·4199801	·462771	2·160895	·9075333	10
10	·4093923	·448718	2·228567	·9123584	50	31	·4149579	·456077	2·192609	·9098406	29	51	·4202441	·463124	2·159247	·9074111	9
11	·4096577	·449068	2·226833	·9122393	49	32	·4152226	·456429	2·190921	·9097199	28	52	·4205080	·463477	2·157601	·9072888	8
12	·4099230	·449417	2·225100	·9121201	48	33	·4154872	·456780	2·189234	·9095990	27	53	·4207719	·463831	2·155957	·9071665	7
13	·4101883	·449767	2·223370	·9120008	47	34	·4157517	·457132	2·187551	·9094781	26	54	·4210358	·464184	2·154315	·9070440	6
14	·4104536	·450117	2·221643	·9118815	46	35	·4160163	·457483	2·185869	·9093572	25	55	·4212996	·464538	2·152675	·9069215	5
15	·4107189	·450467	2·219917	·9117620	45	36	·4162808	·457835	2·184189	·9092361	24	56	·4215634	·464891	2·151037	·9067989	4
16	·4109841	·450817	2·218194	·9116425	44	37	·4165453	·458187	2·182511	·9091150	23	57	·4218272	·465245	2·149402	·9066762	3
17	·4112492	·451167	2·216473	·9115229	43	38	·4168097	·458539	2·180836	·9089938	22	58	·4220909	·465599	2·147768	·9065535	2
18	·4115144	·451517	2·214754	·9114033	42	39	·4170741	·458891	2·179163	·9088725	21	59	·4223546	·465953	2·146136	·9064307	1
19	·4117795	·451867	2·213037	·9112835	41	40	·4173385	·459243	2·177492	·9087511	20	60	·4226183	·466307	2·144506	·9063078	0
20	·4120445	·452217	2·211323	·9111637	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang	Sine.	′

Deg. 65. **Deg. 65.** **Deg. 65.**

25 Deg. 25 Deg 25 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4226183	·466307	2·144506	·9063078	60	21	·4281467	·473765	2·110747	·9037093	39	41	·4333970	·480909	2·079394	·9012031	19
1	·4228819	·466661	2·142879	·9061848	59	22	·4284095	·474122	2·109161	·9035847	38	42	·4336591	·481267	2·077846	·9010770	18
2	·4231455	·467016	2·141253	·9060618	58	23	·4286723	·474478	2·107577	·9034600	37	43	·4339212	·481625	2·076300	·9009508	17
3	·4234090	·467370	2·139630	·9059386	57	24	·4289351	·474834	2·105995	·9033353	36	44	·4341832	·481984	2·074756	·9008246	16
4	·4236725	·467725	2·138008	·9058154	56	25	·4291979	·475191	2·104415	·9032105	35	45	·4344453	·482342	2·073214	·9006982	15
5	·4239360	·468079	2·136389	·9056922	55	26	·4294606	·475548	2·102836	·9030856	34	46	·4347072	·482701	2·071674	·9005718	14
6	·4241994	·468434	2·134771	·9055688	54	27	·4297233	·475904	2·101260	·9029606	33	47	·4349692	·483060	2·070135	·9004453	13
7	·4244628	·468789	2·133155	·9054454	53	28	·4299859	·476261	2·099686	·9028356	32	48	·4352311	·483418	2·068599	·9003188	12
8	·4247262	·469143	2·131542	·9053219	52	29	·4302485	·476618	2·098114	·9027105	31	49	·4354930	·483777	2·067064	·9001921	11
9	·4249895	·469498	2·129930	·9051983	51	30	·4305111	·476975	2·096543	·9025853	30	50	·4357548	·484136	2·065531	·9000654	10
10	·4252528	·469853	2·128321	·9050746	50	31	·4307736	·477332	2·094975	·9024600	29	51	·4360166	·484495	2·064000	·8999386	9
11	·4255161	·470209	2·126713	·9049509	49	32	·4310361	·477689	2·093408	·9023347	28	52	·4362784	·484855	2·062471	·8998117	8
12	·4257793	·470564	2·125108	·9048271	48	33	·4312986	·478047	2·091843	·9022092	27	53	·4365401	·485214	2·060944	·8996848	7
13	·4260425	·470919	2·123504	·9047032	47	34	·4315610	·478404	2·090280	·9020838	26	54	·4368018	·485573	2·059418	·8995578	6
14	·4263056	·471275	2·121903	·9045792	46	35	·4318234	·478762	2·088720	·9019582	25	55	·4370634	·485933	2·057895	·8994307	5
15	·4265687	·471630	2·120303	·9044551	45	36	·4320857	·479119	2·087161	·9018325	24	56	·4373251	·486293	2·056373	·8993035	4
16	·4268318	·471986	2·118705	·9043310	44	37	·4323481	·479477	2·085603	·9017068	23	57	·4375866	·486652	2·054853	·8991763	3
17	·4270949	·472342	2·117110	·9042068	43	38	·4326103	·479835	2·084048	·9015810	22	58	·4378482	·487012	2·053334	·8990489	2
18	·4273579	·472697	2·115516	·9040825	42	39	·4328726	·480193	2·082495	·9014551	21	59	·4381097	·487372	2·051818	·8989215	1
19	·4276208	·473053	2·113924	·9039582	41	40	·4331348	·480551	2·080943	·9013292	20	60	·4383711	·487732	2·050303	·8987940	0
20	·4278838	·473409	2·112334	·9038338	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 64. Deg. 64. Deg. 64.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4383711	·487732	2·050303	·8987940	60
1	·4386326	·488092	2·048791	·8986665	59
2	·4388940	·488453	2·047280	·8985389	58
3	·4391553	·488813	2·045770	·8984112	57
4	·4394166	·489173	2·044263	·8982834	56
5	·4396779	·489534	2 042757	·8981555	55
6	·4399392	·489894	2·041254	·8980276	54
7	·4402004	·490255	2·039751	·8978996	53
8	·4404615	·490616	2·038251	·8977715	52
9	·4407227	·490977	2·036753	·8976433	51
10	·4409838	·491338	2·035256	·8975151	50
11	·4412448	·491699	2·033761	·8973868	49
12	·4415059	·492061	2·032268	·8972584	48
13	·4417668	·492422	2·030776	·8971299	47
14	·4420278	·492783	2·029287	·8970014	46
15	·4422887	·493145	2·027799	·8968727	45
16	·4425496	·493507	2·026313	·8967440	44
17	·4428104	·493868	2·024828	·8966153	43
18	·4430712	·494230	2·023346	·8964864	42
19	·4433319	·494592	2·021865	·8963575	41
20	·4435927	·494954	2·020386	·8962285	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4438534	·495317	2·018908	·8960994	39
22	·4441140	·495679	2·017433	·8959703	38
23	·4443746	·496041	2·015959	·8958411	37
24	·4446352	·496404	2·014486	·8957118	36
25	·4448957	·496766	2·013016	·8955824	35
26	·4451562	·497129	2·011547	·8954529	34
27	·4454167	·497492	2·010080	·8953234	33
28	·4456771	·497855	2·008615	·8951938	32
29	·4459375	·498218	2·007151	·8950641	31
30	·4461978	·498581	2·005689	·8949344	30
31	·4464581	·498944	2·004229	·8948045	29
32	·4467184	·499308	2·002771	·8946746	28
33	·4469786	·499671	2·001314	·8945446	27
34	·4472388	·500035	1·999859	·8944146	26
35	·4474990	·500398	1·998405	·8942844	25
36	·4477591	·500762	1·996953	·8941542	24
37	·4480192	·501126	1·995503	·8940240	23
38	·4482792	·501490	1·994055	·8938936	22
39	·4485392	·501854	1·992608	·8937632	21
40	·4487992	·502218	1·991163	·8936326	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4490591	·502583	1·989720	·8935021	19
42	·4493190	·502947	1·988278	·8933714	18
43	·4495789	·503312	1·986838	·8932406	17
44	·4498387	·503676	1·985400	·8931098	16
45	·4500984	·504041	1·983963	·8929789	15
46	·4503582	·504406	1·982528	·8928480	14
47	·4506179	·504771	1·981095	·8927169	13
48	·4508775	·505136	1·979663	·8925858	12
49	·4511372	·505501	1·978233	·8924546	11
50	·4513967	·505866	1·976805	·8923234	10
51	·4516563	·506232	1·975378	·8921920	9
52	·4519158	·506597	1·973953	·8920606	8
53	·4521753	·506963	1·972529	·8919291	7
54	·4524347	·507329	1·971107	·8917975	6
55	·4526941	·507694	1·969687	·8916659	5
56	·4529535	·508060	1·968268	·8915342	4
57	·4532128	·508426	1·966851	·8914024	3
58	·4534721	·508792	1·965436	·8912705	2
59	·4537313	·509159	1·964022	·8911385	1
60	·4539905	·509525	1·962610	·8910065	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

27 Deg.

27 Deg.

27 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4539905	·509525	1·962610	·8910065	60	21	·4594248	517244	1·933323	·8882166	39	41	·4645845	·524640	1·906066	·8855288	19
1	·4542497	·509891	1·961200	·8908744	59	22	·4596832	·517612	1·931945	·8880830	38	42	·4648420	·525011	1.904719	·8853936	18
2	·4545088	·510258	1·959791	·8907423	58	23	·4599415	·517981	1·930569	·8879492	37	43	·4650996	·525382	1.903373	·8852584	17
3	·4547679	·510625	1·958383	·8906100	57	24	·4601998	·518350	1·929195	·8878154	36	44	·4653571	·525754	1·902029	·8851230	16
4	·4550269	·510991	1·956978	·8904777	56	25	·4604580	·518719	1·927822	·8876815	35	45	·4656145	·526125	1 900687	·8849876	15
5	·4552859	·511358	1·955573	·8903453	55	26	·4607162	·519089	1·926451	·8875475	34	46	·4658719	·526496	1·899346	·8848522	14
6	·4555449	·511725	1·954171	·8902128	54	27	·4609744	·519458	1·925081	·8874134	33	47	·4661293	·526868	1·898006	·8847166	13
7	·4558038	·512093	1·952770	·8900803	53	28	·4612325	·519827	1·923713	·8872793	32	48	·4663866	·527240	1·896668	·8845810	12
8	·4560627	·512460	1·951371	·8899476	52	29	·4614906	·520197	1·922347	·8871451	31	49	·4666439	·527612	1·895332	·8844453	11
9	·4563216	·512827	1·949973	·8898149	51	30	·4617486	·520567	1·920982	·8870108	30	50	·4669012	·527983	1·893997	·8843095	10
10	·4565804	·513195	1·948577	·8896822	50	31	·4620066	·520936	1·919618	·8868765	29	51	·4671584	·528356	1·892663	·8841736	9
11	·4568392	·513562	1·947182	·8895493	49	32	·4622646	·521306	1·918256	·8867420	28	52	·4674156	·528728	1·891331	·8840377	8
12	·4570979	·513930	1·945789	·8894164	48	33	·4625225	·521676	1·916896	·8866075	27	53	·4676727	·529100	1·890000	·8839017	7
13	·4573566	·514298	1·944398	·8892834	47	34	·4627804	·522046	1·915537	·8864730	26	54	·4679298	·529472	1·888671	·8837656	6
14	·4576153	·514665	1·943008	·8891503	46	35	·4630382	·522417	1·914179	·8863383	25	55	·4681869	·529845	1·887343	·8836295	5
15	·4578739	·515033	1·941620	·8890171	45	36	·4632960	·522787	1·912823	·8862036	24	56	·4684439	·530217	1·886017	·8834933	4
16	·4581325	·515401	1·940233	·8888839	44	37	·4635538	·523157	1·911469	·8860688	23	57	·4687009	·530590	1·884692	·8833569	3
17	·4583910	·515770	1·938848	·8887506	43	38	·4638115	·523528	1·910116	·8859339	22	58	·4689578	·530963	1·883369	·8832206	2
18	·4586496	·516138	1·937464	·8886172	42	39	·4640692	·523899	1·908764	·8857989	21	59	·4692147	·531336	1·882047	·8830841	1
19	·4589080	·516506	1·936082	·8884838	41	40	·4643269	·524269	1·907414	·8856639	20	60	·4694716	·531709	1·880726	·8829476	0
20	·4591665	·516875	1·934702	·8883503	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 62.

Deg. 62.

Deg. 62.

28 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4694716	·531709	1·880726	·8829476	60
1	·4697284	·532082	1·879407	·8828110	59
2	·4699852	·532455	1·878089	·8826743	58
3	·4702419	·532829	1·876773	·8825376	57
4	·4704986	·533202	1·875458	·8824007	56
5	·4707553	·533576	1·874145	·8822638	55
6	·4710119	·533950	1·872833	·8821269	54
7	·4712685	·534324	1·871523	·8819898	53
8	·4715250	·534698	1·870214	·8818527	52
9	·4717815	·535072	1·868906	·8817155	51
10	·4720380	·535446	1·867600	·8815782	50
11	·4722944	·535820	1·866295	·8814409	49
12	·4725508	·536195	1·864992	·8813035	48
13	·4728071	·536569	1·863690	·8811660	47
14	·4730634	·536944	1·862389	·8810284	46
15	·4733197	·537319	1·861090	·8808907	45
16	·4735759	·537694	1·859792	·8807530	44
17	·4738321	·538069	1·858496	·8806152	43
18	·4740882	·538444	1·857201	·8804774	42
19	·4743443	·538819	1·855908	·8803394	41
20	·4746004	·539195	1·854615	·8802014	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 61.

28 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4748564	·539570	1·853325	·8800633	39
22	·4751124	·539946	1·852035	·8799251	38
23	·4753683	·540322	1·850747	·8797869	37
24	·4756242	·540698	1·849461	·8796486	36
25	·4758801	·541074	1·848176	·8795102	35
26	·4761359	·541450	1·846892	·8793717	34
27	·4763917	·541826	1·845609	·8792332	33
28	·4766474	·542202	1·844328	·8790946	32
29	·4769031	·542579	1·843049	·8789559	31
30	·4771588	·542955	1·841770	·8788171	30
31	·4774144	·543332	1·840494	·8786783	29
32	·4776700	·543709	1·839218	·8785394	28
33	·4779255	·544086	1·837944	·8784004	27
34	·4781810	·544463	1·836671	·8782613	26
35	·4784364	·544840	1·835399	·8781222	25
36	·4786919	·545217	1·834129	·8779830	24
37	·4789472	·545595	1·832861	·8778437	23
38	·4792026	·545972	1·831593	·8777043	22
39	·4794579	·546350	1·830327	·8775649	21
40	·4797131	·546728	1·829062	·8774254	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 61.

28 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4799683	·547106	1·827799	·8772858	19
42	·4802235	·547484	1·826537	·8771462	18
43	·4804786	·547862	1·825276	·8770064	17
44	·4807337	·548240	1·824017	·8768666	16
45	·4809888	·548618	1·822759	·8767268	15
46	·4812438	·548997	1·821502	·8765868	14
47	·4814987	·549375	1·820247	·8764468	13
48	·4817537	·549754	1·818993	·8763067	12
49	·4820086	·550133	1·817740	·8761665	11
50	·4822634	·550512	1·816489	·8760263	10
51	·4825182	·550891	1·815239	·8758859	9
52	·4827730	·551270	1·813990	·8757455	8
53	·4830277	·551650	1·812743	·8756051	7
54	·4832824	·552029	1·811496	·8754645	6
55	·4835370	·552409	1·810252	·8753239	5
56	·4837916	·552789	1·809008	·8751832	4
57	·4840462	·553168	1·807766	·8750425	3
58	·4843007	·553548	1·806525	·8749016	2
59	·4845552	·553928	1·805286	·8747607	1
60	·4848096	·554309	1·804047	·8746197	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 61.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4848096	·554309	1·804047	·8746197	60
1	·4850640	·554689	1·802810	·8744786	59
2	·4853184	·555069	1·801575	·8743375	58
3	·4855727	·555450	1·800340	·8741963	57
4	·4858270	·555831	1·799107	·8740550	56
5	·4860812	·556211	1·797875	·8739137	55
6	·4863354	·556592	1·796645	·8737722	54
7	·4865895	·556973	1·795416	·8736307	53
8	·4868436	·557355	1·794188	·8734891	52
9	·4870977	·557736	1·792961	·8733475	51
10	·4873517	·558117	1·791736	·8732058	50
11	·4876057	·558499	1·790512	·8730640	49
12	·4878597	·558881	1·789289	·8729221	48
13	·4881136	·559262	1·788067	·8727801	47
14	·4883674	·559644	1·786847	·8726381	46
15	·4886212	·560026	1·785628	·8724960	45
16	·4888750	·560409	1·784410	·8723538	44
17	·4891288	·560791	1·783194	·8722116	43
18	·4893825	·561173	1·781979	·8720693	42
19	·4896361	·561556	1·780765	·8719269	41
20	·4898897	·561939	1·779552	·8717844	40
′	Cosine.	Cotang.	Tang.	Sine.	′

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4901433	·562321	1·778340	·8716419	39
22	·4903968	·562704	1·777130	·8714993	38
23	·4906503	·563087	1·775921	·8713566	37
24	·4909038	·563471	1·774714	·8712138	36
25	·4911572	·563854	1·773507	·8710710	35
26	·4914105	·564237	1·772302	·8709281	34
27	·4916638	·564621	1·771098	·8707851	33
28	·4919171	·565005	1·769895	·8706420	32
29	·4921704	·565388	1·768694	·8704989	31
30	·4924236	·565772	1·767494	·8703557	30
31	·4926767	·566156	1·766295	·8702124	29
32	·4929298	·566541	1·765097	·8700691	28
33	·4931829	·566925	1·763900	·8699256	27
34	·4934359	·567309	1·762705	·8697821	26
35	·4936889	·567694	1·761511	·8696386	25
36	·4939419	·568079	1·760318	·8694949	24
37	·4941948	·568463	1·759126	·8693512	23
38	·4944476	·568848	1·757936	·8692074	22
39	·4947005	·569233	1·756747	·8690636	21
40	·4949532	·569619	1·755559	·8689196	20
′	Cosine.	Cotang.	Tang.	Sine.	′

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4952060	·570004	1·754372	·8687756	19
42	·4954587	·570389	1·753186	·8686315	18
43	·4957113	·570775	1·752002	·8684874	17
44	·4959639	·571161	1·750819	·8683431	16
45	·4962165	·571547	1·749637	·8681988	15
46	·4964690	·571933	1·748456	·8680544	14
47	·4967215	·572319	1·747276	·8679100	13
48	·4969740	·572705	1·746098	·8677655	12
49	·4972264	·573091	1·744921	·8676209	11
50	·4974787	·573478	1·743745	·8674762	10
51	·4977310	·573864	1·742570	·8673314	9
52	·4979833	·574251	1·741396	·8671866	8
53	·4982355	·574638	1·740224	·8670417	7
54	·4984877	·575025	1·739053	·8668967	6
55	·4987399	·575412	1·737883	·8667517	5
56	·4989920	·575799	1·736714	·8666066	4
57	·4992441	·576187	1·735546	·8664614	3
58	·4994961	·576574	1·734380	·8663161	2
59	·4997481	·576962	1·733214	·8661708	1
60	·5000000	·577350	1·732050	·8660254	0
′	Cosine.	Cotang.	Tang.	Sine.	′

30 Deg. **30 Deg.** **30 Deg.**

'	Sine.	Tang.	Cotang.	Cosine.	'	'	Sine.	Tang.	Cotang.	Cosine.	'	'	Sine.	Tang.	Cotang.	Cosine.	'
0	·5000000	·577350	1·732050	·8660254	60	21	·5052809	·585524	1·707871	·8629549	39	41	·5102928	·593363	1·685308	·8600007	19
1	·5002519	·577738	1·730887	·8658799	59	22	·5055319	·585914	1·706732	·8628079	38	42	·5105429	·593756	1·684191	·8598523	18
2	·5005037	·578126	1·729726	·8657344	58	23	·5057828	·586305	1·705595	·8626608	37	43	·5107930	·594150	1·683076	·8597037	17
3	·5007556	·578514	1·728565	·8655887	57	24	·5060338	·586696	1·704458	·8625137	36	44	·5110431	·594543	1·681962	·8595551	16
4	·5010073	·578902	1·727406	·8654430	56	25	·5062846	·587087	1·703323	·8623664	35	45	·5112931	594937	1·680848	·8594064	15
5	·5012591	·579291	1·726247	·8652973	55	26	·5065355	·587478	1·702189	·8622191	34	46	·5115431	·595331	1·679736	·8592576	14
6	·5015107	·579679	1·725090	·8651514	54	27	·5067863	·587870	1·701055	·8620717	33	47	·5117930	·595725	1·678625	·8591088	13
7	·5017624	·580068	1·723934	·8650055	53	28	·5070370	·588261	1·699923	·8619243	32	48	·5120429	·596119	1.677515	·8589599	12
8	·5020140	·580457	1·722779	·8648595	52	29	·5072877	·588653	1·698792	·8617768	31	49	·5122927	·596514	1·676406	·8588109	11
9	·5022655	·580846	1·721626	·8647134	51	30	·5075384	·589045	1·697663	.8616292	30	50	·5125425	·596908	1·675298	·8586619	10
10	·5025170	·581235	1·720473	·8645673	50	31	·5077890	·589436	1·696534	·8614815	29	51	·5127923	·597303	1·674192	·8585127	9
11	·5027685	·581624	1·719322	·8644211	49	32	·5080396	·589828	1·695406	·8613337	28	52	·5130420	·597697	1·673086	·8583635	8
12	·5030199	·582013	1·718172	·8642748	48	33	·5082901	·590221	1·694280	·8611859	27	53	·5132916	·598092	1·671981	·8582143	7
13	·5032713	·582403	1·717023	·8641284	47	34	·5085406	·590613	1·693155	·8610380	26	54	·5135413	·598487	1·670878	·8580649	6
14	·5035227	·582793	1·715875	·8639820	46	35	·5087910	·591005	1·692030	·8608901	25	55	·5137908	·598882	1·669775	·8579155	5
15	·5037740	·583182	1·714728	·8638355	45	36	·5090414	·591398	1·690907	·8607420	24	56	·5140404	·599278	1·668674	·8577660	4
16	·5040252	·583572	1·713582	·8636889	44	37	·5092918	·591791	1·689785	·8605939	23	57	·5142899	·599673	1·667574	·8576164	3
17	·5042765	·583962	1·712438	·8635423	43	38	·5095421	·592183	1·688664	·8604457	22	58	·5145393	·600069	1·666474	·8574668	2
18	·5045276	·584352	1·711294	·8633956	42	39	·5097924	·592576	1·687544	·8602975	21	59	·5147887	·600464	1·665376	·8573171	1
19	·5047788	·584743	1·710152	·8632488	41	40	·5100426	·592969	1·686426	·8601491	20	60	·5150381	·600860	1·664279	·8571673	0
20	·5050298	·585133	1·709011	·8631019	40												
'	Cosine.	Cotang.	Tang.	Sine.	'	'	Cosine.	Cotang.	Tang.	Sine.	'	'	Cosine.	Cotang.	Tang.	Sine.	'

Deg. 59. **Deg. 59.** **Deg 59.**

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5150381	·600860	1·664279	·8571673	60
1	·5152874	·601256	1·663183	·8570174	59
2	·5155367	·601652	1·662088	·8568675	58
3	·5157859	·602049	1·660994	·8567175	57
4	·5160351	·602445	1·659901	·8565674	56
5	·5162842	·602841	1·658809	·8564173	55
6	·5165333	·603238	1·657718	·8562671	54
7	·5167824	·603635	1·656629	·8561168	53
8	·5170314	·604032	1·655540	·8559664	52
9	·5172804	·604429	1·654452	·8558160	51
10	·5175293	·604826	1·653366	·8556655	50
11	·5177782	·605224	1·652280	·8555149	49
12	·5180270	·605621	1·651196	·8553643	48
13	·5182758	·606019	1·650112	·8552135	47
14	·5185246	·606417	1·649030	·8550627	46
15	·5187733	·606814	1·647949	·8549119	45
16	·5190219	·607213	1·646868	·8547609	44
17	·5192705	·607611	1·645789	·8546099	43
18	·5195191	·608009	1·644711	·8544588	42
19	·5197676	·608408	1·643633	·8543077	41
20	·5200161	·608806	1·642557	·8541564	40
	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5202646	·609205	1·641482	·8540051	39
22	·5205130	·609604	1·640408	·8538538	38
23	·5207613	·610003	1·639335	·8537023	37
24	·5210096	·610402	1·638263	·8535508	36
25	·5212579	.610801	1·637191	·8533992	35
26	·5215061	·611201	1·636121	·8532475	34
27	·5217543	·611601	1·635052	·8530958	33
28	·5220024	·612000	1·633984	·8529440	32
29	·5222505	·612400	1·632917	·8527921	31
30	·5224986	·612800	1·631851	·8526402	30
31	·5227466	·613201	1·630786	·8524881	29
32	·5229945	·613601	1·629722	·8523360	28
33	·5232424	·614001	1·628659	·8521839	27
34	·5234903	·614402	1·627597	·8520316	26
35	·5237381	·614803	1·626536	·8518793	25
36	·5239859	·615204	1·625476	·8517269	24
37	·5242336	·615605	1·624417	·8515745	23
38	·5244813	·616006	1·623359	·8514219	22
39	·5247290	·616407	1·622302	·8512693	21
40	·5249766	·616809	1·621246	·8511167	20
′	Cosine	Cotang.	Tang.	Sine.	′

Deg. 58.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5252241	·617210	1·620192	·8509639	19
42	·5254717	·617612	1·619138	·8508111	18
43	·5257191	·618014	1·618085	·8506582	17
44	·5259665	·618416	1·617033	·8505053	16
45	·5262139	·618818	1·615982	·8503522	15
46	·5264613	·619221	1·614932	·8501991	14
47	·5267085	·619623	1·613882	·8500459	13
48	·5269558	·620026	1·612834	·8498927	12
49	·5272030	·620429	1·611787	·8497394	11
50	·5274502	·620832	1·610741	·8495860	10
51	·5276973	·621235	1·609696	·8494325	9
52	·5279443	·621638	1·608652	·8492790	8
53	·5281914	·622041	1·607609	·8491254	7
54	·5284383	·622445	1·606567	·8489717	6
55	·5286853	·622848	1·605526	·8488179	5
56	·5289322	·623252	1·604485	·8486641	4
57	·5291790	·623656	1·603446	·8485102	3
58	·5294258	·624060	1·602408	·8483562	2
59	·5296726	·624465	1·601370	·8482022	1
60	·5299193	·624869	1·600334	·8480481	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5299193	·624869	1·600334	·8480481	60
1	·5301659	·625273	1·599299	·8478939	59
2	·5304125	·625678	1·598264	·8477397	58
3	·5306591	·626083	1·597231	·8475853	57
4	·5309057	·626488	1·596198	·8474309	56
5	·5311521	·626893	1·595167	·8472765	55
6	·5313986	·627298	1·594136	·8471219	54
7	·5316450	·627704	1·593107	·8469673	53
8	·5318913	·628109	1·592078	·8468126	52
9	·5321376	·628515	1·591050	·8466579	51
10	·5323839	·628921	1·590023	·8465030	50
11	·5326301	·629327	1·588997	8463481	49
12	·5328763	·629733	1·587973	·8461932	48
13	·5331224	·630139	1·586949	·8460381	47
14	·5333685	·630546	1·585926	·8458830	46
15	·5336145	·630953	1·584904	·8457278	45
16	·5338605	·631359	1·583883	·8455726	44
17	·5341065	·631766	1·582862	·8454172	43
18	·5343523	·632173	1·581843	·8452618	42
19	·5345982	·632581	1·580825	·8451064	41
20	·5348440	632988	1·579807	·8449508	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5350898	·633395	1·578791	·8447952	39
22	·5353355	·633803	1·577776	·8446395	38
23	·5355812	·634211	1·576761	·8444838	37
24	·5358268	·634619	1·575747	·8443279	36
25	·5360724	·635027	1·574735	·8441720	35
26	·5363179	·635435	1·573723	·8440161	34
27	·5365634	·635844	1·572712	·8438600	33
28	·5368089	·636252	1·571702	·8437039	32
29	·5370543	·636661	1·570693	·8435477	31
30	·5372996	·637070	1·569685	·8433914	30
31	·5375449	·637479	1·568678	·8432351	29
32	·5377902	·637888	1·567672	·8430787	28
33	·5380354	·638297	1·566666	·8429222	27
34	·5382806	·638707	1·565662	·8427657	26
35	·5385257	·639116	1·564659	·8426091	25
36	·5387708	·639526	1·563656	·8424524	24
37	·5390158	·639936	1·562654	·8422956	23
38	·5392608	·640346	1·561654	·8421388	22
39	·5395058	·640756	1·560654	·8419819	21
40	·5397507	·641167	1·559655	·8418249	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5399955	·641577	1·558657	·8416679	19
42	·5402403	·641988	1·557660	·8415108	18
43	·5404851	·642399	1·556663	·8413536	17
44	·5407298	·642810	1·555668	·8411963	16
45	·5409745	·643221	1·554674	·8410390	15
46	·5412191	·643632	1·553680	·8408816	14
47	·5414637	·644044	1·552688	·8407241	13
48	·5417082	·644456	1·551696	·8405666	12
49	.5419527	·644867	1·550705	·8404090	11
50	·5421971	·645279	1·549715	·8402513	10
51	·5424415	·645691	1·548726	·8400936	9
52	·5426859	·646104	1·547738	·8399357	8
53	·5429302	·646516	1·546751	·8397778	7
54	·5431744	·646929	1·545764	·8396199	6
55	·5434187	·647341	1·544779	·8394618	5
56	·5436628	·647754	1·543794	·8393037	4
57	·5439069	·648167	1·542810	·8391455	3
58	·5441510	·648580	1·541828	·8389873	2
59	·5443951	·648994	1·540846	·8388290	1
60	·5446390	·649407	1·539865	·8386706	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5446390	·649407	1·539865	·8386706	60
1	·5448830	·649821	1·538884	·8385121	59
2	·5451269	·650235	1·537905	·8383536	58
3	·5453707	·650649	1·536927	·8381950	57
4	·5456145	·651063	1·535949	·8380363	56
5	·5458583	·651477	1·534972	·8378775	55
6	·5461020	·651891	1·533996	·8377187	54
7	·5463456	·652306	1·533021	·8375598	53
8	·5465892	·652721	1·532047	·8374009	52
9	·5468328	·653136	1·531074	·8372418	51
10	·5470763	·653551	1·530102	·8370827	50
11	·5473198	·653966	1·529130	·8369236	49
12	·5475632	·654381	1·528160	·8367643	48
13	·5478066	·654797	1·527190	·8366050	47
14	·5480499	·655212	1·526221	·8364456	46
15	·5482932	·655628	1·525253	·8362862	45
16	·5485365	·656044	1·524286	·8361266	44
17	·5487797	·656460	1·523320	·8359670	43
18	·5490228	·656877	1·522354	·8358074	42
19	·5492659	·657293	1·521389	·8356476	41
20	·5495090	·657710	1·520426	·8354878	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5497520	·658127	1·519463	·8353279	39
22	·5499950	·658544	1·518501	·8351680	38
23	·5502379	·658961	1 517540	·8350080	37
24	·5504807	·659378	1·516579	·8348479	36
25	·5507236	·659796	1·515620	·8346877	35
26	·5509663	·660213	1·514661	·8345275	34
27	·5512091	·660631	1·513703	·8343672	33
28	·5514518	·661049	1·512746	·8342068	32
29	·5516944	·661467	1·511790	·8340463	31
30	·5519370	·661885	1·510835	·8338858	30
31	·5521795	·662304	1·509880	·8337252	29
32	·5524220	·662722	1·508927	·8335646	28
33	·5526645	·663141	1·507974	·8334038	27
34	·5529069	·663560	1·507022	·8332430	26
35	·5531492	·663979	1·506071	8330822	25
36	·5533915	·664398	1·505121	·8329212	24
37	·5536338	·664817	1·504171	·8327602	23
38	·5538760	·665237	1·503222	·8325991	22
39	·5541182	·665657	1·502275	·8324380	21
40	·5543603	·666076	1·501328	·8322768	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5546024	·666496	1·500382	·8321155	19
42	·5548444	·666917	1·499436	·8319541	18
43	·5550864	·667337	1·498492	·8317927	17
44	·5553283	·667758	1·497548	·8316312	16
45	5555702	·668178	1·496605	·8314696	15
46	·5558121	·668599	1·495663	·8313080	14
47	5560539	·669020	1·494722	·8311463	13
48	·5562956	·669441	1·493782	·8309845	12
49	·5565373	·669863	1·492842	·8308226	11
50	·5567790	·670284	1·491903	·8306607	10
51	·5570206	·670706	1·490965	·8304987	9
52	·5572621	·671128	1·490028	·8303366	8
53	·5575036	·671550	1·489092	·8301745	7
54	·5577451	·671972	1·488157	·8300123	6
55	·5579865	·672394	1·487222	·8298500	5
56	·5582279	·672816	1·486288	·8296877	4
57	·5584692	·673239	1·485355	·8295252	3
58	·5587105	·673662	1·484423	·8293628	2
59	·5589517	·674085	1·483491	·8292002	1
60	·5591929	·674508	1·482561	·8290376	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5591929	·674508	1·482561	·8290376	60
1	·5594340	·674931	1·481631	·8288749	59
2	·5596751	·675355	1·480702	·8287121	58
3	·5599162	·675779	1·479773	·8285493	57
4	·5601572	·676202	1·478846	·8283864	56
5	·5603981	·676626	1·477919	·8282234	55
6	·5606390	·677050	1·476993	·8280603	54
7	·5608798	·677475	1·476068	·8278972	53
8	·5611206	·677899	1·475144	·8277340	52
9	·5613614	·678324	1·474221	·8275708	51
10	·5616021	·678749	1·473298	·8274074	50
11	·5618428	·679174	1·472376	·8272440	49
12	·5620834	·679599	1·471455	·8270806	48
13	·5623239	·680024	1·470535	·8269170	47
14	·5625645	·680450	1·469615	·8267534	46
15	·5628049	·680875	1·468696	·8265897	45
16	·5630453	·681301	1·467778	·8264260	44
17	·5632857	·681727	1·466861	·8262622	43
18	·5635260	·682153	1·465945	·8260983	42
19	·5637663	·682580	1·465029	·8259343	41
20	·5640066	·683006	1·464114	·8257703	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55.

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5642467	·683433	1·463200	·8256062	39
22	·5644869	·683860	1·462287	·8254420	38
23	·5647270	·684287	1·461374	·8252778	37
24	·5649670	·684714	1·460463	·8251135	36
25	·5652070	·685141	1·459552	·8249491	35
26	·5654469	·685569	1·458642	·8247847	34
27	·5656868	·685996	1·457732	·8246202	33
28	·5659267	·686424	1·456824	·8244556	32
29	·5661665	·686852	1·455916	·8242909	31
30	·5664062	·687281	1·455009	·8241262	30
31	·5666459	·687709	1·454102	·8239614	29
32	·5668856	·688137	1·453197	·8237965	28
33	·5671252	·688566	1·452292	·8236316	27
34	·5673648	·688995	1·451388	·8234666	26
35	·5676043	·689424	1·450485	·8233015	25
36	·5678437	·689853	1·449582	·8231364	24
37	·5680832	·690283	1·448680	·8229712	23
38	·5683225	·690712	1·447779	·8228059	22
39	·5685619	·691142	1·446879	·8226405	21
40	·5688011	·691572	1·445980	·8224751	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55.

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5690403	·692002	1·445081	·8223096	19
42	·5692795	·692432	1·444183	·8221440	18
43	·5695187	·692863	1·443286	·8219784	17
44	·5697577	·693293	1·442389	·8218127	16
45	·5699968	·693724	1·441494	·8216469	15
46	·5702357	·694155	1·440599	·8214811	14
47	·5704747	·694586	1·439704	·8213152	13
48	·5707136	·695018	1·438811	·8211492	12
49	·5709524	·695449	1·437918	·8209832	11
50	·5711912	·695881	1·437026	·8208170	10
51	·5714299	·696313	1·436135	·8206509	9
52	·5716686	·696745	1·435245	·8204846	8
53	·5719073	·697177	1·434355	·8203183	7
54	·5721459	·697609	1·433466	·8201519	6
55	·5723844	.698042	1·432578	·8199854	5
56	·5726229	·698474	1·431690	·8198189	4
57	·5728614	·698907	1.430803	·8196523	3
58	·5730998	·699340	1.429917	·8194856	2
59	·5733381	·699774	1.429032	·8193189	1
60	·5735764	·700207	1.428148	·8191520	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55

35 Deg.

′	Sine.	Tang	Cotang.	Cosine.	′
0	·5735764	·700207	1·428148	·8191520	60
1	·5738147	·700641	1·427264	·8189852	59
2	·5740529	·701074	1·426381	·8188182	58
3	·5742911	·701508	1·425498	·8186512	57
4	·5745292	·701943	1·424617	·8184841	56
5	·5747672	·702377	1·423736	·8183169	55
6	·5750053	·702811	1·422856	·8181497	54
7	·5752432	·703246	1·421976	·8179824	53
8	·5754811	·703681	1·421097	·8178151	52
9	·5757190	·704116	1·420220	·8176476	51
10	·5759568	·704551	1·419342	·8174801	50
11	·5761946	·704986	1·418466	·8173125	49
12	·5764323	·705422	1·417590	·8171449	48
13	·5766700	·705858	1·416715	·8169772	47
14	·5769076	·706294	1·415840	·8168094	46
15	·5771452	·706730	1·414967	·8166416	45
16	·5773827	·707166	1·414094	·8164736	44
17	·5776202	·707602	1·413222	·8163056	43
18	·5778576	·708039	1·412350	·8161376	42
19	·5780950	·708476	1·411479	·8159695	41
20	·5783323	·708913	1·410609	·8158013	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·5785696	·709350	1·409740	·8156330	39
22	·5788069	·709787	1·408871	·8154647	38
23	·5790440	·710225	1·408003	·8152963	37
24	·5792812	·710663	1·407136	·8151278	36
25	.5795183	·711100	1·406270	·8149593	35
26	·5797553	·711539	1·405404	·8147906	34
27	·5799923	·711977	1·404539	·8146220	33
28	·5802292	·712415	1·403674	·8144532	32
29	·5804661	·712854	1·402811	·8142844	31
30	·5807030	·713293	1·401948	·8141155	30
31	·5809397	·713732	1·401086	·8139466	29
32	·5811765	·714171	1·400224	·8137775	28
33	·5814132	·714610	1·399363	·8136084	27
34	·5816498	·715050	1·398503	·8134393	26
35	·5818864	·715490	1·397644	·8132701	25
36	·5821230	·715929	1·396785	·8131008	24
37	·5823595	·716369	1·395927	·8129314	23
38	·5825959	·716810	1·395069	·8127620	22
39	·5828323	·717250	1·394213	·8125925	21
40	·5830687	·717691	1·393357	·8124229	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5833050	·718131	1·392501	·8122532	19
42	·5835412	·718572	1·391647	·8120835	18
43	·5837774	·719014	1·390793	·8119137	17
44	·5840136	·719455	1·389940	·8117439	16
45	·5842497	·719897	1·389087	·8115740	15
46	·5844857	·720338	1·388235	·8114040	14
47	·5847217	·720780	1·387384	·8112339	13
48	·5849577	·721222	1·386534	·8110638	12
49	·5851936	·721665	1·385684	·8108936	11
50	·5854294	·722107	1·384835	·8107234	10
51	·5856652	·722550	1·383986	·8105530	9
52	·5859010	·722993	1·383139	·8103826	8
53	·5861367	·723436	1·382292	·8102122	7
54	·5863724	·723879	1·381445	·8100416	6
55	·5866080	·724322	1·380600	·8098710	5
56	·5868435	·724766	1·379755	·8097004	4
57	·5870790	·725210	1·378910	·8095296	3
58	·5873145	·725654	1·378067	·8093588	2
59	·5875499	·726098	1·377224	·8091879	1
60	·5877853	·726542	1·376381	·8090170	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5877853	·726542	1·376381	·8090170	60
1	·5880206	·726987	1·375540	·8088460	59
2	·5882558	·727431	1·374699	·8086749	58
3	·5884910	·727876	1·373859	·8085037	57
4	·5887262	·728321	1·373019	·8083325	56
5	5889613	·728767	1·372180	·8081612	55
6	·5891964	·729212	1·371342	·8079899	54
7	·5894314	·729658	1·370504	·8078185	53
8	·5896663	·730104	1·369667	·8076470	52
9	·5899012	·730550	1·368831	·8074754	51
10	·5901361	·730996	1·367995	·8073038	50
11	·5903709	·731442	1·367161	·8071321	49
12	·5906057	·731889	1·366326	·8069603	48
13	·5908404	·732336	1·365493	·8067885	47
14	·5910750	·732783	1·364660	·8066166	46
15	·5913096	·733230	1·363827	·8064446	45
16	·5915442	·733677	1·362996	·8062726	44
17	·5917787	·734125	1·362165	·8061005	43
18	·5920132	·734573	1·361335	·8059283	42
19	·5922476	·735021	1·360505	·8057560	41
20	·5924819	·735469	1·359676	·8055837	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5927163	·735917	1·358848	·8054113	39
22	·5929505	·736366	1·358020	·8052389	38
23	·5931847	·736814	1·357193	·8050664	37
24	·5934189	·737263	1·356367	·8048938	36
25	·5936530	·737712	1·355541	·8047211	35
26	·5938871	·738162	1·354716	·8045484	34
27	·5941211	·738611	1·353891	·8043756	33
28	·5943550	·739061	1·353068	·8042028	32
29	·5945889	·739511	1·352244	·8040299	31
30	·5948228	·739961	1·351422	·8038569	30
31	·5950566	·740411	1·350600	·8036838	29
32	·5952904	·740861	1·349779	·8035107	28
33	·5955241	·741312	1·348958	·8033375	27
34	·5957577	·741763	1·348139	·8031642	26
35	·5959913	·742214	1·347319	·8029909	25
36	·5962249	·742665	1·346501	·8028175	24
37	·5964584	·743117	1.345683	·8026440	23
38	·5966918	·743568	1.344865	·8024705	22
39	·5969252	·744020	1.344049	·8022969	21
40	·5971586	·744472	1.343233	·8021232	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5973919	·744924	1·342417	·8019495	19
42	·5976251	·745377	1·341602	·8017756	18
43	·5978583	·745829	1·340788	·8016018	17
44	·5980915	·746282	1·339975	·8014278	16
45	·5983246	·746735	1·339162	·8012538	15
46	·5985577	·747188	1·338350	·8010797	14
47	·5987906	·747642	1·337538	·8009056	13
48	·5990236	·748095	1·336727	·8007314	12
49	·5992565	·748549	1·335917	·8005571	11
50	·5994893	·749003	1·335107	·8003827	10
51	·5997221	·749457	1·334298	·8002083	9
52	·5999549	·749911	1·333490	·8000338	8
53	·6001876	·750366	1·332682	·7998593	7
54	·6004202	·750821	1·331875	·7996847	6
55	·6006528	·751276	1·331068	·7995100	5
56	·6008854	·751731	1·330262	·7993352	4
57	·6011179	·752186	1·329457	·7991604	3
58	·6013503	·752642	1·328652	·7989855	2
59	·6015827	·753098	1·327848	·7988105	1
60	·6018150	·753554	1·327044	·7986355	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

37 Deg. 37 Deg. 37 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6018150	·753554	1·327044	·7986355	60	21	·6066824	·763175	1·310314	·7949444	39	41	·6112969	·772423	1·294627	·7914014	19
1	·6020473	·754010	1·326242	·7984604	59	22	·6069136	·763636	1·309523	·7947678	38	42	·6115270	·772887	1·293848	·7912235	18
2	·6022795	·754466	1·325439	·7982853	58	23	·6071447	·764096	1·308734	·7945913	37	43	·6117572	·773352	1·293071	·7910456	17
3	.6025117	·754923	1·324638	·7981100	57	24	·6073758	·764557	1·307945	·7944146	36	44	·6119873	·773817	1·292294	·7908676	16
4	·6027439	·755379	1·323837	·7979347	56	25	·6076069	·765018	1·307157	·7942379	35	45	·6122173	·774282	1·291517	·7906896	15
5	·6029760	·755836	1·323036	·7977594	55	26	·6078379	·765480	1·306369	·7940611	34	46	·6124473	·774748	1·290742	·7905115	14
6	·6032080	·756294	1·322237	·7975839	54	27	·6080689	·765941	1·305582	·7938843	33	47	·6126772	·775213	1·289966	·7903333	13
7	·6034400	·756751	1·321437	·7974084	53	28	·6082998	·766403	1·304796	·7937074	32	48	·6129071	·775679	1·289192	·7901550	12
8	·6036719	·757209	1·320639	·7972329	52	29	·6085306	·766864	1·304010	·7935304	31	49	·6131369	·776145	1·288418	·7899767	11
9	·6039038	·757666	1·319841	·7970572	51	30	·6087614	·767327	1·303225	·7933533	30	50	·6133666	·776611	1·287644	·7897983	10
10	·6041356	·758124	1·319044	·7968815	50	31	·6089922	·767789	1·302440	·7931762	29	51	·6135964	·777078	1·286871	·7896198	9
11	·6043674	·758582	1·318247	·7967058	49	32	·6092229	·768251	1·301656	·7929990	28	52	·6138260	·777544	1·286099	·7894413	8
12	·6045991	·759041	1·317451	·7965299	48	33	·6094535	·768714	1·300873	·7928218	27	53	·6140556	·778011	1·285327	·7892627	7
13	·6048308	·759499	1·316655	·7963540	47	34	·6096841	·769177	1·300090	·7926445	26	54	·6142852	·778478	1·284556	·7890841	6
14	·6050624	·759958	1·315861	·7961780	46	35	·6099147	·769640	1·299308	·7924671	25	55	·6145147	·778946	1·283786	·7889054	5
15	·6052940	·760417	1.315066	·7960020	45	36	·6101452	·770103	1·298526	·7922896	24	56	·6147442	·779413	1·283016	·7887266	4
16	·6055255	·760876	1·314273	·7958259	44	37	·6103756	·770567	1·297745	·7921121	23	57	·6149736	·779881	1·282246	·7885477	3
17	·6057570	·761336	1.313480	·7956497	43	38	·6106060	·771030	1·296964	·7919345	22	58	·6152029	·780349	1·281477	·7883688	2
18	·6059884	·761795	1.312687	·7954735	42	39	·6108363	·771494	1·296185	·7917569	21	59	·6154322	·780817	1·280709	·7881898	1
19	·6062198	·762255	1.311895	.7952972	41	40	·6110666	·771958	1·295405	·7915792	20	60	·6156615	·781285	1·279941	·7880108	0
20	·6064511	·762715	1.311104	·7951208	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 52. Deg. 52. Deg. 52.

38 Deg. 38 Deg. 38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6156615	·781285	1·279941	·7880108	60	21	·6204636	·791170	1·263950	·7842352	39	41	·6250156	·800673	1·248948	·7806123	19
1	·6158907	·781754	1·279174	·7878316	59	22	·6206917	·791643	1·263195	·7840547	38	42	·6252427	·801151	1·248204	·7804304	18
2	·6161198	·782222	1·278407	·7876524	58	23	·6209198	·792116	1·262440	·7838741	37	43	·6254696	·801628	1·247460	·7802485	17
3	·6163489	·782691	1·277641	·7874732	57	24	·6211478	·792590	1·261686	·7836935	36	44	·6256966	·802106	1·246716	·7800665	16
4	·6165780	·783161	1·276876	·7872939	56	25	·6213757	·793064	1·260932	·7835127	35	45	·6259235	·802584	1·245974	·7798845	15
5	·6168069	·783630	1·276111	·7871145	55	26	·6216036	·793537	1·260179	·7833320	34	46	·6261503	·803063	1·245232	·7797024	14
6	·6170359	·784100	1·275347	·7869350	54	27	·6218314	·794012	1·259426	·7831511	33	47	·6263771	·803541	1·244490	·7795202	13
7	·6172648	·784570	1·274583	·7867555	53	28	·6220592	·794486	1·258674	·7829702	32	48	·6266038	·804020	1·243749	·7793380	12
8	·6174936	·785040	1·273820	·7865759	52	29	·6222870	·794961	1·257923	·7827892	31	49	·6268305	·804499	1·243008	·7791557	11
9	·6177224	·785510	1·273057	·7863963	51	30	·6225146	·795435	1·257172	·7826082	30	50	·6270571	·804979	1·242268	·7789733	10
10	·6179511	·785980	1·272295	·7862165	50	31	·6227423	·795911	1·256421	·7824270	29	51	·6272837	·805458	1·241529	·7787909	9
11	·6181798	·786451	1·271534	·7860367	49	32	·6229698	·796386	1·255672	·7822459	28	52	·6275102	·805938	1·240790	·7786084	8
12	·6184084	·786922	1·270773	·7858569	48	33	·6231974	·796861	1·254922	·7820646	27	53	·6277366	·806418	1·240051	·7784258	7
13	·6186370	·787393	1·270013	·7856770	47	34	·6234248	·797337	1·254174	·7818833	26	54	·6279631	·806898	1·239313	·7782431	6
14	·6188655	·787864	1·269253	·7854970	46	35	·6236522	·797813	1·253426	·7817019	25	55	·6281894	·807378	1·238576	·7780604	5
15	·6190939	·788336	1·268494	·7853169	45	36	·6238796	·798289	1·252678	·7815205	24	56	·6284157	·807859	1·237839	·7778777	4
16	·6193224	·788808	1·267735	·7851368	44	37	·6241069	·798765	1·251931	·7813390	23	57	·6286420	·808340	1·237103	·7776949	3
17	·6195507	·789280	1·266977	·7849566	43	38	·6243342	·799242	1·251184	·7811574	22	58	·6288682	·808821	1·236367	·7775120	2
18	·6197790	·789752	1·266219	·7847764	42	39	·6245614	·799719	1·250438	·7809757	21	59	·6290943	·809302	1·235631	·7773290	1
19	·6200073	·790224	1·265462	·7845961	41	40	·6247885	·800196	1·249693	·7807940	20	60	·6293204	·809784	1·234897	·7771460	0
20	·6202355	·790697	1·264706	·7844157	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 51. Deg. 51. Deg. 51.

39 Deg.

'	Sine.	Tang.	Cotang.	Cosine.	'
0	·6293204	·809784	1·234897	·7771460	60
1	·6295464	·810265	1·234162	·7769629	59
2	·6297724	·810747	1·233429	·7767797	58
3	·6299983	·811230	1·232696	·7765965	57
4	·6302242	·811712	1·231963	·7764132	56
5	·6304500	·812195	1·231231	·7762298	55
6	·6306758	·812678	1·230499	·7760464	54
7	·6309015	·813161	1·229768	·7758629	53
8	·6311272	·813644	1·229038	·7756794	52
9	·6313528	·814128	1·228308	·7754957	51
10	·6315784	·814611	1·227578	·7753121	50
11	·6318039	·815095	1·226849	·7751283	49
12	·6320293	·815580	1·226121	·7749445	48
13	·6322547	·816064	1·225393	·7747606	47
14	·6324800	·816549	1·224665	·7745767	46
15	·6327053	·817034	1·223938	·7743926	45
16	·6329306	·817519	1·223212	·7742086	44
17	·6331557	·818004	1·222486	·7740244	43
18	·6333809	·818490	1·221761	·7738402	42
19	·6336059	·818976	1·221036	·7736559	41
20	·6338310	·819462	1·220312	·7734716	40
'	Cosine.	Cotang.	Tang.	Sine.	'

Deg. 50.

39 Deg.

'	Sine.	Tang.	Cotang.	Cosine.	'
21	·6340559	·819948	1·219588	·7732872	39
22	·6342808	·820435	1·218865	·7731027	38
23	·6345057	·820922	1·218142	·7729182	37
24	·6347305	·821409	1·217419	·7727336	36
25	·6349553	·821896	1·216698	·7725489	35
26	·6351800	·822384	1·215976	·7723642	34
27	·6354046	·822871	1·215256	·7721794	33
28	·6356292	·823359	1·214535	·7719945	32
29	·6358537	·823847	1·213816	·7718096	31
30	·6360782	·824336	1·213097	·7716246	30
31	·6363026	·824825	1·212378	·7714395	29
32	·6365270	·825314	1·211660	·7712544	28
33	·6367513	·825803	1·210942	·7710692	27
34	·6369756	·826292	1·210225	·7708840	26
35	·6371998	·826782	1·209508	·7706986	25
36	·6374240	·827271	1·208792	·7705132	24
37	·6376481	·827762	1·208076	·7703278	23
38	·6378721	·828252	1·207361	·7701423	22
39	·6380961	·828742	1·206646	·7699567	21
40	·6383201	·829233	1·205932	·7697710	20
'	Cosine.	Cotang.	Tang.	Sine.	'

Deg. 50.

39 Deg.

'	Sine.	Tang.	Cotang.	Cosine.	'
41	·6385440	·829724	1·205219	·7695853	19
42	·6387678	·830216	1·204505	·7693996	18
43	·6389916	·830707	1·203793	·7692137	17
44	·6392153	·831199	1·203081	·7690278	16
45	·6394390	·831691	1·202369	·7688418	15
46	·6396626	·832183	1·201658	·7686558	14
47	·6398862	·832675	1·200947	·7684697	13
48	·6401097	·833168	1·200237	·7682835	12
49	·6403332	·833661	1·199527	·7680973	11
50	·6405566	·834154	1·198818	·7679110	10
51	·6407799	·834648	1·198109	·7677246	9
52	·6410032	·835141	1·197401	·7675382	8
53	·6412264	·835635	1·196693	·7673517	7
54	·6414496	·836129	1·195986	·7671652	6
55	·6416728	·836624	1·195279	·7669785	5
56	·6418958	·837118	1·194573	·7667918	4
57	·6421189	·837613	1·193867	·7666051	3
58	·6423418	·838108	1·193162	·7664183	2
59	·6425647	·838604	1·192457	·7662314	1
60	·6427876	·839099	1·191753	·7660444	0
'	Cosine.	Cotang.	Tang.	Sine.	'

Deg. 50.

40 Deg. 40 Deg. 40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6427876	·839099	1·191753	·7660444	60	21	·6474551	·849563	1·177075	·7621036	39	41	·6518778	·859629	1·163291	·7583240	19
1	·6430104	·839595	1·191049	·7658574	59	22	·6476767	·850064	1·176382	·7619152	38	42	·6520984	·860135	1·162607	·7581343	18
2	·6432332	·840091	1.190346	·7656704	58	23	·6478984	·850565	1·175688	·7617268	37	43	6523189	·860641	1·161923	·7579446	17
3	·6434559	·840587	1·189643	·7654832	57	24	·6481199	·851066	1·174996	·7615383	36	44	·6525394	·861148	1·161240	·7577548	16
4	·6436785	·841084	1·188941	·7652960	56	25	·6483414	·851568	1·174303	·7613497	35	45	·6527598	·861655	1·160557	·7575650	15
5	·6439011	·841581	1·188239	·7651087	55	26	·6485628	·852070	1·173612	·7611611	34	46	·6529801	·862162	1·159874	·7573751	14
6	·6441236	·842078	1·187538	·7649214	54	27	·6487842	·852572	1·172920	·7609724	33	47	·6532004	·862669	1·159192	·7571851	13
7	·6443461	·842575	1·186837	·7647340	53	28	·6490056	·853075	1·172229	·7607837	32	48	·6534206	·863176	1·158511	·7569951	12
8	·6445685	·843073	1·186136	·7645465	52	29	·6492268	·853577	1·171539	·7605949	31	49	·6536408	·863684	1·157830	·7568050	11
9	·6447909	·843570	1·185437	·7643590	51	30	·6494480	·854080	1·170849	·7604060	30	50	·6538609	·864192	1·157149	·7566148	10
10	·6450132	·844068	1·184737	·7641714	50	31	·6496692	·854583	1·170160	·7602170	29	51	·6540810	·864700	1·156469	·7564246	9
11	·6452355	·844567	1·184038	·7639838	49	32	·6498903	·855087	1·169471	·7600280	28	52	·6543010	·865209	1·155789	·7562343	8
12	·6454577	·845065	1·183340	·7637960	48	33	·6501114	·855591	1·168782	·7598389	27	53	·6545209	·865718	1·155110	·7560439	7
13	·6456798	·845564	1·182642	·7636082	47	34	·6503324	·856095	1·168094	·7596498	26	54	·6547408	·866227	1·154431	·7558535	6
14	·6459019	·846063	1·181944	·7634204	46	35	·6505533	·856599	1·167407	·7594606	25	55	·6549607	·866736	1·153753	·7556630	5
15	·6461240	·846562	1·181247	·7632325	45	36	·6507742	·857103	1·166720	·7592713	24	56	·6551804	·867246	1·153075	·7554724	4
16	·6463460	·847062	1·180551	·7630445	44	37	·6509951	·857608	1·166033	·7590820	23	57	·6554002	·867755	1·152397	·7552818	3
17	·6465679	·847561	1·179855	·7628564	43	38	·6512158	·858113	1·165347	·7588926	22	58	·6556198	·868265	1·151721	·7550911	2
18	·6467898	·848061	1·179159	·7626683	42	39	·6514366	·858618	1·164661	·7587031	21	59	·6558395	·868776	1·151044	·7549004	1
19	·6470116	·848561	1·178464	·7624802	41	40	·6516572	·859124	1·163976	·7585136	20	60	·6560590	·869286	1·150368	·7547096	0
20	·6472334	·849062	1·177769	·7622919	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49. Deg. 49. Deg. 49.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6560590	·869286	1·150368	·7547096	60
1	·6562785	·869797	1·149692	·7545187	59
2	·6564980	·870308	1·149017	·7543278	58
3	·6567174	·870820	1·148342	·7541368	57
4	·6569367	·871331	1·147668	·7539457	56
5	·6571560	·871843	1·146994	·7537546	55
6	·6573752	·872355	1·146321	·7535634	54
7	·6575944	·872868	1·145648	·7533721	53
8	·6578135	·873380	1·144976	·7531808	52
9	·6580326	·873893	1·144304	·7529894	51
10	·6582516	·874406	1·143632	·7527980	50
11	·6584706	·874920	1·142961	·7526065	49
12	·6586895	·875433	1·142290	·7524149	48
13	·6589083	·875947	1·141620	·7522233	47
14	·6591271	·876462	1·140950	·7520316	46
15	·6593458	·876976	1·140281	·7518398	45
16	·6595645	·877491	1·139612	·7516480	44
17	·6597831	·878006	1·138944	·7514561	43
18	·6600017	·878521	1·138276	·7512641	42
19	·6602202	·879037	1·137608	·7510721	41
20	·6604386	·879552	1·136941	·7508800	40
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6606570	·880068	1·136274	·7506879	39
22	·6608754	·880585	1·135608	·7504957	38
23	·6610936	·881101	1·134942	·7503034	37
24	·6613119	·881618	1·134277	·7501111	36
25	·6615300	·882135	1·133612	·7499187	35
26	·6617482	·882653	1·132947	·7497262	34
27	·6619662	·883170	1·132283	·7495337	33
28	·6621842	·883688	1·131620	·7493411	32
29	·6624022	·884206	1·130957	·7491484	31
30	·6626200	·884725	1·130294	·7489557	30
31	·6628379	·885244	1·129632	·7487629	29
32	·6630557	·885763	1·128970	·7485701	28
33	·6632734	·886282	1·128308	·7483772	27
34	·6634910	·886801	1·127647	·7481842	26
35	·6637087	·887321	1·126987	·7479912	25
36	·6639262	·887841	1·126327	·7477981	24
37	·6641437	·888361	1·125667	·7476049	23
38	·6643612	·888882	1·125008	·7474117	22
39	·6645785	·889403	1·124349	·7472184	21
40	·6647959	·889924	1·123690	·7470251	20
′	Cosine.	Cotang.	Tang.	Sine.	′

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6650131	·890445	1·123032	·7468317	19
42	·6652304	·890967	1·122375	·7466382	18
43	·6654475	·891489	1·121718	·7464446	17
44	·6656646	·892011	1·121061	·7462510	16
45	·6658817	·892534	1·120405	·7460574	15
46	·6660987	·893056	1·119749	·7458636	14
47	·6663156	·893579	1·119094	·7456699	13
48	·6665325	·894103	1·118439	·7454760	12
49	·6667493	.894626	1·117784	·7452821	11
50	·6669661	·895150	1·117130	·7450881	10
51	·6671828	·895674	1·116476	·7448941	9
52	·6673994	·896199	1·115823	·7446999	8
53	·6676160	·896723	1·115170	·7445058	7
54	·6678326	·897248	1·114518	·7443115	6
55	·6680490	·897773	1·113866	·7441173	5
56	·6682655	·898299	1·113214	·7439229	4
57	·6684818	·898825	1·112563	·7437285	3
58	·6686981	·899351	1·111912	·7435340	2
59	·6689144	·899877	1·111262	·7433394	1
60	·6691306	·900404	1·110612	·7431448	0
′	Cosine.	Cotang.	Tang.	Sine.	′

42 Deg. **42 Deg.** **42 Deg.**

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6691306	·900404	1·110612	·7431448	60	21	·6736577	·911526	1·097060	·7390435	39	41	·6779459	·922235	1.084322	·7351118	19
1	·6693468	·900930	1·109963	·7429502	59	22	·6738727	·912059	1·096420	·7388475	38	42	·6781597	·922773	1·083689	·7349146	18
2	·6695628	·901458	1·109314	·7427554	58	23	·6740876	·912592	1·095779	·7386515	37	43	·6783734	·923312	1·083057	·7347173	17
3	·6697789	·901985	1·108665	·7425606	57	24	·6743024	·913125	1·095139	·7384553	36	44	·6785871	·923851	1·082425	·7345199	16
4	·6699948	·902513	1·108017	·7423658	56	25	·6745172	·913659	1·094500	·7382592	35	45	·6788007	·924391	1·081793	·7343225	15
5	·6702108	·903041	1·107369	·7421708	55	26	·6747319	·914192	1·093861	·7380629	34	46	·6790143	·924930	1·081162	·7341250	14
6	·6704266	·903569	1·106721	·7419758	54	27	·6749466	·914727	1·093222	·7378666	33	47	·6792278	·925470	1·080532	·7339275	13
7	·6706424	904097	1·106075	·7417808	53	28	·6751612	·915261	1·092584	·7376703	32	48	·6794413	·926010	1·079901	·7337299	12
8	·6708582	·904626	1·105428	·7415857	52	29	·6753757	·915796	1·091946	·7374738	31	49	·6796547	·926550	1·079271	·7335322	11
9	·6710739	·905155	1·104782	·7413905	51	30	·6755902	·916331	1·091308	·7372773	30	50	·6798681	·927091	1·078642	·7333345	10
10	·6712895	·905685	1·104136	·7411953	50	31	·6758046	·916866	1·090671	·7370808	29	51	·6800813	·927632	1·078013	·7331367	9
11	·6715051	·906214	1·103491	·7410000	49	32	·6760190	·917402	1·090034	·7368842	28	52	·6802946	·928173	1·077384	·7329388	8
12	·6717206	·906744	1·102846	·7408046	48	33	·6762333	·917937	1·089398	·7366875	27	53	·6805078	·928715	1·076756	·7327409	7
13	·6719361	·907274	1·102201	·7406092	47	34	·6764476	·918474	1·088762	·7364908	26	54	·6807209	·929257	1·076128	·7325429	6
14	·6721515	·907805	1·101557	·7404137	46	35	·6766618	·919010	1·088126	·7362940	25	55	·6809339	·929799	1·075500	·7323449	5
15	·6723668	·908336	1·100914	·7402181	45	36	·6768760	·919547	1·087491	·7360971	24	56	·6811469	·930342	1·074873	·7321467	4
16	·6725821	·908867	1·100270	·7400225	44	37	·6770901	·920084	1·086857	·7359002	23	57	·6813599	·930884	1·074246	·7319486	3
17	·6727973	·909398	1·099628	·7398268	43	38	·6773041	·920621	1·086222	·7357032	22	58	·6815728	·931428	1·073620	·7317503	2
18	·6730125	·909930	1·098985	·7396311	42	39	·6775181	·921159	1·085588	·7355061	21	59	·6817856	·931971	1·072994	·7315521	1
19	·6732276	·910461	1·098343	·7394353	41	40	·6777320	·921696	1·084955	·7353090	20	60	·6819984	·932515	1·072368	·7313537	0
20	·6734427	·910994	1·097702	·7392394	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47. **Deg. 47.** **Deg. 47.**

43 Deg. **43 Deg.** **43 Deg.**

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6819984	·932515	1·072368	·7313537	60	21	·6864532	·944001	1·059320	·7271740	39	41	·6906721	·955064	1·047049	·7231681	19
1	·6822111	·933059	1·071743	·7311553	59	22	·6866647	·944551	1·058703	·7269743	38	42	·6908824	·955620	1·046440	·7229671	18
2	·6824237	·933603	1·071118	·7309568	58	23	·6868761	·945102	1·058086	·7267745	37	43	·6910927	·956177	1·045831	·7227661	17
3	·6826363	·934147	1·070494	·7307583	57	24	·6870875	·945653	1·057470	·7265747	36	44	·6913029	·956734	1·045222	·7225651	16
4	·6828489	·934692	1·069870	·7305597	56	25	·6872988	·946204	1·056854	·7263748	35	45	·6915131	·957291	1·044613	·7223640	15
5	·6830613	·935238	1·069246	·7303610	55	26	·6875101	·946755	1·056238	·7261748	34	46	·6917232	·957849	1·044005	·7221628	14
6	·6832738	·935783	1·068623	·7301623	54	27	·6877213	·947307	1·055623	·7259748	33	47	·6919332	·958407	1·043397	·7219615	13
7	·6834861	·936329	1·068000	·7299635	53	28	·6879325	·947859	1·055008	·7257747	32	48	·6921432	·958965	1·042790	·7217602	12
8	·6836984	·936875	1·067377	·7297646	52	29	·6881435	·948411	1·054394	·7255746	31	49	·6923531	·959524	1·042183	·7215589	11
9	·6839107	·937421	1·066755	·7295657	51	30	·6883546	·948964	1·053780	·7253744	30	50	·6925630	·960082	1·041576	·7213574	10
10	·6841229	·937968	1·066134	·7293668	50	31	·6885655	·949517	1·053166	·7251741	29	51	·6927728	·960642	1·040970	·7211559	9
11	·6843350	·938515	1·065512	·7291677	49	32	·6887765	·950070	1·052553	·7249738	28	52	·6929825	·961201	1·040364	·7209544	8
12	·6845471	·939062	1·064891	·7289686	48	33	·6889873	·950624	1·051940	·7247734	27	53	·6931922	·961761	1·039758	·7207528	7
13	·6847591	·939610	1·064271	·7287695	47	34	·6891981	·951178	1·051327	·7245729	26	54	·6934018	·962321	1·039153	·7205511	6
14	·6849711	·940157	1·063651	·7285703	46	35	·6894089	·951732	1·050715	·7243724	25	55	·6936114	·962881	1·038548	·7203494	5
15	·6851830	·940706	1·063031	·7283710	45	36	·6896195	·952287	1·050103	·7241719	24	56	·6938209	·963442	1·037944	·7201476	4
16	·6853948	·941254	1·062411	·7281716	44	37	·6898302	·952842	1·049492	·7239712	23	57	·6940304	·964003	1·037340	·7199457	3
17	·6856066	·941803	1·061792	·7279722	43	38	·6900407	·953397	1·048880	·7237705	22	58	·6942398	·964565	1·036736	·7197438	2
18	·6858184	·942352	1·061174	·7277728	42	39	·6902512	·953952	1·048270	·7235698	21	59	·6944491	·965126	1·036133	·7195418	1
19	·6860300	·942901	1·060556	·7275732	41	40	·6904617	·954508	1·047659	·7233690	20	60	·6946584	·965688	1·035530	·7193398	0
20	·6862416	·943451	1·059938	·7273736	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46. **Deg. 46.** **Deg. 46.**

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6946584	·965688	1·035530	·7193398	60
1	·6948676	·966251	1·034927	·7191377	59
2	·6950767	·966813	1·034325	·7189355	58
3	6952858	·967376	1·033723	·7187333	57
4	·6954949	·967939	1·033122	·7185310	56
5	·6957039	·968503	1·032520	·7183287	55
6	·6959128	·969067	1·031919	·7181263	54
7	·6961217	·969631	1·031319	·7179238	53
8	·6963305	·970196	1·030719	·7177213	52
9	·6965392	·970761	1·030119	·7175187	51
10	·6967479	·971326	1·029520	·7173161	50
11	·6969565	·971891	1·028921	·7171134	49
12	·6971651	·972457	1·028322	·7169106	48
13	·6973736	·973023	1·027724	·7167078	47
14	·6975821	·973590	1·027126	·7165049	46
15	·6977905	·974156	1·026528	·7163019	45
16	·6979988	·974724	1·025931	·7160989	44
17	·6982071	·975291	1·025334	·7158959	43
18	·6984153	·975859	1·024738	·7156927	42
19	·6986234	·976427	1·024141	·7154895	41
20	·6988315	·976995	1·023546	·7152863	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6990396	·977564	1·022950	·7150830	39
22	·6992476	·978133	1·022355	·7148796	38
23	·6994555	·978702	1·021760	·7146762	37
24	·6996633	·979272	1·021166	·7144727	36
25	·6998711	·979842	1·020572	·7142691	35
26	·7000789	·980412	1·019978	·7140655	34
27	·7002866	·980983	1·019385	·7138618	33
28	·7004942	·981554	1·018792	·7136581	32
29	·7007018	·982125	1·018199	·7134543	31
30	·7009093	·982697	1·017607	·7132504	30
31	·7011167	·983269	1·017015	·7130465	29
32	·7013241	·983841	1·016423	·7128426	28
33	·7015314	·984414	1·015832	·7126385	27
34	·7017387	·984987	1·015241	·7124344	26
35	·7019459	·985560	1·014651	·7122303	25
36	·7021531	·986133	1·014061	·7120260	24
37	·7023601	.986707	1·013471	·7118218	23
38	·7025672	.987282	1·012881	·7116174	22
39	·7027741	.987856	1·012292	·7114130	21
40	·7029811	.988431	1·011703	·7112086	20
′	Cosine.	Cotang.	Tang.	Sine.	′

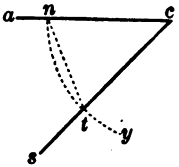
Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·7031879	·989006	1·011115	·7110041	19
42	·7033947	·989582	1·010527	·7107995	18
43	·7036014	·990158	1·009939	·7105948	17
44	·7038081	·990734	1·009352	·7103901	16
45	·7040147	·991311	1·008764	·7101854	15
46	·7042213	·991888	1·008178	·7099806	14
47	·7044278	·992465	1·007591	·7097757	13
48	·7046342	·993042	1·007005	·7095707	12
49	·7048406	·993620	1·006420	·7093657	11
50	·7050469	·994199	1·005834	·7091607	10
51	·7052532	·994777	1·005249	·7089556	9
52	·7054594	·995356	1·004665	·7087504	8
53	·7056655	·995935	1·004080	·7085451	7
54	·7058716	·996515	1·003496	·7083398	6
55	·7060776	·997095	1·002913	·7081345	5
56	·7062835	·997675	1·002329	·7079291	4
57	·7064894	·998256	1·001746	·7077236	3
58	·7066953	·998837	1·001164	·7075180	2
59	·7069011	·999418	1·000581	·7073124	1
60	·7071068	1·00000	1·000000	·7071068	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45.

The table of chords, below, furnishes the means of laying down angles on paper more accurately than by an ordinary protractor. To do this, after having drawn and measured the first side (say *a c*) of the figure that is to be plotted; from its end *c* as a center, describe an arc *n y* of a circle of sufficient extent to subtend the angle at that point. The rad *c n* with which the arc is described should be as great as convenience will permit; and it is to be assumed as unity or 1; and must be decimally divided, and subdivided, to be used as a scale for laying down the chords taken from the table, in which their lengths are given in parts of said rad 1. Having described the arc, find in the table the length of the chord *n t* corresponding to the angle *a c t*. Let us suppose this angle to be 45°; then we find that the tabular chord is .7654 of our rad 1. Therefore from *n* we lay off the chord *n t*, equal to .7654 of our radius-scale; and the line *c s* drawn through the point *t* will form the reqd angle *a c t* of 45°. And so at each angle. The degree of accuracy attained will evidently depend on the length of the rad, and the neatness of the drafting. The method becomes preferable to the common protractor in proportion as the lengths of the sides of the angles exceed the rad of the protractor. With a protractor of 4 to 6 ins rad, and with sides of angles not much exceeding the same limits, the protractor will usually be preferable. The dividers in boxes of instruments are rarely fit for accurate arcs of more than about 6 ins diam. In practice it is not necessary to actually describe the whole arc, but merely the portion near *t*, as well as can be judged by eye. We thus avoid much use of the India-rubber, and dulling of the pencil-point. For larger radii we may dispense with the dividers, and use a straight strip of paper with the length of the rad marked on one edge; and by laying it from *c* toward *s*, and at the same time placing another strip (with one edge divided to a radius-scale) from *n* toward *t*, we can by trial find their exact point of intersection at the required point *t*. In such matters, practice and some ingenuity are very essential to satisfactory results. We cannot devote more space to the subject.



CHORDS TO A RADIUS 1.

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0′	.0000	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	0′
2	.0006	.0180	.0355	.0529	.0704	.0878	.1053	.1227	.1401	.1575	.1749	2
4	.0012	.0186	.0361	.0535	.0710	.0884	.1058	.1233	.1407	.1581	.1755	4
6	.0017	.0192	.0366	.0541	.0715	.0890	.1064	.1238	.1413	.1587	.1761	6
8	.0023	.0198	.0372	.0547	.0721	.0896	.1070	.1244	.1418	.1592	.1766	8
10	.0029	.0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	.1598	.1772	10
12	.0035	.0209	.0384	.0558	.0733	.0907	.1082	.1256	.1430	.1604	.1778	12
14	.0041	.0215	.0390	.0564	.0739	.0913	.1087	.1262	.1436	.1610	.1784	14
16	.0047	.0221	.0396	.0370	.0745	.0919	.1093	.1267	.1442	.1616	.1789	16
18	.0052	.0227	.0401	.0576	.0750	.0925	.1099	.1273	.1447	.1621	.1795	18
20	.0058	.0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	.1627	.1801	20
22	.0064	.0239	.0413	.0588	.0762	.0936	.1111	.1285	.1459	.1633	.1807	22
24	.0070	.0244	.0419	.0593	.0768	.0942	.1116	.1291	.1465	.1639	.1813	24
26	.0076	.0250	.0425	.0599	.0774	.0948	.1122	.1296	.1471	.1645	.1818	26
28	.0081	.0256	.0430	.0605	.0779	.0954	.1128	.1302	.1476	.1650	.1824	28
30	.0087	.0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	.1656	.1830	30
32	.0093	.0268	.0442	.0617	.0791	.0965	.1140	.1314	.1488	.1662	.1836	32
34	.0099	.0273	.0448	.0622	.0797	.0971	.1145	.1320	.1494	.1668	.1842	34
36	.0105	.0279	.0454	.0628	.0802	.0977	.1151	.1325	.1500	.1674	.1847	36
38	.0111	.0285	.0460	.0634	.0808	.0983	.1157	.1331	.1505	.1679	.1853	38
40	.0116	.0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	.1685	.1859	40
42	.0122	.0297	.0471	.0646	.0820	.0994	.1169	.1343	.1517	.1691	.1865	42
44	.0128	.0303	.0477	.0651	.0826	.1000	.1175	.1349	.1523	.1697	.1871	44
46	.0134	.0308	.0483	.0657	.0832	.1006	.1180	.1355	.1529	.1703	.1876	46
48	.0140	.0314	.0489	.0663	.0838	.1012	.1186	.1360	.1534	.1708	.1882	48
50	.0145	.0320	.0494	.0669	.0843	.1018	.1192	.1366	.1540	.1714	.1888	50
52	.0151	.0326	.0500	.0675	.0849	.1023	.1198	.1372	.1546	.1720	.1894	52
54	.0157	.0332	.0506	.0681	.0855	.1029	.1204	.1378	.1552	.1726	.1900	54
56	.0163	.0337	.0512	.0686	.0861	.1035	.1209	.1384	.1558	.1732	.1905	56
58	.0169	.0343	.0518	.0692	.0867	.1041	.1215	.1389	.1563	.1737	.1911	58
60	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	.1917	60

Table of Chords, in parts of a rad 1; for protracting—Continued

M.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	M.
0′	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	0′
2	.1923	.2096	.2270	.2443	.2616	.2789	.2962	.3134	.3307	.3479	2
4	.1928	.2102	.2276	.2449	.2622	.2795	.2968	.3140	.3312	.3484	4
6	.1934	.2108	.2281	.2455	.2628	.2801	.2973	.3146	.3318	.3490	6
8	.1940	.2114	.2287	.2460	.2634	.2807	.2979	.3152	.3324	.3496	8
10	.1946	.2119	.2293	.2466	.2639	.2812	.2985	.3157	.3330	.3502	10
12	.1952	.2125	.2299	.2472	.2645	.2818	.2991	.3163	.3335	.3507	12
14	.1957	.2131	.2305	.2478	.2651	.2824	.2996	.3169	.3341	.3513	14
16	.1963	.2137	.2310	.2484	.2657	.2830	.3002	.3175	.3347	.3519	16
18	.1969	.2143	.2316	.2489	.2662	.2835	.3008	.3180	.3353	.3525	18
20	.1975	.2148	.2322	.2495	.2668	.2841	.3014	.3186	.3358	.3530	20
22	.1981	.2154	.2328	.2501	.2674	.2847	.3019	.3192	.3364	.3536	22
24	.1986	.2160	.2333	.2507	.2680	.2853	.3025	.3198	.3370	.3542	24
26	.1992	.2166	.2339	.2512	.2685	.2858	.3031	.3203	.3376	.3547	26
28	.1998	.2172	.2345	.2518	.2691	.2864	.3037	.3209	.3381	.3553	28
30	.2004	.2177	.2351	.2524	.2697	.2870	.3042	.3215	.3387	.3559	30
32	.2010	.2183	.2357	.2530	.2703	.2876	.3048	.3221	.3393	.3565	32
34	.2015	.2189	.2362	.2536	.2709	.2881	.3054	.3226	.3398	.3570	34
36	.2021	.2195	.2368	.2541	.2714	.2887	.3060	.3282	.3404	.3576	36
38	.2027	.2200	.2374	.2547	.2720	.2898	.3065	.3238	.3410	.3582	38
40	.2033	.2206	.2380	.2553	.2726	.2899	.3071	.3244	.3416	.3587	40
42	.2038	.2212	.2385	.2559	.2732	.2904	.3077	.3249	.3421	.3593	42
44	.2044	.2218	.2391	.2564	.2737	.2910	.3083	.3255	.3427	.3599	44
46	.2050	.2224	.2397	.2570	.2743	.2916	.3088	.3261	.3483	.3605	46
48	.2056	.2229	.2403	.2576	.2749	.2922	.3094	.3267	.3439	.3610	48
50	.2062	.2235	.2409	.2582	.2755	.2927	.3100	.3272	.3444	.3616	50
52	.2067	.2241	.2414	.2587	.2760	.2933	.3106	.3278	.3450	.3622	52
54	.2073	.2247	.2420	.2593	.2766	.2939	.3111	.3284	.3456	.3628	54
56	.2079	.2253	.2426	.2599	.2772	.2945	.3117	.3289	.3462	.3633	56
58	.2085	.2258	.2432	.2605	.2778	.2950	.3123	.3295	.3467	.3639	58
60	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	.3645	60

M.	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	M.
0′	.3645	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	0′
2	.3650	.3822	.3993	.4164	.4334	.4505	.4675	.4844	.5013	.5182	2
4	.3656	.3828	.3999	.4170	.4340	.4510	.4680	.4850	.5019	.5188	4
6	.3662	.3833	.4004	.4175	.4346	.4516	.4686	.4855	.5024	.5193	6
8	.3668	.3839	.4010	.4181	.4352	.4522	.4692	.4861	.5030	.5199	8
10	.3673	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	.5204	10
12	.3679	.3850	.4022	.4192	.4363	.4533	.4703	.4872	.5041	.5210	12
14	.3685	.3856	.4027	.4198	.4369	.4539	.4708	.4878	.5047	.5216	14
16	.3690	.3862	.4033	.4204	.4374	.4544	.4714	.4884	.5053	.5221	16
18	.3696	.3868	.4039	.4209	.4380	.4550	.4720	.4889	.5058	.5227	18
20	.3702	.3873	.4044	.4215	.4386	.4556	.4725	.4895	.5064	.5233	20
22	.3708	.3879	.4050	.4221	.4391	.4561	.4731	.4901	.5070	.5238	22
24	.3713	.3885	.4056	.4226	.4397	.4567	.4737	.4906	.5075	.5244	24
26	.3719	.3890	.4061	.4232	.4403	.4573	.4742	.4912	.5081	.5249	26
28	.3725	.3896	.4067	.4238	.4408	.4578	.4748	.4917	.5086	.5255	28
30	.3730	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	.5261	30
32	.3786	.3908	.4079	.4249	.4420	.4590	.4759	.4929	.5098	.5266	32
34	.3742	.3913	.4084	.4255	.4425	.4595	.4765	.4934	.5103	.5272	34
36	.3748	.3919	.4090	.4261	.4431	.4601	.4771	.4940	.5109	.5277	36
38	.3753	.3925	.4096	.4266	.4437	.4607	.4776	.4946	.5115	.5283	38
40	.3759	.3930	.4101	.4272	.4442	.4612	.4782	.4951	.5120	.5289	40
42	.3765	.3936	.4107	.4278	.4448	.4618	.4788	.4957	.5126	.5294	42
44	.3770	.3942	.4113	.4283	.4454	.4624	.4793	.4963	.5131	.5300	44
46	.3776	.3947	.4118	.4289	.4459	.4629	.4799	.4968	.5137	.5306	46
48	.3782	.3953	.4124	.4295	.4465	.4635	.4805	.4974	.5143	.5311	48
50	.3788	.3959	.4130	.4300	.4471	.4641	.4810	.4979	.5148	.5317	50
52	.3793	.3965	.4135	.4306	.4476	.4646	.4816	.4985	.5154	.5322	52
54	.3799	.3970	.4141	.4312	.4482	.4652	.4822	.4991	.5160	.5328	54
56	.3805	.3976	.4147	.4317	.4488	.4658	.4827	.4996	.5165	.5334	56
58	.3810	.3982	.4153	.4323	.4493	.4663	.4833	.5002	.5171	.5339	58
60	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	.5345	60

Table of chords, in parts of a rad 1; for protracting—Continued.

M.	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	M.
0′	.5345	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	0′
2	.5350	.5518	.5686	.5853	.6020	.6186	.6352	.6517	.6682	.6846	2
4	.5356	.5524	.5691	.5859	.6025	.6191	.6357	.6522	.6687	.6851	4
6	.5362	.5530	.5697	.5864	.6031	.6197	.6363	.6528	.6693	.6857	6
8	.5367	.5535	.5703	.5870	.6036	.6202	.6368	.6533	.6698	.6862	8
10	.5373	.5541	.5708	.5875	.6042	.6208	.6374	.6539	.6704	.6868	10
12	.5378	.5546	.5714	.5881	.6047	.6214	.6379	.6544	.6709	.6873	12
14	.5384	.5552	.5719	.5886	.6053	.6219	.6385	.6550	.6715	.6879	14
16	.5390	.5557	.5725	.5892	.6058	.6225	.6390	.6555	.6720	.6884	16
18	.5395	.5563	.5730	.5897	.6064	.6230	.6396	.6561	.6725	.6890	18
20	.5401	.5569	.5736	.5903	.6070	.6236	.6401	.6566	.6731	.6895	20
22	.5406	.5574	.5742	.5909	.6075	.6241	.6407	.6572	.6736	.6901	22
24	.5412	.5580	.5747	.5914	.6081	.6247	.6412	.6577	.6742	.6906	24
26	.5418	.5585	.5753	.5920	.6086	.6252	.6418	.6583	.6747	.6911	26
28	.5423	.5591	.5758	.5925	.6092	.6258	.6423	.6588	.6753	.6917	28
30	.5429	.5597	.5764	.5931	.6097	.6263	.6429	.6594	.6758	.6922	30
32	.5434	.5602	.5769	.5936	.6103	.6269	.6434	.6599	.6764	.6928	32
34	.5440	.5608	.5775	.5942	.6108	.6274	.6440	.6605	.6769	.6933	34
36	.5446	.5613	.5781	.5947	.6114	.6280	.6445	.6610	.6775	.6939	36
38	.5451	.5619	.5786	.5953	.6119	.6285	.6451	.6616	.6780	.3944	38
40	.5457	.5625	.5792	.5959	.6125	.6291	.6456	.6621	.6786	.6950	40
42	.5462	.5630	.5797	.5964	.6130	.6296	.6462	.6627	.6791	.6955	42
44	.5468	.5636	.5803	.5970	.6136	.6302	.6467	.6632	.6797	.6961	44
46	.5474	.5641	.5808	.5975	.6142	.6307	.6473	.6638	.6802	.6966	46
48	.5479	.5647	.5814	.5981	.6147	.6313	.6478	.6643	.6808	.6971	48
50	.5485	.5652	.5820	.5986	.6153	.6318	.6484	.6649	.6813	.6977	50
52	.5490	.5658	.5825	.5992	.6158	.6324	.6489	.6654	.6819	.6982	52
54	.5496	.5664	.5831	.5997	.6164	.6330	.6495	.6660	.6824	.6988	54
56	.5502	.5669	.5836	.6003	.6169	.6335	.6500	.6665	.6829	.6993	56
58	.5507	.5675	.5842	.6009	.6175	.6341	.6506	.6671	.6835	.6999	58
60	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	.7004	60

M.	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	M.
0′	.7004	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	0′
2	.7010	.7173	.7335	.7498	.7659	.7820	.7980	.8140	.8299	.8458	2
4	.7015	.7178	.7341	.7503	.7664	.7825	.7986	.8145	.8304	.8463	4
6	.7020	.7184	.7346	.7508	.7670	.7831	.7991	.8151	.8310	.8468	6
8	.7026	.7189	.7352	.7514	.7675	.7836	.7996	.8156	.8315	.8473	8
10	.7031	.7195	.7357	.7519	.7681	.7841	.8002	.8161	.8320	.8479	10
12	.7037	.7200	.7362	.7524	.7686	.7847	.8007	.8167	.8326	.8484	12
14	.7042	.7205	.7368	.7530	.7691	.7852	.8012	.8172	.8331	.8489	14
16	.7048	.7211	.7373	.7535	.7697	.7857	.8018	.8177	.8336	.8495	16
18	.7053	.7216	.7379	.7541	.7702	.7863	.8023	.8183	.8341	.8500	18
20	.7059	.7222	.7384	.7546	.7707	.7868	.8028	.8188	.8347	.8505	20
22	.7064	.7227	.7390	.7551	.7713	.7873	.8034	.8193	.8352	.8510	22
24	.7069	.7232	.7395	.7557	.7718	.7879	.8039	.8198	.8357	.8516	24
26	.7075	.7238	.7400	.7562	.7723	.7884	.8044	.8204	.8363	.8521	26
28	.7080	.7243	.7406	.7568	.7729	.7890	.8050	.8209	.8368	.8526	28
30	.7086	.7249	.7411	.7573	.7734	.7895	.8055	.8214	.8373	.8531	30
32	.7091	.7254	.7417	.7578	.7740	.7900	.8060	.8220	.8378	.8537	32
34	.7097	.7260	.7422	.7584	.7745	.7906	.8066	.8225	.8384	.8542	34
36	.7102	.7265	.7427	.7589	.7750	.7911	.8071	.8230	.8389	.8547	36
38	.7108	.7270	.7433	.7595	.7756	.7916	.8076	.8236	.8394	.8552	38
40	.7113	.7276	.7438	.7600	.7761	.7922	.8082	.8241	.8400	.8558	40
42	.7118	.7281	.7443	.7605	.7766	.7927	.8087	.8246	.8405	.8563	42
44	.7124	.7287	.7449	.7611	.7772	.7932	.8092	.8251	.8410	.8568	44
46	.7129	.7292	.7454	.7616	.7777	.7938	.8098	.8257	.8415	.8573	46
48	.7135	.7298	.7460	.7621	.7782	.7943	.8103	.8262	.8421	.8579	48
50	.7140	.7303	.7465	.7627	.7788	.7948	.8108	.8267	.8426	.8584	50
52	.7146	.7308	.7471	.7632	.7793	.7954	.8113	.8273	.8431	.8589	52
54	.7151	.7314	.7476	.7638	.7799	.7959	.8119	.8278	.8437	.8594	54
56	.7156	.7319	.7481	.7643	.7804	.7964	.8124	.8283	.8442	.8600	56
58	.7162	.7325	.7487	.7648	.7809	.7970	.8129	.8289	.8447	.8605	58
60	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	.8610	60

Table of chords, in parts of a rad 1; for protracting.—Continued.

M.	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	M.
0′	.8610	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	0′
2	.8615	.8773	.8929	.9085	.9240	.9395	.9548	.9701	.9854	1.0005	2
4	.8621	.8778	.8934	.9090	.9245	.9400	.9553	.9706	.9859	1.0010	4
6	.8626	.8783	.8940	.9095	.9250	.9405	.9559	.9711	.9864	1.0015	6
8	.8631	.8788	.8945	.9101	.9256	.9410	.9564	.9717	.9869	1.0020	8
10	.8636	.8794	.8950	.9106	.9261	.9415	.9569	.9722	.9874	1.0025	10
12	.8642	.8799	.8955	.9111	.9266	.9420	.9574	.9727	.9879	1.0030	12
14	.8647	.8804	.8960	.9116	.9271	.9425	.9579	.9732	.9884	1.0035	14
16	.8652	.8809	.8966	.9121	.9276	.9430	.9584	.9737	.9889	1.0040	16
18	.8657	.8814	.8971	.9126	.9281	.9436	.9589	.9742	.9894	1.0045	18
20	.8663	.8820	.8976	.9132	.9287	.9441	.9594	.9747	.9899	1.0050	20
22	.8668	.8825	.8981	.9137	.9292	.9446	.9599	.9752	.9904	1.0055	22
24	.8673	.8830	.8986	.9142	.9297	.9451	.9604	.9757	.9909	1.0060	24
26	.8678	.8835	.8992	.9147	.9302	.9456	.9610	.9762	.9914	1.0065	26
28	.8684	.8841	.8997	.9152	.9307	.9461	.9615	.9767	.9919	1.0070	28
30	.8689	.8846	.9002	.9157	.9312	.9466	.9620	.9772	.9924	1.0075	30
32	.8694	.8851	.9007	.9163	.9317	.9472	.9625	.9778	.9929	1.0080	32
34	.8699	.8856	.9012	.9168	.9323	.9477	.9630	.9783	.9934	1.0086	34
36	.8705	.8861	.9018	.9173	.9328	.9482	.9635	.9788	.9939	1.0091	36
38	.8710	.8867	.9023	.9178	.9333	.9487	.9640	.9793	.9945	1.0096	38
40	.8715	.8872	.9028	.9183	.9338	.9492	.9645	.9798	.9950	1.0101	40
42	.8720	.8877	.9033	.9188	.9343	.9497	.9650	.9803	.9955	1.0106	42
44	.8726	.8882	.9038	.9194	.9348	.9502	.9655	.9808	.9960	1.0111	44
46	.8731	.8887	.9044	.9199	.9353	.9507	.9661	.9813	.9965	1.0116	46
48	.8736	.8893	.9049	.9204	.9359	.9512	.9666	.9818	.9970	1.0121	48
50	.8741	.8898	.9054	.9209	.9364	.9518	.9671	.9823	.9975	1.0126	50
52	.8747	.8903	.9059	.9214	.9369	.9523	.9676	.9828	.9980	1.0131	52
54	.8752	.8908	.9064	.9219	.9374	.9528	.9681	.9833	.9985	1.0136	54
56	.8757	.8914	.9069	.9225	.9379	.9533	.9686	.9838	.9990	1.0141	56
58	.8762	.8919	.9075	.9230	.9384	.9538	.9691	.9843	.9995	1.0146	58
60	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	60

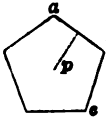
M.	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M.
0′	1.0151	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	0′
2	1.0156	1.0306	1.0455	1.0603	1.0751	1.0898	1.1044	1.1189	1.1333	1.1476	2
4	1.0161	1.0311	1.0460	1.0608	1.0756	1.0903	1.1048	1.1194	1.1338	1.1481	4
6	1.0166	1.0316	1.0465	1.0613	1.0761	1.0907	1.1053	1.1198	1.1342	1.1486	6
8	1.0171	1.0321	1.0470	1.0618	1.0766	1.0912	1.1058	1.1203	1.1347	1.1491	8
10	1.0176	1.0326	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	1.1495	10
12	1.0181	1.0331	1.0480	1.0628	1.0775	1.0922	1.1068	1.1213	1.1357	1.1500	12
14	1.0186	1.0336	1.0485	1.0633	1.0780	1.0927	1.1073	1.1218	1.1362	1.1505	14
16	1.0191	1.0341	1.0490	1.0638	1.0785	1.0932	1.1078	1.1222	1.1366	1.1510	16
18	1.0196	1.0346	1.0495	1.0643	1.0790	1.0937	1.1082	1.1227	1.1371	1.1514	18
20	1.0201	1.0351	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	1.1519	20
22	1.0206	1.0356	1.0504	1.0653	1.0800	1.0946	1.1092	1.1237	1.1381	1.1524	22
24	1.0211	1.0361	1.0509	1.0658	1.0805	1.0951	1.1097	1.1242	1.1386	1.1529	24
26	1.0216	1.0366	1.0514	1.0662	1.0810	1.0956	1.1102	1.1246	1.1390	1.1533	26
28	1.0221	1.0370	1.0519	1.0667	1.0815	1.0961	1.1107	1.1251	1.1395	1.1538	28
30	1.0226	1.0375	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	1.1543	30
32	1.0231	1.0380	1.0529	1.0677	1.0824	1.0971	1.1116	1.1261	1.1405	1.1548	32
34	1.0236	1.0385	1.0534	1.0682	1.0829	1.0976	1.1121	1.1266	1.1409	1.1552	34
36	1.0241	1.0390	1.0539	1.0687	1.0834	1.0980	1.1126	1.1271	1.1414	1.1557	36
38	1.0246	1.0395	1.0544	1.0692	1.0839	1.0985	1.1131	1.1275	1.1419	1.1562	38
40	1.0251	1.0400	1.0549	1.0697	1.0844	1.0990	1.1136	1.1280	1.1424	1.1567	40
42	1.0256	1.0405	1.0554	1.0702	1.0849	1.0995	1.1140	1.1285	1.1429	1.1571	42
44	1.0261	1.0410	1.0559	1.0707	1.0854	1.1000	1.1145	1.1290	1.1433	1.1576	44
46	1.0266	1.0415	1.0564	1.0712	1.0859	1.1005	1.1150	1.1295	1.1438	1.1581	46
48	1.0271	1.0420	1.0569	1.0717	1.0863	1.1010	1.1155	1.1299	1.1443	1.1586	48
50	1.0276	1.0425	1.0574	1.0721	1.0868	1.1014	1.1160	1.1304	1.1448	1.1590	50
52	1.0281	1.0430	1.0579	1.0726	1.0873	1.1019	1.1165	1.1309	1.1452	1.1595	52
54	1.0286	1.0435	1.0584	1.0731	1.0878	1.1024	1.1169	1.1314	1.1457	1.1600	54
56	1.0291	1.0440	1.0589	1.0736	1.0883	1.1029	1.1174	1.1319	1.1462	1.1605	56
58	1.0296	1.0445	1.0593	1.0741	1.0888	1.1034	1.1179	1.1323	1.1467	1.1609	58
60	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	60

Table of Chords, in parts of a rad 1; for protracting—Conti

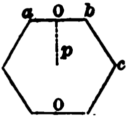
M.	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°
0′	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856
2	1.1619	1.1760	1.1901	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860
4	1.1624	1.1765	1.1906	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865
6	1.1628	1.1770	1.1910	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869
8	1.1633	1.1775	1.1915	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2874
10	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878
12	1.1642	1.1784	1.1924	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882
14	1.1647	1.1789	1.1929	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887
16	1.1652	1.1793	1.1934	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891
18	1.1657	1.1798	1.1938	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896
20	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900
22	1.1666	1.1807	1.1948	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905
24	1.1671	1.1812	1.1952	1.2092	1.2231	1.2368	1.2505	1.2641	1.2775	1.2909
26	1.1676	1.1817	1.1957	1.2097	1.2235	1.2373	1.2509	1.2645	1.2780	1.2914
28	1.1680	1.1821	1.1962	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918
30	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922
32	1.1690	1.1831	1.1971	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927
34	1.1694	1.1836	1.1976	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931
36	1.1699	1.1840	1.1980	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936
38	1.1704	1.1845	1.1985	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940
40	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945
42	1.1713	1.1854	1.1994	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949
44	1.1718	1.1859	1.1999	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954
46	1.1723	1.1864	1.2004	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958
48	1.1727	1.1868	1.2008	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962
50	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967
52	1.1737	1.1878	1.2018	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971
54	1.1742	1.1882	1.2022	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976
56	1.1746	1.1887	1.2027	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1.2980
58	1.1751	1.1892	1.2032	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985
60	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989

M.	81°	82°	83°	84°	85°	86°	87°	88°	89°
0′	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018
2	1.2993	1.3126	1.3257	1.3387	1.3516	1.3644	1.3771	1.3897	1.4022
4	1.2998	1.3130	1.3261	1.3391	1.3520	1.3648	1.3776	1.3902	1.4026
6	1.3002	1.3134	1.3265	1.3396	1.3525	1.3653	1.3780	1.3906	1.4031
8	1.3007	1.3139	1.3270	1.3400	1.3529	1.3657	1.3784	1.3910	1.4035
10	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039
12	1.3015	1.3147	1.3279	1.3409	1.3538	1.3665	1.3792	1.3918	1.4043
14	1.3020	1.3152	1.3283	1.3413	1.3542	1.3670	1.3797	1.3922	1.4047
16	1.3024	1.3156	1.3287	1.3417	1.3546	1.3674	1.3801	1.3927	1.4051
18	1.3029	1.3161	1.3292	1.3421	1.3550	1.3678	1.3805	1.3931	1.4055
20	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060
22	1.3038	1.3169	1.3300	1.3430	1.3559	1.3687	1.3813	1.3939	1.4064
24	1.3042	1.3174	1.3305	1.3434	1.3563	1.3691	1.3818	1.3943	1.4068
26	1.3046	1.3178	1.3309	1.3439	1.3567	1.3695	1.3822	1.3947	1.4072
28	1.3051	1.3183	1.3313	1.3443	1.3572	1.3699	1.3826	1.3952	1.4076
30	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080
32	1.3060	1.3191	1.3322	1.3452	1.3580	1.3708	1.3834	1.3960	1.4084
34	1.3064	1.3196	1.3326	1.3456	1.3585	1.3712	1.3839	1.3964	1.4089
36	1.3068	1.3200	1.3331	1.3460	1.3589	1.3716	1.3843	1.3968	1.4093
38	1.3073	1.3204	1.3335	1.3465	1.3593	1.3721	1.3847	1.3972	1.4097
40	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101
42	1.3082	1.3213	1.3344	1.3473	1.3602	1.3729	1.3855	1.3981	1.4105
44	1.3086	1.3218	1.3348	1.3477	1.3606	1.3733	1.3860	1.3985	1.4109
46	1.3090	1.3222	1.3352	1.3482	1.3610	1.3738	1.3864	1.3989	1.4113
48	1.3095	1.3226	1.3357	1.3486	1.3614	1.3742	1.3868	1.3993	1.4117
50	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122
52	1.3104	1.3235	1.3365	1.3495	1.3623	1.3750	1.3876	1.4002	1.4126
54	1.3108	1.3239	1.3370	1.3499	1.3627	1.3754	1.3881	1.4006	1.4130
56	1.3112	1.3244	1.3374	1.3503	1.3631	1.3759	1.3885	1.4010	1.4134
58	1.3117	1.3248	1.3378	1.3508	1.3636	1.3763	1.3889	1.4014	1.4138
60	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142

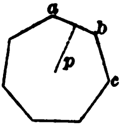
POLYGONS.



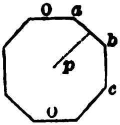
Pentagon.
5 sides.



Hexagon.
6 sides.



Heptagon.
7 sides.



Octagon.
8 sides.

Any straight-sided fig is called a polygon. If all the sides and angles are equal, it is a **regular** polygon; if not, it is **irregular.** Of course the number of polygons is infinite.

Table of Regular Polygons.

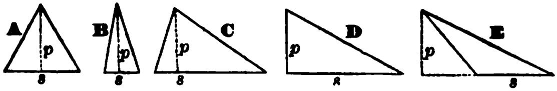
Number of Sides.	Name of Polygon.	**Area** = (square of one side) mult by	**Radius** of circumscribing circle = side mult by	**Interior angle** *a b c* contained between two sides.	**Angle at cen,** subtended by a side.
3	Equilateral triangle.	.433013	.577350	60°	120°
4	Square.	1.000000	.707107	90°	90°
5	Pentagon.	1.720477	.850651	108°	72°
6	Hexagon.	2.598076	1.000000	120°	60°
7	Heptagon.	3.633912	1.152382	128° 34.2857′	51° 25.7143′
8	Octagon.	4.828427	1.306563	135°	45°
9	Nonagon.	6.181824	1.461902	140°	40°
10	Decagon.	7.694209	1.618034	144°	36°
11	Undecagon.	9.365640	1.774733	147° 16.3636′	32° 43.6364′
12	Dodecagon.	11.196152	1.931854	150°	30°

Area of any regular polygon = length of one side, *a b* × perp *p* drawn from cen of fig to cen of side × half the number of sides.

Sum of interior angles, a b c, etc, of any polygon, regular or irregular = 180° × (number of sides − 2).

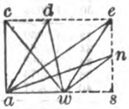
Angle at cen subtended by a side, in any regular polygon = 360° ÷ number of sides.

TRIANGLES.



We speak here of **plane** triangles only; or those having **straight** sides.

A triangle is equilateral when all its sides are equal, as A; **isosceles** when only two sides are equal, as B; **scalene** when all the sides are unequal, as C, D, and E; **acute-angled** when all its angles are acute, or each less than 90°, as A, B, and C; **right-angled** when it contains a right angle, as D; **obtuse-angled** when it contains an obtuse angle, or one greater than 90°, as E.



All the three angles of any triangle are equal to two right angles, or 180°; therefore, if we know two of them, we can find the third by subtracting their sum from 180°. **All triangles which have equal bases,** and equal perp heights, have also equal areas; thus the areas of *a w c*, *a w d*, and *a w e*, are equal to each other. **The area of any triangle** is equal to half that of any parallelogram which has an equal base, and an equal perp height. **The areas of triangles which have equal bases,** but diff perp heights, are to each other as, or in proportion to, their perp heights; thus the triangle *a w n*, with a perp height *s n*, equal to but one-half that (*s e*) of the three other triangles, but with the same base *a w*, has also but half the area of either of those others.

Area of any triangle, Figs A, B, C, D, E, = half the base, S, × the height, or perp dist *p* to the opposite angle. Any side may be taken as the base of a triangle; but the perp height must always be measured from the side so assumed; to do which, the side must sometimes be prolonged, as in Fig E; but the prolongation is not to be considered as a part of the base.

Area of any equilateral triangle = .433013 × square of one side.

To find area, having the three sides.

Add them together; div the sum by 2; from the half sum, subtract each side separately; mult the half sum and the three remainders continuously together; take the sq rt of the prod.

Ex.—The three sides = 20, 30, 40 ft. Here $20 + 30 + 40 = 90$; and $\frac{90}{2} = 45$. And $45 - 20 = 25$; $45 - 30 = 15$; and $45 - 40 = 5$. And $45 \times 25 \times 15 \times 5 = 84375$· and the sq rt of 84375 is 290.47 sq ft, area reqd.

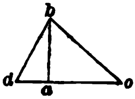
To find area, having one side and the 2 angles at its ends.

Add the 2 angles together; take the sum from 180°; the rem will be the angle opp the given side. Find the nat sine of this angle; also find the nat sines of the other angles, and mult them together. Then as the nat sine of the single angle, is to the prod of the nat sines of the other 2 angles, so is the *square* of the given side to *double* the reqd area.

To find area, having two sides, and the included angle.

Mult together the two sides, and the nat sine of the included angle; div by 2.

Ex.—Sides 650 ft and 960 ft; included angle 69° 20′. By the table we find the nat sine .9356; therefore, $\frac{650 \times 960 \times .9356}{2}$ = 297988.6 square ft area.



To find area, having the three angles and the perp height, *a b*.

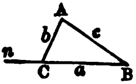
Find the nat sines of the three angles; mult together the sines of the angles *d* and *o*; div the sine of the angle *b* by the prod; mult the quot by the square of the perp height *a b*; div by 2.

To find any side, as *d o*, having the three angles, *d*, *b* and *o*, and the area.

(Sine of *d* × sine of *o*) : sine of *b* : : **twice** the area : **square** of *d o*.

The perp height of an equilateral triangle is equal to one side × .866025. Hence one of its sides is equal to the perp height div by .866025 or to perp height × 1.1547. Or, **to find a side,** mult the sq rt of its area by 1.51967. The side of an equilateral triangle, mult by .658037 = side of a square of the same area; or mult by .742517 it gives the diam of a circle of the same area.

The following apply to any plane triangle, whether oblique or right-angled:

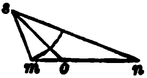


1. The three angles amount to 180°, or two right angles.

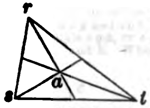
2. Any exterior angle, as A C *n*, is equal to the two interior and opposite ones, A and B.

3. The greater side is opposite the greater angle.

4. The sides are as the sines of the opposite angles. Thus, the side *a* is to the side *b* as the sine of A is to the sine of B.



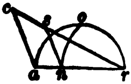
5. If any angle as *s* be bisected by a line *s o*, the two parts *m o*, *o n* of the opposite side *m n* will be to each other as the other two sides *s m*, *s n*; or, *m o* : *o n* :: *s m* : *s n*.



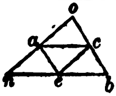
6. If lines be drawn from each angle *r s t* to the center of the opposite side, they will cross each other at one point, *a*, and the short part of each of the lines will be the third part of the whole line. Also, *a* is the **cen of grav** of the triangle.

7. If lines be drawn bisecting the three angles, they will meet at a point perpendicularly equidistant from each side, and consequently the **center of the greatest circle** that can be drawn in the triangle.

8. If a line *s n* be drawn parallel to any side *c a*, the two triangles *r s n*, *r c a*, will be similar.

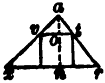


9. To divide any triangle *a c r* into two equal parts by a line *s n* parallel to any one of its sides *c a*. On either one of the other sides, as *a r*, as a diam, describe a semicircle *a o r*; and find its middle *o*. From *r* (opposite *c a*), with radius *r o*, describe the arc *o n*. From *n* draw *n s*, parallel to *c a*.



10. To find the greatest parallelogram that can be drawn in any given triangle *o n b*. Bisect the three sides at *a e c*, and join *a c*, *a e*, *e c*. Then either *a e b c*, *a e c o*, or *a c e n*, each equal to half the triangle, will be the reqd parallelogram. Any of these parallelograms can plainly be converted into a rectangle of equal area, and the greatest that can be drawn in the triangle.

10½. If a line *a c* bisects any two sides *o b*, *o n*, of a triangle, it will be parallel to the third side *n b*, and half as long as it.

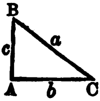


11. To find the greatest square that can be drawn in any triangle *a x r*. From an angle as *a* draw a perp *a n* to the opposite side *x r*, and find its length. Then *o n*, or a side *v t* of the square will = $\frac{x\,r \times a\,n}{x\,r + a\,n}$

Rem.—If the triangle is such that two or three such perps can be drawn, then two or three equal squares may be found.

Right-angled Triangles.

All the foregoing apply also to right-angled triangles; but what follow apply to them only. Call the right angle A, and the others B and C; and call the sides respectively opposite to them a, b, and c. Then is



$$a = \frac{c}{\text{Sine C}} = c \times \text{Sec B} = \frac{b}{\text{Cosine C}} = b \times \text{Sec C} = \sqrt{b^2 + c^2}$$

$$b = a \times \text{Sine B} = a \times \text{Cos C} = c \times \text{Cot C} = c \times \text{Tang B}.$$

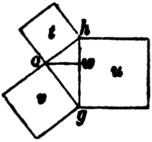
$$c = a \times \text{Sine C} = a \times \text{Cos B} = b \times \text{Tang C}.$$

Also Sine of C $= \frac{c}{a}$; Cos C $= \frac{b}{a}$; Tang C $= \frac{c}{b}$.

And Sine of B $= \frac{b}{a}$; Cos B $= \frac{c}{a}$; Tang B $= \frac{b}{c}$.

And Sine of A or 90° $= 1$. Cos A $= 0$. Tang A $=$ infinity. Sec A $=$ infinity.

1. If from the right angle o a line $o\,w$ be drawn perp to the hypothenuse or long side $h\,g$, then the two small triangles $o\,w\,h$, $o\,w\,g$, and the large one $o\,h\,g$, will be similar. Or $g\,w : w\,o :: w\,o : w\,h$; and $g\,w \times w\,h = w\,o^2$.



2. A line drawn from the right angle to the center of the long side will be half as long as said side.

3. If on the three sides $o\,h$, $o\,g$, $g\,h$ we draw three squares t, u, v, or three circles, or triangles, or any other three figs that are similar, then the area of the largest one is equal to the sum of the areas of the two others.

4. In a triangle whose sides are as 3, 4, and 5 (as are those of the triangle A B C), the angles are very approximately 90°; 53° 7′ 48.38″; and 36° 52′ 11.62″. Their Sines, 1.; .8; and .6. Their Tangs, infinity; 1.3333; and .75.

5. One whose sides are as 7, 7, and 9.9, has very approx one angle of 90° and two of 45° each, near enough for all practical purposes.

PLANE TRIGONOMETRY.

PLANE trigonometry teaches how to find certain unknown parts of plane, or straight-sided triangles, by means of other parts which are known; and thus enables us to measure inaccessible distances, &c. A triangle consists of six parts, namely, three sides, and three angles; and if we know any three of these, (except the three angles, and in the ambiguous case under "Case 2,") we can find the other three. The following four cases include the whole subject; the student should commit them to memory.

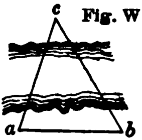


Fig. W

Case 1. Having any two angles, and one side, to find the other sides and angle.

Add the two angles together; and subtract their sum from 180°; the rem will be the third angle. And for the sides, as

Sine of the angle opp the given side : Sine of the angle opp the reqd side : : given side : reqd side.

Use the side thus found, as the given one; and in the same manner find the third side.

Case 2. Having two sides, *b a*, *a c*, Fig X, and the angle *a b c*, opposite to one of them, to find the other side and angles.

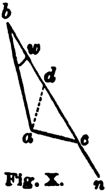


Fig. X.

Side $a\,c$ opp the given angle $a\,b\,c$: The other given side $b\,a$: : Sine of the given angle $a\,b\,c$: Sine of angle $b\,d\,a$ or $b\,c\,a$ opposite the other given side $b\,a$.

Having found the sine, take out the corresponding angle from the table of nat sines, but, in doing so, if the side $a\,c$ opp the given angle is shorter than the other given side $b\,a$, bear in mind that an angle and its supplement have the same sine. Thus, in Fig X, the sine, as found above, is opp the angle $b\,c\,a$ in the table. But $a\,c$, if *shorter than* $b\,a$, can evidently be laid off in the opp direction, $a\,d$, in which case $b\,d\,a$ is the supplement of $b\,c\,a$. If $a\,c$ is as long as, or longer than, $b\,a$, there can be no doubt; for in that case it cannot be drawn toward b, but only toward n, and the angle $b\,c\,a$ will be found at once in the table. opp the sine as found above.

When the two angles, *a b c*, *b c a*, have been found, find the remaining side by Case 1.

For the remaining angle, *b a c*, add together the angle *a b c* first given, and the one, *b c a*, found as above. Deduct their sum from 180°.

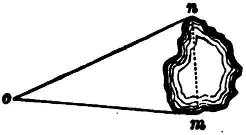
Case 3. Having two sides, and the angle included between them.

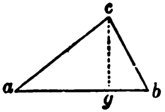
Take the angle from 180°; the rem will be the sum of the two unknown angles. Div this sum by 2; and find the nat tang of the quot. Then as

The sum of the two given sides : Their diff : : Tang of half the sum of the two unknown angles : Tang of half their diff.

Take from the table of nat tang, the angle opposite this last tang. Add this angle to the half sum of the two unknown angles, and it will give the angle opp the longest given side; and subtract it from the same half sum, for the angle opp the shortest given side. Having thus found the angles, find the third side by Case 1.

As a practical example of the use of Case 3, we can ascertain the dist *n m* across a deep pond, by measuring two lines *n o* and *m o*; and the angle *n o m*. From these data we may calculate *n m*; or by drawing the two sides, and the angle on paper, by a scale, we can afterward measure *n m* on the drawing.





Case 4. Having the three sides,

To find the three angles; upon one side *a b* as a base, draw (or suppose to be drawn) a perp *c g* from the opposite angle *c*. Find the diff between the other two sides, *a c* and *c b*: also their sum. Then, as

The base : Sum of the other two sides : : Diff of other two sides : Diff of the two parts *a g* and *b g*, of the base.

Add *half* this diff of the parts, to *half* the base *a b*; the sum will be the longest part *a g*; which taken from the whole base, gives the shortest part *g b*. By this means we get in each of the small triangles *a c g* and *c g b*, two sides, (namely, *a c* and *a g*; and *c b* and *g b*;) and an angle (namely, the right angle *c g a*, or *c g b*) opposite to one of the given sides. Therefore, use Case 2 for finding the angles *a* and *b*. When that is done, take their sum from 180°, for the angle *a c b*.

Or, 2d mode: call **half** the sum of the three sides, *s*; and call the two sides which form either angle, *m* and *n*. Then the nat sine of **half** that angle will be equal to $\sqrt{\frac{(s-m) \times (s-n)}{m \times n}}$.

Ex. 1. To find the dist from *a* to an inaccessible object *c*.

Measure a line *a b*; and from its ends measure the angles *c a b* and *c b a*. Thus having found one side and two angles of the triangle *a b c*, calculate *a c* by means of Case 1. Or if extreme accuracy is not reqd, draw the line *a b* on paper to any convenient scale; then by means of a protractor lay off the angles *c a b*, *c b a*; and draw *a c* and *c b*; then measure *a c* by the same scale.

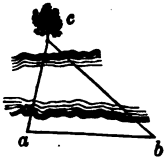


Fig. 1.

Ex. 2. To find the height of a *vertical* object, *n a*.

Place the instrument for measuring angles, at any convenient spot *o*; also meas the dist *o a*; or if *o a* cannot be actually meas'd in consequence of some obstacle, calculate it by the same process as *a c* in Fig 1. Then, first directing the instrument horizontally,* as *o s*, measure the angle of depression, *s o a*, say 12°; also the angle *s o n*, say 30°. These two angles added together, give the angle *a o n*, 42°. Now, in the small triangle *o s a* we have the angle *o s a* equal to 90°, because *a n* is vert, and *o s* hor; and since the three angles of any triangle are equal to 180°, if we subtract the angles *o s a* (90°), and *s o a* (12°) from 180°, the rem (78°) will be the angle *o a s* or *o a n*. Therefore, in the triangle *o n a*, we have one side *o a*; and two angles *a o n*, and *o a n*, to calculate the side *a n* by Case 1.

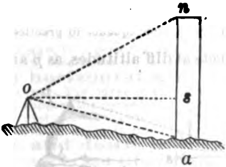
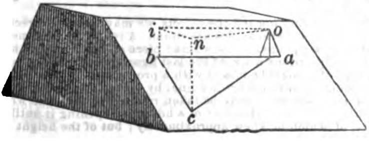


Fig. 2.

---

* **Angles and dists on sloping ground must be measured horizontally.** The graduated hor circle of the instrument evidently measures the angle between two objects horizontally, no matter how much higher one of them may be than the other; one perhaps requiring the telescope of the instrument to be directed upward toward it; and the other downward. If, therefore, the sides of triangles lying upon sloping ground, are not also meas'd hor, there can be no accordance between the two. Thus,



Rem. If, as in Fig 3, it should be necessary to ascertain the *vert* height an from a point o, entirely above it, then both the angles measd at o, namely, son, and soa, will be angles of depression, or *below* the hor line os assumed to measure them from. In this case we have the side oa as before: the angle $noa = soa - son$; and the angle $oan = 180° - \left(osa\ (90°), \text{ and } soa;\right)$ to calculate an by Case I.

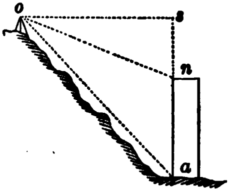


Fig. 3.

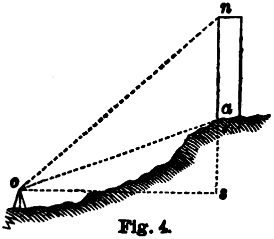


Fig. 4.

Or if, as in Fig 4, the observations are to be taken from a point o, entirely below the object an, then both the angles soa, son, will be angles of elevation, or *above* the assumed hor line os. Here we have in the triangle ona, the given side oa as before; the angle $aon = son - soa$; and the angle $ona = 180° - \left(osn\ (90°), \text{ and } nos,\right)$ to calculate an by Case I.

If the object an, as in Fig 5, instead of being *vert*, is *inclined;* and instead of its vert height, we wish to find its length an, we must first ascertain its angle yti of inclination to the horizon; to which angle each of the angles osn will be equal. To find this angle yti, suspend a plumb-line iy, of any convenient known length, from the object an; and measure also yt horizontally. Then say as

$$yt : iy :: 1 : \text{nat tang of angle } yti.$$

From the table of nat tangs take out the angle yti found opposite this nat tang; and use it for the angles osn or osa; instead of the 90° of Figs 3 and 4. Also when the object inclines, the side ao of the triangle must be measd in line, or in range with the inclination. If the object, as the rock an, Fig 6, is curved or irregular, a pole as may be planted sloping in the direction an; and

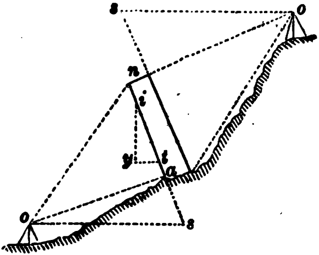
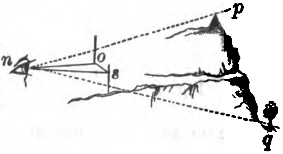


Fig. 5.

---

in the triangle abc, upon sloping ground, the instrument at o, measures the hor angle ion; and not the angle bac. Therefore, the side which corresponds with this hor angle ion, is the hor dist in; and not the sloping dist bc. In other words, when sides and angles are on sloping ground, we do not seek their *actual* measures; but their *hor* ones. This remark applies to all surveying for farms, railroads, triangulations of countries, &c, &c; and the want of a strict attention to it, is one cause of the small errors, almost unavoidable, (and fortunately, of but trifling consequence in practice), which occur in all ordinary field operations.

When a sextant is used, angles between objects at diff altitudes, as p and q, may be measd hor, by first planting two vert rods o and s, in range with the objects; and then taking the hor angle ons, subtended by the rods.



Angles may be measd without any inst, thus: Measure 100 ft toward each object, and drive stakes; measure the dist across from one stake to the other. Half this dist will be the sine of *half* the angle to a rad of 100; and if we move the decimal point two places to the left, we get the *nat sine* of this one half of the angle to a rad of 1, as in the tables. Thus, suppose the dist to be 80.64 feet; then 40.32 is the sine of half the angle; and .4032 will be the nat sine, opposite to which in the table of nat sines we find the angle 23° 47′; which mult by 2, gives 47° 34′, the reqd angle. If obstacles prevent measuring *toward* the objects, we may measure directly *from* them; because, when two lines intersect, the opposite angles are equal. A rough measurement may be made by sticking three pins vert, and a few ins apart, into a small piece of board, nailed hor to the top of a post. The pins would occupy the positions nos, of the last figure. Pencil-lines may then be drawn, connecting the pin-holes; and the angle be measd with a protractor. By nailing a piece of board vert to a tree, and then drawing upon it a short hor line, by means of a pocket carpenters' spirit-level, vert angles of elevation and depression may be taken roughly in the same way. In this way the writer has at times availed himself of the outer door of a house, by opening it until it inted toward some mountain-peak, the dist of which he knew approximately; but of the height of

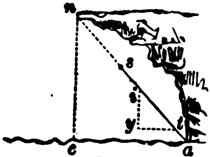


Fig. 6.

its angle $y\,t\,i$ of inclination with the horizon found as before; in which case the dist $a\,n$ is calculated. Or if the *vert* height $c\,n$ is sought, the point c may first be found by sighting upward along a plumb-line held above the head.

Ex. 3. To find the approximate height, $s\,x$, of a mountain,

Of which, perhaps, only the very summit, x, is visible above interposing forests, or other obstacles; but the dist, $m\,i$, of which is known. In this case, first direct the instrument hor, as $m\,h$; and then measure the angle $i\,m\,x$. Then in the triangle $i\,m\,x$ we have one side $m\,i$; the measd angle $i\,m\,x$, and the angle $m\,i\,x$ (90°), to find $i\,x$ by Case 1. But to this $i\,x$ we must add $i\,o$, equal to the height $y\,m$ of the instrument above the ground; and also $o\,s$. Now, $o\,s$ is *apparently* due entirely to the curvature of the earth, which is equal to very nearly 8 ins, or .667 ft in one mile; and increases as the squares of the dists; being 4 times 8 ins in 2 miles; 9 times 8 ins in 3 miles, &c. But this is somewhat diminished by the refraction of the atmosphere; which varies with temperature, moisture, &c; but always tends to make the object x appear higher than it actually is. At an average, this deceptive elevation amounts to about $\frac{1}{7}$th part of the curvature of the earth; and like the latter, it varies with the squares of the dists. Consequently if we subtract $\frac{1}{7}$ part from 8 ins, or .667 ft, we have at once the combined effect of curvature and refraction for one mile, equal to 6.857 ins, or .5714 ft; and for other dists, as shown in the following table, by the use of which we avoid the necessity of making *separate* allowances for curvature and refraction.

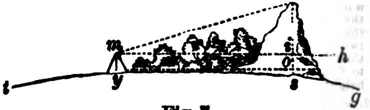


Fig. 7.

Table of allowances to be added for curvature of the earth; and for refraction; combined.

Dist. in yards.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.
100	.002	¼	.036	6	20.6	20	229
150	.004	½	.143	7	28.0	22	277
200	.007	¾	.321	8	36.6	25	357
300	.017	1	.572	9	46.3	30	514
400	.030	1¼	.893	10	57.2	35	700
500	.046	1½	1.29	11	69.2	40	915
600	.066	1¾	1.75	12	82.3	45	1158
700	.090	2	2.29	13	96.6	50	1429
800	.118	2½	3.57	14	112	55	1729
900	.149	3	5.14	15	129	60	2058
1000	.185	3½	7.00	16	146	70	2801
1200	.266	4	9.15	17	165	80	3659
1500	.415	4½	11.6	18	185	90	4631
2000	.738	5	14.3	19	206	100	5717

Hence, if a person whose eye is 5.14 ft, or 112 ft above the sea, sees an object just at the sea's horizon, that object will be about 3 miles, or 14 miles distant from him.

A horizontal line is not a level one, for a straight line cannot be a level one. The curve of the earth, as exemplified in an expanse of quiet water, is level. In Fig 7, if we suppose the curved line $t\,y\,s\,g$ to represent the surface of the sea, then the points $t\,y\,s$ and g are on a level with each other. They need not be equidistant from the center of the earth, for the sea at the poles is about 13 miles nearer it than at the equator; yet its surface is everywhere on a level. **Up, and down,** refer to sea level. **Level** means parallel to the curvature of the sea; and **horizontal** means tangential to a level.

Ex. 4. If the inaccessible vert height $c\,d$, Fig 8,

Is so situated that we cannot reach it at all, then place the instrument for measuring angles, at any convenient spot n; and in range between n and d, plant two staffs, whose tops o and i shall range precisely with n, though they need not be on the same level or hor line with it. Measure $n\,o$: also from n measure the angles $o\,n\,d$ and $o\,n\,c$. Then move the instrument to the precise spot previously

---

which he had no idea. For allowance for curvature and refraction see above Table.

A triangle whose sides are as 3, 4, and 5, is right angled; and one whose sides are as 7; 7; and 9.9; contains 1 right angle; and 2 angles of 45° each. As it is frequently necessary to lay down angles of 45° and 90° on the ground, these proportions may be used for the purpose, by shaping a portion of a tape-line or chain into such a triangle, and driving a stake at each angle.

occupied by the top o of the staff; and from o measure the angles iod and doc. This being done, subtract the angle ioc from 180°; the rem will be the angle con. Consequently in the triangle noc, we have one side no, and two angles, cno and ocn, to find by Case 1 the side oc. Again, take the angle iod from 180°; the remainder will be the angle nod, so that in the triangle dno we have one side no, and the two angles dno and nod, to find by Case 1 the side od. Finally, in the triangle cod, we have two sides co and od, and their included angle cod, to find cd, the reqd vert height.

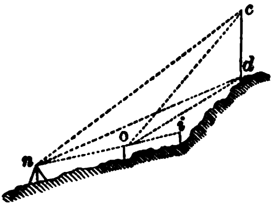


Fig. 8.

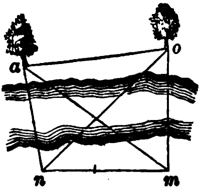


Fig. 9.

Rem. If cd were in a valley, or on a hill, and the observations reqd to be made from either higher or lower ground, the operation would be precisely the same.

Ex. 5. See Ex 10.

To find the dist ao, Fig 9, between two entirely inaccessible objects,

Measure a side nm; at n measure the angles anm and onm; also at m measure the angles omn, and amn. This being done, we have in the triangle anm, one side nm, Fig 9, and the angles anm, and nma; hence, by Case 1, we can calculate the side an. Again, in the triangle omn we have one side nm, and the two angles omn, and mno; hence, by Case 1, we can calculate the side no. This being done, we have in the triangle ano, two sides an, and no; and their included angle ano; hence, by Case 3, we can calculate the side ao, which is the reqd dist. It is plain that in this manner we may obtain also the position or direction of the inaccessible line ao; for we can calculate the angle nao; and can therefrom deduce that of ao; and thus be enabled to run a line parallel to it, if required. By drawing nm on paper by a scale, and laying down the four measd angles, the dist ao may be measd upon the drawing by the same scale.

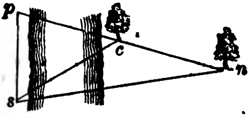


Fig. 10.

If the position of the inaccessible dist cn, Fig 10, be such that we can place a stake p in line with it, we may proceed thus: Place the instrument at any suitable point s, and take the angles psc and csn. Also find the angle cps, and measure the dist ps. Then in the triangle psc find sc by Case 1; again, the exterior angle ncs, being equal to the two interior and opposite angles cps, and psc, we have in the triangle csn, one side and two angles to find cn by Case 1.

Ex. 6. To find a dist ab, Fig 11, of which the ends only are accessible.

From a and b, measure any two lines ac, bc, meeting at c; also measure the angle acb. Then in the triangle abc we have two sides, and the included angle, to find the third side ab by Case 3.

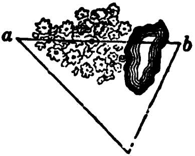


Fig. 11.

Ex. 7. To find the vert height om, of a hill, above a given point i.

Place the instrument at i; measure am. Directing the instrument hor, as an, take the angle nam. Then, since anm is 90° Fig 12, we have one side am, and two angles, nam and anm, to find nm by Case 1. Add no, equal to ai, the height of the instrument. Also, if the hill is a long one, add for curvature of the earth, and for refraction, as explained in Example 3, Fig 7. Or the instrument may be placed at the top of the hill; and an angle of depression measured; instead of the angle of elevation nam.

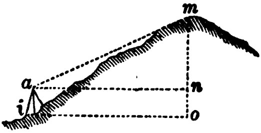


Fig. 12.

Rem. 1. It is plain, that if the height om be previously known, and we wish to ascertain the dist from its summit m to any point i, the same measurement as before, of the angle nam, will enable us to calculate am by Case 1. So in Ex. 2, if the height na be known, the angles measd in that example, will enable us to compute the dist ao; so also in Figs 3, 4, 5, and 7; in all of which the process is so plain as to require no further explanation.

Rem. 2. The height of a vert object **by means of its shadow.** Plant one end of a straight stick vert in the ground; and measure its shadow; also measure the length of the shadow of the object. Then, as the length of the shadow of the stick is to the length of the stick above ground, so is the length of the shadow of the object, to its height. If the object is inclined, the stick must be equally inclined.

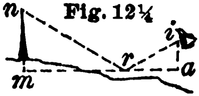


Fig. 12¼

Rem. 3. Or the height of a vert object mn, Fig 12¼, whose distance rm is known, may be found **by its reflection** in a vessel of water, or in a piece of looking glass placed perfectly horizontal at r; for as ra is to the height ai of the eye above the reflector r, so is rm to the height mn of the object above r.

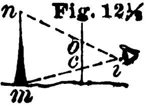


Fig. 12½

Rem. 4. Or let oc, Fig 12½, **be a planted pole,** or a rod held vert by an assistant. Then stand at a proper dist back from it, and keeping the eyes steady, let marks be made at o and c, where the lines of sight in and im strike the rod. Then as ic is to co, so is im to mn.

The following examples may be regarded as substitutes for strict trigonometry: and will at times be useful, in case a table of sines, &c, is not at hand for making trigonometrical calculations.

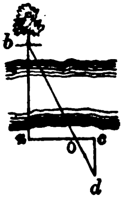


Fig. 13.

Ex. 8. To find the dist ab, of which one end only is accessible.

Drive a stake at any convenient point a; from a lay off any angle bac. In the line ac, at any convenient point c, drive a stake; and from c lay off an angle acd, equal to the angle bac. In the line cd, at any convenient point, as d, drive a stake. Then, standing at d, and looking at b, place a stake o in range with db; and at the same time in the line ac. Measure ao, oc, and cd; then, from the principle of similar triangles, as

$$oc : cd :: ao : ab.$$

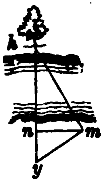


Fig. 14.

Or thus:

Fig 14, nh being the dist, place a stake at n; and lay off the angle hnm 90°. At any convenient dist nm, place a stake m. Make the angle hmy = 90°; and place a stake at y, in range with hn. Measure ny and nm; then, from the principle of similar triangles, as

$$ny : nm :: nm : nh.$$

Or thus, Fig 14. Lay off the angle $hnm = 90°$, placing a stake m, at any convenient dist nm. Measure nm. Also measure the angle nmh. Find nat tang of nmh by Table Mult this nat tang by nm. The prod will be nh.

Or thus. Lay off angle $hnm = 90°$. From m measure the angle nmh, and lay off angle nmy equal to it, placing a stake at y in range with hn. Then is $ny = nh$.

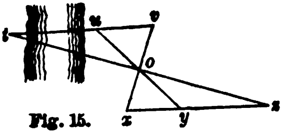


Fig. 15.

Or thus, without measuring any angle;

tu being the dist. Make uv of any convenient length, in range with tu. Measure any vo; and ox equal to it, in range. Measure uo; and oy equal to it in range. Place a stake s in range with both xy, and to. Then will ys be both equal to tu, and parallel to it.

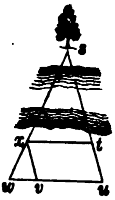
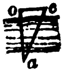


Fig. 16.

Or thus, without measuring any angle.

Drive two stakes t and u, in range with the object s. From t lay off any convenient dist tx, in any direction. From u lay off uw parallel to ts, placing w in range with xs. Make uv equal to tx. Measure wv, vx, and xt. Then, as

$$wv : vx :: xt : ts.$$



Or thus. At a lay off angle $oac = 5° 48'$. Lay off oc at right angles to ao. Measure oc. Then $ao = 10\,oc$, too long only 1 part in 935.6, or 5.643 feet in a mile, or .1069 foot (full 1¼ inches) in 100 feet.

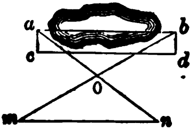


Fig. 17.

Ex. 9. To find the dist ab, of which the ends only are accessible.

From a lay off the angle bac; and from b, the angle abd, each 90°. Make ac and bd equal to each other. Then, $cd = ab$. Or ab may be considered as the dist across the river in Figs 12, 13, or 14; and be ascertained in the same way. Or measure any dist, Fig 17, ao; and make on in line and equal to it. Also measure bo; and make om in line and equal to it. Then will mn be both parallel to ab, and equal to it.

Ex. 10. See Ex. 4. To find the entirely inaccessible dist yz, and also its direction.

At any two convenient points a and b, from each of which y and z can be seen, drive stakes. Then we have the four corners of a four-sided figure, in which are given the directions of three of its sides, and of its two diags. These data enable us to lay out on the ground, the small four-sided fig $acoi$, exactly similar to the large one. Thus, in the line ab place a stake c; and make co parallel to bz; o being at the same time in range of the diag az. Also, from c make ci parallel to by; i being at the same time in range of ay. Then will io be in the same direction as yz, or parallel to it. Measure ac, ab, and io; then evidently, from the principle of similar figures, as

$$ac : ab :: io : yz.$$

If yz were a visible line, such as a fence or road, we could from a divide it into any required portions. Thus, if we wish to place a stake halfway between y and z, first place one halfway between i and o; then standing at a, by means of signals, place a person in range on yz. Or, to find along ab, a point t perp to yz at y, first make $ois = 90°$; and measure as. Then, as

$$oi : as :: yz : at.$$

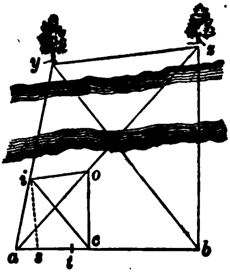


Fig. 18.

Ex. 11. To find the position of a point, n, Fig 19,

By means of two angles a n b *and* b n c, *taken from it to the three objects* a b c, *whose positions and dists apart are known.*

The use of this problem is more frequent in marine than in land surveying. It is chiefly employed for determining the position n of a boat from which soundings are being taken along a coast. As the boat moves from point to point to take fresh soundings, it becomes necessary to make a fresh observation at each point, in order to define its position on the chart. An observation consists in the measurement by a sextant of the two angles anb, bnc, to the signals abc, previously arranged on the shore. When practicable, this method should be rejected; and the observations taken to the boat at the same instant, by two observers on shore, at two of the stations. The boat to show a signal at the proper moment. The most expeditious mode of fixing the point n upon the map, is to draw three lines, forming the two angles, and extended indefinitely, on a piece of transparent paper. Place the paper upon the map, and move it about until the three lines pass through the three stations; then prick through the point n wherever it happens to come.

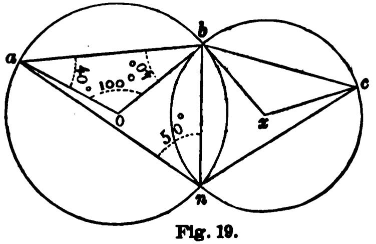
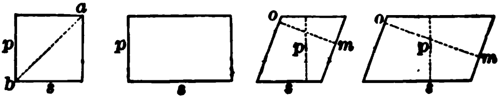


Fig. 19.

Instead of the transparent paper, an instrument called a *station pointer* may be used when there are many points to be fixed.

But the position of the point n can be found more correctly by describing two circles, as in Fig 19, each of which shall pass through n and two of the station points. The question is to find the centers o and x of two such circles. This is very simple. We know that the angle aob at the center of a circle is twice as great as any angle anb at the circumf of the same circle, when both are subtended by the same chord ab. Consequently, if the angle anb, observed from the boat, is say, 50°, the angle aob must be 100°. And, since the three angles of every plane triangle are equal to 180°, the two angles oab and oba are together equal to $180° - 100° = 80°$. And, since the two sides ao and bo are equal (being radii of the same circle), therefore, the angles oab and oba are equal; and each equal to $\frac{80°}{2} = 40°$. Consequently, on the map we have only to lay down at a and b, two angles of 40°; the point o of intersection will be the center of the circle abn. Proceed in the same way with the angle bnc, to find the center x. Then the intersection of the two circles at n will be the point sought.

PARALLELOGRAMS.



Square. **Rectangle.** **Rhombus.** **Rhomboid.**

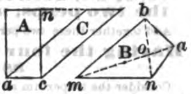
A PARALLELOGRAM is any figure of four straight sides, the opposite ones of which are parallel. There are but four, as in the above figs. The rhombus, like the rhombohedron, Fig 3, p 195, is sometimes called "rhomb." In the square and rhombus all the four sides are equal; in the rectangle and rhomboid only the opposite ones are equal. In any parallelogram the four angles amount to four right angles, or 360°; and any two diagonally opposite angles are equal to each other; hence, having one angle given, the other three can readily be found. In a square, or a rhombus, a diag divides each of two angles into two equal parts; but in the two other parallelograms it does not.

To find the area of any parallelogram.

Multiply any side, as S, by the perp height, or dist p to the opposite side. **Or,** multiply together two sides and nat sine of their included angle.

The diag a b of any square is equal to one side mult by 1.41421; and a side is equal to $\frac{\text{diagonal}}{1.41421}$; **or,** to diag mult by .707107.

The side of a square equal in area to a given circle, is equal to diam × .886227.



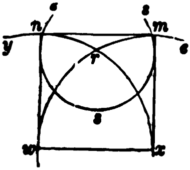
The side of the greatest square, *that can be inscribed in a given circle*, is equal to diam × .707107.

The side of a square mult by 1.51967 gives the **side of an equilateral triangle** of the same area. **All parallelograms** as A and C, **which have equal bases,** *a c*, and equal perp heights *n c*, have also equal areas; and the area of each is twice that of a triangle having the same base, and perp height. **The area of a square inscribed in a circle** is equal to twice the square of the rad.

In every parallelogram, the 4 squares drawn on its sides have a united area equal to that of the two squares drawn on its 2 diags. **If a larger square** be drawn on the diag *a b* of a smaller square, its area will be twice that of said smaller square. **Either diag of any parallelogram** divides it into two equal triangles, and the 2 diags div it into 4 triangles of equal areas. **The two diags of any parallelogram** divide each other into two equal parts. **Any line drawn through the center of a diag** divides the parallelogram into two equal parts.

Remark 1.—The area of any fig whatever as B **that is enclosed by four straight lines,** may be found thus: Mult together the two diags *a m*, *n b*; and the nat sine of the least angle *a o b*; or *n o m*, formed by their intersection. Div the product by 2. This is useful in land surveying, when obstacles, as is often the case, make it difficult to measure the sides of the fig or field; while it may be easy to measure the diags; and after finding their point of intersection *o*, to measure the required angle. **But if the fig is to be drawn,** the **parts** *o a*, *o b*, *o n*, *o m* of the diags must also be measd.

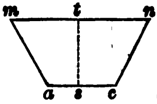
Rem. 2.—The sides of a parallelogram, triangle, and many other figs may be found, when only the area and angles are given, thus: Assume some particular one of its sides to be of the length 1; and calculate what its area would be if that were the case. Then as the sq rt of the area thus found is to this side 1, so is the sq rt of the actual given area, to the corresponding actual side of the fig.

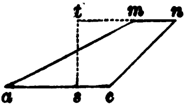


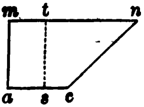
On a given line *w x*, to draw a square, *w x n m*.

From *w* and *x*, with rad *w x*, describe the arcs *x r y* and *w r e*. From their intersection *r*, and with rad equal to ½ of *w x*, describe *s s s*. From *w* and *x* draw *w n* and *x m* tangential to *s s s*, and ending at the other arcs; join *n m*.

TRAPEZOIDS.





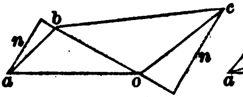


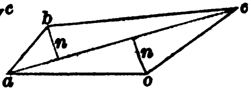
A trapezoid $a c n m$, is any figure with four straight sides, only two of which, as $a c$ and $n m$, are parallel.

To find the area of any trapezoid.

Add together the two parallel sides, $a c$ and $m n$; mult the sum by the perp dist $s t$ between them; div the prod by 2. See the following rules for trapeziums, which are all equally applicable to trapezoids; also see Remarks after Parallelograms.

TRAPEZIUMS.





A trapezium $a b c o$, is any fig with four straight sides, of which no two are parallel.

To find the area of any trapezium, having given the diag $b o$, or $a c$, between either pair of opposite angles; and also the two perps, n, n, from the other two angles.

Add together these two perps; mult the sum by the diag; div the prod by 2.

Having the four sides; and either pair of opposite angles, as $a b c$, $a o c$; or $b a o$, and $b c o$.

Consider the trapezium as divided into two triangles, in each of which are given two sides and the included angle. Find the area of each of these triangles as directed under the preceding head "Triangles," and add them together.

Having the four angles, and either pair of opposite sides.

Begin with one of the sides, and the two angles at its ends. If the sum of these two angles exceeds 180°, subtract each of them from 180°, and make use of the rems instead of the angles themselves. Then consider this side and its two adjacent angles (or the two rems, as the case may be) as those of a triangle; and find its area as directed for that case under the preceding head "Triangle." Do the same with the other given side, and its two adjacent angles, (or their rems, as the case may be.) Subtract the least of the areas thus found, from the greatest; the rem will be the reqd area.

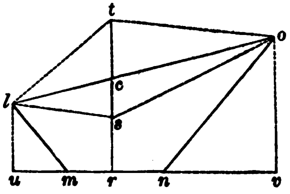
Having three sides; and the two included angles.

Mult together the middle side, and one of the adjacent sides; mult the prod by the nat sine of their included angle; call the result a. Do the same with the middle side and its other adjacent side, and the nat sine of the other included angle; call the result b. Add the two angles together; find the diff between their sum and 180°, whether greater or less; find the nat sine of this diff; mult together the two given sides which are *opposite* one another; mult the prod by the nat sine just found; call the result c. Add together the results a and b; then, if the sum of the two given angles is *less* than 180°, subtract c from the sum of a and b; *half* the rem will be the area of the trapezium. But if the sum of the two given angles be *greater* than 180°, add together the three results a, b, and c; *half* their sum will be the area.

Having the two diagonals, and either angle formed by their intersection.

See Remarks after Parallelograms.

In railroad measurements



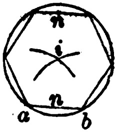
Of excavation and embankment, the trapezium $l m n o$ frequently occurs; as well as the two 5-sided figures $l m n o t$ and $l m n o s$; in all of which $m n$ represents the roadway; $r s$, $r c$, and $r t$ the center-depths or heights; $l u$ and $o v$ the side-depths or heights, as given by the level; $l m$ and $n o$ the side-slopes.

The same general rule for area applies to all three of these figs; namely, mult the extreme hor width $u v$ by *half* the center depth $r s$, $r c$, or $r t$, as the case may be. Also mult one *fourth* of the width of roadway $m n$, by the *sum* of the two side-depths $l u$ and $o v$. Add the two prods together; the sum is the reqd area. This rule applies whether the two side-slopes $m l$ and $n o$ have the same angle of inclination or not. **In railroad work, etc.,** the mid-way hor width, center depth, and side depths of a prismoid are respectively = The half sums of the corresponding end ones, and thus can be found without actual measurement.

To draw a hexagon, each side of which shall be equal to a given line, $a b$.

From a and b, with rad $a b$, describe the two arcs; from their intersection, i, with the same rad, describe a circle; around the circumf of which, step off the same rad.

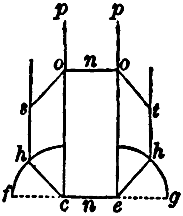
Side of a hexagon = $n n$ × .57735.



To draw an octagon, with each side equal to a given line, $c e$.

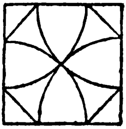
From c and e draw two perps, $c p$, $e p$. Also prolong $c e$ toward f and g; and from c and e, with rad equal $c e$, draw the two quadrants; and find their centers $h h$; join $c h$, and $e h$; draw $h s$ and $h t$ parallel to $e p$; and make each of them equal to $c e$; make $c o$, and $e o$, each equal to $h h$; join $o o$, $o s$, and $o t$.

Side of an octagon = $n n$ × .41421354.



To draw an octagon in a given square.

From each corner of the square, and with a rad equal to half its diag, describe the four arcs; and join the points at which they cut the sides of the square.

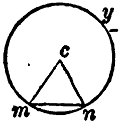


To draw any regular polygon, with each side equal to $m n$.

Div 360 degrees by the number of sides; take the quot from 180°; div the rem by 2. This will give the angle $c m n$, or $c n m$. At m and n lay down these angles by a protractor: the sides of these angles will meet at a point, c, from which describe the circle $m n y$; and around its circumf step off dists equal to $m n$.

In any circle, $m n y$, to draw any regular polygon.

Div 360° by the number of sides; the quot will be the angle $m c n$, at the center. Lay off this angle by a protractor; and its chord $m n$ will be one side; which step off around the circumf.



To reduce any polygon, as $a b c d e f a$, to a triangle of the same area.

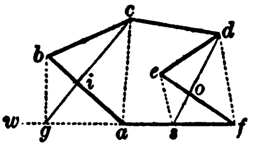


Fig. 1.

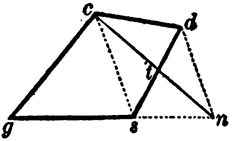
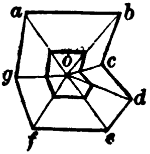


Fig. 2.

If we produce the side $f a$ toward w; and draw $b g$ parallel to $a c$, and join $g c$, we get equal triangles $a c b$, and $a c g$, both on the same base $a c$; and both of the same perp height, inasmuch as they are between the two parallels $a c$ and $g b$. But the part $a c i$ forms a portion of both these triangles, or in other words, *is common to both*. Therefore, if it be taken away from both triangles, the remaining parts, $i c b$ of one of them, and $i g a$ of the other, are also equal. Therefore, if the part $i c b$ be left off from the polygon, and the part $i g a$ be taken into it, the polygon $g f e d c i g$ will have the same area as $a f e d c b a$; but it will have but five sides, while the other has six. Again, if $e s$ be drawn parallel to $d f$, and $d s$ joined, we have upon the same base $e s$, and between the same parallels $e s$ and $d f$, the two equal triangles $e s d$, and $e s f$, with the part $e o s$ common to both; and consequently the remaining part $e o d$ of one, and $o s f$ of the other, are equal. Therefore, if $o s f$ be left off from the polygon, and $e o d$ be taken into it, the new polygon $g s d c g$, Fig 2, will have the same area as $g f e d c g$; but it has but four sides, while the other has five. Finally, if $g s$, Fig 2, be extended toward n; and $d n$ drawn parallel to $c s$; and $c n$ joined, we have on the same base $c s$, and between the same parallels $c s$ and $d n$, the two equal triangles $c s n$, and $c s d$, with the part $c s t$ common to both. Therefore, if we leave out $c d t$, and take in $s t n$, we have the triangle $g n c$ equal to the polygon $g s d c g$, Fig 2; or to $a f e d c b a$, Fig 1.

This simple method is applicable to polygons of any number of sides.

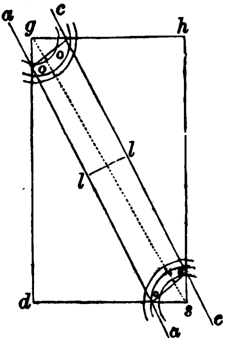


To reduce a large fig, *a b c d e f g*, to a smaller similar one.

From any interior point *o*, which had better be near the center, draw lines to all the angles *a*, *b*, *c*, &c. Join these lines by others parallel to the sides of the fig. If it should be reqd to enlarge a small fig, draw, from any point *o* within it, lines extending beyond its angles; and join these lines by others parallel to the sides of the small fig.

To reduce a map to one on a smaller scale.

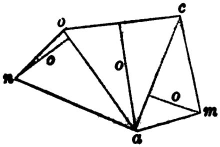
The best method is by dividing the large map into squares by faint lines, with a very soft lead pencil; and then drawing the reduced map upon a sheet of smaller squares. A pair of proportional dividers will assist much in fixing points intermediate of the sides of the squares. If the large map would be injured by drawing and rubbing out the squares, threads may be stretched across it to form the squares.



In a rectangular fig, *g h s d*,

Representing an open panel, to find the points *o o o o* in its sides; and at equal dists from the angles *g*, and *s*; for inserting a diag piece *o o o o*, of a given width *l l*, measured at right angles to its length. From *g* and *s* as centers, describe several concentric arcs, as in the Fig. Draw upon transparent paper, two parallel lines *a a*, *c c*, at a distance apart equal to *l l*; and placing these lines on top of the panel, move them about until it is shown by the arcs that the four dists *g o*, *g o*, *s o*, *s o*, are equal. Instead of the transparent paper, a strip of common paper, of the width *l l* may be used.

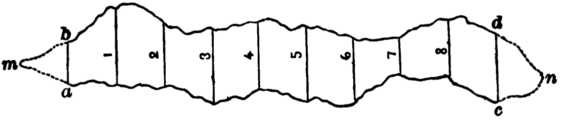
Rem. Many problems which would otherwise be very difficult, may be thus solved with an accuracy sufficient for practical purposes, **by means of transparent paper.**



To find the area of any irregular polygon, *a n b c m*.

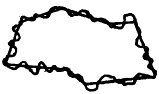
Div it into triangles, as *a n b*, *a m c*, and *a b c*; in each of which find the perp dist *o*, between its base *a b*, *a c*, or *b c*; and the opposite angle *n*, *m*, or *a*; mult each base by its perp dist; add all the prods together; div by 2.

To find approx the area of a long irregular fig, as *a b c d*.



Between its ends *a b*, *c d*, space off *equal* dists, (the shorter they are the more accurate will be the result,) through which draw the intermediate parallel lines 1, 2, 3, &c, across the breadth of the fig. Measure the lengths of these intermediate lines; add them together: to the sum add *half* the sum of the two end breadths *a b* and *c d*. Mult the entire sum by one of the equal spaces between the parallel lines. The prod will be the area. This rule answers as well if either one or both the ends terminate in points, as at *m* and *n*. In the last of these cases, both *a b* and *c d* will be included in the intermediate lines; and half the two end breadths will be 0, or nothing.

To find the area of any irregular figure.



Draw around it lines which shall enclose within them (as nearly as can be judged by the eye) as much space not belonging to the figure as they exclude space belonging to it. The area of the simplified figure thus formed, being in this manner rendered equal to that of the complicated one, may be calculated by dividing it into triangles, &c. By using a piece of fine thread, the proper position for the new boundary lines may be found, before drawing them in.

Areas of irregular figures may be found from a drawing, by laying upon it a piece of transparent paper carefully ruled into small squares, each of a given area, say 10 20, or 100 sq. ft. each; and by first counting the whole squares, and then adding the fractions of squares.

CIRCLES.

A circle is the area included within a curved line of such a character that every point in it is equally distant from a certain point within it, called its center. The curved line itself is called the circumference, or periphery of the circle; or very commonly it is called the circle.

To find the circumference.

Mult diam by 3.1416, which gives too much by only .148 of an inch in a mile. **Or,** as 113 is to 355 so is diam to circumf; too great 1 inch in 186 miles. **Or,** mult diam by $3\frac{1}{7}$; too great by about 1 part in 2485. **Or,** mult area by 12.566, and take sq root of prod.

To find the diam.

Div the circumf by 3.1416; or, as 355 is to 113, so is circumf to diam; or, mult the circumf. by 7: and div the prod by 22, which gives the diam too small by only about one part in 2485; or, mult the area by 1.2732; and take the sq rt of the prod.

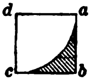
The diam is to the circumf more exactly as 1 to 3.14159265.

To find the area of a circle.

Square the diam; mult this square by .7854; or more accurately by .78539816; or square the circumf; mult this square by .07958: or more accurately by .07957747; or mult half the diam by half the circumf; or refer to the following table of areas of circles. Also area = sq of rad × 3.1416.

The area of a circle is to the area of any circumscribed straight-sided fig, as the circumf of the circle is to the circumf or periphery of the fig. The area of a square inscribed in a circle, is equal to twice the square of the rad. Of a circle in a square, = square × .7854.

It is convenient to remember, in rounding off a square corner *a b c*, by a quarter of a circle, that the shaded area *a b c* is equal to about $\frac{1}{5}$ part (correctly .2146) of the whole square *a b c d*.



To find the diam of a circle equal in area to a given square.

Mult one side of the square by 1.12838.

To find the rad of a circle to circumscribe a given square.

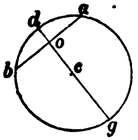
Mult one side by .7071; or take ½ the diag.

To find the side of a square equal in area to a given circle.

Mult the diam by .88623.

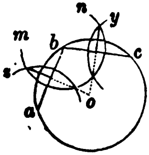
To find the side of the greatest square in a given circle.

Mult diam by .7071. The area of the greatest square that can be inscribed in a circle is equal to twice the square of the rad. The diam × by 1.3468 gives the side of an equilateral triangle of equal area.



To find the center *c*, of a given circle.

Draw any chord *a b*; and from the middle of it *o*, draw at right angles to it, a diam *d g*; find the center *c* of this diam.

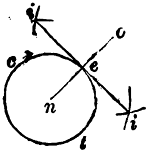


To describe a circle through any three points, *a b c*, not in a straight line.

Join the points by the lines *a b*, *b c*; from the centers of these lines draw the dotted perps meeting, as at *o*, which will be the center of the circle. Or from *b*, with any convenient rad. draw the arc *m n*; and from *a* and *c*, with the same rad, draw arcs *y* and *z*; then two lines drawn through the intersections of these arcs, will meet at the center *o*.

To describe a circle to touch the three angles of a triangle is plainly the same as this.

To inscribe a circle in a triangle draw two lines bisecting any two of the angles. Where these lines meet is the center of the circle.

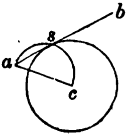


To draw a tangent, *i e i*, to a circle, from any given point, *e*, in its circumf.

Through the center *n*, and the given point *e*, draw *n o*; make *e o* equal to *e n*; from *n* and *o*, with any rad greater than half of *o n*, describe the two pairs of arc *i i*; join their intersections *i i*.

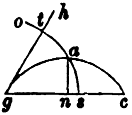
Here, and in the following three figs, the *tangents* are *ordinary* or *geometrical* ones; and may end where we please. But the *trigonometrical* tangent of a given *angle*, must end in a secant.

Or from *e* lay off two equal distances *e c*, *e t*; and draw *i i* parallel to *c t*.



To draw a tang, *a s b*, to a circle, from a point, *a*, which is outside of the circle.

Draw *a c*, and on it describe a semicircle; through the intersection, *s*, draw *a s b*. Here *c* is the center of the circle.

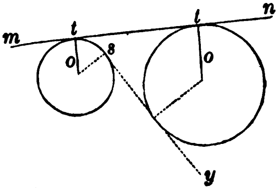


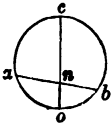
To draw a tang, *g h*, from a circular arc, *g a c*,

Of which *n a* is the rise. With rad *g a*, describe an arc, *s a o*. Make *t o* equal to *s a*. Through *t* draw *g h*.

To draw a tang to two circles.

First draw the line *m n*, just touching the two circles; this gives the *direction* of the tang. Then from the centers of the circles draw the radii, *o o*, perp to *m n*. The points *t t* are the tang points. If the tang is in the position of the dotted line, *s y*, the operation is the same.





If any two chords, as a b, o c, cross each other, then as $o\,n : n\,b :: a\,n : n\,c$. Hence, $n\,b \times a\,n = o\,n \times n\,c$. That is, the product of the two parts of one of the lines, is = the product of the two parts of the other line.

TABLE 1 OF CIRCLES.
Diameters in units and eighths, &c.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
1-64	.049087	.00019	3. ½	10.9956	9.6211	10⅛	31.8086	80.516	19¼	60.4757	291.04
1-32	.098175	.00077	9-16	11.1919	9.9678	¼	32.2013	82.516	⅜	60.8684	294.83
3-64	.147262	.00173	⅝	11.3883	10.321	⅜	32.5940	84.541	½	61.2611	298.65
1-16	.196350	.00307	11-16	11.5846	10.680	½	32.9867	86.590	⅝	61.6538	302.49
3-32	.294524	.00690	¾	11.7810	11.045	⅝	33.3794	88.664	¾	62.0465	306.35
⅛	.392699	.01227	13-16	11.9773	11.416	¾	33.7721	90.763	⅞	62.4392	310.24
5-32	.490874	.01917	⅞	12.1737	11.793	⅞	34.1648	92.886	20.	62.8319	314.16
3-16	589049	.02761	15-16	12.3700	12.177	11.	34.5575	95.033	⅛	63.2246	318.10
7-32	.687223	.03758	4.	12.5664	12.566	⅛	34.9502	97.205	¼	63.6173	322.06
¼	.785398	.04909	1-16	12.7627	12.962	¼	35.3429	99.402	⅜	64.0100	326.05
9-32	.883573	.06213	⅛	12 9591	13.364	⅜	35.7356	101.62	½	64.4026	330.06
5-16	.981748	.07670	3-16	13.1554	13.772	½	36.1283	103.87	⅝	64.7953	334.10
11-32	1.07992	.09281	¼	13.3518	14.186	⅝	36.5210	106.14	¾	65.1880	338.16
⅜	1.17810	.11045	5-16	13.5481	14.607	¾	36.9137	108.43	⅞	65.5807	342.25
13-32	1.27627	.12962	⅜	13.7445	15.033	⅞	37.3064	110.75	21.	65.9734	346.36
7-16	1.37445	.15033	7-16	13.9408	15.466	12.	37.6991	113.10	⅛	66.3661	350.50
15-32	1.47262	.17257	½	14.1372	15.904	⅛	38.0918	115.47	¼	66.7588	354.66
½	1.57080	.19635	9-16	14.3335	16.349	¼	38.4845	117.86	⅜	67.1515	358.84
17-32	1.66897	.22166	⅝	14.5299	16.800	⅜	38.8772	120.28	½	67.5442	363.05
9-16	1.76715	.24850	11-16	14.7262	17.257	½	39.2699	122.72	⅝	67.9369	367.28
19-32	1.86532	.27688	¾	14.9226	17.721	⅝	39.6626	125.19	¾	68.3296	371.54
⅝	1.96350	.30680	13-16	15.1189	18.190	¾	40.0553	127.68	⅞	68.7223	375.83
21-32	2.06167	.33824	⅞	15.3153	18.665	⅞	40.4480	130.19	22.	69.1150	380.13
11-16	2.15984	.37122	15-16	15.5116	19.147	13.	40.8407	132.73	⅛	69.5077	384.46
23-32	2.25802	.40574	5.	15.7080	19.635	⅛	41.2334	135.30	¼	69.9004	388.82
¾	2.35619	.44179	1-16	15.9043	20.129	¼	41.6261	137.89	⅜	70.2931	393.20
25-32	2.45437	.47937	⅛	16.1007	20.629	⅜	42.0188	140.50	½	70.6858	397.61
13-16	2.55254	.51849	3-16	16.2970	21.135	½	42.4115	143.14	⅝	71.0785	402.04
27-32	2.65072	.55914	¼	16.4934	21.648	⅝	42.8042	145.80	¾	71.4712	406.49
⅞	2.74889	.60132	5-16	16.6897	22.166	¾	43.1969	148.49	⅞	71.8639	410.97
29-32	2.84707	.64504	⅜	16.8861	22.691	⅞	43.5896	151.20	23.	72.2566	415.48
15-16	2.94524	.69029	7-16	17.0824	23.221	14.	43.9823	153.94	⅛	72.6493	420.00
31-32	3.04342	.73708	½	17.2788	23.758	⅛	44.3750	156.70	¼	73.0420	424.56
1.	3.14159	.78540	9-16	17.4751	24.301	¼	44.7677	159.48	⅜	73.4347	429.13
1-16	3.33794	.88664	⅝	17.6715	24.850	⅜	45.1604	162.30	½	73.8274	433.74
⅛	3.53429	.99402	11-16	17.8678	25.406	½	45.5531	165.13	⅝	74.2201	438.36
3-16	3.73064	1.1075	¾	18.0642	25.967	⅝	45.9458	167.99	¾	74.6128	443.01
¼	3.92699	1.2272	13-16	18.2605	26.535	¾	46.3385	170.87	⅞	75.0055	447.69
5-16	4.12334	1.3530	⅞	18.4569	27.109	⅞	46.7312	173.78	24.	75.3982	452.39
⅜	4 31969	1.4849	15-16	18.6532	27.688	15.	47.1239	176.71	⅛	75.7909	457.11
7-16	4.51604	1.6230	6.	18.8496	28.274	⅛	47.5166	179.67	¼	76.1836	461.86
½	4.71239	1.7671	⅛	19.2423	29.465	¼	47.9093	182.65	⅜	76.5763	466.64
9-16	4.90874	1.9175	¼	19.6350	30.680	⅜	48.3020	185.66	½	76.9690	471.44
⅝	5.10509	2.0739	⅜	20.0277	31.919	½	48.6947	188.69	⅝	77.3617	476.26
11-16	5.30144	2.2365	½	20.4204	33.183	⅝	49.0874	191.75	¾	77.7544	481.11
¾	5.49779	2.4053	⅝	20.8131	34.472	¾	49.4801	194.83	⅞	78.1471	485.98
13-16	5.69414	2.5802	¾	21.2058	35.785	⅞	49.8728	197.93	25.	78.5398	490.87
⅞	5.89049	2.7612	⅞	21.5984	37.122	16.	50.2655	201.06	⅛	78.9325	495.79
15-16	6.08684	2.9483	7.	21.9911	38.485	⅛	50.6582	204.22	¼	79.3252	500.74
2.	6.28319	3.1416	⅛	22.3838	39.871	¼	51.0509	207.39	⅜	79.7179	505.71
1-16	6.47953	3.3410	¼	22.7765	41.282	⅜	51.4436	210.60	½	80.1106	510.71
⅛	6.67588	3.5466	⅜	23.1692	42.718	½	51.8363	213.82	⅝	80.5033	515.72
3-16	6.87223	3.7583	½	23.5619	44.179	⅝	52.2290	217.08	¾	80.8960	520.77
¼	7.06858	3.9761	⅝	23.9546	45.664	¾	52.6217	220.35	⅞	81.2887	525.84
5-16	7.26493	4.2000	¾	24.3473	47.173	⅞	53.0144	223.65	26.	81.6814	530.93
⅜	7.46128	4.4301	⅞	24.7400	48.707	17.	53.4071	226.98	⅛	82.0741	536.05
7-16	7.65763	4.6664	8.	25.1327	50.265	⅛	53.7998	230.33	¼	82.4668	541.19
½	7.85398	4.9087	⅛	25.5254	51.849	¼	54.1925	233.71	⅜	82.8595	546.35
9-16	8.05033	5.1572	¼	25 9181	53.456	⅜	54.5852	237.10	½	83.2522	551.55
⅝	8.24668	5.4119	⅜	26.3108	55.088	½	54.9779	240.53	⅝	83.6449	556.76
11-16	8.44303	5.6727	½	26.7035	56.745	⅝	55.3706	243.98	¾	84.0376	562.00
¾	8.63938	5.9396	⅝	27.0962	58.426	¾	55.7633	247.45	⅞	84.4303	567.27
13-16	8.83573	6.2126	¾	27.4889	60.132	⅞	56.1560	250.95	27.	84.8230	572.56
⅞	9.03208	6.4918	⅞	27.8816	61.862	18.	56.5487	254.47	⅛	85.2157	577.87
15-16	9.22843	6.7771	9.	28.2743	63.617	⅛	56.9414	258.02	¼	85.6084	583.21
3.	9.42478	7.0686	⅛	28.6670	65.397	¼	57.3341	261.59	⅜	86.0011	588.57
1-16	9.62113	7.3662	¼	29.0597	67.201	⅜	57.7268	265.18	½	86.3938	593.96
⅛	9.81748	7.6699	⅜	29.4524	69.029	½	58.1195	268.80	⅝	86.7865	599.37
3-16	10.0138	7.9798	½	29.8451	70.882	⅝	58.5122	272.45	¾	87.1792	604.81
¼	10.2102	8.2958	⅝	30.2378	72.760	¾	58.9049	276.12	⅞	87.5719	610.27
5-16	10.4065	8.6179	¾	30.6305	74.662	⅞	59.2976	279.81	28.	87.9646	615.75
⅜	10.6029	8.9462	⅞	31.0232	76.589	19.	59.6903	283.53	⅛	88.3573	621.26
7-16	10.7992	9.2806	10.	31.4159	78.540	⅛	60.0830	287.27	¼	88.7500	626.80

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
28⅜	89.1427	632.36	38.	119.381	1134.1	47⅝	149.618	1781.4	57¼	179.856	2574.2
½	89.5354	637.94	⅛	119.773	1141.6	¾	150.011	1790.8	⅜	180.249	2585.4
⅝	89.9281	643.55	¼	120.166	1149.1	⅞	150.404	1800.1	½	180.642	2596.7
¾	90.3208	649.18	⅜	120.559	1156.6	48.	150.796	1809.6	⅝	181.034	2608.0
⅞	90.7135	654.84	½	120.951	1164.2	⅛	151.189	1819.0	¾	181.427	2619.4
29.	91.1062	660.52	⅝	121.344	1171.7	¼	151.582	1828.5	⅞	181.820	2630.7
⅛	91.4989	666.23	¾	121.737	1179.3	⅜	151.975	1837.9	58.	182.212	2642.1
¼	91.8916	671.96	⅞	122.129	1186.9	½	152.367	1847.5	⅛	182.605	2653.5
⅜	92.2843	677.71	39.	122.522	1194.6	⅝	152.760	1857.0	¼	182.998	2664.9
½	92.6770	683.49	⅛	122.915	1202.3	¾	153.153	1866.5	⅜	183.390	2676.4
⅝	93.0697	689.30	¼	123.308	1210.0	⅞	153.545	1876.1	½	183.783	2687.8
¾	93.4624	695.13	⅜	123.700	1217.7	49.	153.938	1885.7	⅝	184.176	2699.3
⅞	93.8551	700.98	½	124.093	1225.4	⅛	154.331	1895.4	¾	184.569	2710.9
30.	94.2478	706.86	⅝	124.486	1233.2	¼	154.723	1905.0	⅞	184.961	2722.4
⅛	94.6405	712.76	¾	124.878	1241.0	⅜	155.116	1914.7	59.	185.354	2734.0
¼	95.0332	718.69	⅞	125.271	1248.8	½	155.509	1924.4	⅛	185.747	2745.6
⅜	95.4259	724.64	40.	125.664	1256.6	⅝	155.902	1934.2	¼	186.139	2757.2
½	95.8186	730.62	⅛	126.056	1264.5	¾	156.294	1943.9	⅜	186.532	2768.8
⅝	96.2113	736.62	¼	126.449	1272.4	⅞	156.687	1953.7	½	186.925	2780.5
¾	96.6040	742.64	⅜	126.842	1280.3	50.	157.080	1963.5	⅝	187.317	2792.2
⅞	96.9967	748.69	½	127.235	1288.2	⅛	157.472	1973.3	¾	187.710	2803.9
31.	97.3894	754.77	⅝	127.627	1296.2	¼	157.865	1983.2	⅞	188.103	2815.7
⅛	97.7821	760.87	¾	128.020	1304.2	⅜	158.258	1993.1	60.	188.496	2827.4
¼	98.1748	766.99	⅞	128.413	1312.2	½	158.650	2003.0	⅛	188.888	2839.2
⅜	98.5675	773.14	41.	128.805	1320.3	⅝	159.043	2012.9	¼	189.281	2851.0
½	98.9602	779.31	⅛	129.198	1328.3	¾	159.436	2022.8	⅜	189.674	2862.9
⅝	99.3529	785.51	¼	129.591	1336.4	⅞	159.829	2032.8	½	190.066	2874.8
¾	99.7456	791.73	⅜	129.983	1344.5	51.	160.221	2042.8	⅝	190.459	2886.6
⅞	100.138	797.98	½	130.376	1352.7	⅛	160.614	2052.8	¾	190.852	2898.6
32.	100.531	804.25	⅝	130.769	1360.8	¼	161.007	2062.9	⅞	191.244	2910.5
⅛	100.924	810.54	¾	131.161	1369.0	⅜	161.399	2073.0	61.	191.637	2922.5
¼	101.316	816.86	⅞	131.554	1377.2	½	161.792	2083.1	⅛	192.030	2934.5
⅜	101.709	823.21	42.	131.947	1385.4	⅝	162.185	2093.2	¼	192.423	2946.5
½	102.102	829.58	⅛	132.340	1393.7	¾	162.577	2103.3	⅜	192.815	2958.5
⅝	102.494	835.97	¼	132.732	1402.0	⅞	162.970	2113.5	½	193.208	2970.6
¾	102.887	842.39	⅜	133.125	1410.3	**52.**	163.363	2123.7	⅝	193.601	2982.7
⅞	103.280	848.83	½	133.518	1418.6	⅛	163.756	2133.9	¾	193.993	2994.8
33.	103.673	855.30	⅝	133.910	1427.0	¼	164.148	2144.2	⅞	194.386	3006.9
⅛	104.065	861.79	¾	134.303	1435.4	⅜	164.541	2154.5	62.	194.779	3019.1
¼	104.458	868.31	⅞	134.696	1443.8	½	164.934	2164.8	⅛	195.171	3031.3
⅜	104.851	874.85	43.	135.088	1452.2	⅝	165.326	2175.1	¼	195.564	3043.5
½	105.243	881.41	⅛	135.481	1460.7	¾	165.719	2185.4	⅜	195.957	3055.7
⅝	105.636	888.00	¼	135.874	1469.1	⅞	166.112	2195.8	½	196.350	3068.0
¾	106.029	894.62	⅜	136.267	1477.6	**53.**	166.504	2206.2	⅝	196.742	3080.3
⅞	106.421	901.26	½	136.659	1486.2	⅛	166.897	2216.6	¾	197.135	3092.6
34.	106.814	907.92	⅝	137.052	1494.7	¼	167.290	2227.0	⅞	197.528	3104.9
⅛	107.207	914.61	¾	137.445	1503.3	⅜	167.683	2237.5	63.	197.920	3117.2
¼	107.600	921.32	⅞	137.837	1511.9	½	168.075	2248.0	⅛	198.313	3129.6
⅜	107.992	928.06	44.	138.230	1520.5	⅝	168.468	2258.5	¼	198.706	3142.0
½	108.385	934.82	⅛	138.623	1529.2	¾	168.861	2269.1	⅜	199.098	3154.5
⅝	108.778	941.61	¼	139.015	1537.9	⅞	169.253	2279.6	½	199.491	3166.9
¾	109.170	948.42	⅜	139.408	1546.6	**54.**	169.646	2290.2	⅝	199.884	3179.4
⅞	109.563	955.25	½	139.801	1555.3	⅛	170.039	2300.8	¾	200.277	3191.9
35.	109.956	962.11	⅝	140.194	1564.0	¼	170.431	2311.5	⅞	200.669	3204.4
⅛	110.348	969.00	¾	140.586	1572.8	⅜	170.824	2322.1	64.	201.062	3217.0
¼	110.741	975.91	⅞	140.979	1581.6	½	171.217	2332.8	⅛	201.455	3229.6
⅜	111.134	982.84	45.	141.372	1590.4	⅝	171.609	2343.5	¼	201.847	3242.2
½	111.527	989.80	⅛	141.764	1599.3	¾	172.002	2354.3	⅜	202.240	3254.8
⅝	111.919	996.78	¼	142.157	1608.2	⅞	172.395	2365.0	½	202.633	3267.5
¾	112.312	1003.8	⅜	142.550	1617.0	55.	172.788	2375.8	⅝	203.025	3280.1
⅞	112.705	1010.8	½	142.942	1626.0	⅛	173.180	2386.6	¾	203.418	3292.8
36.	113.097	1017.9	⅝	143.335	1634.9	¼	173.573	2397.5	⅞	203.811	3305.6
⅛	113.490	1025.0	¾	143.728	1643.9	⅜	173.966	2408.3	65.	204.204	3318.3
¼	113.883	1032.1	⅞	144.121	1652.9	½	174.358	2419.2	⅛	204.596	3331.1
⅜	114.275	1039.2	46.	144.513	1661.9	⅝	174.751	2430.1	¼	204.989	3343.9
½	114.668	1046.3	⅛	144.906	1670.9	¾	175.144	2441.1	⅜	205.382	3356.7
⅝	115.061	1053.5	¼	145.299	1680.0	⅞	175.536	2452.0	½	205.774	3369.6
¾	115.454	1060.7	⅜	145.691	1689.1	56.	175.929	2463.0	⅝	206.167	3382.4
⅞	115.846	1068.0	½	146.084	1698.2	⅛	176.322	2474.0	¾	206.560	3395.3
37.	116.239	1075.2	⅝	146.477	1707.4	¼	176.715	2485.0	⅞	206.952	3408.2
⅛	116.632	1082.5	¾	146.869	1716.5	⅜	177.107	2496.1	66.	207.345	3421.2
¼	117.024	1089.8	⅞	147.262	1725.7	½	177.500	2507.2	⅛	207.738	3434.2
⅜	117.417	1097.1	47.	147.655	1734.9	⅝	177.893	2518.3	¼	208.131	3447.2
½	117.810	1104.5	⅛	148.048	1744.2	¾	178.285	2529.4	⅜	208.523	3460.2
⅝	118.202	1111.8	¼	148.440	1753.5	⅞	178.678	2540.6	½	208.916	3473.2
¾	118.596	1119.2	⅜	148.833	1762.7	57.	179.071	2551.8	⅝	209.309	3486.3
⅞	118.988	1126.7	½	149.226	1772.1	⅛	179.463	2563.0	¾	209.701	3499.4

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
⅞	210.094	3512.5	75¼	236.405	4447.4	83⅝	262.716	5492.4	92.	289.027	6647.6
67.	210.487	3525.7	⅜	236.798	4462.2	¾	263.108	5508.8	⅛	289.419	6665.7
⅛	210.879	3538.8	½	237.190	4477.0	⅞	263.501	5525.3	¼	289.812	6683.8
¼	211.272	3552.0	⅝	237.583	4491.8	84.	263.894	5541.8	⅜	290.205	6701.9
⅜	211.665	3565.2	¾	237.976	4506.7	⅛	264.286	5558.3	½	290.597	6720.1
½	212.058	3578.5	⅞	238.368	4521.5	¼	264.679	5574.8	⅝	290.990	6738.2
⅝	212.450	3591.7	76.	238.761	4536.5	⅜	265.072	5591.4	¾	291.383	6756.4
¾	212.843	3605.0	⅛	239.154	4551.4	½	265.465	5607.9	⅞	291.775	6774.7
⅞	213.236	3618.8	¼	239.546	4566.4	⅝	265.857	5624.5	93.	292.168	6792.9
68.	213.628	3631.7	⅜	239.939	4581.3	¾	266.250	5641.2	⅛	292.561	6811.2
⅛	214.021	3645.0	½	240.332	4596.3	⅞	266.643	5657.8	¼	292.954	6829.5
¼	214.414	3658.4	⅝	240.725	4611.4	85.	267.035	5674.5	⅜	293.346	6847.8
⅜	214.806	3671.8	¾	241.117	4626.4	⅛	267.428	5691.2	½	293.739	6866.1
½	215.199	3685.3	⅞	241.510	4641.5	¼	267.821	5707.9	⅝	294.132	6884.5
⅝	215.592	3698.7	77.	241.903	4656.6	⅜	268.213	5724.7	¾	294.524	6902.9
¾	215.984	3712.2	⅛	242.295	4671.8	½	268.606	5741.5	⅞	294.917	6921.3
⅞	216.377	3725.7	¼	242.688	4686.9	⅝	268.999	5758.3	94.	295.310	6939.8
69.	216.770	3739.3	⅜	243.081	4702.1	¾	269.392	5775.1	⅛	295.702	6958.2
⅛	217.163	3752.8	½	243.473	4717.3	⅞	269.784	5791.9	¼	296.095	6976.7
¼	217.555	3766.4	⅝	243.866	4732.5	86.	270.177	5808.8	⅜	296.488	6995.3
⅜	217.948	3780.0	¾	244.259	4747.8	⅛	270.570	5825.7	½	296.881	7013.8
½	218.341	3793.7	⅞	244.652	4763.1	¼	270.962	5842.6	⅝	297.273	7032.4
⅝	218.733	3807.3	78.	245.044	4778.4	⅜	271.355	5859.6	¾	297.666	7051.0
¾	219.126	3821.0	⅛	245.437	4793.7	½	271.748	5876.5	⅞	298.059	7069.6
⅞	219.519	3834.7	¼	245.830	4809.0	⅝	272.140	5893.5	95.	298.451	7088.2
70.	219.911	3848.5	⅜	246.222	4824.4	¾	272.533	5910.6	⅛	298.844	7106.9
⅛	220.304	3862.2	½	246.615	4839.8	⅞	272.926	5927.6	¼	299.237	7125.6
¼	220.697	3876.0	⅝	247.008	4855.2	87.	273.319	5944.7	⅜	299.629	7144.3
⅜	221.090	3889.8	¾	247.400	4870.7	⅛	273.711	5961.8	½	300.022	7163.0
½	221.482	3903.6	⅞	247.793	4886.2	¼	274.104	5978.9	⅝	300.415	7181.8
⅝	221.875	3917.5	79.	248.186	4901.7	⅜	274.497	5996.0	¾	300.807	7200.6
¾	222.268	3931.4	⅛	248.579	4917.2	½	274.889	6013.2	⅞	301.200	7219.4
⅞	222.660	3945.3	¼	248.971	4932.7	⅝	275.282	6030.4	96.	301.593	7238.2
71.	223.053	3959.2	⅜	249.364	4948.3	¾	275.675	6047.6	⅛	301.986	7257.1
⅛	223.446	3973.1	½	249.757	4963.9	⅞	276.067	6064.9	¼	302.378	7276.0
¼	223.838	3987.1	⅝	250.149	4979.5	88.	276.460	6082.1	⅜	302.771	7294.9
⅜	224.231	4001.1	¾	250.542	4995.2	⅛	276.853	6099.4	½	303.164	7313.8
½	224.624	4015.2	⅞	250.935	5010.9	¼	277.246	6116.7	⅝	303.556	7332.8
⅝	225.017	4029.2	80.	251.327	5026.5	⅜	277.638	6134.1	¾	303.949	7351.8
¾	225.409	4043.3	⅛	251.720	5042.3	½	278.031	6151.4	⅞	304.342	7370.8
⅞	225.802	4057.4	¼	252.113	5058.0	⅝	278.424	6168.8	97.	304.734	7389.8
72.	226.195	4071.5	⅜	252.506	5073.8	¾	278.816	6186.2	⅛	305.127	7408.9
⅛	226.587	4085.7	½	252.898	5089.6	⅞	279.209	6203.7	¼	305.520	7428.0
¼	226.980	4099.8	⅝	253.291	5105.4	89.	279.602	6221.1	⅜	305.913	7447.1
⅜	227.373	4114.0	¾	253.684	5121.2	⅛	279.994	6238.6	½	306.305	7466.2
½	227.765	4128.2	⅞	254.076	5137.1	¼	280.387	6256.1	⅝	306.698	7485.3
⅝	228.158	4142.5	81.	254.469	5153.0	⅜	280.780	6273.7	¾	307.091	7504.5
¾	228.551	4156.8	⅛	254.862	5168.9	½	281.173	6291.2	⅞	307.483	7523.7
⅞	228.944	4171.1	¼	255.254	5184.9	⅝	281.565	6308.8	98.	307.876	7543.0
73.	229.336	4185.4	⅜	255.647	5200.8	¾	281.958	6326.4	⅛	308.269	7562.2
⅛	229.729	4199.7	½	256.040	5216.8	⅞	282.351	6344.1	¼	308.661	7581.5
¼	230.122	4214.1	⅝	256.433	5232.8	90.	282.743	6361.7	⅜	309.054	7600.8
⅜	230.514	4228.5	¾	256.825	5248.9	⅛	283.136	6379.4	½	309.447	7620.1
½	230.907	4242.9	⅞	257.218	5264.9	¼	283.529	6397.1	⅝	309.840	7639.5
⅝	231.300	4257.4	82.	257.611	5281.0	⅜	283.921	6414.9	¾	310.232	7658.9
¾	231.692	4271.8	⅛	258.003	5297.1	½	284.314	6432.6	⅞	310.625	7678.3
⅞	232.085	4286.3	¼	258.396	5313.3	⅝	284.707	6450.4	99.	311.018	7697.7
74.	232.478	4300.8	⅜	258.789	5329.4	¾	285.100	6468.2	⅛	311.410	7717.1
⅛	232.871	4315.4	½	259.181	5345.6	⅞	285.492	6486.0	¼	311.803	7736.6
¼	233 263	4329.9	⅝	259.574	5361.8	91.	285.885	6503.9	⅜	312.196	7756.1
⅜	233.656	4344.5	¾	259.967	5378.1	⅛	286.278	6521.8	½	312.588	7775.6
½	234.049	4359.2	⅞	260.359	5394.3	¼	286.670	6539.7	⅝	312.981	7795.2
⅝	234.441	4373.8	83.	260.752	5410.6	⅜	287.063	6557.6	¾	313.374	7814.8
¾	234.834	4388.5	⅛	261.145	5426.9	½	287.456	6575.5	⅞	313.767	7834.4
⅞	235.227	4403.1	¼	261.538	5443.3	⅝	287.848	6593.5	100.	314.159	7854.0
75.	235.619	4417.9	⅜	261.930	5459.6	¾	288.241	6611.5			
⅛	236.012	4432 6	½	262.323	5476.0	⅞	288.634	6629.6			

TABLE 2 OF CIRCLES.

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
0.1	.314159	.007854	6.3	19.79203	31.17245	12.5	39.26991	122.7185
.2	.628319	.031416	.4	20.10619	32.16991	.6	39.58407	124.6898
.3	.942478	.070686	.5	20.42035	33.18307	.7	39.89823	126.6769
.4	1.256637	.125664	.6	20.73451	34.21194	.8	40.21239	128.6796
.5	1.570796	.196350	.7	21.04867	35.25652	.9	40.52655	130.6981
.6	1.884956	.282743	.8	21.36283	36.31681	13.0	40.84070	132.7323
.7	2.199115	.384845	.9	21.67699	37.39281	.1	41.15486	134.7822
.8	2.513274	.502655	7.0	21.99115	38.48451	.2	41.46902	136.8478
.9	2.827433	.636173	.1	22.30531	39.59192	.3	41.78318	138.9291
1.0	3.141593	.785398	.2	22.61947	40.71504	.4	42.09734	141.0261
.1	3.455752	.950332	.3	22.93363	41.85387	.5	42.41150	143.1388
.2	3.769911	1.13097	.4	23.24779	43.00840	.6	42.72566	145.2672
.3	4.084070	1.32732	.5	23.56194	44.17865	.7	43.03982	147.4114
.4	4.398230	1.53938	.6	23.87610	45.36460	.8	43.35398	149.5712
.5	4.712389	1.76715	.7	24.19026	46.56626	.9	43.66814	151.7468
.6	5.026548	2.01062	.8	24.50442	47.78362	14.0	43.98230	153.9380
.7	5.340708	2.26980	.9	24.81858	49.01670	.1	44.29646	156.1450
.8	5.654867	2.54469	8.0	25.13274	50.26548	.2	44.61062	158.3677
.9	5.969026	2.83529	.1	25.44690	51.52997	.3	44.92477	160.6061
2.0	6.283185	3.14159	.2	25.76106	52.81017	.4	45.23893	162.8602
.1	6.597345	3.46361	.3	26.07522	54.10608	.5	45.55309	165.1300
.2	6.911504	3.80133	.4	26.38938	55.41769	.6	45.86725	167.4155
.3	7.225663	4.15476	.5	26.70354	56.74502	.7	46.18141	169.7167
.4	7.539822	4.52389	.6	27.01770	58.08805	.8	46.49557	172.0336
.5	7.853982	4.90874	.7	27.33186	59.44679	.9	46.80973	174.3662
.6	8.168141	5.30929	.8	27.64602	60.82123	15.0	47.12389	176.7146
.7	8.482300	5.72555	.9	27.96017	62.21139	.1	47.43805	179.0786
.8	8.796459	6.15752	9.0	28.27433	63.61725	.2	47.75221	181.4584
.9	9.110619	6.60520	.1	28.58849	65.03882	.3	48.06637	183.8539
3.0	9.424778	7.06858	.2	28.90265	66.47610	.4	48.38053	186.2650
.1	9.738937	7.54768	.3	29.21681	67.92909	.5	48.69469	188.6919
.2	10.05310	8.04248	.4	29.53097	69.39778	.6	49.00885	191.1345
.3	10.36726	8.55299	.5	29.84513	70.88218	.7	49.32300	193.5928
.4	10.68142	9.07920	.6	30.15929	72.38229	.8	49.63716	196.0668
.5	10.99557	9.62113	.7	30.47345	73.89811	.9	49.95132	198.5565
.6	11.30973	10.17876	.8	30.78761	75.42964	16.0	50.26548	201.0619
.7	11.62389	10.75210	.9	31.10177	76.97687	.1	50.57964	203.5831
.8	11.93805	11.34115	10.0	31.41593	78.53982	.2	50.89380	206.1199
.9	12.25221	11.94591	.1	31.73009	80.11847	.3	51.20796	208.6724
4.0	12.56637	12.56637	.2	32.04425	81.71282	.4	51.52212	211.2407
.1	12.88053	13.20254	.3	32.35840	83.32289	.5	51.83628	213.8246
.2	13.19469	13.85442	.4	32.67256	84.94867	.6	52.15044	216.4243
.3	13.50885	14.52201	.5	32.98672	86.59015	.7	52.46460	219.0397
.4	13.82301	15.20531	.6	33.30088	88.24734	.8	52.77876	221.6708
.5	14.13717	15.90431	.7	33.61504	89.92024	.9	53.09292	224.3176
.6	14.45133	16.61903	.8	33.92920	91.60884	17.0	53.40708	226.9801
.7	14.76549	17.34945	.9	34.24336	93.31316	.1	53.72123	229.6583
.8	15.07964	18.09557	11.0	34.55752	95.03318	.2	54.03539	232.3522
.9	15.39380	18.85741	.1	34.87168	96.76891	.3	54.34955	235.0618
5.0	15.70796	19.63495	.2	35.18584	98.52035	.4	54.66371	237.7871
.1	16.02212	20.42821	.3	35.50000	100.2875	.5	54.97787	240.5282
.2	16.33628	21.23717	.4	35.81416	102.0703	.6	55.29203	243.2849
.3	16.65044	22.06183	.5	36.12832	103.8689	.7	55.60619	246.0574
.4	16.96460	22.90221	.6	36.44247	105.6832	.8	55.92035	248.8456
.5	17.27876	23.75829	.7	36.75663	107.5132	.9	56.23451	251.6494
.6	17.59292	24.63009	.8	37.07079	109.3588	18.0	56.54867	254.4690
.7	17.90708	25.51759	.9	37.38495	111.2202	.1	56.86283	257.3043
.8	18.22124	26.42079	12.0	37.69911	113.0973	.2	57.17699	260.1553
.9	18.53540	27.33971	.1	38.01327	114.9901	.3	57.49115	263.0220
6.0	18.84956	28.27433	.2	38.32743	116.8987	.4	57.80530	265.9044
.1	19.16372	29.22467	.3	38.64159	118.8229	.5	58.11946	268.8025
.2	19.47787	30.19071	.4	38.95575	120.7628	.6	58.43362	271.7163

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
18.7	58.74778	274.6459	24.9	78.22566	486.9547	31.1	97.70353	759.6450
.8	59.06194	277.5911	25.0	78.53982	490.8739	.2	98.01769	764.5380
.9	59.37610	280.5521	.1	78.85398	494.8087	.3	98.33185	769.4467
19.0	59.69026	283.5287	.2	79.16813	498.7592	.4	98.64601	774.3712
.1	60.00442	286.5211	.3	79.48229	502.7255	.5	98.96017	779.3113
.2	60.31858	289.5292	.4	79.79645	506.7075	.6	99.27433	784.2672
.3	60.63274	292.5530	.5	80.11061	510.7052	.7	99.58849	789.2388
.4	60.94690	295.5925	.6	80.42477	514.7185	.8	99.90265	794.2260
.5	61.26106	298.6477	.7	80.73893	518.7476	.9	100.2168	799.2290
.6	61.57522	301.7186	.8	81.05309	522.7924	32.0	100.5310	804.2477
.7	61.88938	304.8052	.9	81.36725	526.8529	.1	100.8451	809.2821
.8	62.20353	307.9075	26.0	81.68141	530.9292	.2	101.1593	814.3322
.9	62.51769	311.0255	.1	81.99557	535.0211	.3	101.4734	819.3980
20.0	62.83185	314.1593	.2	82.30973	539.1287	.4	101.7876	824.4796
.1	63.14601	317.3087	.3	82.62389	543.2521	.5	102.1018	829.5768
.2	63.46017	320.4739	.4	82.93805	547.3911	.6	102.4159	834.6896
.3	63.77433	323.6547	.5	83.25221	551.5459	.7	102.7301	839.8184
.4	64.08849	326.8513	.6	83.56636	555.7163	.8	103.0442	844.9628
.5	64.40265	330.0636	.7	83.88052	559.9025	.9	103.3584	850.1225
.6	64.71681	333.2916	.8	84.19468	564.1044	33.0	103.6726	855.2986
.7	65.03097	336.5353	.9	84.50884	568.3220	.1	103.9867	860.4901
.8	65.34513	339.7947	27.0	84.82300	572.5553	.2	104.3009	865.6973
.9	65.65929	343.0698	.1	85.13716	576.8043	.3	104.6150	870.9202
21.0	65.97345	346.3606	.2	85.45132	581.0690	.4	104.9292	876.1588
.1	66.28760	349.6671	.3	85.76548	585.3494	.5	105.2434	881.4131
.2	66.60176	352.9894	.4	86.07964	589.6455	.6	105.5575	886.6831
.3	66.91592	356.3273	.5	86.39380	593.9574	.7	105.8717	891.9688
.4	67.23008	359.6809	.6	86.70796	598.2849	.8	106.1858	897.2703
.5	67.54424	363.0503	.7	87.02212	602.6282	.9	106.5000	902.5874
.6	67.85840	366.4354	.8	87.33628	606.9871	34.0	106.8142	907.9203
.7	68.17256	369.8361	.9	87.65044	611.3618	.1	107.1283	913.2688
.8	68.48672	373.2526	28.0	87.96459	615.7522	.2	107.4425	918.6331
.9	68.80088	376.6848	.1	88.27875	620.1582	.3	107.7566	924.0131
22.0	69.11504	380.1327	.2	88.59291	624.5800	.4	108.0708	929.4088
.1	69.42920	383.5963	.3	88.90707	629.0175	.5	108.3849	934.8202
.2	69.74336	387.0756	.4	89.22123	633.4707	.6	108.6991	940.2473
.3	70.05752	390.5707	.5	89.53539	637.9397	.7	109.0133	945.6901
.4	70.37168	394.0814	.6	89.84955	642.4243	.8	109.3274	951.1486
.5	70.68583	397.6078	.7	90.16371	646.9246	.9	109.6416	956.6228
.6	70.99999	401.1500	.8	90.47787	651.4407	35.0	109.9557	962.1128
.7	71.31415	404.7078	.9	90.79203	655.9724	.1	110.2699	967.6184
.8	71.62831	408.2814	29.0	91.10619	660.5199	.2	110.5841	973.1397
.9	71.94247	411.8707	.1	91.42035	665.0830	.3	110.8982	978.6768
23.0	72.25663	415.4756	.2	91.73451	669.6619	.4	111.2124	984.2296
.1	72.57079	419.0963	.3	92.04866	674.2565	.5	111.5265	989.7980
.2	72.88495	422.7327	.4	92.36282	678.8668	.6	111.8407	995.3822
.3	73.19911	426.3848	.5	92.67698	683.4928	.7	112.1549	1000.9821
.4	73.51327	430.0526	.6	92.99114	688.1345	.8	112.4690	1006.5977
.5	73.82743	433.7361	.7	93.30530	692.7919	.9	112.7832	1012.2290
.6	74.14159	437.4354	.8	93.61946	697.4650	36.0	113.0973	1017.8760
.7	74.45575	441.1503	.9	93.93362	702.1538	.1	113.4115	1023.5387
.8	74.76991	444.8809	30.0	94.24778	706.8583	.2	113.7257	1029.2172
.9	75.08406	448.6273	.1	94.56194	711.5786	.3	114.0398	1034.9113
24.0	75.39822	452.3893	.2	94.87610	716.3145	.4	114.3540	1040.6212
.1	75.71238	456.1671	.3	95.19026	721.0662	.5	114.6681	1046.3467
.2	76.02654	459.9606	.4	95.50442	725.8336	.6	114.9823	1052.0880
.3	76.34070	463.7698	.5	95.81858	730.6166	.7	115.2965	1057.8449
.4	76.65486	467.5947	.6	96.13274	735.4154	.8	115.6106	1063.6176
.5	76.96902	471.4352	.7	96.44689	740.2299	.9	115.9248	1069.4060
.6	77.28318	475.2916	.8	96.76105	745.0601	37.0	116.2389	1075.2101
.7	77.59734	479.1636	.9	97.07521	749.9060	.1	116.5531	1081.0299
.8	77.91150	483.0513	31.0	97.38937	754.7676	.2	116.8672	1086.8654

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
37.3	117.1814	1092.7166	43.5	136.6593	1486.1697	49.7	156.1372	1940.0041
.4	117.4956	1098.5835	.6	136.9734	1493.0105	.8	156.4513	1947.8189
.5	117.8097	1104.4662	.7	137.2876	1499.8670	.9	156.7655	1955.6493
.6	118.1239	1110.3645	.8	137.6018	1506.7393	50.0	157.0796	1963.4954
.7	118.4380	1116.2786	.9	137.9159	1513.6272	.1	157.3938	1971.3572
.8	118.7522	1122.2083	44.0	138.2301	1520.5308	.2	157.7080	1979.2348
.9	119.0664	1128.1538	.1	138.5442	1527.4502	.3	158.0221	1987.1280
38.0	119.3805	1134.1149	.2	138.8584	1534.3853	.4	158.3363	1995.0370
.1	119.6947	1140.0918	.3	139.1726	1541.3360	.5	158.6504	2002.9617
.2	120.0088	1146.0844	.4	139.4867	1548.3025	.6	158.9646	2010.9020
.3	120.3230	1152.0927	.5	139.8009	1555.2847	.7	159.2787	2018.8581
.4	120.6372	1158.1167	.6	140.1150	1562.2826	.8	159.5929	2026.8299
.5	120.9513	1164.1564	.7	140.4292	1569.2962	.9	159.9071	2034.8174
.6	121.2655	1170.2118	.8	140.7434	1576.3255	51.0	160.2212	2042.8206
.7	121.5796	1176.2830	.9	141.0575	1583.3706	.1	160.5354	2050.8395
.8	121.8938	1182.3698	45.0	141.3717	1590.4313	.2	160.8495	2058.8742
.9	122.2080	1188.4724	.1	141.6858	1597.5077	.3	161.1637	2066.9245
39.0	122.5221	1194.5906	.2	142.0000	1604.5999	.4	161.4779	2074.9905
.1	122.8363	1200.7246	.3	142.3141	1611.7077	.5	161.7920	2083.0723
.2	123.1504	1206.8742	.4	142.6283	1618.8313	.6	162.1062	2091.1697
.3	123.4646	1213.0396	.5	142.9425	1625.9705	.7	162.4203	2099.2829
.4	123.7788	1219.2207	.6	143.2566	1633.1255	.8	162.7345	2107.4118
.5	124.0929	1225.4175	.7	143.5708	1640.2962	.9	163.0487	2115.5563
.6	124.4071	1231.6300	.8	143.8849	1647.4826	52.0	163.3628	2123.7166
.7	124.7212	1237.8582	.9	144.1991	1654.6847	.1	163.6770	2131.8926
.8	125.0354	1244.1021	46.0	144.5133	1661.9025	.2	163.9911	2140.0843
.9	125.3495	1250.3617	.1	144.8274	1669.1360	.3	164.3053	2148.2917
40.0	125.6637	1256.6371	.2	145.1416	1676.3853	.4	164.6195	2156.5149
.1	125.9779	1262.9281	.3	145.4557	1683.6502	.5	164.9336	2164.7537
.2	126.2920	1269.2348	.4	145.7699	1690.9308	.6	165.2478	2173.0082
.3	126.6062	1275.5573	.5	146.0841	1698.2272	.7	165.5619	2181.2785
.4	126.9203	1281.8955	.6	146.3982	1705.5392	.8	165.8761	2189.5644
.5	127.2345	1288.2493	.7	146.7124	1712.8670	.9	166.1903	2197.8661
.6	127.5487	1294.6189	.8	147.0265	1720.2105	53.0	166.5044	2206.1834
.7	127.8628	1301.0042	.9	147.3407	1727.5697	.1	166.8186	2214.5165
.8	128.1770	1307.4052	47.0	147.6549	1734.9445	.2	167.1327	2222.8653
.9	128.4911	1313.8219	.1	147.9690	1742.3351	.3	167.4469	2231.2298
41.0	128.8053	1320.2543	.2	148.2832	1749.7414	.4	167.7610	2239.6100
.1	129.1195	1326.7024	.3	148.5973	1757.1635	.5	168.0752	2248.0059
.2	129.4336	1333.1663	.4	148.9115	1764.6012	.6	168.3894	2256.4175
.3	129.7478	1339.6458	.5	149.2257	1772.0546	.7	168.7035	2264.8448
.4	130.0619	1346.1410	.6	149.5398	1779.5237	.8	169.0177	2273.2879
.5	130.3761	1352.6520	.7	149.8540	1787.0086	.9	169.3318	2281.7466
.6	130.6903	1359.1786	.8	150.1681	1794.5091	54.0	169.6460	2290.2210
.7	131.0044	1365.7210	.9	150.4823	1802.0254	.1	169.9602	2298.7112
.8	131.3186	1372.2791	48.0	150.7964	1809.5574	.2	170.2743	2307.2171
.9	131.6327	1378.8529	.1	151.1106	1817.1050	.3	170.5885	2315.7386
42.0	131.9469	1385.4424	.2	151.4248	1824.6684	.4	170.9026	2324.2759
.1	132.2611	1392.0476	.3	151.7389	1832.2475	.5	171.2168	2332.8289
.2	132.5752	1398.6685	.4	152.0531	1839.8423	.6	171.5310	2341.3976
.3	132.8894	1405.3051	.5	152.3672	1847.4528	.7	171.8451	2349.9820
.4	133.2035	1411.9574	.6	152.6814	1855.0790	.8	172.1593	2358.5821
.5	133.5177	1418.6254	.7	152.9956	1862.7210	.9	172.4734	2367.1979
.6	133.8318	1425.3092	.8	153.3097	1870.3786	55.0	172.7876	2375.8294
.7	134.1460	1432.0086	.9	153.6239	1878.0519	.1	173.1018	2384.4767
.8	134.4602	1438.7238	49.0	153.9380	1885.7410	.2	173.4159	2393.1396
.9	134.7743	1445.4546	.1	154.2522	1893.4457	.3	173.7301	2401.8183
43.0	135.0885	1452.2012	.2	154.5664	1901.1662	.4	174.0442	2410.5126
.1	135.4026	1458.9635	.3	154.8805	1908.9024	.5	174.3584	2419.2227
.2	135.7168	1465.7415	.4	155.1947	1916.6543	.6	174.6726	2427.9485
.3	136.0310	1472.5352	.5	155.5088	1924.4218	.7	174.9867	2436.6899
.4	136.3451	1479.3446	.6	155.8230	1932.2051	.8	175.3009	2445.4471

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
55.9	175.6150	2454.2200	62.1	195.0929	3028.8173	68.3	214.5708	3663.7960
56.0	175.9292	2463.0086	.2	195.4071	3038.5798	.4	214.8849	3674.5324
.1	176.2433	2471.8130	.3	195.7212	3048.3580	.5	215.1991	3685.2845
.2	176.5575	2480.6330	.4	196.0354	3058.1520	.6	215.5133	3696.0523
.3	176.8717	2489.4687	.5	196.3495	3067.9616	.7	215.8274	3706.8359
.4	177.1858	2498.3201	.6	196.6637	3077.7869	.8	216.1416	3717.6351
.5	177.5000	2507.1873	.7	196.9779	3087.6279	.9	216.4557	3728.4500
.6	177.8141	2516.0701	.8	197.2920	3097.4847	69.0	216.7699	3739.2807
.7	178.1283	2524.9687	.9	197.6062	3107.3571	.1	217.0841	3750.1270
.8	178.4425	2533.8830	63.0	197.9203	3117.2453	.2	217.3982	3760.9891
.9	178.7566	2542.8129	.1	198.2345	3127.1492	.3	217.7124	3771.8668
57.0	179.0708	2551.7586	.2	198.5487	3137.0688	.4	218.0265	3782.7603
.1	179.3849	2560.7200	.3	198.8628	3147.0040	.5	218.3407	3793.6695
.2	179.6991	2569.6971	.4	199.1770	3156.9550	.6	218.6548	3804.5944
.3	180.0133	2578.6899	.5	199.4911	3166.9217	.7	218.9690	3815.5350
.4	180.3274	2587.6985	.6	199.8053	3176.9042	.8	219.2832	3826.4918
.5	180.6416	2596.7227	.7	200.1195	3186.9023	.9	219.5973	3837.4633
.6	180.9557	2605.7626	.8	200.4336	3196.9161	70.0	219.9115	3848.4510
.7	181.2699	2614.8183	.9	200.7478	3206.9456	.1	220.2256	3859.4544
.8	181.5841	2623.8896	64.0	201.0619	3216.9909	.2	220.5398	3870.4736
.9	181.8982	2632.9767	.1	201.3761	3227.0518	.3	220.8540	3881.5084
58.0	182.2124	2642.0794	.2	201.6902	3237.1285	.4	221.1681	3892.5590
.1	182.5265	2651.1979	.3	202.0044	3247.2209	.5	221.4823	3903.6252
.2	182.8407	2660.3321	.4	202.3186	3257.3289	.6	221.7964	3914.7072
.3	183.1549	2669.4820	.5	202.6327	3267.4527	.7	222.1106	3925.8049
.4	183.4690	2678.6476	.6	202.9469	3277.5922	.8	222.4248	3936.9182
.5	183.7832	2687.8289	.7	203.2610	3287.7474	.9	222.7389	3948.0473
.6	184.0973	2697.0259	.8	203.5752	3297.9183	71.0	223.0531	3959.1921
.7	184.4115	2706.2386	.9	203.8894	3308.1049	.1	223.3672	3970.3526
.8	184.7256	2715.4670	65.0	204.2035	3318.3072	.2	223.6814	3981.5289
.9	185.0398	2724.7112	.1	204.5177	3328.5253	.3	223.9956	3992.7208
59.0	185.3540	2733.9710	.2	204.8318	3338.7590	.4	224.3097	4003.9284
.1	185.6681	2743.2466	.3	205.1460	3349.0085	.5	224.6239	4015.1518
.2	185.9823	2752.5378	.4	205.4602	3359.2736	.6	224.9380	4026.3908
.3	186.2964	2761.8448	.5	205.7743	3369.5545	.7	225.2522	4037.6456
.4	186.6106	2771.1675	.6	206.0885	3379.8510	.8	225.5664	4048.9160
.5	186.9248	2780.5058	.7	206.4026	3390.1633	.9	225.8805	4060.2022
.6	187.2389	2789.8599	.8	206.7168	3400.4913	72.0	226.1947	4071.5041
.7	187.5531	2799.2297	.9	207.0310	3410.8350	.1	226.5088	4082.8217
.8	187.8672	2808.6152	66.0	207.3451	3421.1944	.2	226.8230	4094.1550
.9	188.1814	2818.0165	.1	207.6593	3431.5695	.3	227.1371	4105.5040
60.0	188.4956	2827.4334	.2	207.9734	3441.9603	.4	227.4513	4116.8687
.1	188.8097	2836.8660	.3	208.2876	3452.3669	.5	227.7655	4128.2491
.2	189.1239	2846.3144	.4	208.6018	3462.7891	.6	228.0796	4139.6452
.3	189.4380	2855.7784	.5	208.9159	3473.2270	.7	228.3938	4151.0571
.4	189.7522	2865.2582	.6	209.2301	3483.6807	.8	228.7079	4162.4846
.5	190.0664	2874.7536	.7	209.5442	3494.1500	.9	229.0221	4173.9279
.6	190.3805	2884.2648	.8	209.8584	3504.6351	73.0	229.3363	4185.3868
.7	190.6947	2893.7917	.9	210.1725	3515.1359	.1	229.6504	4196.8615
.8	191.0088	2903.3343	67.0	210.4867	3525.6524	.2	229.9646	4208.3519
.9	191.3230	2912.8926	.1	210.8009	3536.1845	.3	230.2787	4219.8579
61.0	191.6372	2922.4666	.2	211.1150	3546.7824	.4	230.5929	4231.3797
.1	191.9513	2932.0563	.3	211.4292	3557.2960	.5	230.9071	4242.9172
.2	192.2655	2941.6617	.4	211.7433	3567.8754	.6	231.2212	4254.4704
.3	192.5796	2951.2828	.5	212.0575	3578.4704	.7	231.5354	4266.0394
.4	192.8938	2960.9197	.6	212.3717	3589.0811	.8	231.8495	4277.6240
.5	193.2079	2970.5722	.7	212.6858	3599.7075	.9	232.1637	4289.2243
.6	193.5221	2980.2405	.8	213.0000	3610.3497	74.0	232.4779	4300.8403
.7	193.8363	2989.9244	.9	213.3141	3621.0075	.1	232.7920	4312.4721
.8	194.1504	2999.6241	68.0	213.6283	3631.6811	.2	233.1062	4324.1195
.9	194.4646	3009.3395	.1	213.9425	3642.3704	.3	233.4203	4335.7827
62.0	194.7787	3019.0705	.2	214.2566	3653.0754	.4	233.7345	4347.4616

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
74.5	234.0487	4359.1562	80.7	253.5265	5114.8977	86.9	273.0044	5931.0206
.6	234.3628	4370.8664	.8	253.8407	5127.5819	87.0	273.3186	5944.6787
.7	234.6770	4382.5924	.9	254.1548	5140.2818	.1	273.6327	5958.3525
.8	234.9911	4394.3341	81.0	254.4690	5152.9974	.2	273.9469	5972.0420
.9	235.3053	4406.0916	.1	254.7832	5165.7287	.3	274.2610	5985.7472
75.0	235.6194	4417.8647	.2	255.0973	5178.4757	.4	274.5752	5999.4681
.1	235.9336	4429.6535	.3	255.4115	5191.2384	.5	274.8894	6013.2047
.2	236.2478	4441.4580	.4	255.7256	5204.0168	.6	275.2035	6026.9570
.3	236.5619	4453.2783	.5	256.0398	5216.8110	.7	275.5177	6040.7250
.4	236.8761	4465.1142	.6	256.3540	5229.6208	.8	275.8318	6054.5088
.5	237.1902	4476.9659	.7	256.6681	5242.4463	.9	276.1460	6068.3082
.6	237.5044	4488.8332	.8	256.9823	5255.2876	88.0	276.4602	6082.1234
.7	237.8186	4500.7163	.9	257.2964	5268.1446	.1	276.7743	6095.9542
.8	238.1327	4512.6151	82.0	257.6106	5281.0173	.2	277.0885	6109.8008
.9	238.4469	4524.5296	.1	257.9248	5293.9056	.3	277.4026	6123.6631
76.0	238.7610	4536.4598	.2	258.2389	5306.8097	.4	277.7168	6137.5411
.1	239.0752	4548.4057	.3	258.5531	5319.7295	.5	278.0309	6151.4348
.2	239.3894	4560.3673	.4	258.8672	5332.6650	.6	278.3451	6165.3442
.3	239.7035	4572.3446	.5	259.1814	5345.6162	.7	278.6593	6179.2693
.4	240.0177	4584.3377	.6	259.4956	5358.5832	.8	278.9734	6193.2101
.5	240.3318	4596.3464	.7	259.8097	5371.5658	.9	279.2876	6207.1666
.6	240.6460	4608.3708	.8	260.1239	5384.5641	89.0	279.6017	6221.1389
.7	240.9602	4620.4110	.9	260.4380	5397.5782	.1	279.9159	6235.1268
.8	241.2743	4632.4669	83.0	260.7522	5410.6079	.2	280.2301	6249.1304
.9	241.5885	4644.5384	.1	261.0663	5423.6534	.3	280.5442	6263.1498
77.0	241.9026	4656.6257	.2	261.3805	5436.7146	.4	280.8584	6277.1849
.1	242.2168	4668.7287	.3	261.6947	5449.7915	.5	281.1725	6291.2356
.2	242.5310	4680.8474	.4	262.0088	5462.8840	.6	281.4867	6305.3021
.3	242.8451	4692.9818	.5	262.3230	5475.9923	.7	281.8009	6319.3843
.4	243.1593	4705.1319	.6	262.6371	5489.1163	.8	282.1150	6333.4822
.5	243.4734	4717.2977	.7	262.9513	5502.2561	.9	282.4292	6347.5958
.6	243.7876	4729.4792	.8	263.2655	5515.4115	90.0	282.7433	6361.7251
.7	244.1017	4741.6765	.9	263.5796	5528.5826	.1	283.0575	6375.8701
.8	244.4159	4753.8894	84.0	263.8938	5541.7694	.2	283.3717	6390.0309
.9	244.7301	4766.1181	.1	264.2079	5554.9720	.3	283.6858	6404.2073
78.0	245.0442	4778.3624	.2	264.5221	5568.1902	.4	284.0000	6418.3995
.1	245.3584	4790.6225	.3	264.8363	5581.4242	.5	284.3141	6432.6073
.2	245.6725	4802.8983	.4	265.1504	5594.6739	.6	284.6283	6446.8309
.3	245.9867	4815.1897	.5	265.4646	5607.9392	.7	284.9425	6461.0701
.4	246.3009	4827.4969	.6	265.7787	5621.2203	.8	285.2566	6475.3251
.5	246.6150	4839.8198	.7	266.0929	5634.5171	.9	285.5708	6489.5958
.6	246.9292	4852.1584	.8	266.4071	5647.8296	91.0	285.8849	6503.8822
.7	247.2433	4864.5128	.9	266.7212	5661.1578	.1	286.1991	6518.1843
.8	247.5575	4876.8828	85.0	267.0354	5674.5017	.2	286.5133	6532.5021
.9	247.8717	4889.2685	.1	267.3495	5687.8614	.3	286.8274	6546.8356
79.0	248.1858	4901.6699	.2	267.6637	5701.2367	.4	287.1416	6561.1848
.1	248.5000	4914.0871	.3	267.9779	5714.6277	.5	287.4557	6575.5498
.2	248.8141	4926.5199	.4	268.2920	5728.0345	.6	287.7699	6589.9304
.3	249.1283	4938.9685	.5	268.6062	5741.4569	.7	288.0840	6604.3268
.4	249.4425	4951.4328	.6	268.9203	5754.8951	.8	288.3982	6618.7388
.5	249.7566	4963.9127	.7	269.2345	5768.3490	.9	288.7124	6633.1666
.6	250.0708	4976.4084	.8	269.5486	5781.8185	92.0	289.0265	6647.6101
.7	250.3849	4988.9198	.9	269.8628	5795.3038	.1	289.3407	6662.0692
.8	250.6991	5001.4469	86.0	270.1770	5808.8048	.2	289.6548	6676.5441
.9	251.0133	5013.9897	.1	270.4911	5822.3215	.3	289.9690	6691.0347
80.0	251.3274	5026.5482	.2	270.8053	5835.8539	.4	290.2832	6705.5410
.1	251.6416	5039.1225	.3	271.1194	5849.4020	.5	290.5973	6720.0630
.2	251.9557	5051.7124	.4	271.4336	5862.9659	.6	290.9115	6734.6008
.3	252.2699	5064.3180	.5	271.7478	5876.5454	.7	291.2256	6749.1542
.4	252.5840	5076.9394	.6	272.0619	5890.1407	.8	291.5398	6763.7233
.5	252.8982	5089.5764	.7	272.3761	5903.7516	.9	291.8540	6778.3082
.6	253.2124	5102.2292	.8	272.6902	5917.3783	93.0	292.1681	6792.9087

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
93.1	292.4823	6807.5250	95.5	300.0221	7163.0276	97.8	307.2478	7512.2078
.2	292.7964	6822.1569	.6	300.3363	7178.0366	.9	307.5619	7527.5780
.3	293.1106	6836.8046	.7	300.6504	7193.0612	98.0	307.8761	7542.9640
.4	293.4248	6851.4680	.8	300.9646	7208.1016	.1	308.1902	7558.3656
.5	293.7389	6866.1471	.9	301.2787	7223.1577	.2	308.5044	7573.7830
.6	294.0531	6880.8419	96.0	301.5929	7238.2295	.3	308.8186	7589.2161
.7	294.3672	6895.5524	.1	301.9071	7253.3170	.4	309.1327	7604.6648
.8	294.6814	6910.2786	.2	302.2212	7268.4202	.5	309.4469	7620.1293
.9	294.9956	6925.0205	.3	302.5354	7283.5391	.6	309.7610	7635.6095
94.0	295.3097	6939.7782	.4	302.8495	7298.6737	.7	310.0752	7651.1054
.1	295.6239	6954.5515	.5	303.1637	7313.8240	.8	310.3894	7666.6170
.2	295.9380	6969.3406	.6	303.4779	7328.9901	.9	310.7035	7682.1444
.3	296.2522	6984.1453	.7	303.7920	7344.1718	99.0	311.0177	7697.6874
.4	296.5663	6998.9658	.8	304.1062	7359.3693	.1	311.3318	7713.2461
.5	296.8805	7013.8019	.9	304.4203	7374.5824	.2	311.6460	7728.8206
.6	297.1947	7028.6538	97.0	304.7345	7389.8113	.3	311.9602	7744.4107
.7	297.5088	7043.5214	.1	305.0486	7405.0559	.4	312.2743	7760.0166
.8	297.8230	7058.4047	.2	305.3628	7420.3162	.5	312.5885	7775.6382
.9	298.1371	7073.3037	.3	305.6770	7435.5922	.6	312.9026	7791.2754
95.0	298.4513	7088.2184	.4	305.9911	7450.8839	.7	313.2168	7806.9284
.1	298.7655	7103.1488	.5	306.3053	7466.1913	.8	313.5309	7822.5971
.2	299.0796	7118.0950	.6	306.6194	7481.5144	.9	313.8451	7838.2815
.3	299.3938	7133.0568	.7	306.9336	7496.8532	100.0	314.1593	7853.9816
.4	299.7079	7148.0343						

Circumferences when the diameter has more than one place of decimals.

Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.
.1	.314159	.01	.031416	.001	.003142	.0001	.000314	.00001	.000031
.2	.628319	.02	.062832	.002	.006283	.0002	.000628	.00002	.000063
.3	.942478	.03	.094248	.003	.009425	.0003	.000942	.00003	.000094
.4	1.256637	.04	.125664	.004	.012566	.0004	.001257	.00004	.000126
.5	1.570796	.05	.157080	.005	.015708	.0005	.001571	.00005	.000157
.6	1.884956	.06	.188496	.006	.018850	.0006	.001885	.00006	.000188
.7	2.199115	.07	.219911	.007	.021991	.0007	.002199	.00007	.000220
.8	2.513274	.08	.251327	.008	.025133	.0008	.002513	.00008	.000251
.9	2.827433	.09	.282743	.009	.028274	.0009	.002827	.00009	.000283

Examples.

Diameter = 3.12699

Circumference = Sum of

Circ for dia of	3.1	=	9.738937
"	.02	=	.062832
"	.006	=	.018850
"	.0009	=	.002827
"	.00009	=	.000283
			9.823729

Circumfce = 9.823729

Diameter = Sum of

Dia for circ of	9.738937	=	3.1
	.084792		
"	.062832	=	.02
	.021960		
"	.018850	=	.006
	.003110		
"	.002827	=	.0009
	.000283		
"	.000283	=	.00009
			3.12699

TABLE 3 OF CIRCLES.

Diams in units and twelfths; as in feet and inches.

Dia.		Circumf.	Area.	Dia.		Circumf.	Area.	Dia.		Circumf.	Area.
Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.
				5	0	15.70796	19.63495	10	0	31.41593	78.53982
0	1	.261799	.005454		1	15.96976	20.29491		1	31.67773	79.85427
	2	.523599	.021817		2	16.23156	20.96577		2	31.93953	81.17963
	3	.785398	.049087		3	16.49336	21.64754		3	32.20132	82.51589
	4	1.047198	.087266		4	16.75516	22.34021		4	32.46312	83.86307
	5	1.308997	.136354		5	17.01696	23.04380		5	32.72492	85.22115
	6	1.570796	.196350		6	17.27876	23.75829		6	32.98672	86.59015
	7	1.832596	.267254		7	17.54056	24.48370		7	33.24852	87.97005
	8	2.094395	.349066		8	17.80236	25.22001		8	33.51032	89.36086
	9	2.356195	·441786		9	18.06416	25.96723		9	33.77212	90.76258
	10	2.617994	.545415		10	18.32596	26.72535		10	34.03392	92.17520
	11	2.879793	.659953		11	18.58776	27.49439		11	34.29572	93.59874
1	0	3.14159	.785398	6	0	18.84956	28.27433	11	0	34.55752	95.03318
	1	3.40339	.921752		1	19.11136	29.06519		1	34.81932	96.47853
	2	3.66519	1.06901		2	19.37315	29.86695		2	35.08112	97.93479
	3	3.92699	1.22718		3	19.63495	30.67962		3	35.34292	99.40196
	4	4.18879	1.39626		4	19.89675	31.50319		4	35.60472	100.8800
	5	4.45059	1.57625		5	20.15855	32.33768		5	35.86652	102.3690
	6	4.71239	1.76715		6	20.42035	33.18307		6	36.12832	103.8689
	7	4.97419	1.96895		7	20.68215	34.03937		7	36.39011	105.3797
	8	5.23599	2.18166		8	20.94395	34.90659		8	36.65191	106.9014
	9	5.49779	2.40528		9	21.20575	35.78470		9	36.91371	108.4340
	10	5.75959	2.63981		10	21.46755	36.67373		10	37.17551	109.9776
	11	6.02139	2.88525		11	21.72935	37.57367		11	37.43731	111.5320
2	0	6.28319	3.14159	7	0	21.99115	38.48451	12	0	37.69911	113.0973
	1	6.54498	3.40885		1	22.25295	39.40626		1	37.96091	114.6736
	2	6.80678	3.68701		2	22.51475	40.33892		2	38.22271	116.2607
	3	7.06858	3.97608		3	22.77655	41.28249		3	38.48451	117.8588
	4	7.33038	4.27606		4	23.03835	42.23697		4	38.74631	119.4678
	5	7.59218	4.58694		5	23.30015	43.20235		5	39.00811	121.0877
	6	7.85398	4.90874		6	23.56194	44.17865		6	39.26991	122.7185
	7	8.11578	5.24144		7	23.82374	45.16585		7	39.53171	124.3602
	8	8.37758	5.58505		8	24.08554	46.16396		8	39.79351	126.0128
	9	8.63938	5.93957		9	24.34734	47.17298		9	40.05531	127.6763
	10	8.90118	6.30500		10	24.60914	48.19290		10	40.31711	129.3507
	11	9.16298	6.68134		11	24.87094	49.22374		11	40.57891	131.0360
3	0	9.42478	7.06858	8	0	25.13274	50.26548	13	0	40.84070	132.7323
	1	9.68658	7.46674		1	25.39454	51.31813		1	41.10250	134.4394
	2	9.94838	7.87580		2	25.65634	52.38169		2	41.36430	136.1575
	3	10.21018	8.29577		3	25.91814	53.45616		3	41.62610	137.8865
	4	10.47198	8.72665		4	26.17994	54.54154		4	41.88790	139.6263
	5	10.73377	9.16843		5	26.44174	55.63782		5	42.14970	141.3771
	6	10.99557	9.62113		6	26.70354	56.74502		6	42.41150	143.1388
	7	11.25737	10.08473		7	26.96534	57.86312		7	42.67330	144.9114
	8	11.51917	10.55924		8	27.22714	58.99213		8	42.93510	146.6949
	9	11.78097	11.04466		9	27.48894	60.13205		9	43.19690	148.4893
	10	12.04277	11.54099		10	27.75074	61.28287		10	43.45870	150.2947
	11	12.30457	12.04823		11	28.01253	62.44461		11	43.72050	152.1109
4	0	12.56637	12.56637	9	0	28.27433	63.61725	14	0	43.98230	153.9380
	1	12.82817	13.09542		1	28.53613	64.80080		1	44.24410	155.7761
	2	13.08997	13.63538		2	28.79793	65.99526		2	44.50590	157.6250
	3	13.35177	14.18625		3	29 05973	67.20063		3	44.76770	159.4849
	4	13.61357	14.74803		4	29.32153	68.41691		4	45.02949	161.3557
	5	13.87537	15.32072		5	29.58333	69.64409		5	45.29129	163.2374
	6	14.13717	15.90431		6	29.84513	70.88218		6	45.55309	165.1300
	7	14.39897	16.49882		7	30.10693	72.13119		7	45.81489	167.0335
	8	14.66077	17.10423		8	30.36873	73.39110		8	46.07669	168.9479
	9	14.92257	17.72055		9	30.63053	74.66191		9	46.33849	170.8732
	10	15.18436	18.34777		10	30.89233	75.94364		10	46.60029	172.8094
	11	15.44616	18.98591		11	31.15413	77.23627		11	46.86209	174.7565

TABLE 8 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
15 0	47.12389	176.7146	20 0	62.83185	314.1593	25 0	78.53982	490.8739
1	47.38569	178.6835	1	63.09365	316.7827	1	78.80162	494.1518
2	47.64749	180.6634	2	63.35545	319.4171	2	79.06342	497.4407
3	47.90929	182.6542	3	63.61725	322.0623	3	79.32521	500.7404
4	48.17109	184.6558	4	63.87905	324.7185	4	79.58701	504.0511
5	48.43289	186.6684	5	64.14085	327.3856	5	79.84881	507.3727
6	48.69469	188.6919	6	64.40265	330.0636	6	80.11061	510.7052
7	48.95649	190.7263	7	64.66445	332.7525	7	80.37241	514.0486
8	49.21828	192.7716	8	64.92625	335.4523	8	80.63421	517.4029
9	49.48008	194.8278	9	65.18805	338.1630	9	80.89601	520.7681
10	49.74188	196.8950	10	65.44985	340.8846	10	81.15781	524.1442
11	50.00368	198.9730	11	65.71165	343.6172	11	81.41961	527.5312
16 0	50.26548	201.0619	21 0	65.97345	346.3606	26 0	81.68141	530.9292
1	50.52728	203.1618	1	66.23525	349.1149	1	81.94321	534.3380
2	50.78908	205.2725	2	66.49704	351.8802	2	82.20501	537.7578
3	51.05088	207.3942	3	66.75884	354.6564	3	82.46681	541.1884
4	51.31268	209.5268	4	67.02064	357.4434	4	82.72861	544.6300
5	51.57448	211.6703	5	67.28244	360.2414	5	82.99041	548.0825
6	51.83628	213.8246	6	67.54424	363.0503	6	83.25221	551.5459
7	52.09808	215.9899	7	67.80604	365.8701	7	83.51400	555.0202
8	52.35988	218.1662	8	68.06784	368.7008	8	83.77580	558.5054
9	52.62168	220.3533	9	68.32964	371.5424	9	84.03760	562.0015
10	52.88348	222.5513	10	68.59144	374.3949	10	84.29940	565.5085
11	53.14528	224.7602	11	68.85324	377.2584	11	84.56120	569.0264
17 0	53.40708	226.9801	22 0	69.11504	380.1327	27 0	84.82300	572.5553
1	53.66887	229.2108	1	69.37684	383.0180	1	85.08480	576.0950
2	53.93067	231.4525	2	69.63864	385.9141	2	85.34660	579.6457
3	54.19247	233.7050	3	69.90044	388.8212	3	85.60840	583.2072
4	54.45427	235.9685	4	70.16224	391.7392	4	85.87020	586.7797
5	54.71607	238.2429	5	70.42404	394.6680	5	86.13200	590.3631
6	54.97787	240.5282	6	70.68583	397.6078	6	86.39380	593.9574
7	55.23967	242.8244	7	70.94763	400.5585	7	86.65560	597.5626
8	55.50147	245.1315	8	71.20943	403.5201	8	86.91740	601.1787
9	55.76327	247.4495	9	71.47123	406.4926	9	87.17920	604.8057
10	56.02507	249.7784	10	71.73303	409.4761	10	87.44100	608.4436
11	56.28687	252.1183	11	71.99483	412.4704	11	87.70279	612.0924
18 0	56.54867	254.4690	23 0	72.25663	415.4756	28 0	87.96459	615.7522
1	56.81047	256.8307	1	72.51843	418.4918	1	88.22639	619.4228
2	57.07227	259.2032	2	72.78023	421.5188	2	88.48819	623.1044
3	57.33407	261.5867	3	73.04203	424.5568	3	88.74999	626.7968
4	57.59587	263.9810	4	73.30383	427.6057	4	89.01179	630.5002
5	57.85766	266.3863	5	73.56563	430.6654	5	89.27359	634.2145
6	58.11946	268.8025	6	73.82743	433.7361	6	89.53539	637.9397
7	58.38126	271.2296	7	74.08923	436.8177	7	89.79719	641.6758
8	58.64306	273.6676	8	74.35103	439.9102	8	90.05899	645.4228
9	58.90486	276.1165	9	74.61283	443.0137	9	90.32079	649.1807
10	59.16666	278.5764	10	74.87462	446.1280	10	90.58259	652.9495
11	59.42846	281.0471	11	75.13642	449.2532	11	90.84439	656.7292
19 0	59.69026	283.5287	24 0	75.39822	452.3893	29 0	91.10619	660.5199
1	59.95206	286.0213	1	75.66002	455.5364	1	91.36799	664.3214
2	60.21386	288.5247	2	75.92182	458.6943	2	91.62979	668.1339
3	60.47566	291.0391	3	76.18362	461.8632	3	91.89159	671.9572
4	60.73746	293.5644	4	76.44542	465.0430	4	92.15338	675.7915
5	60.99926	296.1006	5	76.70722	468.2337	5	92.41518	679.6367
6	61.26106	298.6477	6	76.96902	471.4352	6	92.67698	683.4928
7	61.52286	301.2056	7	77.23082	474.6477	7	92.93878	687.3597
8	61.78466	303.7746	8	77.49262	477.8711	8	93.20058	691.2377
9	62.04645	306.3544	9	77.75442	481.1055	9	93.46238	695.1265
10	62.30825	308.9451	10	78.01622	484.3507	10	93.72418	699.0262
11	62.57005	311.5467	11	78.27802	487.6068	11	93.98598	702.9368

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft. In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
30 0	94.24778	706.8583	**35** 0	109.9557	962.1128	**40** 0	125.6637	1256.6371
1	94.50958	710.7908	1	110.2175	966.6997	1	125.9255	1261.8785
2	94.77138	714.7841	2	110.4793	971.2975	2	126.1873	1267.1309
3	95.03318	718.6884	3	110.7411	975.9063	3	126.4491	1272.3941
4	95.29498	722.6536	4	111.0029	980.5260	4	126.7109	1277.6683
5	95.55678	726.6297	5	111.2647	985.1566	5	126.9727	1282.9534
6	95.81858	730.6166	6	111.5265	989.7980	6	127.2345	1288.2493
7	96.08038	734.6145	7	111.7883	994.4504	7	127.4963	1293.5562
8	96.34217	738.6233	8	112.0501	999.1137	8	127.7581	1298.8740
9	96.60397	742.6431	9	112.3119	1003.7879	9	128.0199	1304.2027
10	96.86577	746.6737	10	112.5737	1008.4731	10	128.2817	1309.5424
11	97.12757	750.7152	11	112.8355	1013.1691	11	128.5435	1314.8929
31 0	97.38937	754.7676	**36** 0	113.0973	1017.8760	**41** 0	128.8053	1320.2543
1	97.65117	758.8310	1	113.3591	1022.5939	1	129.0671	1325.6267
2	97.91297	762.9052	2	113.6209	1027.3226	2	129.3289	1331.0099
3	98.17477	766.9904	3	113.8827	1032.0623	3	129.5907	1336.4041
4	98.43657	771.0865	4	114.1445	1036.8128	4	129.8525	1341.8091
5	98.69837	775.1934	5	114.4063	1041.5743	5	130.1143	1347.2251
6	98.96017	779.3113	6	114.6681	1046.3467	6	130.3761	1352.6520
7	99.22197	783.4401	7	114.9299	1051.1300	7	130.6379	1358.0898
8	99.48377	787.5798	8	115.1917	1055.9242	8	130.8997	1363.5385
9	99.74557	791.7304	9	115.4535	1060.7293	9	131.1615	1368.9981
10	100.0074	795.8920	10	115.7153	1065.5453	10	131.4233	1374.4686
11	100.2692	800.0644	11	115.9771	1070.3723	11	131.6851	1379.9500
32 0	100.5310	804.2477	**37** 0	116.2389	1075.2101	**42** 0	131.9469	1385.4424
1	100.7928	808.4420	1	116.5007	1080.0588	1	132.2087	1390.9456
2	101.0546	812.6471	2	116.7625	1084.9185	2	132.4705	1396.4598
3	101.3164	816.8632	3	117.0243	1089.7890	3	132.7323	1401.9848
4	101.5782	821.0901	4	117.2861	1094.6705	4	132.9941	1407.5208
5	101.8400	825.3280	5	117.5479	1099.5629	5	133.2559	1413.0676
6	102.1018	829.5768	6	117.8097	1104.4662	6	133.5177	1418.6254
7	102.3636	833.8365	7	118.0715	1109.3804	7	133.7795	1424.1941
8	102.6254	838.1071	8	118.3333	1114.3055	8	134.0413	1429.7737
9	102.8872	842.3886	9	118.5951	1119.2415	9	134.3031	1435.3642
10	103.1490	846.6810	10	118.8569	1124.1884	10	134.5649	1440.9656
11	103.4108	850.9844	11	119.1187	1129.1462	11	134.8267	1446.5780
33 0	103.6726	855.2986	**38** 0	119.3805	1134.1149	**43** 0	135.0885	1452.2012
1	103.9344	859.6237	1	119.6423	1139.0946	1	135.3503	1457.8353
2	104.1962	863.9598	2	119.9041	1144.0851	2	135.6121	1463.4804
3	104.4580	868.3068	3	120.1659	1149.0866	3	135.8739	1469.1364
4	104.7198	872.6646	4	120.4277	1154.0990	4	136.1357	1474.8032
5	104.9816	877.0334	5	120.6895	1159.1222	5	136.3975	1480.4810
6	105.2434	881.4131	6	120.9513	1164.1564	6	136.6593	1486.1697
7	105.5052	885.8037	7	121.2131	1169.2015	7	136.9211	1491.8693
8	105.7670	890.2052	8	121.4749	1174.2575	8	137.1829	1497.5798
9	106.0288	894.6176	9	121.7367	1179.3244	9	137.4447	1503.3012
10	106.2906	899.0409	10	121.9985	1184.4022	10	137.7065	1509.0335
11	106.5524	903.4751	11	122.2603	1189.4910	11	137.9683	1514.7767
34 0	106.8142	907.9203	**39** 0	122.5221	1194.5906	**44** 0	138.2301	1520.5308
1	107.0759	912.3763	1	122.7839	1199.7011	1	138.4919	1526.2959
2	107.3377	916.8433	2	123.0457	1204.8226	2	138.7537	1532.0718
3	107.5995	921.3211	3	123.3075	1209.9550	3	139.0155	1537.8587
4	107.8613	925.8099	4	123.5693	1215.0982	4	139.2773	1543.6565
5	108.1231	930.3096	5	123.8311	1220.2524	5	139.5391	1549.4651
6	108.3849	934.8202	6	124.0929	1225.4175	6	139.8009	1555.2847
7	108.6467	939.3417	7	124.3547	1230.5935	7	140.0627	1561.1152
8	108.9085	943.8741	8	124.6165	1235.7804	8	140.3245	1566.9566
9	109.1703	948.4174	9	124.8783	1240.9782	9	140.5863	1572.8089
10	109.4321	952.9716	10	125.1401	1246.1869	10	140.8481	1578.6721
11	109.6939	957.5867	11	125.4019	1251.4065	11	141.1099	1584.5462

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
45 0	141.3717	1590.4313	50 0	157.0796	1963.4954	55 0	172.7876	2375.8294
1	141.6335	1596.3272	1	157.3414	1970.0458	1	173.0494	2383.0344
2	141.8953	1602.2341	2	157.6032	1976.6072	2	173.3112	2390.2502
3	142.1571	1608.1518	3	157.8650	1983.1794	3	173.5730	2397.4770
4	142.4189	1614.0805	4	158.1268	1989.7626	4	173.8348	2404.7146
5	142.6807	1620.0201	5	158.3886	1996.3567	5	174.0966	2411.9632
6	142.9425	1625.9705	6	158.6504	2002.9617	6	174.3584	2419.2227
7	143.2043	1631.9319	7	158.9122	2009.5776	7	174.6202	2426.4931
8	143.4661	1637.9042	8	159.1740	2016.2044	8	174.8820	2433.7744
9	143.7279	1643.8874	9	159.4358	2022.8421	9	175.1438	2441.0666
10	143.9897	1649.8816	10	159.6976	2029.4907	10	175.4056	2448.3697
11	144.2515	1655.8866	11	159.9594	2036.1502	11	175.6674	2455.6837
46 0	144.5133	1661.9025	51 0	160.2212	2042.8206	56 0	175.9292	2463.0086
1	144.7751	1667.9294	1	160.4830	2049.5020	1	176.1910	2470.3445
2	145.0369	1673.9671	2	160.7448	2056.1942	2	176.4528	2477.6912
3	145.2987	1680.0158	3	161.0066	2062.8974	3	176.7146	2485.0489
4	145.5605	1686.0753	4	161.2684	2069.6114	4	176.9764	2492.4174
5	145.8223	1692.1458	5	161.5302	2076.3364	5	177.2382	2499.7969
6	146.0841	1698.2272	6	161.7920	2083.0723	6	177.5000	2507.1873
7	146.3459	1704.3195	7	162.0538	2089.8191	7	177.7618	2514.5886
8	146.6077	1710.4227	8	162.3156	2096.5768	8	178.0236	2522.0008
9	146.8695	1716.5368	9	162.5774	2103.3454	9	178.2854	2529.4239
10	147.1313	1722.6618	10	162.8392	2110.1249	10	178.5472	2536.8579
11	147.3931	1728.7977	11	163.1010	2116.9153	11	178.8090	2544.3028
47 0	147.6549	1734.9445	52 0	163.3628	2123.7166	57 0	179.0708	2551.7586
1	147.9167	1741.1023	1	163.6246	2130.5289	1	179.3326	2559.2254
2	148.1785	1747.2709	2	163.8864	2137.3520	2	179.5944	2566.7030
3	148.4403	1753.4505	3	164.1482	2144.1861	3	179.8562	2574.1916
4	148.7021	1759.6410	4	164.4100	2151.0310	4	180.1180	2581.6910
5	148.9639	1765.8423	5	164.6718	2157.8869	5	180.3798	2589.2014
6	149.2257	1772.0546	6	164.9336	2164.7537	6	180.6416	2596.7227
7	149.4875	1778.2778	7	165.1954	2171.6314	7	180.9034	2604.2549
8	149.7492	1784.5119	8	165.4572	2178.5200	8	181.1652	2611.7980
9	150.0110	1790.7569	9	165.7190	2185.4195	9	181.4270	2619.3520
10	150.2728	1797.0128	10	165.9808	2192.3299	10	181.6888	2626.9169
11	150.5346	1803.2796	11	166.2426	2199.2512	11	181.9506	2634.4927
48 0	150.7964	1809.5574	53 0	166.5044	2206.1834	58 0	182.2124	2642.0794
1	151.0582	1815.8460	1	166.7662	2213.1266	1	182.4742	2649.6771
2	151.3200	1822.1456	2	167.0280	2220.0806	2	182.7360	2657.2856
3	151.5818	1828.4560	3	167.2898	2227.0456	3	182.9978	2664.9051
4	151.8436	1834.7774	4	167.5516	2234.0214	4	183.2596	2672.5354
5	152.1054	1841.1096	5	167.8134	2241.0082	5	183.5214	2680.1767
6	152.3672	1847.4528	6	168.0752	2248.0059	6	183.7832	2687.8289
7	152.6290	1853.8069	7	168.3370	2255.0145	7	184.0450	2695.4920
8	152.8908	1860.1719	8	168.5988	2262.0340	8	184.3068	2703.1659
9	153.1526	1866.5478	9	168.8606	2269.0644	9	184.5686	2710.8508
10	153.4144	1872.9346	10	169.1224	2276.1057	10	184.8304	2718.5467
11	153.6762	1879.3324	11	169.3842	2283.1579	11	185.0922	2726.2534
49 0	153.9380	1885.7410	54 0	169.6460	2290.2210	59 0	185.3540	2733.9710
1	154.1998	1892.1605	1	169.9078	2297.2951	1	185.6158	2741.6995
2	154.4616	1898.5910	2	170.1696	2304.3800	2	185.8776	2749.4390
3	154.7234	1905.0323	3	170.4314	2311.4759	3	186.1394	2757.1893
4	154.9852	1911.4846	4	170.6932	2318.5826	4	186.4012	2764.9506
5	155.2470	1917.9478	5	170.9550	2325.7003	5	186.6630	2772.7228
6	155.5088	1924.4218	6	171.2168	2332.8289	6	186.9248	2780.5058
7	155.7706	1930.9068	7	171.4786	2339.9684	7	187.1866	2788.2998
8	156.0324	1937.4027	8	171.7404	2347.1188	8	187.4484	2796.1047
9	156.2942	1943.9095	9	172.0022	2354.2801	9	187.7102	2803.9205
10	156.5560	1950.4273	10	172.2640	2361.4523	10	187.9720	2811.7472
11	156.8178	1956.9559	11	172.5258	2368.6354	11	188.2338	2819.5849

TABLE 8 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
60 0	188.4956	2827.4334	**65** 0	204.2035	3318.3072	**70** 0	219.9115	3848.4510
1	188.7574	2835.2928	1	204.4653	3326.8212	1	220.1733	3857.6194
2	189.0192	2843.1632	2	204.7271	3335.3460	2	220.4351	3866.7988
3	189.2810	2851.0444	3	204.9889	3343.8818	3	220.6969	3875.9890
4	189.5428	2858.9366	4	205.2507	3352.4284	4	220.9587	3885.1902
5	189.8046	2866.8397	5	205.5125	3360.9860	5	221.2205	3894.4022
6	190.0664	2874.7536	6	205.7743	3369.5545	6	221.4823	3903.6252
7	190.3282	2882.6785	7	206.0361	3378.1339	7	221.7441	3912.8591
8	190.5900	2890.6143	8	206.2979	3386.7241	8	222.0059	3922.1039
9	190.8518	2898.5610	9	206.5597	3395.3253	9	222.2677	3931.3596
10	191.1136	2906.5186	10	206.8215	3403.9375	10	222.5295	3940.6262
11	191.3754	2914.4871	11	207.0833	3412.5605	11	222.7913	3949.9037
61 0	191.6372	2922.4666	**66** 0	207.3451	3421.1944	**71** 0	223.0531	3959.1921
1	191.8990	2930.4569	1	207.6069	3429.8392	1	223.3149	3968.4915
2	192.1608	2938.4581	2	207.8687	3438.4950	2	223.5767	3977.8017
3	192.4226	2946.4703	3	208.1305	3447.1616	3	223.8385	3987.1229
4	192.6843	2954.4934	4	208.3923	3455.8392	4	224.1003	3996.4549
5	192.9461	2962.5273	5	208.6541	3464.5277	5	224.3621	4005.7979
6	193.2079	2970.5722	6	208.9159	3473.2270	6	224.6239	4015.1518
7	193.4697	2978.6280	7	209.1777	3481.9373	7	224.8857	4024 5165
8	193.7315	2986.6947	8	209.4395	3490.6585	8	225.1475	4033.8922
9	193.9933	2994.7723	9	209.7013	3499.3906	9	225.4093	4043.2788
10	194.2551	3002.8608	10	209.9631	3508.1336	10	225.6711	4052.6763
11	194.5169	3010.9602	11	210.2249	3516.8875	11	225.9329	4062.0848
62 0	194.7787	3019.0705	**67** 0	210.4867	3525.6524	**72** 0	226.1947	4071.5041
1	195.0405	3027.1918	1	210.7485	3534.4281	1	226.4565	4080.9343
2	195.3023	3035.3239	2	211.0103	3543.2147	2	226.7183	4090.3755
3	195.5641	3043.4670	3	211.2721	3552.0123	3	226.9801	4099.8275
4	195.8259	3051.6209	4	211.5339	3560.8207	4	227.2419	4109.2905
5	196.0877	3059.7858	5	211.7957	3569.6401	5	227.5037	4118.7643
6	196.3495	3067.9616	6	212.0575	3578.4704	6	227.7655	4128.2491
7	196.6113	3076.1483	7	212.3193	3587.3116	7	228.0273	4137.7448
8	196.8731	3084.3459	8	212.5811	3596.1637	8	228.2891	4147.2514
9	197.1349	3092.5544	9	212.8429	3605.0267	9	228.5509	4156.7689
10	197.3967	3100.7738	10	213.1047	3613.9006	10	228.8127	4166.2973
11	197.6585	3109.0041	11	213.3665	3622.7854	11	229.0745	4175.8366
63 0	197.9203	3117.2453	**68** 0	213.6283	3631.6811	**73** 0	229.3363	4185.3868
1	198.1821	3125.4974	1	213.8901	3640.5877	1	229.5981	4194.9479
2	198.4439	3133.7605	2	214.1519	3649.5058	2	229 8599	4204.5200
3	198.7057	3142.0344	3	214.4137	3658.4337	3	230.1217	4214.1029
4	198.9675	3150.3193	4	214.6755	3667.3731	4	230.3835	4223.6968
5	199.2293	3158.6151	5	214.9373	3676.3234	5	230.6453	4233.3016
6	199.4911	3166.9217	6	215.1991	3685.2845	6	230.9071	4242.9172
7	199.7529	3175.2393	7	215.4609	3694.2566	7	231.1689	4252.5438
8	200.0147	3183.5678	8	215.7227	3703.2396	8	231.4307	4262.1813
9	200.2765	3191.9072	9	215.9845	3712.2335	9	231.6925	4271.8297
10	200.5383	3200.2575	10	216.2463	3721.2383	10	231.9543	4281.4890
11	200.8001	3208.6188	11	216.5081	3730.2540	11	232.2161	4291.1592
64 0	201.0619	3216.9909	**69** 0	216.7699	3739.2807	**74** 0	232.4779	4300.8403
1	201.3237	3225.3739	1	217.0317	3748.3182	1	232.7397	4310.5324
2	201.5855	3233.7679	2	217.2935	3757.3666	2	233.0015	4320.2353
3	201.8473	3242.1727	3	217.5553	3766.4260	3	233.2633	4329.9492
4	202.1091	3250.5885	4	217.8171	3775.4962	4	233.5251	4339.6739
5	202.3709	3259.0151	5	218.0789	3784.5774	5	233.7869	4349.4096
6	202.6327	3267.4527	6	218.3407	3793.6695	6	234.0487	4359.1562
7	202.8945	3275.9012	7	218.6025	3802.7725	7	234.3105	4368.9136
8	203.1563	3284.3606	8	218.8643	3811.8864	8	234.5723	4378.6820
9	203.4181	3292.8309	9	219.1261	3821.0112	9	234.8341	4388.4613
10	203.6799	3301.3121	10	219.3879	3830.1469	10	235.0959	4398.2515
11	203 9417	3309.8042	11	219.6497	3839.2935	11	235.3576	4408.0526

TABLE 8 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
75 0	235.6194	4417.8647	**80** 0	251.3274	5026.5482	**85** 0	267.0354	5674.5017
1	235.8812	4427.6876	1	251.5892	5037.0257	1	267.2972	5685.6337
2	236.1430	4437.5214	2	251.8510	5047.5140	2	267.5590	5696.7765
3	236.4048	4447.3662	3	252.1128	5058.0133	3	267.8208	5707.9302
4	236.6666	4457.2218	4	252.3746	5068.5234	4	268.0826	5719.0949
5	236.9284	4467.0884	5	252.6364	5079.0445	5	268.3444	5730.2705
6	237.1902	4476.9659	6	252.8982	5089.5764	6	268.6062	5741.4569
7	237.4520	4486.8543	7	253.1600	5100.1193	7	268.8680	5752.6543
8	237.7138	4496.7536	8	253.4218	5110.6731	8	269.1298	5763.8626
9	237.9756	4506.6637	9	253.6836	5121.2378	9	269.3916	5775.0818
10	238.2374	4516.5849	10	253.9454	5131.8134	10	269.6534	5786.3119
11	238.4992	4526.5169	11	254.2072	5142.3999	11	269.9152	5797.5529
76 0	238.7610	4536.4598	**81** 0	254.4690	5152.9974	**86** 0	270.1770	5808.8048
1	239.0228	4546.4136	1	254.7308	5163.6057	1	270.4388	5820.0676
2	239.2846	4556.3784	2	254.9926	5174.2249	2	270.7006	5831.3414
3	239.5464	4566.3540	3	255.2544	5184.8551	3	270.9624	5842.6260
4	239.8082	4576.3406	4	255.5162	5195.4961	4	271.2242	5853.9216
5	240.0700	4586.3380	5	255.7780	5206.1481	5	271.4860	5865.2280
6	240.3318	4596.3464	6	256.0398	5216.8110	6	271.7478	5876.5454
7	240.5936	4606.3657	7	256.3016	5227.4847	7	272.0096	5887.8737
8	240.8554	4616.3959	8	256.5634	5238.1694	8	272.2714	5899.2129
9	241.1172	4626.4370	9	256.8252	5248.8650	9	272.5332	5910.5630
10	241.3790	4636.4890	10	257.0870	5259.5715	10	272.7950	5921.9240
11	241.6408	4646.5519	11	257.3488	5270.2889	11	273.0568	5933.2959
77 0	241.9026	4656.6257	**82** 0	257.6106	5281.0173	**87** 0	273.3186	5944.6787
1	242.1644	4666.7104	1	257.8724	5291.7565	1	273.5804	5956.0724
2	242.4262	4676.8061	2	258.1342	5302.5066	2	273.8422	5967.4771
3	242.6880	4686.9126	3	258.3960	5313.2677	3	274.1040	5978.8926
4	242.9498	4697.0301	4	258.6578	5324.0396	4	274.3658	5990.3191
5	243.2116	4707.1584	5	258.9196	5334.8225	5	274.6276	6001.7564
6	243.4734	4717.2977	6	259.1814	5345.6162	6	274.8894	6013.2047
7	243.7352	4727.4479	7	259.4432	5356.4209	7	275.1512	6024.6639
8	243.9970	4737.6090	8	259.7050	5367.2365	8	275.4130	6036.1340
9	244.2588	4747.7810	9	259.9668	5378.0630	9	275.6748	6047.6149
10	244.5206	4757.9639	10	260.2286	5388.9004	10	275.9366	6059.1068
11	244.7824	4768.1577	11	260.4904	5399.7487	11	276.1984	6070.6097
78 0	245.0442	4778.3624	**83** 0	260.7522	5410.6079	**88** 0	276.4602	6082.1234
1	245.3060	4788.5781	1	261.0140	5421.4781	1	276.7220	6093.6480
2	245.5678	4798.8046	2	261.2758	5432.3591	2	276.9838	6105.1835
3	245.8296	4809.0420	3	261.5376	5443.2511	3	277.2456	6116.7300
4	246.0914	4819.2904	4	261.7994	5454.1539	4	277.5074	6128.2873
5	246.3532	4829.5497	5	262.0612	5465.0677	5	277.7692	6139.8556
6	246.6150	4839.8198	6	262.3230	5475.9923	6	278.0309	6151.4348
7	246.8768	4850.1009	7	262.5848	5486.9279	7	278.2927	6163.0248
8	247.1386	4860.3929	8	262.8466	5497.8744	8	278.5545	6174.6258
9	247.4004	4870.6958	9	263.1084	5508.8318	9	278.8163	6186.2377
10	247.6622	4881.0096	10	263.3702	5519.8001	10	279.0781	6197.8605
11	247.9240	4891.3343	11	263.6320	5530.7793	11	279.3399	6209.4942
79 0	248.1858	4901.6699	**84** 0	263.8938	5541.7694	**89** 0	279.6017	6221.1389
1	248.4476	4912.0165	1	264.1556	5552.7705	1	279.8635	6232.7944
2	248.7094	4922.3739	2	264.4174	5563.7824	2	280.1253	6244.4608
3	248.9712	4932.7423	3	264.6792	5574.8053	3	280.3871	6256.1382
4	249.2330	4943.1215	4	264.9410	5585.8390	4	280.6489	6267.8264
5	249.4948	4953.5117	5	265.2028	5596.8837	5	280.9107	6279.5256
6	249.7566	4963.9127	6	265.4646	5607.9392	6	281.1725	6291.2356
7	250.0184	4974.3247	7	265.7264	5619.0057	7	281.4343	6302.9566
8	250.2802	4984.7476	8	265.9882	5630.0831	8	281.6961	6314.6885
9	250.5420	4995.1814	9	266.2500	5641.1714	9	281.9579	6326.4313
10	250.8038	5005.6261	10	266.5118	5652.2706	10	282.2197	6338.1850
11	251.0656	5016.0817	11	266.7736	5663.3807	11	282.4815	6349.9496

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.		Circumf.	Area.	Dia.		Circumf.	Area.	Dia.		Circumf.	Area.
Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.
90	0	282.7433	6361.7251	**93**	5	293.4771	6853.9134	**96**	9	303.9491	7351.7686
	1	283.0051	6373.5116		6	293.7389	6866.1471		10	304.2109	7364.4386
	2	283.2669	6385.3089		7	294.0007	6878.3917		11	304.4727	7377.1195
	3	283.5287	6397.1171		8	294.2625	6890.6472	**97**	0	304.7345	7389.8113
	4	283.7905	6408.9363		9	294.5243	6902.9135		1	304.9963	7402.5140
	5	284.0523	6420.7663		10	294.7861	6915.1908		2	305.2581	7415.2277
	6	284.3141	6432.6073		11	295.0479	6927.4791		3	305.5199	7427.9522
	7	284.5759	6444.4592	**94**	0	295.3097	6939.7782		4	305.7817	7440.6877
	8	284.8377	6456.3220		1	295.5715	6952.0882		5	306.0435	7453.4340
	9	285.0995	6468.1957		2	295.8333	6964.4091		6	306.3053	7466.1913
	10	285.3613	6480.0803		3	296.0951	6976.7410		7	306.5671	7478.9595
	11	285.6231	6491.9758		4	296.3569	6989.0837		8	306.8289	7491.7385
91	0	285.8849	6503.8822		5	296.6187	7001.4374		9	307.0907	7504.5285
	1	286.1467	6515.7995		6	296.8805	7013.8019		10	307.3525	7517.3294
	2	286.4085	6527.7278		7	297.1423	7026.1774		11	307.6143	7530.1412
	3	286.6703	6539.6669		8	297.4041	7038.5638	**98**	0	307.8761	7542.9640
	4	286.9321	6551.6169		9	297.6659	7050.9611		1	308.1379	7555.7976
	5	287.1939	6563.5779		10	297.9277	7063.3693		2	308.3997	7568.6421
	6	287.4557	6575.5498		11	298.1895	7075.7884		3	308.6615	7581.4976
	7	287.7175	6587.5325	**95**	0	298.4513	7088.2184		4	308.9233	7594.3639
	8	287.9793	6599.5262		1	298.7131	7100.6593		5	309.1851	7607.2412
	9	288.2411	6611.5308		2	298.9749	7113.1112		6	309.4469	7620.1293
	10	288.5029	6623.5463		3	299.2367	7125.5739		7	309.7087	7633.0284
	11	288.7647	6635.5727		4	299.4985	7138.0476		8	309.9705	7645.9384
92	0	289.0265	6647.6101		5	299.7603	7150.5321		9	310.2323	7658.8593
	1	289.2883	6659.6583		6	300.0221	7163.0276		10	310.4941	7671.7911
	2	289.5501	6671.7174		7	300.2839	7175.5340		11	310.7559	7684.7338
	3	289.8119	6683.7875		8	300.5457	7188.0513	**99**	0	311.0177	7697.6874
	4	290.0737	6695.8684		9	300.8075	7200.5794		1	311.2795	7710.6519
	5	290.3355	6707.9603		10	301.0693	7213.1185		2	311.5413	7723.6274
	6	290.5973	6720.0630		11	301.3311	7225.6686		3	311.8031	7736.6137
	7	290.8591	6732.1767	**96**	0	301.5929	7238.2295		4	312.0649	7749.6109
	8	291.1209	6744.3013		1	301.8547	7250.8013		5	312.3267	7762.6191
	9	291.3827	6756.4368		2	302.1165	7263.3840		6	312.5885	7775.6382
	10	291.6445	6768.5832		3	302.3783	7275.9777		7	312.8503	7788.6681
	11	291.9063	6780.7405		4	302.6401	7288.5822		8	313.1121	7801.7090
93	0	292.1681	6792.9087		5	302.9019	7301.1977		9	313.3739	7814.7608
	1	292.4299	6805.0878		6	303.1637	7313.8240		10	313.6357	7827.8235
	2	292.6917	6817.2779		7	303.4255	7326.4613		11	313.8975	7840.8971
	3	292.9535	6829.4788		8	303.6873	7339.1095	**100**	0	314.1593	7853.9816
	4	293.2153	6841.6907								

Diam, inch.	Circumf, foot	Diam, inch.	Circumf, foot	Diam, inch	Circumf, foot.	Diam, inch.	Circumf, foot.	Diam, inch.	Circumf, foot.
1-64	.004091	7-32	.057269	27-64	.110447	5-8	.163625	53-64	.216803
1-32	.008181	15-64	.061359	7-16	.114537	41-64	.167715	27-32	.220893
3-64	.012272	¼	.065450	29-64	.118628	21-32	.171806	55-64	.224984
1-16	.016362	17-64	.069540	15-32	.122718	43-64	.175896	7-8	.229074
5-64	.020453	9-32	.073631	31-64	.126809	11-16	.179987	57-64	.233165
3-32	.024544	19-64	.077722	½	.130900	45-64	.184078	29-32	.237256
7-64	.028634	5-16	.081812	33-64	.134990	23-32	.188168	59-64	.241346
⅛	.032725	21-64	.085903	17-32	.139081	47-64	.192259	15-16	.245437
9-64	.036816	11-32	.089994	35-64	.143172	¾	.196350	61-64	.249528
5-32	.040906	23-64	.094084	9-16	.147262	49-64	.200440	31-32	.253618
11-64	.044997	⅜	.098175	37-64	.151353	25-32	.204531	63-64	.257709
3-16	.049087	25-64	.102265	19-32	.155443	51-64	.208621	1	.261799
13-64	.053178	13-32	.106356	39-64	.159534	13-16	.212712		

CIRCULAR ARCS.

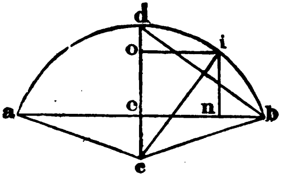


Fig.1

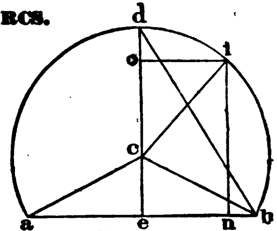


Fig. 2.

Rules for Fig. 1 apply to all arcs equal to, or less than, a semi-circle.
" " Fig. 2 " " " or greater than, a semi-circle.

Chord, *a b*, of whole arc, *a d b*,

$= 2 \times \sqrt{\text{radius}^2 - (\text{radius} - \text{rise})^2}$. Fig. 1.

$= 2 \times \sqrt{\text{radius}^2 - (\text{rise} - \text{radius})^2}$. Fig. 2.

$= 2 \times \sqrt{\text{rise} \times (2 \times \text{radius} - \text{rise})}$. Figs. 1 and 2.

$= 2 \times \text{radius} \times \text{sine of } \tfrac{1}{2}\, a\,c\,b$. Figs. 1 and 2.

$= 2 \times \dfrac{\text{rise}}{\text{tangent of } a\,b\,d.^*}$ Figs. 1 and 2.

$= 2 \times d\,b\,§ \times \text{cosine of } a\,b\,d.^*$ Figs. 1 and 2.

$= 2 \times \sqrt{d\,b^2 - \text{rise}^2}$. Figs. 1 and 2.§

$=$ approximately $8 \times d\,b\,§ - 3 \times$ length of arc *a d b* ¶. Fig. 1.

Length, *a d b*,

$= 2\,\pi\,\text{radius} \times \dfrac{\text{arc } a\,d\,b \text{ in degrees}}{360}$. Figs. 1 and 2;

$= .01745 \times \text{radius} \times \text{arc } a\,d\,b$ in degrees. Figs. 1 and 2.

$=$ circumference of circle $-$ length of *small* arc subtending angle *a c b*. Fig. 2.

$=$ approximately $\dfrac{8 \times d\,b\,§ - \text{chord } a\,b.^{**}}{3}$ Fig. 1.

---

* *a b d* is $= \tfrac{1}{4}$ of the angle *a c b*, subtended by the arc. In Fig. 2 the latter angle exceeds 180°.

§ $d\,b =$ chord of *d i b*, or of half $a\,d\,b = \sqrt{\text{rise}^2 + (\tfrac{1}{2}\,a\,b)^2}$. Figs. 1 and 2.

¶ If rise =	multiply the result by	If rise =	multiply the result by
.5 chord,	1.036	.25 chord,	1.0044
.4 "	1.0193	.2 "	1.0021
.333 "	1.0114	.125 "	1.00036
.3 "	1.0083	.1 "	1.00015

** If rise =	multiply the result by	If rise =	multiply the result by
.5 chord	1.012	.25 chord	1.0015
.4 "	1.0065	.2 "	1.0007
.333 "	1.0038	.125 "	1.00012
.3 "	1.0028	.1 "	1.00005

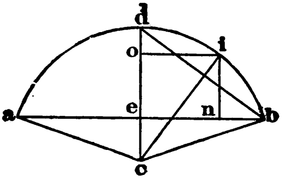


Fig. 1

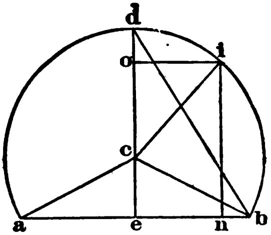


Fig. 2.

Continued from p. 179.

Rules for Fig. 1 apply to all arcs equal to or less than a semi-circle.
" " Fig. 2 " " " or greater than a semi-circle.

Radius, *c a*, *c d*, *c i* or *c b*,

$= \frac{(\frac{1}{2}\,ab)^2 + \text{rise}^2}{2 \times \text{rise}}$, Figs. 1 and 2.

$= \frac{db^2\,\S}{2 \times \text{rise}}$, Figs. 1 and 2.

$= \frac{\frac{1}{2}\,ab}{\text{sine of } \frac{1}{2}\,acb}$, Figs. 1 and 2. $= \frac{\frac{1}{2}\,db\,\S}{\text{sine of } \frac{1}{2}\,bcd\,\|}$, Figs. 1 and 2.

$= \frac{\text{rise } de}{1 - \text{cosine of } \frac{1}{2}\,acb}$, Fig. 1. $= \frac{\text{rise } de}{1 + \text{cosine of } \frac{1}{2}\,acb\,\dagger}$, Fig. 2.

Rise, or middle ordinate, *d e*,

$= \text{radius} - \sqrt{\text{radius}^2 - (\frac{1}{2}\,ab)^2}$, Fig. 1.

$= \text{radius} + \sqrt{\text{radius}^2 - (\frac{1}{2}\,ab)^2}$, Fig. 2.

$= \text{radius} \times (1 - \text{cosine of } bcd\,\|)$, Fig. 1.

$= \text{radius} \times (1 + \text{cosine of } bcd\,\|)$,† Fig. 2.

$= \frac{db^2\,\S}{2 \times \text{radius}}$, Figs. 1 and 2.

$= \frac{1}{2}\,ab \times \text{tangent of } abd$,* Figs. 1 and 2.

$= \text{approximately } \frac{(\frac{1}{2}\,ab)^2}{2 \times \text{radius}}$, Fig. 1.

When radius = chord *a b*, the result is 6.7 parts in 100 too short.
" " = 3 × chord *a b*, the result is 0.7 parts in 100 too short.

Side ordinate, as *n i*,

$= \sqrt{\text{radius}^2 - en^2} + \text{rise} - \text{radius}$, Figs. 1 and 2.

$= \text{approximately } \frac{an \times nb}{2 \times \text{radius}}$. Fig. 1.¶

---

* *a b d* is = ¼ of the angle *a c b*, subtended by the *arc*.

† Strictly, this should read 1 *minus* cosine; but the cosines of angles between 90° and 270° must then be regarded as *minus* or *negative*. Our rule, therefore, amounts to the same thing.

§ *d b* = chord of *d i b*, or of half *a d b*, $= \sqrt{\text{rise}^2 + (\frac{1}{2}\,ab)^2}$. Figs. 1 and 2.

‖ *b c d* = half the angle *a c b* subtended by the *arc*. In Fig. 2, the latter angle exceeds 180°.

¶ When radius = chord *a b*, this makes *d e* 6.7 parts in 100 too short.
" " = 3 × chord *a b*, this makes *d e* 0.7 parts in 100 too short.
The proportionate error is greater with the side ordinates.

Angle, *a c b*, subtended by arc, *a d b*.

An angle and its supplement (as *b c e* and *b e d*, Fig. 2) have the same *sine*, the same *cosine* and the same *tangent*.

Caution. The following sines, etc., are those of only *half a c b*.

Sine of $\frac{1}{2}\, a\, c\, b = \frac{\frac{1}{2}\, a\, b}{\text{radius}}$, Figs. 1 and 2.

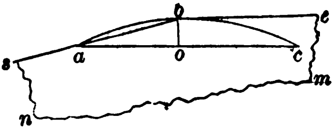
Cosine of $\frac{1}{2}\, a\, c\, b = \frac{\text{radius} - \text{rise}}{\text{radius}}$, Fig. 1; $= \frac{\text{rise} - \text{radius}}{\text{radius}}$, Fig. 2.

Tangent of $\frac{1}{2}\, a\, c\, b = \frac{\frac{1}{2}\, a\, b}{\text{radius} - \text{rise}}$, Fig. 1; $= \frac{\frac{1}{2}\, a\, b}{\text{rise} - \text{radius}}$, Fig. 2.

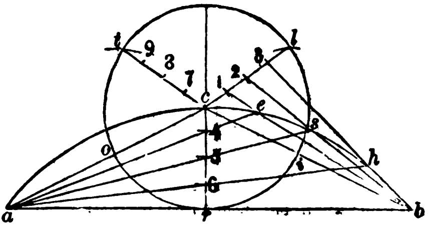
Versed sine of $\frac{1}{2}\, a\, c\, b = \frac{\text{rise}}{\text{radius}}$, Figs. 1 and 2.

To describe the arc of a circle too large for the dividers.

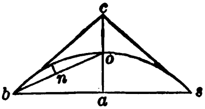
1st Method. Let *a c* be the chord, and *o b* the height, of the required arc, as laid down *on the drawing*. On a separate strip of paper, *s e m n*, draw *a c*. *o b*. and *a b* also *b e*, parallel to the chord *a c*. It is well to make *b s*, and *b e*, each a little longer than *a b*. Then cut off the paper carefully along the lines *s b* and *b e*, so as to leave remaining only the strip *s a b e m n*. Now, if the straight sides *s b* and *b e* be applied to the drawing, so that any parts of them shall touch at the same time the points *a* and *b*, or *b* and *c*, the point *b* on the strip will be in the circumference of the arc, and may be pricked off. Thus, any number of points in the arc may be found, and afterward united to form the curve.



2d Method. Draw the span *a b*; the rise *r c*; and *a c*, *b c*. From *c* with radius *c r* describe a circle. Make each of the arcs *o t* and *i l* equal to *r o* or *r i*; and draw *c t*. *c l*. Divide *c t*, *c l*, *c r*, each into half as many equal parts as the curve is to be divided into. Draw the lines *b* 1, *b* 2, *b* 3; and *a* 4, *a* 5, *a* 6, extended to meet the first ones at *e*, *s*, *h*. Then *e*, *s*, *h*, are points in one half the curve. Then for the other half, draw similar lines from *a* to 7, 8, 9; and others from *b* to meet them, as before. Trace the curve by hand.



Remark.—It may frequently be of use to remember, that in any arc *b o s*, not exceeding 29°, or in other words, *whose chord* bs *is at least sixteen times its rise*, the **middle ordinate** *a o*, will be one-half of *a c*, quite near enough for many purposes; *b c* and *s c* being tangents to the arc.[†] And vice versa, if in such an arc we make *o c* equal *a o*, then will *c* be, very nearly, the point at which tangents from the ends of the arc will meet. Also the **middle ordinate** ***n*, of the half arc** *o b*, or *o s*, will be approximately ¼ of *a o*, the middle ordinate of the whole arc. Indeed, this last observation will apply near enough for many approximate uses even if the arc be as great as 45°; for if in that case we take ¼ of *o a* for the ordinate *n*, *n* will then be but 1 part in 103 too small; and therefore the principle may often be used in drawings, for finding points in a curve of too great radius to be drawn by the dividers; for in the same manner, ¼ of *n* will be the middle ordinate for the arc *n b* or *n o;* and so on to any extent. Below will be found **a table by which the rise or middle ordinate of a half arc** can be obtained with greater accuracy when required for more exact drawings.



CIRCULAR ARCS IN FREQUENT USE.

The fifth column is of use for finding points for drawing arcs too large for the beam-compass, on the principle given above. In even the largest office drawings it will not be necessary to use more than the first three decimals of the fifth column; and after the arc is subdivided into parts smaller than about 35° each, the first two decimals .25 will generally suffice. Original.

Rise in parts of chord.	Degrees in whole arc.		For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by	Rise in parts of chord.	Degrees in whole arc.		For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by
	°	′					°	′			
1-50	9	9.75	313.	1.00107	.2501	1/8	56	8.70	8.5	1.04116	.2538
1-45	10	10.75	253.625	1.00132	.2501	1-7	63	46.90	6.625	1.05356	.2549
1-40	11	26.98	200.5	1.00167	.2502	.155	68	53.63	5.70291	1.06288	.2557
1-35	13	4.92	153.625	1.00219	.2502	1-6	73	44.39	5.	1.07250	.2566
1-30	15	15.38	113.	1.00296	.2503	.18	79	11.73	4.35803	1.08426	.2576
1-25	18	17.74	78.625	1.00426	.2504	1-5	87	12.34	3.625	1.10347	.2593
1-20	22	50.54	50.5	1.00665	.2506	.207107	90		3.41422	1.11072	.2599
1-19	24	2.16	45.625	1.00737	.2507	.225	96	54.67	2.96913	1.12997	.2615
1-18	25	21.65	41.	1.00821	.2508	¼	106	15 61	2.5	1.15912	.2639
1-17	26	50.36	36.625	1.00920	.2509	.275	115	14.59	2.15289	1.19082	.2665
1-16	28	30.00	32.5	1.01038	.2510	.3	123	51 30	1.88889	1.22495	.2692
1-15	30	22.71	28.625	1.01181	.2511	⅓	134	45.62	1.625	1.27401	.2729
1-14	32	31.22	25.	1.01355	.2513	.365	144	30.98	1.43827	1.32413	.2766
1-13	34	59.08	21.625	1.01571	.2515	.4	154	38 35	1.28125	1.38322	.2808
1-12	37	50.96	18.5	1.01842	.2517	.425	161	27.52	1.19204	1.42764	.2838
1-11	41	13.16	15.625	1.02189	.2520	.45	167	56.93	1.11728	1.47377	.2868
1-10	45	14.38	13.	1.02646	.2525	.475	174	7.49	1.05402	1.52152	.2899
1-9	50	6.91	10.625	1.03260	.2530	.5	180		1.	1.57080	.2929

[†] At 29° *o c* thus found will be but about 3 parts too short in 100.

Lengths of circular arcs. If arc exceeds a semicircle, see p 184

Knowing its chord and height, divide the height by the chord. Find in the column of heights the number equal to this quotient. Take out the corresponding number from the column of lengths. Multiply this last number by the length of the given chord.

TABLE OF CIRCULAR ARCS.

No errors.

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.
.001	1.00002	.076	1.01533	.151	1.05973	.226	1.13108	.301	1.22636
.002	1.00002	.077	1.01573	.152	1.06051	.227	1.13219	.302	1.22778
.003	1.00003	.078	1.01614	.153	1.06130	.228	1.13331	.303	1.22920
.004	1.00004	.079	1.01656	.154	1.06209	.229	1.13444	.304	1.23063
.005	1.00007	.080	1.01698	.155	1.06288	.230	1.13557	.305	1.23206
.006	1.00010	.081	1.01741	.156	1.06368	.231	1.13671	.306	1.23349
.007	1.00013	.082	1.01784	.157	1.06449	.232	1.13786	.307	1.23492
.008	1.00017	.083	1.01828	.158	1.06530	.233	1.13900	.308	1.23636
.009	1.00022	.084	1.01872	.159	1.06611	.234	1.14015	.309	1.23781
.010	1.00027	.085	1.01916	.160	1.06693	.235	1.14131	.310	1.23926
.011	1.00032	.086	1.01961	.161	1.06775	.236	1.14247	.311	1.24070
.012	1.00038	.087	1.02006	.162	1.06858	.237	1.14363	.312	1.24216
.013	1.00045	.088	1.02052	.163	1.06941	.238	1.14480	.313	1.24361
.014	1.00053	.089	1.02098	.164	1.07025	.239	1.14597	.314	1.24507
.015	1.00061	.090	1.02146	.165	1.07109	.240	1.14714	.315	1.24654
.016	1.00069	.091	1.02192	.166	1.07194	.241	1.14832	.316	1.24801
017	1.00078	.092	1.02240	.167	1.07279	.242	1.14951	.317	1.24948
.018	1.00087	.093	1.02289	.168	1.07365	.243	1.15070	.318	1.25095
.019	1.00097	.094	1.02339	.169	1.07451	.244	1.15189	.319	1.25243
.020	1.00107	.095	1.02389	.170	1.07537	.245	1.15308	.320	1.25391
.021	1.00117	.096	1.02440	.171	1.07624	.246	1.15428	.321	1.25540
.022	1.00128	097	1.02491	.172	1.07711	.247	1.15549	.322	1.25689
.023	1.00140	.098	1.02542	.173	1.07799	.248	1.15670	.323	1.25838
.024	1.00153	.099	1.02593	.174	1.07888	.249	1.15791	.324	1.25988
.025	1 00167	.100	1.02646	.175	1.07977	.250	1.15912	.325	1.26138
.026	1.00182	.101	1.02698	.176	1.08066	.251	1.16034	.326	1.26288
.027	1.00196	.102	1.02752	.177	1.08156	.252	1.16156	.327	1.26437
.028	1.00210	.103	1.02806	.178	1.08246	.253	1.16279	.328	1.26588
.029	1.00225	.104	1.02860	.179	1.08337	.254	1.16402	.329	1.26740
.030	1.00240	.105	1.02914	.180	1.08428	.255	1.16526	.330	1.26892
.031	1.00256	.106	1.02970	.181	1.08519	.256	1.16650	.331	1.27044
.032	1.00272	.107	1.03026	.182	1.08611	.257	1.16774	.332	1.27196
.033	1.00289	.108	1.03082	.183	1.08704	.258	1.16899	.333	1.27349
.034	1.00307	.109	1.03139	.184	1.08797	.259	1.17024	.334	1.27502
.035	1.00327	.110	1.03196	.185	1.08890	.260	1.17150	.335	1.27656
.036	1.00345	.111	1.03254	.186	1.08984	.261	1.17276	.336	1.27810
.037	1.00364	.112	1.03312	.187	1.09079	.262	1.17403	.337	1.27964
.038	1.00384	.113	1.03371	.188	1.09174	.263	1.17530	.338	1.28118
.039	1.00405	.114	1.03430	.189	1.09269	.264	1.17657	.339	1.28273
.040	1.00426	.115	1.03490	.190	1.09365	.265	1.17784	.340	1.28428
.041	1.00447	.116	1.03551	.191	1.09461	.266	1.17912	.341	1.28583
.042	1.00469	.117	1.03611	.192	1.09557	.267	1.18040	.342	1.28739
.043	1.00492	.118	1.03672	.193	1.09654	.268	1.18169	.343	1.28895
.044	1.00515	.119	1.03734	.194	1.09752	.269	1.18299	.344	1.29052
.045	1.00539	.120	1.03797	.195	1.09850	.270	1.18429	.345	1.29209
.046	1.00563	.121	1.03860	.196	1.09949	.271	1.18559	.346	1.29366
.047	1.00587	.122	1.03923	.197	1.10048	.272	1.18689	.347	1.29523
.048	1.00612	.123	1.03987	.198	1.10147	.273	1.18820	.348	1.29681
.049	1.00638	.124	1.04051	.199	1.10247	.274	1.18951	.349	1.29839
.050	1.00665	.125	1.04116	.200	1.10347	.275	1.19082	.350	1.29997
.051	1.00692	.126	1.04181	.201	1.10447	.276	1.19214	.351	1.30156
.052	1 00720	.127	1.04247	.202	1.10548	.277	1.19346	.352	1.30315
053	1 00748	.128	1.04313	.203	1.10650	.278	1.19479	.353	1.30474
.054	1.00776	.129	1.04380	.204	1.10752	.279	1.19612	.354	1.30634
.055	1.00805	.130	1.04447	.205	1.10855	.280	1.19746	.355	1.30794
.056	1.00834	.131	1 04515	.206	1.10958	.281	1.19880	.356	1.30954
.057	1 00864	.132	1.04584	.207	1.11062	.282	1.20014	.357	1.31115
.058	1.00895	.133	1.04652	.208	1.11165	.283	1.20149	.358	1.31276
.059	1.00926	134	1.04722	.209	1.11269	.284	1.20284	.359	1.31437
.060	1.00957	.135	1.04792	.210	1.11374	.285	1.20419	.360	1.31599
.061	1.00989	.136	1.04862	.211	1.11479	.286	1.20555	.361	1.31761
.062	1.01021	.137	1.04932	.212	1.11584	.287	1.20691	.362	1.31923
.063	1.01054	.138	1.05003	.213	1.11690	.288	1.20827	.363	1.32086
.064	1.01088	.139	1.05075	.214	1.11796	.289	1.20964	.364	1.32249
.065	1.01123	.140	1.05147	.215	1.11904	.290	1.21102	.365	1.32413
.066	1.01158	.141	1.05220	.216	1.12011	.291	1.21239	366	1.32577
.067	1.01193	.142	1.05293	.217	1.12118	.292	1.21377	.367	1.32741
.068	1.01228	.143	1.05367	.218	1.12225	.293	1.21515	.368	1.32905
.069	1.01264	.144	1.05441	.219	1.12334	.294	1.21654	.369	1.33069
.070	1.01302	.145	1.05516	.220	1.12444	.295	1.21794	.370	1.33234
.071	1.01338	.146	1.05591	.221	1.12554	.296	1.21933	.371	1.33399
.072	1.01376	.147	1.05667	.222	1.12664	.297	1.22073	.372	1.33564
.073	1.01414	.148	1.05743	.223	1.12774	.298	1.22213	373	1.33730
.074	1.01453	.149	1.05819	.224	1.12885	.299	1.22354	.374	1.33896
.075	1.01493	.150	1.05896	.225	1 12997	.300	1.22495	.375	1.34063

TABLE OF CIRCULAR ARCS—(CONTINUED.)

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.
.376	1.34229	.401	1.38496	.426	1.42945	.451	1.47565	.476	1.52346
.377	1.34396	.402	1.38671	.427	1.43127	.452	1.47753	.477	1.52541
.378	1.34563	.403	1.38846	.428	1.43309	.453	1.47942	.478	1.52736
.379	1.34731	.404	1.39021	.429	1.43491	.454	1.48131	.479	1.52931
.380	1.34899	.405	1.39196	.430	1.43673	.455	1.48320	.480	1.53126
.381	1.35068	.406	1.39372	.431	1.43856	.456	1.48509	.481	1.53322
.382	1.35237	.407	1.39548	.432	1.44039	.457	1.48699	.482	1.53518
.383	1.35406	.408	1.39724	.433	1.44222	.458	1.48889	.483	1.53714
.384	1.35575	.409	1.39900	.434	1.44405	.459	1.49079	.484	1.53910
.385	1.35744	.410	1.40077	.435	1.44589	.460	1.49269	.485	1.54106
.386	1.35914	.411	1.40254	.436	1.44773	.461	1.49460	.486	1.54302
.387	1.36084	.412	1.40432	.437	1.44957	.462	1.49651	.487	1.54499
.388	1.36254	.413	1.40610	.438	1.45142	.463	1.49842	.488	1.54696
.389	1.36425	.414	1.40788	.439	1.45327	.464	1.50033	.489	1.54893
.390	1.36596	.415	1.40966	.440	1.45512	.465	1.50224	.490	1.55091
.391	1.36767	.416	1.41145	.441	1.45697	.466	1.50416	.491	1.55289
.392	1.36939	.417	1.41324	.442	1.45883	.467	1.50608	.492	1.55487
.393	1.37111	.418	1.41503	.443	1.46069	.468	1.50800	.493	1.55685
.394	1.37283	.419	1.41682	.444	1.46255	.469	1.50992	.494	1.55884
.395	1.37455	.420	1.41861	.445	1.46441	.470	1.51185	.495	1.56083
.396	1.37628	.421	1.42041	.446	1.46628	.471	1.51378	.496	1.56282
.397	1.37801	.422	1.42221	.447	1.46815	.472	1.51571	.497	1.56481
.398	1.37974	.423	1.42402	.448	1.47002	.473	1.51764	.498	1.56681
.399	1.38148	.424	1.42583	.449	1.47189	.474	1.51958	.499	1.56881
.400	1.38322	.425	1.42764	.450	1.47377	.475	1.52152	.500	1.57080

If the arc is greater than a semicircle, then, as directed at top of p 187, find the diam of the circle. Then find its circumf. From diam take ht of arc. The rem will be ht of the smaller arc of the circle. By rule at top of p 183 find the length of this smaller arc. Subtract it from circumf.

The length of 1 degree of a circular arc is equal to .017453 292 520 × its radius.
" " " 1 minute " " " " " .000290 888 209 × " "
" " " 1 second " " " " " .000004 848 137 × " "

An arc of 1° of the **earth's great circle** is but 4.6356 feet longer than its chord. Its length is 69.16 land or statute miles. Earth's equatorial rad = 3962.5705 miles. Polar 3949.67.

An arc of 1°, rad 1 mile, is 92.1534 feet; a minute is 1.5359 feet; a second is .0256 of a foot or very nearly 5-sixteenths of an inch. **Arc of 1°, rad 100 ft** = 1.74533 feet.

To find the length of a circular arc by the following table.

Knowing the rad of the circle, and the measure of the arc in deg, min, &c.

Rule. Add together the lengths in the table found respectively opposite to the deg, min, &c, of the arc. Mult the sum by the rad of the circle.

Ex. In a circle of 12.43 feet rad, is an arc of 13 deg, 27 min, 8 sec. How long is the arc?

Here, opposite 13 deg in the table, we find, .2268928
" 27 min " " " .0078540
" 8 sec " " " .0000388

Sum = .2347856

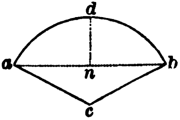
And .2347856 × 12.43 or rad = 2.918385 feet, the reqd length of arc.

LENGTHS OF CIRCULAR ARCS TO RAD 1.

Deg.	Length.	Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	.0174533	61	1.0646508	121	2.1118484	1	.0002909	1	.0000048
2	.0349066	62	1.0821041	122	2.1293017	2	.0005818	2	.0000097
3	.0523599	63	1.0995574	123	2.1467550	3	.0008727	3	.0000145
4	.0698132	64	1.1170107	124	2.1642083	4	.0011636	4	.0000194
5	.0872665	65	1.1344640	125	2.1816616	5	.0014544	5	.0000242
6	.1047198	66	1.1519173	126	2.1991149	6	.0017453	6	.0000291
7	.1221730	67	1.1693706	127	2.2165682	7	.0020362	7	.0000339
8	.1396263	68	1.1868239	128	2.2340214	8	.0023271	8	.0000388
9	.1570796	69	1.2042772	129	2.2514747	9	.0026180	9	.0000436
10	.1745329	70	1.2217305	130	2.2689280	10	.0029089	10	.0000485
11	.1919862	71	1.2391838	131	2.2863813	11	.0031998	11	.0000533
12	.2094395	72	1.2566371	132	2.3038346	12	.0034907	12	.0000582
13	.2268928	73	1.2740904	133	2.3212879	13	.0037815	13	.0000630
14	.2443461	74	1.2915436	134	2.3387412	14	.0040724	14	.0000679
15	.2617994	75	1.3089969	135	2.3561945	15	.0043633	15	.0000727
16	.2792527	76	1.3264502	136	2.3736478	16	.0046542	16	.0000776
17	.2967060	77	1.3439035	137	2.3911011	17	.0049451	17	.0000824
18	.3141593	78	1.3613568	138	2.4085544	18	.0052360	18	.0000873
19	.3316126	79	1.3788101	139	2.4260077	19	.0055269	19	.0000921
20	.3490659	80	1.3962634	140	2.4434610	20	.0058178	20	.0000970
21	.3665191	81	1.4137167	141	2.4609142	21	.0061087	21	.0001018
22	.3839724	82	1.4311700	142	2.4783675	22	.0063995	22	.0001067
23	.4014257	83	1.4486233	143	2.4958208	23	.0066904	23	.0001115
24	.4188790	84	1.4660766	144	2.5132741	24	.0069813	24	.0001164
25	.4363323	85	1.4835299	145	2.5307274	25	.0072722	25	.0001212
26	.4537856	86	1.5009832	146	2.5481807	26	.0075631	26	.0001261
27	.4712389	87	1.5184364	147	2.5656340	27	.0078540	27	.0001309
28	.4886922	88	1.5358897	148	2.5830873	28	.0081449	28	.0001357
29	.5061455	89	1.5533430	149	2.6005406	29	.0084358	29	.0001406
30	.5235988	90	1.5707963	150	2.6179939	30	.0087266	30	.0001454
31	.5410521	91	1.5882496	151	2.6354472	31	.0090175	31	.0001503
32	.5585054	92	1.6057029	152	2.6529005	32	.0093084	32	.0001551
33	.5759587	93	1.6231562	153	2.6703538	33	.0095993	33	.0001600
34	.5934119	94	1.6406095	154	2.6878070	34	.0098902	34	.0001648
35	.6108652	95	1.6580628	155	2.7052603	35	.0101811	35	.0001697
36	.6283185	96	1.6755161	156	2.7227136	36	.0104720	36	.0001745
37	.6457718	97	1.6929694	157	2.7401669	37	.0107629	37	.0001794
38	.6632251	98	1.7104227	158	2.7576202	38	.0110538	38	.0001842
39	.6806784	99	1.7278760	159	2.7750735	39	.0113446	39	.0001891
40	.6981317	100	1.7453293	160	2.7925268	40	.0116355	40	.0001939
41	.7155850	101	1.7627825	161	2.8099801	41	.0119264	41	.0001988
42	.7330383	102	1.7802358	162	2.8274334	42	.0122173	42	.0002036
43	.7504916	103	1.7976891	163	2.8448867	43	.0125082	43	.0002085
44	.7679449	104	1.8151424	164	2.8623400	44	.0127991	44	.0002133
45	.7853982	105	1.8325957	165	2.8797933	45	.0130900	45	.0002182
46	.8028515	106	1.8500490	166	2.8972466	46	.0133809	46	.0002230
47	.8203047	107	1.8675023	167	2.9146999	47	.0136717	47	.0002279
48	.8377580	108	1.8849556	168	2.9321531	48	.0139626	48	.0002327
49	.8552113	109	1.9024089	169	2.9496064	49	.0142535	49	.0002376
50	.8726646	110	1.9198622	170	2.9670597	50	.0145444	50	.0002424
51	.8901179	111	1.9373155	171	2.9845130	51	.0148353	51	.0002473
52	.9075712	112	1.9547688	172	3.0019663	52	.0151262	52	.0002521
53	.9250245	113	1.9722221	173	3.0194196	53	.0154171	53	.0002570
54	.9424778	114	1.9896753	174	3.0368729	54	.0157080	54	.0002618
55	.9599311	115	2.0071286	175	3.0543262	55	.0159989	55	.0002666
56	.9773844	116	2.0245819	176	3.0717795	56	.0162897	56	.0002715
57	.9948377	117	2.0420352	177	3.0892328	57	.0165806	57	.0002763
58	1.0122910	118	2.0594885	178	3.1066861	58	.0168715	58	.0002812
59	1.0297443	119	2.0769418	179	3.1241394	59	.0171624	59	.0002860
60	1.0471976	120	2.0943951	180	3.1415927	60	.0174533	60	.0002909

CIRCULAR SECTORS, RINGS, SEGMENTS, ETC.

Fig. A.

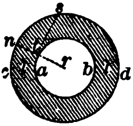


Area of a circular sector, *a d b c*, Fig A,

$$= \frac{\text{arc } a\,d\,b}{2} \times \text{radius } c\,a.$$

$$= \text{area of entire circle} \times \frac{\text{arc } a\,d\,b \text{ in degrees.}}{360}$$

Fig. B.



Area of a circular ring, Fig. B,

= area of larger circle, *c d*, — area of smaller one, *a b*.

= .7854 × (sum of diams. *c d* + *a b*) × (diff. of diams. *c d* — *a b*.)

= 1.5708 × thickness *c a* × sum of diameters *c d* and *a b*.

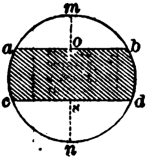
To find the radius of a circle which shall have the same area as a given circular ring *c s d a b*, Fig. B,

Draw any radius *n r* of the outer circle; and from where said radius cuts the inner circle at *t*, draw *t s* at right angles to it. Then will *t s* be the required radius.

Breadth, *c a* = *b d*, of a circular ring, Fig. B,

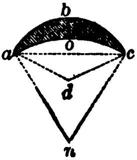
= ½ difference of diameters *c d* and *a b*.

$$= \tfrac{1}{2}\left(\text{diameter } c\,d - \sqrt{1.2732 \text{ area of circle } a\,b.}\right)$$



Area of a circular zone *a b c d*,

= area of circle *m n*—areas of segments *a m b* and *c n d*, (for areas of segments, see below.)



A circular lune is a crescent-shaped figure, comprised between two arcs *a b c* and *a o c* of circles of different radii, *a d* and *a n*.

Area of a circular lune *a b c o*

= area of segment *a b c* — area of segment *a o c*, (for areas of segments see below.)

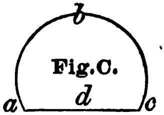
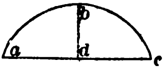


Fig. C.

Fig. D.



To find the area of a circular segment, *a b c d*, Figs. C, D.

Area of Segment *a d b n*, Fig. A (at top of page)

= Area of Sector *a d b c* — Area of Triangle *a b c*.

= ½ (Arc *a d b* × radius *a c* — *c n* × chord *a b*).

Having the area of a segment required to be cut off from a given circle, to find its chord and rise.

Divide the area by the square of the diameter of the circle; look for the quotient in the column of areas in the table of areas, opposite; take out from the table the corresponding number in the column of rises. Multiply this number by the diameter. The product will be the required rise. Then

$$\text{chord} = 2 \times \sqrt{(\text{diameter} - \text{rise}) \times \text{rise}}$$

TABLE OF AREAS OF CIRCULAR SEGMENTS, Figs C, D.

If the segment exceeds a semicircle, its area is = area of circle — area of a segment whose rise is = (diam of circle — rise of given segment). **Diam of circle** = (square of half chord ÷ rise) + rise, whether the segment exceeds a semicircle or not.

Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam mult by
.001	.000042	.064	.021168	.127	.057991	.190	.103900	.253	.156149
.002	.000119	.065	.021660	.128	.058658	.191	.104686	.254	.157019
.003	.000219	.066	.022155	.129	.059328	.192	.105472	.255	.157891
.004	.000337	.067	.022653	.130	.059999	.193	.106261	.256	.158763
.005	.000471	.068	.023155	.131	.060673	.194	.107051	.257	.159636
.006	.000619	.069	.023660	.132	.061349	.195	.107843	.258	.160511
.007	.000779	.070	.024168	.133	.062027	.196	.108636	.259	.161386
.008	.000952	.071	.024680	.134	.062707	.197	.109431	.260	.162263
.009	.001135	.072	.025196	.135	.063389	.198	.110227	.261	.163141
.010	.001329	.073	.025714	.136	.064074	.199	.111025	.262	.164020
.011	.001533	.074	.026236	.137	.064761	.200	.111824	.263	.164900
.012	.001746	.075	.026761	.138	.065449	.201	.112625	.264	.165781
.013	.001969	.076	.027290	.139	.066140	.202	.113427	.265	.166663
.014	.002199	.077	.027821	.140	.066833	.203	.114231	.266	.167546
.015	.002438	078	.028356	.141	.067528	.204	.115036	.267	.168431
.016	.002685	.079	.028894	.142	.068225	.205	.115842	.268	.169316
.017	.002940	.080	.029435	.143	.068924	.206	.116651	.269	.170202
.018	.003202	.081	.029979	.144	.069626	.207	.117460	.270	.171090
.019	.003472	.082	.030526	.145	.070329	.208	.118271	.271	.171978
020	.003749	.083	.031077	.146	.071034	.209	.119084	.272	.172868
.021	.004032	.084	.031630	.147	.071741	.210	.119898	.273	.173758
.022	.004322	.085	.032186	.148	.072450	.211	.120713	.274	.174650
.023	.004619	.086	.032746	.149	.073162	.212	.121530	.275	.175542
.024	.004922	.087	.033308	.150	.073875	.213	.122348	.276	.176436
.025	.005231	.088	.033873	.151	.074590	.214	.123167	.277	.177330
.026	.005546	.089	.034441	.152	.075307	.215	.123988	.278	.178226
.027	.005867	.090	.035012	.153	.076026	.216	.124811	.279	.179122
.028	.006194	.091	.035586	.154	.076747	.217	.125634	.280	.180020
.029	.006527	.092	.036162	.155	.077470	.218	.126459	.281	.180918
.030	.006866	.093	.036742	.156	.078194	.219	.127286	.282	.181818
.031	.007209	.094	.037324	.157	.078921	.220	.128114	.283	.182718
.032	.007559	.095	.037909	.158	.079650	.221	.128943	.284	.183619
.033	.007918	.096	.038497	.159	.080380	.222	.129773	.285	.184522
.034	.008278	.097	.039087	.160	.081112	.223	.130605	.286	.185425
.035	.008638	.098	.039681	.161	.081847	.224	.131438	.287	.186329
.036	.009008	.099	.040277	.162	.082582	.225	.132273	.288	.187235
.037	.009383	.100	.040875	.163	.083320	.226	.133109	.289	.188141
.038	.009764	.101	.041477	.164	.084060	.227	.133946	.290	.189048
.039	.010148	.102	.042081	.165	.084801	.228	.134784	.291	.189956
.040	.010538	.103	.042687	.166	.085545	.229	.135624	.292	.190865
041	.010932	.104	.043296	.167	.086290	.230	.136465	.293	.191774
.042	.011331	.105	.043908	.168	.087037	.231	.137307	.294	.192685
.043	.011734	.106	.044523	.169	.087785	.232	.138151	.295	.193597
.044	.012142	.107	.045140	.170	.088536	.233	.138996	.296	.194509
.045	.012555	.108	.045759	.171	.089288	.234	.139842	.297	.195423
.046	.012971	.109	.046381	.172	.090042	.235	.140689	.298	.196337
.047	.013393	.110	.047006	.173	.090797	.236	.141538	.299	.197252
.048	.013818	.111	.047633	.174	.091555	.237	.142388	.300	.198168
.049	.014248	.112	.048262	.175	.092314	.238	.143239	.301	.199085
.050	.014681	.113	.048894	.176	.093074	.239	.144091	.302	.200003
.051	.015119	.114	.049529	.177	.093837	.240	.144945	.303	.200922
.052	.015561	.115	.050165	.178	.094601	.241	.145800	.304	.201841
.053	.016008	.116	.050805	.179	.095367	.242	.146656	.305	.202762
.054	.016458	.117	.051446	.180	.096135	.243	.147513	.306	.203683
.055	.016912	.118	.052090	.181	.096904	.244	.148371	.307	.204605
.056	.017369	.119	.052737	.182	.097675	.245	.149231	.308	.205525
.057	.017831	.120	.053385	.183	.098447	.246	.150091	.309	.206452
.058	.018297	.121	.054037	.184	.099221	.247	.150953	.310	.207376
.059	.018766	.122	.054690	.185	.099997	.248	.151816	.311	.208302
.060	.019239	.123	.055346	.186	.100774	.249	.152681	.312	.209228
.061	.019716	.124	.056004	.187	.101553	.250	.153546	.313	.210155
.062	.020197	.125	.056664	.188	.102334	.251	.154413	.314	.211083
.063	.020681	.126	.057327	.189	.103116	.252	.155281	.315	.212011

TABLE OF AREAS OF CIRCULAR SEGMENTS—(CONTINUED.)

Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam mult by
.316	.212941	.353	.247845	.390	.283593	.427	.319959	.464	.356730
.317	.213871	.354	.248801	.391	.284569	.428	.320949	.465	.357728
.318	.214802	.355	.249758	.392	.285545	.429	.321938	.466	.358725
.319	.215734	.356	.250715	.393	.286521	.430	.322928	.467	.359723
.320	.216666	.357	.251673	.394	.287499	.431	.323919	.468	.360721
.321	.217600	.358	.252632	.395	.288476	.432	.324909	.469	.361719
.322	.218534	.359	.253591	.396	.289454	.433	.325900	.470	.362717
.323	.219469	.360	.254551	.397	.290432	.434	.326891	.471	.363715
.324	.220404	.361	.255511	.398	.291411	.435	.327883	.472	.364714
.325	.221341	.362	.256472	.399	.292390	.436	.328874	.473	.365712
.326	.222278	.363	.257433	.400	.293370	.437	.329866	.474	.366711
.327	.223216	.364	.258395	.401	.294350	.438	.330858	.475	.367710
.328	.224154	.365	.259358	.402	.295330	.439	.331851	.476	.368708
.329	.225094	.366	.260321	.403	.296311	.440	.332843	.477	.369707
.330	.226034	.367	.261285	.404	.297292	441	.333836	.478	.370706
.331	.226974	.368	.262249	.405	.298274	.442	.334829	.479	.371705
.332	.227916	.369	.263214	.406	.299256	.443	.335823	.480	.372704
.333	.228858	.370	.264179	.407	.300238	.444	.336816	.481	.373704
.334	.229801	.371	.265145	.408	.301221	.445	.337810	.482	.374703
.335	.230745	.372	.266111	.409	.302204	.446	.338804	.483	.375702
.336	.231689	.373	.267078	.410	.303187	.447	.339799	.484	.376702
.337	.232634	.374	.268046	.411	.304171	.448	.340793	.485	.377701
.338	.233580	.375	.269014	.412	.305156	.449	.341788	.486	.378701
.339	.234526	.376	.269982	.413	.306140	.450	.342783	.487	.379701
.340	.235473	.377	.270951	.414	.307125	.451	.343778	.488	.380700
.341	.236421	.378	.271921	.415	.308110	.452	.344773	.489	.381700
.342	.237369	.379	.272891	.416	.309096	.453	.345768	.490	.382700
.343	.238319	.380	.273861	.417	.310082	.454	.346764	.491	.383700
.344	.239268	.381	.274832	.418	.311068	.455	.347760	.492	384699
.345	.240219	.382	.275804	.419	.312055	.456	.348756	.493	.385699
.346	.241170	.383	.276776	.420	.313042	.457	.349752	.494	.386699
.347	.242122	.384	.277748	.421	.314029	.458	.350749	.495	.387699
.348	.243074	.385	.278721	.422	.315017	.459	.351745	.496	.388699
.349	.244027	.386	.279695	.423	.316005	.460	.352742	.497	.389699
.350	.244980	.387	.280669	.424	.316993	.461	.353739	.498	.390699
.351	.245935	.388	.281643	.425	.317981	.462	.354736	.499	.391699
.352	.246890	.389	.282618	.426	.318970	.463	.355733	.500	.392699

ELLIPSE (page 189).

Focal distance $= fg = 2\sqrt{co^2 - bo^2}$;*

$$cf = gw = \frac{cw - fg}{2} = \frac{cw}{2} - \sqrt{co^2 - bo^2}.$$

---

* Because $co = \frac{cw}{2} = \frac{fc + cg}{2} = \frac{fb + bg}{2} = fb.$

THE ELLIPSE.

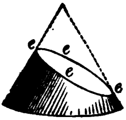


Fig. 1.

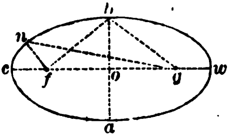


Fig. 2.

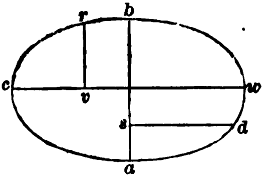


Fig. 3.

An ellipse is a curve, *e e e e*, Fig 1, formed by an oblique section of either a cone or a cylinder, passing through its curved surface, without cutting the base. Its nature is such that if two lines, as *n f* and *n g*, Fig. 2, be drawn from any point *n* in its periphery or circumf. to two certain points *f* and *g*, in its long diam *c w*, (and called the foci of the ellipse,) their sum will be equal to that of any other two lines, as *b f*, and *b g* drawn from any other point, as *b*, in the circumf, to the foci *f* and *g*; also the sum of any two such lines will be equal to the long diam *c w*. The line *c w* dividing the ellipse into two equal parts lengthwise, is called its transverse, or major axis, or long diam; and *a b*, which divides it equally at right-angles to *c w*, is called the conjugate, or minor axis, or short diam. To find the position of the foci of an ellipse, from either end, as *b*, of the short diam, measure off the dists *b f* and *b g*, Fig 2, each equal to *o c*, or one half the long diam.

The parameter of an ellipse is a certain length obtained thus; as the long diam : short diam : : short diam : parameter. Any line *r v*, or *s d*, Fig 3, drawn from the circumf, to, and at right angles to, either diam, is called an *ordinate;* and the parts *c v* and *v w*, *b s* and *s a*, of that diam, between the ord and the circumf, are called *abscissæ*, or *abscisses.*

To find the length of any ordinate, *r v* or *s d*, drawn to either diam, *c w* or *b a*.

Knowing the abscis, *c v* or *s a*, and the two diams, *c w*, *b a*;

$c\,w^2 : b\,a^2 :: c\,v \times v\,w : r\,v^2.$ $b\,a^2 : c\,w^2 :: b\,s \times s\,a : s\,d^2.$

To find the circumf of an ellipse.

Mathematicians have furnished practical men with no simple working rule for this purpose. The so-called approximate rules do not deserve the name. They are as follows, D being the long diam: and *d* the short one.

RULE 1. Circumf $= 3.1416\,\dfrac{D+d}{2}$; RULE 2. $3.1416\sqrt{\left(\dfrac{D^2+d^2}{2}\right)}$; RULE 3. $2.2215\sqrt{D^2+d^2}$: this is the same as Rule 2, but in a diff shape. RULE 4. $2\times\sqrt{D^2+1.4674\,d^2}$. Now, in an ellipse whose long and short diams are 10 and 2, the circumf is actually 21, very approximately; but rule 1 gives it = 18.85; rule 2, or 3, = 22.65; and rule 4, = 20.58. Again, if the diams be 10 and 6, the circumf actually = 25.59; but rule 4 gives 24.72. These examples show that none of the rules usually given are reliable. The following one by the writer, is sufficiently exact for ordinary purposes; not being in error probably more than 1 part in 1000. When D is not more than 5 times as long as *d*.

$$\text{Circumf} = 3.1416\sqrt{\frac{D^2+d^2}{2}-\frac{(D-d)^2}{8.8}}$$

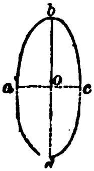
If D exceeds 5 times *d*, then instead of dividing $(D-d)^2$ by 8.8, div it by the number in this table.

D =	Div by
6 *d*	9
7 *d*	9.2
8 *d*	9.3
9 *d*	9.35
10 *d*	9.4
12 *d*	9.5
14 *d*	9.6
16 *d*	9.68
18 *d*	9.75
20 *d*	9.8
25 *d*	9.87
30 *d*	9.92
40 *d*	9.98
50 *d*	10.04
60 *d*	10.10
70 *d*	10.17
80 *d*	10.25
100 *d*	10.35

The following rule originated with Mr. M. Arnold Pears, of New South Wales, Australia, and was by him kindly communicated to the author. Although not more accurate than our own, it is much neater.

$$\text{Circumf} = 3.1416\,d + 2(D-d) - \frac{d(D-d)}{\sqrt{(D+d)\times(D+2d)}}$$

The following table of semi-elliptic arcs was prepared by our rule.



To use this table, div the height or rise of the arc, by its span or chord. The quot will be the height of an arc whose span is 1. Find this quot in the column of heights; and take out the corresponding number from the col. of lengths. Mult this number by the actual span. The prod will be the reqd length.

When the height becomes .500 of the chord (as at the end of the table) the ellipse becomes a circle. When the height exceeds .500 of the chord, as in *a b c*, then take *a o*, or half the chord, as the rise; and div this rise by the long diam *b d*, for the quot to be looked for in the col of heights; and to be mult by long diam. We thus get the arc *b a d*, which is evidently equal to *a b c*.

TABLE OF LENGTHS OF SEMI-ELLIPTIC ARCS. (Original.)

Height + span.	Length = span × by	Height + span.	Length = span × by	Height + span.	Length = span × by	Height + span.	Length = span × by
.005	1.000	.130	1.079	.255	1.219	.380	1.390
.01	1.001	.135	1.084	.260	1.226	.385	1.397
.015	1.002	.140	1.089	.265	1.233	.390	1.404
.02	1.003	.145	1.094	.270	1.239	.395	1.412
.025	1.004	.150	1.099	.275	1.245	.400	1.419
.03	1.006	.155	1.104	.280	1.252	.405	1.426
.035	1.008	.160	1.109	.285	1.259	.410	1.434
.04	1.011	.165	1.115	.290	1.265	.415	1.441
.045	1.014	.170	1.120	.295	1.272	.420	1.449
.05	1.017	.175	1.125	.300	1.279	.425	1.456
.055	1.020	.180	1.131	.305	1.285	.430	1.464
.06	1.023	.185	1.137	.310	1.292	.435	1.471
.065	1.026	.190	1.142	.315	1.298	.440	1.479
.07	1.029	.195	1.147	.320	1.305	.445	1.486
.075	1.032	.200	1.153	.325	1.312	.450	1.494
.08	1.036	.205	1.159	.330	1.319	.455	1.501
.085	1.039	.210	1.165	.335	1.325	.460	1.509
.09	1.043	.215	1.171	.340	1.332	.465	1.517
.095	1.046	.220	1.177	.345	1.339	.470	1.524
.100	1.051	.225	1.183	.350	1.346	.475	1.532
.105	1.055	.230	1.189	.355	1.353	.480	1.540
.110	1.059	.235	1.196	.360	1.361	.485	1.547
.115	1.064	.240	1.202	.365	1.368	.490	1.555
.120	1.069	.245	1.207	.370	1.375	.495	1.563
.125	1.074	.250	1.213	.375	1.382	.500	1.571

Area of an ellipse = prod of diams × .7854. Ex. D = 10; $d = 6$. Then 10 × 6 × .7854 = 47.124 area. The area of an ellipse is a mean proportional between the areas of two circles, described on its two diams; therefore it may be found by mult together the areas of those two circles; and taking the sq rt of the prod. The area of an ellipse is therefore always greater than that of the circular section of the cylinder from which it may be supposed to be derived.

Diam of circ of same area as a given ellipse = $\sqrt{\text{Long diam} \times \text{short diam.}}$

To find the area of an elliptic segment whose base is parallel to either diam. Div the height of the segment, by that diam of which said height is a part. From the table of circular segments take out the tabular area opposite the quot. Mult together this area, the long diam, and the short diam.

To draw an ellipse. Having its long and short diams $a\,b$ and $c\,d$, Fig. 4.

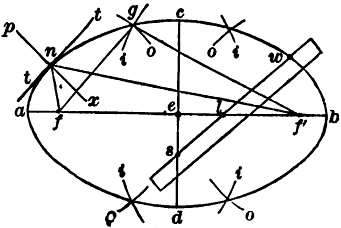


Fig. 4.

RULE 1. From either end of the short diam., as c, lay off the dists. $c\,f$, $c\,f'$, each equal to $e\,a$, or to one-half of the long diam. The points f, f' are the foci of the ellipse. Prepare a string, $f'\,n\,f$, or $f'\,g\,f$, with a loop at each end; the total length of string from end to end of loop, being equal to the long diam. Place pins at f and f'; and placing the loops over them, trace the curve by a pencil, which in every position, as at n, or g, keeps the string $f'\,n\,f$, or $f'\,g\,f$ stretched all the time.

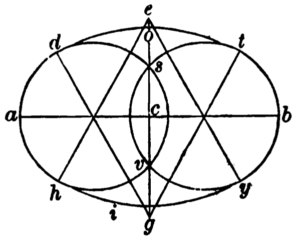
Note. Owing to the difficulty of keeping the string equally stretched, this method is not as satisfactory as the following.

RULE 2. On the edge of a strip of paper $w\,s$, mark $w\,l$ equal to half the short diam.; and $w\,s$ equal half the long diam. Then in whatever position this strip be placed, keeping l on the long diam., and s on the short diam., w will mark a point in the circumf. of the ellipse. We may thus obtain as many such points as we please; and then draw the curve through them by hand.

RULE 3. From the two foci f and f', Fig. 4, with a rad. equal to any part whatever of the long diam. describe 4 short arcs, $o\ o\ o\ o$; also with a rad. equal to the remaining part of the long diam., describe 4 other arcs, $i\ i\ i\ i$. The intersections of these four pairs of arcs, will give four points in the circumf. In this manner any number of such points may be found, and the curve be drawn by hand.

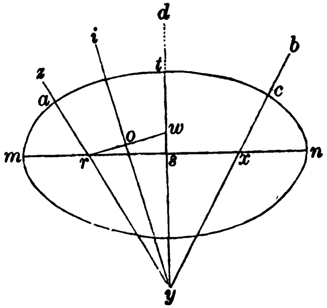
To draw a tangent $t\,t$, at any point n of an ellipse. Draw $n\,f$ and $n\,f'$, to the foci; bisect the angle $f\,n\,f'$ by the line $x\,p$; draw $t\,n\,t$ at right angles to $x\,p$.

To draw a joint $n\,p$, of an elliptic arch, from any point n, in he arch. Proceed as in the foregoing rule for a tangent, only omitting $t\,t$; $n\,p$ will be the quired joint.



To draw an oval, or false ellipse.

When only the long diam $a\,b$ is given, the following will give agreeable curves, of which the span $a\,b$ will not exceed about three times the rise $c\,o$. On $a\,b$ describe two intersecting circles of any rad; through their intersections s, v, draw $e\,g$; make $s\,g$ and $v\,e$ each equal to the diam of one of the circles. Through the centers of the circles, draw $e\,y$, $e\,h$, $g\,d$, $g\,t$. From e describe $h\,i\,y$; and from g describe $d\,o\,t$.



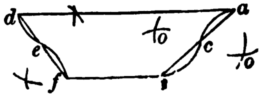
When the span, $m\,n$, and the rise, $s\,t$, are both given.

Make any $t\,w$ and $m\,r$, equal to each other, but each less than $t\,s$. Draw $r\,w$; and through its center o draw the perp $t\,o\,y$. Draw $y\,r\,z$. Make $n\,x$ equal $m\,r$, and draw $y\,x\,b$. From x and r describe $n\,c$ and $m\,a$; and from y describe $a\,t\,c$. By making $s\,d$ equal to $s\,y$, we obtain the center for the other side of the oval.

The beauty of the curve will depend upon what portion of $t\,s$ is taken for $m\,r$ and $t\,w$. When an oval is very flat, more than three centers are required for drawing a graceful curve; but the finding of these centers is quite as troublesome as to draw the correct ellipse.

On the given line, $a\,s$, to draw a cyma recta, $a\,c\,s$.

Find the center c, of $a\,s$. From a, c, and s, with one-half of $a\,s$ as rad, draw the four small arcs at o, o. The intersections o, o, are the centers for drawing the cyma, with the same rad. By reversing the position of the arcs, we obtain the *cyma reversa*, or *ogee*, $d\,e\,f$.



THE PARABOLA.

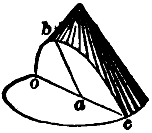


Fig. 1.

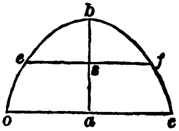


Fig. 2.

The common or conic parabola,

o b c, **Fig 1, is a** curve formed by cutting a cone in a direction *b a*, parallel to its side. The curved line *o b c* itself is called the *perimeter* of the parabola; the line *o c* is called its *base*; *b a* its height or *axis*; *b* its *apex* or *vertex*; any line *e s*, or *o a*, Fig 2, drawn from the curve, to, and at right angles to, the axis, is an *ordinate*; and the part *s b*, or *a b*, of the axis, between the ordinate and the apex *b*, is an *abscissa*. The *focus* of a parabola is that point in the axis, where the abscissa *b s*, is equal to one-half of the ord *e s*. The dist from apex to focus, called the *focal dist*, is found thus: square any ord, as *o a*; div this square by the abscissa *b a* of that ord; div the quot by 4. The nature of the parabola is such that its abscissas, as *b s*, *b a*, &c, are to each other as, or in proportion to, the squares of their respective ords *e s*, *o a*, &c; that is, as $bs : ba :: es^2 : oa^2$; or $bs : es^2 :: ba : oa^2$. If the square of any ord be divided by its abscissa, the quot will be a constant quantity; that is, it will be equal to the square of any other ord divided by its abscissa. This quot or constant quantity is also equal to a certain quantity called the *parameter* of the parabola. Therefore the parameter may be found by squaring *e s*, or *o a*, (one-*half* of the base,) and dividing said square by the height *b s*, or *b a*, as the case may be. If the square of any ord be divided by the parameter, the quot will be the abscissa of that ord.

To find the length of a parabolic curve.

The approximate rule given by various pocket-books, is as follows:

$$\text{Length} = 2 \times \sqrt{(\tfrac{1}{2}\text{ base})^2 + 1\tfrac{1}{3}\text{ times the }(\text{Height}^2)}$$

Where the height does not exceed 1-10th of the base, this rule may, for practical purposes, be called exact. With ht = ¼ base, it gives about ½ per cent too much; ht = ½ base, about 3⅓ per cent; ht = base, about 8⅓ per cent; ht = twice the base, about 12⅓ per cent; ht = 10 × base, or more, about 15½ per cent

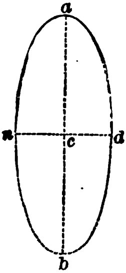


Fig. 4.

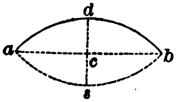


Fig. 3.

The following by the writer is correct within perhaps 1 part in 800, in all cases; and will therefore answer for many purposes.

Let *a d b*, Fig 3, or *n a d*, Fig 4, be the parabola, in which are given the base *a b* or *n d*; and the height *c d* or *c a*. Imagine the complete fig *a d b s*, or *n a d b*, to be drawn; and in *either* case, assume its *long* diam *a b* to be the chord or base; and one-half the short diam, or *c d*, to be the height, of a circular arc. Find the length of this circular arc, by means of the rule and table given for that purpose. Then div the chord or base *a b*, or *n d* of the parabola, by its height *c d* or *c a*. Look for the quot in the column of bases in the following table, and take from the table the corresponding multiplier. Mult the length of the circular arc by this; the prod will be the length of arc *a d b*, or *n a d*, as the case may be. For bases of parabolas less than .05 of the height, or greater than 10 times the height, the multiplier is 1, and is very approximate; or in other words, the parabola will be of almost exactly the same length as the circular arc.

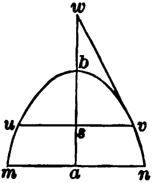


Fig. 5.

To find the area of a parabola *m a n b*.

Mult its base *m n*, Fig 5, by its height *a b*; and take ⅔ds of the prod. The area of any segment, as *u b v*, whose base *u v* is parallel to *m n*, is found in the same way, using *u v* and *s b*, instead of *m n* and *a b*.

To find the area of a parabolic zone, or frustum, as *m n u v*.

Rule 1. First find by the preceding rule the area of the whole parabola *m b n*; then that of the segment *u b v*; and subtract the last from the first.

Rule 2. From the cube of *m n*, take the cube of *u v*; call the diff *c*. From the square of *m n*, take the square of *u v*; call the diff *s*. Div *c* by *s*. Mult the quot by ⅔ds of the height *a s*.

Table for Lengths of Parabolic Curves. See opp page. (Original.)

Base.	Mult.	Base.	Mult.	Base.	Mult.	Base.	Mult.
.05	1.000	1.10	.999	2.15	.949	3.20	.983
.10	1.001	1.15	.997	2.20	.951	3.30	.984
.15	1.002	1.20	.995	2.25	.954	3.40	.985
.20	1.004	1.25	.993	2.30	.956	3.50	.986
.25	1.006	1.30	.990	2.35	.958	3.60	.987
.30	1.007	1.35	.987	2.40	.960	3.70	.988
.35	1.007	1.40	.984	2.45	.962	3.80	.989
.40	1.008	1.45	.980	2.50	.963	3.90	.990
.45	1.009	1.50	.977	2.55	.965	4.00	.991
.50	1.010	1.55	.974	2.60	.967	4.25	.992
.55	1.010	1.60	.970	2.65	.969	4.50	.993
.60	1.010	1.65	.966	2.70	.970	4.75	.994
.65	1.011	1.70	.963	2.75	.972	5.00	.995
.70	1.011	1.75	.960	2.80	.973	5.25	.996
.75	1.010	1.80	.957	2.85	.975	5.50	.997
.80	1.009	1.85	.953	2.90	.976	5.75	.998
.85	1.008	1.90	.950	2.95	.978	6.00	.998
.90	1.006	1.95	.946	3.00	.979	7.00	.999
.95	1.004	2.00	.942	3.05	.980	8.00	1.000
1.00	1.002	2.05	.944	3.10	.981	10.00	1.000
1.05	1.001	2.10	.946	3.15	.982		

To draw a parabola, having base *c s* and height *e o*.

e o s, Fig 6. Make *o t* equal to the height *e o*. Draw *c t* and *s t*; and divide each of them into any number of equal parts; numbering them as in the Fig. Join 1, 1; 2, 2; 3, 3, &c; then draw the curve by hand. It will be observed that the intersections of the lines 1, 1; 2, 2, &c, do not give points in the curve; but a portion of each of those lines forms a tangent to the curve. By increasing the number of divisions on *c t* and *s t*, an almost perfect curve is formed, scarcely requiring to be touched up by hand. In practice it is best first to draw only the center portions of the two lines which cross each other just above *o*; and from them to work downward; actually drawing only that small portion of each successive lower line, which is necessary to indicate the curve.

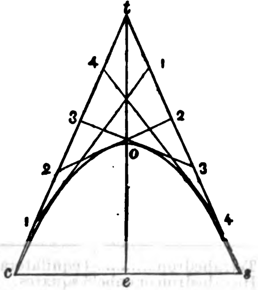


Fig. 6.

Or the parabola may be drawn thus:

Let *b c*, Fig 7, be the base; and *a d* the height. Draw the rectangle *b n m c*; div each half of the base into any number of equal parts, and number them from the center each way. Div *n b*, and *m c* into the same number of equal parts; and number them from the top, downward. From the points on *b c* draw vert lines; and from those at the sides draw lines to *d*. Then the intersections of lines 1, 1; 2, 2, &c, will form points in the parabola. As in the preceding case, it is not necessary to draw the entire lines; but merely portions of them, as shown between *d* and *c*.

Or a parabola may be drawn by first div the height *a b*, Fig 5, into any number of parts, either equal or unequal; and then calculating the ordinates *u s*, &c; thus, as the height *a b* : square of half base *a m* : : any absciss *b s* : square of its ord *u s*. Take the sq rt for *u s*.

Rem.—When the height of a parabola is not greater than 1-10th part its base, the curve coincides so *very* closely with that of a circular arc, that in the preparation of drawings for suspension bridges, &c., the circular arc may be employed; or if no great accuracy is reqd, the circle may be used even when the height is as great as one-eighth of the base.

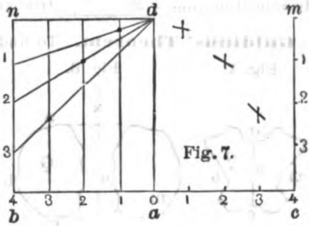


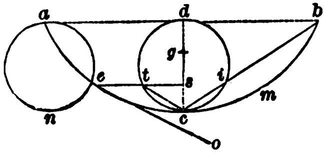
Fig. 7.

To draw a tangent *w v*, Fig. 5, to a parabola, from any point *v*.

Draw *v s* perp to axis *a b*; prolong *a b* until *b w* equals *s b*. Join *w v*.

The Cycloid,

$a c b$, is the curve described by a point a in the circumference of a circle, $a n$, during one complete revolution of the circle, rolled along a straight line $a b$; which is called the base of the cycloid.



The vertex of the cycloid is at c.

Base, $a b$, = circumference of generating circle $a n$
= diameter, $c d$, of generating circle $\times \pi = 3.1416 c d$.

Axis, or height, $c d = a n$.

Length, $a c b, = 4 c d$.

Area, $a c b d = 3 \times$ area of generating circle, $a n$

$$= 3 \frac{c d^2 \pi}{4} = c d^2 \times \tfrac{3}{4}\pi = c d^2 \times 2.3562.$$

Center of gravity of *surface* at g. $c g = \frac{7}{12} c d$. Center of gravity of *cycloid* (curved line $a c b$) in axis $c d$ at a point (as s) distant $\frac{1}{3} c d$ from c.

To draw a tangent, $e o$, from any point e in a cycloid; draw $e s$ at right angles to the axis $c d$; on $c d$ describe the generating circle $d c t$; join $t c$; from e draw $e o$ parallel to $t c$. The cycloid is the **curve of quickest descent;** so that a body would fall from b to c along the curve $b m c$, in less time than along the inclined plane $b i c$, or any other line.

SOLIDS.

THE REGULAR BODIES.

A regular body, or regular polyhedron, is one which has all its sides, and its solid angles, respectively similar and equal to each other. There are but five such bodies, as follows:

Name.	Bounded by	Surface (= sum of surfaces of all the faces). Multiply the square of the length of one edge by	Volume. Multiply the cube of the length of one edge by
Tetrahedron	4 equilateral triangles.	1.7320	.1178
Hexahedron or cube	6 squares.	6.	1.
Octahedron	8 equilateral triangles.	3.4641	.4714
Dodecahedron.........	12 " pentagons.	20.6458	7.6631
Icosahedron............	20 " triangles.	8.6602	2.1817

Guldinus' Theorem. To find the **volume of any body** (as the irregular mass $a b c m$, Fig A, or the ring $a b c m$, Fig B), generated by a complete or partial revolution of any figure (as $a b c a$) around one of its sides (as $a c$, Fig A), or around any other axis (as $x y$, Fig B).

Fig. A.

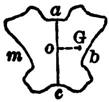
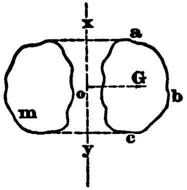


Fig. B.



Volume = surface $a b c a \times$ length of arc described by its center of gravity G.

If the revolution is complete, the arc described is = circumference = radius o G* $\times 2\pi$ = radius o G* $\times$ 6.283186; and **Volume** = surface $a b c a \times$ radius o G* $\times$ 6.283186.

If the revolution is incomplete,

$$\text{complete revolution} : \text{incomplete revolution} :: \text{circumference found as above} : \text{arc described}$$

---

* Measured perpendicularly to the axis of revolution.

PARALLELOPIPEDS.

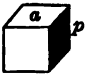


Fig. 1.

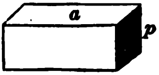


Fig. 2.

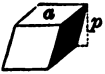


Fig. 3.

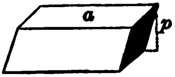


Fig. 4.

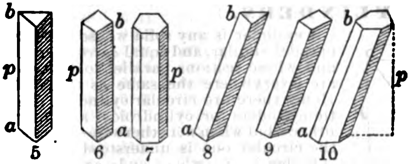
A parallelopiped is any solid contained within six sides, all of which are parallelograms; and those of each opposite pair, parallel to each other. We show but four of them; corresponding to the four parallelograms; namely, the *cube*, Fig 1, which has all its sides equal squares, and all its angles right angles; the *right rectangular prism*, Fig 2, has all its angles right angles, each pair of opposite faces equal, but not all of its faces equal; the *Rhombohedron*, Fig 3, which has all its sides equal rhombuses, and which, like the rhombus, p 157, is sometimes called "rhomb"; the *Rhombic* prism, Fig 4; its faces, rhombuses, or rhomboids, each pair of opposite faces equal, but not all its faces equal. All parallelopipeds are prisms.

Volume of any parallelopiped = area of any face, as a, × *perpendicular* distance, p, to the opposite face.

Volume of a cube = cube of length of one edge,
= 1.90985 × volume of inscribed sphere,
= 1.27324 × " " cylinder,
= 3.81972 × " " cone.

Diagonal of a cube = diameter of circumscribing sphere,
= 1.7320508 × length of one edge of cube.

PRISMS.

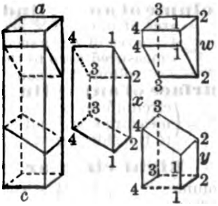


A prism is any solid whose two *ends* are parallel, similar, and equal; and whose *sides are parallelograms*, as Figs 5 to 10. Consequently the foregoing parallelopipeds are prisms. A *right* prism is one whose sides are perpendicular to its ends, as 5, 6, 7; when not so, the prism is *oblique*, as 8, 9, 10. When all the sides of the figures which form the ends are equal, and the angles included between those sides are also equal, the prism is said to be *regular*: otherwise, *irregular*.

Volume of any prism (whether regular or irregular, right or oblique)
= area of one end × *perpendicular* distance, p, to the other end,
= area of cross section perpendicular to the sides × actual length, $a\,b$, Figs 5 to 10,
= 3 × volume of pyramid whose base and height are = those of the prism.

To find the volume of any frustum of any prism,

Whose cross section, perpendicular to its sides, is either any triangle; any parallelogram; a square, (as in Fig 10¼) or a *regular* polygon of any number of sides; no matter how the two ends of the frustum may be inclined with regard to each other; or whether one, or neither of them, is parallel to the base of the original prism.



Figs. 10¼.

$$\textbf{Volume of frustum} = \frac{\text{sum of lengths of \textit{parallel} edges, } \overline{1\,1} + \overline{2\,2} + \overline{3\,3} + \overline{4\,4}}{\text{number of such edges (4 in Fig 10¼)}} \times \text{area of cross section \textit{perpendicular} to such edges.}$$

This rule may be used for ascertaining beforehand, the quantity of earth to be removed from a "borrow pit." The irregular surface of the ground is first staked out in squares; (the tape-line being stretched *horizontally*, when measuring off their sides). These squares should be of such a size that without material error each of them may be considered to be a plane surface, either horizontal or inclined. The depth of the horizontal bottom of the pit being determined on, and the levels being taken at every corner of the squares, we are thereby furnished with the lengths of the four parallel vertical edges of each of the resulting frustums of earth. In Figs 10¼ *y* may be supposed to represent one of these frustums.

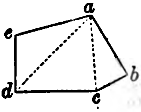


Fig. 10½.

If the frustum is that of an *irregular* 4-sided, or polygonal prism, first divide its cross section perpendicular to its sides, into triangles, by lines drawn from any one of its angles, as *a*, Fig 10½. Calculate the area of each of these triangles separately; then consider the entire frustum to be made up of so many triangular ones; calculate the volume of each of these by the preceding rule for triangular frustums; and add them together, for the volume of the entire frustum.

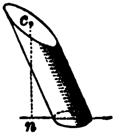


Fig. 10¾.

Volume of any frustum of any prism,

Or of a cylinder. Consider either end to be the base; and find its area. Also find the center of gravity *c* of the other end, and the *perpendicular* distance *n c*, from the base to said center of gravity. Then **Volume of frustum** = area of base × *n c*, Fig 10¾.

The slant end, *c*, is an ellipse. Its area is greater than that of the circular end.

Surface of any prism, Figs 5 to 10, whether right or oblique, regular or irregular

$$= \left(\begin{array}{c}\text{circumference measured}\\ \text{perpendicular to the sides}\end{array} \times \text{actual length, } a\,b\right) + \begin{array}{c}\text{sum of the areas}\\ \text{of the two ends.}\end{array}$$

CYLINDERS.

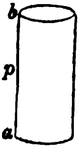


Fig. 11.

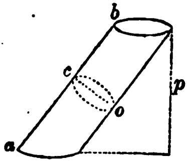


Fig. 12.

A cylinder is any solid whose ends are parallel, similar, and equal *curved* figures; and whose sections parallel to the ends are everywhere the same as the ends. Hence there are circular cylinders, elliptic cylinders (or cylindroids) and many others; but when not otherwise expressed, the circular one is understood. A *right* cylinder is one whose ends are perpendicular to its sides, as Fig. 11; when otherwise, it is *oblique*, as Fig 12. If the ends of a right circular cylinder be cut so as to make it oblique, it becomes an elliptic one; because then both its ends, and all sections parallel to them, are ellipses. An oblique circular cylinder seldom occurs; it may be conceived of by imagining the two ends of Fig 12 to be circles, united by straight lines forming its curved sides

A cylinder is a prism having an infinite number of sides.

Volume of any cylinder (whether circular or elliptic, &c, right or oblique)

= area of one end × *perpendicular* distance, *p*, to the other end,

$$= \left\{\begin{array}{c}\text{area of cross section}\\ \text{measured perp to the sides}\end{array}\right. \times \text{actual length, } a\,b\text{, Figs 11 and 12,}$$

= 3 × volume of a cone whose base and height are = those of the cylinder.

Surface of any cylinder (whether circular or elliptic, &c, right or oblique)

$$= \left(\begin{array}{l}\text{circumference}\\ \text{measured perpendicularly}\\ \text{to the sides, as at } c\,o\text{, Fig 12,}\end{array} \times \text{actual length, } a\,b\right) + \begin{array}{c}\text{sum of the areas}\\ \text{of the two ends.}\end{array}$$

Right circular cylinder whose height = diameter.

Volume = 1½ × volume of inscribed sphere.
Curved surface = surface of inscribed sphere.
Area of one end = ¼ surface of inscribed sphere = ¼ curved surface.
Entire surface = 1½ × surface of inscribed sphere = 1½ × curved surface.

Contents for one foot in length, in Cub Ft, and in U. S. Gallons of 231 cub ins, or 7.4805 Galls to a Cub Ft. **A cub ft of water weighs** about 62½ lbs; and a gallon about 8⅓ lbs. **Diams 2, 3, or 10 times as great,** give 4, 9, or 100 times the content.

Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft in length. Cub. Feet. Also area in sq. ft.	For 1 ft in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.
¼	.0208	.0003	.0025	¾	.5625	.2485	1.859	19.	1.583	1.969	14.73
5-16	.0260	.0005	.0040	7.	.5833	.2673	1.999	½	1.625	2.074	15.51
⅜	.0313	.0008	.0057	¼	.6042	.2867	2.145	20.	1.667	2.182	16.32
7-16	.0365	.0010	.0078	½	.6250	.3068	2.295	½	1.708	2.292	17.15
½	.0417	.0014	.0102	¾	.6458	.3276	2.450	21.	1.750	2.405	17.99
9-16	.0469	.0017	.0129	8.	.6667	.3491	2.611	½	1.792	2.521	18.86
⅝	.0521	.0021	.0159	¼	.6875	.3712	2.777	22.	1.833	2.640	19.75
11-16	.0573	.0026	.0193	½	.7083	.3941	2.948	½	1.875	2.761	20.66
¾	.0625	.0031	.0230	¾	.7292	.4176	3.125	23.	1.917	2.885	21.58
13-16	.0677	.0036	.0269	9.	.7500	.4418	3.305	½	1.958	3.012	22.53
⅞	.0729	.0042	.0312	¼	.7708	.4667	3.491	24.	2.000	3.142	23.50
15-16	.0781	.0048	.0359	½	.7917	.4922	3.682	25.	2.083	3.409	25.50
1.	.0833	.0055	.0408	¾	.8125	.5185	3.879	26.	2.167	3.687	27.58
¼	.1042	.0085	.0638	10.	.8333	.5454	4.080	27.	2.250	3.976	29.74
½	.1250	.0123	.0918	¼	.8542	.5730	4.286	28.	2.333	4.276	31.99
¾	.1458	.0167	.1249	½	.8750	.6013	4.498	29.	2.417	4.587	34.31
2.	.1667	.0218	.1632	¾	.8958	.6303	4.715	30.	2.500	4.909	36.72
¼	.1875	.0276	.2066	11.	.9167	.6600	4.937	31.	2.583	5.241	39.21
½	.2083	.0341	.2550	¼	.9375	.6903	5.164	32.	2.667	5.585	41.78
¾	.2292	.0412	.3085	½	.9583	.7213	5.396	33.	2.750	5.940	44.43
3.	.2500	.0491	.3672	¾	.9792	.7530	5.633	34.	2.833	6.305	47.13
¼	.2708	.0576	.4309	12.	1 Foot.	.7854	5.875	35.	2.917	6.681	49.98
½	.2917	.0668	.4998	½	1.042	.8522	6.375	36.	3.000	7.069	52.88
¾	.3125	.0767	.5738	13.	1.083	.9218	6.895	37.	3.083	7.467	55.86
4.	.3333	.0873	.6528	½	1.125	.9940	7.436	38.	3.167	7.876	58.92
¼	.3542	.0985	.7369	14.	1.167	1.069	7.997	39.	3.250	8.296	62.06
½	.3750	.1104	.8263	½	1.208	1.147	8.578	40.	3.333	8.727	65.28
¾	.3958	.1231	.9206	15.	1.250	1.227	9.180	41.	3.417	9.168	68.58
5.	.4167	.1364	1.020	½	1.292	1.310	9.801	42.	3.500	9.621	71.97
¼	.4375	.1503	1.125	16.	1.333	1.396	10.44	43.	3.583	10.085	75.44
½	.4583	.1650	1.234	½	1.375	1.485	11.11	44.	3.667	10.559	78.99
¾	.4792	.1803	1.349	17.	1.417	1.576	11.79	45.	3.750	11.045	82.62
6.	.5000	.1963	1.469	½	1.458	1.670	12.49	46.	3.833	11.541	86.33
¼	.5208	.2131	1.594	18.	1.500	1.767	13.22	47.	3.917	12.048	90.13
½	.5417	.2304	1.724	½	1.542	1.867	13.96	48.	4.000	12.566	94.00

Table continued, but with the diams in feet.

Diam. Feet.	Cub. Feet.	U. S. Galls.	Diam. Feet.	Cub. Feet.	U. S. Galls.	Dia. Feet.	Cub. Feet.	U. S. Galls.	Dia. Feet.	Cub. Feet.	U. S. Galls.
4	12.57	94.0	7	38.48	287.9	12	113.1	846.0	24	452.4	3384
¼	14.19	106.1	¼	41.28	308.8	13	132.7	992.9	25	490.9	3672
½	15.90	119.0	½	44.18	330.5	14	153.9	1152.	26	530.9	3972
¾	17.72	132.6	¾	47.17	352.9	15	176.7	1322.	27	572.6	4283
5	19.63	146.9	8	50.27	376.0	16	201.1	1504.	28	615.8	4606
¼	21.65	161.9	½	56.75	424.5	17	227.0	1698.	29	660.5	4941
½	23.76	177.7	9	63.62	475.9	18	254.5	1904.	30	706.9	5288
¾	25.97	194.2	½	70.88	530.2	19	283.5	2121.	31	754.8	5646
6	28.27	211.5	10	78.54	587.5	20	314.2	2350.	32	804.2	6016
¼	30.68	229.5	½	86.59	647.7	21	346.4	2591.	33	855.3	6398
½	33.18	248.2	11	95.03	710.9	22	380.1	2844.	34	907.9	6792
¾	35.78	267.7	½	103.87	777.0	23	415.5	3108.	35	962.1	7197

CONTENTS AND LININGS OF WELLS.

For diams twice as great as those in the table, for the cub yds of digging, take out those opposite *one half* of the greater diam; and mult them by 4. Thus, for the cub yds in each foot of depth of a well 31 feet in diam, first take out from the table those opposite the diam of 15½ feet; namely, 6.989. Then 6.989 × 4 = 27.956 cub yds reqd for the 31 ft diam. But for the stone lining or walling, bricks or plastering, mult the tabular quantity opposite *half* the greater diam, by 2. Thus, the perches of stone walling for each foot of depth of a well of 31 ft diam, will be 2.073 × 2 = 4.146. If the wall is more or less than one foot thick, within usual moderate limits, it will generally be near enough for practice to assume that the number of perches, or of bricks, will increase or decrease in the same proportion.

The size of the bricks is taken at 8¼ × 4 × 2 inches; and to be laid dry, or without mortar. In practice an addition of about 5 per cent should be made for waste. The brick lining is supposed to be 1 brick thick, or 8¼ ins.

CAUTION. — Be careful to observe that the diams to be used for the digging, are greater than those for the walling, bricks, or plastering. No errors.

Diam. in Feet.	For each foot of depth.				Diam. in Feet.	For each foot of depth.			
	For this col use the Diameter of the Digging.	For these three cols use the diam in clear of the lining.				For this col use the Diameter of the Digging.	For these three cols use the diam in clear of the lining.		
	Cub Yds. of Digging.	Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.		Cub Yds. of Digging.	Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.
1.	.0291	.2513	57	.3491	¼	5.107	1.791	750	4.625
¼	.0455	.2827	71	.4364	½	5.301	1.822	764	4.713
½	.0654	.3142	85	.5236	¾	5.500	1.854	778	4.800
¾	.0891	.3456	99	.6109	14.	5.701	1.885	792	4.887
2.	.1164	.3770	114	.6982	¼	5.907	1.916	806	4.974
¼	.1473	.4084	128	.7855	½	6.116	1.948	820	5.062
½	.1818	.4398	142	.8727	¾	6.329	1.979	834	5.149
¾	.2200	.4712	156	.9600	15.	6.545	2.011	849	5.236
3.	.2618	.5027	170	1.047	¼	6.765	2.042	863	5.323
¼	.3073	.5341	184	1.135	½	6.989	2.073	877	5.411
½	.3563	.5655	198	1.222	¾	7.216	2.105	891	5.498
¾	.4091	.5969	212	1.309	16.	7.447	2.136	905	5.585
4.	.4654	.6283	227	1.396	¼	7.681	2.168	919	5.673
¼	.5254	.6597	241	1.484	½	7.919	2.199	933	5.760
½	.5890	.6912	255	1.571	¾	8.161	2.231	948	5.847
¾	.6563	.7226	269	1.658	17.	8.407	2.262	962	5.934
5.	.7272	.7540	283	1.745	¼	8.656	2.293	976	6.022
¼	.8018	.7854	297	1.833	½	8.908	2.325	990	6.109
½	.8799	.8168	311	1.920	¾	9.165	2.356	1004	6.196
¾	.9617	.8482	326	2.007	18.	9.425	2.388	1018	6.283
6.	1.047	.8796	340	2.095	¼	9.688	2.419	1032	6.371
¼	1.136	.9111	354	2.182	½	9.956	2.450	1046	6.458
½	1.229	.9425	368	2.269	¾	10.23	2.482	1061	6.545
¾	1.325	.9739	382	2.356	19.	10.50	2.513	1075	6.633
7.	1.425	1.005	396	2.444	¼	10.78	2.545	1089	6.720
¼	1.529	1.037	410	2.531	½	11.06	2.576	1103	6.807
½	1.636	1.068	425	2.618	¾	11.35	2.608	1117	6.894
¾	1.747	1.100	439	2.705	20.	11.64	2.639	1131	6.982
8.	1.862	1.131	453	2.793	¼	11.93	2.670	1145	7.069
¼	1.980	1.162	467	2.880	½	12.22	2.702	1160	7.156
½	2.102	1.194	481	2.967	¾	12.52	2.733	1174	7.243
¾	2.227	1.225	495	3.054	21.	12.83	2.765	1188	7.331
9.	2.356	1.257	509	3.142	¼	13.14	2.796	1202	7.418
¼	2.489	1.288	523	3.229	½	13.45	2.827	1216	7.505
½	2.625	1.319	538	3.316	¾	13.76	2.859	1230	7.592
¾	2.765	1.351	552	3.404	22.	14.08	2.890	1244	7.680
10.	2.909	1.382	566	3.491	¼	14.40	2.922	1259	7.767
¼	3.056	1.414	580	3.578	½	14.73	2.953	1273	7.854
½	3.207	1.445	594	3.665	¾	15.06	2.985	1287	7.942
¾	3.362	1.477	608	3.753	23.	15.39	3.016	1301	8.029
11.	3.520	1.508	622	3.840	¼	15.72	3.047	1315	8.116
¼	3.682	1.539	637	3.927	½	16.06	3.079	1329	8.203
½	3.847	1.571	651	4.014	¾	16.41	3.110	1343	8.291
¾	4.016	1.602	665	4.102	24.	16.76	3.142	1357	8.378
12.	4.189	1.634	679	4.189	¼	17.11	3.173	1372	8.465
¼	4.365	1.665	693	4.276	½	17.46	3 204	1386	8.552
½	4.545	1.696	707	4.364	¾	17.82	3,236	1400	8.640
¾	4.729	1.728	721	4.451	25.	18.18	3.267	1414	8.727
13.	4.916	1.759	736	4.538					

A cub yd = 202 U. S. galls.

If **perches** are named in a contract, it is necessary, in order to prevent fraud, to specify the number of cub feet contained in the perch; for stone-quarriers have one perch, stone-masons another, &c. Engineers, on this account, contract by the *cubic yard*. The perch should be done away with entirely; Perches of 25 cub ft × .926 = cub yds; and cub yds ÷ .926 = pers of 25 cub ft.

CIRCULAR CYLINDRIC UNGULAS.

I. When the cutting plane does not cut the base. Figs 13, 14.

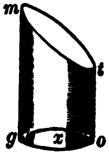


FIG. 13.

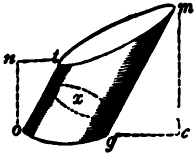


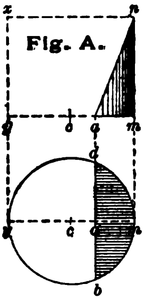
FIG. 14.

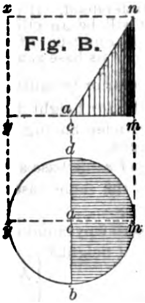
Volume of ungula } = area of base $o\,g$ × ½ sum of greatest & least perp heights, $o\,n$, $c\,m$,
= { area of cross sec meas'd perp to sides, as x, × ½ sum of greatest and least lengths, $g\,m$, $o\,t$, meas'd along the sides.

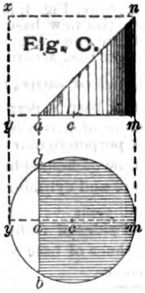
Area of *curved* surface } = { circumf meas'd perp to sides, as at x, × half sum of greatest and least lengths, gm, $o\,t$, meas'd along the sides.

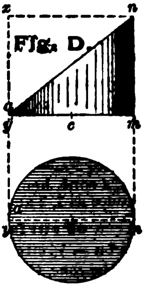
Add areas of ends if required.
For areas of sections perpendicular to the sides, see Circles.
For areas of sections oblique to the sides, see The Ellipse.

II. When the cutting plane touches the base. Figs A to D.









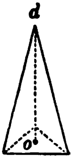
Volume (whether right or oblique)

Fig **A** = $(\frac{2}{3}\,ab^3 - ac \times$ area $a\,d\,m\,b$ of base$)\,\frac{m\,n}{a\,m}$.
Fig **B** = $\frac{2}{3}\,c\,b^2 \times m\,n$.
Fig **C** = $(\frac{2}{3}\,ab^3 + ac \times$ area $a\,d\,m\,b$ of base$)\,\frac{m\,n}{a\,m}$.
} $m\,n$ meas'd *perp* to base
Fig **D** = ½ area of circle $y\,m$ × $m\,n$
= ½ volume of cylinder $x\,y\,m\,n$.

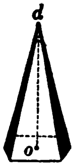
Curved surface (*right* ungula only)

Fig **A** = $(ab \times my - ac \times$ length of arc $d\,m\,b$ $)\,\frac{mn}{am}$.
Fig **B** = $my \times mn$.
Fig **C** = $(ab \times my + ac \times$ length of arc $d\,m\,b$ $)\,\frac{mn}{am}$.
Fig **D** = ½ circumference of base $my \times mn$
= ½ curved surface of cylinder $x\,y\,m\,n$.

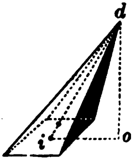
PYRAMIDS AND CONES.



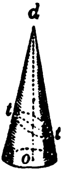
1



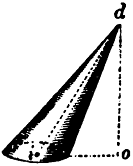
2



3



4



5

A pyramid, Figs. 1, 2, 3, is any solid which has, for its base, a plane figure of any number of sides, and, for its sides, plane triangles all terminating at one point *d*, called its *apex*, or top. When the base is a regular figure, the pyramid is *regular*, otherwise *irregular*.

A cone, Figs. 4 and 5, is a solid, of which the base is a curved figure; and which may be considered as made or generated by a line, of which one end is stationary at a certain point *d*, called the apex or top, while the line is being carried around the circumference of the base, which may be a circle, ellipse, or other curve. A cone may also be regarded as a pyramid with an infinite number of sides.

The axis of a pyramid or cone, is a straight line *d o* in Figs. 1, 2, 4; and *d i* in Figs. 3 and 5, from the apex *d*, to the center of gravity of the base. When the axis is perpendicular to the base, as in Figs. 1, 2, 4, the solid is said to be a *right* one; when otherwise, as Figs 3, 5, an *oblique* one. When the word cone is used alone, the right circular cone, Fig. 4, is understood. If such a cone be cut, as at *t t*, obliquely to its base, the new base *t t* will be an ellipse; and the cone *d t t* becomes an oblique elliptic one. Fig. 5 will represent either an oblique circular cone, or an oblique elliptic one, according as its base is a circle or an ellipse.

Volume of pyramid or cone, regular or irregular, right or oblique.

Volume = ⅓ area of base × *perpendicular* height *d o*, Figs. 1 to 5.
= ⅓ volume of prism or cylinder having same area of base and same perpendicular height.
= ½ volume of hemisphere of same base and same height.

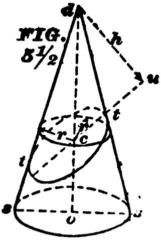
Or, a cone, hemisphere and cylinder, of the same base and same height, have volumes as 1, 2 and 3.

Area of surface of sides of right regular pyramid or right circular cone.

Area = ½ circumference of base × slant height.*
In the *cone*, this becomes
Area = $\frac{\text{area of base}}{\text{radius of base}}$ × slant height.

} Add area of base if required.

FIG. 5½



Area of surface of oblique elliptic cone, *d t t*, Fig. 5½, cut from a right circular cone, *d s s*. From the point *c* where the axis *d o* of the right circular cone cuts the elliptic base *t t*, measure a perpendicular, *r*, in any direction, to the curved surface of the cone. Let v = the volume of oblique elliptic cone, *d t t*; let a = the area of its elliptic base *t t*, and let h = the height *d u* measured perpendicularly to said base. Then

$$\textbf{Curved surface} = \frac{a\,h}{r} = \frac{3\,v}{r}.$$

Add area of base if required

No measurement has been devised for the surface of an **oblique circular cone.**

---

* In the *pyramid*, this slant height must be measured along the middle of one of the sides, and not along one of the edges.

To find the surface of an irregular pyramid.

Whether right or oblique, each side must be calculated as a separate triangle (see p. 148); and the several areas added together. Add the area of base if required.

FRUSTUMS OF PYRAMIDS AND CONES.

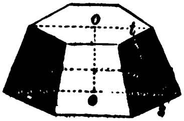


Fig. 6.

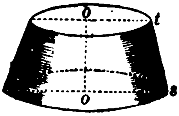


Fig. 7.

Frustum of pyramid (Fig. 6) or of cone (Fig. 7) with base and top parallel.

Volume (regular or irregular, right or oblique)

$$= \frac{1}{3} \times \begin{matrix}\text{perpendicular}\\ \text{height } o\,o\end{matrix} \times \left(\begin{matrix}\text{area}\\ \text{of top}\end{matrix} + \begin{matrix}\text{area}\\ \text{of base}\end{matrix} + \sqrt{\begin{matrix}\text{area}\\ \text{of top}\end{matrix} \times \begin{matrix}\text{area}\\ \text{of base}\end{matrix}}\right)$$

$$= \frac{1}{6} \times \begin{matrix}\text{perpendicular}\\ \text{height } o\,o\end{matrix} \times \left(\begin{matrix}\text{area}\\ \text{of top}\end{matrix} + \begin{matrix}\text{area}\\ \text{of base}\end{matrix} + \begin{matrix}4 \times \text{area of a section}\\ \text{parallel to, and midway}\\ \text{between, base and top}\end{matrix}\right)$$

= (for frustum of right or oblique *circular cone only*; See Fig. 7)

$$\frac{1}{3} \times \begin{matrix}\text{perpendicular}\\ \text{height } o\,o\end{matrix} \times 3.1416 \times \left(o\,t^2 + o\,s^2 + o\,t \,.\, o\,s\right)$$

Surface of frustum of *right regular* pyramid or cone, with top and base parallel, Figs. 6 and 7.

$$= \frac{1}{2}\left(\begin{matrix}\text{circumference}\\ \text{of top}\end{matrix} + \begin{matrix}\text{circumference}\\ \text{of base}\end{matrix}\right) \times \begin{matrix}\text{slant}\\ \text{height } t\,s\end{matrix}\,{}^{*}$$

Add areas of top and base if required.

In the frustum of a right circular cone, this becomes

$$\text{Surface} = \pi \left(\begin{matrix}\text{radius}\\ \text{of top}\end{matrix} + \begin{matrix}\text{radius}\\ \text{of base}\end{matrix}\right) \times \begin{matrix}\text{slant}^{*}\\ \text{height } s\,t\end{matrix}$$

$(\pi = 3.1416)$. Add areas of top and base if required.

Frustum of irregular or oblique pyramid or cone. Surface = sum of surfaces of sides, each of which must be treated as a trapezoid.

---

[*] In the frustum of the *pyramid* (Fig 6), this slant height must be measured along the middle of one of the sides (as at *t s*), and not along one of the edges.

PRISMOIDS.

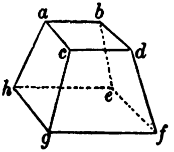


Fig. 1.

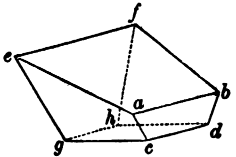


Fig. 2.

A prismoid is sometimes defined as a solid having for its ends two parallel plane figures, connected by other plane figures on which, and through every point of which, a straight line may be drawn from one of the two parallel ends to the other. These connecting planes may be parallelograms or not, and parallel to each other or not.

This definition would include the cube and all other parallelopipeds; the prism; the cylinder (considered as a prism having an infinite number of sides); the pyramid and cone (in which one of the two parallel ends, *i e* the one forming the apex, is considered to be infinitely small), and their frustums with top and base parallel; and the wedge.

But the use of the term **prismoid** is frequently **restricted** to six-sided solids, in which the two parallel ends are unequal quadrangles; and the connecting planes, trapezoids; as in Figs. 1 and 2; and, by some writers, to cases where the parallel quadrangular ends are *rectangles*.

The following "**prismoidal formula**" applies to all the foregoing solids, and to others, as noted below.

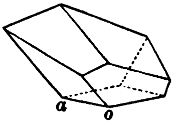
Let A = the area of one of the two parallel ends.
a = " " the other of the two parallel ends.
M = " " a cross section midway between, and parallel to, the two parallel ends.
L = the perpendicular distance between the two parallel ends.

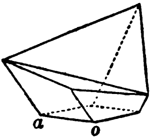
Then

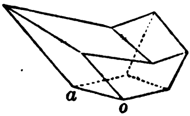
$$\textbf{Volume} = L \times \frac{A + a + 4M}{6}$$

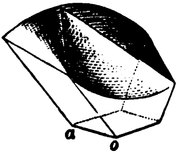
$$= L \times \text{mean area of cross section.}$$

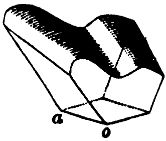
The following six figures represent a few of the irregular solids which fall under the above broad definition of "prismoid," and to which the prismoidal formula applies. They may be regarded as one-chain lengths of railroad cuttings; *a o* being the length, or perpendicular (horizontal) distance between the two parallel (vertical) ends.

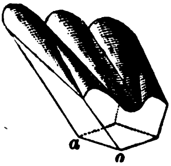




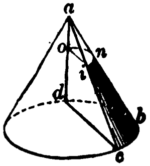


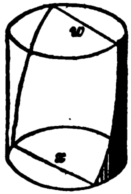






The prismoidal formula applies also to the sphere, hemisphere, and other spherical segments; also to any sections such as *a b c d*, and *o n i d b c*, of the





cone, in which the sides *a d*, *a c*, or *o d*, *i c*, are *straight*; as they are only when the cutting plane *a d c passes through the apex* or top *a*. **Also to the cylinder** when a plane *parallel to the sides* passes through both ends; but not if the plane *w s* is *oblique*, as in the fig., though never erring more than 1 in 142. **In this last case** we must imagine the plane to be extended until it cuts the side of the cylinder likewise extended; and then by page 199 find the solidity of the ungula thus formed. Then find the solidity of the small ungula above *w*, also thus formed, and subtract it from the large one.

This very extended applicability of the prismoidal formula was first discovered, and made known, by Ellwood Morris, C. E., of Philadelphia, in 1840.

WEDGES.

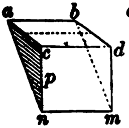


Fig. 8.

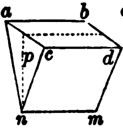


Fig. 9.

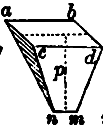


Fig. 10.

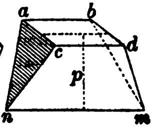


Fig. 11.

A wedge

Is usually defined to be a solid, Figs. 8 and 9, generated by a plane triangle, *a n c*, moving parallel to itself, in a straight line. This definition requires that the two triangular ends of the wedge should be parallel; but a wedge may be shaped as in Fig. 10 or 11. We would therefore propose the following definition, which embraces all the figs.; besides various modifications of them. A solid of five plane faces; one of which is a parallelogram *a b c d*, two opposite sides of which, as *a c* and *b d*, are united by means of two triangular faces *a c n*, and *b d m*, to an edge or line *n m*, parallel to the other opposite sides *a b* and *c d*. The parallelogram *a b c d* may be either rectangular, or not; the two triangular faces may be similar, or not; and the same with regard to the other two faces. The following rule applies equally to all:

$$\begin{array}{c}\textbf{Volume}\\ \textbf{of wedge}\end{array} = \tfrac{1}{6} \times \begin{array}{c}\text{Sum of lengths}\\ \text{of the 3 edges}\\ ab + cd + nm\end{array} \times \begin{array}{c}\text{perp ht } p \text{ from}\\ \text{edge to back}\end{array} \times \begin{array}{c}\text{width of}\\ \text{back } (a\,b\,c\,d)\\ \text{meas'd perp to } a\,b\end{array}$$

SPHERES OR GLOBES.

A Sphere

Is a solid generated by the revolution of a semicircle around its diameter. Every point in the surface of a sphere is equidistant from a certain point called the center. Any line passing entirely through a sphere, and through its center, is called its *axis*, or diameter. Any circle described on the surface of a sphere, from the center of the sphere as the center of the circle, is called a *great circle* of that sphere; in other words any entire circumference of a sphere is a great circle. A sphere has a greater content or solidity than any other solid with the same amount of surface; so that if the shape of a sphere be any way changed, its content will be reduced. The intersection of a sphere with any plane is a circle.

Volume of sphere

$= \frac{4}{3}\pi\ \text{radius}^3 \qquad = 4.1888\ \text{radius}^3$

$= \frac{1}{6}\pi\ \text{diameter}^3 \qquad = 0.5236\ \text{diameter}^3$

$= \frac{1}{6}\frac{\text{circumference}^3}{\pi^2} \qquad = 0.01689\ \text{circumference}^3$

$= \frac{1}{6}$ diameter × area of surface

$= \frac{2}{3}$ diameter × area of great circle

$= \frac{2}{3}$ volume of circumscribing cylinder

$=$ 0.5236 volume of circumscribing cube.

Area of surface of sphere

$= 4\pi\ \text{radius}^2 \qquad = 12.5664\ \text{radius}^2$

$= \pi\ \text{diameter}^2 \qquad = 3.1416\ \text{diameter}^2$

$= \frac{\text{circumference}^2}{\pi} \qquad = 0.3183\ \text{circumference}^2$

$=$ diameter × circumference

$=$ 4 × area of great circle

$=$ area of circle whose diameter is equal to twice diameter of sphere.

$=$ curved surface of circumscribing cylinder

$= \frac{6 \times \text{volume}}{\text{diameter.}}$

Radius of sphere

$= \sqrt[3]{\frac{3}{4}\frac{\text{volume}}{\pi}} \qquad = 0.62035\ \sqrt[3]{\text{volume}}$

$= \sqrt{\frac{\text{Area of surface}}{4\pi}} \qquad = \sqrt{.07958 \times \text{area of surface}}$

Circumference of sphere

$= \sqrt[3]{6\pi^2\ \text{volume}} \qquad = \sqrt[3]{59.2176\ \text{volume}}$

$= \sqrt{\pi\ \text{area of surface}} \qquad = \sqrt{3.1416\ \text{area of surface}}$

$= \frac{\text{area of surface}}{\text{diameter.}}$

SPHERES. (ORIGINAL.)

Some errors of 1 in the last figure only.

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
1-64	.00077		13-32	18.190	7.2949	⅜	170.87	210.03	⅛	921.33	2629.6
1-32	.00307	.00002	7-16	18.666	7.5829	½	176.71	220.89	¼	934.83	2687.6
3-64	.00690	.00005	15-32	19.147	7.8783	⅝	182.66	232.13	⅜	948.43	2746.5
1-16	.01227	.00013	½	19.635	8.1813	¾	188.69	243.73	½	962.12	2806.2
3-32	.02761	.00043	17-32	20.129	8.4919	⅞	194.83	255.72	⅝	975.91	2866.8
⅛	.04909	.00102	9-16	20.629	8.8103	8.	201.06	268.08	¾	989.80	2928.2
5-32	.07670	.00200	19-32	21.135	9.1366	⅛	207.39	280.85	⅞	1003.8	2990.5
3-16	.11045	.00345	⅝	21.648	9.4708	¼	213.82	294.01	18.	1017.9	3053.6
7-32	.15033	.00548	21-32	22.166	9.8131	⅜	220.36	307.58	⅛	1032.1	3117.7
¼	.19635	.00818	11-16	22.691	10.164	½	226.98	321.56	¼	1046.4	3182.6
9-32	.24851	.01165	23-32	23.222	10.522	⅝	233.71	335.95	⅜	1060.8	3248.5
5-16	.30680	.01598	¾	23.758	10.889	¾	240.53	350.77	½	1075.2	3315.3
11-32	.37123	.02127	25-32	24.302	11.265	⅞	247.45	366.02	⅝	1089.8	3382.9
⅜	.44179	.02761	13-16	24.850	11.649	9.	254.47	381.70	¾	1104.5	3451.5
13-32	.51848	.03511	27-32	25.405	12.041	⅛	261.59	397.83	⅞	1119.3	3521.0
7-16	.60132	.04385	⅞	25.967	12.443	¼	268.81	414.41	19.	1134.1	3591.4
15-32	.69028	.05393	29-32	26.535	12.853	⅜	276.12	431.44	⅛	1149.1	3662.8
½	.78540	.06545	15-16	27.109	13.272	½	283.53	448.92	¼	1164.2	3735.0
17-32	.88664	.07850	31-32	27.688	13.700	⅝	291.04	466.87	⅜	1179.3	3808.2
9-16	.99403	.09319	**3.**	28.274	14.137	¾	298.65	485.31	½	1194.6	3882.5
19-32	1.1075	.10960	1-16	29.465	15.039	⅞	306.36	504.21	⅝	1210.0	3957.6
⅝	1.2272	.12783	⅛	30.680	15.979	10.	314.16	523.60	¾	1225.4	4033.7
21-32	1.3530	.14798	3-16	31.919	16.957	⅛	322.06	543.48	⅞	1241.0	4110.8
11-16	1.4849	.17014	¼	33.183	17.974	¼	330.06	563.86	20.	1256.7	4188.8
23-32	1.6230	.19442	5-16	34.472	19.031	⅜	338.16	584.74	⅛	1272.4	4267.8
¾	1.7671	.22089	⅜	35.784	20.129	½	346.36	606.13	¼	1288.3	4347.8
25-32	1.9175	.24967	7-16	37.122	21.268	⅝	354.66	628.04	⅜	1304.2	4428.8
13-16	2.0739	.28084	½	38.484	22.449	¾	363.05	650.46	½	1320.3	4510.9
27-32	2.2365	.31451	9-16	39.872	23.674	⅞	371.54	673.42	⅝	1336.4	4593.9
⅞	2.4053	.35077	⅝	41.283	24.942	11.	380.13	696.91	¾	1352.7	4677.9
29-32	2.5802	.38971	11-16	42.719	26.254	⅛	388.83	720.95	⅞	1369.0	4763.0
15-16	2.7611	.43143	¾	44.179	27.611	¼	397.61	745.51	21.	1385.5	4849.1
31-32	2.9483	.47603	13-16	45.664	29.016	⅜	406.49	770.64	⅛	1402.0	4936.2
1.	3.1416	.52360	⅞	47.173	30.466	½	415.48	796.33	¼	1418.6	5024.3
1-32	3.3410	.57424	15-16	48.708	31.965	⅝	424.56	822.58	⅜	1435.4	5113.5
1-16	3.5466	.62804	**4.**	50.265	33.510	¾	433.73	849.40	½	1452.2	5203.7
3-32	3.7583	.68511	1-16	51.848	35.106	⅞	443.01	876.79	⅝	1469.2	5295.1
⅛	3.9761	.74551	⅛	53.456	36.751	12.	452.39	904.78	¾	1486.2	5387.4
5-32	4.2000	.80939	3-16	55.089	38.448	⅛	461.87	933.34	⅞	1503.3	5480.8
3-16	4.4301	.87681	¼	56.745	40.195	¼	471.44	962.52	22.	1520.5	5575.3
7-32	4.6664	.94786	5-16	58.427	41.994	⅜	481.11	992.28	⅛	1537 9	5670.8
¼	4.9088	1.0227	⅜	60.133	43.847	½	490.87	1022.7	¼	1555.3	5767.6
9-32	5.1573	1.1013	7-16	61.863	45.752	⅝	500.73	1053.6	⅜	1572.8	5865.2
5-16	5.4119	1.1839	½	63.617	47.713	¾	510.71	1085.3	½	1590.4	5964.1
11-32	5.6728	1.2704	9-16	65.397	49.729	⅞	520.77	1117.5	⅝	1608.2	6064.1
⅜	5.9396	1.3611	⅝	67.201	51.801	13.	530.93	1150.3	¾	1626.0	6165.2
13-32	6.2126	1.4561	11-16	69.030	53.929	⅛	541.19	1183.8	⅞	1643.9	6267.3
7-16	6.4919	1.5553	¾	70.883	56.116	¼	551.55	1218.0	23.	1661.9	6370.6
15-32	6.7771	1.6590	13-16	72.759	58.359	⅜	562.00	1252.7	⅛	1680.0	6475.0
½	7.0686	1.7671	⅞	74.663	60.663	½	572.55	1288.3	¼	1698.2	6580.6
17-32	7.3663	1.8799	15-16	76.589	63.026	⅝	583.20	1324.4	⅜	1716.5	6687.3
9-16	7.6699	1.9974	**5.**	78.540	65.450	¾	593.95	1361.2	½	1735.0	6795.2
19-32	7.9798	2.1196	1-16	80.516	67.935	⅞	604.80	1398.6	⅝	1753.5	6904 2
⅝	8.2957	2.2468	⅛	82.516	70.482	14.	615.75	1436.8	¾	1772.1	7014.3
21-32	8.6180	2.3789	3-16	84.541	73.092	⅛	626.80	1475.6	⅞	1790.8	7125.6
11-16	8.9461	2.5161	¼	86.591	75.767	¼	637.95	1515.1	24.	1809.6	7238.2
23-32	9.2805	2.6586	5-16	88.664	78.505	⅜	649.17	1555.3	⅛	1828.5	7351.9
¾	9.6211	2.8062	⅜	90.763	81.308	½	660.52	1596.3	¼	1847.5	7466.7
25-32	9.9678	2.9592	7-16	92.887	84.178	⅝	671.95	1637.9	⅜	1866.6	7583.0
13-16	10.321	3.1177	½	95.033	87.113	¾	683.49	1680.3	½	1885.8	7700.1
27-32	10.680	3.2818	9-16	97.205	90.118	⅞	695.13	1723.3	⅝	1905.1	7818.6
⅞	11.044	3.4514	⅝	99.401	93.189	15.	706.85	1767.2	¾	1924.4	7938.3
29-32	11.416	3.6270	11-16	101.62	96.331	⅛	718.69	1811.7	⅞	1943.9	8059.2
15-16	11.793	3.8083	¾	103.87	99.541	¼	730.63	1857.0	25.	1963.5	8181.3
31-32	12.177	3.9956	13-16	106.14	102.82	⅜	742.65	1903.0	⅛	1983.2	8304.7
2.	12.566	4.1888	⅞	108.44	106.18	½	754.77	1949.8	¼	2002.9	8429.2
1-32	12.962	4.3882	15-16	110.75	109.60	⅝	767.00	1997.4	⅜	2022.9	8554.9
1-16	13.364	4.5939	**6.**	113.10	113.10	¾	779.32	2045.7	½	2042.8	8682.0
3-32	13.772	4.8060	⅛	117.87	120.31	⅞	791.73	2094.8	⅝	2062.9	8810.3
⅛	14.186	5.0243	¼	122.72	127.83	16.	804.25	2144.7	¾	2083.0	8939.9
5-32	14.607	5.2493	⅜	127.68	135.66	⅛	816.85	2195.3	⅞	2103.4	9070.6
3-16	15.033	5.4809	½	132.73	143.79	¼	829.57	2246.8	26.	2123.7	9202.8
7-32	15.466	5.7190	⅝	137.89	152.25	⅜	842.40	2299.1	⅛	2144.2	9336.2
¼	15.904	5.9641	¾	143.14	161.03	½	855.29	2352.1	¼	2164.7	9470.8
9-32	16.349	6.2161	⅞	148.49	170.14	⅝	868.31	2406.0	⅜	2185.5	9606.7
5-16	16.800	6.4751	**7.**	153.94	179.59	¾	881.42	2460.6	½	2206.2	9744.0
11-32	17.258	6.7412	⅛	159.49	189.39	⅞	894.62	2516.1	⅝	2227.1	9882.5
⅜	17.721	7.0144	¼	165.13	199.53	17.	907.93	2572.4	¾	2248.0	10022

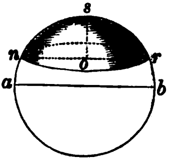
SPHERES—(CONTINUED.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
⅞	2269.1	10164	⅝	4214.1	25724	⅜	6756.5	52222	⅛	9896.0	92570
27.	2290.2	10306	¾	4243.0	25988	½	6792.9	52645	¼	9940.2	93190
⅛	2311.5	10450	⅞	4271.8	26254	⅝	6829.5	53071	⅜	9984.4	93812
¼	2332.8	10595	37.	4300.9	26522	¾	6866.1	53499	½	10029	94438
⅜	2354.3	10741	⅛	4330.0	26792	⅞	6902.9	53929	⅝	10073	95066
½	2375.8	10889	¼	4359.2	27063	47.	6939.9	54362	¾	10118	95697
⅝	2397.5	11038	⅜	4388.5	27337	⅛	6976.8	54797	⅞	10163	96330
¾	2419.2	11189	½	4417.9	27612	¼	7013.9	55234	57.	10207	96967
⅞	2441.1	11341	⅝	4447.5	27889	⅜	7050.9	55674	⅛	10252	97606
28.	2463.0	11494	¾	4477.1	28168	½	7088.3	56115	¼	10297	98248
⅛	2485.1	11649	⅞	4506.8	28449	⅝	7125.6	56559	⅜	10342	98893
¼	2507.2	11805	38.	4536.5	28731	¾	7163.1	57006	½	10387	99541
⅜	2529.5	11962	⅛	4566.5	29016	⅞	7200.7	57455	⅝	10432	100191
½	2551.8	12121	¼	4596.4	29302	48.	7238.3	57906	¾	10478	100845
⅝	2574.3	12281	⅜	4626.5	29590	⅛	7276.0	58360	⅞	10523	101501
¾	2596.7	12443	½	4656.7	29880	¼	7313.9	58815	58.	10568	102161
⅞	2619.4	12606	⅝	4686.9	30173	⅜	7351.9	59274	⅛	10614	102823
29.	2642.1	12770	¾	4717.3	30466	½	7389.9	59734	¼	10660	103488
⅛	2665.0	12936	⅞	4747.9	30762	⅝	7428.0	60197	⅜	10706	104155
¼	2687.8	13103	39.	4778.4	31059	¾	7466.3	60663	½	10751	104826
⅜	2710.9	13272	⅛	4809.0	31359	⅞	7504.5	61131	⅝	10798	105499
½	2734.0	13442	¼	4839.9	31661	49.	7543.1	61601	¾	10844	106175
⅝	2757.3	13614	⅜	4870.8	31964	⅛	7581.6	62074	⅞	10890	106854
¾	2780.5	13787	½	4901.7	32270	¼	7620.1	62549	59.	10936	107536
⅞	2804.0	13961	⅝	4932.7	32577	⅜	7658.9	63026	⅛	10983	108221
30.	2827.4	14137	¾	4964.0	32886	½	7697.7	63506	¼	11029	108909
⅛	2851.1	14315	⅞	4995.3	33197	⅝	7736.7	63989	⅜	11076	109600
¼	2874.8	14494	40.	5026.5	33510	¾	7775.7	64474	½	11122	110294
⅜	2898.7	14674	⅛	5058.1	33826	⅞	7814.8	64961	⅝	11169	110990
½	2922.5	14856	¼	5089.6	34143	50.	7854.0	65450	¾	11216	111690
⅝	2946.6	15039	⅜	5121.3	34462	⅛	7893.3	65941	⅞	11263	112392
¾	2970.6	15224	½	5153.1	34783	¼	7932.8	66436	60.	11310	113098
⅞	2994.9	15411	⅝	5184.9	35106	⅜	7972.2	66934	⅛	11357	113806
31.	3019.1	15599	¾	5216.8	35431	½	8011.8	67433	¼	11404	114518
⅛	3043.6	15788	⅞	5248.9	35758	⅝	8051.6	67935	⅜	11452	115232
¼	3068.0	15979	41.	5281.1	36087	¾	8091.4	68439	½	11499	115949
⅜	3092.7	16172	⅛	5313.3	36418	⅞	8131.3	68946	⅝	11547	116669
½	3117.3	16366	¼	5345.6	36751	51.	8171.2	69456	¾	11595	117392
⅝	3142.1	16561	⅜	5378.1	37086	⅛	8211.4	69967	⅞	11642	118118
¾	3166.9	16758	½	5410.7	37423	¼	8251.6	70482	61.	11690	118847
⅞	3192.0	16957	⅝	5443.3	37763	⅜	8292.0	70999	⅛	11738	119579
32.	3217.0	17157	¾	5476.0	38104	½	8332.3	71519	¼	11786	120315
⅛	3242.2	17359	⅞	5508.9	38448	⅝	8372.8	72040	⅜	11834	121053
¼	3267.4	17563	42.	5541.9	38792	¾	8413.4	72565	½	11882	121794
⅜	3292.9	17768	⅛	5574.9	39140	⅞	8454.1	73092	⅝	11931	122538
½	3318.3	17974	¼	5608.0	39490	52.	8494.8	73622	¾	11980	123286
⅝	3343.9	18182	⅜	5641.3	39841	⅛	8535.8	74154	⅞	12028	124036
¾	3369.6	18392	½	5674.5	40194	¼	8576.8	74689	62.	12076	124789
⅞	3395.4	18604	⅝	5708.0	40551	⅜	8617.8	75226	⅛	12126	125545
33.	3421.2	18817	¾	5741.5	40908	½	8658.9	75767	¼	12174	126305
⅛	3447.3	19032	⅞	5775.2	41268	⅝	8700.4	76309	⅜	12223	127067
¼	3473.3	19248	43.	5808.8	41630	¾	8741.7	76854	½	12272	127832
⅜	3499.5	19466	⅛	5842.7	41994	⅞	8783.2	77401	⅝	12322	128601
½	3525.7	19685	¼	5876.5	42360	53.	8824.8	77952	¾	12371	129373
⅝	3552.1	19907	⅜	5910.7	42729	⅛	8866.4	78505	⅞	12420	130147
¾	3578.5	20129	½	5944.7	43099	¼	8908.2	79060	63.	12469	130925
⅞	3605.1	20354	⅝	5978.9	43472	⅜	8950.1	79617	⅛	12519	131706
34.	3631.7	20580	¾	6013.2	43846	½	8992.0	80178	¼	12568	132490
⅛	3658.5	20808	⅞	6047.7	44224	⅝	9034.1	80741	⅜	12618	133277
¼	3685.3	21037	44.	6082.1	44602	¾	9076.4	81308	½	12668	134067
⅜	3712.3	21268	⅛	6116.8	44984	⅞	9118.5	81876	⅝	12718	134860
½	3739.3	21501	¼	6151.5	45367	54.	9160.8	82448	¾	12768	135657
⅝	3766.5	21736	⅜	6186.3	45753	⅛	9203.3	83021	⅞	12818	136456
¾	3793.7	21972	½	6221.2	46141	¼	9246.0	83598	64.	12868	137259
⅞	3821.1	22210	⅝	6256.1	46530	⅜	9288.5	84177	⅛	12918	138065
35.	3848.5	22449	¾	6291.2	46922	½	9331.2	84760	¼	12969	138874
⅛	3876.1	22691	⅞	6326.5	47317	⅝	9374.1	85344	⅜	13019	139686
¼	3903.7	22934	45.	6361.7	47713	¾	9417.2	85931	½	13070	140501
⅜	3931.5	23179	⅛	6397.2	48112	⅞	9460.2	86521	⅝	13121	141320
½	3959.2	23425	¼	6432.7	48513	55.	9503.2	87114	¾	13172	142142
⅝	3987.2	23674	⅜	6468.3	48916	⅛	9546.5	87709	⅞	13222	142966
¾	4015.2	23924	½	6503.9	49321	¼	9590.0	88307	65.	13273	143794
⅞	4043.3	24176	⅝	6539.7	49729	⅜	9633.3	88908	⅛	13324	144625
36.	4071.5	24429	¾	6575.5	50139	½	9676.8	89511	¼	13376	145460
⅛	4099.9	24685	⅞	6611.6	50551	⅝	9720.6	90117	⅜	13427	146297
¼	4128.3	24942	46.	6647.6	50965	¾	9764.4	90726	½	13478	147138
⅜	4156.9	25201	⅛	6683.7	51382	⅞	9808.1	91338	⅝	13530	147982
½	4185.5	25461	¼	6720.0	51801	56.	9852.0	91953	¾	13582	148828

SPHERES—(Continued.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
⅞	13633	149680	½	17437	216505	⅛	21708	300743	¾	26446	404406
66.	13685	150533	⅝	17496	217597	¼	21773	302100	⅞	26518	406060
⅛	13737	151390	¾	17554	218693	⅜	21839	303463	92.	26590	407721
¼	13789	152251	⅞	17613	219792	½	21904	304831	⅛	26663	409384
⅜	13841	153114	75.	17672	220894	⅝	21970	306201	¼	26735	411054
½	13893	153980	⅛	17731	222001	¾	22036	307576	⅜	26808	412726
⅝	13946	154850	¼	17790	223111	⅞	22102	308957	½	26880	414405
¾	13998	155724	⅜	17849	224224	84.	22167	310340	⅝	26953	416086
⅞	14050	156600	½	17908	225341	⅛	22234	311728	¾	27026	417774
67.	14103	157480	⅝	17968	226463	¼	22300	313118	⅞	27099	419464
⅛	14156	158363	¾	18027	227588	⅜	22366	314514	93.	27172	421161
¼	14208	159250	⅞	18087	228716	½	22432	315915	⅛	27245	422862
⅜	14261	160139	76.	18146	229848	⅝	22499	317318	¼	27318	424567
½	14314	161032	⅛	18206	230984	¾	22565	318726	⅜	27391	426277
⅝	14367	161927	¼	18266	232124	⅞	22632	320140	½	27464	427991
¾	14420	162827	⅜	18326	233267	85.	22698	321556	⅝	27538	429710
⅞	14474	163731	½	18386	234414	⅛	22765	322977	¾	27612	431433
68.	14527	164637	⅝	18446	235566	¼	22832	324402	⅞	27686	433160
⅛	14580	165547	¾	18506	236719	⅜	22899	325831	94.	27759	434894
¼	14634	166460	⅞	18566	237879	½	22966	327264	⅛	27833	436630
⅜	14688	167376	77.	18626	239041	⅝	23034	328702	¼	27907	438373
½	14741	168295	⅛	18687	240206	¾	23101	330142	⅜	27981	440118
⅝	14795	169218	¼	18748	241376	⅞	23168	331588	½	28055	441871
¾	14849	170145	⅜	18809	242551	86.	23235	333039	⅝	28130	443625
⅞	14903	171074	½	18869	243728	⅛	23303	334492	¾	28204	445387
69.	14957	172007	⅝	18930	244908	¼	23371	335951	⅞	28278	447151
⅛	15012	172944	¾	18992	246093	⅜	23439	337414	95.	28353	448920
¼	15066	173883	⅞	19053	247283	½	23506	338882	⅛	28428	450695
⅜	15120	174828	78.	19114	248475	⅝	23575	340352	¼	28503	452475
½	15175	175774	⅛	19175	249672	¾	23643	341829	⅜	28577	454259
⅝	15230	176723	¼	19237	250873	⅞	23711	343307	½	28652	456047
¾	15284	177677	⅜	19298	252077	87.	23779	344792	⅝	28727	457839
⅞	15339	178635	½	19360	253284	⅛	23847	346281	¾	28802	459638
70.	15394	179595	⅝	19422	254496	¼	23916	347772	⅞	28878	461439
⅛	15449	180559	¾	19483	255713	⅜	23984	349269	96.	28953	463248
¼	15504	181525	⅞	19545	256932	½	24053	350771	⅛	29028	465059
⅜	15560	182497	79.	19607	258155	⅝	24122	352277	¼	29104	466875
½	15615	183471	⅛	19669	259383	¾	24191	353785	⅜	29180	468697
⅝	15670	184449	¼	19732	260618	⅞	24260	355301	½	29255	470524
¾	15726	185430	⅜	19794	261848	88.	24328	356819	⅝	29331	472354
⅞	15782	186414	½	19856	263088	⅛	24398	358342	¾	29407	474189
71.	15837	187402	⅝	19919	264330	¼	24467	359869	⅞	29483	476029
⅛	15893	188394	¾	19981	265577	⅜	24536	361400	97.	29559	477874
¼	15949	189389	⅞	20044	266829	½	24606	362935	⅛	29636	479725
⅜	16005	190387	80.	20106	268083	⅝	24676	364476	¼	29712	481579
½	16061	191389	⅛	20170	269342	¾	24745	366019	⅜	29788	483438
⅝	16117	192395	¼	20232	270604	⅞	24815	367568	½	29865	485302
¾	16174	193404	⅜	20296	271871	89.	24885	369122	⅝	29942	487171
⅞	16230	194417	½	20358	273141	⅛	24955	370678	¾	30018	489045
72.	16286	195433	⅝	20422	274416	¼	25025	372240	⅞	30095	490924
⅛	16343	196453	¾	20485	275694	⅜	25095	373806	98.	30172	492806
¼	16400	197476	⅞	20549	276977	½	25165	375378	⅛	30249	494695
⅜	16456	198502	81.	20612	278263	⅝	25236	376954	¼	30326	496588
½	16513	199532	⅛	20676	279553	¾	25306	378531	⅜	30404	498486
⅝	16570	200566	¼	20740	280847	⅞	25376	380115	½	30481	500388
¾	16628	201604	⅜	20804	282145	90.	25447	381704	⅝	30558	502296
⅞	16685	202645	½	20867	283447	⅛	25518	383297	¾	30636	504208
73.	16742	203689	⅝	20932	284754	¼	25589	384894	⅞	30713	506125
⅛	16799	204737	¾	20996	286064	⅜	25660	386496	99.	30791	508047
¼	16857	205789	⅞	21060	287378	½	25730	388102	⅛	30869	509975
⅜	16914	206844	82.	21124	288696	⅝	25802	389711	¼	30947	511906
½	16972	207903	⅛	21189	290019	¾	25873	391327	⅜	31025	513843
⅝	17030	208966	¼	21253	291345	⅞	25944	392945	½	31103	515785
¾	17088	210032	⅜	21318	292674	91.	26016	394570	⅝	31181	517730
⅞	17146	211102	½	21382	294010	⅛	26087	396197	¾	31259	519682
74.	17204	212175	⅝	21448	295347	¼	26159	397831	⅞	31338	521638
⅛	17262	213252	¾	21512	296691	⅜	26230	399468	100.	31416	523598
¼	17320	214333	⅞	21578	298036	½	26302	401109			
⅜	17379	215417	83.	21642	299388	⅝	26374	402756			

To find the solidity of a spherical segment.



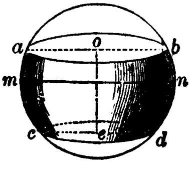
RULE 1. Square the rad *o n*, of its base; mult this square by 3; to the prod add the square of its height *o s*; mult the sum by the height *o s*: and mult this last prod by .5236.

RULE 2. Mult the diam *a b of the sphere* by 3; from the prod take twice the height *o s* of the segment; mult the rem by the square of the height *o s*; and mult this prod by .5236.

The solidity of a sphere being ⅔ds that of its circumscribing cylinder, if we add to any solidity in the table, its half, we obtain that of a cylinder of the same diam as the sphere, and whose height equals its diam.

To find the curved surface of a spherical segment.

RULE 1. Mult the diam *a b* of the sphere from which the segment is cut, by 3.1416; mult the prod by the height *o s* of the seg. Add area of base if reqd. REM. Having the diam *n r* of the seg, and its height *o s*, the diam *a b* of the sphere may be found thus: Div the square of half the diam *n r*, by its height *o s*; to the quot add the height *o s*. RULE 2. The *curved* surf of either a segment, last Fig, or of a zone, (next Fig,) bears the same proportion to the surf of the whole sphere, that the height of the seg or zone bears to the diam of the sphere. Therefore, first find the surf of the whole sphere, either by rule or from the preceding table; mult it by the height of the seg or zone; div the prod by diam of sphere. RULE 3. Mult the circumf of the sphere by the height *o s* of the seg.

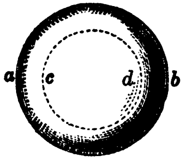


To find the solidity of a spherical zone.

Add together the square of the rad *e d*, the square of rad *o b*, and ⅓d of the square of the perp height *e o*; mult the sum by 1.5708; and mult this prod by the height *e o*. *

To find the curved surface of a spherical zone.

RULE 1. Mult together the diam *m n* of the sphere; the height *e o* of the zone, and the number 3.1416. Or see preceding Rule 2 for surf of segments. Rule 2. Mult the circumf of the sphere, by the height of the zone. *



To find the solidity of a hollow spherical shell.

Take from the foregoing table the solidities of two spheres having the diams *a b*, and *c d*. Subtract the least from the greatest. Here *a c* or *b d* is the thickness of the shell.

THE ELLIPSOID, OR SPHEROID,

Is a solid generated by the revolution of an ellipse around either its long or its short diam. When around the long (or **transverse**) diam, as at *a*, Fig 1, it is an **oblong** or **prolate** spheroid; when around the short (or **conjugate**) one, as at *m*, in Fig 2, it is **oblate.**

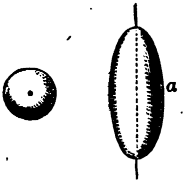


Fig. 1.

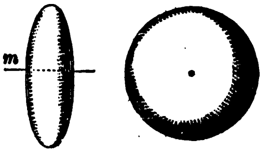


Fig. 2.

For the solidity in either case, mult the fixed diam or axis by the square of the revolving one; and mult the prod by .5236.

---

* This rule applies, whether the zone includes the equator (as in our figure), or not, as in the earth's temperate zones.

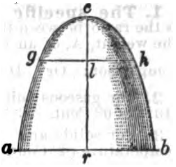
THE PARABOLOID, OR PARABOLIC CONOID,

next Fig, is a solid generated by the revolution of a parabola *a c b*, around its axis, *c r*.

For its solidity mult the area of its base, by half its height, *r c*. Or mult together the square of the rad *a r* of the base; the height *r c*; and the number 1.5708.

For the solidity of a frustum,

a b g h, the ends of which are perp to the axis *r c*; add together the squares of the two diams *a b* and *g h*; mult the sum by the height *r l*; mult the prod by the decimal .3927.



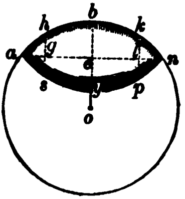
To find the surface of a paraboloid,

Mult the rad *a r* of its base, by 6.2832; div the prod by 12 times the square of the height *r c*; call the quot *p*. Then add together the square of the rad *a r*, and 4 times the square of the height *r c*. Cube the sum; take the sq rt of this cube; from the sq rt subtract the cube of the rad *a r*. Mult the rem by *p*.

Either the solidity, or the surface of a frustum, *a b g h*, when *g h* is parallel to *a b*, may be found by calculating for the whole paraboloid, and for the upper portion *c g h*, as two separate paraboloids, and taking their diff.

THE CIRCULAR SPINDLE,

is a solid *a b n y* generated by the revolution of a circular segment *a b n e a*, around its chord *a n* as an axis.



To find its solidity.

Rule 1. First find the area of *a b e*, or half the generating circular segment. Then to the square of *a e*, add the square of *b e*; div the sum by *b y*; from the quot take *b e*; mult the rem by the area of *a e b*; call the prod *p*. Cube *a e*; div the cube by 3; from the quot take *p*. Mult the rem by 12.5664.

Rule 2. When the dist *o e* is known, from the center of the circle to the cen of the spindle, then mult that dist *o e*, by the area of *a b e*; call the prod *p*; cube *a e*; div the cube by 3; from the quot take *p*; mult the rem by 12.5664.

To find its surface.

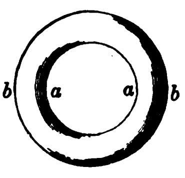
Rule 1. First find the length of the circular arc *a b n*; and mult it by the dist *o e* from the center of the circle to the center of the spindle. Call the prod *p*. Next mult the length *a n* of the spindle, by the rad *o b* of the circle. From the prod take *p*; mult the rem by 6.2832.

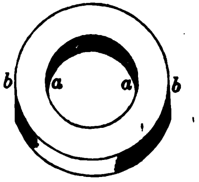
Rule 2. First find the length of the arc *a b n*. Square *a e*; also square *b e*; add these squares together; div their sum by *b y*; call the quot *s*; and mult it by *a n*; call the prod *p*. Next from *s* take *b e*: mult the rem by the length of the arc *a b n*. Subtract the prod from *p*; mult the rem by 6.2832.

To find the solidity of a middle zone of a circular spindle,

As *h s k p*

$$\left(\left(ae^2-\frac{ge^2}{3}\right)\times ge\right)-\left(oe\times\text{area of } g\,h\,l\,k\right)\Bigg)\times 6.2832.$$





CIRCULAR RINGS.

Volume = area of cross section of bar of which ring is made × ½ sum of inner and outer diameters, *a a* and *b b* × 3.141593.

Surface = circumference of bar of which ring is made × ½ sum of inner and outer diameters, *a a* and *b b* × 3.141593.

14

SPECIFIC GRAVITY.

1. **The specific gravity, or relative density, D***, of a substance, is the ratio between the weight, W, of any given volume of that substance and the weight, A, of an equal volume of some substance adopted as a standard of comparison. Or: $D = \frac{W}{A}$.

2. For gaseous substances, **the standard substance** is air, at a temperature of 0° Cent. = 32° Fahr., with barometer at 760 millimeters = 29.922 inches.

3. For solids and liquids, the standard substance is distilled water, at its temperature (4° Cent. = 39.2° Fahr.) of maximum density.

4. For all ordinary purposes of civil engineering, **any clear fresh water, at any ordinary temperature,** may be used. Even with water at 30° Cent., = 86° Fahr., the result is only 4 parts in 1000 too great.

5. When a body is immersed in water, the upward force, or "**buoyancy,**" exerted upon it by the water, or the "**loss of weight**" of the body, due to its immersion, is equal to the weight of the water displaced by the immersion of the body †; or, if

W = the weight of the body in air,
w = its weight in water,
D = its relative density or specific gravity,
A = the weight of water displaced;

then $A = W - w$; and $D = \frac{W}{A} = \frac{W}{W - w}$.

6. Since the volume, V, of a body, of given weight, W, is inversely as its density, or specific gravity, D; the specific gravity is equal also to the ratio between the volume V_s of an equal weight of the standard substance, to the volume, V, of the body in question; or $D = \frac{V_s}{V}$.

7. The specific gravities of **substances heavier than water** are ordinarily determined by weighing a mass of the substance, first in air (obtaining its weight, W), and then when the mass is completely submerged in water (obtaining its diminished weight, w). Then $D = \frac{W}{W - w}$, as in ¶ 5.

8. **If the body is lighter than water,** it must be entirely immersed, and held down against its tendency to rise. Its weight, w, in water, or its upward tendency, is then a *negative* quantity, and means must be provided for measuring it, as by making it act upward against the scale pan. We then have, $A = W - (-w) = W + w$; or

Loss due to immersion = weight of body in air, *plus* its buoyancy.

9. Or, first allow the body to float upon the water, and note the resulting displacement, v, of water, as by the rise of its surface level in a prismatic vessel. Then immerse the body completely, and again note the displacement, V. Now v, the volume displaced by the body when floating, and V, the volume displaced by the body when completely immersed, are proportional respectively to the weight, W, of the body, and to the weight, $W - w$, of a mass of water of equal volume with the body. Hence $D = \frac{W}{W - w} = \frac{v}{V}$.

10. Or, attach to the light body, b, a heavier body, or sinker, S, of such density and mass that both bodies together will sink in water. Let W be the weight of the light body, b, in air; Q the weight of both bodies in air, and q their combined weight in water. Then $Q - q$ = the weight of a mass of water of equal volume with the two bodies, and $Q - W$ = the weight, S, of the sinker in air. By immersing the sinker alone, find the weight, k, of water equal in volume to the sinker alone, = loss of weight in sinker, due to immersion. Then, for the weight, A, of water of equal volume with the light body, b, or for

---

* Strictly speaking, "specific gravity" refers to *weight*, and "relative density" to *mass* (see Mechanics, Art. 14 *a*); but, as specific gravity and density are numerically equal, they are often treated as identical.

† See Hydrostatics, Art. 18.

the loss of weight of b, due to immersion, we have $A = Q - q - k$; and, for the specific gravity, D, of the light body, b, we have $D = \frac{W}{Q - q - k} = \frac{W}{W - w}$ where w = the (unknown) buoyancy of b.

11. **A granular body,** as a mass of saw-dust, gravel, sand, cement, etc., or a porous body, as a mass of wood, cinder, concrete, sandstone, etc., is a composite body, consisting partly of solid matter and partly of air. Thus, a cubic foot of quartz sand weighs about 100 lbs.; while a cubic foot of quartz weighs about 165 lbs.

12. **The specific gravity of porous substances** is usually taken as that of the composite mass of solid and air. Thus, a wood, weighing (with its contained air) 62.5 lbs. per cubic foot, or the same as water, is said to have a specific gravity of 1. The absorption of water, when such bodies are immersed for the purpose of determining their specific gravities, may be prevented by a thin coat of varnish.

13. **The specific gravity of granular substances** is sometimes taken as that of the solid part alone. Thus, Portland cements ordinarily weigh (in air) from 75 to 90 lbs. per cubic foot, which would correspond to specific gravities of from 1.20 to 1.44; but the specific gravity of the solid portion ranges from 3.00 to 3.25; and the latter figures are usually taken as representing the specific gravities.

14. In determining the specific gravities of substances (such as cement) which are **soluble in water** or otherwise affected by it, the substances are weighed in some liquid (such as benzine, turpentine or alcohol) which will not affect them, instead of in water. The result, so obtained, must then be multiplied by the ratio between the density of the liquid and that of water.

15. **The specific gravity of a liquid** is most directly determined by weighing equal volumes of the liquid and of water.

16. Or weigh, in the liquid, some body, whose weight, W, in air, and whose specific gravity, d, are known. Let w' = its weight in the liquid. Then, for the specific gravity, D, of the liquid, we have

$$W : W - w' = d : D; \text{ or } D = \frac{d\,(W - w')}{W}.$$

17. Or, let the body, in ¶ 16 (weighing W in air), weigh w in water, and (as before) w' in the liquid in question. Then, since specific gravity of water = 1, we have

$$W - w : W - w' = 1 : D; \text{ or } D = \frac{W - w'}{W - w}.$$

18. The specific gravities of liquids are commonly obtained by observing the depth to which some standard instrument (called a **hydrometer**) sinks when allowed to float upon the surface of the liquid. The greater the depth, the less the specific gravity of the liquid. In **Beaumé's hydrometer** the depth of immersion is shown by a scale upon the instrument. The graduations of the scale are arbitrary. For liquids heavier than water, 0° corresponds to a specific gravity of 1, and 76° to a specific gravity of 2. For liquids lighter than water, 10° correspond to a specific gravity of 1, and 60° to a specific gravity of 0.745.

19. **In Twaddell's hydrometer,** used for liquids heavier than water,

$$\text{specific gravity} = \frac{5 \times \text{No. of degrees} + 1{,}000}{1{,}000}$$

Thus, if the reading be 90°,

$$\text{specific gravity} = \frac{5 \times 90 + 1{,}000}{1{,}000} = \frac{1{,}450}{1{,}000} = 1.45.$$

20. **In Nicholson's hydrometer,** largely used also for solids, the specific gravity is deduced from the weights required to produce a standard depth of immersion. It consists of a hollow metal float, from which rises a thin but stiff wire carrying a shallow dish, which always remains above water. From the float is suspended a loaded dish, which, like the float, is always submerged. On the wire supporting the upper dish is a standard mark, which, in observations, is always brought to the surface of the water. The specific gravity is then determined by means of the weights carried in the two dishes respectively.

21. The determination of the **specific gravities of gaseous substances** requires the skill of expert chemists.

Table of specific gravities, and weights.

In this table, the sp gr of air, and gases also, are compared with that of water, instead of that of air; which last is usual.

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Air, atmospheric; at 60° Fah, and under the pressure of one atmosphere or 14.7 lbs per sq inch, weighs $\frac{1}{815}$ part as much as water at 60°	.00123	.0765
Alcohol, pure	.793	49.43
“ of commerce	.834	52.1
“ proof spirit	.916	57.2
Ash, perfectly dry. average	.752	47.
1000 ft board measure weighs 1.748 tons.		
Ash, American white, dry “	.61	38.
1000 ft board measure weighs 1.414 tons.		
Alabaster, falsely so called; but really Marbles	2.7	168.
“ real; a compact white plaster of Paris average	2.31	144.
Aluminium	2.6	162.
Antimony, cast, 6.66 to 6.74 average	6.70	418.
“ native “	6.67	416.
Anthracite. See Coal, below.		
Asphaltum, 1 to 1.8 “	1.4	87.3
Basalt. See Limestones, quarried “	2.9	181.
Bath Stone, Oolite “	2.1	131.
Bismuth, cast. Also native “	9.74	607.
Bitumen, solid. See Asphaltum.		
Brass, (Copper and Zinc,) cast, 7.8 to 8.4 “	8.1	504.
“ rolled “	8.4	524.
Bronze. Copper 8 parts; Tin 1. (Gun metal.) 8.4 to 8.6 “	8.5	529.
Brick, best pressed	....	150.
“ common hard	....	125.
“ soft, inferior	....	100.
Brickwork. See Masonry.		
Boxwood, dry “	.96	60
Calcite, transparent “	2.722	169.9
Carbonic Acid Gas, is 1½ times as heavy as air “	.00187	
Cement. (See ¶ 13.)		
“ Portland, 3.00 to 3.25	3.12	75 to 90
“ Natural, 2.75 to 3.00	2.87	50 to 56
Chalk, 2.2 to 2.8. See Limestones, quarried	2.50	156.
Charcoal, of pines and oaks		15 to 30
Cherry, perfectly dry	0.67	42.
Chestnut, perfectly dry	0.66	41.
Coal. See also page 215.		
Anthracite, 1.3 to 1.7	1.50	81 to 106
“ piled loose		47 to 58
Bituminous, 1.2 to 1.4	1.30	78 to 88
“ piled loose		44 to 54
Coke	1.00	62.5
“ piled loose		23 to 32
In coking, coals swell from 25 to 50 per cent.		
Copper, cast, 8.6 to 8.8	8.7	542.
“ rolled, 8.8 to 9.0	8.9	555.
Crystal, pure Quartz. See Quartz.		
Cork	.25	15.6
Diamond, 3.44 to 3.55; usually 3.51 to 3.55	3.53	
Earth; common loam, perfectly dry, loose	....	72 to 80
“ “ “ “ “ shaken	....	82 to 92
“ “ “ “ “ moderately rammed	....	90 to 100
“ “ “ slightly moist, loose	....	70 to 76
“ “ “ more moist, “	....	66 to 68
“ “ “ “ shaken	....	75 to 90
“ “ “ “ moderately packed	....	90 to 100
“ “ “ as a soft flowing mud	....	104 to 112
“ “ “ as a soft mud, well pressed into a box	....	110 to 120
Ether	.716	44.6
Elm, perfectly dry. average	.56	35.
1000 ft board measure weighs 1.302 tons.		
Ebony, dry “	1.22	76.1
Emerald, 2.63 to 2.76 “	2.7	
Fat “	.93	58.
Flint “	2.6	162.
Feldspar, 2.5 to 2.8 “	2.65	166.
Garnet, 3.5 to 4.3; Precious, 4.1 to 4.3 “	4.2	
Glass, 2.5 to 3.45 “	2.98	186.
“ common window “	2.52	157.
“ Millville, New Jersey. Thick flooring glass “	2.53	158.
Granite, 2.56 to 2.88. See Limestone, 160 to 180 “	2.72	170.

Table of specific gravities, and weights—(Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Gneiss, common, 2.62 to 2.76	2.69	168.
“ in loose piles “		96.
“ Hornblendic “	2.8	175.
“ “ quarried, in loose piles “		100.
Gypsum, Plaster of Paris, 2.24 to 2.30 “	2.27	141.6
“ in irregular lumps “		82.
“ ground, loose, per struck bushel, 70 “		56.
“ “ well shaken, “ “ 80 “		64.
“ “ Calcined, loose, per struck bush, 65 to 75 “		52 to 60
Greenstone, trap, 2.8 to 3.2 “	3.	187.
“ “ quarried, in loose piles “		107.
Gravel, about the same as sand, which see.		
Gold, cast, pure, or 24 carat “	19.258	1204.
“ native, pure, 19.3 to 19.34 “	19.32	1206.
“ “ frequently containing silver, 15.6 to 19.3 “		
“ pure, hammered, 19.4 to 19.6 “	19.5	1217.
Gutta Percha “	.98	61.1
Hornblende, black, 3.1 to 3.4 “	3.25	203.
Hydrogen Gas, is 14½ times lighter than air; and 16 times lighter than oxygen average		.00559
Hemlock, perfectly dry. “	.4	25.
1000 feet board measure weighs .930 ton.		
Hickory, perfectly dry. “	.85	53.
1000 feet board measure weighs 1.971 tons.		
Iron, and steel.		
“ Pig and cast iron and cast steel	7.2	450.
“ Wrought iron and steel, and wire, 7.6 to 7.9	7.75	475 to 494
Ivory average	1.82	114.
Ice, .917 to .922 “	.92	57.4
India rubber “	.93	58.
Lignum vitæ, dry “	1.33	83.
Lard “	.95	59.3
Lead, of commerce, 11.30 to 11.47; either rolled or cast “	11.38	709.6
Limestones and Marbles, 2.4 to 2.86, 150 to 178.8	2.6	164.4
“ “ “ ordinarily about	2.7	168.
“ “ “ quarried in irregular fragments, 1 cub yard solid, makes about 1.9 cub yds perfectly loose; or about 1¾ yds piled. In this last case, 571 of the pile is solid; and the remaining .429 part of it is voids piled		96.
Lime, quick, ground, loose, per struck bushel 62 to 70 lbs		53.
“ “ “ well shaken, “ “ ....80 “		64.
“ “ “ thoroughly shaken, “ ....93¾ “		75.
Mahogany, Spanish, dry* average	.85	53.
“ Honduras, dry “	.56	35.
Maple, dry* “	.79	49.
Marbles, see Limestones.		
Masonry, of granite or limestones, well dressed throughout		165.
“ “ “ well-scabbled mortar rubble. About $\frac{1}{5}$ of the mass will be mortar		154.
“ “ “ well-scabbled dry rubble		138.
“ “ “ roughly scabbled mortar rubble. About ¼ to ⅓ part will be mortar		150.
“ “ “ roughly scabbled dry rubble		125.
At 155 lbs per cub ft, a cub yard weighs 1.868 tons; and 14.45 cub ft, 1 ton.		
Masonry of sandstone; about ⅛ part less than the foregoing.		
“ “ brickwork, pressed brick, fine joints average		140.
“ “ “ medium quality “		125.
“ “ “ coarse; inferior soft bricks “		100.
At 125 lbs per cub ft, a cub yard weighs 1.507 tons; and 17.92 cub ft, 1 ton.		
Mercury, at 32° Fah	13.62	849.
“ 60° “	13.58	846.
“ 212° “	13.38	836.
Mica, 2.75 to 3.1	2.93	183.
Mortar, hardened, 1.4 to 1.9	1.65	103.
Mud, dry, close		80 to 110
“ wet, moderately pressed		110 to 130
“ wet, fluid		104 to 120

---

* **Green timbers** usually weigh from one-fifth to nearly one-half more than dry; and ordinary building timbers when tolerably seasoned about one-sixth more than perfectly dry.

Table of specific gravities, and weights—(Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Naphtha	.848	52.9
Nitrogen Gas is about $\frac{1}{35}$ part lighter than air	....	.0744
Oak, live, perfectly dry, .88 to 1.02 * average	.95	59.8
" white, " " .66 to .88 "	.77	48.
" red, black, &c* "	....	32 to 45
Oils, whale; olive "	.92	57.3
" of turpentine "	.87	54.3
Oolites, or Roestones, 1.9 to 2.5 "	2.2	137.
Oxygen Gas, a little more than $\frac{1}{10}$ part heavier than air	.00136	.0846
Petroleum	.878	54.8
Peat, dry, unpressed	....	20 to 30
Pine, white, perfectly dry, .35 to .45*	.40	25.
1000 ft board measure weighs .930 ton.*		
" yellow, Northern, .48 to .62	.55	34.3
1000 ft board measure weighs 1.276 tons.*		
" " Southern, .64 to .80	.72	45.
1000 ft board measure weighs 1.674 tons.*		
Pine, heart of long-leafed Southern yellow, unseas.	1.04	65.
1000 ft board measure weighs 2.418 tons.		
Pitch	1.15	71.7
Plaster of Paris; see Gypsum.		
Powder, slightly shaken	1.	62.3
Porphyry, 2.66 to 2.8	2.73	170.
Platinum 21 to 22	21.5	1342.
" native, in grains 16 to 19	17.5	
Quartz, common, pure 2.64 to 2.67	2.65	165.
" " finely pulverized, loose		90.
" " " " well shaken		105.
" " " " well packed		112.
" quarried, loose. One measure solid, makes full 1¾ broken and piled		94.
Ruby and Sapphire, 3.8 to 4.0	3.9	
Rosin	1.1	68.6
Salt		50 to 70
Sand, pure quartz, perfectly dry, loose		90 to 106
" " " " " slightly shaken		92 to 110
" " " rammed, dry		100 to 120
Natural sand consists of grains of different sizes, and weighs more, per unit of volume, than a sand sifted from it and having grains of uniform size. Sharp sand with very large and very small grains may weigh as much as		117.
Sand is very retentive of moisture, and, when in large bulk, its natural moisture may diminish its weight from 5 to 10 per cent.		
" perfectly wet, voids full of water		118 to 120
Sandstones, fit for building, dry, 2.1 to 2.73, ...131 to 171.	2.41	151.
" quarried, and piled, 1 measure solid, makes about 1¾ piled	....	86.
Serpentines, good 2.5 to 2.65	2.6	162.
Snow, fresh fallen		5 to 12
" moistened, and compacted by rain		15 to 50
Sycamore, perfectly dry.	.59	37.
1000 ft board measure weighs 1.376 tons.		
Shales, red or black 2.4 to 2.8 average	2.6	162.
" quarried, in piles "	....	92.
Slate 2.7 to 2.9 "	2.8	175.
Silver "	10.5	655.
Soapstone, or Steatite 2.65 to 2.8 "	2.73	170.
Steel, 7.7 to 7.9. The heaviest contains least carbon "	7.85	490.
Steel is not heavier than the iron from which it is made; unless the iron had impurities which were expelled during its conversion into steel.		
Sulphur average	2.	125.
Spruce, perfectly dry. "	.4	25.
1000 ft board measure weighs .930 ton.		
Spelter, or Zinc 6.8 to 7.2. "	7.00	437.5
Sapphire; and Ruby, 3.8 to 4. "	3.9	
Tallow "	.94	58.6
Tar "	1.	62.4
Trap, compact, 2.8 to 3.2 "	3.	187.
" quarried; in piles "	....	107.
Topaz. 3.45 to 3.65 "	3.55	

---

* **Green timbers** usually weigh from one-fifth to nearly one-half more than dry; and ordinary building timbers when tolerably seasoned about one-sixth more than perfectly dry.

Table of specific gravities, and weights — (Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**		Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Tin, cast, 7.2 to 7.5	average..	7.35	459.
Turf, or Peat, dry, unpressed			20 to 30
Water. See page 326.			62.417
Wax, bees	average..	.97	60.5
Wines, .993 to 1.04	" ..	.998	62.3
Walnut, black, perfectly dry. 1000 ft board measure weighs 1.414 tons.	" ..	.61	38.
Zinc, or Spelter, 6.8 to 7.2	" ..	7.00	437.5
Zircon, 4.0 to 4.9	" ..	4.45	

Space occupied by coal. In cubic feet per ton of 2240 pounds.

Pennsylvania Anthracite.

	Broken.	Egg.	Stove.	Nut.	Pea.	Buckwheat.	Average.
Hard white ash *	39.4	38.6	39.2	39.8	40.5	41.1	39.8
		39.6	39.6	39.6	39.8	39.8	39.6
Free-burning white ash *	39.6	39.0	39.6	40.2	40.8	41.5	40.2
		39.6	39.6	41.2	41.9	42.4	40.7
Shamokin *	39.0	39.3	39.9	40.5	41.2	41.9	40.6
		39.9	42.6	45.7	46.5	47.7	43.6
Schuylkill white ash *		39.6	40.3	40.9	41.6	42.3	40.9
" red " *		40.0	40.5	41.1	41.7	42.3	41.1
Lykens Valley *	44.2	44.8	45.2	45.7	46.2	46.7	45.7
		44.3	44.3	45.0	46.1	46.5	45.1
Wyoming free-burning †		40.0	39.8	39.4			39.7
Lehigh †	39.4	38.8	38.5	38.4	42.1	41.4	40.0
Lehigh; Reading C. & I. Co. *	38.5	38.8	40.1	40.3	40.3	40.5	39.7

Lehigh: † Lump, 40.5; cupola, 40.3; dust, 39.1.

Bituminous.

From Coxe Bros. & Co. †		From Jour. U. S. Ass'n Charcoal Iron Workers. Vol. III, 1882.§			
Pittsburg	48.2	Pittsburg	47.1	Clover Hill, Va	49.0
Erie	46.6	Cumberland, max	42.3	Richmond, Va.	
Hocking Valley	45.4	" min	41.2	(Midlothian)	41.0
Ohio Cannel	45.5	Blossburg, Pa	42.2	Cannelton, Ind	47.0
Indiana Block	51.1			Pictou, N. S.	45.0
Illinois	47.4			Sydney, Cape Breton	47.0

	Logarithm.
1 cubic foot per ton of 2240 pounds =	
0.89286 cubic foot per ton of 2000 pounds	$\bar{1}$.950 7820
2240 (exact) pounds per cubic foot	3.350 2480
1 cubic foot per ton of 2000 pounds =	
1.12 (exact) cubic feet per ton of 2240 pounds	0.049 2180
2000 (exact) pounds per cubic foot	3.301 0300
1 pound per cubic foot =	
2240 (exact) cubic feet per ton of 2240 pounds	3.350 2480
2000 " " " 2000 "	3.301 0300

---

* From Edwin F. Smith, Sup't & Eng'r, Canal Div., Phila. and Reading R. R.

† From very careful weighings in the Chicago yards of Coxe Bros. & Co. Note the irregular variation with size of anthracite in Coxe Bros.' figures.

§ Quoted from *The Mining Record*. On the authority of "many years' experience" of "a prominent retail dealer in Philadelphia," the Journal gives also figures requiring from 4 to 13 per cent. less volume per ton than those here quoted from the Journal and from other authorities.

WEIGHTS AND MEASURES.

United States and British measures of length and weight, of the same denomination, may, *for all ordinary purposes*, be considered as equal; but the **liquid and dry** measures of the same denomination differ widely in the two countries. **The standard measure of length** of both countries is theoretically that of a pendulum vibrating seconds at the level of the sea, in the latitude of London, in a vacuum, with Fahrenheit's thermometer at 62°. The length of such a pendulum is supposed to be divided into 39.1393 equal parts, called inches; and 36 of these inches were adopted as the standard yard of both countries. But the Parliamentary standard having been destroyed by fire, in 1834, it was found to be impossible to restore it by measurement of a pendulum. The present British Imperial yard, as determined, at a temperature of 62° Fahrenheit, by the standard preserved in the Houses of Parliament, is the standard of the United States Coast and Geodetic Survey, and is recognized as standard throughout the country and by the Departments of the Government, although not so declared by Act of Congress. The yard between the 27th and 63d inches of a scale made for the U. S. Coast Survey by Troughton, of London, in 1814, is found to be of this standard length when at a temperature of 59°.62 Fahrenheit; but at 62° is too long by 0.00083 inch, or about 1 part in 43373, or 1.46 inch per mile, or 0.0277 inch in 100 feet.

The Coast Survey now uses, for purposes of comparison, two measures presented by the British Government in 1855, as copies of the Imperial standard, namely:

"Bronze standard, No. 11;" of standard length at 62°.25 Fahr.
"Malleable iron standard, No. 57;" " " " 62°.10 "

See Appendix No. 12, Report of U. S. Coast and Geodetic Survey for 1877.

The legal standard of weight of the United States is the **Troy pound of the Mint** at Philadelphia. This standard, containing 5760 grains, is an exact copy of the **Imperial Troy pound of Great Britain.** The avoirdupois or commercial pound of the United States, containing 7000 grains, and derived from the standard Troy pound of the Mint, is found to agree within one thousandth of a grain with the British avoirdupois pound. The U. S. Coast Survey therefore declares the weights of the two countries identical.

The Ton. In Revised Statutes of the United States, 2d Edition, 1878, Title XXXIV, Collection of Duties upon Imports, Chapter Six, Appraisal, says:

"Sec. 2951. Wherever the word 'ton' is used in this chapter, in reference to weight, it shall be construed as meaning twenty-hundredweight, each hundredweight being one hundred and twelve pounds avoirdupois."

This appears to be the only U. S. Government regulation on the subject.

The ton of 2240 lbs (often called a **gross ton** or **long ton**) is commonly used in buying and selling iron ore, pig iron, steel rails and other manufactured iron and steel. Coke and many other articles are bought and sold by the **net ton** or **short ton** of 2000 lbs. The bloom ton had 2464 lbs, = 2240 lbs + 2 hundredweight of 112 lbs each; and the pig iron ton had 2268 lbs, = 2240 lbs + a "sandage" of 28 lbs, or one "quarter," to allow for sand adhering to the pigs, but some furnace men allowed only 14 lbs. In electric traction work the ton means 2000 lbs.

As a measure, the ton, or tun, is defined as 252 gallons, as 40 cubic feet of round or rough timber or in ship measurement, or as 50 feet of hewn timber. 252 U. S. gallons of water weigh about 2100 lbs; 252 Imperial gallons about 2500 lbs; 50 cub ft yellow pine about 2500 lbs.

The metric system* was legalized in the United States in

---

* The metric system, as compared with the English, has much the same advantages and disadvantages that our American decimal coinage has in comparison with the English monetary system of pounds, shillings and pence. It will enormously facilitate all calculations, but, like all other improvements, it will necessarily cause some inconvenience while the change is being made. The metric system has also this further and very great advantage, that it bids fair to become universal among civilized nations.

1866, but has not been made obligatory. The government has since furnished very exact metric standards to the several States. The use of the metric system has been permitted in Great Britain, beginning with August 6, 1897, and in Russia, beginning with 1900. Its use is now at least permissive in most civilized nations.

The metric unit of length is the metre, or meter, which was intended to be one ten-millionth $\left(\frac{1}{10,000,000}\right)$ of the earth's quadrant, *i. e.*, of that portion of a meridian embraced between either pole and the equator. This length was measured, and a set of metrical standards of weight and measure were prepared in accordance with the result, and deposited among the archives of France at Paris (Mètre des Archives. Kilogramme des Archives, etc.). It has since been discovered that errors occurred in the calculations for ascertaining the length of the quadrant; but the standards nevertheless remain as originally prepared.

The metric measures of surface and of capacity are the squares and cubes of the meter and of its (decimal) fractions and multiples.

The metric unit of weight is the gramme or gram, which is the weight of a milliliter or cubic centimeter * of pure water at its temperature of maximum density, about 4.5° Centigrade or 40° Fahrenheit.

By the concurrent action of the principal governments of the world, an **International Bureau of Weights and Measures** has been established, with its seat near Paris. It has prepared two ingots of pure platinum-iridium, from one of which a number of standard kilograms (1000 grams) have been made, and from the other a number of standard meter bars, both derived from the standards of the Archives of France. Of these copies, certain ones were selected as international standards, and the others were distributed to the different governments. Those sent to the United States are in the keeping of the U. S. Coast Survey.

The determination of the **equivalent of the meter in English measure** is a very difficult matter. The standard *meter* is measured *from end to end* of a *platinum* bar and at the *freezing* point; whereas the standard *yard* is measured *between two lines* drawn on a *silver* scale inlaid in a *bronze* bar, and *at* 62° *Fahrenheit.* **The United States Coast Survey** † adopts, as the length of the meter at 62° Fahrenheit, the value determined by Capt. A. R. Clarke and Col. Sir Henry James, at the office of the British Ordnance Survey, in 1866, viz.: 39.370432 inches (= 3.2808666 + feet = 1.0936222 + yards); but the **lawful equivalent,** established by Congress, is 39.37 inches (= 3.28083 feet = 1.093611 yards). This value is as accurate as any that can be deduced from existing data.

The gram weighs, by Prof. W. H. Miller's determination,‡ 15.43234874 grains. An examination made at the International Bureau of Weights and Measures in 1884 makes it 15.43235639 grains. The **legal value** in the United States is 15.432 grains.

---

* 1 centimeter = $\frac{1}{100}$ meter = 0.3937 inch. 1 milliliter ($\frac{1}{1000}$ liter) or cubic centimeter = 0.061 + cubic inches.

† Appendix No. 22 to report of 1876, page 6.

‡ Philosophical Transactions, 1856, pp. 893, etc.

Approximate Values of Foreign Coins, in U. S. Money.

The **references** (1, 2, 3, and 4) are to foot-notes on next page.

From Circular of U. S. Treasury Department, Bureau of the Mint, Jan. 1, 1887; from "Question Monétaire," by H. Costes, Paris, 1884; and from our 10th edition.

Argentine Repub.—Peso = 100 Centavos, 96.5 cts.[2] [3] Argentino = 5 Pesos, $4.82.
Austria.—Florin = 100 Kreutzer, 47.7 cts.,[2] 35.9 cts.[3] Ducat, $2.29. Maria Theresa Thaler, or Levantin, 1780, $1.00.[2] Rix Thaler, 97 cts.[4] Souverain, $3.57.[4]
Belgium.[1]—Franc = 100 centimes, 17.9 cts.,[2] 19.3 cts.[3]
Bolivia.—Boliviano = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Once, $14.95. Dollar, 96 cts.[4]
Brazil.—Milreis = 1000 Reis, 50.2 cts.,[2] 54.6 cts.[3]
Canada.—English and U. S. coins. Also Pound, $4.[4]
Central America.[4]—Doubloon, $14.50 to $15.65. Reale, average 5¾ cts. See Honduras.
Ceylon.—Rupee, same as India.
Chili.—Peso = 10 Dineros or Decimos = 100 Centavos, 96.5 cts.,[2] 91.2 cts.[3] Condor = 2 Doubloons = 5 Escudos = 10 Pesos. Dollar, 93 cts.[4]
Cuba.—Peso, 93.2 cts.[3] Doubloon, $5.02.
Denmark.—Crown = 100 Ore, 25.7 cts.,[2] 26.8 cts.[3] Ducat, $1.81.[4] Skilling, ¾ ct.[4]
Ecuador.—Sucre, 72.7 cts.[3] Doubloon, $3.86. Condor, $9.65. Dollar, 93 cts.[4] Reale, 9 cts.[4]
Egypt.—Pound = 100 Piastres = 4000 Paras, $4.94,3.[3]
Finland.—Markka = 100 Penni, 19.1 cts.[2] 10 Markkaa, $1.93.
France.[1]—Franc = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3] Napoleon, $3.84.[4] Livre, 18.5 cts.[4] Sous, 1 ct.[4]
Germany.—Mark = 100 Pfennigs, 21.4 cts.,[2] 23.8 cts.[3] Augustus (Saxony), $3.98.[4] Carolin (Bavaria), $4.93.[4] Crown (Baden, Bavaria, N. Germany), $1.06.[4] Ducat (Hamburg, Hanover), $2.28.[4] Florin (Prussia, Hanover), 55 cts.[4] Groschen, 2.4 cts.[4] Kreutzer (Prussia), .7 ct. Maximilian (Bavaria), $3.30.[4] Rix Thaler (Hamburg, Hanover), $1.10[4] (Baden, Brunswick), $1.00[4] (Prussia, N. Germany, Bremen, Saxony, Hanover), 69 cts.[4]
Great Britain.—Pound Sterling or Sovereign (£) = 20 Shillings = 240 Pence, $4.86.65.[3] Guinea = 21 Shillings Crown = 5 Shillings. Shilling (*s*), 22.4 cts.,[2] 24.3 cts. ($\frac{1}{20}$ pound sterling). Penny (*d*), 2 cts.
Greece.[1]—Drachma = 100 Lepta, 17 cts.,[2] 19.3 cts.[3]
Hayti.—Gourde of 100 cents, 96.5 cts.[2] [3]
Honduras.—Dollar or Piastre of 100 cents, $1.01. See Central America.
India.—Rupee = 16 Annas, 45.9 cts.,[2] 34.6 cts.[3] Mohur = 15 Rupees, $7.10. Star Pagoda (Madras), $1.81.[4]
Italy, etc.[1]—Lira = 100 Centesimi, 17.9 cts.,[2] 19.3 cts.[3] Carlin (Sardinia), $8.21.[4] Crown (Sicily), 96 cts.[4] Livre (Sardinia), 18.5 cts.[4] (Tuscany, Venice), 16 cts.[4] Ounce (Sicily), $2.50.[4] Paolo (Rome), 10 cts.[4] Pistola (Rome), $3.37.[4] Scudo[4] (Piedmont), $1.36 (Genoa), $1.28 (Rome), $1.00 (Naples, Sicily), 95 cts. (Sardinia), 92 cts. Teston (Rome), 30 cts.[4] Zecchino (Rome), $2.27.[4]
Japan.—Yen = 100 Sen (gold), 99.7 cts.[3] (silver), $1.04[2], 78.4 cts.[3]
Liberia.—Dollar, $1.00.[3] [4]
Mexico.—Dollar, Peso, or Piastre = 100 Centavos (gold), 98.3 cts. (silver), $1.05,[2] 79 cts.[3] Once or Doubloon = 16 Pesos, $15.74.
Netherlands.—Florin of 100 cents, 40.5 cts.,[2] 40.2 cts.[3] Ducatoon, $1.32.[4] Guilder, 40 cts.[4] Rix Dollar, $1.05.[4] Stiver, 2 cts.[4]
New Granada.—Doubloon, $15.34.[4]
Norway.—Crown = 100 Ore = 30 Skillings, 25.7 cts.,[2] 26.8 cts.[3]
Paraguay.—Piastre = 8 Reals, 90 cts.
Persia.—Thoman = 5 Sachib-Kerans = 10 Banabats = 25 Abassis = 100 Scahis, $2.29.
Peru.—Sol = 10 Dineros = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Dollar, 93 cts.[4]
Portugal.—Milreis = 10 Testoons = 1000 Reis, $1.08.[3] Crown = 10 Milreis. Moidore, $6.50.[4]
Russia.—Rouble = 2 Poltinniks = 4 Tchetvertaks = 5 Abassis = 10 Griviniks = 20 Pietaks = 100 Kopecks, 77 cts.,[2] 58.2 cts.[3] Imperial = 10 Roubles, $7.72. Ducat = 3 Roubles, $2.39.
Sandwich Islands.—Dollar, $1.00.[4]
Sicily.—See Italy.
Spain.—Peseta or Pistareen = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3] Doubloon (new) = 10 Escudos = 100 Reals, $5.02. Duro = 2 Escudos,[4] $1.00.[2] Doubloon (old), $15.65.[4] Pistole = 2 Crowns, $3.90.[4] Piastre, $1.04.[4] Reale Plate, 10 cts.[4] Reale vellon, 5 cts.[4]

1, 2, 3, 4. See foot-notes, next page.

(Foreign Coins *(ontinued.* **Small figures** (1, 2, 3, 4) *refer to foot notes.)*

Sweden.—Crown = 100 Ore, 25.7 cts.,[2] 26.8 cts.[3] Ducat, $2.20.[4] Rix Dollar, $1.05.[4]
Switzerland.[1]—Franc = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3]
Tripoli.—Mahbub = 20 Piastres, 65.6 cts.[3]
Tunis.—Piastre = 16 Karobs, 12 cts.[2] 10 Piastres, $1.16.6.
Turkey.—Piastre = 40 Paras, 4.4 cts.[3] Zecchin, $1.40.[4]
United States of Colombia.—Peso = 10 Dineros or Decimos = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Condor = 10 Pesos, $9.65. Dollar, 93 5 cts.[4]
Uruguay.—Peso = 100 Centavos or Centesimos (gold), $1.03 (silver), 96.5 cts.[2]
Venezuela.—Bolivar = 2 Decimos, 17.9 cts.,[2] 19.3 cts.[3] Venezolano = 5 Bolivars.

Sizes and Weights of United States Coins.*

		Diameter.	Thickness.	Legal weight of coin.	
		Inch.	Inch.	Grains.	Grams.
Gold, 10 per cent. alloy:					
Double eagle	$20	1.35	.077	516	33.436
Eagle	10	1.05	.060	258	16.718
Half eagle	5	.85	.046	129	8.359
Three dollars	3	.8	.034	77.4	5.015
Quarter eagle	2.50	.75	.034	64.5	4.179
Dollar	1.00	.55	.018	25.8	1.672
Silver, 10 per cent. alloy:					
Trade dollar	100 cts.	1.5	.082	420.	27.215
Standard dollar	100 "	1.5	.080	412.5	26.729
Half dollar	50 "	1.2	.067	192.9	12.5
Quarter dollar	25 "	.95	.045	96.45	6.25
Twenty cents	20 "	.875	.047	77.16	5.
Dime	10 "	.7	.032	38.58	2.5
Half dime	5 "	.6	.023	19.2	1.244
Three cents	3 "	.55	.018	11.52	.746
Minor.					
5 cents, 75% copper, 25% nickel		.8	.062	77.16	5.0
3 " " " " "		.725	.034	30.	1.944
2 " 95% " 5% tin and zinc.		.9	.060	96.	6.22
1 " " " " "		.75	.043	48.	3.11

Perfectly pure gold is worth $1 per 23.22 grs = $20.67183 per troy oz = $18.84151 per avoir oz. **Standard** (U. S. coin) is worth $18.60465 per troy oz = $16.95736 per avoir oz. It consists of 9 parts by weight of pure gold, to 1 part alloy. Its value is that of the pure gold only; the cost of the alloy and of the coinage being borne by Government. **A cubic foot of pure gold weighs** about 1204 avoir ℔s; and is worth $362963. A cubic inch weighs about 11.148 avoir oz; and is worth $210.04.

Pure gold is called fine, or 24 **carat** gold; and when alloyed, the alloy is supposed to be divided into 24 parts by weight, and according as 10, 15, or 20, &c, of these parts are pure gold, the alloy is said to be 10, 15, or 20, &c, carat.

The average fineness of California native gold, by some thousands of assays at the U. S. Mint in Philada., is 88.5 parts gold, 11.5 silver. Some from Georgia, 99 per cent. gold.

Pure silver fluctuates in value: thus, during 1878–1879 it ranged between $1.05 and $1.18 per troy oz., or $.957 and $1.076 per avoir. oz. A cubic inch weighs about 5.528 troy, or 6.065 avoir. ounces.

---

[1] France, Belgium, Italy, Switzerland, and Greece form the Latin Union. Their coins are alike in diameter, weight, and fineness.

[2] = 19.3 times the value of a single coin in francs as given by Costes.

[3] Par of exchange, or equivalent value in terms of U.S. gold dollar.—Treasury Circular.

[4] From our 10th edition.

* Thirteenth Annual Report of the Director of the Mint. 1885; pp. 106 and 148.

Troy Weight. U. S. and British.

24 grains	1 pennyweight, dwt.
20 pennyweights	1 ounce = 480 grains.
12 ounces	1 pound = 240 dwts. = 5760 grains.

Troy weight is used for gold and silver.

A carat of the jewellers, for precious stones is, in the U. S. = 3.2 grs.; in London, 3.17 grs.; in Paris, 3.18 grains., divided into 4 jewellers' grs. **In troy, apothecaries' and avoirdupois, the grain is the same.**

Apothecaries' Weight. U. S. and British.

20 grains	1 scruple.
3 scruples	1 dram = 60 grains.
8 drams	1 ounce = 24 scruples = 480 grains.
12 ounces	1 pound = 96 drams = 288 scruples = 5760 grains.

In troy and apothecaries' weights, the grain, ounce and pound are the same.

Avoirdupois or Commercial Weight. U. S. and British.

27.34375 grains	1 dram.
16 drams	1 ounce = 437½ grains.
16 ounces	1 pound = 256 drams = 7000 grains.
28 pounds	1 quarter = 448 ounces.
4 quarters	1 hundredweight = 112 ℔s.
20 hundredweights	1 ton = 80 quarters = 2240 ℔s.

A stone = 14 pounds. **A quintal** = 100 pounds avoir.

The standard of the avoirdupois pound, which is the one in common commercial use, is the weight of 27.7015 cub ins of pure distilled water, at its maximum density at about 39°.2 Fahr, in latitude of London, at the level of the sea; barometer at 30 ins. But this involves an error of about 1 part in 1362, for the 1℔ of water = 27.68122 cub ins.

A troy lb = .82286 avoir ℔. An avoir ℔ = 1.21528 troy ℔, or apoth.

A troy oz. = 1.09714 avoir. oz. An avoir. oz. = .911458 troy oz., or apoth.

Long Measure. U. S. and British.

12 inches	1 foot = .3047973 metre.
3 feet	1 yard = 36 ins = .9143919 metre.
5½ yards	1 rod, pole, or perch = 16½ feet = 198 ins.
40 rods	1 furlong = 220 yards = 660 feet.
8 furlongs	1 statute, or land mile = 320 rods = 1760 yds = 5280 ft = 63360 ins.
3 miles	1 league = 24 furlongs = 960 rods = 5280 yds = 15840 ft.

A point = $\frac{1}{72}$ inch. **A line** = 6 points = $\frac{1}{12}$ inch. **A palm** = 3 ins. **A hand** = 4 ins. **A span** = 9 ins. **A fathom** = 6 feet. **A cable's length** = 120 fathoms = 720 feet. **A Gunter's surveying chain** is 66 feet, or 4 rods long. It has 100 links, 7.92 inches long. 80 Gunter's chains = 1 mile.

A nautical mile, geographical mile, sea mile, or knot. is variously defined as being = the length of

		metres	feet	statute miles
1 min of longitude at the equator	=	1855.345	6087.15	1.15287
1 " latitude " "	=	1842.787	6045.95	1.14507
1 " " " pole	=	1861.655	6107.85	1.15679
1 " " at lat 45°	=	1852.181	6076.76	1.15090
1 " a great circle of a *true sphere* whose *surface area* is equal to that of the earth	=	value adopted by **U. S. Coast and Geodetic Survey** 1853.248	6080.27	1.15157
British **Admiralty knot**	=	1853.169	6080.00	1.15152

The above lengths of minutes, in metres and feet, are those published by the U. S. Coast and Geodetic Survey in Appendix No 12, Report for 1881, and are calculated from Clarke's spheroid, which is now the standard of that Survey.

At the equator 1° of lat = 68.70 land miles; at lat 20° = 68.78; at 40° = 69.00; at 60° = 69.23; at 80° = 69.39; at 90° = 69.41.

Lengths of a Degree of Longitude in different Latitudes, and at the level of the Sea. These lengths are in common land or statute miles, of 5280 ft. Since the figure of the earth has never been *precisely* ascertained, these are but close approximations. Intermediate ones may be found correctly by simple proportion. 1° of longitude corresponds to 4 mins of civil or clock time; 1 min of longitude to 4 secs of time.

Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.
0	69.16	14	67.12	28	61.11	42	51.47	56	38.76	70	23.72
2	69.12	16	66.50	30	59.94	44	49.83	58	36.74	72	21.43
4	68.99	18	65.80	32	58.70	46	48.12	60	34.67	74	19.12
6	68.78	20	65.02	34	57.39	48	46.36	62	32.55	76	16.78
8	68.49	22	64.15	36	56.01	50	44.54	64	30.40	78	14.42
10	68.12	24	63.21	38	54.56	52	42.67	66	28.21	80	12.05
12	67.66	26	62.20	40	53.05	54	40.74	68	25.98	82	9.66

Inches reduced to Decimals of a Foot.

No errors.

Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.
0	.0000	**2**	.1667	**4**	.3333	**6**	.5000	**8**	.6667	**10**	.8333
1-32	.0026		.1693		.3359		.5026		.6693		.8359
1-16	.0052		.1719		.3385		.5052		.6719		.8385
3-32	.0078		.1745		.3411		.5078		.6745		.8411
⅛	.0104	⅛	.1771	⅛	.3438	⅛	.5104	⅛	.6771	⅛	.8438
5-32	.0130		.1797		.3464		.5130		.6797		.8464
3-16	.0156		.1823		.3490		.5156		.6823		.8490
7-32	.0182		.1849		.3516		.5182		.6849		.8516
¼	.0208	¼	.1875	¼	.3542	¼	.5208	¼	.6875	¼	.8542
9-32	.0234		.1901		.3568		.5234		.6901		.8568
5-16	.0260		.1927		.3594		.5260		.6927		.8594
11-32	.0286		.1953		.3620		.5286		.6953		.8620
⅜	.0313	⅜	.1979	⅜	.3646	⅜	.5313	⅜	.6979	⅜	.8646
13-32	.0339		.2005		.3672		.5339		.7005		.8672
7-16	.0365		.2031		.3698		.5365		.7031		.8698
15-32	.0391		.2057		.3724		.5391		.7057		.8724
½	.0417	½	.2083	½	.3750	½	.5417	½	.7083	½	.8750
17-32	.0443		.2109		.3776		.5443		.7109		.8776
9-16	.0469		.2135		.3802		.5469		.7135		.8802
19-32	.0495		.2161		.3828		.5495		.7161		.8828
⅝	.0521	⅝	.2188	⅝	.3854	⅝	.5521	⅝	.7188	⅝	.8854
21-32	.0547		.2214		.3880		.5547		.7214		.8880
11-16	.0573		.2240		.3306		.5573		.7240		.8906
23-32	.0599		.2266		.3932		.5599		.7266		.8932
¾	.0625	¾	.2292	¾	.3958	¾	.5625	¾	.7292	¾	.8958
25-32	.0651		.2318		.3984		.5651		.7318		.8984
13-16	.0677		.2344		.4010		.5677		.7344		.9010
27-32	.0703		.2370		.4036		.5703		.7370		.9036
⅞	.0729	⅞	.2396	⅞	.4063	⅞	.5729	⅞	.7396	⅞	.9063
29-32	.0755		.2422		.4089		.5755		.7422		.9089
15-16	.0781		.2448		.4115		.5781		.7448		.9115
31-32	.0807		.2474		.4141		.5807		.7474		.9141
1	.0833	**3**	.2500	**5**	.4167	**7**	.5833	**9**	.7500	**11**	.9167
1-32	.0859		.2526		.4193		.5859		.7526		.9193
1-16	.0885		.2552		.4219		.5885		.7552		.9219
3-32	.0911		.2578		.4245		.5911		.7578		.9245
⅛	.0938	⅛	.2604	⅛	.4271	⅛	.5938	⅛	.7604	⅛	.9271
5-32	.0964		.2630		.4297		.5964		.7630		.9297
3-16	.0990		.2656		.4323		.5990		.7656		.9323
7-32	.1016		.2382		.4349		.6016		.7682		.9349
¼	.1042	¼	.2708	¼	.4375	¼	.6042	¼	.7708	¼	.9375
9-32	.1068		.2734		.4401		.6068		.7734		.9401
5-16	.1094		.2760		.4427		.6094		.7760		.9427
11-32	.1120		.2786		.4453		.6120		.7786		.9453
⅜	.1146	⅜	.2813	⅜	.4479	⅜	.6146	⅜	.7813	⅜	.9479
13-32	.1172		.2839		.4505		.6172		.7839		.9505
7-16	.1198		.2865		.4531		.6198		.7865		.9631
15-32	.1224		.2891		.4557		.6224		.7891		.9557
½	.1250	½	.2917	½	.4583	½	.6250	½	.7917	½	.9583
17-32	.1276		.2943		.4609		.6276		.7943		.9609
9-16	.1302		.2969		.4635		.6302		.7969		.9635
19-32	.1328		.2995		.4661		.6328		.7995		.9661
⅝	.1354	⅝	.3021	⅝	.4688	⅝	.6354	⅝	.8021	⅝	.9688
21-32	.1380		.3047		.4714		.6380		.8047		.9714
11-16	.1406		.3073		.4740		.6406		.8073		.9740
23-32	.1432		.3099		.4766		.6432		.8099		.9766
¾	.1458	¾	.3125	¾	.4792	¾	.6458	¾	.8125	¾	.9792
25-32	.1484		.3151		.4818		.6484		.8151		.9818
13-16	.1510		.3177		.4844		.6510		.8177		.9844
27-32	.1536		.3203		.4870		.6536		.8203		.9870
⅞	.1563	⅞	.3229	⅞	.4896	⅞	.6563	⅞	.8229	⅞	.9896
29-32	.1589		.3255		.4922		.6589		.8255		.9922
15-16	.1615		.3281		.4948		.6815		.8281		.9948
31-32	.1641		.3307		.4974		.6641		.8307		.9974

Square, or Land Measure.

U. S. and British.

144 square inches.................. 1 sq foot. 100 sq ft = 1 **square.**
9 sq feet.......................... 1 sq yard = 1296 sq ins.
30¼ sq yards....................... 1 sq rod = 272¼ sq feet.
40 sq rods......................... 1 rood = 1210 sq yds = 10890 sq feet.
4 roods............................ 1 acre = 160 rods = 4840 sq yds = 43560 sq feet.

A section of land is 1 mile sq, or 27878400 sq ft; or 3097600 sq yds; or 640 acres. **An acre** contains 10 sq Gunter's chains. **A sq acre** is 208.710 feet; a sq half acre, 147.581 ft; and a sq quarter acre, 104.355 ft on each side. **A circular acre** is 235.504 feet: a circular half acre = 166.527 ft; and a circular quarter acre = 117.752 ft diam. **A circular inch** is a circle of 1 inch diam; a sq ft = 183.346 cir ins. **Also 1 sq inch** = 1.27324 cir ins; and 1 cir inch = .7854 of a sq inch.

Cubic, or Solid Measure.

U. S. and British.

1728 cubic inches.......................... 1 cubic, or solid foot.
27 cubic feet.............................. 1 cubic, or solid yard.

A cord of wood = 128 cub ft; being 4 ft × 4 ft × 8 ft. **A perch of masonry** actually contains 24¾ cub ft; being 16½ ft × 1½ ft × 1 ft. It is generally taken at 25 cub ft; but by some at 22, &c; and there is every probability that a payer will be cheated unless the number of cubic ft be distinctly agreed upon in his contract. It is gradually falling into disuse among engineers; and the cub yd is very properly taking its place. **To reduce cub yds** to perches of 25 cub ft, mult by 1.080; and to reduce perches to cub yds, mult by .926. **The Brit rod** of brickwork, of house-builders, is 16½ feet square, by 14 inches (1½ English bricks) thick = 272¼ sq ft of 14 inch wall. It is conventionally taken at 272 sq ft; which gives 317⅓ cub ft. In Brit engineering works the rod is 306 cub ft, or 11⅓ cub yds. The Montreal, (Canada,) **toise** = 261½ cub ft; or 9.6852 cub yds, or 10.46 perches of 25 cub ft. The Canadian **chaldron** = 58.64 cub ft. **A ton** (2240 lbs) of Pennsylvania anthracite, when broken for domestic use, occupies from 41 to 43 cub ft of space; the mean of which is equal to 1.556 cub yds: or a cube of 3.476 ft on each edge. Bituminous coal 44 to 48 cub ft; mean equal to 1.704 cub yd; or a cube of 3.583 ft on each edge. Coke 80 cub ft.

A cubic foot is equal to

1728 cub ins, or 3300.23 spherical ins.
.037037 cub yard, or 1.90985 spherical ft.
.002832 myriolitre, or decastere.
.028316 kilolitre, or cubic metre, or stere.
.283161 hectolitre, or decistere.
2.83161 decalitres, or centisteres.
28.3161 litres, or cub decimetres.
283.161 decilitres.
2831.61 centilitres.
28316.1 millilitres. or cub centimetres.
.803564 U.S. struck bushel of 2150.42 cub ins, or 1.24445 cub ft.
.779013 Brit bushel of 2218.191 cub ins, or 1.28368 cub ft.
3.21426 U. S. pecks.
3.11605 Brit pecks.
7.48052 U. S. liquid galls of 231 cub ins.
6.42851 U. S. dry galls.
6.23210 Brit galls of 277.274 cub ins.
29.92208 U. S. liquid quarts.
25.71405 U. S. dry quarts.
24.92842 Brit quarts.
59.84416 U. S. liquid pints.
51.42809 U. S. dry pints.
49.85684 Brit pints.
239.37662 U. S. gills.
199.42737 Brit gills.
.26667 flour barrel of 3 struck bushels.
.23748 U. S. liquid barrel of 31½ galls.

A cubic inch is equal to

16.38663 millilitres; or 1.638663 centilitres; or .1638663 decilitre; or .01638663 litre; or to .0005787 cub ft; or to .138528 U. S. gill; or 1.90985 spherical ins.

A cubic yard is equal to

27 cub feet, or to 201.974 U. S. galls.
46656 cub ins.
.0764534 myriolitre.
.764534 kilolitre, or cub metre.
7.64534 hectolitres.
7.2 flour barrels of 3 struck bushels.
76.4534 decalitres.
764.534 litres, or cub decimetres.
7645.34 decilitres.
21.69623 U. S. bushels (struck).
21.03336 Brit bushels.

A sphere 1 foot in diameter, contains

.01939 cub yard.
.5236 cub foot.
904.781 cub inches.
.42075 U. S. bushel.
1.6830 U. S. pecks.
13.4639 U. S. dry quarts.
26.9278 U. S. dry pints.
3.9168 U. S. liquid gallons.
15.6672 U. S. liquid quarts.
31.3344 U. S. liquid pints.
125.3376 U. S. liquid gills.
3.2631 Brit imp gallons.
13.0525 Brit imp quarts.
26.1050 Brit imp pints.
104.4201 Brit imp gills.
14.8263 litres.
1.48263 decalitres.
.148263 hectolitres.

A sphere 1 inch in diameter, contains

.000303 cub foot.
.5236 cub inch.
.07253 U. S. gill.
.06043 Brit gill.
8.580 millilitre.
.8580 centilitre.
.08580 decilitre.

A cylinder 1 foot in diameter, and 1 foot high, contains

.02909 cub yard.
.7854 cub foot.
1357.1712 cub inches.
.63112 U. S. dry bushels.
2.5245 U. S. dry pecks.
20.1958 U. S. dry quarts.
40.3916 U. S. dry pints.
5.8752 U. S. liquid gallons.
23.5008 U. S. liquid quarts.
47.0016 U. S. liquid pints.
188.0064 U. S. liquid gills.
4.8947 Brit imp gallons.
19.5788 Brit imp quarts.
39.1575 Brit imp pints.
156.6302 Brit imp gills.
222.396 decilitres.
22.2395 litres.
2.22395 decalitres.
.222395 hectolitre.

A cylinder 1 inch in diameter, and 1 foot high, contains

.005454 cub foot.
9.4248 cub inches.
.2805 U. S. dry pint.
.3264 liquid pint.
1.3056 U. S. gill.
.2719 Brit imp pint.
1.0877 Brit imp gill.
15.4441 centilitres.
1.54441 decilitres.
.154441 litres.

Liquid Measure. U. S. only.

The basis of this measure in the U. S. is the old Brit wine gallon of 231 cub ins; or 8.33888 lbs avoir of pure water, at its max density of about 39°.2 Fahr; the barom at 30 ins. A cylinder 7 ins diam. and 6 ins high, contains 230.904 cub ins, or almost precisely a gallon; as does also a cube of 6.1358 ins on an edge. Also a gallon = .13368 of a cub ft; and a cub ft contains 7.48052 galls; nearly 7½ galls. **This basis however involves an error** of about 1 part in 1362, for the water actually weighs 8.345008 lbs.

		cub ins.		
4 gills	1 pint	= 28.875.	63 gallons	1 hogshead.
2 pints	1 quart	= 57.750 = 8 gills.	2 hogsheads	1 pipe, or butt.
4 quarts	1 gallon	= 231. = 8 pints = 32 gills.	2 pipes	1 tun.

In the U. S. and Great Brit. **1 barrel of wine** or brandy = 31½ galls; in Pennsylvania, a half barrel, 16 galls; a double barrel, 64 galls; **a puncheon,** 84 galls; **a tierce,** 42 galls. A liquid measure barrel of 31½ galls contains 4.211 cub ft = a cube of 1.615 ft on an edge; or 3.38 U. S. struck bushels. **A gill** = 7.21875 cub ins. **The following cylinders** contain some of these measures very approximately.

cub ins.	Diam. Ins.	Height. Ins.		Diam. Ins.	Height. Ins.
Gill (7.21875)	1¾	3	Gallon	7	6
½ pint	2¼	3⅝	2 gallons	7	12
Pint	3½	3	8 gallons	14	12
Quart	3½	6	10 gallons	14	15

Apothecaries' or Wine Measure.

Measure.	Symbol.	Pints.	Fluid ounces.	Fluid drachms.	Minims.	Cubic inches.	Weight of water.‡	
							Pounds, av.	Grains.
1 Gallon	Cong *	8	128	1024	61440	231	8.345	58415
1 Pint	O †	1	16	128	7680	28.875	1.043	7301.9
1 Fluid ounce	f℥	....	1	8	480	1.8047	Ounces, av. 1.043	456.4
1 Fluid drachm	fʒ	....	....	1	60	0.2256	....	57.05
1 Minim	♏	....	....	...	1	0.0038	....	0.95

To reduce U. S. liquid measures to Brit ones of the same denomination, divide by 1.20032; or near enough for common use, by 1.2; or to reduce Brit to U. S. multiply by 1.2.

Dry Measure.

U. S. only.

The basis of this is the old British Winchester struck bushel of 2150.42 cub ins; or 77.627413 pounds avoir of pure water at its max density. Its dimensions by law are 18½ ins inner diam; 19½ ins outer diam; and 8 ins deep; and when heaped, the cone is not to be less than 6 ins high; which makes a heaped bushel equal to 1¼ struck ones; or to 1.55556 cub ft.

		Edge of a cube of equal capacity.
2 pints	1 quart, = 67.2006 cub ins = 1.16365 liquid qt.	4.066 ins.
4 quarts	1 gallon, = 8 pints, = 268.8025 cub ins, = 1.16365 liq gal.	6.454 "
2 gallons	1 peck, = 16 pints, = 8 quarts, = 537.6050 cub ins	8.131 "
4 pecks	1 struck bushel, = 64 pints, = 32 quarts, = 8 gals, = 2150.4200 cub ins.	12.908 "

---

* Abbreviation of Latin, Congius.

† Abbreviation of Latin, Octarius.

‡ At its maximum density, 62.425 pounds per cubic foot, corresponding to a temperature of 4° Centigrade = 39.2° Fahrenheit.

A struck bushel = 1.24445 cub ft. A cub ft = .80356 of a struck bushel. The dry **flour barrel** = 3.75 cub ft; = 3 struck bushels. The dry barrel is not, however, a legalized measure; and no great attention is given to its capacity; consequently, barrels vary considerably. A barrel of flour contains by law, 196 lbs. In ordering by the barrel, the amount of its contents should be specified in pounds or galls.

To reduce U. S. dry measures to Brit imp ones of the same name, div by 1.031516; and to reduce Brit ones to U. S. mult by 1.031516; or for common purposes use 1.032.

British Imperial Measure, both liquid and dry.

This system is established throughout Great Britain, to the exclusion of the old ones. Its basis is the imperial gallon of 277.274 cub ins, or 10 lbs avoir of pure water at the temp of 62° Fahr, when the barom is at 30 ins. **This basis involves an error** of about 1 part in 1836, for 10 lbs of the water = only 277.123 cub ins.

	Avoir lbs. of water.		Cub. ins.	Cub. ft.	Edge of a cube of equal capacity. Inches.
4 gills 1 pint	1.25		34.6592	....	3.2605
2 pints 1 quart	2.50		69.3185	....	4.1079
2 quarts 1 pottle	5.		138.637	....	5.1756
2 pottles 1 gallon	10.		277.274	....	6.5208
2 gallons 1 peck	20.	Dry meas.	554.548	....	8.2157
4 pecks 1 bushel	80.		2218.192	1.2837	13.0417
4 bushels 1 coomb	320.		8872.768	5.1347	
2 coombs 1 quarter	640.		17745.536	10.2694	

The imp gall = .16046 cub ft; and 1 cub ft = 6.23210 galls.

Measure.	Symbol.	Pints.	Fluid ounces.	Fluid drachms.	Minims.	Cubic inches.	Weight of water.‡	
							Pounds, av.	Grains.
1 Gallon	C *	8	160	1280	76800	277.274	10	70000
1 Pint	O †	1	20	160	9600	36.659	1.25	8750
							Ounces, av.	
1 Fluid ounce	fl. oz.	....	1	8	480	1.733	1	437.5
1 Fluid drachm	fl. dr.	....	....	1	60	0.217	....	54.6875
1 Minim	min.	....	....	....	1	0.0036	....	0.9114

The weight of water affords an easy way to find the cubic contents of a vessel.

To obtain the size of commercial measures by means of the weight of water.

At the common temperature of from 70° to 75° Fah, a cub foot of fresh water weighs very approximately 62¼ lbs avoir. A cubic half foot, (6 ins on each edge,) 7.78125 lbs. A cub quarter foot, (3 ins on each edge,) .97266 lb. A cub yard, 1680.75 lbs; or .75034 ton. A cub half yd, (18 ins on each edge,) 210.094 lbs; or .0938 ton. A cub inch, .036024 lb; or .576384 ounce; or 9.2222 drams; or 252.170 grains. An inch square, and one foot long, .432292 lb. Also 1 lb = 27.75903 cub ins, or a cube of 3.028 ins on an edge. An ounce, 1.735 cub ins; a ton, 35.984 cub ft, all near enough for common use.

Original.

Liquid Measures.

	Lbs Avoir. of Water.
U. S. Gill	.26005*
U. S. Pint	1.0402
U. S. Quart	2.0804
U. S. Gallon 8 lbs 5$\frac{1}{7}$ oz	8.3216
U. S. Wine Barrel, 31½ Gall	262.1310

Dry Measures.

U. S. Pint	1.2104
U. S. Quart	2.4208
U. S. Gallon	9.6834
U. S. Peck	19.3668
U. S. Bushel, struck	77.4670

* Or 4 ounces; 2 drams; 15.6625 grs.

Liquid and Dry.

	Lbs Avoir. of Water.
British Imp Gill	.31214*
" " Pint	1.24858
" " Quart	2.49715
" " Gallon	9.9886
" " Peck	19.9772
" " Bushel	79.9088

* 4.9942; or very nearly 5 ounces.

Metric Measures.

Centilitre	.02198†
Decilitre	.2198‡
Litre	2.1981
Decalitre, or Centistere	21.9808
Metre, or Stere	2198.0786

† Or 5.6271 drams; or 153.866 grs.
‡ 3.5169 ounces.

---

* Abbreviation of Latin, Congius.
† Abbreviation of Latin, Octarius.
‡ At the standard temperature, 62° Fahrenheit = about 16.7° Centigrade.

Metric Measures of Length.

By U. S. and British Standard.

	Ins.	Ft.	Yds.	Miles.
Millimetre*	.039370	.003281	....	....
Centimetre†	.39370428	.032809	....	....
Decimetre	3.9370428	.3280869	.1093623	....
Metre‡	39.370428	3.280869	1.093623	....
Decametre	393.70428	32.80869	10.93623	....
Hectometre	Road	328.0869	109.3623	.0621375
Kilometre	measures.	3280.869	1093.623	.6213750
Myriametre		32808.69	10936.23	6.213750

* Nearly the $\frac{1}{25}$ part of an inch. † Full ⅜ inch.

‡ Very nearly 3 ft, 3⅜ ins, which is too long by only 1 part in 8616.

Metric Square Measure.

By U. S. and British Standard.

	Sq. Ins.	Sq. Feet.	Sq. Yds.	Acres.
Sq Millimetre	.001550	.00001076	.0000012	....
Sq Centimetre	.155003	.00107641	.0001196	....
Sq Decimetre	15.5003	.10764101	.0119601	....
Sq Metre, or Centiare	1550.03	10.764101	1.19601	.000247
Sq Decametre, or Are	155003	1076.4101	119.6011	.024711
Decare (not used)	....	10764.101	1196.011	.247110
Hectare	....	107641.01	11960.11	2.47110
Sq Kilometre	.3861090 **sq miles.**	10764101	1196011.	247.110
Sq Myriametre	38.61090 "	....	....	24711.0

Metric Cubic or Solid Measure.

According to U. S. Standard.

Only those marked "Brit" are British.

	Cub Ins.			
Millilitre, or cub Centimetre	.0610254	Liquid.	.0084537 gill.	
		"	.0070428 Brit gill.	
		Dry.	.0018162 dry pint.	
Centilitre	.610254	Liquid.	.084537 gill.	
		"	.070428 Brit gill.	
		Dry.	.018162 dry pint.	
Decilitre	6.10254	Liquid.	.84537 gill = .21134 pint.	
		"	.70428 Brit gill = .17607 Brit pint.	
		Dry.	.18162 dry pint.	
Litre, or cubic Decimetre	61.0254	Liquid.	1.05671 quart = 2.1134 pints.	
		"	.88036 Brit quart = 1.7607 Brit pints.	
		Dry.	.11351 peck = .9081 dry qt = 1.8162 dry pt.	
Decalitre, or Centistere	610.254 **Cub Ft.** .353156	Liquid.	2.64179 U. S. liquid gal.	
		"	2.20090 Brit gal.	
		Dry.	.283783 bush = 1.1351 peck = 9.081 dry qts.	
Hectolitre, or Decistere	3.53156	Liquid.	26.4179 U. S. liquid gal.	
		"	22.0090 Brit gal.	
		Dry.	2.83783 bush.	
Kilolitre, or Cubic Metre, or Stere	35.3156	Liquid.	264.179 U. S. liquid gal.	Cub yds, 1.3080.
		"	220.090 Brit gal.	
		Dry.	28.3783 bush.	
Myriolitre, or Decastere	353.156	Liquid.	2641.79 U. S. liquid gal.	Cub yds, 13.080.
		Dry.	283.783 bush.	

Metric Weights, reduced to common Commercial or Avoir Weight, of 1 pound = 16 ounces, or 7000 grains.

	Grains.
Milligramme	.015432
Centigramme	.15432
Decigramme	1.5432
Gramme	15.432
By law a 5-cent nickel = 5 grammes	Pounds av.
Decagramme	.022046
Hectogramme	.22046
Kilogramme	2.2046
Myriogramme	22.046
Quintal*	220.46
Tonneau; Millier; or Tonne	2204.6

The gramme is the basis of French weights; and is the weight of a cub centimetre of distilled water at its max density, at sea level, in lat of Paris; barom 29.922 ins.

French Measures of the "Systeme Usuel."

This system was in use from about 1812 to 1840, when it was forbidden by law to use even its names. This was done in order to expedite the general use of the tables which we have before given. But as the Systeme Usuel appears in books published during the above interval, we add a table of some of its values.

Measures of Length.

	Yards.	Feet.	Inches.
Ligne usuel, or line	....	....	.09113
Pouce usuel, or inch, = 12 lignes	....	.09113	1.09362
Pied usuel, or foot, = 12 pouces	.36454	1.09362	13.12344
Aune usuel, or ell	1.31236	3.93708	47.245
Toise usuel, = 6 pieds	2.18727	6.56181	78.74172

Weights, Usuel.	
Grain usuel	.8375 grains.
Gros usuel	60.297 "
Once usuel	1.10258 avoir oz.
Marc usuel	.55129 avoir lb.
Livre usuel, or pound,	1.10258 avoir lb.

Cubic, or Solid, Usuel.	
Litron usuel, or 1 litre	= 1.7608 British pint.
Boisseau usuel	2.7512 British gals.

Before 1812, or before the "Systeme usuel," the Old System, "Systeme Ancien," was in use.

French Measures of the "Systeme Ancien."

Lineal.	Square.			Cubic.		
	Sq. ins.	Sq. ft.	Sq. yds.	C. ins.	C. ft.	C. yds.
Point ancien, .0148 ins						
Ligne ancien, .0888 ins	.00789	....	....	.0007		
Pouce ancien, 1.06577 ins = .0888 ft	1.1359	....	....	1.2106		
Pied ancien, 12.7892 ins = 1.06577 ft	....	1.1359	....	......	1.2106	
Aune ancien, 46.8939 ins = 3.90782 ft = 1.30261 yds						
Toise ancien, = 6.3946 ft = 2.1315 yds	....	40.8908	4.5434	......	261.482	9.6845
League = 2282 toises = 2.7637 miles						

There is, however, much confusion about these old measures. Different measures had the same name in different provinces.

---

* The *avoirdupois* quintal is 100 avoirdupois pounds.

Russian.

Foot; same as U. S. or British foot. **Sachine** = 7 feet. **Verst** = 500 sachine = 3500 feet = 1166⅔ yards = .6629 mile. **Pood** = 36.114 lbs avoirdupois.

Spanish.

The castellano of Spain and New Granada, for weighing gold, is variously estimated, from 71.07 to 71.04 grains. At 71.055 grains, (the mean between the two,) an avoirdupois, or common commercial ounce contains 6.1572 castellano; and a lb avoirdupois contains 98.515. Also a troy ounce = 6.7553 castellano; and a troy lb = 81.064 castellano. Three U. S. gold dollars weigh about 1.1 castellano.

The Spanish mark, or marco, for precious metals, in South America, may be taken in practice, as .5065 of a lb avoirdupois. In Spain, .5076 lb. In other parts of Europe, it has a great number of values; most of them, however, being between .5 and .54 of a pound avoirdupois. The .5065 of a lb = 3545½ grains; and .5076 lb = 3553.2 grains. 1 marco = 50 castellanos = 400 tomine = 4800 *Spanish gold*-grains.

The arroba has various values in different parts of Spain. That of Castile, or Madrid, is 25.4025 lbs avoirdupois; **the tonelada** of Castile = 2032.2 lbs avoirdupois; **the quintal** = 101.61 lbs avoirdupois; **the libra** = 1.0161 lbs avoirdupois; **the cantara** of wine, &c. of Castile = 4.263 U. S. gallons; that of Havana = 4.1 gallons.

The vara of Castile = 32.8748 inches, or almost precisely 82⅞ inches; or 2 feet 8⅞ inches. **The fanegada** of land since 1801 = 1.5871 acres = 69134.08 square feet. **The fanega** of corn, &c = 1.59914 U. S. struck bushels. In California, **the vara by law** = 33.372 U. S. inches; and **the legua** = 5000 varas; or 2.6335 U. S. miles.

CONVERSION TABLES OF UNITS OF MEASURES, WEIGHTS, Etc., Etc., Introduction.

In the following tables **repeating decimals** are indicated by a *. Thus: 1.2*01 = 1.2010101 . . . ; 333.*5657 = 333.56575657 . . . ;

Exact values are indicated by a †. Thus: 1 ft = 12† ins; 1 meter = 0.001† kilometer.

Negative characteristics are indicated by the letter n. Thus: n 1.285 5573 = $\overline{1}$.285 5573 = 0.285 5573 — 1 = 9.285 5573 — 10. For the use of logarithms, see pp. 70, etc.

Gunter's and Engineers' **chains and links** are distinguished by the letters G and E respectively. Thus: 1 chain G = 66† ft; 1 link E = 1† ft.

Table A contains the equations taken as the basis of Tables 1 to 36 inclusive.

In Table C we give equivalents and numbers in common use, in order to facilitate the transformation of equations, as explained below.

Each table has been calculated independently by at least two persons, and their results compared and corrected. One of these results was used as copy for the compositor, and his proofs were compared with the other.

The calculations comprised many times the number of equations here given. To have published any considerable proportion of those calculated would have made far too bulky a table for an Engineer's Pocket Book. With a little practice, however, the reader can easily find other equations by means of those given. In doing this, the logarithms given in Table C will be found useful.

Thus, suppose we desire the length of the meter, in rods. In Table 1 (Length) or in Table C (Equivalents in Common Use), we find

1 meter = 3.28083 feet, Log 0.515 9842

and in Table 1 we find

1 foot = 0.*0606 rod, Log n 2.782 5161

Adding logs, we find

1 meter = 0.19884 rod, Log n 1.298 5003

Again: 1 meter per second = ? miles per day.

Table 18 (Length per Time, or Velocity) gives

1 meter per second = 2.23693 miles per hour. Log 0.349 6528

and Table 5 (Time) or Table C gives

1 day = 24 hours Log 1.380 2112

Adding logs, we obtain

1 meter per second = 53.686 miles per day 1.729 8640

Again:

1 cu ft per sq mile per sec = ? U. S. gals per acre per day.

1 cu ft per sq mile per sec = $\frac{1{,}728 \times 86{,}400}{231 \times 640}$ gals per acre per day.

Table C gives

Log 1,728 3.237 5437

Add Log 86,400 4.936 5137

1 cu ft per acre per sec = x cu ins per acre per sec 8.174 0574

Subtract sum of $\left\{\begin{array}{l}\text{Log 231} \quad 2.363\ 6120 \\ \text{and Log 640} \quad 2.806\ 1800\end{array}\right.$ 5.169 7920

1 cu ft per sq mile per sec = 1009.87 gals per acre per day 3.004 2654

In a few cases, of frequent occurrence, we give two tables, each containing the reciprocals of the values given in the other. Thus, we give Weights of Volumes (as 1 cu ft = ? lbs) and Volumes of Weights (as 1 lb = ? cu ft) of Water; Volume per Surface to Length (as 1 gal per acre = ? ins.) and Length to Volume per Surface (as 1 inch = ? gals per acre), etc., etc. Where only one of two such reciprocal values is given, the other may easily be found from it. Thus, (But see CAUTION, next page.)

Log 1 0.000 0000

1 ft per sec = 720 ins per minute. Log 2.857 3325

1 sec per ft = 0.0013*888 min per inch. Log n 3.142 6675

* Repeating decimals. † Exact values. n, Negative characteristic. E, Engineer's. G, Gunter's.

CONVERSION TABLES, Introduction (Continued).

Caution. It will be noticed that, although a body moving 1 foot per second (or 720 inches per minute) requires 1 sec to travel 1 ft (or 0.0013*888 min to travel 1 inch), and although 1 ft per sec and 1 sec per ft are also *numerically* equal, they are not really identical, but reciprocal. Thus, also,

1 dime per dollar = 1 : 10
but 1 dollar per dime = 10 : 1.

In selecting equations for publication, we have in general given preference to those which cannot easily be found by means of others in the same table, or by means of simpler tables. Thus, having given, in Table 12 (Weight per Surface), the equivalent of 1 kilogram per sq centimeter, in pounds per sq *inch*, we omit the equivalent in pounds per sq *foot*, which is readily found, by using Table 2 (Surface), from the given equivalent in pounds per sq *inch*.

Again, the equivalents of a pressure of 1 pound per sq inch, in ounces per sq inch, and in pounds per sq foot, are not given in Table 12 (Weight per Surface); because they are easily deduced from 1 pound per sq inch, by means of Table 4 (Weight) and Table 2 (Surface), respectively.

The ratios between the values of the coins of different nations being subject to change, the equivalents in Tables 25 to 30 inclusive (Values) can at best be only rudely approximate. Their logarithms are therefore given to only three places of decimals, although the calculations were made by means of seven-place logarithms. Conversions of such approximate equivalents may be conveniently made by means of our two-page table of 5-place logarithms, pp. 78 and 79, omitting the last two decimal places. Indeed, the two-page table, which obviates the necessity of turning over pages, may often be used to advantage in dealing with quantities more accurately known.

List of Tables.

PAGE
A Fundamental Equivalents, ... 230
B Abbreviations, ... 230
C Equivalents and Numbers in Common Use, ... 231
D Prefixes used in the Metric System, ... 231
1 Length, ... 232
2 Surface, ... 233
3 Volume, ... 234
4 Weight, ... 235
5 Time, ... 236
6 Work and Heat, ... 237
7 Length per Length (Grades, etc.), ... 237
8 Surface per Length (Widths), ... 238
9 Volume per Length (Surfaces), ... 238
10 Weight per Length, ... 239
11 Volume per Surface (Depths), ... 239
12 Weight per Surface (Unit pressures), ... 240
13 Weight per Volume (Specific gravity), ... 241
14 Water, Weights of Volumes of, — 241
15 Water, Volumes of Weights of, — 242
16 Air, Weights of Volumes of, — 242
17 Air, Volumes of Weights of, — 242
18 Length per Time (Velocity), ... 242
19 Surface per Time, ... 243
20 Volume per Time (Discharges), ... 243
21 Weight per Time, ... 244
22 Work per Time (Power), ... 244
23 Work per Volume, ... 245
24 Work per Weight, ... 246
25 Value per Length, ... 246
26 Value per Surface, ... 247
27 Value per Volume, ... 248
28 Value per Weight, ... 248
29 Value per Time (Wages), ... 249
30 Value per Work, ... 250
31 Length per Time per Time (Acceleration), ... 250
32 Length to Volume per Surface, ... 250
33 Volume per Surface to Length, ... 251
34 Volume per Surface per Time, ... 251
35 Volume per Surface per Time to Velocity, ... 252
36 **Velocity to Volume per Surface** per Time, ... 253

*** Repeating decimals. † Exact values. n, Negative characteristic. E, Engineer's. G, Gunter's.**

Table A.

Fundamental Equivalents.

(*See Notes below.*)

			Logarithm
1 meter =	39.37	inches (a)	1.595 1654
1 gram =	15.43235	grains (a)	1.188 4320
1 cu centimeter water weighs	1	gram (a)	0.000 0000
1 cu meter air weighs	1.293052	kilograms (b)	0.111 6159
1 inch mercury =	13.5956	inches water (c)	1.133 4112
1 atmosphere =	760	mm of mercury	2.880 8136
1 pound Fahr (B. T. U.) =	778	foot-pounds (d)	2.890 9796
g = acceleration of gravity =	9.81	meters per second per second (e)	0.991 6690
£ 1 =	$4.86	(f)	0.686 6363
1 franc =	$0.193	(f)	n 1.285 5573
1 mark =	$0.238	(f)	n 1.376 5770

(a) See p 216.

(b) See Density of air, p 320.

(c) 1 inch mercury = 13.5956 inches water. Value adopted by International Bureau of Weights and Measures. Lat. 45°. Sea level. Temperature of mercury 0° C. = 32° Fahr.

(d) 1 pound Fahr = 778 foot-pounds. According to very careful determinations by Prof. Henry A. Rowland (Proceedings, Am. Acad. Arts and Sciences for 1880) and generally accepted as standard.

(e) $g = 9.81$ meters per sec per sec. Generally accepted average. At latitude 45° and at sea level, $g = 9.806056$ m. For other latitudes, L, and elevations h, in centimeters;

g in centimeters per sec per sec = approx $980.6056 - 2.5028 \cos 2L - 0.000003\,h$.

(f) According to circular of U. S. Treasury Department, Bureau of the Mint, Jan'y 1, 1887, £1 = $4.8665 gold; 1 franc = $0.193; 1 mark = $0.238.

Table B.

Abbreviations.

a	acre
bu	bushel
C	Centigrade
c	centime
cc	cubic centimeter
cg	centigram
ch	chain
cl	centiliter
cm	centimeter
cs	centistere
ct	cent
cu	cubic
d	day
d	penny or pence
dcg	decigram
dcl	deciliter
dcm	decimeter
dcs	decistere
dkl	dekaliter
dkm	dekameter
dks	dekastere
dwt	pennyweight
F	Fahrenheit
ft	foot or feet
fur	furlong
g	acceleration of gravity
g	gramme
gal	gallon
gr	grains
hec	hectare
h	hour
hg	hectogram
hl	hectoliter or decistere
hm	hectometer
H.P.	horse-power
Imp	Imperial
in	inch
J	joule
kg	kilogram
kgm	kilogrammeter
kl	kiloliter
km	kilometer
kw	kilowatt
£	pound (money)
li	liter
℔	pound (weight)
liq	liquid
m	meter
mi	mile
min	minute
mlg	milligram
mll	milliliter
mm	millimeter
myg	myriagram
pf	pfennig
ql	quintal
s	shilling
sec	second
sect	section
sq	square
st	stere
w	Watt
yd	yard
$	dollar

CONVERSION TABLES, Introduction (Continued).

Table C.

Equivalents and Numbers in Common Use.

(Arranged numerically.)

			Logarithm
1 cu cm of water weighs	1	gram*	0.000 0000
1 meter =	1.093611	yards	0.038 8629
	2		0.301 0300
1 kilogram =	2.20462	pounds	0.343 3340
1 yard =	3	feet	0.477 1213
1 circumference =	π	diameters	0.497 1499
1 meter =	3.280833	feet	0.515 9842
1 gallon =	4	quarts	0.602 0600
1 link G =	7.92	inches	0.898 7252
1 sq yard =	9	sq feet	0.954 2425
g = acceleration of gravity =	9.81	meters per sec per sec	0.991 6690
1 meter =	10	decimeters	1.000 0000
1 foot =	12	inches	1.079 1812
1 gram =	15.43235	grains†	1.188 4320
1 pound =	16	ounces	1.204 1200
1 rod =	16.5	feet	1.217 4839
1 day =	24	hours	1.380 2112
1 cu yard =	27	cu feet	1.431 3638
1 yard =	36	inches	1.556 3025
1 meter =	39.37	inches†	1.595 1654
1 hour =	60	minutes	1.778 1513
1 chain G =	66	feet	1.819 5439
1 cwt =	112	pounds	2.049 2180
1 sq foot =	144	sq inches	2.158 3625
1 gallon U. S. =	231	cu inches	2.363 6120
1 ounce =	437.5	grains	2.640 9781
1 horse-power =	550	foot-pounds per sec	2.740 3627
1 sq mile =	640	acres	2.806 1800
1 sq yard =	1,296	sq inches	3.112 6050
1 day =	1,440	minutes	3.158 3625
1 cu foot =	1,728	cu inches	3.237 5437
1 mile =	1,760	yards	3.245 5127
1 ton =	2,240	pounds	3.350 2480
1 hour =	3,600	seconds	3.556 3025
1 acre =	4,840	sq yards	3.684 8454
1 mile =	5,280	feet	3.722 6339
1 pound =	7,000	grains	3.845 0980
1 horse-power =	33,000	foot-pounds per min	4.518 5139
1 acre =	43,560	sq feet	4.639 0879
1 cu yard =	46,656	cu inches	4.668 9075
1 mile =	63,360	inches	4.801 8152
1 day =	86,400	seconds	4.936 5137

Table D.

Prefixes used in the Metric System.

1 Myria-meter, gram, etc.,	=	10,000 meters, grams, etc.	4.000 0000
1 Kilo- " " "	=	1,000 " " "	3.000 0000
1 Hecto- " " "	=	100 " " "	2.000 0000
1 Deka- " " "	=	10 " " "	1.000 0000
1 .... meter, gram, etc.	=	1 meter, gram, etc.	0.000 0000
1 Deci- " " "	=	0.1 " " "	n 1.000 0000
1 Centi- " " "	=	0.01 " " "	n 2.000 0000
1 Milli- " " "	=	0.001 " " "	n 3.000 0000

---

* At maximum density, = 4° Centigrade, = 39.2° Fahr.

† See page 217.

CONVERSION TABLES.

(1) LENGTH.

1 Chain E =

			LOGARITHM.
100 †	feet		2.000 0000
6.*0606	rods		0.782 5161
1.*5151	chains G		0.180 4561
0.018*93	mile	n	2.277 3661
30.4801	meters		1.484 0158

1 Chain G =

66 †	feet		1.819 5439
4 †	rods		0.602 0600
0.66†	chain E	n	1.819 5439
0.0125†	mile	n	2.096 9100
20.1168	meters		1.303 5597

1 Fathom =

6 †	feet		0.778 1513

1 Foot =

12 †	inches		1.079 1812
0.*333	yard	n	1.522 8787
0.*0606	rod	n	2.782 5161
0.0*1515	chain G	n	2.180 4561
0.00018*93	mile	n	4.277 3661
0.30480	meter	n	1.484 0158

1 Furlong =

0.125†	mile	n	1.096 9100

1 Inch =

1,000 †	mils		3.000 0000
0.08*33	foot	n	2.920 8188
0.02*777	yard	n	2.443 6975
2.540005	centimeters		0.404 8346

(1) LENGTH.—(Continued.)

1 Link G =

			LOGARITHM.
7.92†	inches		0.898 7252
0.66†	foot	n	1.819 5439

1 Mil =

0.001†	inch	n	3.000 0000
0.02540	millimeter	n	2.404 8346

1 Mile =

63,360 †	inches		4.801 8152
8,000 †	links G		3.903 0900
5,280 †	feet		3.722 6339
1,760 †	yards		3.245 5127
320 †	rods		2.505 1500
80 †	chains G		1.903 0900
52.8†	chains E		1.722 6339
8 †	furlongs		0.903 0900
1.60935	kilometers		0.206 6497

1 Rod =

25 †	links G		1.397 9400
16.5†	feet		1.217 4839
0.25†	chain G	n	1.397 9400
0.003125†	mile	n	3.494 8500

1 Yard =

36 †	inches		1.556 3025
3 †	feet		0.477 1213
0.00056*81	mile	n	4.754 4873
0.914402	meter	n	1.961 1371

1 Mikron =

0.001†	millimeter	n	3.000 0000

* Repeating decimals. † Exact values. n. Negative characteristic. E. Engineer's. G. Gunter's.

(1) LENGTH.—(Continued.)

			LOGARITHM.
1 Millimeter =			
39.37	mils		1.595 1654
0.03937	inch	n	2.595 1654
1,000 †	mikrons		3.000 0000
0.001†	meter	n	3.000 0000
1 Centimeter =			
0.3937	inch	n	1.595 1654
0.0328083	foot	n	2.515 9842
10 †	millimeters		1.000 0000
0.01†	meter	n	2.000 0000
1 Decimeter =			
3.937	inches		0.595 1654
0.1†	meter	n	1.000 0000
1 Meter =			
39.37	inches (see p. 230)		1.595 1654
3.28083	feet		0.515 9842
1.09361	yards		0.038 8629
1,000 †	millimeters		3.000 0000
100 †	centimeters		2.000 0000
10 †	decimeters		1.000 0000
0.001†	kilometer	n	3.000 0000
1 Dekameter =			
10 †	meters		1.000 0000
1 Hectometer =			
100 †	meters		2.000 0000
1 Kilometer =			
3,280.83	feet		3.515 9842
1,093.61	yards		3.038 8629
0.62137	mile	n	1.793 3503
1,000 †	meters		3.000 0000

(2) SURFACE.

			LOGARITHM.
1 Acre =			
100,000 †	sq links G		5.000 0000
43,560 †	sq feet		4.639 0879
4,840 †	sq yards		3.684 8454
208.710	feet square		2.319 5440
160 †	sq rods		2.204 1200
10 †	sq chains G		1.000 0000
3.16228	chains G square		0.500 0000
0.0015625†	sq mile	n	3.193 8200
4,046.87	sq meters		3.607 1196
0.404687	hectare	n	1.607 1196
1 Sq Foot =			
144 †	sq inches		2.158 3625
0.*111	sq yard	n	1.045 7575
0.0929034	sq meter	n	2.968 0316
1 Sq Inch =			
0.0069*44	sq foot	n	3.841 6375
6.45163	sq centimeters		0.809 6692
1,273,240	circular mils		6.104 9101
1 Sq Mil =			
0.000001†	sq inch	n	6.000 0000
0.000645163	sq millimeter	n	4.809 6692
1 Sq Mile =			
27,878,400 †	sq feet		7.445 2678
3,097,600 †	sq yards		6.491 0253
640 †	acres		2.806 1800
1 †	section		0.000 0000
259.000	hectares		2.413 2996
2.59000	sq kilometers		0.413 2996
1 Section =			
1 †	sq mile. (See sq mile.)		
1 Half-section =			
3,200 †	sq chains G		3.505 1500
1.29500	sq kilometers		0.112 2696

* Repeating decimals. † Exact values. n. Negative characteristic. E. Engineer's. G. Gunter's.

(2) SURFACE.—(Continued.)

1 Sq Yard =

			Logarithm.
1,296 †	sq inches		3.112 6050
9 †	sq feet		0.954 2425
0.836131	sq meter	n	1.922 2742

1 Sq Millimeter =

1,550.0	sq mils		3.190 3308
0.0015500	sq inch	n	3.190 3308
0.01†	sq centimeter	n	2.000 0000

1 Sq Centimeter =

0.15500	sq inch	n	1.190 3308
0.0001†	sq meter	n	4.000 0000

1 Sq Meter =

1,550.0	sq inches		3.190 3308
10.7639	sq feet		1.031 9683
1.19598	sq yard		0.077 7258
10,000 †	sq centimeters		4.000 0000

1 Sq Hectometer =

1 .... hectare. (See 1 hectare.)

1 Sq Kilometer =

247.105	acres		2.392 8804
0.386101	sq mile	n	1.586 7005
1,000,000 †	sq meters		6.000 0000
100 †	hectares		2.000 0000

1 Hectare =

107,639	sq feet		5.031 9683
2.47104	acres		0.392 8804
0.003861	sq mile	n	3.586 7005
10,000 †	sq meters		4.000 0000
100 †	meters square		2.000 0000
0.01†	sq kilometer	n	2.000 0000

(3) VOLUME.

1 Bushel U S =

			Logarithm.
1.24446	cu feet		0.094 9796

1 Bushel British =

1.28368	cu feet		0.108 4555

1 Cord of Wood =

4 × 4 × 8 †	feet		
128 †	cu feet		2.107 2100

1 Cu Foot =

1,728 †	cu inches		3.237 5437
$\frac{1728}{231}$ = 7.48052	gallons U S liquid		0.873 9317
6.23210	gallons Imp		0.794 6345
0.*037	cu yard	n	2.568 6362
28.317	liters		1.452 0475
0.028317	cu meter	n	2.452 0475

1 Cu Inch =

16.3872	cu centimeters		1.214 5038

1 Cu Yard =

46,656 †	cu inches		4.668 9075
27 †	cu feet		1.431 3638
0.00061983	acre-foot	n	4.792 2759
0.76456	cu meter	n	1.883 4113

1 U S Gallon =

231 †	cu inches		2.363 6120
8 †	U S pints liquid		0.903 0900
0.833111	Imp gallon	n	1.920 7028
$\frac{231}{1728}$ = 0.133681	cu foot	n	1.126 0683
3.78543	liters		0.578 1158
0.00378543	cu meter or stere	n	3.578 1158

1 Imp Gallon =

277.274	cu inches		2.442 9092
1.20032	U S gallons liquid		0.079 2972
0.160459	cu foot	n	1.205 3655
4.54374	liters		0.657 4130

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(3) VOLUME.—(Continued.)

			Logarithm.
1 Ounce, Fluid =			
1.8047	cu inches		0.256 4050
29.5739	cu centimeters		1.470 9088
1 Perch =			
24.75†	cu feet		1.393 5752
16.5 × 1.5 × 1† feet			
0.916667	cu yard	n	1.962 2114
0.700846	cu meter	n	1.845 6227
1 Cu Millimeter =			
0.000061023	cu inch	n	5.785 4962
0.001†	cu centimeter	n	3.000 0000
1 Cu Centimeter or Milliliter =			
0.0610234	cu inch	n	2.785 4962
1,000 †	cu millimeters		3.000 0000
0.001†	liter	n	3.000 0000
1 Centiliter =			
10 †	milliliters		1.000 0000
0.01†	liter	n	2.000 0000
1 Deciliter =			
0.1†	liter	n	1.000 0000
1 Liter or Cu Decimeter =			
61.0234	cu inches		1.785 4962
0.26417	U S gallon liquid	n	1.421 8842
0.220083	Imp gallon	n	1.342 5870
0.0353145	cu foot	n	2.547 9525
1,000 †	cu cm or milliliters		3.000 0000
0.001†	cu meter or stere	n	3.000 0000
1 Dekaliter or Centistere =			
10 †	liters		1.000 0000
0.01†	cu meter	n	2.000 0000

(3) VOLUME.—(Continued.)

			Logarithm.
1 Hectoliter or Dekastere =			
100 †	liters		2.000 0000
1 Cu Meter, Stere, or Kiloliter =			
61,023.4	cu inches		4.785 4962
264.170	gallons U S liquid		2.421 8842
220.083	gallons British		2.342 5870
35.3145	cu feet		1.547 9525
1.30794	cu yards		0.116 5887
0.000810708	acre-foot	n	4.908 8646
1,000 †	liters		3.000 0000
1 Acre-foot =			
325,851	gallons U S liquid		5.513 0196
271,472	gallons Imp		5.433 7224
43,560 †	cu feet		4.639 0879
1,613.*333	cu yards		3.207 7241
1,233.49	cu meters		3.091 1354

(4) WEIGHT (AND FORCE).

Avoirdupois and Metric.

1 Grain =			
0.0647989	gram	n	2.811 5680
1 Ounce =			
437.5†	grains		2.640 9781
0.0625†	pound	n	2.795 8800
28.3496	grams		1.452 5461
1 Pound =			
7,000 †	grains		3.845 0980
16 †	ounces		1.204 1200
453.593	grams		2.656 6660
0.453593	kilogram	n	1.656 6660

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(4) WEIGHT (AND FORCE).—(Continued.)

Avoirdupois and Metric.

		Logarithm.
1 Ton (2,240 Pounds) =		
2,240 †	pounds	3.350 2480
1,016.05	kilograms	3.006 9141
1.01605	metric tons	0.006 9141
1 Milligram =		
0.001†	gram	n 3.000 0000
1 Centigram =		
0.01†	gram	n 2.000 0000
1 Decigram =		
0.1†	gram	n 1.000 0000
1 Gram =		
15.43235	grains (see p. 230)	1.188 4320
0.001†	kilogram	n 3.000 0000
1 Dekagram =		
10 †	grams	1.000 0000
1 Hectogram =		
100 †	grams	2.000 0000
1 Kilogram =		
2.20462	pounds	0.343 3340
1,000 †	grams	3.000 0000
1 Myriagram =		
10 †	kilograms	1.000 0000
1 Quintal (Metric) =		
100 †	kilograms	2.000 0000
1 Tonne or Metric Ton =		
2,204.62	pounds	3.343 3340
1.10231	tons (2,000 lbs)	0.042 3040
0.984206	ton (2,240 lbs)	n 1.993 0860
1,000 †	kilograms	3.000 0000

(5) TIME.

1 year = 365† days. 1 leap year = 366† days.

		Logarithm.
1 Leap Year =		
8,784 †	hours	3.943 6923
366 †	days	2.563 4811
1 Year =		
8,760 †	hours	3.942 5041
365 †	days	2.562 2929
1 Day =		
86,400 †	seconds	4.936 5137
1,440 †	minutes	3.158 3625
24 †	hours	1.380 2112
0.142857	week	n 1.154 9020
0.00273973	year	n 3.437 7071
0.00273224	leap year	n 3.436 5189
1 Hour =		
3,600 †	seconds	3.556 3025
60 †	minutes	1.778 1513
0.041*666	day	n 2.619 7888
0.000114155	year	n 4.057 4959
1 Minute =		
60 †	seconds	1.778 1513
0.01*66	hour	n 2.221 8487
0.00069*44	day	n 4.841 6375
1 Second =		
0.01*66	minute	n 2.221 8487
0.0002*77	hour	n 4.443 6975
0.000011574	day	n 5.063 4863
1 Week =		
168 †	hours	2.225 3093
7 †	days	0.845 0980

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(6) WORK AND HEAT.

For Power, see Table (22).

1 Foot-pound =

			LOGARITHM.
0.00128535	pound °F	n	3.109 0204
13,825.5	gram cm.		4.140 6818
1.3562844	joules		0.132 3508
0.138255	kilogrammeter	n	1.140 6818
0.00184340	metric H P second	n	3.265 6205
0.000323902	kilogram °C	n	4.510 4139
0.000376746	watt-hour	n	4.576 0483

1 H P (Horse-power) Hour =

1,980,000 †	foot-pounds		6.296 6652
2,544.99	pound °F		3.405 6856
2,685,443	joules		6.429 0160
273,746	kilogrammeters		5.437 3470
641.326	kilogram °C		2.807 0791

1 Inch-pound =

1,152.13	gram cm.		3.061 5006

1 B. T. U. (British Thermal Unit). See Pound Degree Fahrenheit.

1 Pound Degree Fahrenheit =

1 †	British Thermal Unit		0.000 0000
778	foot-pounds		2.890 9796
107.563	kilogrammeters		2.031 6514
0.251996	kilogram °C	n	1.401 3935

1 Joule =

0.737308	foot-pound	n	1.867 6492
0.101937	kilogrammeter	n	1.008 3310

1 Kilogrammeter =

7.23300	foot-pounds		0.859 3182
0.0131509	H P second	n	2.118 9555
0.00929691	pound °F	n	3.968 3386
9.81	joules		0.991 6690
0.01*33	metric H P second	n	2.124 9387
0.00234278	kilogram °C	n	3.369 7321
0.002725	watt-hour	n	3.435 3665

(6) WORK AND HEAT.—(Continued.)

1 Calorie. See Kilogram Degree Centigrade.

1 Kilogram Degree Centigrade =

			LOGARITHM.
3,087.35	foot-pounds		3.489 5861
3.96832	pound °F		0.598 6065
426.843	kilogrammeters		2.630 2679

1 Watt-hour =

2,654.31	foot-pounds		3.423 9517
3.41171	pound °F		0.532 9721
3,600 †	joules		3.556 3025
366.972	kilogrammeters		2.564 6335
0.859737	kilogram °C	n	1.934 3656
0.00135916	metric H P hour	n	3.133 2697

1 Metric H P (Horse-power) Hour =

1,952,910	foot-pounds		6.290 6820
2,648,700	joules		6.423 0328
270,000 †	kilogrammeters		5.431 3638
632.551	kilogram °C		2.801 0959

(7) LENGTH PER LENGTH. (GRADES, ETC.)

1 Foot per Mile = 1:5280 † =

0.00018*93		n	4.277 3661
0.002*27	inch per foot	n	3.356 5473

1 Inch per Foot = 1:12 =

0.08*33 (tan 4° 46′ = 0.083386)		n	2.920 8188
0.0069*44	foot per inch	n	3.841 6376
440 †	feet per mile		2.643 4527

1 Inch per Mile =

0.00157828	foot per 100 feet	n	3.198 1849

1 Mile per Inch =

63,360 †			4.801 8152
21,120 †	yards per foot		4.324 6939

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(8) SURFACE PER LENGTH. (WIDTHS.)

			LOGARITHM.
1 Acre per Chain E =			
435.6†	feet		2.639 0879
0.0825†	sq mile per mile	n	2.916 4539
0.132771	sq km per km	n	1.123 1038
1 Acre per Chain G =			
660 †	feet		2.819 5439
66,000 †	sq feet per 100 feet		4.819 5439
0.125†	sq mile per mile	n	1.096 9100
0.201168	sq km per km	n	1.303 5599
1 Acre per Foot =			
43,560 †	feet		4.639 0879
4,356,000 †	sq feet per 100 feet		6.639 0879
8.25†	sq miles per mile		0.916 4539
13.2771	sq km per km		1.123 1038
1 Acre per Mile =			
8.25†	feet		0.916 4539
825 †	sq feet per 100 feet		2.916 4539
1,188 †	sq inches per foot		3.074 8165
0.251461	hectare per km	n	1.400 4699
1 Sq Foot per Mile =			
0.002*27	inch = $\frac{1}{440}$† inch	n	3.356 5473
0.00018*93	foot = $\frac{1}{5280}$† foot	n	4.277 3661
0.0577274	sq meter per km	n	2.761 3819
1 Sq Inch per Foot =			
36.*666	sq feet per mile		1.564 2714
2.11667	sq meters per km		0.325 6534
1 Sq Inch per Inch =			
0.08*33	sq foot per foot	n	2.920 8187
0.0069*44	sq foot per inch	n	3.841 6375
0.*0101	acre per mile	n	2.004 3648
25.4001	sq m per km		1.404 8346
1 Sq Mile per Mile =			
0.*1212	acre per foot	n	1.083 5461
1.60935	sq km per km		0.206 6499

(8) SURFACE PER LENGTH. (WIDTHS.)

—(Continued.)

			LOGARITHM.
1 Sq Mile per Chain E =			
440 †	sq feet per inch		2.643 4527
1 Sq Meter per Kilometer =			
17.3228	sq feet per mile		1.238 6180
0.47244	sq inch per foot	n	1.674 3466
1 Hectare per Kilometer =			
10 †	meters		1.000 0000
3.97677	acres per mile		0.599 5301

(9) VOLUME PER LENGTH. (SURFACES.)

1 Cu Foot per Foot =			
1 †	sq foot		0.000 0000
195.*55	cu yards per mile		2.291 2701
92.9034	liters per meter		1.968 0317
1 Cu Foot per Inch =			
12 †	sq feet		1.079 1812
0.*444	cu yard per foot	n	1.647 8174
1,114.84	liters per meter		3.047 2129
1 Cu Foot per Mile =			
0.00018*93	sq ft = $\frac{1}{5280}$† sq ft	n	4.277 3661
0.0*2727	sq in = $\frac{144}{5280}$† sq in	n	2.435 7286
0.3*2727	cu inch per foot	n	1.514 9098
0.0175953	liter per meter	n	2.245 3978
1 Cu Inch per Foot =			
0.08*3	sq inch = $\frac{1}{12}$† sq inch	n	2.920 8188
3.0*55	cu feet per mile		0.485 0902
0.0537637	liter per meter	n	2.730 4880
1 Cu Inch per Inch =			
1 †	sq inch		0.000 0000
36.*66	cu feet per mile		1.564 2715
0.645163	liter per meter	n	1.809 6692

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(9) VOLUME PER LENGTH. (SURFACES.) —(Continued.)

		LOGARITHM.
1 Cu Yard per Foot =		
3 †	sq yards	0.477 1218
2.25†	cu feet per inch	0.352 1826
2508.39	liters per meter	3.399 3955
1 Cu Yard per Inch =		
46,656 †	sq inches	4.668 9075
1 Cu Yard per Mile =		
0.00056818	sq yd = $\frac{1}{1760}$† sq yd	n 4.754 4878
8.8*3636	cu inches per foot	0.946 2736
0.47507	liter per meter	n 1.676 7616
1 U S Gallon per Foot =		
19.25†	sq inches	1.284 4307
705.833	cu feet per mile	2.848 7022
12.4194	liters per meter	1.094 1000
1 U S Gallon per Inch =		
231 †	sq inches	2.363 6120
1.60417	cu feet per foot	0.205 2495
149.033	liters per meter	2.173 2812
1 Liter per Meter =		
0.001†	sq meter	n 3.000 0000
18.6	cu inches per foot	1.269 5120
56.8332	cu feet per mile	1.754 6022

(10) WEIGHT PER LENGTH.

1 Pound per Foot =		
1.48816	kg per meter	0.172 6502
1 Pound per Inch =		
17.8579	kg per meter	1.251 8314
1 Pound per Mile =		
0.281849	gram per meter	n 1.450 0163

(10) WEIGHT PER LENGTH.—(Continued.)

		LOGARITHM.
1 Gram per Meter =		
3.5480	pounds per mile	0.549 9837
1 Kilogram per Meter =		
0.67197	pound per foot	n 1.827 3498
1 Kilogram per Kilometer =		
3.548	pounds per mile	0.549 9837

(11) VOLUME PER SURFACE. (DEPTHS.)

1 Cu Foot per Acre =		
0.000275482	inch	n 4.440 0938
23.7037	cu yards per sq mile	1.374 8162
0.0699726	cu meter per hectare	n 2.844 9279
1 Cu Foot per Sq Foot =		
1 †	foot	0.000 0000
1,613.*333	cu yards per acre	3.207 7241
0.05195	U S gal per sq inch	n 2.715 5692
1 Cu Foot per Sq Mile =		
2.7†	cu inches per acre	0.431 3638
0.0109332	cu meter per sq km	n 2.038 7479
1 Cu Inch per Sq Foot =		
25.2083	cu feet per acre	1.401 5442
597.531	cu yards per sq mile	2.776 3603
0.176389	liter per sq meter	n 1.246 4722
1 Cu Inch per Sq Inch =		
1 †	inch	0.000 0000
3,630 †	cu feet per acre	3.559 9067
0.02540	cu meter per sq meter	n 2.404 8346

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(11) VOLUME PER SURFACE. (DEPTHS.)
—(Continued.)

1 Cu Yard per Acre =

			Logarithm.
1.07107	cu inches per sq foot		0.029 8196
0.188926	liter per sq meter	n	1.276 2917

1 Cu Yard per Sq Foot =

27 †	feet		1.431 3638
8.22962	cu meters per sq meter		0.915 3797

1 Cu Yard per Sq Mile =

72.9†	cu inches per acre		1.862 7275
0.295197	cu meter per sq km	n	1.470 1117

1 U S Gallon per Acre =

85.*555	cu feet per sq mile		1.932 2483
0.935397	cu meter per sq km	n	1.970 9962

1 U S Gallon per Sq Foot =

5,823.13	cu feet per acre		3.765 1562
40.7459	liters per sq meter		1.610 0842

1 Liter per Sq Meter =

0.001†	meter	n	3.000 0000
142.913	cu feet per acre		2.155 0721
0.0245423	U S gal per sq foot	n	2.389 9159

1 Liter per Hectare =

9.14644	cu feet per sq mile		0.961 2521
0.106906	U S gal per acre	n	1.029 0038

1 Cu Meter per Sq Meter =

1 †	meter		0.000 0000
24.5423	U S gal per sq foot		1.389 9158

1 Cu Meter per Sq Kilometer =

0.001†	mm	n	3.000 0000
91.4644	cu feet per sq mile		1.961 2521
1.06906	U S gal per acre		0.029 0038

(12) WEIGHT PER SURFACE. (UNIT PRESSURES.)

Avoirdupois and Metric.

1 Atmosphere =

			Logarithm.
760	mm of mercury		2.880 8136
29.9212	inches of mercury		1.475 9790
33.9007	feet of water		1.530 2090
14.697	pounds per sq inch		1.167 2280
1.033296	kg per sq cm		0.014 2248
1.033296	metric atmospheres		0.014 2248

1 Metric Atmosphere =

1	kg per sq cm		0.000 0000
0.967777	atmosphere	n	1.985 7752

1 Pound per Sq Inch =

2.30665	feet of water		0.362 9810
0.0680412	atmosphere	n	2.832 7720
0.0703067	kg per sq cm	n	2.846 9968
2.03589	inches of mercury		0.308 7510

1 Pound per Sq Foot =

0.0141380	inch of mercury	n	2.150 3885
0.0160184	foot of water	n	2.204 6185

1 Kilogram per Sq Centimeter =

1	metric atmosphere		0.000 0000
14.2234	pounds per sq inch		1.153 0032
0.967777	atmosphere	n	1.985 7752
32.8083	feet of water		1.515 9842
28.9570	inches of mercury		1.461 7542

1 Foot of Water =

0.0294979	atmosphere	n	2.469 7910
0.882612	inch of mercury	n	1.945 7700
0.433530	pound per sq inch	n	1.637 0190
30.4801	grams per sq cm		1.484 0158

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(12) WEIGHT PER SURFACE.
(UNIT PRESSURES.)—(Continued.)

Avoirdupois and Metric.

		LOGARITHM.
1 Foot of Mercury =		
0.401053	atmosphere	n 1.603 2022
0.414407	kg per sq cm	n 1.617 4270
1 Inch of Water =		
2.54001	grams per sq cm	0.404 8346
1 Inch of Mercury =		
0.03342	atmosphere	n 2.524 0210
1.133	feet of water	0.054 2299
0.491189	pound per sq inch	n 1.691 2490
34.5339	grams per sq cm	1.538 2458

(13) WEIGHT PER VOLUME.
(SPECIFIC GRAVITY.)

Avoirdupois and Metric.

1 Pound per Cu Inch =		
27.6797	grams per cu cm	1.442 1621
1 Pound per U S Gallon =		
0.119826	kg per liter	n 1.078 5501
1 Pound per Imp Gallon =		
0.0998281	kg per liter	n 2.999 2530
1 Pound per Cu Foot =		
16.0184	kg per cu meter	1.204 6184
1 Pound per Cu Yard =		
0.593273	kg per cu meter	n 1.773 2546
1 Kilogram per Cu Meter =		
0.0624283	pound per cu foot	n 2.795 3816
1 Gram per Cu Centimeter =		
62.4283	pound per cu foot	1.795 3816

(14) WEIGHTS OF VOLUMES OF WATER.

Avoirdupois and Metric.

Water at maximum density = 1 gram per cu centimeter.

		LOGARITHM.
1 Cu Inch Weighs		
0.578040	ounce	n 1.761 9578
0.0361275	pound	n 2.557 8378
16.3872	grams	1.214 5038
1 Gallon U S Liquid Weighs		
8.34545	pounds	0.921 4498
3.78543	kilograms	0.578 1158
1 Gallon Imp Weighs		
10.0172	pounds	1.000 7469
4.54373	kilograms	0.657 4129
1 Cu Foot Weighs		
62.4283	pounds	1.795 3815
28.3170	kilograms	1.452 0475
1 Cu Yard Weighs		
1,685.56	pounds	3.226 7453
764.559	kilograms	2.883 4113
1 Cu Centimeter or Milliliter Weighs		
0.0352739	ounce	n 2.547 4540
1 †	gram	0.000 0000
1 Liter or Cu Decimeter Weighs		
2.20462	pounds	0.343 3340
1 †	kilogram	0.000 0000
1 Cu Meter, Stere, or Kiloliter Weighs		
2,204.62	pounds	3.343 3340
1,000 †	kilograms	3.000 0000

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(15) VOLUMES OF WEIGHTS OF WATER.

Avoirdupois and Metric.

Water at maximum density = 1 gram per cu centimeter.

		LOGARITHM.
1 Pound Measures		
27.6798	cu inches	1.442 1621
0.119825	gallon U S	n 1.078 5501
0.0998281	gallon Imp.	n 2.999 2530
0.0160184	cu foot	n 2.204 6184
0.453592	liter	n 1.656 6660
1 Gram Measures		
0.0610233	cu inch	n 2.785 4962
1 †	cu centimeter	0.000 0000
1 Kilogram Measures		
61.0234	cu inches	1.785 4962
0.264170	gallon U S	n 1.421 8842
0.220083	gallon Imp.	n 1.342 5871
0.0353144	cu foot	n 2.547 9525
1 †	liter	0.000 0000

(16) WEIGHTS OF VOLUMES OF AIR.

Avoirdupois and Metric. (See p. 320.)

1 Cu Inch Weighs		
0.327003	grain	n 1.514 5518
21.1895	milligram	1.326 1198
1 Cu Foot Weighs		
565.061	grains	2.752 0955
0.0807230	pound	n 2.906 9975
36.6154	grams	1.563 6635
1 Cu Yard Weighs		
2.17952	pounds	0.338 3613
988.615	grams	2.995 0273
1 Cu Meter Weighs		
2.85069	pounds	0.454 9500
1.293052	kilograms (see p. 230)	0.111 6159

(17) VOLUMES OF WEIGHTS OF AIR.

Avoirdupois and Metric. (See p. 320.)

		LOGARITHM.
1 Grain Measures		
3.05808	cu inches	0.485 4482
0.0501132	liter	n 2.699 9520
1 Pound Measures		
21,406.5	cu inches	4.330 5462
12.3880	cu feet	1.093 0025
0.458816	cu yard	n 1.661 6387
0.350792	cu meter	n 1.545 0500
1 Kilogram Measures		
47,193.3	cu inches	4.673 8802
27.3109	cu feet	1.436 3365
1.01152	cu yards	0.004 9727
0.773364	cu meter	n 1.888 3840

(18) LENGTH PER TIME. (VELOCITY.)

1 Foot per Second =		
0.6*8181	mile per hour	n 1.833 6686
1.09728	km per hour	0.040 3183
1 Inch per Second =		
300 †	feet per hour	2.477 1213
152.400	cm per minute	2.182 9859
1 Mile per Hour =		
1.4*66	feet per second	0.166 3314
0.447041	meter per second	n 1.650 3472
1 Mile per Minute =		
88 †	feet per second	1.944 4826
96.5608	km per hour	1.984 8010
1 Mile per Second =		
96.5608	km per minute	1.984 8010
1 Meter per Second =		
196.85	feet per minute	2.294 1354
2.23693	miles per hour	0.349 6528

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(18) LENGTH PER TIME. (VELOCITY.) —(Continued.)

		LOGARITHM.
1 Kilometer per Day =		
0.0379726	foot per second	n 2.579 4705
0.0258904	mile per hour	n 2.413 1391
1 Kilometer per Hour =		
0.911343	foot per second	n 1.959 6817
14.9129	miles per day	1.173 5615
1 Centimeter per Second =		
1.96850	feet per minute	0.294 1355

(19) SURFACE PER TIME.

1 Acre per Day =		
1,815 †	sq feet per hour	3.258 8766
1 Acre per Hour =		
12.1†	sq feet per second	1.082 7854
0.03750†	sq mile per day	n 2.574 0313
1.12413	sq meters per second	0.050 8170
1 Acre per Second =		
5.625†	sq miles per hour	0.750 1225
14.5687	sq km per hour	1.163 4222
1 Sq Foot per Hour =		
2.22968	sq meters per day	0.348 2429
1 Sq Foot per Second =		
0.0826446	acre per hour	n 2.917 2147
0.00309917	sq mile per day	n 3.491 2460
334.452	sq meters per hour	2.524 3341
1 Sq Inch per Second =		
25 †	sq feet per hour	1.397 9400
0.0137741	acre per day	n 2.139 0633
2.32259	sq meters per hour	0.365 9718

(19) SURFACE PER TIME.—(Continued.)

		LOGARITHM.
1 Sq Mile per Day =		
322.*666	sq feet per second	2.508 7541
26.*666	acres per hour	1.425 9688
0.107917	sq km per hour	n 1.033 0883
1 Sq Mile per Hour =		
7,744 †	sq feet per second	3.888 9653
15,360 †	acres per day	4.186 3912
719.444	sq meters per second	2.856 9970
1 Sq Meter per Second =		
21.3498	acres per day	1.329 3942
1 Sq Kilometer per Day =		
124.582	sq feet per second	2.095 4545
10.2960	acres per hour	1.012 6692
1 Sq Kilometer per Second =		
1,389.96	sq miles per hour	3.143 0030
1 Sq Centimeter per Second =		
3.87499	sq feet per hour	0.588 2708

(20) VOLUME PER TIME. (DISCHARGES.)

1 Cu Foot per Day =		
0.311688	U S gal per hour	n 1.493 7205
1.17987	liters per hour	0.071 8363
1 Cu Foot per Hour =		
179.532	U S gals per day	2.254 1429
0.679608	cu meter per day	n 1.832 2587
1 Cu Foot per Minute =		
448.831	U S gals per hour	2.652 0830
1.69902	cu meters per hour	0.230 1988
1 Cu Foot per Second =		
646,317	U S gals per day	5.810 4453
448.831	U S gals per minute	2.652 0828
1.98347	acre-feet per day	0.297 4258
101.941	cu meters per hour	2.008 3500
about 40	miner's inches	1.602 0600

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(20) VOLUME PER TIME. (DISCHARGES.)

—(Continued.)

		LOGARITHM.
1 Cu Inch per Second =		
50 †	cu feet per day	1.698 9700
15.5844	U S gal per hour	1.192 6905
58.9938	liters per hour	1.770 8063
1 Cu Yard per Hour =		
18.3494	cu meters per day	1.263 6225
1 Cu Yard per Second =		
53.5537	acre-feet per day	1.728 7896
1,000,000 U S Gallons per Day =		
1.54723	cu feet per second	0.189 5546
43.813	liters per second	1.641 6021
1 U S Gallon per Hour =		
3.208*33	cu feet per day	0.506 2795
0.0908504	cu meter per day	n 2.958 3270
1 U S Gallon per Minute =		
8.02084	cu feet per hour	0.904 2196
0.002228	cu foot per second	n 3.347 9171
0.00441919	acre-foot per day	n 3.645 3429
1 U S Gallon per Second =		
481.25†	cu feet per hour	2.682 3708
0.26*5151	acre-foot per day	n 1.423 4941
13.6276	cu meters per hour	1.134 4183
1 Miner's Inch =		
about 0.025	cu ft per sec	n 2.397 9400
1 Cu Centimeter per Second =		
3.05117	cu feet per day	0.484 4662
1 Liter per Second =		
127.132	cu feet per hour	2.104 2550
1 Cu Meter per Day =		
1.47144	cu feet per hour	0.167 7413

(20) VOLUME PER TIME. (DISCHARGES.)

—(Continued.)

		LOGARITHM.
1 Cu Meter per Hour =		
6,340.09	U S gal per day	3.802 0954
0.00980957	cu foot per second	n 3.991 6500
1 Cu Meter per Second =		
2.91855	acre-feet per hour	0.465 1671
1 Acre-foot per Day =		
1,815 †	cu feet per hour	3.258 8767
3.77143	U S gal per second	0.576 5059
0.0142765	cu meter per second	n 2.154 6217
1 Acre-foot per Hour =		
726 †	cu feet per minute	2.860 9366
90.5143	U S gal per second	1.956 7171
0.342636	cu meter per second	n 1.534 8329

(21) WEIGHT PER TIME.

Avoirdupois and Metric.

1 Pound per Second =		
27.2156	kg per minute	1.434 8173
1 Kilogram per Hour =		
52.9109	pounds per day	1.723 5452
1 Kilogram per Second =		
7,936.6	pounds per hour	3.899 6365

(22) WORK PER TIME. (POWER.)

For Work, see Table (6).

1 H P =		
5,694,120	foot-gallons per day	6.755 4265
33,000 †	foot-lbs per minute	4.518 5139
550 †	foot-lbs per second	2.740 3627
745.956	joules per second	2.872 7134
0.706941	pound F per second	n 1.849 3839
1.01387	metric H P	0.005 9832
0.178146	kg C per second	n 1.250 7764

* Repeating decimals; thus, 0.0*1515 = 0.0151515... † Exact values. n. Negative characteristic.

(22) WORK PER TIME. (POWER.)—(Continued.)

		Logarithm
1 Foot-pound per Second =		
0.00*1818	H P	n 3.259 6373
0.00184340	metric H P	n 3.265 6205
1 Million Foot-gallons Water per Day =		
0.175620	H P	n 1.244 5734
1 Watt =		
1 †	joule per second	0.000 0000
0.737308	foot-lb per second	n 1.867 6492
0.00134056	H P	n 3.127 2865
0.00135916	metric H P	n 3.133 2697
1 Kilowatt =		
1,000 †	watts. (See watt.)	
1 Metric H P =		
735.75	watts	2.866 7303
542.475	foot-lbs per second	2.734 3795
75 †	kgm per second	1.875 0613
0.986318	H P	n 1.994 0168
1 Joule per Second =		
0.00134056	H P	n 3.127 2865
0.00135916	metric H P	n 3.133 2697
366.973	kgm per hour	2.564 6336
1 Kilogrammeter per Second =		
0.0131509	H P	n 2.118 9555
0.01*33	metric H P	n 2.124 9387

(23) WORK PER VOLUME.

		Logarithm
1 Foot-pound per Cu Foot =		
0.000171826	pound F per gal U S	n 4.235 0886
4.88241	kgm per cu meter	0.688 6343
1 Foot-pound per Cu Inch =		
8.4368	kgm per liter	0.926 1780

(23) WORK PER VOLUME.—(Continued.)

		Logarithm
1 Foot-pound per Gallon U S =		
36.5230	kgm per cu meter	1.562 5660
1 H P Hour per Cu Foot =		
340.215	pounds F per gal U S	2.531 7538
22.6481	kg C per liter	1.355 0316
1 H P Hour per Cu Yard =		
73,333.*333	foot-lbs per cu foot	4.865 3014
12.6006	pounds F per gal U S	1.100 3901
358.044	kgm per liter	2.553 9358
1 H P Hour per Gallon U S =		
0.267835	metric H P h per liter	n 1.427 8674
1 Pound F per Cu Foot =		
0.0106091	H P hour per cu yard	n 2.025 6782
104.003	foot-lbs per gal U S	2.017 0478
8.89909	kg C per cu meter	0.949 3459
1 Pound F per Gallon U S =		
0.0793615	H P hour per cu yard	n 2.899 6099
66.5699	kg C per cu meter	1.823 2776
1 Kilogram Centigrade per Liter =		
50.5929	foot-lbs per cu inch	1.704 0899
0.0441538	H P hour per cu foot	n 2.644 9684
1 Kilogram Centigrade per Cu Meter =		
11.6870	foot-lbs per gal U S	1.067 7019
1 Kilogrammeter per Liter =		
204.817	foot-lbs per cu foot	2.311 3657
0.00279296	H P hour per cu yard	n 3.446 0643
0.0351929	pound F per gal U S	n 2.546 4544
1 Kilogrammeter per Cu Meter =		
0.027380	foot-lb per gal U S	n 2.437 4340

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(24) WORK PER WEIGHT.

		Logarithm.
1 Kilogrammeter per Gram =		
5.96515	H P seconds per pound	0.775 6215
1 Foot-pound per Pound =		
0.304801	kgm per kg	n 1.484 0158
1 Pound F per Pound =		
0.237135	kgm per gram	n 1.374 9954

(25) VALUE PER LENGTH.

		Logarithm.
1 Dollar per Foot =		
12.35	shillings per yard	1.092
0.1379	mark per cm	n 1.139
0.170	franc per cm	n 1.230
1 Penny per Foot =		
0.0607	dollar per yard	n 2.783
106.92	dollars per mile	2.029
440 †	shillings per mile	2.643
0.344	franc per meter	n 1.537
0.279	mark per meter	n 1.446
1 Shilling per Foot =		
0.729	dollar per yard	n 1.863
4.13	centimes per cm	0.616
3.35	marks per meter	0.525
1 Cent per Inch =		
0.12†	dollar per foot	n 1.079
0.494	shilling per foot	n 1.694
2.04	francs per meter	0.310
1.654	marks per meter	0.219
1 Penny per Inch =		
0.7290	dollar per yard	n 1.863
4.13	centimes per cm	0.616
3.35	pfennigs per cm	0.525

(25) VALUE PER LENGTH.—(Continued.)

		Logarithm.
1 Cent per Yard =		
17.60†	dollars per mile	1.246
72.43	shillings per mile	1.860
0.0566	franc per meter	n 2.753
0.0459	mark per meter	n 2.662
1 Penny per Yard =		
35.64	dollars per mile	1.552
7.*333	pounds per mile	0.865
0.1148	franc per meter	n 1.060
0.09305	mark per meter	n 2.969
1 Shilling per Yard =		
8.1	cents per foot	0.908
1.377	francs per meter	0.139
1.117	marks per meter	0.048
1 Dollar per Mile =		
0.018*93	cent per foot	n 2.277
0.009353	penny per foot	n 3.971
0.0007795	shilling per foot	n 4.892
3.22	francs per km	0.508
2.61	marks per km	0.417
1 Shilling per Mile =		
0.0046	cent per foot	n 3.663
0.002*2727	penny per foot	n 3.357
0.7823	franc per km	n 1.893
0.634	mark per km	n 1.802
1 Franc per Meter =		
0.0588	dollar per foot	n 2.770
2.905	pence per foot	0.463
0.242	shilling per foot	n 1.384
310.60	dollars per mile	2.492
1,278	shillings per mile	3.107

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(25) VALUE PER LENGTH.—(Continued.)

		LOGARITHM.
1 Mark per Meter =		
0.0726	dollar per foot	n 2.861
3.58	pence per foot	0.554
0.2985	shilling per foot	n 1.475
383	dollars per mile	2.583
1,576	shillings per mile	3.198
1 Franc per Kilometer =		
0.8106	dollar per mile	n 1.492
15.34	pence per mile	1.186
1.28	shillings per mile	0.107
1 Mark per Kilometer =		
0.383	dollar per mile	n 1.583
18.92	pence per mile	1.277
1.576	shillings per mile	0.198

(26) VALUE PER SURFACE.

1 Dollar per Acre =		
0.002295	cent per sq foot	n 3.361
0.00113	penny per sq foot	n 3.054
0.000850	shilling per sq yard	n 4.930
12.8	francs per hectare	1.107
10.38	marks per hectare	1.016
1 Cent per Sq Foot =		
0.09†	dollar per sq yard	n 2.954
4.44	pence per sq yard	0.648
0.557	franc per sq meter	n 1.746
0.452	mark per sq meter	n 1.655
1 Shilling per Sq Foot =		
10,585	dollars per acre	4.025
108 †	pence per sq yard	2.033
13.55	francs per sq meter	1.132
11.0	marks per sq meter	1.041

(26) VALUE PER SURFACE.—(Continued.)

		LOGARITHM.
1 Cent per Sq Inch =		
12.96†	dollars per sq yard	1.113
71.1	pence per sq foot	1.852
53.3	shillings per sq yard	1.727
80.3	francs per sq meter	1.905
65.1	marks per sq meter	1.814
1 Dollar per Sq Yard =		
0.457	shilling per sq foot	n 1.660
996	pounds per acre	2.998
6.20	francs per sq meter	0.792
5.02	marks per sq meter	0.701
1 Penny per Sq Yard =		
98.01	dollars per acre	1.991
0.125	franc per sq meter	n 1.099
0.102	mark per sq meter	n 1.008
1 Shilling per Sq Yard =		
1,176	dollars per acre	3.070
1.506	francs per sq meter	0.178
1.22	marks per sq meter	0.087
1 Franc per Sq Meter =		
1.79	cents per sq foot	0.254
0.885	penny per sq foot	n 1.947
0.664	shilling per sq yard	n 1.822
1 Mark per Sq Meter =		
2.21	cents per sq foot	0.345
1.09	pence per sq foot	0.038
0.819	shilling per sq yard	n 1.913
1 Franc per Hectare =		
0.0781	dollar per acre	n 2.893
0.321	shilling per acre	n 1.507
1 Mark per Hectare =		
0.0963	dollar per acre	n 2.984
0.396	shilling per acre	n 1.598

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(27) VALUE PER VOLUME.

		LOGARITHM.
1 Dollar per Cu Foot =		
111.1	shillings per cu yard	2.046
13.36	cents per U S gal	1.126
0.183	franc per liter	n 1.262
0.1483	mark per liter	n 1.171
1 Shilling per Cu Foot =		
6.56	dollars per cu yard	0.817
324 †	pence per cu yard	2.511
0.0445	franc per liter	n 2.648
0.03605	mark per liter	n 2.557
1 Cent per Cu Inch =		
17.28†	dollars per cu foot	1.238
71.11	shillings per cu foot	1.852
0.316	centime per cc	n 1.500
0.2565	pfennig per cc	n 1.409
1 Penny per Cu Inch =		
944.78	dollars per cu yard	2.975
3,888 †	shillings per cu yard	3.590
6.40	francs per liter	0.806
5.19	marks per liter	0.715
1 Shilling per Cu Inch =		
56.13	dollars per gal U S	1.749
0.07683	franc per cc	n 2.886
0.0624	mark per cc	n 2.795
1 Dollar per Cu Yard =		
3.*703	cents per cu foot	0.569
0.1523	shilling per cu foot	n 1.183
6.777	francs per cu meter	0.831
5.495	marks per cu meter	0.740
1 Shilling per Cu Yard =		
0.9000	cent per cu foot	n 1.954
0.*444	penny per cu foot	n 1.648
0.165	centime per liter	n 1.217
0.1336	pfennig per liter	n 1.126

(27) VALUE PER VOLUME.—(Continued.)

		LOGARITHM.
1 Penny per Gallon Imp =		
1.687	cents per gal U S	0.227
2.309	centimes per liter	0.363
1.87	pfennigs per liter	0.272
1 Shilling per Gallon Imp =		
0.2025	dollar per gal U S	n 1.306
0.277	franc per liter	n 1.443
0.225	mark per liter	n 1.352
1 Dollar per Gallon U S =		
4.94	shillings per gal Imp	0.694
1.369	francs per liter	0.136
1.11	marks per liter	0.045
1 Franc per Cu Meter =		
0.1477	dollar per cu yard	n 1.169
7.287	pence per cu yard	0.863
0.607	shilling per cu yard	n 1.783
1 Mark per Cu Meter =		
0.182	dollar per cu yard	n 1.260
8.985	pence per cu yard	0.954
0.7488	shilling per cu yard	n 1.874

(28) VALUE PER WEIGHT.

Avoirdupois and Metric.

1 Dollar per Pound =		
11.43	francs per kg	1.058
9.263	marks per kg	0.967
1 Penny per Pound =		
0.231	franc per kg	n 1.364
0.1875	mark per kg	n 1.273
1 Shilling per Pound =		
2.775	francs per kg	0.443
2.25	marks per kg	0.352

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(28) VALUE PER WEIGHT.—(Continued.)

Avoirdupois and Metric.

1 Franc per Kilogram =

		LOGARITHM.
0.08754	dollar per pound	n 2.942
0.3603	shilling per pound	n 1.557

1 Mark per Kilogram =

0.108	dollar per pound	n 1.033
0.444	shilling per pound	n 1.648

(29) VALUE PER TIME.

1 Dollar per Day =

4.1*66	cents per hour	0.620
2.058	pence per hour	0.313
0.216	franc per hour	n 1.334
0.175	mark per hour	n 1.243

1 Penny per Day =

0.08437	cent per hour	n 2.926
0.437	centime per hour	n 1.641
0.355	pfennig per hour	n 1.550

1 Shilling per Day =

1.012	cents per hour	0.005
0.5†	penny per hour	n 1.699
0.0525	franc per hour	n 2.720
0.0426	mark per hour	n 2.629

1 Franc per Day =

0.0338	mark per hour	n 2.529
0.8042	cent per hour	n 1.905
0.397	penny per hour	n 1.599

1 Mark per Day =

0.0514	franc per hour	n 2.711
0.9917	cent per hour	n 1.996
0.49	penny per hour	n 1.690

(29) VALUE PER TIME.—(Continued.)

1 Cent per Hour =

		LOGARITHM.
0.9877	shilling per day	n 1.995
1.244	francs per day	0.095
1.008	marks per day	0.004

1 Dollar per Hour =

0.0137	penny per second	n 2.137
0.144	centime per second	n 1.158
0.1168	pfennig per second	n 1.067

1 Penny per Hour =

0.486	dollar per day	n 1.687
2 †	shillings per day	0.301
2.52	francs per day	0.401
2.04	marks per day	0.310

1 Shilling per Hour =

0.035	centime per second	n 2.544
0.0284	pfennig per second	n 2.453
5.83	dollars per day	0.766

1 Franc per Hour =

19.46	marks per day	1.289
4.63	dollars per day	0.666
19.06	shillings per day	1.280

1 Mark per Hour =

29.6	francs per day	1.471
5.712	dollars per day	0.757
23.5	shillings per day	1.371

1 Cent per Second =

36 †	dollars per hour	1.556
1,778	pence per hour	3.250
148.2	shillings per hour	2.171
186.5	francs per hour	2.271
151.3	marks per hour	2.180

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(29) VALUE PER TIME.—(Continued.)

1 Penny per Second =

		LOGARITHM.
72.90	dollars per hour	1.863
300 †	shillings per hour	2.477
378	francs per hour	2.577
306	marks per hour	2.486

1 Centime per Second =

36 †	francs per hour	1.556
29.2	marks per hour	1.465
6.95	dollars per hour	0.842
28.6	shillings per hour	1.456

1 Pfennig per Second =

44.4	francs per hour	1.647
36 †	marks per hour	1.556
8.568	dollars per hour	0.933
35.26	shillings per hour	1.547

(30) VALUE PER WORK.

1 Cent per Foot-pound =

0.375	franc per kgm	n 1.574
0.304	mark per kgm	n 1.483

1 Penny per Foot-pound =

0.7589	franc per kgm	n 1.880
0.615	mark per kgm	n 1.789

1 Shilling per Foot-pound =

9.107	francs per kgm	0.959
7.38	marks per kgm	0.868

1 Franc per Kilogrammeter =

2.668	cents per foot-lb	0.426
1.318	pence per foot-lb	0.120

1 Mark per Kilogrammeter =

3.29	cents per foot-lb	0.517
1.625	pence per foot-lb	0.211

(31) LENGTH PER TIME PER TIME.
(ACCELERATION.)

Acceleration of Gravity = g =

		LOGARITHM.
32.1850	ft per sec per sec	1.507 6532
9.81	m per sec per sec	0.991 6690

1 Foot per Second per Second =

0.0310704	g in ft per sec per sec	n 2.492 3468

1 Meter per Second per Second =

0.101937	g in cm per sec per sec	n 1.008 3310

(32) LENGTH TO VOLUME PER SURFACE.

1 Inch in Depth =

27,154.3	U S gal per acre	4.433 8384
3,630 †	cu feet per acre	3.559 9067
0.623376	U S gal per sq foot	n 1.794 7505
254.0005	cu meters per hectare	2.404 8346

1 Foot in Depth =

43,560 †	cu feet per acre	4.639 0879
67.3247	U S gal per sq yard	1.828 1742
3,048.006	cu meters per hectare	3.484 0158

1 Centimeter in Depth =

10 †	liters per sq meter	1.000 0000
1,429.13	cu feet per acre	3.155 0721
0.245423	U S gal per sq foot	n 1.389 9159

1 Meter in Depth =

1,000 †	liters per sq meter	3.000 0000
142,913	cu feet per acre	5.155 0721
24.5423	U S gal per sq foot	1.389 9159

---

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(33) VOLUME PER SURFACE TO LENGTH.

		LOGARITHM.
1 Cu Foot per Acre =		
0.000275482	inch in depth	n 4.440 0933
0.000699726	cm in depth	n 4.844 9279
1 Cu Inch per Sq Foot =		
0.0069*44	inch in depth	n 3.841 6375
0.0176389	cm in depth	n 2.246 4721
1 Cu Yard per Acre =		
0.00743802	inch in depth	n 3.871 4571
0.0188926	cm in depth	n 2.276 2917
1 Gallon U S per Sq Foot =		
1.60417	inches in depth	0.205 2495
4.07459	cm in depth	0.610 0841
1 Liter per Sq Meter =		
1 †	mm in depth	0.000 0000
0.03937	inch in depth	n 2.595 1654

(34) VOLUME PER SURFACE PER TIME.

1 Cu Foot per Acre per Day =		
0.00292	cu m per hec per h	n 3.464 7167
1 Cu Foot per Acre per Hour =		
1.32987	gal U S per sq mi per sec	0.123 8092
1.67934	cu m per hec per d	0.225 1391
1 Cu Foot per Acre per Second =		
0.618225	gal U S per sq ft per h	n 1.791 1463
251.901	cu m per hec per h	2.401 2304
1 Cu Foot per Sq Mile per Day =		
0.000487013	gal U S per a per h	n 4.687 5405
0.00455551	li per hec per h	n 3.658 5367
1 Cu Foot per Sq Mile per Hour =		
0.280519	gal U S per a per d	n 1.447 9629
0.262397	cu m per sq km per d	n 1.418 9591

(34) VOLUME PER SURFACE PER TIME.

—(Continued.)

		LOGARITHM.
1 Cu Foot per Sq Mile per Second =		
42.0779	gal U S per a per h	1.624 0542
944.63	cu m per sq km per d	2.975 2616
1 Gallon U S per Acre per Day =		
3.56482	cu ft per sq mi per h	0.552 0371
0.0389749	cu m per sq km per h	n 2.590 7850
1 Gallon U S per Acre per Hour =		
0.0237654	cu ft per sq mi per sec	n 2.375 9458
22.4495	cu m per sq km per d	1.351 2074
1 Gallon U S per Acre per Second =		
0.0110480	cu ft per sq ft per h	n 2.043 2829
56.1239	cu m per sq km per min	1.749 1475
1 Gallon U S per Sq Foot per Day =		
242.630	cu ft per a per h	2.384 9450
1.69775	li per sq m per h	0.229 8730
1 Gallon U S per Sq Foot per Hour =		
1.61754	cu ft per a per sec	0.208 8537
0.0113183	li per sq m per sec	n 2.053 7817
1 Gallon U S per Sq Foot per Second =		
146.685	cu m per sq m per h	2.166 3867
1 Gallon U S per Sq Inch per Day =		
0.802084	cu ft per sq ft per h	n 1.904 2196
0.244476	cu m per sq m per h	n 1.388 2354
1 Gallon U S per Sq Inch per Hour =		
462 †	cu ft per sq ft per d	2.664 6420
140.818	cu m per sq m per d	2.148 6578
1 Liter per Sq Meter per Second =		
283.464	cu ft per sq ft per d	2.452 4978
88.3524	gal U S per sq ft per h	1.946 2183

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

(34) VOLUME PER SURFACE PER TIME.—(Continued.)

		LOGARITHM.
1 Cu Meter per Sq Meter per Day =		
0.000284055	gal U S per sq ft per sec.	n 4.453 4021
1 Cu Meter per Hectare per Day =		
0.595471	cu ft per a per h	n 1.774 8609
1 Cu Meter per Hectare per Hour =		
342.991	cu ft per a per d	2.535 2833
1 Cu Meter per Hectare per Second =		
51,448.7	cu ft per a per h	4.711 3746
1 Cu Meter per Sq Kilometer per Day =		
3.81102	cu ft per sq mi per h	0.581 0409
1 Cu Meter per Sq Kilometer per Hour =		
2,195.15	cu ft per sq mi per d	3.341 4633
25.6575	gal U S per a per d	1.409 2150
1 Cu Meter per Sq Kilometer per Second =		
329,272	cu ft per sq mi per h	5.517 5546
64.1439	gal U S per a per min	1.807 1551

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.

		LOGARITHM.
1,000,000 Cu Feet per Acre per Day =		
0.000265704	foot per second	n 4.424 3984
0.00809868	cm per second	n 3.908 4142
1,000,000 Cu Feet per Acre per Hour =		
0.0063769	foot per second	n 3.804 6096
0.194368	cm per second	n 1.288 6254
1 Cu Foot per Acre per Second =		
1.98347	feet per day	0.297 4258
0.604563	meter per day	n 1.781 4416

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.—(Continued.)

		LOGARITHM.
1,000,000 Cu Feet per Sq Mile per Second =		
129.132	feet per hour	2.111 0346
944.63	meters per day	2.975 2616
1 Cu Inch per Sq Foot per Second =		
25 †	inches per hour	1.397 9400
50 †	feet per day	1.698 9700
15.240	meters per day	1.182 9858
1,000,000 Gallons U S per Sq Mile per Day =		
0.00239757	inch per hour	n 3.379 7704
1,000,000 Gallons U S per Sq Mile per Hour =		
0.115083	foot per day	n 1.061 0116
0.0350774	meter per day	n 2.545 0274
1,000,000 Gallons U S per Sq Mile per Second =		
5.26161	meters per hour	0.721 1187
1,000,000 Gallons U S per Acre per Day =		
0.000035519	foot per second	n 5.550 4667
0.00108264	cm per second	n 3.034 4825
1,000,000 Gallons U S per Acre per Hour =		
0.000852468	foot per second	n 4.930 6779
0.0259833	cm per second	n 2.414 6937
1 Gallon U S per Acre per Second =		
0.132576	inch per hour	n 1.122 4641
0.265152	foot per day	n 1.423 4941
8.08183	meters per day	0.907 5099
1,000,000 Gallons U S per Sq Foot per Day =		
1.54723	feet per second	0.189 5546
47.1596	cm per second	1.673 5705

* Repeating decimals; thus, 0.0*1515 = 0.0151515 . . . † Exact values. n. Negative characteristic.

(35) VOLUME PER SURFACE PER TIME TO VELOCITY.—(Continued.)

1 Gallon U S per Sq Foot per Hour =

		LOGARITHM.
38.5†	inches per day	1.585 4607
0.977902	meter per day	n 1.990 2954

1 Gallon U S per Sq Foot per Second =

		LOGARITHM.
146.685	meters per hour	2.166 3867

1 Cu Meter per Sq Meter per Second =

		LOGARITHM.
11,811	feet per hour	4.072 2867

1 Cu Meter per Hectare per Second =

		LOGARITHM.
340.157	inches per day	2.531 6791

(36) VELOCITY TO VOLUME PER SURFACE PER TIME.

1 Inch per Second =

		LOGARITHM.
2,244.16	U S gal per sq ft per h	3.351 0531
91.4402	cu m per sq m per h	1.961 1372

(36) VELOCITY TO VOLUME PER SURFACE PER TIME. —(Continued.)

1 Inch per Hour =

		LOGARITHM.
87,120 †	cu ft per acre per d	4.940 1179
645.*333	cu ft per sq mi per sec	2.809 7841
14.9610	U S gal per sq ft per d	1.174 9617
7.54286	U S gal per a per sec	0.877 5358
6,096.01	cu m per hec per d	3.785 0458

1 Foot per Second =

		LOGARITHM.
26,929.9	U S gal per sq ft per h	4.430 2343
1,097.28	cu m per sq m per h	3.040 3184

1 Foot per Hour =

		LOGARITHM.
7,744.0†	cu ft per sq mi per sec	3.888 9653
179.532	U S gal per sq ft per d	2.254 1429
90.5143	U S gal per a per sec	1.956 7171
12.10†	cu ft per a per sec	1.082 7854
0.84667	cu m per hec per sec	n 1.927 7133

1 Centimeter per Second =

		LOGARITHM.
118.11	cu ft per sq ft per h	2.072 2867

1 Meter per Hour =

		LOGARITHM.
39.6981	cu ft per a per sec	1.598 7695
589.016	U S gal per sq ft per d	2.770 1271

---

* Repeating decimals; thus, 0.0*1515 = 0.0151515 ... † Exact values. n. Negative characteristic.

TABLE OF ACRES REQUIRED per mile, and per 100 feet, for different widths.

Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.
1	.121	.002	26	3.15	.060	52	6.30	.119	78	9.45	.179
2	.242	.005	27	3.27	.062	53	6.42	.122	79	9.58	.181
3	.364	.007	28	3.39	.064	54	6.55	.124	80	9.70	.184
4	.485	.009	29	3.52	.067	55	6.67	.126	81	9.82	.186
5	.606	.011	30	3.64	.069	56	6.79	.129	82	9.94	.188
6	.727	.014	31	3.76	.071	57	6.91	.131	½	10.	.189
7	.848	.016	32	3.88	.073	¾	7.	.133	83	10.1	.190
8	.970	.018	33	4.00	.076	58	7.03	.133	84	10.2	.193
¼	1.	.019	34	4.12	.078	59	7.15	.135	85	10.3	.195
9	1.09	.021	35	4.24	.080	60	7.27	.138	86	10.4	.197
10	1.21	.023	36	4.36	.083	61	7.39	.140	87	10.5	.200
11	1.33	.025	37	4.48	.085	62	7.52	.142	88	10.7	.202
12	1.46	.028	38	4.61	.087	63	7.64	.145	89	10.8	.204
13	1.58	.030	39	4.73	.090	64	7.76	.147	90	10.9	.207
14	1.70	.032	40	4.85	.092	65	7.88	.149	¾	11.	.209
15	1.82	.034	41	4.97	.094	66	8.	.151	91	11.0	.209
16	1.94	.037	¼	5.	.094	67	8.12	.154	92	11.2	.211
½	2.	.038	42	5.09	.096	68	8.24	.156	93	11.3	.213
17	2.06	.039	43	5.21	.099	69	8.36	.158	94	11.4	.216
18	2.18	.041	44	5.33	.101	70	8.48	.161	95	11.5	.218
19	2.30	.044	45	5.45	.103	71	8.61	.163	96	11.6	.220
20	2.42	.046	46	5.58	.106	72	8.73	.165	97	11.8	.223
21	2.55	.048	47	5.70	.108	73	8.85	.168	98	11.9	.225
22	2.67	.051	48	5.82	.110	74	8.97	.170	99	12.	.227
23	2.79	.053	49	5.94	.112	¼	9.	.170	100	12.1	.230
24	2.91	.055	½	6.	.114	75	9.09	.172			
¾	3.	.057	50	6.06	.115	76	9.21	.174			
25	3.03	.057	51	6.18	.117	77	9.33	.177			

Table of grades per mile, and per 100 feet measured horizontally, and corresponding to different angles of inclination.

Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.
0	1	1.536	.0291	0	45	69.11	1.3090	1	58	181.3	3.4341	3	26	316.8	5.9994
	2	3.072	.0582		46	70.64	1.3381	2	0	184.4	3.4924		28	319.8	6.0579
	3	4.608	.0873		47	72.18	1.3672		2	187.5	3.5506		30	322.9	6.1163
	4	6.144	.1164		48	73.72	1.3963		4	190.6	3.6087		32	326.0	6.1747
	5	7.680	.1455		49	75.26	1.4254		6	193.6	3.6669		34	329.1	6.2330
	6	9.216	.1746		50	76.80	1.4545		8	196.7	3.7250		36	332.2	6.2914
	7	10.75	.2037		51	78.33	1.4837		10	199.8	3.7833		38	335.3	6.3498
	8	12.29	.2328		52	79.87	1.5128		12	202.8	3.8416		40	338.4	6.4083
	9	13.82	.2619		53	81.40	1.5419		14	205.9	3.8999		42	341.4	6.4664
	10	15.36	.2909		54	82.94	1.5710		16	208.9	3.9581		44	344.5	6.5246
	11	16.90	.3200		55	84.47	1.6000		18	212.0	4.0163		46	347.6	6.5832
	12	18.43	.3491		56	86.01	1.6291		20	215.1	4.0746		48	350.7	6.6418
	13	19.96	.3782		57	87.54	1.6583		22	218.1	4.1329		50	353.8	6.7004
	14	21.50	.4073		58	89.08	1.6873		24	221.2	4.1911		52	356.8	6.7583
	15	23.04	.4364		59	90.62	1.7164		26	224.3	4.2494		54	359.9	6.8163
	16	24.58	.4655	1		92.16	1.7455		28	227.4	4.3076		56	363.0	6.8751
	17	26.11	.4946		2	95.23	1.8038		30	230.5	4.3659		58	366.1	6.9339
	18	27.64	.5237		4	98.30	1.8620		32	233.5	4.4242	4		369.2	6.9926
	19	29.17	.5528		6	101.4	1.9202		34	236.6	4.4826		5	376.9	7.1384
	20	30.72	.5818		8	104.5	1.9784		36	239.7	4.5409		10	384.6	7.2842
	21	32.26	.6109		10	107.5	2.0366		38	242.8	4.5993		15	392.3	7.4300
	22	33.80	.6400		12	110.6	2.0948		40	245.9	4.6576		20	400.1	7.5767
	23	35.33	.6691		14	113.6	2.1530		42	248.9	4.7159		25	407.8	7.7234
	24	36.86	.6982		16	116.7	2.2112		44	252.0	4.7742		30	415.5	7.8701
	25	38.40	.7273		18	119.8	2.2694		46	255.1	4.8325		35	423.2	8.0163
	26	39.94	.7564		20	122.9	2.3277		48	258.2	4.8908		40	431.0	8.1625
	27	41.47	.7855		22	126.0	2.3859		50	261.3	4.9492		45	438.7	8.3087
	28	43.01	.8146		24	129.1	2.4441		52	264.3	5.0075		50	446.5	8.4554
	29	44.54	.8436		26	132.1	2.5023		54	267.4	5.0658		55	454.2	8.6021
	30	46.08	.8727		28	135.2	2.5604		56	270.5	5.1241	5		461.9	8.7489
	31	47.62	.9018		30	138.3	2.6186		58	273.6	5.1824		5	469.6	8.8951
	32	49.16	.9309		32	141.3	2.6768	3		276.7	5.2407		10	477.4	9.0413
	33	50.69	.9600		34	144.4	2.7350		2	279.7	5.2990		15	485.1	9.1875
	34	52.23	.9891		36	147.4	2.7932		4	282.8	5.3573		20	492.9	9.3347
	35	53 76	1.0182		38	150.5	2.8514		6	285.9	5.4158		25	500.6	9.4819
	36	55.30	1.0472		40	153.6	2.9097		8	289.0	5.4742		30	508.4	9.6292
	37	56 83	1.0763		42	156.6	2.9679		10	292.1	5.5326		35	516.1	9.7755
	38	58.37	1.1054		44	159.7	3.0262		12	295.1	5.5909		40	523.9	9.9218
	39	59.90	1.1345		46	162.8	3.0844		14	298.2	5.6493		45	531.6	10.068
	40	61.44	1.1636		48	165.9	3.1427		16	301.3	5.7077		50	539.4	10.215
	41	62.97	1.1927		50	169.0	3.2010		18	304.4	5.7660		55	547.2	10.362
	42	64.51	1.2218		52	172.0	3.2592		20	307.5	5.8244	6		555.	10.510
	43	66.04	1.2509		54	175.1	3.3175		22	310.5	5.8827				
	44	67.57	1.2800		56	178.2	3.3758		24	313.6	5.9410				

On a turnpike road 1° 38′, or about 1 in 35, or 151 feet per mile, is the greatest slope that will allow horses to trot down rapidly with safety. In crossing mountains, this is often increased to 3°, or even to 5°. It should never exceed 2½°, except when absolutely necessary.

Any hor dist is = sloping dist × cosine ang of slope.
" **sloping dist** is = hor dist ÷ cosine " " "
" **vert height** is = hor dist × tangent " " "
or = sloping dist × sine " " "

A grade of *n* feet rise per 100 feet is usually called a grade of *n* **per cent.**

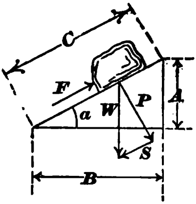
SLOPES IN FEET PER 100 FT. HORIZONTAL.

The fractions of minutes are given only to 34 feet in 100.

A clinometer graduated by the 3d column, and numbered by the first one, will give at sight the slopes in feet per 100 feet. No errors. Original.

Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.	
	Feet.	Deg.	Min.		Feet.	Deg.	Min.		Feet.	Deg.	Min.
1	100.005	0	34.4	35	105.948	19	17	69	121.495	34	36
2	100.020	1	8.7	36	106.283	19	48	70	122.066	35	0
3	100.045	1	43.1	37	106.626	20	18	71	122.642	35	23
4	100.080	2	17.5	38	106.977	20	48	72	123.223	35	45
5	100.125	2	51.8	39	107.336	21	18	73	123.810	36	8
6	100.180	3	26.0	40	107.703	21	48	74	124.403	36	30
7	100.245	4	0.3	41	108.079	22	18	75	125.000	36	52
8	100.319	4	34.4	42	108.462	22	47	76	125.603	37	14
9	100.404	5	8.6	43	108.853	23	16	77	126.210	37	36
10	100.499	5	42.6	44	109.252	23	45	78	126.823	37	57
11	100.603	6	16.6	45	109.659	24	14	79	127.440	38	19
12	100.717	6	50.6	46	110.073	24	42	80	128.062	38	40
13	100.841	7	24.4	47	110.494	25	10	81	128.690	39	1
14	100.975	7	58.2	48	110.923	25	38	82	129.321	39	21
15	101.119	8	31.9	49	111.359	26	6	83	129.958	39	42
16	101.272	9	5.4	50	111.803	26	34	84	130.599	40	2
17	101.435	9	38.9	51	112.254	27	1	85	131.244	40	22
18	101.607	10	12.2	52	112.712	27	28	86	131.894	40	42
19	101.789	10	45.5	53	113.177	27	55	87	132.548	41	1
20	101.980	11	18.6	54	113.649	28	22	88	133.207	41	21
21	102.181	11	51.6	55	114.127	28	49	89	133.869	41	40
22	102.391	12	24.5	56	114.612	29	15	90	134.536	41	59
23	102.611	12	57.2	57	115.104	29	41	91	135.207	42	18
24	102.840	13	29.8	58	115.603	30	7	92	135.882	42	37
25	103.078	14	2.2	59	116.108	30	32	93	136.561	42	55
26	103.325	14	34.5	60	116.619	30	58	94	137.244	43	14
27	103.581	15	6.6	61	117.137	31	23	95	137.931	43	32
28	103.846	15	38.5	62	117.661	31	48	96	138.622	43	50
29	104.120	16	10 3	63	118.191	32	13	97	139.316	44	8
30	104.403	16	42.0	64	118.727	32	37	98	140.014	44	25
31	104.695	17	13.4	65	119.269	33	1	99	140.716	44	43
32	104.995	17	44.7	66	119.817	33	25	100	141.421	45	00
33	105.304	18	15.8	67	120.370	33	49	101	142.130	45	17
34	105.622	18	46.7	68	120.930	34	13	102	142.843	45	34

In describing **railroad grades,** it is usual, as in our tables, to refer the rise, A, to the corresponding *horizontal* length, B. We then have $\frac{\text{rise}}{\text{length}} = \frac{A}{B} =$ the *tangent* of the angle, *a*, between the plane and the horizontal. If the rise, A, be referred to the *sloping* length, C, we have $\frac{\text{rise}}{\text{length}} = \frac{A}{C} =$ *sine* A; and this fraction is proportional to the component, S, of the weight, W, in the direction of the slope. Thus, on a grade where rise, A, = 0.1 × sloping length, C, we have sin $a = 0.1$, and S = 0.1 W. The *tangent* of *a* is only *approximately* proportional to S; but the steepest grades, surmounted by traction only, even on electric railways, rarely, if ever, exceed from 13 to 15 per cent; and, on these, the error, due to using tan *a* instead of sin *a*, is less than a difference of 0.2 per cent in the grade, and about = 1 per cent of the true value of S. For steeper grades, such as those of rack railways, it should always be specified whether the rise refers to the horizontal or to the sloping measurement.



Transverse slopes, such as those of earthwork, are sometimes, like railroad grades, stated in $\frac{\text{ft vertical}}{\text{ft horizontal}}$, $\frac{A}{B}$, but usually in $\frac{\text{ft horizontal}}{\text{ft vertical}}$, as $\frac{B}{A}$. Here $\frac{B}{A}$ is the cotangent of the angle, *a*, with the horizontal, or the tangent of the angle (90°–*a*) with the vertical. Thus stated, a slope of 2 to 1 means a slope of 2 rizontal to 1 vertical.

Table of grades per mile; or per 100 feet measured horizontally.

Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.
1	.01894	39	.73864	77	1.45833	115	2.17803
2	.03788	40	.75758	78	1.47727	116	2.19697
3	.05682	41	.77652	79	1.49621	117	2.21591
4	.07576	42	.79545	80	1.51515	118	2.23485
5	.09470	43	.81439	81	1.53409	119	2.25379
6	-11364	44	.83333	82	1.55303	120	2.27273
7	.13258	45	.85227	83	1.57197	121	2.29167
8	.15152	46	.87121	84	1.59091	122	2.31061
9	.17045	47	.89015	85	1.60985	123	2.32955
10	.18939	48	.90909	86	1.62879	124	2.34848
11	.20833	49	.92803	87	1.64773	125	2.36742
12	.22727	50	.94697	88	1.66666	126	2.38636
13	.24621	51	.96591	89	1.68561	127	2.40530
14	.26515	52	.98485	90	1.70455	128	2.42424
15	.28409	53	1.00379	91	1.72348	129	2.44318
16	.30303	54	1.02273	92	1.74242	130	2.46212
17	.32197	55	1.04167	93	1.76136	131	2.48106
18	.34091	56	1.06061	94	1.78030	132	2.50000
19	.35985	57	1.07955	95	1.79924	133	2.51894
20	.37879	58	1.09848	96	1.81818	134	2.53788
21	.39773	59	1.11742	97	1.83712	135	2.55682
22	.41667	60	1.13636	98	1.85606	136	2.57576
23	.43561	61	1.15530	99	1.87500	137	2.59470
24	.45455	62	1.17424	100	1.89394	138	2.61364
25	.47348	63	1.19318	101	1.91288	139	2.63258
26	.49242	64	1.21212	102	1.93182	140	2.65152
27	.51136	65	1.23106	103	1.95076	141	2.67045
28	.53030	66	1.25000	104	1.96970	142	2.68939
29	.54924	67	1.26894	105	1.98864	143	2.70833
30	.56818	68	1.28788	106	2.00758	144	2.72727
31	.58712	69	1.30682	107	2.02652	145	2.74621
32	.60606	70	1.32576	108	2.04545	146	2.76515
33	.62500	71	1.34470	109	2.06439	147	2.78409
34	.64394	72	1.36364	110	2.08333	148	2.80303
35	.66288	73	1.38258	111	2.10227	149	2.82197
36	.68182	74	1.40152	112	2.12121	150.	2.84091
37	.70076	75	1.42045	113	2.14015	151	2.85985
38	.71970	76	1.43939	114	2.15909	152	2.87879

If the grade per mile should consist of feet *and tenths*, add to the grade per 100 feet in the foregoing table, that corresponding to the number of tenths taken from the table below; thus, for a grade of 43.7 feet per mile, we have .81439 + .01326 = .82765 feet per 100 feet.

Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.
.05	.00094	.4	.00758	.7	.01326
.1	.00189	.45	.00852	.75	.01420
.15	.00283	.5	.00947	.8	.01515
.2	.00379	.55	.01041	.85	.01609
.25	.00473	.6	.01136	.9	.01705
.3	.00568	.65	.01230	.95	.01799
.35	.00662				

TABLE OF HEADS OF WATER CORRESPONDING TO GIVEN PRESSURES.

Water at maximum density, 62.425 lbs. per cubic foot = 1 gram per cubic centimeter; corresponding to a temperature of 4° Centigrade = 39.2° Fahrenheit.

Head in feet = 2.306768 × pressure in lbs. per square inch.
" " = 0.0160192 × pressure in lbs. per square foot.

Heads corresponding to pressures not given in the table can be found by these formulæ, or taken from the table by simple proportion.

Pressure.		Head.	Pressure.		Head.	Pressure.		Head.
lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.
1	144	2.3068	51	7344	117.645	101	14544	232.984
2	288	4.6135	52	7488	119.952	102	14688	235.290
3	432	6.9203	53	7632	122.259	103	14832	237.597
4	576	9.2271	54	7776	124.565	104	14976	239.904
5	720	11.5338	55	7920	126.872	105	15120	242.211
6	864	13.8406	56	8064	129.179	106	15264	244.517
7	1008	16.1474	57	8208	131.486	107	15408	246.824
8	1152	18.4541	58	8352	133.793	108	15552	249.131
9	1296	20.7609	59	8496	136.099	109	15696	251.438
10	1440	23.0677	60	8640	138.406	110	15840	253.744
11	1584	25.3744	61	8784	140.713	111	15984	256.051
12	1728	27.6812	62	8928	143.020	112	16128	258.358
13	1872	29.9880	63	9072	145.326	113	16272	260.665
14	2016	32.2948	64	9216	147.633	114	16416	262.972
15	2160	34.6015	65	9360	149.940	115	16560	265.278
16	2304	36.9083	66	9504	152.247	116	16704	267.585
17	2448	39.2151	67	9648	154.553	117	16848	269.892
18	2592	41.5218	68	9792	156.860	118	16992	272.199
19	2736	43.8286	69	9936	159.167	119	17136	274.505
20	2880	46.1354	70	10080	161.474	120	17280	276.812
21	3024	48.4421	71	10224	163.781	121	17424	279.119
22	3168	50.7489	72	10368	166.087	122	17568	281.426
23	3312	53.0557	73	10512	168.394	123	17712	283.732
24	3456	55.3624	74	10656	170.701	124	17856	286.039
25	3600	57.6692	75	10800	173.008	125	18000	288.346
26	3744	59.9760	76	10944	175.314	126	18144	290.653
27	3888	62.2827	77	11088	177.621	127	18288	292.960
28	4032	64.5895	78	11232	179.928	128	18432	295.266
29	4176	66.8963	79	11376	182.235	129	18576	297.573
30	4320	69.2030	80	11520	184.541	130	18720	299.880
31	4464	71.5098	81	11664	186.848	131	18864	302.187
32	4608	73.8166	82	11808	189.155	132	19008	304.493
33	4752	76.1233	83	11952	191.462	133	19152	306.800
34	4896	78.4301	84	12096	193.769	134	19296	309.107
35	5040	80.7369	85	12240	196.075	135	19440	311.414
36	5184	83.0436	86	12384	198.382	136	19584	313.720
37	5328	85.3504	87	12528	200.689	137	19728	316.027
38	5472	87.6572	88	12672	202.996	138	19872	318.334
39	5616	89.9640	89	12816	205.302	139	20016	320.641
40	5760	92.2707	90	12960	207.609	140	20160	322.948
41	5904	94.5775	91	13104	209.916	141	20304	325.254
42	6048	96.8843	92	13248	212.223	142	20448	327.561
43	6192	99.1910	93	13392	214.529	143	20592	329.868
44	6336	101.4978	94	13536	216.836	144	20736	332.175
45	6480	103.8046	95	13680	219.143	145	20880	334.481
46	6624	106.1113	96	13824	221.450	146	21024	336.788
47	6768	108.4181	97	13968	223.756	147	21168	339.095
48	6912	110.7249	98	14112	226.063	148	21312	341.402
49	7056	113.0316	99	14256	228.370	149	21456	343.708
50	7200	115.3384	100	14400	230.677	150	21600	346.015

TABLE OF PRESSURES CORRESPONDING TO GIVEN HEADS OF WATER.

Water at maximum density, 62.425 lbs. per cubic foot = 1 gram per cubic centimeter; corresponding to a temperature of 4° Centigrade = 39.2° Fahrenheit.

Pressure in lbs. per square inch = 0.433507 × head in feet.
Pressure in lbs. per square foot = 62.425 × head in feet.

Pressures corresponding to heads not given in the table can be found by these formulæ, or taken from the table by simple proportion.

Head.	Pressure.		Head.	Pressure.	
Inches.	lbs. per sq. in.	lbs. per sq. ft.	Inches.	lbs. per sq. in.	lbs. per sq. ft.
1	0.036126	5.202083	7	0.252879	36.414583
2	0.072251	10.404167	8	0.289005	41.616667
3	0.108377	15.606250	9	0.325130	46.818750
4	0.144502	20.808333	10	0.361256	52.020833
5	0.180628	26.010417	11	0.397381	57.222917
6	0.216753	31.212500	12	0.433507	62.425000

Head.	Pressure.		Head.	Pressure.		Head.	Pressure.	
Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.
1	0.4335	62.425	38	16.4733	2372.150	75	32.5130	4681.875
2	0.8670	124.850	39	16.9068	2434.575	76	32.9465	4744.300
3	1.3005	187.275	40	17.3403	2497.000	77	33.3800	4806.725
4	1.7340	249.700	41	17.7738	2559.425	78	33.8135	4869.150
5	2.1675	312.125	42	18.2073	2621.850	79	34.2471	4931.575
6	2.6010	374.550	43	18.6408	2684.275	80	34.6806	4994.000
7	3.0345	436.975	44	19.0743	2746.700	81	35.1141	5056.425
8	3.4681	499.400	45	19.5078	2809.125	82	35.5476	5118.850
9	3.9016	561.825	46	19.9413	2871.550	83	35.9811	5181.275
10	4.3351	624.250	47	20.3748	2933.975	84	36.4146	5243.700
11	4.7686	686.675	48	20.8083	2996.400	85	36.8481	5306.125
12	5.2021	749.100	49	21.2418	3058.825	86	37.2816	5368.550
13	5.6356	811.525	50	21.6753	3121.250	87	37.7151	5430.975
14	6.0691	873.950	51	22.1089	3183.675	88	38.1486	5493.400
15	6.5026	936.375	52	22.5424	3246.100	89	38.5821	5555.825
16	6.9361	998.800	53	22.9759	3308.525	90	39.0156	5618.250
17	7.3696	1061.225	54	23.4094	3370.950	91	39.4491	5680.675
18	7.8031	1123.650	55	23.8429	3433.375	92	39.8826	5743.100
19	8.2366	1186.075	56	24.2764	3495.800	93	40.3162	5805.525
20	8.6701	1248.500	57	24.7099	3558.225	94	40.7497	5867.950
21	9.1036	1310.925	58	25.1434	3620.650	95	41.1832	5930.375
22	9.5372	1373.350	59	25.5769	3683.075	96	41.6167	5992.800
23	9.9707	1435.775	60	26.0104	3745.500	97	42.0502	6055.225
24	10.4042	1498.200	61	26.4439	3807.925	98	42.4837	6117.650
25	10.8377	1560.625	62	26.8774	3870.350	99	42.9172	6180.075
26	11.2712	1623.050	63	27.3109	3932.775	100	43.3507	6242.500
27	11.7047	1685.475	64	27.7444	3995.200	101	43.7842	6304.925
28	12.1382	1747.900	65	28.1780	4057.625	102	44.2177	6367.350
29	12.5717	1810.325	66	28.6115	4120.050	103	44.6512	6429.775
30	13.0052	1872.750	67	29.0450	4182.475	104	45.0847	6492.200
31	13.4387	1935.175	68	29.4785	4244.900	105	45.5182	6554.625
32	13.8722	1997.600	69	29.9120	4307.325	106	45.9517	6617.050
33	14.3057	2060.025	70	30.3455	4369.750	107	46.3852	6679.475
34	14.7392	2122.450	71	30.7790	4432.175	108	46.8188	6741.900
35	15.1727	2184.875	72	31.2125	4494.600	109	47.2523	6804.325
36	15.6063	2247.300	73	31.6460	4557.025	110	47.6858	6866.750
37	16.0398	2309.725	74	32.0795	4619.450	111	48.1193	6929.175

TABLE OF PRESSURES (Continued).

Head.	Pressure.		Head.	Pressure.		Head.	Pressure.	
Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.	Feet.	lbs. per sq. in.	lbs. per sq. ft.
112	48.5528	6991.600	144	62.4250	8989.200	176	76.2972	10986.800
113	48.9863	7054.025	145	62.8585	9051.625	177	76.7307	11049.225
114	49.4198	7116.450	146	63.2920	9114.050	178	77.1642	11111.650
115	49.8533	7178.875	147	63.7255	9176.475	179	77.5978	11174.075
116	50.2868	7241.300	148	64.1590	9238.900	180	78.0313	11236.500
117	50.7203	7303.725	149	64.5925	9301.325	181	78.4648	11298.925
118	51.1538	7366.150	150	65.0260	9363.750	182	78.8983	11361.350
119	51.5873	7428.575	151	65.4596	9426.175	183	79.3318	11423.775
120	52.0208	7491.000	152	65.8931	9488.600	184	79.7653	11486.200
121	52.4543	7553.425	153	66.3266	9551.025	185	80.1988	11548.625
122	52.8879	7615.850	154	66.7601	9613.450	186	80.6323	11611.050
123	53.3214	7678.275	155	67.1936	9675.875	187	81.0658	11673.475
124	53.7549	7740.700	156	67.6271	9738.300	188	81.4993	11735.900
125	54.1884	7803.125	157	68.0606	9800.725	189	81.9328	11798.325
126	54.6219	7865.550	158	68.4941	9863.150	190	82.3663	11860.750
127	55.0554	7927.975	159	68.9276	9925.575	191	82.7998	11923.175
128	55.4889	7990.400	160	69.3611	9988.000	192	83.2333	11985.600
129	55.9224	8052.825	161	69.7946	10050.425	193	83.6669	12048.025
130	56.3559	8115.250	162	70.2281	10112.850	194	84.1004	12110.450
131	56.7894	8177.675	163	70.6616	10175.275	195	84.5339	12172.875
132	57.2229	8240.100	164	71.0951	10237.700	196	84.9674	12235.300
133	57.6564	8302.525	165	71.5287	10300.125	197	85.4009	12297.725
134	58.0899	8364.950	166	71.9622	10362.550	198	85.8344	12360.150
135	58.5234	8427.375	167	72.3957	10424.975	199	86.2679	12422.575
136	58.9570	8489.800	168	72.8292	10487.400	200	86.7014	12485.000
137	59.3905	8552.225	169	73.2627	10549.825	201	87.1349	12547.425
138	59.8240	8614.650	170	73.6962	10612.250	202	87.5684	12609.850
139	60.2575	8677.075	171	74.1297	10674.675	203	88.0019	12672.275
140	60.6910	8739.500	172	74.5632	10737.100	204	88.4354	12734.700
141	61.1245	8801.925	173	74.9967	10799.525	205	88.8689	12797.125
142	61.5580	8864.350	174	75.4302	10861.950	206	89.3024	12859.550
143	61.9915	8926.775	175	75.8637	10924.375	207	89.7359	12921.975

Table showing the total pressure against a vertical plane one foot wide, extending from the surface of the water to the depth named in the first column.

Water at its maximum density, 62.425 lbs per cubic foot = 1 gram per cubic centimeter, corresponding to a temperature of 4° Cent. = 39.2° Fahr.

Total pressure in pounds = 31.2125 × square of depth in feet.

Depth. Feet.	Total pressure. Pounds.	Depth. Feet.	Total pressure. Pounds.	Depth. Feet.	Total pressure. Pounds.	Depth. Feet.	Total pressure. Pounds.
1	31.21	17	9020	33	33990	49	74941
2	124.85	18	10113	34	36082	50	78031
3	280.9	19	11268	35	38235	51	81184
4	499.4	20	12485	36	40451	52	84399
5	780.3	21	13765	37	42730	53	87676
6	1124	22	15107	38	45071	54	91016
7	1529	23	16511	39	47474	55	94418
8	1998	24	17978	40	49940	60	112365
9	2528	25	19508	41	52468	65	131873
10	3121	26	21100	42	55059	70	152941
11	3777	27	22754	43	57712	75	175570
12	4495	28	24471	44	60427	80	199760
13	5275	29	26250	45	63205	85	225510
14	6118	30	28091	46	66046	90	252821
15	7023	31	29995	47	68948	95	281693
16	7990	32	31962	48	71914	100	312125

TABLE OF DISCHARGES IN CUBIC FEET PER SECOND CORRESPONDING TO GIVEN DISCHARGES IN U. S. GALLONS PER 24 HOURS.

U. S. gallon = 231 cubic inches.
Discharge in cubic feet per second = 1.54723 × discharge in *millions* of U. S. gallons per 24 hours.

Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals. per 24 hrs.	Cubic feet per second.	Millions of U. S. gals. per 24 hrs.	Cubic feet per second.
.010	.0154723	13	20.1140	43	66.5308	72	111.400
.020	.0309446	14	21.6612	44	68.0781	73	112.948
.030	.0464169	15	23.2084	45	69.6253	74	114.495
.040	.0618891	16	24.7557	46	71.1725	75	116.042
.050	.0773614	17	26.3029	47	72.7197	76	117.589
.060	.0928337	18	27.8501	48	74.2670	77	119.137
.070	.108306	19	29.3973	49	75.8142	78	120.684
.080	.123778	20	30.9446	50	77.3614	79	122.231
.090	.139251	21	32.4918	51	78.9087	80	123.778
.100	.154723	22	34.0390	52	80.4559	81	125.326
.200	.309446	23	35.5863	53	82.0031	82	126.873
.300	.464169	24	37.1335	54	83.5503	83	128.420
.400	.618891	25	38.6807	55	85.0976	84	129.967
.500	.773614	26	40.2279	56	86.6448	85	131.514
.600	.928337	27	41.7752	57	88.1920	86	133.062
.700	1.08306	28	43.3224	58	89.7393	87	134.609
.800	1.23778	29	44.8696	59	91.2865	88	136.156
.900	1.39251	30	46.4169	60	92.8337	89	137.703
1	1.54723	31	47.9641	61	94.3809	90	139.251
2	3.09446	32	49.5113	62	95.9282	91	140.798
3	4.64169	33	51.0585	63	97.4754	92	142.345
4	6.18891	34	52.6058	64	99.0226	93	143.892
5	7.73614	35	54.1530	65	100.570	94	145.439
6	9.28337	36	55.7002	66	102.117	95	146.987
7	10.8306	37	57.2475	67	103.664	96	148.534
8	12.3778	38	58.7947	68	105.212	97	150.081
9	13.9251	39	60.3419	69	106.759	98	151.628
10	15.4723	40	61.8891	70	108.306	99	153.176
11	17.0195	41	63.4364	71	109.853	100	154.723
12	18.5667	42	64.9836				

TABLE OF DISCHARGES IN CUBIC FEET PER SECOND CORRESPONDING TO GIVEN DISCHARGES IN IMPERIAL GALLONS PER 24 HOURS.

Imperial gallon = 277.274 cubic inches.
Discharge in cubic feet per second = 1.85717 × discharge in Imperial gallons per 24 hours.

Millions of Imp. gals. per 24 hrs.	Cubic feet per second.	Millions of Imp. gals. per 24 hrs.	Cubic feet per second.	Millions of Imp. gals. per 24 hrs.	Cubic feet per second.	Millions of Imp. gals. per 24 hrs.	Cubic feet per second.
.010	.0185717	13	24.1432	43	79.8583	72	133.7162
.020	.0371434	14	26.0004	44	81.7155	73	135.5734
.030	.0557151	15	27.8576	45	83.5727	74	137.4306
.040	.0742868	16	29.7147	46	85.4298	75	139.2878
.050	.0928585	17	31.5719	47	87.2870	76	141.1449
.060	.111430	18	33.4291	48	89.1442	77	143.0021
.070	.130002	19	35.2862	49	91.0013	78	144.8593
.080	.148574	20	37.1434	50	92.8585	79	146.7164
.090	.167145	21	39.0006	51	94.7157	80	148.5736
.100	.185717	22	40.8577	52	96.5728	81	150.4308
.200	.371434	23	42.7149	53	98.4300	82	152.2879
.300	.557151	24	44.5721	54	100.2872	83	154.1451
.400	.742868	25	46.4293	55	102.1444	84	156.0023
.500	.928585	26	48.2864	56	104.0015	85	157.8595
.600	1.11430	27	50.1436	57	105.8587	86	159.7166
.700	1.30002	28	52.0008	58	107.7159	87	161.5738
.800	1.48574	29	53.8579	59	109.5730	88	163.4310
.900	1.67145	30	55.7151	60	111.4302	89	165.2881
1	1.85717	31	57.5723	61	113.2874	90	167.1453
2	3.71434	32	59.4294	62	115.1445	91	169.0025
3	5.57151	33	61.2866	63	117.0017	92	170.8596
4	7.42868	34	63.1438	64	118.8589	93	172.7168
5	9.28585	35	65.0010	65	120.7160	94	174.5740
6	11.1430	36	66.8581	66	122.5732	95	176.4312
7	13.0002	37	68.7153	67	124.4304	96	178.2883
8	14.8574	38	70.5725	68	126.2876	97	180.1455
9	16.7145	39	72.4296	69	128.1447	98	182.0027
10	18.5717	40	74.2868	70	130.0019	99	183.8598
11	20.4289	41	76.1440	71	131.8591	100	185.7170
12	22.2860	42	78.0011				

TABLE OF DISCHARGES IN GALLONS PER 24 HOURS CORRESPONDING TO GIVEN DISCHARGES IN CUBIC FEET PER SECOND.

U. S. gallon = 231 cubic inches. Imperial gallon = 277.274 cubic inches.
Discharge in U. S. gallons per 24 hours = 646317 × discharge in cubic feet per second.
Discharge in Imperial gallons per 24 hours = 538454 × discharge in cubic feet per second.

Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.	Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
1	0.646317	0.538454	53	34.254795	28.538044
2	1.292634	1.076907	54	34.901112	29.076498
3	1.938951	1.615361	55	35.547428	29.614951
4	2.585268	2.153815	56	36.193745	30.153405
5	3.231584	2.692268	57	36.840062	30.691859
6	3.877901	3.230722	58	37.486379	31.230312
7	4.524218	3.769176	59	38.132696	31.768766
8	5.170535	4.307629	60	38.779013	32.307220
9	5.816852	4.846083	61	39.425330	32.845673
10	6.463169	5.384537	62	40.071647	33.384127
11	7.109486	5.922990	63	40.717963	33.922581
12	7.755803	6.461444	64	41.364280	34.461034
13	8.402119	6.999898	65	42.010597	34.999488
14	9.048436	7.538351	66	42.656914	35.537942
15	9.694753	8.076805	67	43.303231	36.076395
16	10.341070	8.615259	68	43.949548	36.614849
17	10.987387	9.153712	69	44.595865	37.153303
18	11.633704	9.692166	70	45.242182	37.691756
19	12.280021	10.230620	71	45.888498	38.230210
20	12.926338	10.769073	72	46.534815	38.768664
21	13.572654	11.307527	73	47.181132	39.307117
22	14.218971	11.845981	74	47.827449	39.845571
23	14.865288	12.384434	75	48.473766	40.384025
24	15.511605	12.922888	76	49.120083	40.922478
25	16.157922	13.461342	77	49.766400	41.460932
26	16.804239	13.999795	78	50.412717	41.999385
27	17.450556	14.538249	79	51.059034	42.537839
28	18.096873	15.076702	80	51.705350	43.076293
29	18.743190	15.615156	81	52.351667	43.614746
30	19.389506	16.153610	82	52.997984	44.153200
31	20.035823	16.692063	83	53.644301	44.691654
32	20.682140	17.230517	84	54.290618	45.230107
33	21.328457	17.768971	85	54.936935	45.768561
34	21.974774	18.307424	86	55.583252	46.307015
35	22.621091	18.845878	87	56.229569	46.845468
36	23.267408	19.384332	88	56.875885	47.383922
37	23.913725	19.922785	89	57.522202	47.922376
38	24.560041	20.461239	90	58.168519	48.460829
39	25.206358	20.999693	91	58.814836	48.999283
40	25.852675	21.538146	92	59.461153	49.537737
41	26.498992	22.076600	93	60.107470	50.076190
42	27.145309	22.615054	94	60.753787	50.614644
43	27.791626	23.153507	95	61.400104	51.153098
44	28.437943	23.691961	96	62.046420	51.691551
45	29.084260	24.230415	97	62.692737	52.230005
46	29.730576	24.768868	98	63.339054	52.768459
47	30.376893	25.307322	99	63.985371	53.306912
48	31.023210	25.845776	100	64.631688	53.845366
49	31.669527	26.384229	101	65.278005	54.383820
50	32.315844	26.922683	102	65.924322	54.922273
51	32.962161	27.461137	103	66.570639	55.460727
52	33.608478	27.999590	104	67.216956	55.999181

TABLE OF DISCHARGES (Continued).

Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.	Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
105	67.863272	56.537634	167	107.934919	89.921761
106	68.509589	57.076088	168	108.581236	90.460215
107	69.155906	57.614542	169	109.227553	90.998669
108	69.802223	58.152995	170	109.873870	91.537122
109	70.448540	58.691449	171	110.520186	92.075576
110	71.094857	59.229903	172	111.166503	92.614030
111	71.741174	59.768356	173	111.812820	93.152483
112	72.387491	60.306810	174	112.459137	93.690937
113	73.033807	60.845264	175	113.105454	94.229391
114	73.680124	61.383717	176	113.751771	94.767844
115	74.326441	61.922171	177	114.398088	95.306298
116	74.972758	62.460625	178	115.044405	95.844751
117	75.619075	62.999078	179	115.690722	96.383205
118	76.265392	63.537532	180	116.337038	96.921659
119	76.911709	64.075986	181	116.983355	97.460112
120	77.558026	64.614439	182	117.629672	97.998566
121	78.204342	65.152893	183	118.275989	98.537020
122	78.850659	65.691347	184	118.922306	99.075473
123	79.496976	66.229800	185	119.568623	99.613927
124	80.143293	66.768254	186	120.214940	100.152381
125	80.789610	67.306708	187	120.861257	100.690834
126	81.435927	67.845161	188	121.507573	101.229288
127	82.082244	68.383615	189	122.153890	101.767742
128	82.728561	68.922068	190	122.800207	102.306195
129	83.374878	69.460522	191	123.446524	102.844649
130	84.021194	69.998976	192	124.092841	103.383103
131	84.667511	70.537429	193	124.739158	103.921556
132	85.313828	71.075883	194	125.385475	104.460010
133	85.960145	71.614337	195	126.031792	105.098464
134	86.606462	72.152790	196	126.678108	105.536917
135	87.252779	72.691244	197	127.324425	106.075371
136	87.899096	73.229698	198	127.970742	106.613825
137	88.545413	73.768151	199	128.617059	107.152278
138	89.191729	74.306605	200	129.263376	107.690732
139	89.838046	74.845059	201	129.909693	108.229186
140	90.484363	75.383512	202	130.556010	108.767639
141	91.130680	75.921966	203	131.202327	109.306093
142	91.776997	76.460420	204	131.848644	109.844547
143	92.423314	76.998873	205	132.494960	110.383000
144	93.069631	77.537327	206	133.141277	110.921454
145	93.715948	78.075781	207	133.787594	111.459908
146	94.362264	78.614234	208	134.433911	111.998361
147	95.008581	79.152688	209	135.080228	112.536815
148	95.654898	79.691142	210	135.726545	113.075269
149	96.301215	80.229595	211	136.372862	113.613722
150	96.947532	80.768049	212	137.019179	114.152176
151	97.593849	81.306503	213	137.665495	114.690630
152	98.240166	81.844956	214	138.311812	115.229083
153	98.886483	82.383410	215	138.958129	115.767537
154	99.532800	82.921864	216	139.604446	116.305991
155	100.179116	83.460317	217	140.250763	116.844444
156	100.825433	83.998771	218	140.897080	117.382898
157	101.471750	84.537225	219	141.543397	117.921352
158	102.118067	85.075678	220	142.189714	118.459805
159	102.764384	85.614132	221	142.836030	118.998259
160	103.410701	86.152586	222	143.482347	119.536713
161	104.057018	86.691039	223	144.128664	120.075166
162	104.703335	87.229493	224	144.774981	120.613620
163	105.349651	87.767947	225	145.421298	121.152074
164	105.995968	88.306400	226	146.067615	121.690527
165	106.642285	88.844854	227	146.713932	122.228981
166	107.288602	89.383308	228	147.360249	122.767434

TABLE OF DISCHARGES (Continued).

Cub. ft per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.	Cub. ft. per sec.	Millions of U. S. gallons per 24 hours.	Millions of Imperial gallons per 24 hours.
229	148.006566	123.305888	240	155.116051	129.228878
230	148.652882	123.844342	241	155.762368	129.767332
231	149.299199	124.382795	242	156.408685	130.305786
232	149.945516	124.921249	243	157.055002	130.844239
233	150.591833	125.459703	244	157.701319	131.382693
234	151.238150	125.998156	245	158.347636	131.921147
235	151.884467	126.536610	246	158.993952	132.459600
236	152.530784	127.075064	247	159.640269	132.998054
237	153.177101	127.613517	248	160.286586	133.536508
238	153.823417	128.151971	249	160.932903	134.074961
239	154.469734	128.690425	250	161.579220	134.613415

TIME.

60 seconds,*† marked s, = 1 minute
60 minutes,† " m, = 1 hour = 3600 seconds
24 hours, " h, = 1 day = 1440 minutes = 86400 seconds
7 days, " d, = 1 week = 168 hours = 10080 minutes

ARC	TIME
1°	= 4 minutes
1′	= 4 seconds
1″	= 0.066... second

TIME	ARC
24 hours	= 360°
1 hour	= 15°
1 minute	= 0° 15′
1 second	= 0° 0′ 15″

Methods of reckoning time. Astronomers distinguish between mean solar time, true or apparent solar time, and sidereal time.

At a standard meridian (see page 267) **mean solar time** is the same as ordinary clock time. At any point not on a standard meridian, *standard* time is the local mean solar time of the meridian adopted as standard for such point; and *local* time is = time at a standard meridian *plus* correction for longitude from that meridian if the place is *east* of the meridian, and *vice versa*. For the amount of such correction, see second table above. **A true or apparent solar day** is the interval of time between two successive culminations of the sun, *i.e.*, between two successive transits or passages of the sun across the meridian of the same point on the earth; but, since these intervals are unequal, they do not correspond with the uniform movement of clock time. A fictitious or imaginary sun, called the "mean sun," is therefore supposed to move along the equator in such a way that the interval between its culminations is constant. This interval is called a day, or mean solar day, and is the average of the lengths of all the *apparent* solar days in a year. Apparent and mean time agree at four points in the year, viz., about the middle of April and of June, September 1 and December 24. The sun is sometimes behind and sometimes in advance of the mean sun, and is called "slow" or "fast" accordingly. The sun is "slow" in winter, the maximum being about February 11, when it passes any standard meridian, or "souths" (making *apparent* noon), about 14m, 28s, *after* noon by a correct clock. The sun is "fast," or in advance of the clock, in May and in the fall, with a maximum, about November 2, of about 16m, 20s.

The difference between apparent and mean time is called the **equation of time.** It can be obtained from the Nautical Almanac, or, approximately, by taking the mean between the times of sunrise and sunset, as given in ordinary almanacs.

As *solar* time is measured by the apparent daily motion of the *sun*, so *sidereal* time is measured by that of the fixed *stars*, or, more strictly speaking, by the motion of the vernal equinox which is the point where the sun crosses the equator in the spring.

---

* The second was formerly divided into 60 equal parts called thirds (marked ‴); but it is now divided decimally.

† The old and confusing practice of designating minutes, seconds and thirds of time (see footnote *) as ′, ″ and ‴, is no longer in vogue. Days, hours, minutes and seconds are now designated by d, h, m, and s, respectively, thus: 2d, 20h, 48m, 55.43 s., and the symbols ′ and ″ designate minutes and seconds of *arc*.

A sidereal day is the interval of time between two successive passages of the vernal equinox (or, practically, of any star) past the meridian of a given point on the earth. It is, practically, the time required for one complete revolution of the earth on its axis, relatively to the stars.

The length of the sideral day is 23 h, 56 m, 4.09 s, of mean solar time, or 3 m, 55.91 s of mean solar time less than the mean solar day of 24 hours. In other words, a star will, on any night, appear to set 3 m, 55.91 s earlier by a correct clock than it did on the preceding night. Hence, substantially, the number of sidereal days in a year is greater by 1 than the number of solar days.

The sidereal day, like the solar day, is divided into 24 hours. These hours are, of course, shorter than those of the solar day in the same proportion as the sidereal day is shorter than the solar day. They are counted from 0 to 24, commencing with sidereal noon, or the instant when the vernal equinox passes the upper meridian.

The civil day (= 24 hours of clock or mean solar time) commences at midnight; and **the astronomical solar day** at noon on the civil day of the same date. Thus, on a standard meridian, Thursday, May 9, 2 A. M. civil time, is Wednesday, May 8, 14 h, astronomical time; but Thursday, May 9, 2 P. M., civil time, is Thursday, May 9, 2 h, astronomical time.

The civil month is the ordinary and arbitrary month of the calendar, varying in length from 28 to 31 mean solar days.

A sidereal month is the time required for the moon to perform an entire revolution with reference to the stars. Its mean length, in mean solar time, is about 27 d, 7 h, 43 m, 12 s.

A lunation, or synodic month is the time from new moon to new moon. Its mean length is about 29 d, 12 h, 44 m, 3 s.

The tropical or natural year is the time during which the earth describes the circuit from either equinox to the same again. Its mean length, in mean solar time, is now about 365 d, 5 h, 48 m, 49 s.

The sidereal year is the time during which the earth describes its orbit with reference to the stars. Its mean length, in mean solar time, is about 365 d, 6 h, 9 m, 10 s.

The civil year is that arbitrary or conventional and variable division of time comprised between the 1st of January and the 31st of the following December, both inclusive. It contains ordinarily 365 mean solar days of 24 hours, but each year whose number is divisible by 4 contains 366 days, and is called a **leap year**, except that those years whose numbers end in 00 and are not multiples of 400 are not leap years.

To regulate a watch by the stars. The author, after having regulated his chronometer for a year by this method only, differed but a few seconds from the actual time as deduced from careful solar observations. Select a window, facing west if possible, and commanding a view of a roof-crest or other fixed horizontal line, preferably about 40° above the horizon, in order to avoid disturbance due to refraction, and distant say 50 feet or more. Note the time when any bright fixed star (not a planet) passes the range formed between the roof, etc., and any fixed horizontal line about the window frame, as a pin fixed in either jamb. The sight in the window, and the watch, must be illuminated. The star will pass the range 3 m. 55.91 s. earlier on each succeeding evening. Those stars which are nearest the equator appear to move the fastest, and are therefore best suited to the purpose. If the first observation of a given star be made as late as midnight, that same star will answer for about three months, until at last it will begin to pass the range in daylight. Before this happens, transfer the time to another star which sets later. By thus tabulating, throughout the year, about half a dozen stars which follow each other at nearly equal intervals of time, we may provide a standard by means of which correct clock time may be ascertained on any clear night. Experimenting in this way with two of the best **chronometers**, the author found that their *rates* varied, at times, as much as from three to eight seconds per day.

An average man takes **two steps** (one right, one left) **per second.** Hence, march music usually takes one second per measure (or "bar"). Modern **watches** usually **tick** five times, and **clocks** either one, two, or four times, per second.

STANDARD RAILWAY TIME, ADOPTED 1883.

The following arrangement of standard time was recommended by the General and Southern Time Conventions of the railroads of the United States and Canada, held respectively in St. Louis, Mo., and New York city, April, 1883, and in Chicago, Ill., and New York city, in October, 1883, and went into effect on most of the railroads of the United States and Canada, November 18th, 1883. Most of the principal cities of the United States have made their respective local times to correspond with it. This system was proposed by Mr. W. F. Allen, Secretary of the Time Conventions, and its adoption was largely due to his efforts. We are indebted to Mr. Allen for documents from which the following has been condensed. Five standards of time, or five "times," have been adopted for the United States and Canada. These are, respectively, the mean times of the 60th, 75th, 90th, 105th, and 120th meridians west of Greenwich, England. As each of these meridians, in the above order, is 15° west of its predecessor, its time is one hour slower. Thus, when it is noon on the 90th meridian, it is 1 P. M. on the 75th, and 11 A. M. on the 105th. The following gives the name adopted for the standard time of each meridian, and the conventional color adopted, and uniformly adhered to, by Mr. Allen, for the purpose of designating it and its time, &c, on the maps published under his auspices:

Longitude west from Greenwich.	Name of Standard Time.	Conventional color.
60°	Intercolonial.	Brown.
75°	Eastern.	Red.
90°	Central.	Blue.
105°	Mountain.	Green.
120°	Pacific.	Yellow.

Theoretically, each meridian may be said to give the time for a strip of country 15° wide, running north and south, and having the meridian for its center. Thus the meridian on which the change of time between two standard meridians is supposed to take place, lies half-way between them. But it would, of course, not be practicable for the railroads to use an imaginary line in passing from one time standard to another. The changes are made at prominent stations forming the termini of two or more lines; or, as in the case of the long Pacific roads, at the ends of divisions. As far as practicable, points at which changes of time had previously been made, were selected as the changing points under the new system. Detroit, Mich., Pittsburgh, Pa., Wheeling and Parkersburg, W. Va., and Augusta, Ga., although not situated upon the same meridian, are points of change between *eastern* and *central* standard times. A train arriving at Pittsburgh from the east at noon, and leaving for the west 10 minutes after its arrival, leaves (by the figures shown upon its time-table, and by the watches of its train hands) not at 10 minutes after 12, but at 10 minutes after 11.

The necessity for making the changes of time at principal points, instead of on a true meridian line, necessitates also some "overlapping" of the times, or of their colors on the map. Thus, most of the roads between Buffalo and Detroit, on the *north* side of Lake Erie, run by "eastern," or "red," time; while those on the *south* side of the Lake, between Buffalo and Toledo, immediately opposite to and directly south of them, run by "central" or "blue" time.

If the changes of time were made at the meridians midway between the standard ones, it would not be necessary for any town to change its time more than 30 minutes. As it is, somewhat greater changes had to be made at a few points. Thus, standard time at Detroit is 32 minutes ahead, and at Savannah 36 minutes back, of mean local time.

In most cases the necessary change was made upon the railways by simply setting clocks and watches ahead or back the necessary number of minutes, and without making any change in time-tables.

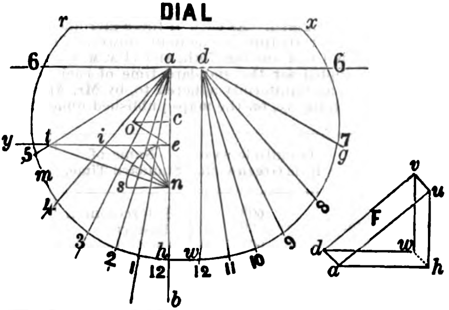
Halifax, and a few adjacent cities, use the time of the 60th meridian, that being the nearest one to them; but the *railroads* in the same district have adopted the 75th meridian, or eastern, time; so that, for railroad purposes, intercolonial time has never come into force.

In 1873 there were 71 time standards in use on the railroads of the United States and Canada. At the time of the adoption of the present system this number had been reduced, by consolidation of roads, &c, to 53. By its adoption, the number became 5, or, practically, 4, owing to the adoption of eastern time by the intercolonial roads, as already explained.

DIALLING.

To make a horizontal Sun-dial,

Draw a line *a b*; and at right angles to it, draw 66. From any convenient point, as *c*, in *a b*, draw the perp *c o*. Make the angle *c a o* equal to the lat of the place; also the angle *c o e* equal to the same; join *o e*. Make *e n* equal to *o e*; and from *n* as a center, with the rad *e n*, describe a quadrant *e s*; and div it into 6 equal parts. Draw *e y*, parallel to 6, 6; and from *n*, through the 5 points on the quadrant, draw lines *n t*, *n i*, &c, terminating in *e y*. From *a* draw lines *a* 5, *a* 4, &c, passing through *t*, *i*, &c. From any convenient point, as *c*, describe an arc *r m h*, as a kind of finish or border to half the dial. All the lines may now be effaced, except the hour lines *a* 6, *a* 5, *a* 4, &c, to *a* 12, or *a h*; unless, as is generally the case, the dial is to be divided to quarters of an hour at least. In this case each of the divisions on the quadrant *e s*, must be subdivided into 4 equal parts; and lines drawn from *n*, through the points of subdivision, terminating in *e y*. The quarter-hour lines must be drawn from *a*, as were the hour lines. Subdivisions of 5 min may be made in the same way; but these, as well as single min, may usually be laid off around the border, by eye. About 8 or 10 times the size of our Fig will be a convenient one for an ordinary dial. To draw the other half of the Fig, make *a d* equal to the intended thickness of the gnomon, or style, of the dial; and draw *d* 12, parallel, and equal to *a* 12; and draw the arc *x g w*, precisely similar to the arc *r m h*. Between *x* and *w*, on the arc *x g w*, space off divisions equal to those on the arc *r m h*; and number them for the hours, as in the Fig. The style F, of metal or stone, (wood is too liable to warp,) will be triangular; its thickness must throughout be equal to *a d* or *h w*; its base must cover the space *a d h w*; its point will be at *a d*; and its perp height *h u*, over *h w*, must be such that lines *v d*, *u a*, drawn from its top, down to *a* and *d*, will make the angles *u a h*, *v d w*, each equal to the lat of the place. Its thickness, if of metal, may conveniently be from ⅛ to ¼ inch; or if of stone, an inch or two, or more, according to the size of the dial. Usually, for neatness of appearance, the back *h u v w* of the style is hollowed inward. The upper edges, *u a*, *v d*, which cast the shadows, must be sharp and straight. The dial must be fixed in place hor, or perfectly level; *a h* and *d w* must be placed truly north and south; *a d* being south, and *h w* north. The dial gives only sun or solar time; but clock time can be found by means of the "fast or slow of the sun," as given by all almanacs. If by the almanac the sun is 5 min, &c, fast, the dial will be the same; and the clock or watch, to be correct, must be 5 min slower than it; and vice versa.



To make a Vertical Sun-Dial.

Proceed as directed above, except that the angles *c a o* and *c o e* on the drawing, and the angle *u a h* or *v d w* of the style, must be equal to the *co*-latitude (= difference between the latitude and 90°) of the place, and the hours must be numbered the opposite way from those in the above figure; *i e*, from *h* to *y* number 12, 11, 10, 9, 8, 7; and from *w* to *g* number 12, 1, 2, 3, 4, 5. The dial plate must be placed vertically, in the position shown in the figure, facing exactly south, and with *a h* and *d w* vertical.

BOARD MEASURE.

Remark on following table. The table extends to 12 ins by 24 ins, but it is easy to find for greater sizes; thus, for example, the board measure in a piece of 19 by 22, will be twice that of a piece of 19 by 11, or 17.42 × 2 = 34.84 ft board meas; or that of 19¼ by 22, will be that of 10¼ by 22 added to that of 9 by 22, or 18.79 + 16.50 = 35.29. A foot of board meas is equal to 1 foot square and 1 inch thick, or to 144 cub ins. Hence 1 cub ft = 12 ft board meas.

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)
1000 ft board measure = 83¼ cub ft.

THICKNESS IN INCHES.

Width in Inches.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	Width in Inches.
	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	
¼	.0208	.0260	.0313	.0365	.0417	.0469	.0521	.0573	.0625	¼
½	.0417	.0521	.0625	.0729	.0833	.0938	.1042	.1146	.1250	½
¾	.0625	.0781	.0938	.1094	.1250	.1406	.1563	.1719	.1875	¾
1.	.0833	.1042	.1250	.1458	.1667	.1875	.2083	.2292	.2500	1.
¼	.1042	.1302	.1563	.1823	.2083	.2344	.2604	.2865	.3125	¼
½	.1250	.1563	.1875	.2188	.2500	.2813	.3125	.3438	.3750	½
¾	.1458	.1823	.2187	.2552	.2917	.3281	.3646	.4010	.4375	¾
2.	.1667	.2083	.2500	.2917	.3333	.3750	.4166	.4583	.5000	2.
¼	.1875	.2344	.2813	.3281	.3750	.4219	.4688	.5156	.5625	¼
½	.2083	.2604	.3125	.3646	.4167	.4688	.5208	.5729	.6250	½
¾	.2292	.2865	.3438	.4010	.4583	.5156	.5729	.6302	.6875	¾
3.	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	3.
¼	.2708	.3385	.4063	.4739	.5416	.6094	.6771	.7448	.8125	¼
½	.2917	.3646	.4375	.5104	.5833	.6563	.7292	.8021	.8750	½
¾	.3125	.3906	.4689	.5469	.6250	.7031	.7813	.8594	.9375	¾
4.	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1.000	4.
¼	.3542	.4427	.5312	.6198	.7083	.7969	.8854	.9740	1.063	¼
½	.3750	.4688	.5625	.6563	.7500	.8438	.9375	1.031	1.125	½
¾	.3958	.4948	.5938	.6927	.7917	.8906	.9896	1.086	1.188	¾
5.	.4167	.5208	.6250	.7292	.8333	.9375	1.042	1.146	1.250	5.
¼	.4375	.5469	.6563	.7656	.8750	.9844	1.094	1.203	1.313	¼
½	.4583	.5729	.6875	.8020	.9167	1.031	1.146	1.260	1.375	½
¾	.4792	.5990	.7188	.8385	.9583	1.078	1.198	1.318	1.438	¾
6.	.5000	.6250	.7500	.8750	1.000	1.125	1.250	1.375	1.500	6.
¼	.5208	.6510	.7813	.9115	1.042	1.172	1.302	1.432	1.563	¼
½	.5417	.6771	.8125	.9479	1.083	1.219	1.354	1.490	1.625	½
¾	.5625	.7031	.8438	.9844	1.125	1.266	1.406	1.547	1.688	¾
7.	.5833	.7292	.8750	1.021	1.167	1.312	1.458	1.604	1.750	7.
¼	.6042	.7552	.9063	1.057	1.208	1.359	1.510	1.661	1.813	¼
½	.6250	.7813	.9375	1.094	1.250	1.406	1.563	1.719	1.875	½
¾	.6458	.8073	.9688	1.130	1.292	1.453	1.615	1.776	1.938	¾
8.	.6667	.8333	1.000	1.167	1.333	1.500	1.667	1.833	2.000	8.
¼	.6875	.8594	1.031	1.203	1.375	1.547	1.719	1.891	2.063	¼
½	.7083	.8854	1.063	1.240	1.417	1.594	1.771	1.948	2.125	½
¾	.7292	.9114	1.094	1.276	1.458	1.641	1.823	2.005	2.188	¾
9.	.7500	.9375	1.125	1.313	1.500	1.688	1.875	2.062	2.250	9.
¼	.7708	.9635	1.156	1.349	1.542	1.734	1.927	2.120	2.313	¼
½	.7917	.9895	1.188	1.385	1.583	1.781	1.979	2.177	2.375	½
¾	.8125	1.016	1.219	1.422	1.625	1.828	2.031	2.234	2.438	¾
10.	.8333	1.042	1.250	1.458	1.667	1.875	2.083	2.292	2.500	10.
¼	.8542	1.068	1.281	1.495	1.708	1.922	2.135	2.349	2.563	¼
½	.8750	1.094	1.313	1.531	1.750	1.969	2.188	2.406	2.625	½
¾	.8958	1.120	1.344	1.568	1.792	2.016	2.240	2.463	2.688	¾
11.	.9167	1.146	1.375	1.604	1.833	2.063	2.292	2.521	2.750	11.
¼	.9375	1.172	1.406	1.641	1.875	2.109	2.344	2.578	2.813	¼
½	.9583	1.198	1.438	1.677	1.917	2.156	2.396	2.635	2.875	½
¾	.9792	1.224	1.469	1.714	1 958	2.203	2.448	2.693	2.938	¾
12.	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	12.
½	1.042	1.302	1.563	1.823	2.083	2.344	2.604	2.865	3.125	½
13.	1.083	1.354	1.625	1.896	2.167	2.438	2.708	2.979	3.250	13.
½	1.125	1.406	1.688	1.969	2.250	2.531	2.813	3.094	3.375	½
14.	1.167	1.458	1.750	2.042	2 333	2.625	2.917	3.208	3.500	14.
½	1.208	1.510	1.813	2.115	2.417	2.719	3.021	3.322	3.625	½
15.	1.250	1.563	1.875	2.188	2.500	2.813	3.125	3.438	3.750	15.
½	1.292	1.615	1.938	2.260	2.583	2.906	3.229	3.552	3.875	½
16.	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	16.
½	1.375	1.719	2.063	2.406	2.750	3.094	3.438	3.781	4.125	½
17.	1.417	1.771	2.125	2.479	2.833	3.188	3.542	3.896	4.250	17.
½	1.458	1.823	2.187	2.552	2.917	3.281	3.646	4.010	4.375	½
18.	1.500	1.875	2.250	2.625	3.000	3.375	3.750	4.125	4.500	18.
19.	1.583	1.979	2.375	2.771	3.167	3.563	3.958	4.354	4.750	19.
20.	1.667	2.083	2.500	2.917	3.333	3.750	4.167	4.583	5.000	20.
21.	1.750	2.188	2.625	3.063	3.500	3.938	4.375	4 812	5.250	21.
22.	1.833	2.292	2.750	3.208	3.667	4.125	4.583	5.042	5.500	22.
23.	1.917	2.396	2.875	3.354	3.833	4.313	4.792	5.270	5.750	23.
24.	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)									Width in Inches.
	THICKNESS IN INCHES.									
	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M	Ft.Bd.M.	
¼	.0677	.0729	.0781	.0833	.0885	.0938	.0990	.1042	.1094	¼
½	.1354	.1457	.1562	.1667	.1770	.1875	.1979	.2083	.2188	½
¾	.2031	.2187	.2344	.2500	.2656	.2813	.2969	.3125	.3281	¾
1.	.2708	.2917	.3125	.3333	.3542	.3750	.3958	.4167	.4375	1.
¼	.3385	.3646	.3906	.4167	.4427	.4688	.4948	.5208	.5469	¼
½	.4063	.4375	.4688	.5000	.5313	.5625	.5938	.6250	.6563	½
¾	.4740	.5104	.5469	.5833	.6198	.6563	.6927	.7292	.7656	¾
2.	.5417	.5833	.6250	.6667	.7083	.7500	.7917	.8333	.8750	2.
¼	.6094	.6563	.7031	.7500	.7969	.8438	.8906	.9375	.9844	¼
½	.6771	.7292	.7813	.8333	.8854	.9375	.9896	1.042	1.094	½
¾	.7448	.8021	.8594	.9167	.9740	1.031	1.089	1.146	1.203	¾
3.	.8125	.8750	.9375	1.000	1.062	1.125	1.188	1.250	1.313	3.
¼	.8802	.9479	1.016	1.083	1.151	1.219	1.286	1.354	1.422	¼
½	.9479	1.021	1.094	1.167	1.240	1.313	1.385	1.458	1.531	½
¾	1.016	1.094	1.172	1.250	1.327	1.406	1.484	1.563	1.641	¾
4.	1.083	1.167	1.250	1.333	1.416	1.500	1.583	1.667	1.750	4.
¼	1.151	1.240	1.328	1.417	1.504	1.594	1.682	1.771	1.859	¼
½	1.219	1.313	1.406	1.500	1.593	1.688	1.781	1.875	1.969	½
¾	1.286	1.384	1.484	1.583	1.681	1.781	1.880	1.979	2.078	¾
5.	1.354	1.457	1.566	1.666	1.770	1.875	1.979	2.083	2.188	5.
¼	1.422	1.530	1.644	1.750	1.858	1.969	2.078	2.188	2.297	¼
½	1.490	1.603	1.722	1.833	1.947	2.063	2.177	2.292	2.406	½
¾	1.557	1.676	1.800	1.917	2.035	2.156	2.276	2.396	2.516	¾
6.	1.625	1.750	1.875	2.000	2.125	2.250	2.375	2.500	2.625	6.
¼	1.693	1.823	1.953	2.083	2.214	2.344	2.474	2.604	2.734	¼
½	1.760	1.896	2.031	2.167	2.302	2.438	2.573	2.708	2.843	½
¾	1.828	1.969	2.109	2.250	2.391	2.531	2.672	2.813	2.953	¾
7.	1.896	2.042	2.188	2.333	2.479	2.625	2.771	2.917	3.063	7.
¼	1.964	2.115	2.266	2.416	2.568	2.719	2.870	3.021	3.172	¼
½	2.031	2.187	2.344	2.500	2.656	2.813	2.969	3.125	3.281	½
¾	2.099	2.260	2.422	2.583	2.745	2.906	3.068	3.229	3.391	¾
8.	2.167	2.333	2.500	2.667	2.833	3.000	3.167	3.333	3.500	8.
¼	2.234	2.406	2.578	2.750	2.922	3.094	3.266	3.438	3.609	¼
½	2.302	2.479	2.656	2.833	3.010	3.188	3.365	3.542	3.718	½
¾	2.370	2.552	2.734	2.916	3.099	3.281	3.464	3.646	3.828	¾
9.	2.438	2.625	2.813	3.000	3.187	3.375	3.563	3.750	3.938	9.
¼	2.505	2.698	2.891	3.083	3.276	3.469	3.661	3.854	4.047	¼
½	2.573	2.771	2.969	3.167	3.365	3.563	3.760	3.958	4.156	½
¾	2.641	2.844	3.047	3.250	3.453	3.656	3.859	4.063	4.266	¾
10.	2.708	2.917	3.125	3.333	3.542	3.750	3.958	4.167	4.375	10.
¼	2.776	2.990	3.203	3.416	3.630	3.844	4.057	4.271	4.484	¼
½	2.844	3.063	3.281	3.500	3.719	3.938	4.156	4.375	4.594	½
¾	2.911	3.135	3.359	3.583	3.807	4.031	4.255	4.479	4.703	¾
11.	2.979	3.208	3.438	3.666	3.896	4.125	4.354	4.583	4.813	11.
¼	3.047	3.281	3.516	3.750	3.984	4.219	4.453	4.688	4.922	¼
½	3.115	3.354	3.594	3.833	4.073	4.313	4.552	4.792	5.031	½
¾	3.182	3.427	3.672	3.916	4.161	4.406	4.651	4.896	5.141	¾
12.	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000	5.250	12.
½	3.385	3.646	3.906	4.167	4.427	4.688	4.948	5.208	5.469	½
13.	3.521	3.792	4.063	4.333	4.604	4.875	5.146	5.417	5.688	13.
½	3.656	3.938	4.219	4.500	4.781	5.063	5.344	5.625	5.906	½
14.	3.792	4.083	4.375	4.667	4.958	5.250	5.542	5.833	6.125	14.
½	3.927	4.229	4.531	4.833	5.135	5.438	5.740	6.042	6.344	½
15.	4.063	4.375	4.688	5.000	5.313	5.625	5.938	6.250	6.563	15.
½	4.198	4.521	4.844	5.166	5.490	5.818	6.135	6.458	6.781	½
16.	4.333	4.667	5.000	5.333	5.667	6.000	6.333	6.667	7.000	16.
½.	4.469	4.813	5.156	5.500	5.844	6.188	6.531	6.875	7.219	½
17.	4.604	4.958	5.313	5.667	6.021	6.375	6.729	7.083	7.438	17.
½	4.740	5.104	5.469	5.833	6.198	6.563	6.927	7.292	7.656	½
18.	4.875	5.250	5.625	6.000	6.375	6.750	7.125	7.500	7.875	18.
19.	5.146	5.542	5.938	6.383	6.729	7.125	7.521	7.917	8.313	19.
20.	5.417	5.833	6.250	6.667	7.083	7.500	7.917	8.333	8.750	20.
21.	5.688	6.125	6.563	7.000	7.438	7.875	8.313	8.750	9.188	21.
22.	5.958	6.417	6.875	7.333	7.792	8.250	8.708	9.167	9.625	22.
23.	6.229	6.708	7.188	7.667	8.145	8.625	9.104	9.583	10.06	23.
24.	6.500	7.000	7.500	8.000	8.500	9.000	9.500	10.00	10.50	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)									Width in Inches.
	THICKNESS IN INCHES.									
	5½	5¾	6	6¼	6½	6¾	7	7¼	7½	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	
¼	.1146	.1198	.1250	.1302	.1354	.1406	.1458	.1510	.1563	¼
½	.2292	.2396	.2500	.2604	.2708	.2813	.2917	.3021	.3125	½
¾	.3438	.3594	.3750	.3906	.4063	.4219	.4375	.4531	.4688	¾
1.	.4583	.4792	.5000	.5208	.5417	.5625	.5833	.6042	.6250	1.
¼	.5729	.5990	.6250	.6510	.6771	.7031	.7292	.7552	.7813	¼
½	.6875	.7188	.7500	.7812	.8125	.8438	.8750	.9062	.9375	½
¾	.8021	.8385	.8750	.9115	.9479	.9844	1.020	1.057	1.094	¾
2.	.9167	.9583	1.000	1.042	1.083	1.125	1.167	1.208	1.250	2.
¼	1.031	1.078	1.125	1.172	1.219	1.266	1.313	1.359	1.406	¼
½	1.146	1.198	1.250	1.302	1.354	1.406	1.458	1.510	1.563	½
¾	1.260	1.318	1.375	1.432	1.490	1.547	1.604	1.661	1.719	¾
3.	1.375	1.438	1.500	1.562	1.625	1.688	1.750	1.813	1.875	3.
¼	1.490	1.557	1.625	1.693	1.760	1.828	1.896	1.964	2.031	¼
½	1.604	1.677	1.750	1.823	1.896	1.969	2.042	2.115	2.188	½
¾	1.719	1.797	1.875	1.953	2.031	2.109	2.188	2.266	2.344	¾
4.	1.833	1.917	2.000	2.083	2.167	2.250	2.333	2.417	2 500	4.
¼	1.948	2.036	2.125	2.214	2.302	2.391	2.479	2.568	2.656	¼
½	2.063	2.156	2.250	2.344	2.438	2.531	2.625	2.719	2.813	½
¾	2.177	2.276	2.375	2.474	2.573	2.672	2.771	2.870	2.969	¾
5.	2.292	2.396	2.500	2.604	2.708	2.813	2.917	3.021	3.125	5.
¼	2.406	2.516	2.625	2.734	2 844	2.953	3.063	3.172	3.281	¼
½	2.521	2.635	2.750	2.865	2.979	3.094	3.208	3.323	3.438	½
¾	2.635	2.755	2.875	2.995	3.115	3.234	3.354	3.474	3.594	¾
6.	2.750	2.875	3.000	3.125	3.250	3.375	3.500	3.625	3.750	6.
¼	2.865	2.995	3.125	3.255	3.385	3.516	3.646	3.776	3.906	¼
½	2.979	3.115	3.250	3.385	3.521	3.656	3.792	3.927	4.063	½
¾	3.094	3.234	3.375	3.516	3.656	3.797	3.938	4.078	4.219	¾
7.	3.208	3.354	3.500	3.646	3.792	3.938	4.083	4.229	4.375	7.
¼	3.323	3.474	3.625	3.776	3.927	4.078	4.229	4.380	4.531	¼
½	3.438	3.594	3.750	3.906	4.063	4.219	4.375	4.531	4.688	½
¾	3.552	3.714	3.875	4.036	4.198	4.359	4.521	4.682	4.844	¾
8.	3.667	3.833	4.000	4.167	4.333	4.500	4.667	4.833	5.000	8.
¼	3.781	3.953	4.125	4.297	4.469	4.641	4.813	4.984	5.156	¼
½	3.896	4.073	4.250	4.427	4.604	4.781	4.957	5.135	5.313	½
¾	4.010	4.193	4.375	4.557	4.740	4.922	5.103	5.286	5.469	¾
9.	4.125	4.313	4.500	4.687	4.875	5.063	5.249	5.438	5.625	9.
¼	4.240	4.432	4.625	4.818	5.010	5.203	5.395	5.589	5.781	¼
½	4.354	4.552	4.750	4.948	5.146	5.344	5.541	5.740	5.938	½
¾	4.469	4.672	4.875	5.078	5.281	5.484	5.687	5.891	6.094	¾
10.	4.583	4.792	5.000	5.208	5.417	5.625	5.833	6.042	6.250	10.
¼	4.698	4.911	5.125	5.339	5.552	5.766	5.979	6.193	6.406	¼
½	4.813	5.031	5.250	5.469	5.688	5.906	6.125	6.344	6.563	½
¾	4.927	5.151	5.375	5.599	5.823	6.047	6.271	6.495	6.719	¾
11.	5.042	5.271	5 500	5.729	5.958	6.188	6.417	6.646	6.876	11.
¼	5.156	5.391	5.625	5.859	6.094	6.328	6.563	6.797	7.031	¼
½	5.271	5.510	5.750	5.990	6.229	6.469	6.708	6.948	7.188	½
¾	5.385	5.630	5.875	6.120	6.365	6.609	6.854	7.099	7.344	¾
12.	5.500	5.750	6.000	6.250	6.500	6.750	7.000	7.250	7.500	12.
½	5.729	5.990	6.250	6.510	6.771	7.031	7.292	7.552	7.813	½
13.	5.958	6.229	6.500	6.771	7.042	7.313	7.583	7.854	8.125	13.
½	6.188	6.469	6.750	7.031	7.313	7.594	7.875	8.156	8.438	½
14.	6.417	6.708	7.000	7.292	7.583	7.875	8.167	8.458	8.750	14.
½	6.646	6.948	7.250	7.552	7.854	8.156	8.458	8.760	9.063	½
15.	6.875	7.188	7.500	7.812	8.125	8.438	8.750	9.063	9.375	15.
½	7.104	7.427	7.750	8.073	8.396	8.719	9.042	9.365	9.688	½
16.	7.333	7.667	8.000	8.333	8.667	9.000	9.333	9.667	10.00	16.
½	7.563	7.906	8.250	8.594	8.938	9.281	9.625	9.969	10.31	½
17.	7.792	8.146	8.500	8.854	9.208	9.563	9.917	10.27	10.63	17.
½	8.021	8.385	8.750	9.115	9.479	9.844	10.21	10.57	10.94	½
18.	8.250	8.625	9.000	9.375	9.750	10.13	10.50	10.88	11.25	18.
19.	8.708	9.104	9.500	9.896	10.29	10.69	11.08	11.48	11.88	19.
20.	9.167	9.583	10.00	10.42	10.83	11.25	11.67	12.08	12.50	20.
21.	9.625	10.06	10.50	10.94	11.38	11.81	12.25	12.69	13.13	21.
22.	10.08	10.54	11.00	11.46	11.92	12.38	12.83	13.29	13.75	22.
23.	10.54	11.02	11.50	11.98	12.46	12.94	13.42	13.90	14.38	23.
24.	11.00	11.50	12.00	12.50	13.00	13.50	14.00	14.50	15.00	24.

Table of Board Measure — (Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

THICKNESS IN INCHES.

Width in Inches.	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	Width in Inches.
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	
¼	.1615	.1667	.1719	.1771	.1823	.1875	.1927	.1979	.2031	¼
½	.3229	.3333	.3438	.3542	.3646	.3750	.3854	.3958	.4063	½
¾	.4844	.5000	.5156	.5313	.5467	.5625	.5781	.5938	.6094	¾
1.	.6458	.6667	.6875	.7083	.7292	.7500	.7708	.7917	.8125	1.
¼	.8073	.8333	.8594	.8854	.9115	.9375	.9635	.9896	1.016	¼
½	.9688	1.000	1.031	1.063	1.094	1.125	1.156	1.188	1.219	½
¾	1.130	1.167	1.203	1.240	1.276	1.313	1.349	1.385	1.422	¾
2.	1.292	1.333	1.375	1.417	1.458	1.500	1.542	1.583	1.625	2.
¼	1.453	1.500	1.547	1.594	1.641	1.688	1.734	1.781	1.828	¼
½	1.615	1.667	1.719	1.771	1.822	1.875	1.927	1.979	2.031	½
¾	1.776	1.833	1.891	1.948	2.005	2.063	2.120	2.177	2.234	¾
3.	1.938	2.000	2.063	2.125	2.188	2.250	2.313	2.375	2.438	3.
¼	2.099	2.167	2.234	2.302	2.370	2.438	2.505	2.573	2.641	¼
½	2.260	2.333	2.406	2.479	2.552	2.625	2.698	2.771	2.844	½
¾	2.422	2.500	2.578	2.656	2.734	2.813	2.891	2.969	3.047	¾
4.	2.583	2.667	2.750	2.833	2.917	3.000	3.083	3.167	3.250	4.
¼	2.745	2.833	2.922	3.010	3.099	3.188	3.276	3.365	3.453	¼
½	2.906	3.000	3.094	3.188	3.281	3.375	3.469	3.563	3.656	½
¾	3.068	3.167	3.266	3.365	3.464	3.563	3.661	3.760	3.859	¾
5.	3.229	3.333	3.438	3.542	3.646	3.750	3.854	3.958	4.063	5.
¼	3.391	3.500	3.609	3.719	3.828	3.938	4.047	4.156	4.266	¼
½	3.552	3.667	3.781	3.896	4.010	4.125	4.240	4.354	4.469	½
¾	3.714	3.833	3.953	4.073	4.193	4.313	4.432	4.552	4.672	¾
6.	3.875	4.000	4.125	4.250	4.375	4.500	4.625	4.750	4.875	6.
¼	4.036	4.167	4.297	4.427	4.557	4.688	4.818	4.948	5.078	¼
½	4.198	4.333	4.469	4.604	4.740	4.875	5.010	5.146	5.281	½
¾	4.359	4.500	4.641	4.781	4.922	5.063	5.203	5.344	5.484	¾
7.	4.521	4.667	4.813	4.958	5.104	5.250	5.360	5.542	5.688	7.
¼	4.682	4.833	4.984	5.135	5.286	5.438	5.590	5.740	5.891	¼
½	4.844	5.000	5.156	5.313	5.469	5.625	5.782	5.938	6.094	½
¾	5.005	5.167	5.328	5.490	5.651	5.813	5.975	6.135	6.297	¾
8.	5.167	5.333	5.500	5.667	5.833	6.000	6.167	6.333	6.500	8.
¼	5.328	5.500	5.672	5.844	6.016	6.188	6.359	6.531	6.703	¼
½	5.490	5.667	5.844	6.021	6.198	6.375	6.552	6.729	6.906	½
¾	5.651	5.833	6.016	6.198	6.380	6.563	6.745	6.927	7.109	¾
9.	5.813	6.000	6.188	6.375	6 563	6.750	6.938	7.125	7.313	9.
¼	5.974	6.167	6.359	6.552	6.745	6.938	7.130	7.323	7.516	¼
½	6.135	6.333	6.531	6.729	6.927	7.125	7.323	7.521	7.719	½
¾	6.297	6.500	6.703	6.906	7.109	7.313	7.516	7.719	7.922	¾
10.	6.458	6.667	6.875	7.083	7.292	7.500	7.708	7.917	8.125	10.
¼	6.620	6.833	7.047	7.260	7.474	7.688	7.901	8.115	8.328	¼
½	6.781	7.000	7.219	7.438	7.656	7.875	8.094	8.313	8.531	½
¾	6.943	7.167	7.391	7.615	7.839	8.063	8.286	8.510	8.734	¾
11.	7.104	7.333	7.563	7.792	8.021	8.250	8.479	8.708	8.938	11.
¼	7.266	7.500	7.735	7.969	8.203	8.438	8.672	8.906	9.141	¼
½	7.427	7.667	7.906	8.146	8.386	8.625	8.865	9.104	9.344	½
¾	7.589	7.833	8.078	8.323	8.568	8.813	9.057	9.302	9.547	¾
12.	7.750	8.000	8.250	8.500	8.750	9.000	9.250	9.500	9.750	12.
½	8.073	8.333	8.594	8.854	9.115	9.375	9.635	9.896	10.16	½
13.	8.396	8.666	8.938	9.208	9.479	9.750	10.02	10.29	10.56	13.
½	8.719	9.000	9.281	9.563	9.844	10.13	10.41	10.69	10.97	½
14.	9.042	9.333	9.625	9.917	10.21	10.50	10.79	11.08	11.38	14.
½	9.365	9.666	9.969	10.27	10.57	10.88	11.18	11.48	11.78	½
15.	9.688	10.000	10.31	10.63	10.94	11.25	11.56	11.88	12.19	15.
½	10.01	10.33	10.66	10.98	11.30	11.63	11.95	12.27	12.59	½
16.	10.33	10.67	11.00	11.33	11.67	12.00	12.33	12.67	13.00	16.
½	10.66	11.00	11.34	11.69	12.03	12.38	12.72	13.06	13.41	½
17.	10.98	11.33	11.69	12.04	12.40	12.75	13.10	13.46	13.81	17.
½	11.30	11.66	12.03	12.40	12.76	13.13	13.49	13.85	14.22	½
18.	11.63	12.00	12.38	12.75	13.13	13.50	13.88	14.25	14.63	18.
19.	12.27	12.67	13.06	13.46	13.85	14.25	14.65	15.04	15.44	19.
20.	12.92	13.33	13.75	14.17	14.58	15.00	15.42	15.83	16.25	20.
21.	13.56	14.00	14.44	14.88	15.31	15.75	16.19	16.63	17.06	21.
22.	14.21	14.66	15.13	15.58	16.04	16.50	16.96	17.42	17.88	22.
23.	14.85	15.33	15.81	16.29	16.77	17.25	17.73	18.21	18.69	23.
24.	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	24.

Table of Board Measure—(Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	THICKNESS IN INCHES.									Width in Inches.
	10	10¼	10½	10¾	11	11¼	11½	11¾	12	
	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	
¼	.2083	.2135	.2188	.2240	.2292	.2344	.2396	.2448	.2500	¼
½	.4167	.4271	.4375	.4479	.4583	.4688	.4792	.4896	.5000	½
¾	.6250	.6406	.6563	.6719	.6875	.7031	.7188	.7344	.7500	¾
1.	.8333	.8542	.8750	.8958	.9167	.9375	.9583	.9792	1.000	1.
¼	1.042	1.068	1.094	1.120	1.146	1.172	1.198	1.224	1.250	¼
½	1.250	1.821	1.313	1.344	1.375	1.406	1.438	1.469	1.500	½
¾	1.458	1.495	1.531	1.568	1.604	1.641	1.677	1.714	1.750	¾
2.	1.667	1.708	1.750	1.792	1.833	1.875	1.917	1.958	2.000	2.
¼	1.875	1.922	1.969	2.016	2.063	2.109	2.156	2.203	2.250	¼
½	2.083	2.135	2.188	2.240	2.292	2.344	2.396	2.448	2.500	½
¾	2.292	2.349	2.406	2.464	2.521	2.578	2.635	2.693	2.750	¾
3.	2.500	2.563	2.625	2.688	2.750	2.813	2.875	2.938	3.000	3.
¼	2.708	2.776	2.844	2.911	2.979	3.047	3.115	3.182	3.250	¼
½	2.917	2.990	3.063	3.135	3.208	3.281	3.354	3.427	3.500	½
¾	3.125	3.203	3.281	3.359	3.438	3.516	3.594	3.672	3.750	¾
4.	3.333	3.417	3.500	3.583	3.667	3.750	3.833	3.917	4.000	4.
¼	3.542	3.630	3.719	3.807	3.896	3.984	4.073	4.161	4.250	¼
½	3.750	3.844	3.938	4.031	4.125	4.219	4.313	4.406	4.500	½
¾	3.958	4.057	4.156	4.255	4.354	4.453	4.552	4.651	4.750	¾
5.	4.167	4.271	4.375	4.479	4.583	4.688	4.791	4.896	5.000	5.
¼	4.375	4.484	4.594	4.703	4.813	4.922	5.031	5.141	5.250	¼
½	4.583	4.698	4.813	4.927	5.042	5.156	5.270	5.385	5.500	½
¾	4.792	4.911	5.031	5.151	5.271	5.391	5.510	5.630	5.750	¾
6.	5.000	5.125	5.250	5.375	5.500	5.625	5.750	5.875	6.000	6.
¼	5.208	5.339	5.469	5.599	5.729	5.859	5.990	6.120	6.250	¼
½	5.417	5.552	5.688	5.823	5.958	6.094	6.229	6.365	6.500	½
¾	5.625	5.766	5.906	6.047	6.188	6.328	6.469	6.609	6.750	¾
7.	5.833	5.979	6.125	6.271	6.417	6.563	6.708	6.854	7.000	7.
¼	6.042	6.193	6.344	6.495	6.646	6.797	6.948	7.099	7.250	¼
½	6.250	6.406	6.563	6.719	6.875	7.031	7.188	7.344	7.500	½
¾	6.458	6 620	6.781	6.943	7.104	7.266	7.427	7.589	7.750	¾
8.	6.667	6.833	7.000	7.167	7.333	7.500	7.667	7.833	8.000	8.
¼	6.875	7.047	7.219	7.391	7.563	7.734	7.906	8.078	8.250	¼
½	7.083	7.260	7.438	7.615	7.792	7.969	8.146	8.323	8.500	½
¾	7.292	7.474	7.656	7.839	8.021	8.203	8.385	8.568	8.750	¾
9.	7.500	7.688	7.875	8.063	8.250	8.438	8.625	8.813	9.000	9.
¼	7.708	7.901	8.094	8.286	8.479	8.672	8.865	9.057	9.250	¼
½	7.917	8.115	8.313	8.510	8.709	8.906	9.104	9.302	9.500	½
¾	8.125	8.328	8.531	8.734	8.939	9.141	9.344	9.547	9.750	¾
10.	8.333	8.542	8.750	8.958	9.167	9.375	9.583	9.792	10.00	10.
¼	8.542	8.755	8.969	9.182	9.396	9.609	9.823	10.04	10.25	¼
½	8.750	8.969	9.188	9.406	9.625	9.844	10.06	10.28	10.50	½
¾	8.958	9.182	9.406	9.630	9.854	10.08	10.30	10.53	10.75	¾
11.	9.167	9.396	9.625	9.854	10.08	10 31	10.54	10.77	11.00	11.
¼	9.375	9.609	9.844	10 08	10.31	10.55	10.78	11.02	11.25	¼
½	9.583	9.823	10.06	10.30	10.54	10.78	11.02	11.26	11.50	½
¾	9.792	10.04	10.28	10.53	10.77	11.02	11.26	11.51	11.75	¾
12.	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.
½	10.42	10.68	10.94	11.20	11.46	11.72	11.98	12.24	12.50	½
13.	10.83	11.10	11.38	11.65	11.92	12.19	12.46	12.73	13.00	13.
½	11.25	11.53	11.81	12.09	12.38	12.66	12.94	13.22	13.50	½
14.	11.67	11.96	12.25	12.54	12.83	13.13	13.42	13.71	14.00	14.
½	12.08	12.39	12.69	12.99	13.29	13.59	13.90	14.20	14.50	½
15.	12.50	12.81	13.13	13.44	13.75	14.06	14.38	14.69	15.00	15.
½	12.92	13.24	13.56	13.89	14.21	14.53	14.85	15.18	15.50	½
16.	13.33	13.67	14.00	14.33	14.67	15.00	15.33	15.67	16.00	16.
½	13.75	14.09	14.44	14.78	15.13	15.47	15.81	16.16	16.50	½
17.	14.17	14.52	14.88	15.23	15.58	15.94	16.29	16.65	17.00	17.
½	14.58	14.95	15.31	15.77	16.04	16.41	16.77	17.14	17.50	½
18.	15.00	15.38	15.75	16.13	16.50	16.88	17.25	17.63	18.00	18.
19.	15.83	16.23	16.63	17.02	17.42	17.81	18.21	18.60	19.00	19.
20.	16.67	17.08	17.50	17.92	18.33	18.75	19.17	19.58	20.00	20.
21.	17.50	17.94	18.38	18.81	19.25	19.69	20.13	20.56	21.00	21.
22.	18.33	18.79	19.25	19.71	20.17	20.63	21.08	21.54	22.00	22.
23.	19.17	19.65	20.13	20.60	21.08	21.56	22.04	22.52	23.00	23.
24.	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.

LAND SURVEYING.

In surveying a tract of ground, the sides which compose its outline are designated by numbers in the order in which they occur. That end of each side which first presents itself in the course of the survey, may be called its *near* end; and the other its *far* end. The number of each side is placed at its far end. Thus, in Fig. 1, the survey being supposed to commence at the corner 6, and to follow the direction of the arrows, the first side is 6, 1; and its number is placed at its far end at 1; and so of the rest. Let N S be a meridian line, that is, a north and south line; and E W an east and west line. Then in any side which runs northwardly, whether northeast, as side 2; or northwest, as sides 5 and 1; or due north; the distance in a due north direction between its near end and its far end, is called its *northing;* thus, *a* 1 is the northing of side 1; 1 *b* the northing of side 2; 4 *c* of side 5. In like manner, if any side runs in a southwardly direction, whether southeastwardly, as side 3; or southwestwardly, as sides 4 and 6; or due south; the corresponding distance in a due south direction between its near end and its far end, is called its *southing;* thus, *d* 3 is the southing of side 3; 3 *e* of side 4; *f* 6 of side 6. Both northings and southings are included in the general term *Difference of Latitude* of a side; or, more commonly but erroneously, its *latitude.* The distance due east, or due west, between the near and the far end of any side, is in like manner called the *easting,* or *westing,* of that side, as the case may be; thus, 6 *a* is the westing of side 1; 5 *f* of side 6; *c* 5 of side 5; *e* 4 of side 4; and *b* 2 is the easting of side 2; 2 *d* of side 3. Both eastings and westings are included in the general term *Departure* of a side; implying that the side *departs* so far from a north or south direction. We may say that a side norths, wests, southeasts, &c. We shall call the northings, southings, &c. the Ns, Ss, Es, and Ws; the latitudes, lats; and the departures, deps.

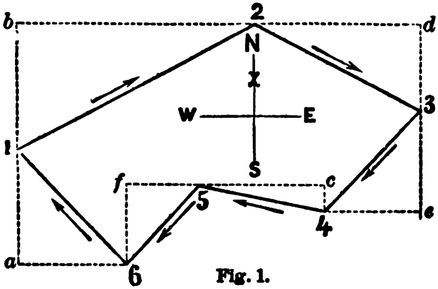


Fig. 1.

Perfect accuracy is unattainable in any operation involving the measurements of angles and distances.* That work is accurate enough, which cannot be made more so without an expenditure more than commensurate with the object to be gained. There is no great difficulty in confining the uncertainty within about one-half per cent. of the content, and this probably never prevents a transfer in farm transactions. But errors always become apparent when we come to *work out* the field notes; and since the map or plot of the survey, and the calculations for ascertaining the content, should be consistent within themselves, we do what is usually called *correcting* the errors, but what in fact is simply *humoring them in,* no matter how scientific the process may appear. We distribute them all around the survey. Two methods are used for this purpose, both based upon precisely the same principle; one by means of drawing; the other, more exact but much more troublesome, by calculation. The graphic method, in the hands of a correct draftsman, is sufficiently exact for all ordinary purposes. Add all the sides in feet together; and divide the sum by their number, for the average length. Divide this average by 8; the quotient will be the proper scale in feet per inch. In other words, take about 8 ins. to represent an average side. We shall take it for granted that an engineer does not consider it accurate work to measure his angles to the nearest quarter of a degree, which is the usual practice among land-surveyors. They can, by means of the engineer's transit, now in universal use on our public works, be readily measured within a minute or two; and being thus much more accurate than the compass courses, (which cannot be read off so closely, and which are moreover subject to many sources of error,) they serve to correct the latter in the office. The noting of the courses, however, should not be confined to the nearest quarters of a degree, but should be read as closely as the observer can guess at the minutes. The back courses also should be taken at every corner, as an additional check, and for the detection of local attraction. It is well in taking the compass bearings, to adopt as a rule, always to point the *north* of the compass-box toward the object whose bearing is to be taken, and to read off from the *north* end of the needle. A person who uses indifferently the N and the S of the box, and of the needle, will be very liable to make mistakes. It is best to measure the *least* angle (shown by dotted arcs, Fig 2,) at the corners; whether it be exterior, as that at corner 5; or interior, as all the others; because it is always less than 180°; so that there is less danger of reading it off incorrectly, than if it exceeded 180°; taking it for granted that the transit instrument is graduated from the same zero to 180° each way; if it is graduated from zero to 360° the precaution is useless. When the small angle is exterior, subtract it from 360° for the interior one.

---

* A 100 ft. chain may vary its length 5 feet per mile, between winter and summer, by mere change of temperature; and this alone will make a difference of about 1 acre in 533. The student should practise plotting from perfectly accurate data; as from the example in table. p. 281, or

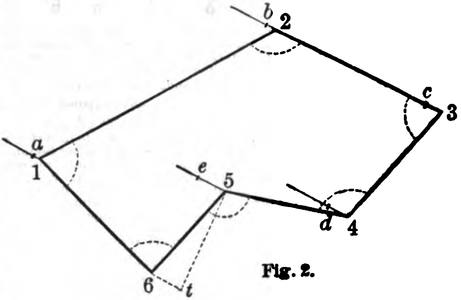


Fig. 2.

Supposing the field work to be finished, and that we require a plot from which the contents may be obtained mechanically, by dividing it into triangles, (the bases and heights of which may be measured by scale, and their areas calculated one by one,) a protraction of it may be made at once from the field notes, either by using the *angles*, or by first correcting the *bearings* by means of the angles, and then using them. The last is the best, because in the first the protractor must be moved to each angle; whereas in the last it will remain stationary while all the bearings are being pricked off. Every movement of it increases the liability to errors. The manner of correcting the bearings is explained on the next page.

In either case the protracted plot will certainly not close precisely; not only in consequence of errors in the field work, but also in the protracting itself. Thus the last side, No 6, Fig 2, instead of closing in at corner 6, will end somewhere else, say, for instance, at *t*; the dist *t* 6 being the *closing error*, which, however, as represented in Fig 2, is more than ten times as great, proportionally to the size of the survey, as would be allowable in practice. Now to humor-in this error, rule through every corner a short line parallel to *t* 6; and, in all cases, in the direction *from t* (wherever it may be) *to* the starting point 6. Add all the sides together; and measure *t* 6 by the scale of the plot. Then beginning at corner 1, at the far end of side 1, say, as the

Sum of all the sides : Total closing error *t* 6 :: Side 1 : Error for side 1.

Lay off this error from 1 to *a*. Then at corner 2, say, as the

Sum of all the sides : Total closing error *t* 6 :: Sum of sides 1 and 2 : Error for side 2.

Which error lay off from 2 to *b*; and so at each of the corners; always using, as the third term, the sum of the sides between the starting point and the given corner. Finally, join the points *a*, *b*, *c*, *d*, *e*, 6; and the plot is finished.

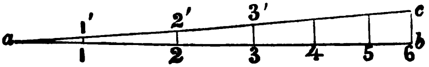
The correction has evidently changed the length of every side; lengthening some and shortening others. It has also changed the angles. The new lengths and angles may with tolerable accuracy be found by means of the scale and protractor; and be marked on the plot instead of the old ones.

---

from those to be found in books on surveying. This is the only way in which he can learn what is meant by accurate work. His semicircular protractor should be about 9 to 12 ins in diam, and graduated to 10 min. His straight edge and triangle should be of metal; we prefer German silver, which does not rust as steel does; and they should be made with *scrupulous accuracy* by a skilful instrument-maker. A very fine needle, with a sealing-wax head, should be used for pricking off dists and angles; it must be held vertically; and the eye of the draftsman must be directly over it. The lead pencil should be hard (Faber's No. 4 is good for protracting), and must be kept to a sharp point by rubbing on a fine file, after using a knife for removing the wood. The scale should be at least as long as the longest side of the plot, and should be made at the edge of a strip of the same paper as the plot is drawn on. This will obviate to a considerable extent, errors arising from contraction and expansion. Unfortunately, a sheet of paper does not contract and expand in the same proportion lengthwise and crosswise, thus preventing the paper scale from being a perfect corrective. In plots of common farm surveys, &c, however, the errors from this source may be neglected. For such plots as may be protracted, divided, and computed within a time too short to admit of appreciable change, the ordinary scales of wood, ivory, or metal may be used; but satisfactory accuracy cannot be obtained with them on plots requiring several days, if the air be meanwhile alternately moist and dry, or subject to considerable variations in temperature. What is called *parchment* paper is worse in this respect than good ordinary drawing-paper.

With the foregoing precautions we may work from a drawing, with as much accuracy as is usually attained in the field work.

When the plot has many sides, this calculating the error for each of them becomes tedious; and since, in a well-performed survey and protraction, the entire error will be but a very small quantity, (it should not *exceed* about $\frac{1}{400}$ part of the periphery,) it may usually be divided among the sides by merely placing about ¼, ½, and ¾ of it at corners about ¼, ½, and ¾ way around the plot; and at intermediate corners proportion it by eye. Or calculation may be avoided entirely by drawing a line *a b* of a length equal to the united lengths of all the sides; dividing it into distances *a*, 1; 1, 2; &c, equal to the respective sides. Make *b c* equal to the entire closing error; join *a c*; and draw 1, 1'; 2, 2'; &c, which will give the error at each corner.



When the plot is thus completed, it may be divided by fine pencil lines into triangles, whose bases and heights may be measured by the scale, in order to compute the contents. With care in both the survey and the drawing, the error should not *exceed* about $\frac{1}{200}$ part of the true area. At least two distinct sets of triangles should be drawn and computed, as a guard against mistakes; and if the two sets differ in calculated contents more than about $\frac{1}{200}$ part, they have not been as carefully prepared as they should have been. The closing error due to imperfect field-work, may be accurately calculated, as we shall show, and laid down on the paper before beginning the plot; thus furnishing a perfect test of the accuracy of the protraction work, which, if correctly done, will not close at the point of beginning, but at the point which indicates the error. But this *calculation* of the error, by a little additional trouble, furnishes data also for dividing it by calculation among the diff sides; besides the means of drawing the plot *correctly* at once, without the use of a protractor; thus enabling us to make the subsequent measurements and computations of the triangles with more certainty.

We shall now describe this process, but would recommend that even when it is employed, and especially in complicated surveys, a rough plot should first be made and corrected, by the first of the two mechanical methods already alluded to. It will prove to be of great service in using the method by calculation, inasmuch as it furnishes an eye check to vexatious mistakes which are otherwise apt to occur: for, although the principles involved are extremely simple, and easily remembered when once understood, yet the continual changes in the directions of the sides will, without great care, cause us to use Ns instead of Ss; Es instead of Ws, &c.

We suppose, then, that such a rough plot has been prepared, and that the angles, bearings, and distances, as taken from the field book, are figured upon it in *lead pencil*.

Add together the *interior* angles formed at all the corners: call their sum *a*. Mult the number of sides by 180°; from the prod subtract 360°: if the remainder is equal to the sum *a*, it is a proof that the angles have been correctly measured.* This, however, will rarely if ever occur; there will always be some discrepancy; but if the field work has been performed with moderate care, this will not *exceed* about two min for each angle. In this case div it *in equal parts* among all the angles, adding or subtracting, as the case may be, unless it amounts to less than a min to each angle, when it may be entirely disregarded in common farm surveys. The corrected angles may then be marked on the plot in ink, and the pencilled figures erased. We will suppose the corrected ones to be as shown in Fig 3.

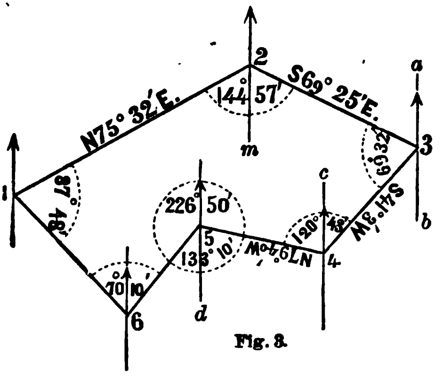


Fig. 3.

Next, by means of these corrected angles, correct the bearings also, thus, Fig 3; Select some side (the longer the better) from the two ends of which the bearing and the reverse bearing agreed; thus showing that that bearing was probably not influenced by local attraction. Let side 2 be the one so selected; assume its bearing, N 75° 32' E, as taken on the ground, to be correct; through either end of it, as at its far end 2, draw the short meridian line; parallel to which draw others through every corner. Now, having the bearing of side 2, N 75° 32' E, and requiring that of side 3, it is plain that the reverse bearing from corner 2 is S 75° 32' W; and that therefore the angle 1, 2, *m*, is 75° 32'. Therefore, if we take 75° 32' from the entire corrected angle 1, 2, 3, or 144° 57', the rem 69° 25' will be the angle *m* 23; consequently the bearing of side 3 *must* be S 69° 25' E. For finding the bearing of side 4, we now have the angle 23 *a* of the reverse bearing of side 3, also equal to 69° 25'; and if we add this to the entire corrected angle 234, or to 69° 32', we have the angle *a* 34 = 69° 25' + 69° 32' = 138° 57'; which taken from 180°, leaves the angle *b* 34 = 41° 3'; consequently the bearing of side 4 *must* be S 41° 3' W. For the bearing of side 5 we now have the angle 34 *c* = 41° 3', which taken from the corrected angle 345, or 120° 48', leaves the angle *c* 45 = 79° 40'; consequently the bearing of side 5 *must* be N 79° 40' W. At corner 5, for the bearing of side 6, we have the angle 45 *d* = 79° 40', which taken from 133° 10', leaves the angle *d* 56 = 53° 30'; consequently the bearing of side 6 *must* be S 53° 30' W. And so with each of the sides, nothing but careful observation is necessary to see how the several angles are to be employed at each corner. Rules are sometimes given for this purpose, but unless frequently used, they are soon forgotten. The plot mechanically prepared obviates the necessity for such rules, inasmuch as the principle of proceeding thereby becomes merely a matter of sight, and tends greatly to prevent error from using the wrong bearings; while the protractor will at once detect any serious mistakes as to the angles, and thus prevent their being carried farther along. After having obtained all the corrected bearings, they may be figured on the plot instead of those taken in the field. They will, however, require a still further correction after a while, since they will be affected by the adjustment of the closing error.

---

* Because in every straight-lined figure the sum of all its interior angles is equal to twice as many right angles as the figure has sides, minus 4 right angles, or 360°.

We now proceed to calculate the closing error *s* 6 of Fig 2, which is done on the principle that in a correct survey the northings will be equal to the southings, and the eastings to the westings. Prepare a table of 7 columns, as below, and in the first 3 cols place the numbers of the sides, and their corrected courses; also the dists or lengths of the sides, as measured on the rough plot, if such a one has been prepared; but if not, then as measured on the ground. Let them be as follows:

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015.5			304.
2	N 75° 32′ E	1202	300.3		1163.9	
3	S 69° 25′ E	1110		390.2	1039.2	
4	S 41° 3′ W	850		641.		558.2
5	N 79° 40′ W	802	143.9			789.
6	S 53° 30′ W	705		419.3		566.7
			1459.7 1450.5	1450.5	2203.1	2217.9 2203.1
			9.2	Error in Lat.	Error in Dep.	14.8

Now, by means of the Table of Sines, etc., find the N, S, E, W, of the several sides, and place them in the corresponding four columns. Thus, for side 1, which is 1060 feet long, with bearing N 16° 40′ W; $\cos 16° 40' = 0.9580$; $\sin 16° 40' = 0.2868$.

Here $N = 1060 \times 0.9580 = 1015.5$; and $W = 1060 \times 0.2868 = 304$. Proceed thus with all. Add up the four cols; find the diff between the N and S cols; and also between the E and W ones. In this instance we find that the Ns are 9.2 feet greater than the Ss; and that the Ws are 14.8 ft greater than the Es; in other words, there is a closing error which would cause a *correct* protraction of our first three cols, to terminate 9.2 feet too far north of the starting point; and 14.8 feet too far west of it. So that by placing this error upon the paper before beginning to protract, we should have a test for the accuracy of the protracting work; but, as before remarked, a little more trouble will now enable us to div the error proportionally among all the Ns, Ss, Es, and Ws, and thereby give us data for drawing the plot correctly at once, without using a protractor at all.

To divide the errors, prepare a table precisely the same as the foregoing, except that the hor spaces are farther apart; and that the addings-up of the *old* N, S, E, W columns are omitted. The additions here noticed are made subsequently.

The new table is on the next page.

REMARK. **The bearing and the reverse bearing** from the two ends of a line will not read precisely the same angle; and the difference varies with the latitude and with the length of the line, but not in the same proportion with either. It is, however, generally too small to be detected by the needle, being, according to Gummere, only three quarters of a minute in a line one mile long in lat 40°. In higher lats it is more, and in lower ones less. It is caused by the fact that meridians or north and south lines are not truly parallel to each other; but would if extended meet at the poles.

Hence the only bearing that can be run in a straight line, with strict accuracy, is a true N and S one; except on the very equator, where alone a due E and W one will also be straight. But **a true curved E and W line** may be found anywhere with sufficient accuracy for the surveyor's purposes thus. Having first by means of the N star p 284, or otherwise got a true N and S bearing at the starting point, lay off from it 90°, for a true E and W bearing *at that point*. This E and W bearing will be tangent to the true E and W curve. Run this tangent carefully; and at intervals (say at the end of each mile) lay off from it (towards the N if in N lat, or vice versa) an offset whose length in *feet* is equal to the proper one from the following table, multiplied by the *square* of the distance in *miles* from the starting point. These offsets will mark points in the true E and W curve.

Latitude N or S.	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
Offsets in ft one mile from starting point.	.058	.118	.179	.243	.311	.385	.467	.559	.667	.795	.952	1.15	1.43

Or, any offset in ft = .6666 × Total Dist in miles^2 × Nat Tang of Lat.

A rhumb line is any one that crosses a meridian obliquely, that is, is neither due N and S, nor E and W.

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015.5 1.7			304.0 2.7
			1013.8...	....	....	... 301.3
2	N 75° 32′ E	1202	300.3 1.9		1163.9 3.1	
			298.4	....	... 1167.0	
3	S 69° 25′ E	1110		390.2 1.8	1039.2 2.9	
				392 ...	... 1042.1	
4	S 41° 3′ W	850		641.0 1.3		558.2 2.2
				642.3...	....	... 556.0
5	N 79° 40′ W	802	143.9 1.3			789.0 2.1
			142.6...	....	....	... 786.9
6	S 58° 30′ W	705		419.3 1.1		566.7 1.8
				420.4...	....	564.9
		5729 Sum of Sides.	1454.8 Cor'd Ns.	1454.7 Cor'd Ss.	2209.1 Cor'd Es.	2209.1 Cor'd Ws.

Now we have already found by the old table that the Ns and the Ws are too long; consequently they must be shortened; while the Ss, and Es, must be lengthened; all in the following proportions: As the

Sum of all the sides : Any given side :: Total err of lat or dep : Err of lat, or dep, of given side.

Thus, commencing with the lat of side 1, we have, as

Sum of all the sides. : Side 1. :: Total lat err. : Lat err of side 1.
5729 : 1060 :: 9.2 : 1.7

Now as the lat of side 1 is north, it must be shortened; hence it becomes = 1015.5 — 1.7 = 1013.8, as figured out in the new table. Again we have for the departure of side 1,

Sum of all the sides. : Side 1. :: Total dep err. : Dep err of side 1.
5729 : 1060 :: 14.8 : 2.7

Now as the dep of side 1 is west, it must be shortened; hence it becomes 304 — 2.7 = 301.3, as figured out in the new table.

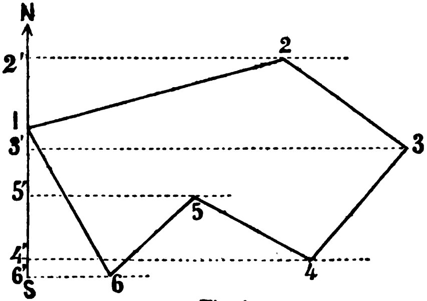


Fig. 4.

Proceeding thus with each side, we obtain all the corrected lats and deps as shown in the new table; where they are connected with their respective sides by dotted lines; but in practice it is better to cross out the original ones when the calculation is finished and proved. If we now add up the 4 cols of corrected N, S, E, W, we find that the Ns = the Ss; and the Es = the Ws; thus proving that the work is right. There is, it is true, a discrepancy of .1 of a ft between the Ns, and the Ss; but this is owing to our carrying out the corrections to only one decimal place; and is too small to be regarded. Discrepancies of 3 or 4 tenths of a foot will sometimes occur from this cause; but may be neglected. The corrected lats and deps must evidently change the bearing and distance of every s; but without knowing either of these, we can now plot the survey by means of the corrected lats and deps alone. The principal is self-evident, explaining itself. First draw a meridian line N S, Fig. 4; and upon it fix on a point 1, to represent the *extreme west* * *corner* of the survey.

Then from the point 1, prick off by scale, northward, the dist 1, 2′=the corrected northing 298.4 of side 2, taken from the last table; from 2′ southward prick off the dist 2′, 3′, the corrected southing 392 of side 3; from 3′ southward prick off 3′, 4′,=southing 642.3 of side 4; from 4′ northward prick off 4′, 5′=northing 142.6 of side 5; from 5′ prick off southward 5′, 6′=southing of side 6†. Then from the points 2′, 3′, 4′, 5′, 6′, draw indefinite lines due eastward, or at right angles to the meridian line. Make by scale, 2′, 2=corrected departure of side 2; and join 1, 2. Make 3′, 3=dep of side 2+dep of side 3; and join 2, 3; make 4′, 4=3′, 3−dep of side 4; and join 3, 4; make 5′, 5 =4′, 4−dep of side 5; and join 4, 5; make 6′, 6=5′, 5−dep of side 6; and join 5, 6.‡ Finally join 6, 1; and the plot is complete. If scrupulous accuracy is not required, the contents may be found by the mechanical method of triangles; the bearings, by the protractor; and the lengths of the sides by the scale; all with an approximation sufficient for ordinary purposes; and perhaps quite as close as by the method by calculation, when, as is customary, the bearings are taken only to the nearest quarter of a degree. We have already said that with a scale of feet per inch $=\frac{\text{average length of sides}}{8}$. the error of area need not *exceed* the $\frac{1}{200}$th part.

But if it is required to calculate the area of the corrected survey with rigorous exactness, it may be done on the following principle, (see Fig 5.) If a meridian line N S be supposed to be drawn through the extreme west corner 1 of a survey; and lines (called *middle distances*) drawn (as the dotted ones in the Fig) at right angles to said meridian, from the *center* of each side of the survey; then if each of the middle dists of such sides as have northings, be mult by the corrected northing of its corresponding side; and if each of the middle dists of such sides as have southings, be mult by the corrected southing of its corresponding side; if we add all the north prods into one sum; and all the south prods into another sum; and subtract the least of these sums from the greatest, the rem will be the area

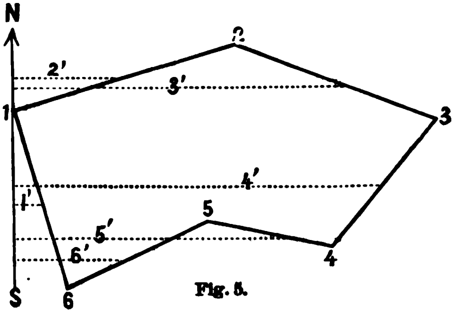


Fig. 5.

---

* The extreme east corner would answer as well, with a slight change in the subsequent operations, as will become evident.

† Instead of pricking off these northings and southings in succession, *from each other*, it will be more correct in practice to prepare first a table showing how far each of the points 2′,3′, &c, is north or south from 1. This being done, the points can be pricked off north or south from 1, without *moving the scale each time;* and of course with greater accuracy. Such a table is readily formed. Rule it as below; and in the first three columns place the numbers of the sides (starting with side 2 from point 1;) and their respective corrected northings and southings. The formation of the 4th and 5th cols by means of the 3d and 4th ones, explains itself. Its accuracy is proved by the final result being 0.

Side.	N. lat.	S. lat.	Dist N or S from Point 1.	
			N.	S.
2	298.4		298.4	
3		392.		93.6
4		642.3		735.9
5	142.6			593.3
6		420.4		1013.7
1	1013.8		000.0	000.0

‡ A similar table should be prepared beforehand for the dists of the points 2, 3, 4, &c, east from the meridian line. It is done in the same manner, but requires one col less, as all the dists are on the same side of the mer line. Thus, starting from point 1, with side 2:

Side.	E. dep.	W. dep.	Dist east from meridian line.
2	1167.0		1167.0
3	1042.1		2209.1
4		556.0	1653.1
5		786.9	866.2
6		564.9	301.3
1		301.3	000.0

This work likewise proves itself by the final result being 0.

of the survey.* The corrected northings and southings we have already found; as also the eastings and westings. The middle dists are found by means of the latter, by employing their *halves*; adding half eastings, and subtracting half westings. Thus it is evident that the middle dist 2′ of side 2, is equal to half the easting of side 2. To this add the other half easting of side 2, and a half easting of side 3; and the sum is plainly equal to the middle dist 3′ of side 3. To this add the other half easting of side 3, and subtract a half westing of side 4, for the middle dist 4′ of side 4. From this subtract the other half westing of side 4, and a half westing of side 5, for the middle dist 5′ of side 5; and so on. The actual calculation may be made thus:

Half easting of side 2 = $\frac{1167}{2}$ = 583.5 E = mid dist of side 2.
583.5 E

Half easting of side 3 = $\frac{1042.1}{2}$ = 1167.0 E
521.0 E

1688.0 E = mid dist of side 3.
521.0 E

Half westing of side 4 = $\frac{556}{2}$ = 2209.0 E
278.0 W

1931.0 E = mid dist of side 4.
278.0 W

Half westing of side 5 = $\frac{786.9}{2}$ = 1653.0 E
393.5 W

1259.5 E = mid dist of side 5.
393.5 W

Half westing of side 6 = $\frac{564.9}{2}$ = 866.0 E
282.4 W

583.6 E = mid dist of side 6.
282.4 W

Half westing of side 1 = $\frac{301.3}{2}$ = 301.2 E
150.6 W

150.6 E = mid dist of side 1.

The work always proves itself by the last two results being equal.

Next make a table like the following, in the first 4 cols of which place the numbers of the sides, the middle dists, the northings, and southings. Mult each middle dist by its corresponding northing or southing, and place the products in their proper col. Add up each col; subtract the least from the

Side.	Middle dist.	Northing.	Southing.	North prod.	South prod.
1	150.6	1013.8		152678	
2	583.5	298.4		174116	
3	1688		392		661696
4	1931		642.3		1240281
5	1259.5	142.6		179605	
6	583.6		420.4		245345
				506399	2147322 506399

43560)1640923(37.67 Acres.

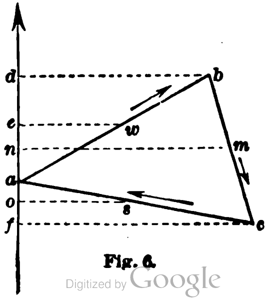


Fig. 6.

* *Proof.* To illustrate the principle upon which this rule is based, let *a b*, *b c*, and *c a*, Fig 6, represent in order the 3 sides of the triangular plot of a survey, with a meridian line *d f* drawn through the extreme west corner, *a*. Let lines *b d* and *c f* be drawn from each corner, perp to the meridian line; also from the middle of each side draw lines *w e*, *m n*, *s o*, also perp to meridian; and representing the middle dists of the sides. Then since the sides are regarded in the order *a b*, *b c*, *c a*, it is plain that *a d* represents the northing of the side *a b*; *f a* the northing of *c a*; and *d f* the southing of *b c*. Now if we mult the *northing a d* of the side *a b*, by its mid dist *e w*, the prod is the area of the triangle *a b d*. In like manner the *northing f a* of the side *c a*, mult by its mid dist *s o*, gives the area of the triangle *a c f*. Again, the *southing d f* of the side *b c*, mult by its mid dist *m n*, gives the area of the entire fig *d b c f d*. If from this area we subtract the areas of the two triangles *a b d*, and *a c f*, the rem is evidently the area of the plot *a b c*. h any other plot, however complicated.

greatest. The rem will be the area of the survey in sq ft; which, div by 43560, (the number of sq ft in an acre,) will be the area in acres; in this instance, 37.67 ac.

It now remains only to calculate the corrected bearings and lengths of the sides of the survey, all of which are necessarily changed by the adoption of the corrected lats and deps. To find the bearing of any side, div its departure (E or W) by its lat (N or S); in the table of nat tang, find the quot; the angle opposite it is the reqd angle of bearing. Thus, for the course of side 1, we have $\frac{301.3\text{ W}}{1013.8\text{ N}}$ = .2972 = nat tang; opposite which in the table is the reqd angle, 16° 33′; the bearing, therefore, is N 16° 33′ W.

Again: for the dist or length of any side, from the table of nat cosines take the cos opposite to the angle of the corrected bearing; divide the corrected lat (N or S) of the side by the cos. Thus for the dist of side 1, we find opposite 16° 33′, the cos .9586. And

$$\underset{\text{Lat.}}{1013.8} + \underset{\text{Cos.}}{.9586} = 1057.6 \text{ the reqd dist.}$$

The following table contains all the corrections of the foregoing survey; consequently, if the bearings and dists are correctly plotted, they will close perfectly. The young assistant is advised to practise doing this, as well as dividing the plot into triangles, and computing the content. In this manner he will soon learn what degree of care is necessary to insure accurate results.

Side.	Bearing.	Dist. Ft.
1	N 16° 33′ W	1057.6
2	N 75° 39′ E	1204.0
3	S 60° 22′ E	1113.3
4	S 40° 53′ W	849.6
5	N 79° 44′ W	800.1
6	S 53° 21′ W	704.3

The following **hints** may often be of service.

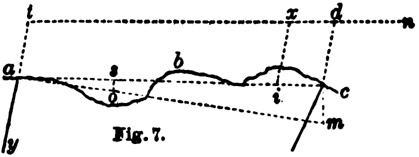


Fig. 7.

1st. Avoid taking bearings and dists along a circuitous boundary line like *a b c*, Fig 7; but run the straight line *a c*; and at right angles to it, measure offsets to the crooked line. 2d. Wishing to survey a straight line from *a* to *c*, but being unable to direct the instrument precisely toward *c*, on account of intervening woods, or other obstacles; first run a trial line, as *a m*, as nearly in the proper direction as can be guessed at. Measure *m c*, and say, as *a m* is to *m c*, so is 100 ft to ? Lay off *a o* equal to 100 ft, and *o s* equal to ?; and run the final line *a s c*. Or, if *m c* is quite small, calculate offsets like *o s* for every 100 ft along *a m*, and thus avoid the necessity for running a second line. 3d. When *c* is visible from *a*, but the intervening ground difficult to measure along, on account of marshes, &c, extend the side *y a* to good ground at *t*; then, making the angle *y t d* equal to *y a c*, run the line *t n* to that point *d* at which the angle *n d c* is found by trial to be equal to the angle *a t d*. It will rarely be necessary to make more than one trial for this point *d*; for, suppose it to be made at *x*, see where it strikes *a c* at *i*; measure *i c*, and continue from *x*, making *x d* = *i c*. 4th. In case of a very irregular piece of land, or a lake, Fig 8, surround it by straight lines. Survey these, and at right angles to them, measure offsets to the crooked boundary. 5th. Surveying a straight line from *w* toward *y*, Fig 9, an obstacle, *o*, is met. To pass it, lay off a right angle *w t u*; measure any *t u*; make *t u v* = 90°; measure *u v*; make *u v i* = 90°; make *v i* = *t u*; make *v i y* = 90°. Then is *t i* = *u v*; and *i y* is in the straight line. Or, with less trouble, at *g* make *t g a* = 60°; measure any *g a*; make *g a s* = 60°; and *a s* = *g a*: make *a s i* = 60°. Then is *g s* = *g a* or *a s*; and *i s*, continued toward *y*, is in the straight line. 6th. Being between two objects, *m* and *n*, and wishing to place myself in range with them, I lay a straight rod *c b* on the ground, and point it to one of the objects *m*; then going to the end *c*, I find that it does not point to the other object. By successive trials, I find the position *e d* in which it points to both objects, and consequently is in range with them.

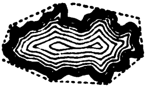


Fig. 8.

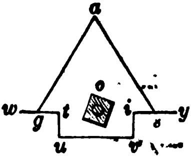


Fig. 9.

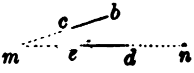


Fig. 10.

CHAINING.

Chains. Engineers have abandoned the Gunter's chain of 66 ft, divided into 100 links of 7.92 ins each. They now use a chain of 100 ft, with 100 links of 1 ft each, and calculate areas in sq ft, the number of which, divided by 43,560, reduces to acres and decimals, instead of to acres, roods, and perches. Gunter's chain is used on U. S. Government land surveys.

Chains are commonly made of iron or steel wire. Each link is bent at each of its ends, to form an eye, by which it is connected with the adjacent links, either directly, as in the Grumman patent chain, or, more commonly, by from 1 to 3 small wire links. The wear of these links is a fruitful source of inaccuracy, inasmuch as even a very slight wear of each link considerably increases the length of the chain. Hence, chains should be compared with some standard, such as a target rod, every few days while in use. For transportation, the lengths are folded on each other, making a compact and sheaf-like bundle.

Tapes. With improved facilities for the manufacture of steel tape, the chain is going out of use. The tape, being much lighter, requires much less pull, and, as there are no links to wear, its length is much more nearly constant than that of the chain. It is replacing, to some extent, the base-measuring rod for accurate geodetic work. Steel tapes are made in continuous lengths up to 500, 600, and even 1000 ft, but those of 100 ft are the most commonly used. Very long tapes are liable to breakage in handling. Even the shorter lengths, unless handled carefully, are apt to kink and break. Breaks are difficult to mend, and the repaired joint is seldom satisfactory; whereas a kink in a wire chain seldom involves more than a temporary change of length. Being run over by a car or wagon will often kink steel tapes very badly, if it does not break them.[*] However, the lightness, neatness, and reliability of the tape offset these disadvantages, which, indeed, the surveyor soon learns to overcome.

Tapes for general field work are usually narrow (from 0.10 to 0.25 in) and thick (from 0.013 to 0.025 in),[†] and are graduated by means of small brass and copper rivets, spaced, in general, 5 ft apart, 1 ft apart in the 10 ft at each end, and 0.1 ft apart in the ft at each end. They are usually mounted on reels.

Tapes for city work are wider (from 0.25 to 0.5 in) and thinner (from 0.007 to 0.010 in)[†] and are graduated (usually to 0.01 ft) throughout their length by means of lines and numerals etched on the steel.

Pins are ordinarily of wire, pointed at the lower end, and bent to a ring at the upper end. They can be forced into almost any ground that is not exceedingly stony. A steel ring, like a large key ring, is often used for carrying the pins. Each pin should have a strip of bright red flannel tied to its top, in order that it may be readily found, among the grass, etc., by the rear chainman.

Corrections for Sag and Stretch. The following diagram[‡] (see p. 283) gives the correction for a steel tape weighing 0.75 lb per 100 ft.[†]

---

[*] The Nichols Engineering & Contracting Co., Chicago, guarantees that its tapes will not be injured by being run over by wagons.

[†] The sizes of tapes, as made by different manufacturers, vary greatly. In applying the corrections, therefore, the width and thickness of the tape to be used should be carefully measured, and its weight per ft computed.

[‡] Deduced from diagrams constructed by Mr. J. O. Clarke, Proceedings Engineers' Club of Philadelphia, April, 1901, Vol. XVIII, No. 2. from the formula:

Stretch in feet ~~Stretch~~ $= \frac{PS}{EA}$

where

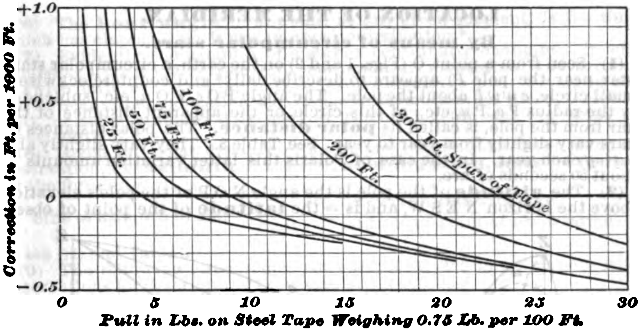
P = pull on tape, in lbs.
S = span of tape, in ~~ins.~~ feet
E = modulus of elasticity for steel = 27,500,000 lbs per sq in.
A = area of cross-section of tape weighing 0.75 lb per 100 ft.
= 0.0022 square ins,

and from the equation of the parabola, according to which

shortening by sag $= \frac{W^2 S^3}{24 P^2}$ in feet

where W = weight of tape in pounds per foot.

Except for very light pulls, this last formula gives practically the same results as the equation of the catenary, which is absolutely correct, but much more cumbersome.



The diagram shows that a span of 100 ft of tape weighing 0.75 ℔ per 100 ft, requires a pull of 11.2 ℔s to reduce the correction to zero; a span of 50 ft, 7.3 ℔s, etc. At these tensions (which are called normal pulls), the opposite effects of sag and of stretch are equal. At higher tensions, the lengthening due to stretch exceeds the shortening due to sag, and vice versa.

Tapes of other weights require pulls proportional to their weights or to their areas of cross-section. Thus, a tape, of any length, weighing 1 ℔ per 100 ft, would require, for any given correction, a pull of $x = \frac{100}{75} y = 1\frac{1}{3} y$, where y = the pull for the same correction on the standard tape, weighing 0.75 ℔ per 100 ft.

Conversely; given a pull of 10 ℔s on a 50 ft span of a tape weighing 0.6 ℔ per 100 ft; required the correction. To produce the same error in the tape weighing 0.75 ℔ per 100 ft would require a pull of $y = 10 \times \frac{75}{60} = 12.5$ ℔s. Referring to the diagram at 12.5 ℔s on the curve for a 50 ft span, we find correction = —0.16. This is the proper correction for either the heavier tape with 12.5 ℔s or for the lighter tape with 10 ℔s pull.

Corrections for temperature. Tapes are usually graduated so as to be of standard length at 62° Fahr. For ordinary steel tape, the correction for temperature is about 0.0000065 ft per ft per degree Fahr.

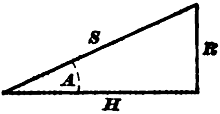
Corrections for temperature are uncertain, since the temperature of the tape cannot be determined with any accuracy. Measurements requiring great accuracy should therefore be made in cloudy weather, or at night, and the tape and the thermometer should be kept off the ground.

When measuring over **sloping ground,** in ordinary work, the chain or tape should be held as nearly horizontal as possible, transferring the position of the raised end to the ground by means of a plumb line. Where the ground is steep, it becomes necessary to use a short length of tape, as the down-hill chainman could not otherwise hold his end high enough; or the tape may be held parallel with the slope, and the distance corrected by the following formulas:

$\frac{H}{S} = \cos A$; $H = S. \cos A$; $S = \frac{H}{\cos A} = H. \sec A$;

$\frac{R}{H} = \tan A$; $R = H. \tan A$;

$\frac{R}{S} = \sin A$; $R = S. \sin A$.



LOCATION OF THE MERIDIAN.

By means of circumpolar stars.

(1) Seen from a point O (Figs. 1 and 2) on the earth, a circumpolar star *e* (star near the pole *P*) appears to describe daily[*] and counterclockwise a small circle, *e u w l*, about the pole. The angle P O *e*, P O *u*, etc., subtended by the radius P *e*, P *u*, etc., of this circle, or the apparent distance of the star from the pole, is called its **polar distance.** The polar distances of stars vary slightly from year to year. See Table 3. They vary slightly also *during* each year. In the case of Polaris this latter variation amounts to about 50 seconds of arc.

(2) The **altitude** of the pole is the angle N O P of the pole's elevation above the horizon N E S W, and is = the **latitude** of the point of observation. **Declination** = angular distance north or south from the celestial equator. Thus, declination of pole = 90°. Declination of any star = 90°—its polar distance.

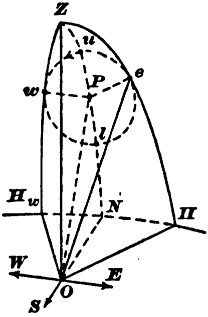


FIG. 1.

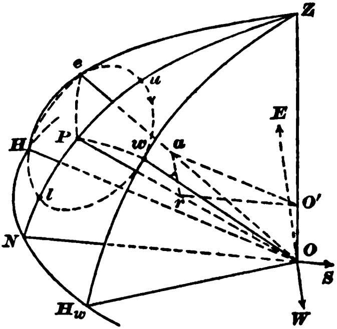


FIG. 2.

(3) Let Z *e* H be an arc of a vertical circle[†] passing through a circumpolar star, *e*, and let H be the point where this arc meets the horizon N E S W. Then the angle N Z H at the zenith Z, or N O H at the point O of observation, between the plane N Z O of the meridian and the plane H Z O of the star's vertical circle (or the arc N H), is called the **azimuth**[‡] of the star. If this angle N O H be laid off from O H, on the ground, the line O N will be in the plane of the **meridian** N Z S, or will be a **north-and-south line.**[‖]

(4) When a star is on the meridian Z N of the observer, above or below the pole P, as at *u* or *l*, it is said to be at its **upper or lower culmination,** respectively. Its azimuth is then = 0, the line O H coinciding with the meridian line O N.

(5) When the star has reached its greatest distance east or west from the pole, as at *e* or *w*, it is said to be at its **eastern or western elongation.**[§]

---

[*] In 23 h. 56.1 m.

[†] A great circle is that section of the surface of a sphere which is formed by a plane passing through the center of the sphere. A vertical circle is a great circle passing through the zenith Z.

[‡] Astronomers usually reckon azimuth from the south point around through the west, north, and east points, to south again; but for our purpose it is evidently much more convenient to reckon it from the north point, and either to the east or to the west, as the case may be.

[‖] The point N, on the horizon, is called the **north point,** and must not be confounded with the north *pole* P.

[§] As seen from the equator, a star, at either elongation, is, like the pole itself, on the horizon; and the two lines P *e*, P *w*, joining it with the pole, form a single straight line perpendicular to the meridian, and lying in the

(6) The **hour angle** of any star, at any given moment, is the time which has elapsed since it was in upper culmination.*

(7) Evidently the azimuth of a star is continually changing. In circumpolar stars it varies from 0° to maximum (at elongation) and back to 0° twice daily, as the star appears to revolve about the pole; but when the star is near either elongation the change in azimuth takes place so slowly that, for some minutes, it is scarcely perceptible, the star appearing to travel vertically.

(8) Given the polar distance of a star and the latitude of the point of observation, the **azimuth** of the star, **at elongation, may be found** by the formula.†

$$\text{Sine of azimuth of star} = \frac{\text{sine of polar distance of star}}{\text{cosine of latitude of point of observation}}$$

or see (11) and Table 3.

(9) The following circumpolar stars are of service in connection with observations for determining the meridian. See Fig. 3.

Constellation	Letter	Called
Ursa minor (Little bear)	α (alpha)	Polaris
Ursa major (Great bear)	ε (epsilon)	Alioth
" " (" ")	ζ (zeta)	Mizar
Cassiopeia	δ (delta)	Delta ‡

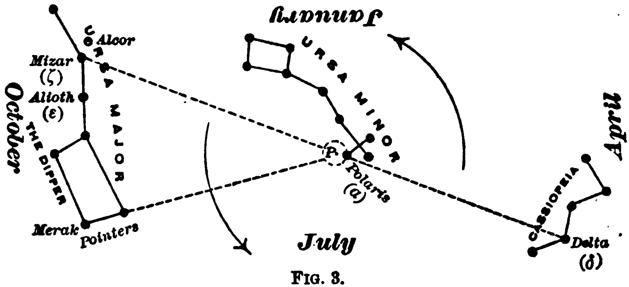


FIG. 3.

(10) **Polaris,** or the **north star,** is fortunately placed for the determination of the meridian, its polar distance being only about 1¼°. See Table 3. Fig. 3 shows the circumpolar stars as they appear about midnight in July; inverted, as in January; with the left side uppermost, as in April; and, with the right side uppermost, as in October.‖

---

horizon. The azimuth of the star is then = its polar distance. But in other latitudes P *e* and P *w* form acute angles with the meridian, as shown, and these angles decrease, and the azimuth of the star at elongation increases, as the latitude increases.

* In lat. 40° N., the hour angle, $ZPe = ZPw$, of Polaris, at elongation, is = 5 h. 55 m. of solar time. **Caution.** It will be noticed that, except for an observer at the equator, the elongations do *not* occur at 90° from the meridian.

† In the spherical triangle Z P *e*, we have:

$$\frac{\sin eZP}{\sin ZeP} = \frac{\sin Pe}{\sin PZ}$$

But, since $ZeP = 90°$, $\sin ZeP = 1$. Also, $\sin PZ = \cos(90° - PZ)$, and eZP = azimuth of *e*.

$$\text{Hence, sin azimuth of } e = \frac{\sin Pe}{\sin PZ} = \frac{\text{sin polar distance } POe}{\text{cos latitude}}$$

‡ δ Cassiopeia is here called Delta, for brevity.

‖ **Polaris is easily found** by means of the two well-known stars called the "**pointers**" in "the dipper," Fig. 3, which forms the hinder

(11) Table 8 gives the polar distances of Polaris and their log sines for January 1 in each third year from 1900 to 1930 inclusive, the log cosines of each fifth degree of latitude from 20° to 50°, and the corresponding azimuths of Polaris at elongation. Intermediate values may be taken by interpolation.*

(12) By observation of Polaris at elongation. This method has the convenience, that at and near elongation the star appears to travel vertically for some minutes, its azimuth, during that time, remaining practically constant; but during certain parts of the year (see Table 1), the elongations of Polaris take place in daylight; so that this method cannot then be used.‡ See (18), (19), (22). Nor can it be used at any time in places south of about 4° N. lat., because there Polaris is not visible.

(13) The approximate times of elongation of Polaris for certain dates, in 1900, are given in Table 1, with instructions for finding the times for other dates. Or, watch Polaris in connection with any of those stars which are nearly in line with it and the pole, as Delta, Mizar, and Alioth. See Fig. 3. The time of elongation is approximated, with sufficient closeness for the determination of the azimuth, by the cessation of apparent horizontal motion during the observation.

(14) From fifteen to thirty minutes before the time of elongation, have the transit, see (21), set up and carefully centered over a stake previously driven and marked with a center point. The transit must be in adjustment, especially in regard to the second adjustment, p. 294, or that of the horizontal axis, by which the line of collimation is made to describe a vertical plane when the transit is leveled and the telescope is swung upward or downward.

(15) Means must be provided for illuminating the cross-hairs of the transit.‡ This may be done by means of a bull's eye, or a dark lantern, so held as not to throw its light into the eye of the observer; or, better, by means of a piece of tin plate, cut and perforated as in Fig. 4, bent at an angle of 45°, as in Fig. 5, and painted white on the surface next to the telescope. The ring, formed by bending the long strip, is placed around the object end of the telescope. A light, screened from the view of the observer, is then held, at one side of the instrument, in such a way that its rays, falling upon the oblique and whitened surface of the tin plate, are reflected directly into the telescope.

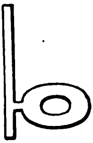


FIG. 4.

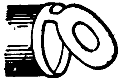


FIG. 5.

(16) Bring the vertical hair to cut Polaris, and, by means of the tangent screw, follow the star as it appears to move, to the *right* if approaching *eastern* elongation, and *vice versa*, keeping the hair upon the star, as nearly as may be. As elongation is approached, the star will appear to move more and more slowly. When it appears to travel vertically along the hair, it has practically reached elongation, and the vertical plane of the transit, *with the vertical hair cutting the star*, is in the plane of the star's vertical circle. Depress the telescope, and fix a point in the line of sight, preferably 300 feet or more distant from the transit.† Immediately reverse the transit, (swinging it horizontally through an arc of 180°), sight to the star again,

---

portion of the "great bear" (Ursa major), a line drawn through these two stars passing near Polaris. As the stars in the handle of the dipper form the tail of the great bear, as shown on celestial maps, so Polaris and the stars near it form the tail of the little bear (Ursa minor.) Polaris is also nearly midway and in line between Delta and Mizar. Polaris forms, with three other and less brilliant stars, a quite symmetrical cross, with Polaris at the end of the right arm. In Fig. 3 this cross is inverted. Its height is about 5°, or = the distance between the pointers.

* Part of a table computed by the Surveying Class of 1882-3, School of Engineering, Vanderbilt University, Nashville, Tenn., and published by Prof. Olin H. Landreth.

† The stake must be illuminated. This may be done by throwing light upon that side of the stake which faces the transit, or, better, by holding a sheet of white paper behind the stake, with a lantern behind the paper. In the latter case, the cross-hairs of the transit, as well as the stake, and the knife-blade or pencil-point with which the assistant marks it, show out dark against the illuminated surface of the paper.

‡ See Note, page 290.

again depress, and, if the line of sight then coincides perfectly with the mark first set, both are in the plane of the star's vertical circle. If not, note where the line of sight does strike, and make a third mark, midway between the two. The line of sight, when directed to this third mark, is in the required plane, from which the azimuth, found as in (8), has yet to be laid off to the meridian, to the *left* from *eastern* elongation, and *vice versa.*

(17) To avoid driving the distant stake and marking it during the night, a fixed target at any convenient point may be used, and the horizontal angle formed between the line of sight to the star and that to the target merely noted, for use in ascertaining and laying off the azimuth of the target.

(18) **By observation of Polaris at culmination.** Owing to its greater difficulty, this method will generally be used only when that by elongation is impracticable. It consists in watching Polaris in connection with another circumpolar star (such as Mizar * or Delta) until Polaris is seen in the same vertical plane with such star, and then waiting a short and known time T, as follows,† until Polaris reaches culmination, whereupon Polaris is sighted and the line of collimation is in the meridian. At their upper culminations, Mizar and Delta are too near the zenith to be conveniently observed at latitudes north of about 25° and 30° respectively. At their lower culminations they are too near the horizon to be used to advantage at places much below about 38° of N. latitude. In general, Delta is conveniently observed at lower culmination from February to August, and Mizar during the rest of the year.

	Mizar T =	Delta T =
In 1900	2.6 mins	3.4 mins
In 1910	6.5 mins	7.2 mins
Mean annual increase, 1900–1910 .	0.39 min	0.38 min

(19) **By observation of Polaris at any point in its path.** Table 1 gives the mean solar times of upper culmination of Polaris on the 1st of each month in 1900, and directions for ascertaining the times on other dates; and Table 2 gives the azimuths of Polaris corresponding to different values of its hour angle in civil or mean solar time, for different latitudes from 30° to 50°, and for the years 1901 and 1906. For hour angles and latitudes intermediate of those in the table, the azimuths may be taken by interpolation. See Caution and formula, p. 290.

(20) The *local* time ‡ of observation must be accurately known, and the time of the preceding upper culmination (as obtained from Table 1) deducted from it. The difference is the hour angle. If the hour angle, thus found, is 11 h. 58 m. or less, the star is west of the meridian. If it is greater than 11 h. 58 m., the star is east of the meridian. In that case deduct the hour angle from 23 h. 56 m. and enter the table with the *remainder* as the hour angle. See Fig. 1.

(21) Where great accuracy is not required, Polaris may be observed by means of a plumb-line and sight. A brick, stone, or other heavy object will answer perfectly as a plumb-bob. It should hang in a pail of water. A compass sight, or any other device with an accurately straight slit about 1/16 inch wide, may be used. The sight must remain always perfectly vertical, but must be adjustable horizontally for a few feet east and west. The plumb-line and sight should be at least 15 feet apart, and so placed that the star and plumb-line can be seen together through the sight, throughout the observation. The plumb-line must be illuminated. It is well to arrange all these matters on an evening preceding that of the observation. When the star reaches elongation, the sight must be fastened in range with the plumb-line and the star. From the line thus obtained, lay off the azimuth; to the *west* for *eastern* elongation, and *vice versa.*

(22) **By any star at equal altitudes.** This method, applicable to south as well as to north latitudes, consists in observing a star when it is at any two equal altitudes, E. and W. of the meridian, thus locating, on the horizon, two points of equal and opposite azimuth. The meridian will be midway between the two points.

---

* Mizar will be recognized by the small star Alcor, close to it.

† Deduced from values calculated in astronomical time (p. 266) by the U. S. Coast and Geodetic Survey.

‡ **Local time** agrees with standard time (p. 267) on the standard meridians only. For other points *add* to standard time 4 minutes for each degree of longitude *east* of a standard meridian, and *vice versa.*

(23) **By equal shadows from the sun.** Fig. 6. Approximate. At the solstices (about June 21 and December 21) the path *a b c d* traversed before and after noon, by the end of the solar shadow O *a*, etc., of a vertical object O, or by the shadow of a knot tied in a plumb-line suspended over O, will intersect a circular arc *a* N *d*, described about O, at equal distances, *a m*, *m d*, from the meridian O N. The observations should be made within two hours before and after noon. At the vernal equinox (March 21) the line thus located will then be west, and at the autumnal equinox (Sept. 21) east, of the meridian, by less than 2½ minutes of arc. For intermediate dates the error is nearly proportional to the time elapsed. It is well to draw several arcs of different radii, O *a*, O *b*, etc., note two points where the path of the shadow intersects each arc, and take the mean of all the results. A small piece of tin plate, with a hole pierced through it, may be placed with the hole vertically over O; and the bright spot, formed by the light shining through the hole, used in place of the end of the shadow.

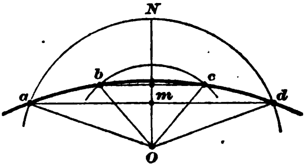


FIG. 6.

Table 1.

Approximate civil times of elongation and culmination of Polaris in lat. 40° N., long. 90° W. from Greenwich, on the first of each month, in 1900.

The times given in this table are mean solar or local times.

P. M. times (between noon and midnight) are printed in **bold-face.**

In latitude 25°, W. elongations occur later and E. earlier } by nearly 2 mins.
In latitude 50°, W. " " earlier and E. later }

The correction for longitude amounts to scarcely a minute of time in any part of the United States.

For other days of the month, deduct 3.94 min. for each succeeding day.

In general, the times are a little later each year. In 1904 they will be about 5½ minutes later, but in 1905, only about 3 minutes later, than in 1900. This discrepancy is due to the occurrence of leap-year in 1904.

Inasmuch as this table serves chiefly to put the observer on guard, and as he should be at his post from 15 to 30 minutes in advance of these times, the gradual increase in the times is of little consequence. The position of the star at elongation is determined by observation.

At culmination, where the change in azimuth is most rapid, an error in time of 2 minutes in observing Polaris involves an error of about 1 minute of azimuth.

At elongation,

an error in time of	will make an error in azimuth of
20 minutes	less than 30 seconds
10 minutes	less than 6 "
5 minutes	less than 2 "
1 minute	about 0.06 second

In all latitudes, each culmination follows the preceding one by 11 h. 58 m. mean solar time. The times between successive elongations vary with the latitudes.

Elongations. (E, eastern: W, western.) **1900.**

Jan. 1. W.	Feb. 1. W.	Mar. 1. W.	Apr. 1. W.	May 1. E.	Jun. 1. E.
12.31 A. M.	**10.30 P. M.**	**8.40 P. M.**	**6.38 P. M.**	4.50 A. M.	2.49 A. M.
July 1. E.	Aug. 1. E.	Sept. 1. E.	Oct. 1. E.	Nov. 1. W.	Dec. 1. W.
12.51 A. M.	**10.46 P. M.**	**8.45 P. M.**	**6.47 P. M.**	4.33 A. M.	2.35 A. M.

Culminations. (U, upper; L, lower.) **1900.**

Jan. 1. U.	Feb. 1. L.	Mar. 1. L.	Apr. 1. L.	May 1. L.	June 1. L.
6.38 P. M.	4.38 A. M.	2.47 A. M.	12.45 A. M.	**10.43 P. M.**	**8.42 P. M.**
July 1. L.	Aug. 1. U.	Sept. 1. U.	Oct. 1. U.	Nov. 1. U.	Dec. 1. U.
.44 P. M.	4.45 A. M.	2.43 A. M.	12.46 A. M.	**10.40 P. M.**	**8.42 P. M.**

Table 2.

AZIMUTHS OF POLARIS.

Hour Angle Mean solar time		Azimuth for latitude					Hour Angle Mean solar time		Azimuth for latitude				
1901	1906	30°	35°	40°	45°	50°	1901	1906	30°	35°	40°	45°	50°
h m	h m	° ′	° ′	° ′	° ′	° ′	h m	h m	° ′	° ′	° ′	° ′	° ′
0 4	0 4	0 2	0 2	0 2	0 2	0 2	6 57	6 27	1 21	1 26	1 32	1 39	1 49
0 8	0 9	0 3	0 3	0 4	0 4	0 4	7 12	6 52	1 20	1 24	1 30	1 37	1 47
0 13	0 13	0 5	0 5	0 5	0 6	0 6	7 25	7 9	1 18	1 22	1 28	1 35	1 44
0 17	0 17	0 6	0 7	0 7	0 8	0 8	7 38	7 25	1 16	1 20	1 26	1 33	1 42
0 21	0 21	0 8	0 8	0 9	0 10	0 11	7 48	7 38	1 14	1 19	1 24	1 31	1 39
0 25	0 26	0 9	0 10	0 11	0 12	0 13	7 57	7 47	1 13	1 17	1 22	1 29	1 37
0 29	0 30	0 11	0 12	0 12	0 14	0 15	8 5	7 57	1 11	1 15	1 20	1 27	1 35
0 33	0 34	0 12	0 13	0 14	0 16	0 17	8 13	8 6	1 10	1 14	1 18	1 25	1 33
0 38	0 38	0 14	0 15	0 16	0 18	0 19	8 20	8 13	1 8	1 12	1 17	1 23	1 31
0 42	0 43	0 15	0 17	0 18	0 20	0 21	8 27	8 21	1 7	1 11	1 15	1 21	1 29
0 46	0 47	0 17	0 18	0 19	0 21	0 23	8 34	8 28	1 5	1 9	1 13	1 19	1 27
0 50	0 51	0 19	0 20	0 21	0 23	0 25	8 40	8 35	1 3	1 7	1 12	1 18	1 25
0 54	0 55	0 20	0 22	0 23	0 25	0 27	8 47	8 42	1 2	1 6	1 10	1 16	1 23
0 59	1 0	0 22	0 23	0 25	0 27	0 29	8 53	8 48	1 0	1 4	1 8	1 14	1 21
1 3	1 4	0 23	0 25	0 26	0 29	0 32	8 58	8 53	0 59	1 3	1 7	1 12	1 19
1 8	1 9	0 25	0 26	0 28	0 31	0 34	9 4	8 59	0 58	1 1	1 5	1 10	1 17
1 13	1 15	0 27	0 29	0 31	0 34	0 37	9 9	9 5	0 56	1 0	1 3	1 8	1 15
1 19	1 20	0 29	0 31	0 33	0 36	0 39	9 15	9 11	0 55	0 58	1 2	1 7	1 13
1 24	1 25	0 31	0 33	0 35	0 38	0 42	9 20	9 16	0 53	0 56	1 0	1 5	1 11
1 29	1 31	0 32	0 35	0 37	0 40	0 44	9 25	9 22	0 51	0 54	0 58	1 3	1 9
1 34	1 37	0 34	0 37	0 39	0 43	0 47	9 31	9 27	0 50	0 53	0 56	1 1	1 7
1 40	1 42	0 36	0 39	0 41	0 45	0 49	9 36	9 33	0 49	0 52	0 55	0 59	1 5
1 45	1 47	0 38	0 41	0 43	0 47	0 51	9 41	9 38	0 47	0 50	0 53	0 57	1 2
1 50	1 53	0 39	0 42	0 45	0 49	0 54	9 47	9 44	0 45	0 48	0 51	0 55	1 0
1 56	1 58	0 41	0 44	0 47	0 51	0 56	9 52	9 49	0 44	0 46	0 49	0 53	0 58
2 1	2 3	0 43	0 46	0 49	0 53	0 59	9 57	9 55	0 42	0 44	0 47	0 51	0 56
2 6	2 9	0 45	0 48	0 51	0 55	1 1	10 2	10 0	0 40	0 42	0 45	0 49	0 54
2 11	2 14	0 46	0 50	0 53	0 57	1 3	10 8	10 5	0 39	0 41	0 43	0 47	0 51
2 17	2 20	0 48	0 51	0 54	0 59	1 5	10 13	10 11	0 37	0 39	0 41	0 45	0 49
2 22	2 25	0 50	0 53	0 56	1 1	1 8	10 18	10 16	0 35	0 37	0 40	0 43	0 47
2 27	2 31	0 51	0 54	0 58	1 3	1 10	10 24	10 21	0 33	0 35	0 38	0 41	0 44
2 33	2 36	0 53	0 56	1 0	1 5	1 12	10 29	10 27	0 32	0 34	0 36	0 39	0 42
2 38	2 42	0 54	0 58	1 2	1 7	1 14	10 34	10 33	0 30	0 32	0 34	0 37	0 40
2 43	2 47	0 56	0 59	1 3	1 9	1 16	10 39	10 38	0 28	0 29	0 31	0 34	0 37
2 49	2 53	0 57	1 1	1 5	1 11	1 18	10 45	10 43	0 26	0 27	0 29	0 32	0 35
2 54	2 59	0 59	1 3	1 7	1 13	1 20	10 50	10 49	0 24	0 25	0 27	0 29	0 32
3 0	3 5	1 0	1 4	1 8	1 15	1 22	10 55	10 54	0 23	0 24	0 25	0 27	0 30
3 5	3 10	1 2	1 6	1 10	1 16	1 24	10 59	10 58	0 21	0 22	0 24	0 26	0 28
3 11	3 16	1 3	1 7	1 12	1 18	1 26	11 4	11 3	0 20	0 21	0 22	0 24	0 26
3 18	3 23	1 5	1 9	1 13	1 20	1 28	11 8	11 7	0 18	0 19	0 20	0 22	0 25
3 24	3 30	1 6	1 10	1 15	1 22	1 30	11 12	11 11	0 17	0 18	0 19	0 20	0 22
3 31	3 37	1 8	1 12	1 17	1 24	1 32	11 16	11 15	0 15	0 16	0 17	0 18	0 20
3 38	3 45	1 9	1 14	1 19	1 26	1 34	11 20	11 20	0 14	0 14	0 15	0 16	0 18
3 45	3 52	1 11	1 15	1 20	1 27	1 36	11 25	11 24	0 12	0 13	0 14	0 15	0 16
3 53	4 1	1 12	1 17	1 22	1 29	1 38	11 29	11 28	0 11	0 11	0 12	0 13	0 14
4 1	4 11	1 14	1 18	1 24	1 31	1 40	11 33	11 32	0 9	0 9	0 10	0 11	0 12
4 10	4 20	1 15	1 20	1 25	1 33	1 42	11 37	11 37	0 8	0 8	0 8	0 9	0 10
4 20	4 33	1 17	1 22	1 27	1 35	1 44	11 41	11 41	0 6	0 6	0 7	0 7	0 8
4 33	4 49	1 19	1 24	1 29	1 37	1 47	11 45	11 45	0 5	0 5	0 5	0 5	0 6
4 46	5 6	1 20	1 25	1 31	1 39	1 49	11 50	11 49	0 3	0 3	0 3	0 4	0 4
5 1	5 31	1 22	1 27	1 33	1 41	1 51	11 54	11 54	0 1	0 1	0 2	0 2	0 2

When the star is near elongation (hour angles between 5 h and 7 h). a considerable change in hour angle corresponds to but a small change in azimuth. At such times it will usually be better to use the method employing the elongation.

Table 3.

POLARIS. POLAR DISTANCES, AND AZIMUTH AT ELONGATION.

Year	Polar Dist. of Polaris	Log sin pol dist.	Azimuth at Elongation, in Latitude						
			20°	25°	30°	35°	40°	45°	50°
	° ′ ″		° ′	° ′	° ′	° ′	° ′	° ′	° ′
1900	1 13 33	8.33 027	1 18.3	1 21.1	1 24.9	1 29.8	1 36.1	1 44.1	1 54.4
1903	1 12 37	8.32 472	1 17.3	1 20.1	1 23.8	1 28.7	1 34.8	1 42.7	1 53.0
1906	1 11 41	8.31 910	1 16.3	1 19.1	1 22.8	1 27.6	1 33.6	1 41.4	1 51.5
1909	1 10 45	8.31 341	1 15.3	1 18.1	1 21.7	1 26.4	1 32.3	1 40.1	1 50.1
1912	1 9 49	8.30 765	1 14.3	1 17.0	1 20.6	1 25.2	1 31.1	1 38.7	1 48.6
1915	1 8 53	8.30 181	1 13.3	1 16.0	1 19.5	1 24.1	1 29.9	1 37.5	1 47.2
1918	1 7 58	8.29 594	1 12.3	1 15.0	1 18.5	1 23.0	1 28.7	1 36.1	1 45.7
1921	1 7 2	8.28 999	1 11.4	1 14.0	1 17.4	1 21.9	1 27.5	1 34.8	1 44.3
1924	1 6 7	8.28 401	1 10.4	1 13.0	1 16.3	1 20.7	1 26.3	1 33.5	1 42.9
1927	1 5 12	8.27 794	1 9.4	1 11.9	1 15.3	1 19.6	1 25.1	1 32.2	1 41.4
1930	1 4 16	8.27 169	1 8.4	1 10.9	1 14.2	1 18.5	1 23.9	1 30.9	1 40.0
	Log cos lat		9.97 299	9.95 728	9.93 753	9.91 337	9.88 425	9.84 949	9.80 807

Owing to changes in the position of Polaris during the year, the positions given in the table may at times be in error by as much as a minute. The error is greater in the higher latitudes.

Having the north polar distance, p, of a star, and the latitude, L, of the point of observation, we have, declination of star $= \delta = 90° - p$; and the **azimuth,** a, of the star, corresponding to any hour angle, h, **may be found** by the following formulas:

$$\text{Tan M} = \frac{\cot p}{\cos h} = \frac{\tan \delta}{\cos h}. \quad \text{Then} \quad \text{Tan } a = \frac{\cos \text{M} \,.\, \tan h}{\cos(\text{L} - \text{M})}.$$

The declinations, δ, of Polaris are given in the U. S. Ephemeris or Nautical Almanac. From these the polar distances may be obtained more accurately than from our Table 3.

Caution. When it is desired to determine the meridian within one minute of arc, it is well to use more than one method and compare the results. For example, observe Polaris both E. and W. of the meridian, and a star at equal altitudes south of the zenith.

NOTE.—If Polaris be found during twilight, in the morning or evening, observations of it may be made without artificial illumination of the cross-hairs. For times of elongation, see Table 1.

Conversion of Arc into Time, and vice versa.

ARC	TIME	TIME	ARC
1°	= 4 minutes	24 hours	= 360°
1′	= 4 seconds	1 hour	= 15°
1″	= 0.066... second	1 minute	= 0° 15′
		1 second	= 0° 0′ 15″

THE ENGINEER'S TRANSIT.

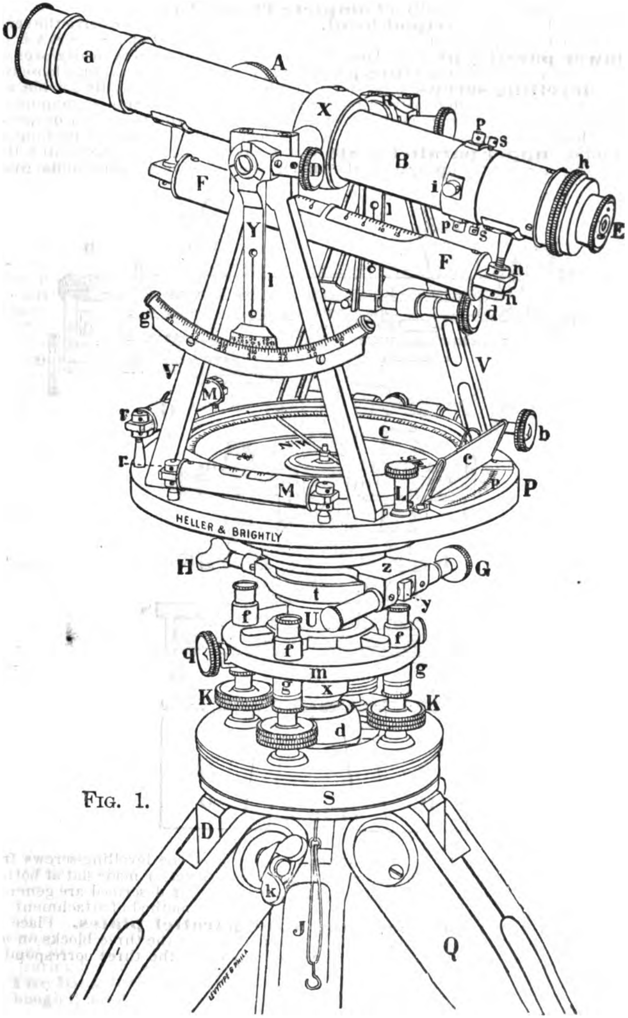


Fig. 1.

THE details of the transit, like those of the level, are differently arranged by diff makers, and to suit particular purposes. We describe it in its modern form, as made by Heller and Brightly, of Philada. Without the **long bubble-tube** F F, Fig 1, under the telescope, and the **graduated arc** *g*, it is their **plain transit.** With these appendages, or rather with a graduated *circle* in place of the *arc*, it becomes virtually a **Complete Theodolite.**

B D D, Fig 2, is the **tripod-head.** The screw-threads at *v* receive the screw of a wooden tripod-head-cover when the instrument is out of use. S S A is the **lower parallel plate.** After the transit has been set very nearly over the center of a stake, the **shifting-plate,** *d d c c*, enables us, by slightly loosening the **levelling-screws** K, to shift the upper parts horizontally a trifle, and thus bring the plumb-bob exactly over the center with less trouble than by the older method of pushing one or two of the legs further into the ground, or spreading them more or less. The screws, K, are then tightened, thereby pushing upward the **upper parallel plate** *m m m x x*, and with it the half-ball *b*, thus pressing *c c* tightly up against the under side of S. The plumb-line passes

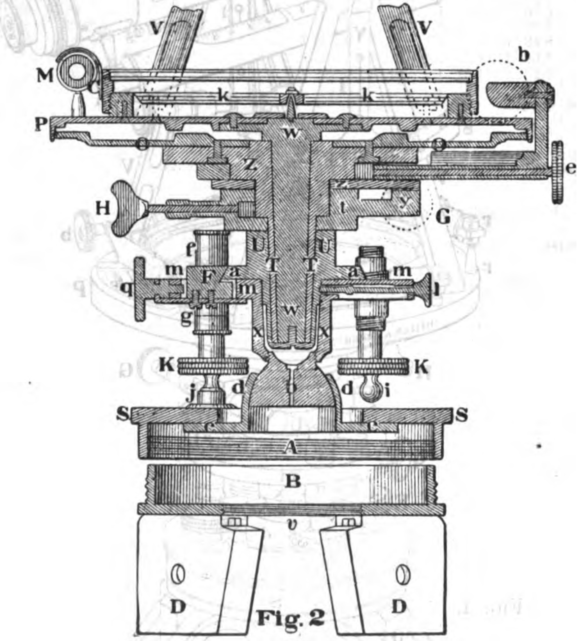


Fig. 2

through the vert hole in *b*. Screw-caps, *f*, *g*, protect the levelling-screws from dust, &c. The feet, *i*, of the screws, work in loose sockets, *j*, made flat at bottom, to preserve S from being indented. The parts thus far described are generally left attached to the legs at all times. Fig 1 shows the method of attachment.

To set the upper parts upon the parallel plates. Place the lower end of U U in *x x*, holding the instrument so that the three blocks on *m m* (of which the one shown at F is movable) may enter the three corresponding recesses in *a*, thus allowing *a* to bear fully on *m*, upon which the upper parts then rest. (The inner end of the spring-catch, *l*, in the meantime enters a groove around U, just below *a*, and prevents the upper parts from falling off, if the instrument is now carried over the shoulder.) Revolve the upper parts horizontally a trifle, in either direction, until they are stopped by the striking of a small lug on *a* against one of the blocks F. The recesses in *a* are now clear of the blocks. Tighten *q*, thereby pushing inward the movable block F, which clamps the bevelled flange *a* between it and the two fixed blocks on *m m*, and confines the spindle U to the fixed parallel plates. It remains so clamped while the instrument is being used.

To remove the upper parts from the parallel plates. Loosen *q*, bring the recesses in *a* opposite the blocks F. Hold back *l*, and lift the upper parts, which are then held together by the broad head of the screw inserted into the foot of the spindle *w*.

T T is the **outer revolving spindle,** cast in one with the **supporting-plate** Z Z, to which is fastened the **graduated limb** O O. The limb extends beyond the compass-box, and thus admits of larger graduations than would otherwise be obtainable. *w w* is the **inner revolving spindle.** At its top it has a broad flange, to which is fastened the vernier plate P. To the latter are fastened the compass-box C, the two bubble-tubes M M, the standards V V, supporting the telescope, &c. Each bubble-tube is supported and adjusted by four capstan-head nuts, two at each end. The bent strip, curving over the tube, protects the glass from accidental blows in swinging the telescope.

Control of motions of graduated limb O O and vernier plate P.—The tangent-screw G and a spiral spring (not shown) opposite to it are fixed to the graduated limb O O, and hold between them a projection *y* from the loose collar *t*, which is thus confined to the limb and made to travel with it. The clamp-screw H passes through the collar *t* and presses against the small lug shown at its inner end. When H is tightened, this lug is pressed against the fixed spindle U U, to which the graduated limb is thus made fast. A slow motion may, however, still be given to the limb by means of the tangent-screw G.

The motion of the vernier plate P over the graduated limb O O is similarly governed by the tangent-screw *b* and its spiral spring (not shown), fixed to the vernier plate P, and the clamp-screw *e*, which passes through the collar *z*, and presses against the small lug shown at its inner end. In Heller and Brightly's instruments, the screw *b* is provided with means for taking up its "wear," or "lost-motion."

There are two verniers. One is shown at *p*, Fig 1. Both may be read, and their mean taken, when great accuracy is required. Ivory reflectors, *c*, facilitate their reading. Before the instrument is moved from one place to another, the **compass-needle,** *k*, Fig 2, should always be pressed up against the glass cover of the compass-box by means of the upright milled-head screw seen on the vernier-plate in Fig 1, just to the right of the nearest standard. The pivot-point is thus protected from injury.

R, Fig 1, is a ring with a clamp (the latter not shown) for holding the telescope in any required position. It is best to let the *eye*-end, E, of the telescope revolve *downward*, as otherwise the shade on O, if in use, may fall off. The tangent-screw, *d*, moves a vert arm attached to R, and is thus used for slightly changing the elevation of the telescope. In the arm is a slit like that seen in the vernier-arm *l*. By means of the screw D, the movable vernier-arm Y may be clamped at any desired point on the vertical limb *g*. When 0° of the vernier is placed at 30° on the arc *g*, and the index of the opposite arm is placed over a small notch on the horizontal brace (not seen in our figs) of the standards, the two slits will be opposite each other, and may be used for laying off offsets, &c, at right-angles to the line of sight.

One end, R, of the telescope axis rests in a movable box, under which is a screw. By means of the screw, the box may be raised or lowered, and the axis thus adjusted for very slight derangements of the standards. For E, B, O, and A, see *Level*, p 306. *a* is a dust-guard for the object-slide.

Stadia Hairs. Immediately behind the capstan-screw, *p*, Fig 1, is seen a smaller one. This and a similar one on the opposite side of the telescope, work in a ring inside the telescope, and hold the ring in position. Across the ring are stretched two additional horizontal hairs, called stadia hairs, placed at such a distance apart, vertically, that they will subtend say 10 divisions of a graduated rod placed 100 ft from the instrument, 15 divisions at 150 ft, &c. They are thus used for measuring hor and sloping distances.

The long bubble-tube, F F, Fig 1, enables us to use the transit as a level, although it is not so well adapted as the latter to this purpose.

To adjust a plain Transit.

When either a level or a transit is purchased, it is a good precaution (but one which the writer has never seen alluded to) to first screw the object-glass firmly home to its place; and then make a short continuous scratch upon the ring of the glass, and upon its slide; so as to be able to see at any time when at work, that the glass is always in the same position with regard to the slide. For if, after all the adjustments are completed, the position of the glass should become changed, (as it is apt to be if unscrewed, and afterward not screwed up to the same precise spot,) the adjustments may thereby become materially deranged; especially if the object-glass is eccentric, or not truly ground, which is often the case. Such scratches should be prepared by the maker. In making adjustments, as well as when using a transit or level, be careful that the eye-glass and object-glass are so drawn out that there shall be no parallax. The eye-glass must first be drawn out so as to obtain perfect distinctness of the cross-hairs; it must not be disturbed afterward; but the object-glass must be moved for different distances.

First. to ascertain that the bubble-tubes, M M, are placed parallel to the vernier-plate, *and that therefore when both bubbles are in the centers of their tubes the axis of the inst is vert.* By means of the four levelling-screws, K, bring both bubbles to the centers of their tubes in one position of the inst; then turn the upper parts of the inst *half-way* round. If the bubbles do not remain in the center, correct *half* the error by means of the two capstan-nuts *r r;* and the other half by the levelling-screws K. Repeat the trial until both bubbles remain in the center while the inst is being turned entirely around on its spindle.

Second, to see that the standards have suffered no derangement; that is, that they are of equal height and perpendicular to the vernier-plate, as they always are when they leave the maker's hands. Level the inst perfectly; then direct the intersection of the hairs to some point of a high object (as the top of a steeple) near by; clamp the inst by means of screws H and *e*, and lower the telescope until the intersection strikes some point of a low object. (If there is none such drive a stake or chain-pin, &c, in the line.) Then unclamp either H or *e*, and turn the upper parts of the inst half-way round; fix the intersection again upon the high point; clamp; lower the telescope to the low point. If the intersection still strikes the low point, the standards are in order. If not, correct *one-half* of the difference by means of the adjusting-block and screw at the end, R, of the telescope axis, Fig. 1, and repeat the trial *de novo*, resetting the stake or chain-pin at each trial. If the inst has no adjusting-block for the axis, it should be returned to the maker for correction of any derangement of the standards.

A transit may be used *for running straight lines*, even if the standards become slightly bent, by the process described at the end of the fourth adjustment.

Third, to see that the cross-hairs are truly vert and hor when the inst is level. When the telescope *inverts*, the cross-hairs are nearer the eye-end than when it shows objects erect. The maker takes care to place the cross-hairs at right-angles to each other in their ring, or diaphragm; and generally he so places the ring in the telescope, that when levelled, they shall be vert and hor. Sometimes, however, this is neglected; or the ring may by accident become turned a little. To be certain that one hair is vert, (in which case the other must, by construction, be hor,) after having adjusted the bubble-tubes, level the instrument carefully, and take sight with the telescope at a plumb-line, or other vert straight edge. If the vert hair coincides with this object, it is, *so far*, in adjustment; but if not, then loosen *slightly* only *two adjacent* screws of the four, *p p i i*, Fig 1; and with a knife, key, or other small instrument, tap very gently against the screw-heads, so as to turn the ring a little in the telescope; persevering until the hair becomes truly vertical. When this is done, tighten the screws. In the absence of a plumb-line, or vert straight edge, sight the cross-hair at a very small distinct point; and see if the hair still cuts that point, when the telescope is raised or lowered by revolving it on its axis.

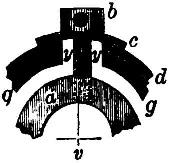


Fig. 3.

The mode of performing the foregoing will be readily understood from this Fig, which represents a section across the top part of the telescope, and at the cross-hairs. The hair-ring, or diaphragm, *a;* vert hair, *v;* telescope tube, *g;* ring outside of telescope tube, *d; b* is one of the four capstan-headed screws which hold the hair-ring, *a*, in its place, and also serve to adjust it. The lower ends of these screws work in the thickness of the hair-ring; so that when they are loosened somewhat, they do not lose their hold on the ring. Small loose washers, *c*, are placed under the heads *b* of the screws. A space *y y* is left around each screw where it passes through the telescope tube, to allow the screws and ring together to be moved a little sideways when the screws *b* are slightly loosened.

Fourth, to see that the vertical hair is in the line of collimation. Plant the tripod firmly upon the ground, as at *a*. Level the inst; clamp it; and direct the vert hair by means of tangent-screw G (figs. 1 and 2) upon some convenient object *b*; or if there is none such, drive a thin stake, or a chain-pin. Then revolving the telescope vert on its axis, observe some object, as *c*, where the vert hair now strikes; or if there is none, place a second pin. Unclamp the instrument by the clamp-screw H; and turn the whole upper part of it around until the vert hair again strikes *b*. Clamp again; and again revolve the telescope vert on its axis. If the vert hair now strikes *c*, as it did before, it shows that *c* is really at *o*; and that *b*, *a*, *c*, *are in the same straight line;* and therefore this adjustment is in order. If not, observe where it does strike, say at *m*, (the dist *a m* being taken equal to *a c*,) and place a pin there also. Measure *m c*; and place a pin at *v*, in the line *m c*, making *m v* = one-fourth of *m c*. Also put a pin at *o*, halfway between *m* and *c*, or in range with *a* and *b*. By means of the two hor screws that move the ring carrying the cross-hairs, adjust the vert hair until it cuts *v*. Now repeat the *entire* operation; and persevere until the telescope, after being directed to *b*, shall strike the same object *o*, *both times*, when revolved on its axis. See whether the movement of the ring in this 4th adjustment has disturbed the verticality of the hair. If it has, repeat the 3d adjustment. Then repeat the 4th, if necessary; and so on until both adjustments are found to be right at the same time. Thus a straight line may be run, even if the hairs are out of adjustment; but with somewhat more trouble. For at each station, as at *a*, two back-sights, and two fore-sights, *a c* and *a m*, may be taken, as when making the adjustment; and the point *o*, half-way between *c* and *m*, will be in the straight line. The inst may then be moved to *o*, and the two back-sights be taken to *a*; and so on.

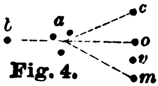
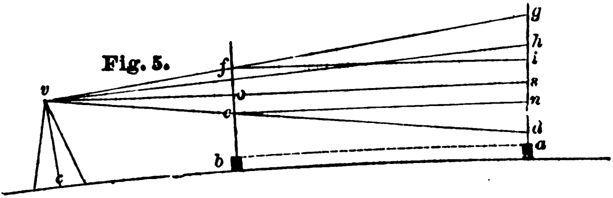


Fig. 4.

Angles measured by the transit, whether vert or hor, will evidently not be affected by the hairs being out of adjustment, provided either that the vert hair is truly vert, or that we use the *intersection* of the hairs when measuring.

The foregoing are all the adjustments needed, unless the transit is required for levelling, in which case the following one must be attended to:

Fig. 5.



To adjust the long bubble-tube, F F, Fig. 1, we first place the line of sight of the telescope hor, and then make the bubble-tube hor, so that the two are parallel. Drive two pegs, *a* and *b* Fig. 5, with their tops at precisely the same level (see Rem. p. 296) and at least about 100 ft. apart; 300 or more will be better. Plant the inst firmly, in range with them, as at *c*, making *b c* an aliquot part of *a b*, and as short as will permit focusing on a rod at *b*. The inst need not be leveled. Suppose the line of sight to cut *e* and *d*. Take the readings *b e* and *a d*. Their diff is $b\,e - a\,d = a\,n - a\,d = d\,n$; and $a\,b : a\,c :: d\,n : d\,s$; *s* being the height of the target at *a* when the readings (*a s*, *b o*) on the two stakes are equal. $a\,s = a\,d + d\,s = a\,d + \frac{d\,n \times a\,c}{a\,b}$. If the reading on *a* *exceeds* that on *b* (as when the line of sight is *v f g*) the diff of readings is $= a\,g - b\,f = a\,g - a\,i = g\,i$; and $a\,s = a\,g - g\,s = a\,g - \frac{g\,i \times a\,c}{a\,b}$. Sight to *s*, bring the bubble to the cen of its tube by means of the two small nuts *n n* at one end of the tube, Fig. 1, and assume that the telescope and tube are parallel.* The zeros of the vert circle, and of its vernier, may now be adjusted, if they require it, by loosening the vernier screws and then moving the vernier until the two coincide.

---

* This neglects a small error due to the curvature of the earth; for a *hor* line at *v* is *v h*, tangential to the curved (or "level") surface of still water at *v*, whereas *v s* is tangential to water surf at a point midway between *a* and *b*. Hence if the telescope at *v* points to *s* it will not be parallel to the level bubble-tube. To allow for this, and for the refraction by the air, which *diminishes* the error, raise the target on *a* to a point *h* above *s*. $h\,s = .0000000205 \times$ square of *a c* in ft; but when *a c* is 650 ft, *h s* is only about one tenth of an inch and barely covers the apparent thickness of the cross-hair in the telescope.

Rem. If no level is at hand for levelling the two pegs *a* and *b*, it may be done by the transit itself, thus: Carefully level the two short bubbles, by means of the levelling-screws K. Drive a peg *m*, from 100 to 300 feet from the instrument *o*. Then placing a target-rod on *m*, clamp the target tight at whatever height, as *ev*, the hor hair happens to cut it; it being of no importance whether the telescope is level or not; although it might as well be as nearly so as can conveniently be guessed at. Clamp the telescope in its position by the clamp-ring R, Fig. 1. Revolve the inst a considerable way round; say nearly or quite half way. Place another peg *n*, *at precisely the same dist* from the instrument that *m* is; and continue to drive it until the hor hair cuts the target placed on it, and still kept clamped to the rod, at the same height as when it was on *m*. When this is done, the tops of the two pegs are on a level with each other, and are ready to be used as before directed.

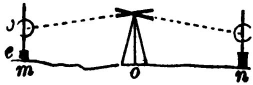
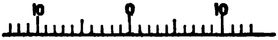


Fig. 6.

When a transit is intended to be used for surveying farms, &c, or for retracing lines of old surveys, it is very useful to set the compass so as to allow for the "variation" during the interval between the two surveys. For this purpose a "**variation-vernier**" is added to such transits; and also to the compass.

When the graduations of a transit are figured, or numbered, so as to read both ways from zero, thus,  the vernier also is made double; that is, it also is graduated and numbered from its zero both ways. In this case, if the angle is measured from zero toward the right hand, the reading must be made from the right hand half of the vernier; and vice versa. If the figuring is single, or only in one direction, from zero to 360°, then only the single vernier is necessary, as the angles are then measured only in the direction that the figuring counts. Engineers differ in their preferences for various manners of figuring the graduations. The writer prefers from zero each way to 180°, with two double verniers.

To replace cross-hairs in a level, or transit. Take out the tube from the eye end of the telescope. Looking in, notice which side of the cross-hair diaphragm is turned toward the eye end. Then loosen the four screws which hold the diaphragm, so as to let the latter fall out of the telescope. Fasten on new hairs with beeswax, varnish, glue, or gum-arabic water, &c. This requires care. Then, to return the diaphragm to its place, press firmly into one of the screw-holes on the circumf of the diaphragm itself, the end of a piece of stick, long enough to reach easily into the telescope as far as to where the diaphragm belongs. By this stick, as a handle, insert the diaphragm edgewise to its place in the telescope, and hold it there until two *opposite* screws are put in place and screwed. Then draw the stick out of the hole in the diaphragm; and with it turn the diaphragm until the same side presents itself toward the eye end as before; then put in the other two screws.

The so-called cross-hairs are actually spider-web, so fine as to be barely visible to the naked eye. Heller & Brightly use very fine platina wire, which is much better. Human hair is entirely too coarse.

To replace a spirit-level, or bubble-glass. Detach the level from the instrument; draw off its sliding ends; push out the broken glass vial, and the cement which held it; insert the new one, with the proper side up (the upper side is always marked with a file by the maker); wrapping some paper around its ends, if it fits loosely. Finally, put a little putty, or melted beeswax over the ends of the vial, to secure it against moving in its tube.

In purchasing instruments, especially when they are to be used far from a maker, it is advisable to provide extras of such parts as may be easily broken or lost; such as glass compass-covers, and needles; adjusting pins; level vials; magnifiers, &c.

Theodolite adjustments are performed like those of the level and transit.

1st. That of the cross-hairs; the same as in the level.

2d. The long bubble-tube of the telescope; also as in the level.

3d. The two short bubble-tubes: as in the transit.

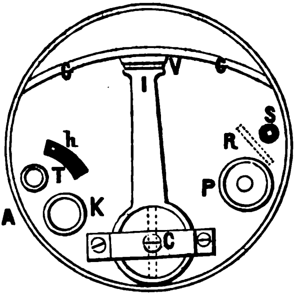
4th The vernier of the vert limb; as in the transit with a vert circle.

5th. To see that the vert hair travels vertically; as in the fourth adjustment of the transit. In some theodolites, no adjustment is provided for this; but in large ones it is provided for by screws under the feet of the standards.

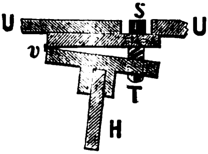
Sometimes a second telescope is added; it is placed below the hor limb, and is called a *watcher*. It has its own clamp, and tangent-screw. Its use is to ascertain whether the zero of that limb has moved during the measurement of hor angles. When, previously to beginning the measurement, the zero and upper telescope are directed toward the first object, point the lower telescope to any small distant object, and then clamp it. During the subsequent measurement, look through it, from time to time, to be sure that it still strikes that object; thus proving that no slipping has occurred.

THE BOX OR POCKET SEXTANT.

THE portability of the pocket sextant, and the fact that it reads to single minutes, render it at times very useful to the engineer. By it, angles can be measured while in a boat, or on horseback; and in many situations which preclude the use of a transit. It is useful for obtaining latitudes, by aid of an artificial horizon. When closed, it resembles a cylindrical brass box, about 3 inches in diameter, and 1½ inches deep. This box is in two parts: by unscrewing which, then inverting one part, and then screwing them together again, the lower part becomes a handle for holding the instrument. Looking down upon its top when thus arranged, we see, as in this figure, a movable arm I C, called the **index**, which turns on a center at C, and carries the vernier V at its other end. G G is the graduated arc or **limb**. It actually subtends about 73°, but is divided into about 146°. Its zero is at one end. Its graduations are not shown in the Fig.



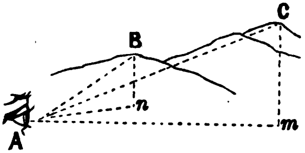
Attached to the index is a small movable lens, (not shown in the figure,) likewise revolving around C, for reading the fine divisions of the limb. When measuring an angle, the index is moved by turning the milled-head P of a pinion, which works in a rack placed within the box. The eye is applied to a circular hole at the side of the box, near A. A small telescope, about 3 inches long, accompanies the instrument; but may generally be dispensed with. When so, the eye-hole at A should be partially closed by a slide which has a very small eye-hole in it; and which is moved by the pin *h*, moving in the curved slot. Another slide, at the side of the box, carries a dark glass for covering the eye-hole when observing the sun. When the telescope is used, it is fastened on by the milled-head screw T. The top part shown in our figure, can be separated from the cylindrical part, by removing 3 or 4 small screws around its edge; and the interior can then be examined, and cleaned if necessary. Like nautical, and other sextants, this one has two principal glasses, both of them mirrors. One, the **index-glass**, is attached to the underside of the index, at C; its upper edge being indicated by the two dotted lines. The other, the **horizon-glass**, (because, when measuring the vert angles of celestial bodies, it is directed toward the horizon,) is also within the box; the position of its upper edge being shown by the dotted lines at R. The horizon-glass is silvered only half-way down; so that one of the observed objects may be seen directly through its lower half, while the image of the other object is seen in the upper half, reflected from the index-glass. That the instrument may be in adjustment, ready for use, these two glasses must be at right angles to the plane of the instrument; that is, to the under side of the top of the box, to which they are attached; and must also be parallel to each other, when the zeros of the vernier and of the limb coincide. The index-glass is already permanently fixed by the maker, and requires no other adjustment. But the horizon-glass has two adjustments, which are made by a key like that of a watch, and having a milled-head K. It is screwed into the top of the box, so as to be always at hand for use. When needed, it is unscrewed. This key fits upon two small square-heads, (like that for winding a watch;) one of which is shown at S; while the other is near it, but on the SIDE of the box. These squares are the heads of two small screws. If the horizon glass H should, as in this sketch, (where it is shown endwise,) not be at right angles to the top U U of the box, it is brought right by turning the square-head S of the screw S T; and if, after being so far rectified, it still is not parallel to the index-glass when the zeros coincide, it is moved a little backward or forward by the square head at the side.



To adjust a box sextant, bring the two zeros to coincide precisely; then look through the eye-hole, and the lower or unsilvered part of the horizon-glass, at some distant object. If the instrument is in adjustment, the object thus seen directly, will coincide precisely with its reflected image, seen at the same time, at the same spot. But if it is not in adjustment, the two will appear separated either hor or vert, or both, thus, * *; in which case apply the key K to the square-head S; and by turning it slightly in whichever direction may be necessary, *still looking at the object and its image*, bring the two into a hor position, or on a level with each other, thus, * *. Then apply the key to the square-head in the side of the box; and by turning it slightly, bring the two to coincide perfectly. The instrument is then adjusted.

In some instruments, the hor glass has a *hinge* at *v*, to allow it play while being adjusted by the single screw S T; but others dispense with this hinge, and use *two* screws like S on top of the box, in addition to the one in the side.

If a sextant is used for measuring vert angles by means of an **artificial horizon,** the actual altitude will be but one-half of that read off on the limb; because we then read at once both the actual and the reflected angle. The great objection to the sextant for engineering purposes, is that it does not measure angles horizontally, as the transit does; unless when the observer, and the two objects happen to be in the same hor plane. Thus an observer with a sextant at A, if measuring the angle subtended by the mountain-peaks B and C, must hold the graduated plane of the sextant in the plane of A B C; and must actually measure the angle B A C; whereas what he wants is the hor angle *n* A *m*. This is greater than B A C, because the dists A *n* and A *m* are shorter than A B and A C. The transit gives the hor angle *n* A *m*, because its graduated plane is first fixed hor by the levelling-screws: and the subsequent measurement of the angle is not affected by his directing merely the line of sight upward, to any extent, in order to fix it upon B and C. For more on this subject; and for a method of partially obviating this objection to the sextant, see the note to Example 2, Case 4, of "Trigonometry."



The nautical sextant, used on ships, is constructed on the same principle as the box sextant; and its adjustments are very similar. In it, also, the index-glass is permanently fixed by the maker; and the horizon-glass has the two adjustments of the box sextant. It also has its dark glasses for looking at the sun; and a small sight-hole, to be used when the telescope is dispensed with.

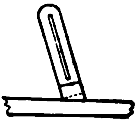
THE COMPASS.

To adjust a Compass.

The first adjustment is that of the bubbles. Plant firmly; and level the instrument, in any position; that is, bring the bubbles to the centers of their tubes. Then turn the instrument half-way round. If the bubbles then remain at the centers, they are in adjustment; but if not, correct *one-half* the diff in each bubble, by means of the adjusting-screws of the tubes. Level the instrument again; turn it half round; and if the bubbles still do not remain at the center, the adjusting-screws must be again moved a little, so as to rectify half the remaining diff. Generally several trials must be thus made, until the bubbles will remain at the cente while the compass is being turned entirely around.

Second adjustment. Level the compass, and then see that the needle is hor; and if not, make it so by means of the small piece of wire which is wrapped around it; sliding the wire toward the high end. A needle thus horizontally adjusted at one place, will not remain so if removed far north or south from that place. If carried to the north, the north end will dip down; and if to the south, the south end will do so. The sliding wire is intended to counteract this.

Third adjustment. This is always fixed right at first by the maker; that is, the sights, or slits for sighting through, are placed at right angles to the compass plate; so that when the latter is levelled by the bubbles, the sights are vert. To test whether they are so, hang up a plumb-line; and having levelled the compass, take sight at the line, and see if the slits coincide with it. If one or both slits should prove to be out of plumb, as shown to an exaggerated extent in this sketch, it should be unscrewed from the compass, and a portion of its foot on the high side be filed or ground off, as per the dotted line; or as a temporary expedient, a small wedge may be placed under the low side, so as to raise it.

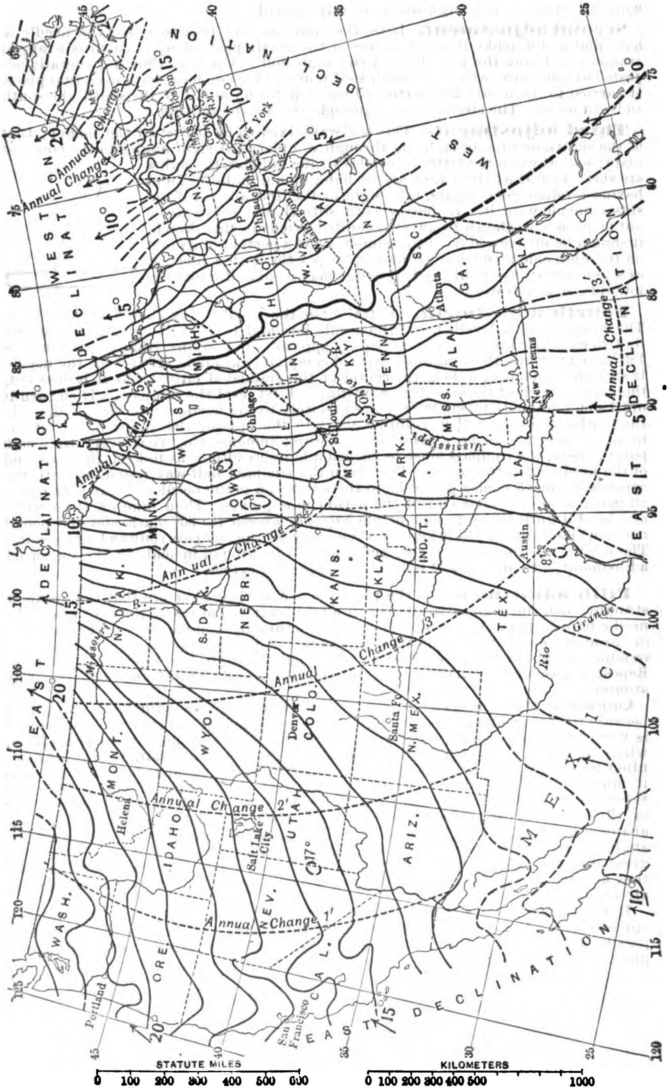


Fourth adjustment, to straighten the needle, if it should become bent. The compass being levelled, and the needle hor, and loose on its pivot, see whether its two ends continue to point to exactly opposite graduations, (that is, graduations 180° apart;) while the compass is turned completely around. If it does, the needle is straight; and its pin is in the center of the graduated circle; but if it does not, then one or both of these require adjusting. First level the compass. Then turn it until some graduation (say 90°) comes precisely to the north end of the needle. If the south end does not then point precisely to the opposite 90° division, lift off the needle, and bend the *pivot-point* until it does; remembering that every time said point is bent, the compass must be turned a hairsbreadth so as to keep the *north* end of the needle at its 90° mark. Then turn the compass half-way round, or until the *opposite* 90° mark comes precisely to the north end of the needle. Make a fine pencil mark where the *south* end of the needle now points. Then take off the needle, and bend it until its south end points *half-way between* its 90° mark and the pencil mark, while its north end is kept at 90° by moving the compass round a hairsbreadth. The needle will then be straight, and must not be altered in making the following adjustment, although it will not yet cut opposite degrees.

Fifth adjustment, of the pivot-pin. After being certain that the needle is straight, turn the compass around until a part is arrived at where the two ends of the needle happen to cut opposite degrees. Then turn the compass *quarter way* around, or through 90°. If the needle then cuts opposite degrees, the pivot-point is already in adjustment; but if the needle does not so cut, bend the *pivot-point* until it does. Repeat, if necessary, until the needle cuts opposite degrees while being turned entirely around.

Care and nicety of observation are necessary in making these adjustments properly; because the entire error to be rectified is, in itself, a minute quantity; and the novice is very apt to increase his trouble by not knowing how to use his **magnifier,** when looking at the end of the needle and the corresponding graduations. The magnifier must always be held with its *center directly over* the point to be examined; and it must be held parallel to the graduated circle. Otherwise annoying errors of several minutes will be made in a single observation; and the accumulation of two or three such errors, arising from a cause unknown to him, may compel him to abandon the adjustments in despair. This suggestion applies also to the reading of angles taken by the transit, &c; although the errors are not then likely to be so great as in the case of the compass. In purchasing a magnifier for a compass, see that no part of it, as hinges, or rivets, are made of iron; for such would change the direction of the needle.

If the sight-slits of a compass are not fixed by the maker in line with the two opposite zeros, the engineer cannot remedy the defect. This can be ascertained by passing a piece of fine thread through the slits, and observing whether it stands precisely over the zeros.


WEST DECLINATION
EAST DECLINATION
Annual Change
0° 5° 10° 15° 20° 25°
5' 4' 3' 2' 1'
Boston
New York
Philadelphia
Washington
Atlanta
New Orleans
Chicago
St. Louis
Austin
Denver
Santa Fe
Salt Lake City
Helena
Portland
San Francisco
Missouri R.
Mississippi R.
Rio Grande
MEXICO
STATUTE MILES
0 100 200 300 400 500 600
KILOMETERS
0 100 200 300 400 500 1000

MAGNETIC DECLINATION AND VARIATION.

Ordinarily, the needle does not lie exactly in the meridian, i. e., in a true N and S line. Its angle of deviation from that line is called the **magnetic declination.**

The magnetic declination varies (at a given time) from place to place, and (at a given place) from time to time. The change in declination, from place to place, or from time to time, is called the **magnetic variation.**

Variations with Place.

Apart from local disturbances, such as those due to iron in the soil or in mines, ferruginous gravel, trap rock, etc., the declination varies, from place to place, according to the general laws of terrestrial magnetism. Thus, in the United States (see Isogonic Chart* opposite) there is an **agonic line,** or line of no declination, marked 0°, entering the country, at present, in Michigan, and leaving it in South Carolina. Along this line, the needle normally points approximately N and S; but at points not on or near the agonic line, the north end of the needle tends toward that line. In other words, at points east of the agonic line, the declination is west, and vice versa; and this declination increases with the distance of the point from the agonic line, reaching at present 21° W in Maine, and 23° E in the State of Washington. The lines on the chart, joining points of equal declination, are called **isogonic lines.** **Where the location of these lines is somewhat uncertain, they are shown broken.**

Variations with Time.

Secular† variation. At present, at any given point in the United States, the declination becomes more westward (W declinations increasing, and E declinations decreasing) at the rate of from 1′ to 5′ per year. This is called the **secular variation,** and the amount of secular variation in one year is called the **annual change.** The chart shows dotted lines of equal annual change, from 1′ to 5′, generally crossing the isogonic lines nearly at right angles. Beyond the limits of the chart, on each side, is found the line of no annual change. Along this line, the W declination (on the Atlantic) and the E declination (on the Pacific) have reached their maxima, and are reversing to E and W respectively. The secular variation involves a westward movement of the agonic line (line of no declination), and of the line of no annual change (line of maximum declination), at the rate of about 3′ to 4′ of longitude per annum.

Diurnal variation. At any given place, the needle swings back and forth once daily, through an arc sometimes as great as 12′ or 15′, reaching its extreme eastern position generally between 8 and 8.30 A.M., its extreme western position generally between 1.15 and 1.30 P.M., and its mean position, or normal declination, generally between 10 and 11.30 A.M. and between 7 and 8 P.M. The eastern position is reached about half an hour earlier in summer and half an hour later in winter than at its average epoch; and the western position about a quarter hour earlier in summer and a quarter hour later in winter than at its average epoch.

The total arc described, between extreme eastern and extreme western position, or between morning and afternoon, is called the **diurnal range.** At Phila. the range varies from 5′ in midwinter to 12′ in midsummer; at Key West, from 3′ to 8′; and at Los Angeles from 4′ to 9′. In years of least sun-spot activity the range is about 20 per cent less, and in years of greatest sun-spot activity about 30 per cent greater, than the average range; but the diurnal variation is quite irregular, especially in the seasons of small diurnal range.

Other variations. The needle is subject also to slight monthly and annual variations, and to the influence of magnetic storms, which frequently cause deflections of 15′ or more from the normal position of the needle.

See Manual of Surveying Instructions, General Land Office, Washington, 1894, pages 84–101, and Magnetic Declination in the

---

*** Reduced from a large chart for 1902, published by the U. S. Coast & Geodetic Survey, and based upon 4700 carefully selected values obtained by the skilled observers of the Survey and by others. In regions where the positions of the lines could not be satisfactorily determined by these means, use was made of the large mass of data compiled by Mr. Henry Gannett for the General Land Office.**

† Secular, occupying a very long period of time.

United States, by Henry Gannett, in 17th Annual Report of U. S. Geological Survey, 1895-6.

Electricity, either atmospheric, or excited by rubbing the glass cover of the compass box, sometimes gives trouble. It may be removed by touching the glass with the moist tongue or finger.

DEMAGNETIZATION.

The needle, if of *soft* metal, sometimes loses part of its magnetism, and consequently does not work well. It may be restored by simply drawing the north pole of a common magnet (either straight or horseshoe) about a dozen times, from the center to the end of the south half of the needle; and the south pole, in the same way, along the north half; pressing the magnet gently upon the needle. After each stroke, remove the magnet several inches from the needle, while bringing it back to the center for making another stroke. Each half of the needle in turn, while being thus operated on, should be held flat upon a smooth hard surface. Sluggish action of the needle is, however, more generally produced by the dulling or other injury of the point of the pivot. Remagnetizing will throw the needle out of balance; which must be counteracted by the sliding wire.

In order to prevent mistakes by reading sometimes from one end, and sometimes from the other end of the needle, it is best to ALWAYS point the N of the compass-box toward the object whose bearing is to be taken; and to read off from the north end of the needle. This is also more accurate.

CONTOUR LINES.

A CONTOUR LINE is a curved hor one, every point in which represents the same level; thus each of the contour lines 88*c*, 91*c*, 94*c*, &c, Fig 1, indicates that every point in the ground through which it is traced is at the same level; and that that level or height is everywhere 88, 91, or 94 ft above a certain other level or height called *datum;* to which all others are referred.

Frequently the level of the starting point of a survey is taken as being O, or zero, or datum; and if we are sure of meeting with no points lower than it, this answers every purpose. But if there is a probability of many lower points, it is better to assume the starting point to be so far above a certain supposed datum, that none of these lower points shall become minus quantities, or *below* said supposed datum or zero. The only object in this is to avoid the liability to error which arises when some of the levels are +, or plus; and some —, or minus. Hence we may assume the level of the starting point to be 10, 100, 1000, &c, ft above datum, according to circumstances.

The vert dists between each two contour lines are supposed to be equal; and in railroad surveys through well-known districts, where the engineer knows that his actual line of survey will not require to be much changed, the dist may be 1 or 2 ft only; and the lines need not be laid down for widths greater than 100 or 200 ft on each side of his center-stakes. But in regions of which the topography is comparatively unknown; and where consequently unexpected obstacles may occur which require the line to be materially changed for a considerable dist back, the observations should extend to greater widths; and for expedition the vertical dists apart may be increased to 3, 5, or even 10 ft, depending on the character of the country, &c. Also, when a survey is made for a topographical map of a State, or of a county, vert dists of 5 or 10 ft will generally suffice.

Let the line A B, Fig 1, starting from O, represent three stations (S 1, S 2, S 3,) of the center line of a railroad survey; and let the numbers 100, 103, 101, 104, along that line denote the heights at the stakes above datum, as determined by levelling. Then the use of the contour lines is to show in the office what would be the effect changing the surveyed center line A B, by moving any part of it to the right or left hand.* Thus, if it should be moved 100 ft to the left, the starting point O would be on ground about 6 ft higher than at present; inasmuch as its level would then be about 106 ft above datum, instead of 100. Station 1 would be about 7 ft higher, or 110 ft instead of 103 Station 2 would be about 7 ft higher, or 108 ft instead of 101. If the line be thrown to the right, it will plainly be on lower ground.

The field observations for contour lines are sometimes made with the spirit-level; but more frequently by a slope-man, with a straight 12-ft graduated rod, and a slope instrument, or clinometer. At each station he lays his rod upon the ground, as nearly at right angles to the center line A B as he can judge by eye; and placing the slope instrument upon it, he takes the angle of the slope of the ground to the nearest ¼ of a degree. He also observes how far beyond the rod the slope continues the same; and with the rod he measures the dist. Then laying down the rod at that point also, he takes the next slope, and measures its length; and so on as far as may be judged necessary. His notes are entered in his field-book as shown in Fig 2; the angles of the slopes being written above the lines, and their lengths below; and should be accompanied by such remarks as the locality suggests; such as woods, rocks, marsh, sand, field, garden, across small run, &c, &c.

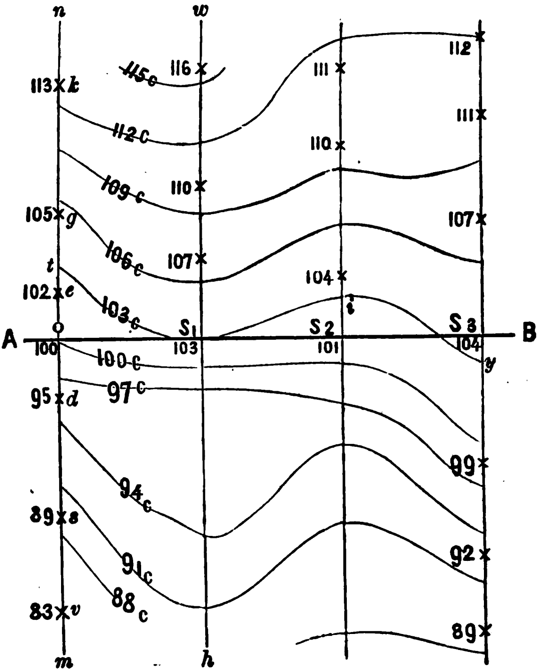


Fig. 1.

---

* In thus using the words right and left we are supposed to have our backs turned to the starting point of the survey. **In a river, the right bank** or shore is that which is on the right hand as we **descend** it, that is, in speaking of its right or left bank, we are supposed to have our backs turned towards its head, or origin; and so with a survey

It is not absolutely necessary to represent the slopes roughly in the field-book, as in Fig 2; for by using the sign + to signify "up;" — "down;" and = "level," the slopes may be written in a straight line, as in Fig 2½.

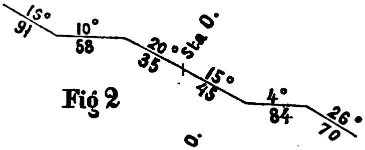


Fig 2

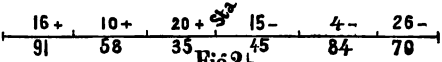


Fig 2½

The notes having been taken, the preparation of the contour lines by means of them, is of course office-work; and is usually done at the same time as the drawing of the map, &c. The field observations at each station are then separately drawn by protractor and scale, as shown in Fig 3 for the starting point O. The scale should not be less than about $\frac{1}{12}$ inch to a ft, if anything like accuracy is aimed at. Suppose that at said station the slopes to the right, taken in their order, are, as in Fig 2, 15°, 4°, and 26°; and those to the left, 20°, 10°, and 16°; and their lengths as in the same Fig. Draw a hor line *h o*, Fig 3; and consider the center of it to be the station-stake. From this point as a center, lay off these angles with a protractor, as shown on the arcs in Fig 3. Then beginning say on the right hand, with a parallel ruler draw the first dist *a c*, at its proper slope of 15°; and of its proper length, 45 ft, by scale. Then the same with *c y* and *y t*. Do the same with those on the left hand. We then have a *cross-section* of the ground at Sta O. Then on the map, as in Fig 1, draw a line as *m n*, or *h w*, at right angles to the line of road, and passing through the station-stake. On this line lay down the *hor* dists *a d*, *d s*, *s v*, *a e*, *e g*, *g k*, marking them with a small star, as is done and lettered in Fig 1, at Sta O.

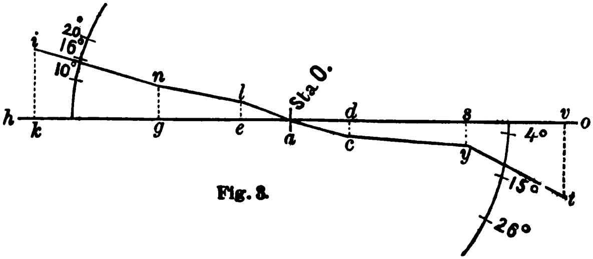


Fig. 3.

When extreme accuracy is pretended to, these hor dists must be found by measure on Fig 3; but as a general rule it will be near enough, when the slopes do not exceed 10°, to assume them to be the same as the sloping dists measured in the field. Next ascertain how high each of the points *c y t l n i* is above datum. Thus, measure by scale the vert dist *d c*. Suppose it is found to be 5 ft; or in other words, that *c* is 5 ft *below* station-stake O. Then since the level at stake O is 100 ft above datum, that at *c* must be 5 ft *less*, or 100 — 5 = 95 ft above datum; which may be marked in light lead-pencil figures on the map, as at *d*, Fig 1. Next for the point *y*, suppose we find *s y* to be 11 ft, or *y* to be 11 ft *below* stake O; then its height above datum must be 100 — 11 = 89; which also write in pencil, as at *s*. Proceed in the same way with *t*. Next going to the left hand of the station-stake, we find *e l* to be say 2 ft; but *l* is *above* the level of the station-stake, therefore its height above datum is 100 + 2 = 102 ft, as figured at *e* on the map. Let *n g* be 5 ft; then is *n*, 100 + 5 = 105 ft above datum, as marked at *g*; and so on at each station. When this has been done at several stations, we may draw in the contour lines of that portion by hand thus: Suppose they are to represent vert heights of 3 ft. Beginning at Station O (of which the height above datum is 100 ft) to lay down a contour line 103 ft above datum, we see at once that the height of 103 ft must be at *t*, or at ⅓ the dist from *e* to *g*. Make a light lead-pencil dot at *t*; and then go to the next Station 1. Here we see that the height of 103 ft coincides with the station-stake itself; place a dot there, and go to Sta 2. The level at this stake is 101; therefore the contour for 103 ft must evidently be 2 ft higher, or at *i*, $\frac{2}{5}$ of the dist from Sta 2 to +104; therefore make a dot at *i*. Then go to Sta 3. Here the level being 104 above datum, the contour of 103 must be at *y*, or $\frac{1}{5}$ of the dist from Sta 3 to +99; put a dot at *y*. Finally draw by hand a curving line through *t*, S 1, *i*, and *y*; and the contour line of 103 ft is done. All the others are prepared in the same way, one by one. The level of each must be figured upon it at short intervals along the map, as at 103 *c*, 106 *c*, &c.

Or, instead of first placing the + points on the map, to denote the slope dists actually measured upon the ground, we may at once, and with less trouble, find and show those only which represent the points *t*, S 1, *i*, *y*, &c, of the contours themselves. Thus, say that at any given station-stake, Fig 4, the level is 104; that the cross-section *c s* of the ground has been prepared as before; and that we want the hor dists from the stake, to contour lines for 94, 97, 100 ft, &c, 3 ft apart vert.

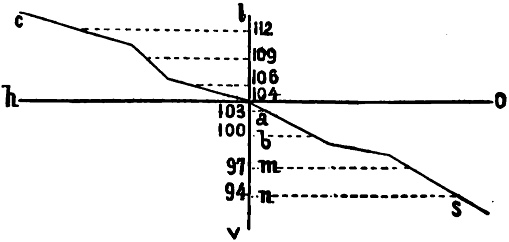


Fig. 4.

Draw a vert line *v l*, through the station-stake, and on it by scale mark levels of 94, 97, 100, &c. ft. This is readily done, inasmuch as we have the level 104 of the stake already given. Through these levels draw the hor lines *a*, *b*, *m*, *n*, &c. to the ground-slopes. Then these lines, measured by the scale, plainly give the required dists.

When the ground is very irregular transversely, the cross-sections must be taken in the field nearer together than 100 ft. The preparation of contour lines will be greatly facilitated by the use of paper ruled into small squares of not less than about $\frac{1}{10}$ inch to a side, for drawing the cross-sections upon.

When the ground is very steep, it is usual to shade such portions of the map to represent hill-side. The closer together the contours come, the steeper of course is the ground between them; and the shading should be proportionally darker at such portions. But for *working* maps it is best to omit the shading.

In surveys of wide districts, the transit instrument with a graduated vertical circle or arc, *g*, p. 291, is used for measuring the angles of slope, instead of the common slope-instrument.

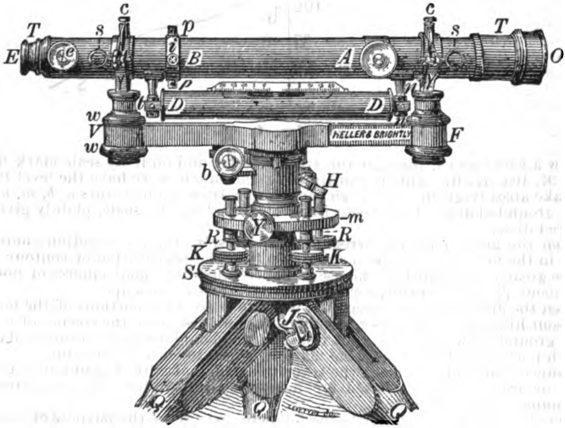
In many cases, notes similar to the following will serve the purpose of contour lines on railroad surveys.

Sta 60.................. — 3. 1 R. + 2. 1 L.
61................. + 2. 2 R. — 1. 3 L.
62.................. = 1. R. + 4. 2 L.
63..................... " "

Which means that at station 60, the slope of the ground on the right, as nearly as he can judge by eye, or by his hand-level, is about 3 ft downward, for 1 chain, or 100 ft; and on the left, about 2 ft upward in 1 chain. At 61, 2 ft up, in 2 chains to the right; and 1 ft down in 3 chains to the left. At 62, level for 1 chain to the right; and ascending 4 ft in 2 chains to the left. At 63, the same as at 62. At some spots it will be well to add a sketch of a cross-section, like Fig 2; only, instead of the angles, use ft of rise or fall, to indicate the slopes, as judged by eye, or by a hand-level. By this method, the result at every station will be somewhat in error; but these small errors will balance each other so nearly that the total may be regarded as sufficiently correct for all the purposes of a preliminary estimate of the cost of a road. When the final stakes for guiding the workmen are placed, the slopes should be carefully taken, in order to calculate the quantity of excavation accurately for payment.

THE LEVEL.

ALTHOUGH the levels of different makers vary somewhat in their details, still their principal parts will be understood from the following figure. The telescope T T rests upon two supports YY, called Ys; out of which it can be lifted, first removing the pins *s s* which confine the semicircular *clips c c*, and then opening the clips. The pins should be tied to the Ys, by pieces of string, to prevent their being lost. The *slide* of the *object-glass* O, is moved backward or forward by a rack and pinion, by means of the milled head A. The slide of the *eye-glass* E, is moved in the same way by the milled head *e*. A cylindrical tube of brass, called a *shade*, is usually furnished with each level. It is intended to be slid on to the object-end O of the telescope, to prevent the glare of the sun upon the object-glass, when the sun is low. At B is an outer ring encircling the telescope, and carrying 4 small capstan-headed screws; two of which, *p p*, are at top and bottom; while the other two, of which *i* is one, are at the sides, and at right angles to *p p*. Inside of this outer ring is another, inside of the telescope, and which has stretched across it two spider-webs, usually called the CROSS-HAIRS. These are much finer than they appear to be, being considerably magnified. They are at right angles to each other; and, in levelling, one is kept vert, and the other hor. They are liable at times to be thrown out of this position by a partial revolution of the telescope, when carrying the level, or when setting the tripod down suddenly upon the ground; but since, in levelling, the *intersection* of the hairs is directed to the target-rod, this derangement does not affect the accuracy of the work. Still it is well to keep them nearly vert and hor, by keeping the BUBBLE-TUBE D D as nearly directly over the bar V F as can be judged by eye. This enables the leveller to see that the rod-man holds his rod nearly vert, which is absolutely essential for correct levelling. If perfect verticality is desired, as is sometimes the case, when staking out work, it may be obtained (*if the instrument is in perfect adjustment, and levelled*) by sighting at a plumb-line, or other vert object, and then turning the telescope a little in its Ys, so as to bring the hair to correspond. When this is done, a short continuous scratch may be made on the telescope and Y, to save that trouble in future. Heller & Brightly, however, provide their levels with a small projection inside of the Ys, and a corresponding stop on the telescope, the contact of which insures the verticality of the hair. Should the hairs be broken by accident, they may be replaced as directed hereafter.



The small holes around the heads of the 4 small capstan-screws *p*, *i*, just referred to, are for admitting the end of a small steel pin, or lever, for turning them. If first the upper screw *p* be loosened, and then the lower one tightened, the interior ring will be lowered, and the horizontal hair with it. But on looking through the telescope they will *appear* to be raised. If first the lower one be loosened, and the upper one tightened, the hor hair will be actually raised, but apparently lowered. This is because the glasses in the eye-piece E reverse the apparent position of objects *inside* of the telescope; which effect is obviated, as regards *exterior* objects, by means of the object-glass O. This must be remembered when adjusting the cross-hairs; for if a hair appears to strike too high, it must be raised still higher; if it appears to be already too far to the right or left, it must be actually moved still more in the same direction.

This remark, however, does not apply to telescopes which make objects appear inverted.

There is no danger of injuring the hairs by these motions, inasmuch as the four screws act against the ring only, and do not come in contact with the hairs themselves.

Under the telescope is the BUBBLE-TUBE D D. One end of this tube can be raised or lowered slightly by means of the two capstan-headed nuts *n n*, one of which must be loosened before the other is tightened. On top of the bubble-tube are scratches for showing when the bubble is central in the tube. Frequently these scratches, or marks, are made on a strip of brass placed above the tube, as in our fig. There are several of them, to allow for the lengthening or shortening of the bubble by changes of temperature. At the other end of the bubble-tube are two small capstan-screws, placed on opposite sides horizontally. The circular head of one of them is shown near *t*. By means of these two screws, that end of the tube can be slightly moved hor, or to right or left. Under the bubble-tube is the BAR V F; at one end of which, as at V, are two large capstan-nuts *w w*, which operate upon a stout interior screw which forms a prolongation of the Y. The holes in these nuts are larger than the others, as they require a larger lever for turning them. If the lower nut is loosened and the upper one tightened, the Y above is raised; and that end of the telescope becomes farther removed from the bar; and vice versa. Some makers place a similar screw and nuts under both Ys; while others dispense with the nuts entirely, and substitute beneath one end of the bar a large circular milled head, to be turned by the fingers. This, however, is exposed to accidental alteration, which should be avoided.

When the portions above *m* are put upon *m*, and fastened by the screw Y, all the upper part may be swung round hor, in either direction, by loosening the **clamp-screw** H; or such motion may be prevented by tightening that screw. It frequently happens, after the telescope has been sighted very nearly upon an object, and then clamped by H, that we wish to bring the cross-hairs to coincide more precisely with the object than we can readily do by turning the telescope *by hand;* and in this case we use the **tangent-screw** *b*, by means of which a slight but steady motion may be given after the instrument is clamped. For fuller remarks on the clamp and tangent-screws, see "Transit."

The **parallel plates** *m* and S are operated by four **levelling-screws**; three of which are seen in the figure, at K K. The screws work in sockets R; which, as well as the screws, extend above the upper plate. When the instrument is placed on the ground for levelling, it is well to set it so that the lower parallel plate S shall be as nearly horizontal as can be roughly judged by eye; in order to avoid much turning of the levelling screws K K in making the upper plate *m* hor. The lower plate S, and the brass parts below it, are together called the **tripod-head**; and, in connection with three wooden legs Q Q Q, constitute the tripod. In the figure are seen the heads of wing-nuts J which confine the legs to the tripod-head. Under the center of the tripod-head should always be placed a small ring, from which a plumb-bob may be suspended. This is not needed in ordinary levelling, but becomes useful when ranging center-stakes, &c.

To adjust a Level.

This is a quite simple operation, but requires a little patience. Be careful to avoid *straining* any of the screws. The large Y nuts *w w* sometimes require some force to *start* them; but it should be applied by pressure, and not by blows. Before beginning to adjust, attend to the object-glass, as directed in the first sentence under "To adjust a plain transit."

Three adjustments are necessary; *and must be made in the following order:*

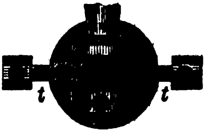
First, that of the cross-hairs; to secure that their intersection shall continue to strike the same point of a distant object, while the telescope is being turned round a complete revolution in its Ys. This is called adjusting the **line of collimation,** or sometimes, the *line of sight;* but it is not strictly the line of sight until all the adjustments are finished; for until then, the line of collimation will not serve for taking levelling sights. **If cross-hairs break,** see p 296.

Second, that of the bubble-tube D D, to place it parallel to the line of collimation. previously adjusted; so that when the bubble stands at the centre of its tube, indicating that it is level, we know that our sight through the telescope is hor. To replace **broken bubble tube,** see p 296.

Third, that of the Ys, by which the telescope and bubble-tube are supported; so that the bubble-tube, and line of sight, shall be perp to the vert axis of the instrument; so as to *remain* hor while the telescope is pointed to objects in diff directions, as when taking back and fore sights.

To make the first adjustment, or that of the cross-hairs, plant the tripod *firmly* upon the ground. In this adjustment it is not necessary to level the instrument. Open the clips of the Ys; unclamp; draw out the eye-glass E, until the cross-hairs are seen *perfectly clear;* sight the telescope toward some clear distant point of an object; or still better, toward some straight line, whether vert or not. Move the object-glass O, by means of the milled head A, so that the object shall be clearly seen, **without parallax,** that is, without any apparent dancing about of the cross-hairs, if the eye is moved a little up or down or sideways. To secure this, the object-glass alone is moved to suit different distances; the eye-glass is not to be changed after it is once properly fixed upon the cross-hairs. The neglect of parallax is a source of frequent errors in levelling. Clamp; and, by means of the tangent-screw *b*, bring either one of the cross-hairs to coincide *precisely* with the object. Then gently, and without jarring, revolve the telescope half-way round in its Ys. When this is done, if the hair still coincides precisely with the object, it is in adjustment; and we proceed to try the other hair. But if it does not coincide, then by means of the 4 screws *p*, *i*, move the ring which carries the hairs, so as to rectify; as nearly as can be judged by eye, only *one-half* of the error; remembering that the ring must be moved in the direction opposite to what *appears* to be the right one; unless the telescope is an inverting one. Then turn the telescope back again to its former position; and again by the tangent-screw bring the cross-hair to coincide with the object. Then again turn the telescope half-way round as before. The hair will now be found to be more nearly in its right place, but, in all probability, not precisely so; inasmuch as it is difficult to estimate one-half the error accurately by eye. Therefore a little more alteration of the ring must be made; and it may be necessary to repeat the operation several times, before the adjustment is perfect. Afterward treat the other hair in precisely the same manner. When both are adjusted, their intersection will strike the same precise spot while the telescope is being turned *entirely* round in its Ys. This must be tried before the adjustment can be pronounced perfect; because at times the adjustment of the second hair, slightly deranges that of the first one; especially if both were much *out* in the beginning.

To make the second adjustment, or to place the bubble-tube parallel to the line of collimation. This consists of two distinct adjustments, one vert, and one hor. The first of these is effected by means of the two nuts *n n* on the vert screw at one end of the tube; and the second by the two hor screws at the other end, *t*, of the tube. Looking at the bubble-tube endwise, from *t* in the foregoing Fig, its two hor adjusting-screws *t t* are seen as in this sketch. The larger capstan-headed nut *below*, has nothing to do with the adjustments; it merely holds the end of the tube in its place.



To make the vert adjustment of the bubble-tube, by means of the two nuts *nn*. Place the telescope over a diagonal pair of the levelling-screws K K; and clamp it there. Open the clips of the Ys; and by means of the levelling-screws bring the bubble to the center of its tube. Lift the telescope gently out of the Ys, turn it end for end, and put it back again in its reversed position. This being done, if the bubble still remains at the center of its tube, this adjustment is in order; but if it moves toward one end, that end is too high, and must be lowered; or else the other end must be raised. First, correct *half* the error by means of the levelling-screws K K, and then the remaining half by means of the two small capstan-headed nuts *n n*. To *raise* the end *n*, first loosen the upper nut and then tighten the lower one; to do which, turn each nut so that the *near* side moves toward your *right*. To *lower* it, first loosen the lower nut, then tighten the upper one, moving the *near* side of each nut toward your *left*. Having thus brought the bubble to the middle again, again lift the telescope out of its Ys; turn it end for end, and replace it. The bubble will now settle nearer the center than it did before, but will probably require still further adjustment. If so, correct *half* the remaining error by the levelling-screws, and half by the nuts, as before; and so continue to repeat the operation until the bubble remains at the center in both positions. For another method, see "To adjust the long bubble-tube," p 295.

Horizontal adjustment of bubble-tube; to see that its axis is in the same plane th that of the telescope, as it usually is in new instruments. It is not easily deranged, except by blows. Have the bubble-tube, as nearly as may be, directly under the telescope, or over the center of the bar V F. Bring the telescope over two of the levelling-screws K K; clamp it there; center the bubble with said screws; turn the telescope in its Ys, say about ¼ inch, bringing the bubble-tube out from over the center of the bar, first on one side, then on the other. If the bubble stays centered while so swung out, this adjustment is correct. If it runs toward *opposite* ends of its tube when swung out on opposite sides of the center, move the end *t* of the tube by the two horizontal screws *t t* until the bubble stays centered when the tube is swung out on either side. If the bubble runs toward the *same* end of its tube *on both sides*, the tube is not truly cylindrical, but slightly conical,* so that if the telescope is turned in its Ys the bubble will leave the center, even when the horizontal adjustment is correct. It is known to be correct, in such tubes, if the bubble runs *the same distance* from the center when swung out the same distance on each side.

Having made the horizontal adjustment, turn the telescope back in its Ys until the bubble-tube is over the bar. Repeat the *vertical* adjustment (p 308), which may have become deranged in making this horizontal one. Persevere until both adjustments are found to be correct at the same time.

To make the third adjustment, or to adjust the heights of the Ys, so as to make the line of collimation parallel to the bar V F, or perp to the vert axis of the instrument. The other adjustments being made, fasten down the clips of the Ys. Make the instrument nearly level by means of all four of the levelling-screws K. Place the telescope over two of the levelling-screws which stand diagonally; and leave it there unclamped. Then bring the bubble to the center of its tube, by the two levelling-screws. Swing the upper part of the instrument half-way around, so that the telescope shall again stand over the same two screws; but end for end. This done, if the bubble leaves the center, bring it *half-way* back by the large capstan nuts *w*, *w*; and the other half by the two levelling-screws. Remember that to raise the Y, and the end of the bubble over *w*, *w*, the lower *w* must be loosened; and the upper one tightened; and vice versa. Now place the telescope over the other diagonal pair of levelling-screws; and repeat the whole operation with them. Having completed it, again try with the first pair; and so keep on until the bubble remains at the center of its tube, in every position of the telescope.

Correct levelling may be performed even if all the foregoing adjustments are out of order; provided each fore-sight be taken *at precisely the same distance from the instrument as the back-sight is*. But a good leveller will keep his instrument always in adjustment; and will test the adjustments at least once a day when at work. As much, however, depends upon the rodman, or target-man, as upon the leveller. A rodman who is careless about holding the rod vert, or about reading the sights correctly, should be discharged without mercy.

The levelling-screws in many instruments become very hard to turn if dirty. Clean with water and a tooth-brush. Use no oil on field instruments.

Forms for level note-books. When the distance is short, so as not to require two sets of books, the following is perhaps as good as any.

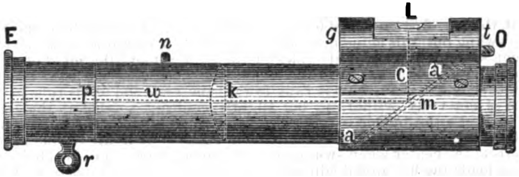
No. of Station.	Back sights.	Fore sights.	Diff.	Level.	Grade.	Cut.	Fill.

But on public works generally the original field-books have only the first five cols After the grades have been determined by means of the profile drawn from these, the results are placed in another book, which has only the first col and the last four. In both cases, the right-hand page is reserved for memoranda. The writer considers it best, both with the level and with the transit, to consider the term "STATION" to apply to the whole dist between two consecutive stakes; and that its number shall be that written on the last stake. Thus, with the transit, Station 6 means the dist from stake 5 to stake 6; that it has a bearing or course of so and so; and its length is so and so. And with the level, Station 6 also means the dist from stake 5 to stake 6; the back-sight for that dist being taken at stake 5, and the fore-sight on stake 6; and that the level, grade, cut, or fill is that at stake 6. The starting-point of the survey, whether a stake, or any thing else, we call and mark simply 0.

---

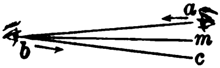
* This defect can be remedied only by removing the tube and inserting a correctly-shaped one, and this is best done by an instrument-maker; but correct work can be done in spite of it, thus: Make all the adjustments as nearly correct as possible. Level the instrument. By turning the telescope in its Ys, make the vertical hair coincide with a plumb-line or other vertical line, and make a short continuous knife-scratch on the collar nearest the object-glass, and on the adjoining Y. Lift the telescope out of its Ys, turn it end for end, replace it in its Ys; again bring the upright hair vertical, and make on the other Y a scratch coinciding with that on the collar. Then, in levelling or in adjusting, always see that the scratch on the collar coincides with that on the adjoining Y when the bubble-tube is under the telescope.

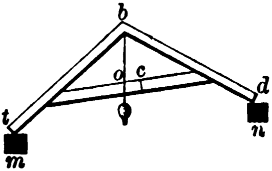
THE HAND-LEVEL.



THIS very useful little instrument, as arranged by Professor Locke, of Cincinnati, is but about five or six inches long. Simply holding it in one hand, and looking through it in any direction, we can ascertain at once, approximately, what objects are at the same level with the eye. E is the eye end: and O the object end. L is a small level, enclosed in a kind of brass boxing *t g*, the bottom of which is open, with a corresponding opening under it, through the top of the main tube E O. Immediately at the bottom of the small level L, is a cross-wire, stretched across said opening, and carried by a small plate, which, for adjusting the wire, can be pushed backward a trifle by tightening the screw *t*, or pushed forward by a small spring within the boxing, near *g*, when the screw *t* is loosened. At *m* is a small semicircular mirror *a a*, silvered on the back *m*. This is placed at an angle of 45°, and occupies one-half the width of the tube E O. Through the forementioned openings, the images of the cross-wire and of the level-bubble are reflected down on the unsilvered face *a a* of the mirror, and thence to the eye, as shown by the single dotted lines *c* and *w*; and when the instrument is adjusted, and held level, the wire will appear to be at the center of the bubble. At *k* is one-half of a plano-convex lens, at the inner end of a short tube *k p*, which may be moved backward or forward by a pin *n*, projecting through a short slit in the main tube. By this means the image of the cross-wire is rendered distinct; and the half lens must be moved until, when viewing an object, the wire shall show no parallax; but appear steady against the object when the eye is slightly moved up or down. At each end of the tube E O is a circular piece of plain glass for excluding dust.

To adjust the hand-level, first fix two precisely level marks, say from 50 feet to 100 yards apart. This being done, rest the instrument against one of the level marks, and take sight at the other. If, then, the wire does not appear to be precisely at the center of the bubble, move it slightly backward or forward, as the case may be, by the screw *t*, until it does so appear. The two level marks may be fixed by means of the hand-level itself, even if it is entirely out of adjustment, thus: First, by the pin *n* arrange the half lens *k*, so as to show the wire distinctly and without parallax. Then holding the level steadily, at any selected object, as *a*, so that the wire appears to cut the center of the bubble, see where it cuts any other convenient object, as *b*. Then go to *b*, and from it, in like manner, sight back toward *a*. If the instrument is in adjustment, the wire will cut *a*; but if not, it will strike either above it or below it, as at *c*. In either case, make a mark *m*, half-way between *c* and *a*. Then *b* and *m* will be the two level marks required. With care, these adjustments, when once made, will remain in order for years. The instrument generally has a small ring *r*, for hanging it around the neck: it is not adapted to very accurate work, but admirably so for exploring a route. The height of a bare hill can be found by beginning at the foot, and sighting ahead at any little chance object which the cross-wire may strike, as a pebble, twig, &c; then going forward, stand at that object, and fix the wire on another one still farther on, and so to the top. At each observation we plainly rise a height equal to that of the eye, say $5\frac{1}{4}$ feet, or whatever it may be. Whether going up or down it, if the hill is covered with grass, bushes, &c, a target rod must be used for the fore-sights; and the constant height of the eye may be regarded as the back-sight at each station. An attachment may be made for screwing the level to a small ball and socket on top of a cane, or of a longer stick, for occasional use, when rather more accuracy is desired.





To adjust a builder's plumb-level, *t b d*; stand it upon any two supports *m* and *n*, and mark where the plumb-line cuts at *o*. Then reverse it, placing the foot *t* upon *n*, and *d* upon *m*, and mark where the line now cuts at *c*. Half-way between *o* and *c* make the permanent mark. Whenever the line cuts this, the feet *t* and *d* are on a level.

To adjust a slope-instrument, or clinometer. As usually made, the bubble-tube is attached to the movable bar by a screw near each end, and the head of one of the screws conceals a small slot in the bar, which allows a slight vert motion to the screw when loose, and with it to that end of the tube. Therefore, in order to adjust the bubble, this screw is first loosened a little, and then moved up or down a trifle, as may be reqd. It is then tightened again.

LEVELLING BY THE BAROMETER.

1. Many circumstances combine to render the results of this kind of levelling unreliable where great accuracy is required. This fact was most conclusively proved by the observations made by Captain T. J. Cram, of the U. S. Coast Survey. See Report of U. S. C. S., vol. for 1854. It is difficult to read off from an aneroid (the kind of barom generally employed for engineering purposes) to within from two to five or six ft, depending on its size. The moisture or dryness of the air affects the results; also winds, the vicinity of mountains, and the daily atmospheric tides, which cause incessant and irregular fluctuations in the barom. A barom hanging quietly in a room will often vary $\frac{1}{10}$ of an inch within a few hours, corresponding to a diff of elevation of nearly 100 ft. No formula can possibly be devised that shall embrace these sources of error. The variations dependent upon temperature, latitude, &c, are in some measure provided for; so that with *very delicate* instruments, a *skilful* observer may measure the diff of altitude of two points close together, such as the bottom and top of a steeple, with a tolerable confidence that he is within two or three feet of the truth. But if as short an interval as even a few hours elapses between his two observations, such changes may occur in the condition of the atmosphere that he *may* make the top of the steeple to be lower than its bottom; or at least, cannot feel by any means certain that he is not ten or twenty ft in error; and this may occur without any *perceptible* change in the atmosphere. Whenever practicable, therefore, there should be a person at each station, to observe at both points at the same time. Single observations at points many miles apart. and made on different days, and in different states of the atmosphere, are of little value. In such cases the mean of many observations, extending over several days, weeks, or months, and made when the air is apparently undisturbed, will give tolerable approximations to the truth. In the tropics the range of the atmospheric pres is much less than in other regions, seldom exceeding ½ inch at any one spot; also more regular in time, and, therefore, less productive of error. Still, the barometer, especially either the aneroid, or Bourdon's metallic, may be rendered highly useful to the civil engineer, in cases where great accuracy is not demanded. By hurrying from point to point, and especially by repeating, he can form a judgment as to which of two summits is the lowest. Or a careful observer, keeping some miles ahead of a surveying party, may materially lessen their labors, especially in a rough country, by selecting the general route for them in advance. The accounts of the agreement within a few inches, in the measurements of high mountains, by diff observers, at diff periods; and those of ascertaining accurately the grades of a railroad, by means of an aneroid, while riding in a car, will be believed by those only who are ignorant of the subject. Such results can happen only by chance.

When possible, the observations at different places should be taken at the same time of day, as some check upon the effects of the daily atmospheric tides; and in very important cases, a memorandum should be made of the year, month, day, and hour, as well as of the state of the weather, direction of the wind, latitude of the place, &c, to be referred to an expert, if necessary.

The effects of latitude are not included in any of our formulas. When reqd they may be found in the table page 314. Several other corrections must be made when great accuracy is aimed at; but they require extensive tables.

In rapid railroad exploring, however, such refinements may be neglected, inasmuch as no approach to such accuracy is to be expected; but on the contrary, errors of from 1 to 10 or more feet in 100 of height, will frequently occur.

As a very rough average we may assume that the barometer falls $\frac{1}{10}$ inch for every 90 feet that we ascend above the level of the sea, up to 1000 ft. But in fact its rate of fall decreases continually as we rise; so that at one mile high it falls $\frac{1}{10}$ inch for about 106 ft rise. Table 2 shows the true rate.

To ascertain the diff of height between two points.

RULE 1. Take readings of the barom and therm (Fah) **in the shade** at both stations. Add together the two readings of the barom, and div their sum by 2, for their mean; which call *b*. Do the same with the two readings of the thermom, and call the mean *t*. Subtract the least reading of the barom from the greatest; and call the diff *d*. Then mult together this diff *d*; the number from the next Table No. 1, opposite *t*; and the constant number 30. Div the prod by *b*. Or

$$\begin{matrix}\text{Height}\\ \text{in feet}\end{matrix} = \frac{\begin{matrix}\text{Diff } (d) \text{ of}\\ \text{barom}\end{matrix} \times \begin{matrix}\text{Tabular number opposite}\\ \text{mean } (t) \text{ of thermom}\end{matrix} \times \text{Constant } 30.}{\text{mean } (b) \text{ of barom.}}$$

EXAMPLE. Reading of the barom at lower station, 26.64 ins; and at the upper sta 20.82 ins. Thermom at lowest sta, 70°; at upper sta, 40°. What is the diff in height of the two stations? Here,

Barom, 26.64
" 20.82
2)47.46
23.73 mean of bar, or *b*.

Also,

Therm, 70°
" 40°
2)110
55° mean of therm, or *t*.

The tabular number opposite 55°, is 917.2.

Again, $\overset{\text{Bar.}}{26.64} - \overset{\text{Bar.}}{20.82} = 5.82$, diff of bar; or *d*. Hence,

$$\begin{matrix}\text{Height}\\ \text{in feet}\end{matrix} = \frac{\overset{d,}{5.82} \times \overset{\text{Tab No.}}{917.2} \times \overset{\text{Con.}}{30}}{23.73 \text{ (or } b)} = \frac{160143.12}{23.73} = 6748.5 \text{ ft; answer.}$$

Then correct for latitude, if more accuracy is reqd, by rule on next page.

The screw at the back of an aneroid is for adjusting the index by a standard barom. After this has been done it must by no means be meddled with. In some instruments specially made to order with that intention, this screw may be used also for turning the index back, after having risen to an elevation so great that the index has reached the extreme limit of the graduated arc. After thus turning it back, the indications of the index at greater heights must be added to that attained when it was turned back.

TABLE 1. For Rule 1.

Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.
0°	801.1	30°	864.4	60°	927.7	90°	991.0
1	803.2	31	866.5	61	929.8	91	993.1
2	805.3	32	868.6	62	931.9	92	995.2
3	807.4	33	870.7	63	934.0	93	997.3
4	809.5	34	872.8	64	936.1	94	999.4
5	811.7	35	874.9	65	938.2	95	1001.6
6	813.8	36	877.0	66	940.3	96	1003.7
7	815.9	37	879.2	67	942.4	97	1005.8
8	818.0	38	881.3	68	944.5	98	1007.9
9	820.1	39	883.4	69	946.7	99	1010.0
10	822.2	40	885.4	70	948.8	100	1012.1
11	824.3	41	887.5	71	950.9	101	1014.2
12	826.4	42	889.6	72	953.0	102	1016.3
13	828.5	43	891.7	73	955.1	103	1018.4
14	830.6	44	893.8	74	957.2	104	1020.5
15	832.8	45	896.0	75	959.3	105	1022.7
16	834.9	46	898.1	76	961.4	106	1024.8
17	837.0	47	900.2	77	963.5	107	1026.9
18	839.1	48	902.3	78	965.6	108	1029.0
19	841.2	49	904.5	79	967.7	109	1031.1
20	843.3	50	906.6	80	969.9	110	1033.2
21	845.4	51	908.7	81	972.0	111	1035.3
22	847.5	52	910.8	82	974.1	112	1037.4
23	849.6	53	913.0	83	976.2	113	1039.5
24	851.8	54	915.1	84	978.3	114	1041.6
25	853.9	55	917.2	85	980.4	115	1043.8
26	856.0	56	919.3	86	982.6	116	1045.9
27	858.1	57	921.4	87	984.7	117	1048.0
28	860.2	58	923.5	88	986.8	118	1050.1
29	862.3	59	925.6	89	988.9	119	1052.2

RULE 2. **Belville's short approx rule** is the one best adapted to rapid field use, namely, add together the two readings of the barom only. Also find the diff between said two readings; then, **as the sum of the two readings is to their diff, so is 55000 feet to the reqd altitude.**

Correction for latitude is usually omitted where great accuracy is not required. To apply it, first find the altitude by the rule, as before. Then divide it by the number in the following table opposite the latitude of the place. (If the two places are in different latitudes, use their mean.) *Add* the quotient to the altitude if the latitude is *less* than 45°. *Subtract* it if the latitude is *more* than 45°. No correction required for latitude 45°.

Table of corrections for latitude.

Lat.		Lat.		Lat.		Lat.		Lat.		Lat.	
0°	352	14°	399	28°	630	42°	3367	54°	1140	68°	490
2	354	16	416	30	705	44	10101	56	941	70	460
4	356	18	436	32	804	45	∞	58	804	72	436
6	360	20	460	34	941	46	10101	60	705	74	416
8	367	22	490	36	1140	48	3367	62	630	76	399
10	375	24	527	38	1458	50	2028	64	572	78	386
12	386	26	572	40	2028	52	1458	66	527	80	375

Levelling by Barometer; or by the boiling point.

RULE 3. The following table, No. 2, enables us to measure heights either by means of boiling water, or by the barom. The third column shows the approximate altitude above sea-level corresponding to diff heights, or readings of the barom; and to the diff degrees of Fahrenheit's thermom, at which water boils in the open air. Thus when the barom, under undisturbed conditions of the atmosphere, stands at 24.08 inches, or when pure rain or distilled water boils at the temp of 201° Fah; the place is about 5764 ft above the level of the sea, as shown by the table. It is therefore very easy to find the *diff* of altitude of two places. Thus: take out from table No 2, the altitudes opposite to the two boiling temperatures; or to the two barom readings. Subtract the one opposite the lower reading, from that opposite the upper reading. The rem will be the reqd height, as a rough approximation. To correct this, add together the two therm readings; and div the sum by 2, for their mean. From table for temperature, p 316, take out the number opposite this mean. Mult the approximate height just found, by this tabular number. Then correct for lat if reqd.

Ex. The same as preceding; namely, barom at lower sta, 26.64; and at upper sta, 20.82. Thermom at lower sta, 70° Fah; and at the upper one, 40°. What is the diff of height of the two stations?

	Alt.
Here the tabular altitudes are, for 20.82............	9579
and for 26.64............	3115
	6464 ft, approx height.

To correct this, we have $\frac{70^\circ + 40^\circ}{2} = \frac{110^\circ}{2} = 55^\circ$ mean; and in table p 316, opp to 55°, we find 1.048. Therefore $6464 \times 1.048 = 6774$ ft, the reqd height.

This is about 26 ft more than by Rule 1; or nearly .4 of a ft in each 100 ft.

At 70° Fah, pure water will boil at 1° less of temp, for an average of about 550 ft of elevation above sea-level, up to a height of ½ a mile. At the height of 1 mile, 1° of boiling temp will correspond to about 560 ft of elevation. In table p 315 the mean of the temps at the two stations is assumed to be 32° Fah; at which no correction for temp is necessary in using the table; hence the tabular number opposite 32°, in table p 316, is 1.

This diff produced in the temp of the *boiling point*, by change of elevation, must not be confounded with that of the *atmosphere*, due to the same cause. The air becomes cooler as we ascend above sea-level, at the rate (very roughly) of about 1° Fah for every 200 ft near sea-level, to 350 ft at the height of 1 mile.

The following table, No. 2, (so far as it relates to the barom,) was deduced by the writer from the standard work on the barom by Lieut.-Col. R. S. Williamson, U. S. army.*

---

* Published by permission of Government in 1868 by Van Nostrand, N. Y.

TABLE 2.

Levelling by Barometer; or by the boiling point.

Assumed temp in the shade 32° Fah. If not 32°, mult barom alt as per Table, p 316

Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.
184°	16.79	15221	.3	19.66	11083	.6	22.93	7048	.9	26.59	3164
.1	16.83	15159	.4	19.70	11029	.7	22.98	6991	206	26.64	3115
.2	16.86	15112	.5	19.74	10976	.8	23.02	6945	.1	26.69	3066
.3	16.90	15050	.6	19.78	10923	.9	23.07	6888	.2	26.75	3007
.4	16 93	15003	.7	19.82	10870	199	23.11	6843	.3	26.80	2958
.5	16 97	14941	.8	19.87	10804	.1	23.16	6786	.4	26.86	2899
.6	17.00	14895	.9	19.92	10738	.2	23.21	6729	.5	26.91	2850
.7	17.04	14833	192	19.96	10685	.3	23.26	6673	.6	26.97	2792
.8	17.08	14772	.1	20.00	10633	.4	23.31	6617	.7	27.02	2743
.9	17.12	14710	.2	20.05	10567	.5	23.36	6560	.8	27.08	2685
185	17.16	14649	.3	20.10	10502	.6	23.40	6516	.9	27.13	2637
.1	17.20	14588	.4	20.14	10450	.7	23 45	6460	207	27.18	2589
.2	17.23	14543	.5	20.18	10398	.8	23.49	6415	.1	27.23	2540
.3	17.27	14482	.6	20.22	10346	.9	23.54	6359	.2	27.29	2483
.4	17.31	14421	.7	20.27	10281	200	23.59	6304	.3	27.34	2435
.5	17.35	14361	.8	20.31	10230	.1	23 64	6248	.4	27.40	2377
.6	17.38	14315	.9	20.35	10178	.2	23.69	6193	.5	27.45	2329
.7	17.42	14255	193	20.39	10127	.3	23.74	6137	.6	27.51	2272
.8	17.46	14195	.1	20.43	10075	.4	23.79	6082	.7	27.56	2224
.9	17.50	14135	.2	20.48	10011	.5	23 84	6027	.8	27.62	2167
186	17.54	14075	.3	20.53	9947	.6	23.89	5972	.9	27.67	2120
.1	17.58	14015	.4	20.57	9896	.7	23.94	5917	208	27.73	2063
.2	17.62	13956	.5	20.61	9845	.8	23.98	5874	.1	27.78	2016
.3	17.66	13896	.6	20.65	9794	.9	24.03	5819	.2	27.84	1959
.4	17.70	13837	.7	20.69	9743	201	24.08	5764	.3	27.89	1912
.5	17.74	13778	.8	20.73	9693	.1	24.13	5710	.4	27.95	1856
.6	17.78	13718	.9	20.77	9642	.2	24.18	5656	.5	28.00	1809
.7	17.82	13660	194	20.82	9579	.3	24.23	5602	.6	28.06	1753
.8	17.86	13601	.1	20.87	9516	.4	24.28	5547	.7	28.11	1706
.9	17.90	13542	.2	20.91	9466	.5	24.33	5494	.8	28.17	1650
187	17.93	13498	.3	20.96	9403	.6	24.38	5440	.9	28.23	1596
.1	17.97	13440	.4	21.00	9353	.7	24.43	5386	209	28.29	1539
.2	18.00	13396	.5	21.05	9291	.8	24.48	5332	.1	28.35	1483
.3	18.04	13338	.6	21.09	9241	.9	24.53	5279	.2	28.40	1437
.4	18.08	13280	.7	21.14	9179	202	24.58	5225	.3	28.45	1391
.5	18.12	13222	.8	21.18	9130	.1	24.63	5172	.4	28.51	1336
.6	18.16	13164	.9	21.22	9080	.2	24.68	5119	.5	28.56	1290
.7	18.20	13106	195	21.26	9031	.3	24.73	5066	.6	28.62	1235
.8	18.24	13049	.1	21.31	8969	.4	24.78	5013	.7	28.67	1189
.9	18.28	12991	.2	21.35	8920	.5	24.83	4960	.8	28.73	1134
188	18.32	12934	.3	21.40	8859	.6	24.88	4907	.9	28.79	1079
.1	18.36	12877	.4	21.44	8810	.7	24.93	4855	210	28.85	1025
.2	18.40	12820	.5	21.49	8749	.8	24.98	4802	.1	28.91	970
.3	18.44	12763	.6	21.53	8700	.9	25.03	4750	.2	28.97	916
.4	18.48	12706	.7	21.58	8639	203	25.08	4697	.3	29.03	862
.5	18.52	12649	.8	21.62	8590	.1	25.13	4645	.4	29.09	808
.6	18.56	12593	.9	21.67	8530	.2	25.18	4593	.5	29.15	754
.7	18.60	12536	196	21.71	8481	.3	25.23	4541	.6	29.20	709
.8	18.64	12480	.1	21.76	8421	.4	25.28	4489	.7	29.25	664
.9	18.68	12424	.2	21.81	8361	.5	25.33	4437	.8	29.31	610
189	18.72	12367	.3	21.86	8301	.6	25.38	4386	.9	29.36	565
.1	18.76	12311	.4	21.90	8253	.7	25.43	4334	211	29.42	512
.2	18.80	12256	.5	21.95	8193	.8	25.49	4272	.1	29.48	458
.3	18.84	12200	.6	21.99	8145	.9	25.54	4221	.2	29.54	405
.4	18.88	12144	.7	22.04	8086	204	25.59	4169	.3	29.60	352
.5	18.92	12089	.8	22.08	8038	.1	25.64	4118	.4	29.65	306
.6	18.96	12033	.9	22.13	7979	.2	25.70	4057	.5	29.71	255
.7	19.00	11978	197	22.17	7932	.3	25.76	3996	.6	29.77	202
.8	19.04	11923	.1	22.22	7873	.4	25.81	3945	.7	29 83	149
.9	19.08	11868	.2	22.27	7814	.5	25.86	3894	.8	29.88	105
190	19.13	11799	.3	22.32	7755	.6	25.91	3844	.9	29.94	52
.1	19.17	11745	.4	22.36	7708	.7	25.96	3793	212	30.00	sea lev=0
.2	19.21	11690	.5	22.41	7649	.8	26.01	3742		Below sea level.	
.3	19.25	11635	.6	22.45	7602	.9	26.06	3692	.1	30.06	— 52
.4	19.29	11581	.7	22.50	7544	205	26.11	3642	.2	30.12	—104
.5	19.33	11527	.8	22.54	7498	.1	26.17	3582	.3	30.18	—156
.6	19.37	11472	.9	22.59	7439	.2	26.22	3532	.4	30.24	—209
.7	19.41	11418	198	22.64	7381	.3	26.28	3472	.5	30.30	—261
.8	19.45	11364	.1	22.69	7324	.4	26.33	3422	.6	30.35	—304
.9	19.49	11310	.2	22.74	7266	.5	26.38	3372	.7	30.41	—356
191	19.54	11243	.3	22.79	7208	.6	26.43	3322	.8	30.47	—408
.1	19.58	11190	.4	22.84	7151	.7	26.48	3273	.9	30.53	—459
.2	19.62	11136	.5	22.89	7093	.8	26.54	3213	213	30.59	—511

Corrections for temperature; to be used in connection with Rule 3, when greater accuracy is necessary. Also in connection with Table 2 when the temp is not 32°.

Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by
Zero.	.933	28°	.992	56°	1.050	84°	1.108
2°	.937	30	.996	58	1.054	86	1.112
4	.942	32	1.000	60	1.058	88	1.117
6	.946	34	1.004	62	1.062	90	1.121
8	.950	36	1.008	64	1.066	92	1.125
10	.954	38	1.012	66	1.071	94	1.129
12	.958	40	1.016	68	1.075	96	1.133
14	.962	42	1.020	70	1.079	98	1.138
16	.967	44	1.024	72	1.083	100	1.142
18	.971	46	1.028	74	1.087	102	1.146
20	.975	48	1.032	76	1.091	104	1.150
22	.979	50	1.036	78	1.096	106	1.154
24	.983	52	1.041	80	1.100	108	1.158
26	.987	54	1.046	82	1.104	110	1.163

SOUND.

The velocity of sound in quiet open air, has been experimentally determined to be very approximately 1090 feet per second, when the temperature is at freezing point, or 32° Fahsenheit. For every degree Fahrenheit of increase of temperature, the velocity increases by from ½ foot to 1¼ feet per second, according to different authorities. Taking the increase at 1 foot per second for each degree (which agrees closely with theoretical calculations), we have

at	— 30°	Fahr	1030	feet	per sec	= 0.1951	mile	per sec	= 1	mile in	5.13	seconds.
"	— 20°	"	1040	"	"	= 0.1970	"	"	= 1	"	5.08	"
"	— 10°	"	1050	"	"	= 0.1989	"	"	= 1	"	5.03	"
"	0	"	1060	"	"	= 0.2008	"	"	= 1	"	4.98	"
"	10°	"	1070	"	"	= 0.2027	"	"	= 1	"	4.93	"
"	20°	"	1080	"	"	= 0.2045	"	"	= 1	"	4.88	"
"	32°	"	1092	"	"	= 0.2068	"	"	= 1	"	4.83	"
"	40°	"	1100	"	"	= 0.2083	"	"	= 1	"	4.80	"
"	50°	"	1110	"	"	= 0.2102	"	"	= 1	"	4.78	"
"	60°	"	1120	"	"	= 0.2121	"	"	= 1	"	4.73	"
"	70°	"	1130	"	"	= 0.2140	"	"	= 1	"	4.68	"
"	80°	"	1140	"	"	= 0.2159	"	"	= 1	"	4.63	"
"	90°	"	1150	"	"	= 0.2178	"	"	= 1	"	4.59	"
"	100°	"	1160	"	"	= 0.2197	"	"	= 1	"	4.55	"
"	110°	"	1170	"	"	= 0.2216	"	"	= 1	"	4.51	"
"	120°	"	1180	"	"	= 0.2235	"	"	= 1	"	4.47	"

If the air is calm, fog or rain does not appreciably affect the result; but winds do. Very loud sounds appear to travel somewhat faster than low ones. The watchword of sentinels has been heard across still water, on a calm night, 10½ miles; and a cannon 20 miles. Separate sounds, at intervals of $\frac{1}{16}$ of a second, cannot be distinguished, but appear to be connected. The distances at which a speaker can be understood, in front, on one side, and behind him. are about as 4, 3, and 1.

Dr. Charles M. Cresson informs the writer that, by repeated trials, he found that in a Philadelphia gas main 20 inches diameter and 16000 feet long, laid and covered in the earth, but empty of gas, and having one horizontal bend of 90°, and of 40 feet radius. the sound of a pistol-shot travelled 16000 feet in precisely 16 seconds, or 1000 feet per second. The arrival of the sound was barely audible; but was rendered very apparent to the eye by its blowing off a diaphragm of tissue-paper placed over the end of the main.

Two boats anchored some distance apart may serve as a base line for triangulating objects along the coast; the distance between them being first found by firing guns on board one of them.

In water the velocity is about 4708 feet per second, or about 4 times that in air. **In woods,** it is from 10 to 16 times; and **in metals,** from 4 to 16 times greater than in air, according to some authorities.

Approximate expansion of solids by heat; and their melting points by Fahrenheit's thermometer.‡

	For 1 degree.		For 180 degrees.*		Melting point in Deg.‡
	1 part in	⅛ inch in	1 part in	⅛ inch in	
Fire-brick	365220	3804 ft.	2029	21.14 ft.	
Granite from	187560	1954	1042	10.85	
Granite to	228060	2375	1267	13.20	
Glass rod	221400	2306	1230	12.81	
Glass tube	214200	2231	1190	12.40	
" crown	211500	2203	1175	12.24	
" plate	209700	2184	1165	12.13	
Platina	208800	2175	1160	12.08	4593
Marble, granular, white, dry	173000	1802	961	10.00	
" " " moist	128000	1333	711	7.41	
" " black, compact	405000	4219	2250	23.44	
Antimony	166500	1722	925	9.63	955
Cast iron	162000‖	1688	900	9.38	1920 to
Slate	173000	1802	961	10.00	2800
Steel	151200	1575	840	8.75	2370 to
" blistered	159840	1665	888	9.25	2550
" untempered	167400	1744	930	9.69	
" tempered yellow	131400	1369	730	7.60	
" hardened	146880	1530	816	8.50	
" annealed	147600	1537	820	8.54	
Iron, rolled	149940‖	1562	833	8.68	3000 to
" soft, forged	147420	1536	819	8.53	3500
" wire	146340	1524	813	8.55	
Bismuth	129600	1350	720	7.50	506
Gold, annealed	123120	1282	684	7.12	2016
Copper average	104400	1088	580	6.04	2000
Sandstone†	103320	1076	574	5.98	
Brass average	97740	1018	543	5.66	1873
" wire	94140	981	523	5.45	
Silver	95040	990	528	5.50	1861
Tin average	87840	915	488	5.08	444
Lead average	63180	658	351	3.66	612
Pewter	78840	821	438	4.56	
Zinc (most of all metals)	61920	645	344	3.58	680 to 773
White pine	440530	4588	2447	25.49	

Heat of a common wood-fire, variously estimated from 800 to 1140 deg. That of a **charcoal** one about 2200°: **coal** about 2400°.

Each 12° to 15° of heat produce in **wrot iron** an expansion equal to that produced by a tension of about 1 ton per sq inch of section; varying with the quality of the iron.

---

* By adding $\frac{1}{10}$ part to the lengths in the two cols under 180°, we get the lengths corresponding to a number of degrees $\frac{1}{11}$ less than 180°; or to 163°.63 deg; which may be taken as about the *extremes* of temp in the colder portions of the United States. In the Middle States the extremes rarely reach 135°, or ¼ part less than 180°.

No dependence whatever is to be placed on results obtained by Wedgewood's pyrometer.

† The table shows that the contraction and **expansion of stone** will cause open joints in winter; and crushing of the mortar in summer, at the ends of long coping-stones.

‡ **The melting points are quite uncertain.** We give the mean of the best authorities. Assuming that with a change of temp of about 163°, wrought iron will alter its length 1 part in 916; this in a mile amounts to 5.764 ft, or about 5 ft 9⅙ ins; and in 100 ft to .109 of a foot; or 1⅓ ins; so that a diff of 5 ft, or more, can readily result from measuring a mile in winter and in summer with the same chain; and a 25 ft rail will change its length full ⅓ of an inch.

‖ Hence **wrought expands** by heat about one eleventh part more than cast; whereas under tension within elastic limits cast stretches twice as much as wrt.

THERMOMETERS.

Below about 35° below zero of Fah, the mercurial thermometer and barometer become too irregular to be depended on. Mercury begins to freeze at about —40° Fah. Below —40° pure alcohol is used.

To change degrees of Fahrenheit to the corresponding degrees of Centigrade; take a Fah reading 32° *lower* than the given one; mult this lower reading by 5; divide the prod by 9. Thus: +14° Fah = (14 —32) × 5 ÷ 9 = —18 × 5 ÷ 9 = —10° Cent. Again, —13° Fah = (—13 —32) × 5 ÷ 9 = —45 × 5 ÷ 9 = —25° Cent.

To change Fah to Réau; take a Fah reading 32° *lower* than the given one; mult this lower reading by 4; div the prod by 9. Thus: +14° Fah = (14 —32) × 4 ÷ 9 = —18 × 4 ÷ 9 = —8° Réau. Again, —13° Fah = (—13 —32) × 4 ÷ 9 = —45 × 4 ÷ 9 = —20° Réau.

To change Cent to Fah; mult the Cent reading by 9; divide by 5. Take a Fah reading 32° *higher* than the quot. Thus: +10° Cent = (10 × 9 ÷ 5) + 32 = 18 + 32 = +50° Fah. Again, —20° Cent = (—20 × 9 ÷ 5) + 32 = —4° Fah.

To change Cent to Réau; mult by 4; div by 5. Thus: +10° Cent = 10 × 4 ÷ 5 = +8° Réau. Again, —10° Cent = —10 × 4 ÷ 5 = —8° Réau.

To change Réau to Fah; mult by 9; div by 4. Take a Fah reading 32° *higher*. Thus: +16° Réau = (16 × 9 ÷ 4) + 32 = 36 + 32 = +68° Fah. Again, —8° Réau = (—8 × 9 ÷ 4) + 32 = —18 + 32 = +14° Fah.

To change Réau to Cent; mult by 5; div by 4. Thus: +8° Réau = + 8° × 5 ÷ 4 = +10° Cent. Again, —8° Réau = —8 × 5 ÷ 4 = —10° Cent.

TABLE 1. Fahrenheit compared with Centigrade and Réaumur. In *this* table the Cent and Réau readings are given to the nearest decimal.

F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
212	100	80.0	158	70.0	56.0	104	40.0	32.0	50	10.0	8.0	—3	—19.4	—15.6
211	99.4	79.6	157	69.4	55.6	103	39.4	31.6	49	9.4	7.6	—4	—20.0	—16.0
210	98.9	79.1	156	68.9	55.1	102	38.9	31.1	48	8.9	7.1	—5	—20.6	—16.4
209	98.3	78.7	155	68.3	54.7	101	38.3	30.7	47	8.3	6.7	—6	—21.1	—16.9
208	97.8	78.2	154	67.8	54.2	100	37.8	30.2	46	7.8	6.2	—7	—21.7	—17.3
207	97.2	77.8	153	67.2	53.8	99	37.2	29.8	45	7.2	5.8	—8	—22.2	—17.8
206	96.7	77.3	152	66.7	53.3	98	36.7	29.3	44	6.7	5.3	—9	—22.8	—18.2
205	96.1	76.9	151	66.1	52.9	97	36.1	28.9	43	6.1	4.9	—10	—23.3	—18.7
204	95.6	76.4	150	65.6	52.4	96	35.6	28.4	42	5.6	4.4	—11	—23.9	—19.1
203	95.0	76.0	149	65.0	52.0	95	35.0	28.0	41	5.0	4.0	—12	—24.4	—19.6
202	94.4	75.6	148	64.4	51.6	94	34.4	27.6	40	4.4	3.6	—13	—25.0	—20.0
201	93.9	75.1	147	63.9	51.1	93	33.9	27.1	39	3.9	3.1	—14	—25.6	—20.4
200	93.3	74.7	146	63.3	50.7	92	33.3	26.7	38	3.3	2.7	—15	—26.1	—20.9
199	92.8	74.2	145	62.8	50.2	91	32.8	26.2	37	2.8	2.2	—16	—26.7	—21.3
198	92.2	73.8	144	62.2	49.8	90	32.2	25.8	36	2.2	1.8	—17	—27.2	—21.8
197	91.7	73.3	143	61.7	49.3	89	31.7	25.3	35	1.7	1.3	—18	—27.8	—22.2
196	91.1	72.9	142	61.1	48.9	88	31.1	24.9	34	1.1	0.9	—19	—28.3	—22.7
195	90.6	72.4	141	60.6	48.4	87	30.6	24.4	33	0.6	0.4	—20	—28.9	—23.1
194	90.0	72.0	140	60.0	48.0	86	30.0	24.0	32	0.0	0.0	—21	—29.4	—23.6
193	89.4	71.6	139	59.4	47.6	85	29.4	23.6	31	—0.6	—0.4	—22	—30.0	—24.0
192	88.9	71.1	138	58.9	47.1	84	28.9	23.1	30	—1.1	—0.9	—23	—30.6	—24.4
191	88.3	70.7	137	58.3	46.7	83	28.3	22.7	29	—1.7	—1.3	—24	—31.1	—24.9
190	87.8	70.2	136	57.8	46.2	82	27.8	22.2	28	—2.2	—1.8	—25	—31.7	—25.3
189	87.2	69.8	135	57.2	45.8	81	27.2	21.8	27	—2.8	—2.2	—26	—32.2	—25.8
188	86.7	69.3	134	56.7	45.3	80	26.7	21.3	26	—3.3	—2.7	—27	—32.8	—26.2
187	86.1	68.9	133	56.1	44.9	79	26.1	20.9	25	—3.9	—3.1	—28	—33.3	—26.7
186	85.6	68.4	132	55.6	44.4	78	25.6	20.4	24	—4.4	—3.6	—29	—33.9	—27.1
185	85.0	68.0	131	55.0	44.0	77	25.0	20.0	23	—5.0	—4.0	—30	—34.4	—27.6
184	84.4	67.6	130	54.4	43.6	76	24.4	19.6	22	—5.6	—4.4	—31	—35.0	—28.0
183	83.9	67.1	129	53.9	43.1	75	23.9	19.1	21	—6.1	—4.9	—32	—35.6	—28.4
182	83.3	66.7	128	53.3	42.7	74	23.3	18.7	20	—6.7	—5.3	—33	—36.1	—28.9
181	82.8	66.2	127	52.8	42.2	73	22.8	18.2	19	—7.2	—5.8	—34	—36.7	—29.3
180	82.2	65.8	126	52.2	41.8	72	22.2	17.8	18	—7.8	—6.2	—35	—37.2	—29.8
179	81.7	65.3	125	51.7	41.3	71	21.7	17.3	17	—8.3	—6.7	—36	—37.8	—30.2
178	81.1	64.9	124	51.1	40.9	70	21.1	16.9	16	—8.9	—7.1	—37	—38.3	—30.7
177	80.6	64.4	123	50.6	40.4	69	20.6	16.4	15	—9.4	—7.6	—38	—38.9	—31.1
176	80.0	64.0	122	50.0	40.0	68	20.0	16.0	14	—10·0	—8.0	—39	—39.4	—31.6
175	79.4	63.6	121	49.4	39.6	67	19.4	15.6	13	—10.6	—8.4	—40	—40.0	—32.0
174	78.9	63.1	120	48.9	39.1	66	18.9	15.1	12	—11.1	—8.9	—41	—40.6	—32.4
173	78.3	62.7	119	48.3	38.7	65	18.3	14.7	11	—11.7	—9.3	—42	—41.1	—32.9
172	77.8	62.2	118	47.8	38.2	64	17.8	14.2	10	—12.2	—9.8	—43	—41.7	—33.3
171	77.2	61.8	117	47.2	37.8	63	17.2	13.8	9	—12.8	—10.2	—44	—42.2	—33.8
170	76.7	61.3	116	46.7	37.3	62	16.7	13.3	8	—13.3	—10.7	—45	—42.8	—34.2
169	76.1	60.9	115	46.1	36.9	61	16.1	12.9	7	—13.9	—11.1	—46	—43.3	—34.7
168	75.6	60.4	114	45.6	36.4	60	15.6	12.4	6	—14.4	—11.6	—47	—43.9	—35.1
167	75.0	60.0	113	45.0	36.0	59	15.0	12.0	5	—15.0	—12.0	—48	—44.4	—35.6
166	74.4	59.6	112	44.4	35.6	58	14.4	11.6	4	—15.6	—12.4	—49	—45.0	—36.0
165	73.9	59.1	111	43.9	35.1	57	13.9	11.1	3	—16.1	—12.9	—50	—45.6	—36.4
164	73.3	58.7	110	43.3	34.7	56	13.3	10.7	2	—16.7	—13.3	—51	—46.1	—36.9
163	72.8	58.2	109	42.8	34.2	55	12.8	10.2	1	—17.2	—13.8	—52	—46.7	—37.3
162	72.2	57.8	108	42.2	33.8	54	12.2	9.8	0	—17.8	—14.2	—53	—47.2	—37.8
161	71.7	57.3	107	41.7	33.3	53	11.7	9.3	—1	—18.3	—14.7	—54	—47.8	—38.2
0	71.1	56.9	106	41.1	32.9	52	11.1	8.9	—2	—18.9	—15.1	—55	—48.3	—38.7
9	70.6	56.4	105	40.6	32.4	51	10.6	8.4						

TABLE 2. Centigrade compared with Fahrenheit and Réaumur.

C.	F.	R.	C.	F.	R.	C.	F.	R.	C.	F.	R.
°	°	°	°	°	°	°	°	°	°	°	°
	Exact.	Exact.		Exact.	Exact.		Exact.	Exact.		Exact.	Exact.
100	212.0	80.0	62	143.6	49.6	24	75.2	19.2	—14	6.8	—11.2
99	210.2	79.2	61	141.8	48.8	23	73.4	18.4	—15	5.0	—12.0
98	208.4	78.4	60	140.0	48.0	22	71.6	17.6	—16	3.2	—12.8
97	206.6	77.6	59	138.2	47.2	21	69.8	16.8	—17	1.4	—13.6
96	204.8	76.8	58	136.4	46.4	20	68.0	16.0	—18	—0.4	—14.4
95	203.0	76.0	57	134.6	45.6	19	66.2	15.2	—19	—2.2	—15.2
94	201.2	75.2	56	132.8	44.8	18	64.4	14.4	—20	—4.0	—16.0
93	199.4	74.4	55	131.0	44.0	17	62.6	13.6	—21	—5.8	—16.8
92	197.6	73.6	54	129.2	43.2	16	60.8	12.8	—22	—7.6	—17.6
91	195.8	72.8	53	127.4	42.4	15	59.0	12.0	—23	—9.4	—18.4
90	194.0	72.0	52	125.6	41.6	14	57.2	11.2	—24	—11.2	—19.2
89	192.2	71.2	51	123.8	40.8	13	55.4	10.4	—25	—13.0	—20.0
88	190.4	70.4	50	122.0	40.0	12	53.6	9.6	—26	—14.8	—20.8
87	188.6	69.6	49	120.2	39.2	11	51.8	8.8	—27	—16.6	—21.6
86	186.8	68.8	48	118.4	38.4	10	50.0	8.0	—28	—18.4	—22.4
85	185.0	68.0	47	116.6	37.6	9	48.2	7.2	—29	—20.2	—23.2
84	183.2	67.2	46	114.8	36.8	8	46.4	6.4	—30	—22.0	—24.0
83	181.4	66.4	45	113.0	36.0	7	44.6	5.6	—31	—23.8	—24.8
82	179.6	65.6	44	111.2	35.2	6	42.8	4.8	—32	—25.6	—25.6
81	177.8	64.8	43	109.4	34.4	5	41.0	4.0	—33	—27.4	—26.4
80	176.0	64.0	42	107.6	33.6	4	39.2	3.2	—34	—29.2	—27.2
79	174.2	63.2	41	105.8	32.8	3	37.4	2.4	—35	—31.0	—28.0
78	172.4	62.4	40	104.0	32.0	2	35.6	1.6	—36	—32.8	—28.8
77	170.6	61.6	39	102.2	31.2	1	33.8	0.8	—37	—34.6	—29.6
76	168.8	60.8	38	100.4	30.4	0	32.0	0.0	—38	—36.4	—30.4
75	167.0	60.0	37	98.6	29.6	—1	30.2	—0.8	—39	—38.2	—31.2
74	165.2	59.2	36	96.8	28.8	—2	28.4	—1.6	—40	—40.0	—32.0
73	163.4	58.4	35	95.0	28.0	—3	26.6	—2.4	—41	—41.8	—32.8
72	161.6	57.6	34	93.2	27.2	—4	24.8	—3.2	—42	—43.6	—33.6
71	159.8	56.8	33	91.4	26.4	—5	23.0	—4.0	—43	—45.4	—34.4
70	158.0	56.0	32	89.6	25.6	—6	21.2	—4.8	—44	—47.2	—35.2
69	156.2	55.2	31	87.8	24.8	—7	19.4	—5.6	—45	—49.0	—36.0
68	154.4	54.4	30	86.0	24.0	—8	17.6	—6.4	—46	—50.8	—36.8
67	152.6	53.6	29	84.2	23.2	—9	15.8	—7.2	—47	—52.6	—37.6
66	150.8	52.8	28	82.4	22.4	—10	14.0	—8.0	—48	—54.4	—38.4
65	149.0	52.0	27	80.6	21.6	—11	12.2	—8.8	—49	—56.2	—39.2
64	147.2	51.2	26	78.8	20.8	—12	10.4	—9.6	—50	—58.0	—40.0
63	145.4	50.4	25	77.0	20.0	—13	8.6	—10.4			

TABLE 3. Réaumur compared with Fahrenheit and Centigrade.

R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.
°	°	°	°	°	°	°	°	°	°	°	°
	Exact.	Exact.		Exact.	Exact.		Exact.	Exact.		Exact.	Exact.
80	212.00	100.00	49	142.25	61.25	19	74.75	23.75	—11	7.25	—13.75
79	209.75	98.75	48	140.00	60.00	18	72.50	22.50	—12	5.00	—15.00
78	207.50	97.50	47	137.75	58.75	17	70.25	21.25	—13	2.75	—16.25
77	205.25	96.25	46	135.50	57.50	16	68.00	20.00	—14	0.50	—17.50
76	203.00	95.00	45	133.25	56.25	15	65.75	18.75	—15	—1.75	—18.75
75	200.75	93.75	44	131.00	55.00	14	63.50	17.50	—16	—4.00	—20.00
74	198.50	92.50	43	128.75	53.75	13	61.25	16.25	—17	—6.25	—21.25
73	196.25	91.25	42	126.50	52.50	12	59.00	15.00	—18	—8.50	—22.50
72	194.00	90.00	41	124.25	51.25	11	56.75	13.75	—19	—10.75	—23.75
71	191.75	88.75	40	122.00	50.00	10	54.50	12.50	—20	—13.00	—25.00
70	189.50	87.50	39	119.75	48.75	9	52.25	11.25	—21	—15.25	—26.25
69	187.25	86.25	38	117.50	47.50	8	50.00	10.00	—22	—17.50	—27.50
68	185.00	85.00	37	115.25	46.25	7	47.75	8.75	—23	—19.75	—28.75
67	182.75	83.75	36	113.00	45.00	6	45.50	7.50	—24	—22.00	—30.00
66	180.50	82.50	35	110.75	43.75	5	43.25	6.25	—25	—24.25	—31.25
65	178.25	81.25	34	108.50	42.50	4	41.00	5.00	—26	—26.50	—32.50
64	176.00	80.00	33	106.25	41.25	3	38.75	3.75	—27	—28.75	—33.75
63	173.75	78.75	32	104.00	40.00	2	36.50	2.50	—28	—31.00	—35.00
62	171.50	77.50	31	101.75	38.75	1	34.25	1.25	—29	—33.25	—36.25
61	169.25	76.25	30	99.50	37.50	0	32.00	0.00	—30	—35.50	—37.50
60	167.00	75.00	29	97.25	36.25	—1	29.75	—1.25	—31	—37.75	—38.75
59	164.75	73.75	28	95.00	35.00	—2	27.50	—2.50	—32	—40.00	—40.00
58	162.50	72.50	27	92.75	33.75	—3	25.25	—3.75	—33	—42.25	—41.25
57	160.25	71.25	26	90.50	32.50	—4	23.00	—5.00	—34	—44.50	—42.50
56	158.00	70.00	25	88.25	31.25	—5	20.75	—6.25	—35	—46.75	—43.75
55	155.75	68.75	24	86.00	30.00	—6	18.50	—7.50	—36	—49.00	—45.00
54	153.50	67.50	23	83.75	28.75	—7	16.25	—8.75	—37	—51.25	—46.25
53	151.25	66.25	22	81.50	27.50	—8	14.00	—10.00	—38	—53.50	—47.50
52	149.00	65.00	21	79.25	26.25	—9	11.75	—11.25	—39	—55.75	—48.75
51	146.75	63.75	20	77.00	25.00	—10	9.50	—12.50	—40	—58.00	—50.00
50	144.50	62.50									

AIR.—ATMOSPHERE.

The atmosphere is known to extend to at least 45 miles above the earth. It is a mixture of about 79 measures of nitrogen gas and 21 of oxygen gas; or about 77 nitrogen, 23 oxygen, by weight. It generally contains, however, a trace of water, and of carbonic acid and carburetted hydrogen gases, and still less ammonia.

Density of air. Under "normal" or "standard" conditions (sea level, lat 45°, barometer 760 mm = 29.922 ins, temperature 0° C = 32° F) **dry air weighs** 1.292673 kilograms per cubic meter[*] = 2.17888 lbs avoir per cubic yard. For other lats and elevations—

$$\text{Density, in kg per cu m,} = 1.292673 \times \frac{R}{R + 2h} \times (1 - 0.002837 \cos 2 \text{ lat})\,[*]$$

where R = earth's mean radius = 6,366,198 meters; h = elevation above sea level, in meters. For other temperatures, see below.

Under normal conditions, but with 0.04 parts carbonic acid (CO_2) in 100 parts of air, density = 1.293052 kg per cu m.[†] = 2.17952 lbs avoir per cu yd.[‡]

The atmospheric pressure, at any given place, may vary 2 inches or more from day to day. **The average pressure,** at sea level[§] varies from about 745 to 770 millimeters of mercury according to the latitude and locality. 760 millimeters[*] is generally accepted as the mean atmospheric pressure, and called **an atmosphere.** The "**metric atmosphere,**" taken arbitrarily at 1 kilogram per square centimeter, is in general use in Continental Europe. The pressure diminishes as the altitude increases.[†] Therefore, a pump in a high region will not lift water to as great a height as in a low one. The pressure of air, like that of water, is, at any given point, equal in all directions.

It is often stated that the **temperature of the atmosphere** lowers at the rate of 1° Fah for each 300 feet of ascent **above the earth's surface;** but this is liable to many exceptions, and varies much with local causes. Actual observation in balloons seems to show that, up to the first 1000 feet, 1° in about 200 feet is nearer the truth; at 2000 feet, 1° in 250 feet; at 4000 feet, 1° in 300 feet; and, at a mile, 1° in 350 feet.

In breathing, a grown person at rest requires from 0.25 to 0.35 of a cubic foot of air per minute; which, when breathed, vitiates from 3.5 to 5 cubic feet. When walking, or hard at work, he breathes and vitiates two or three times as much. About 5 cubic feet of fresh air per person per minute are required for the perfect ventilation of rooms in winter; 8 in summer. Hospitals 40 to 80.

Beneath the general level of the surface of the earth, in temperate regions, a tolerably uniform **temperature** of about 50° to 60° Fah exists at the depth of about 50 to 60 feet; and increases about 1° for each additional 50 to 60 feet; all subject, however, to considerable deviations owing to many local causes. In the Rose Bridge Colliery, England, at the depth of 2424 feet, the temperature of the coal is 93.5° Fah; and at the bottom of a boring 4169 feet deep, near Berlin, the temperature is 119°.

The air is a very slow conductor of heat; hence hollow walls serve to retain the heat in dwellings; besides keeping them dry. **It rushes into a vacuum** near sea level with a velocity of about 1157 feet per second; or 13.3 miles per minute; or about as fast as sound ordinarily travels through quiet air. See Sound.

Like all other elastic fluids, air expands equally with equal increases of temperature. Every increase of 5° Fah, expands the bulk of any of them slightly more than 1 per cent of that which it has at 0° Fah; or 500° about doubles its bulk at zero. The bulk of any of them diminishes inversely in proportion to the total **pressure** to which it is subjected.

This holds good with air at least up to pressures of about 750 lbs per square inch, or 50 times its natural pressure; the air in this case occupying one-fiftieth of its natural bulk. In like manner the bulk will increase as the total pressure is diminished. Substances which follow these laws, are said to be **perfectly**

---

[*] H. V. Regnault, Mémoires de l'Académie Royale des Sciences de l'Institut de France, Tome XXI, 1847. Translation in abstract, Journal of Franklin Institute, Phila., June, 1848.

[†] Travaux et Mémoires du Bureau International des Poids et Mesures, Tome 1, page A 54. Smithsonian Meteorological Tables, 1893, published in Smithsonian Miscellaneous Collections, Vol. XXXV, 1897.

[‡] See Conversion Tables.

[§] See Leveling by the Barometer.

elastic. Under a pressure of about 5½ tons per square inch, air would become as dense as water. Since the air at the surface of the earth is pressed 14¾ lbs per square inch by the atmosphere above it, and since this is equal to the weight of a column of water 1 inch square and 34 feet high, it follows that at the depths of 34, 68, 102 feet, &c, **below water,** air will be compressed into ½, ⅓, ¼, &c, of its bulk at the surface.

In a diving-bell, men, after some experience, can readily work for several hours at a depth of 51 feet, or under a pressure of 2½ atmospheres; or 37½ lbs per square inch. But at 90 feet deep, or under 3.64 atmospheres, or nearly 55 lbs per square inch, they can work for but about an hour, without serious suffering from paralysis, or even danger of death. Still, at the St. Louis bridge, work was done at a depth of 110½ feet; pressure 63.7 lbs per square inch.

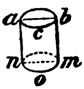
The dew point is that temp (varying) at which the air deposits its vapor.

The greatest heat of the air in the sun probably never exceeds 145° Fah; nor the greatest cold —74° at night. About 130° above, and 40° below zero, are the extremes in the U. S. east of the Mississippi; and 65° below in the N. W.; all at common ground level. It is stated, however, that —81° has been observed in N. E. Siberia; and +101° Fah in the shade in Paris; and +153° in the sun at Greenwich Observatory, both in July, 1881. It has frequently exceeded +100° Fah in the shade in Philadelphia during recent years.

WIND.

The relation between the velocity of wind, and its pressure against an obstacle placed either at right angles to its course, or inclined to it, has not been well determined; and still less so its pressure against curved surfaces. The pressure against a large surface is probably proportionally greater than against a small one. It is generally supposed to vary nearly as the squares of the velocities; and when the obstacle is at right angles to its direction, the pressure in lbs per square foot of exposed surface is considered to be equal to the square of the velocity in miles per hour, divided by 200. On this basis, which is probably quite defective, the following table, as given by Smeaton, is prepared.

Vel. in Miles per Hour.	Vel. in Ft. per Sec.	Pres. in Lbs. per Sq. Ft.	Remarks.
1	1.467	.005	Hardly perceptible.
2	2.933	.020	Pleasant.
3	4.400	.045	
4	5.867	.080	
5	7.33	.125	
10	14.67	.5	
12½	18.33	.781	Fresh breeze.
15	22.	1.125	
20	29.33	2.	
25	36.67	3.125	Brisk wind.
30	44.	4.5	Strong wind.
40	58.67	8.	High wind.
50	73.33	12.5	Storm.
60	88.	18.	Violent storm.
80	117.3	32.	Hurricane.
100	146.7	50.	Violent hurricane, uprooting large trees.



The pres against a semicylindrical surface *a c b n o m* is about half that against the flat surf *a b n m*.

Tredgold recommends to allow 40 lbs per sq ft of roof for the pres of wind against it; but as roofs are constructed with a slope, and consequently do not receive the full force of the wind, this is plainly too much.* Moreover, only one-half of a roof is usually exposed, even thus partially, to the wind. Probably the force in such cases varies approximately as the sines of the angles of slopes. According to observations in Liverpool, in 1860, a wind of 38 miles per hour, produced a pres of 14 lbs per sq ft against an object perp to it: and one of 70 miles per hour, (the severest gale on record at that city,) 42 lbs per sq foot. These would make the pres per sq ft, more nearly equal to the square of the vel in miles per hour, div by 100; or nearly twice as great as given in Smeaton's table. We should ourselves give the preference to the Liverpool observations. A very violent gale in Scotland, registered by an excellent anemometer, or wind-gauge, 45 lbs per sq ft. It is stated that as high as 55 lbs has been observed at Glasgow. High winds often *lift* roofs.

The gauge at Girard College, Philada, broke under a strain of 42 lbs per sq ft; a tornado passing at the moment, within ¼ mile.

By inversion of Smeaton's rule, if the force in lbs per sq ft, be mult by 200, the sq rt of the prod will give the vel in miles per hour. Smeaton's rule is used by the U. S. Signal Service.

---

* The writer thinks 8 lbs per sq foot of *ordinary double-sloping roofs*, or 16 lbs for *shed-roofs*, sufficient allowance for pres of wind.

RAIN AND SNOW.

The annual precipitation* at any given place varies greatly from year to year, the ratio between maximum and minimum being frequently greater than 2 : 1. **Beware of averages.** In estimating *floods*, take the *maximum* falls, and in estimating *water supply*, the *minimum*, not only per annum, but for short periods. In estimating water supply, make deductions for evaporation and leakage.

Maxima and minima deduced from observations covering only 4 or 5 years are apt to be misleading. Data covering even 10 or more years may just miss including a very severe flood or drought. Records of from 15 to 20 years may usually be accepted as sufficient.

Table 1. Average Precipitation* in the United States, in ins.

(From Bulletin C of U. S. Department of Agriculture, compiled to end of 1891.)

State.	Spr.	Sum.	Aut.	Win.	Ann'l	State.	Spr.	Sum.	Aut.	Win.	Ann'l
Alabama	14.9	18.8	10.0	14.9	53.6	Montana	4.2	4.9	2.6	2.8	14.0
Arizona	1.3	4.3	2.2	3.1	10.9	Nebraska	8.9	10.9	4.9	2.2	26.9
Arkansas	14.3	12.5	11.0	12.8	50.6	Nevada	2.8	0.8	1.3	3.2	7.6
California	6.2	0.3	3.5	11.9	21.9	N. Hampshire	9.8	12.2	11.4	10.7	44.1
Colorado	4.2	5.5	2.8	2.3	14.8	New Jersey	11.7	13.3	11.2	11.1	47.3
Connecticut	11.1	12.5	11.7	11.5	46.8	New Mexico	1.4	5.8	3.5	2.0	12.7
Delaware	10.2	11.0	10.0	9.6	40.8	New York	8.5	10.4	9.7	7.9	36.5
Dist. Columbia	11.0	12.4	9.4	9.0	41.8	N. Carolina	12.9	16.6	12.0	12.2	53.7
Florida	10.2	21.4	14.2	9.1	54.9	N. Dakota	4.6	8.0	2.8	1.7	17.1
Georgia	12.4	15.6	10.7	12.7	51.4	Ohio	10.0	11.9	9.0	9.1	40.0
Idaho	4.4	2.1	3.6	7.0	17.1	Oregon	9.8	2.7	10.5	21.0	44.0
Illinois	10.2	11.2	9.0	7.7	38.1	Pennsylvania	10.3	12.7	10.0	9.5	42.5
Indiana	11.0	11.7	9.7	10.3	42.7	Rhode Island	11.9	10.7	11.7	12.4	46.7
Indian T'y	10.6	11.0	8.9	5.7	36.2	S. Carolina	9.8	16.2	9.7	9.7	45.4
Iowa	8.3	12.4	8.1	4.1	32.9	S. Dakota	7.2	9.7	3.5	2.5	22.9
Kansas	8.9	11.9	6 7	3.5	31.0	Tennessee	13.5	12.5	10.2	14.5	50.7
Kentucky	12.4	12.5	9.7	11.8	46.4	Texas	8.1	8.6	7.6	6.0	30.3
Louisiana	13.7	15.0	10.8	14.4	53.9	Utah	3.4	1.5	2.2	3.5	10.6
Maine	11.1	10.5	12.3	11.1	45.0	Vermont	9.2	12.2	11.4	9.3	42.1
Maryland	11.4	12.4	10.7	9.5	44.0	Virginia	10.9	12.5	9.5	9.7	42.6
Massachusetts	11.6	11.4	11.9	11.7	46.6	Washington	8.6	3.9	10.5	16.8	39.8
Michigan	7.9	9.7	9.2	7.0	33.8	W. Virginia	10.9	12.9	9.0	10.0	42.8
Minnesota	6 5	10.8	5.8	3.1	26.2	Wisconsin	7.8	11.6	7.8	5.2	32.5
Mississippi	14.9	12.6	10.1	15.4	53.0	Wyoming	4.3	3.5	2.2	1.6	11.6
Missouri	10.0	12.4	9.1	6.5	38.0	United States	9.2	10.3	8.3	8.6	36.3

At Philadelphia, in 1869, during which occurred the greatest drought known there for at least 50 years, 43.21 inches fell; August 13, 1873, 7.3 inches in 1 day; August, 1867, 15.8 inches in 1 month; July, 1842, 6 inches in 2 hours; 9 inches per month not more than 7 or 8 times in 25 years. From 1825 to 1893, greatest in one year, 61 inches, in 1867; least, 30 inches, in 1825 and 1880. At Norristown, Pennsylvania, in 1865, the writer saw evidence that at least 9 inches fell within 5 hours. At Genoa, Italy, on one occasion, 32 inches fell in 24 hours; at Geneva, Switzerland, 6 inches in 3 hours; at Marseilles, France, 13 inches in 14 hours; in Chicago, Sept., 1878, .97 inch in 7 minutes.

Near London, England, the mean total fall for many years is 23 inches. On one occasion, 6 inches fell in 1½ hours! In the mountain districts of the English lakes, the fall is enormous; reaching in some years to 180 or 240 inches; or from 15 to 20 feet! while, in the adjacent neighborhood, it is but 40 to 60 inches. At Liverpool, the average is 34 inches; at Edinburgh, 30; Glasgow, 22; Ireland, 36; Madras, 47; Calcutta, 60; maximum for 16 years, 82; Delhi, 21; Gibraltar, 30; Adelaide, Australia, 23; West Indies, 36 to 96; Rome, 39. On the Khassya hills north of Calcutta, 500 inches, or 41 feet 8 inches, have fallen in the 6 rainy months! In other mountainous districts of India, annual falls of 10 to 20 feet are common.

A moderate steady rain, continuing 24 hours, will yield a depth of about an inch.

As a general rule, more rain falls in warm than in cold countries; and more in elevated regions than in low ones. Local peculiarities, however, sometimes reverse this; and also cause great differences in the amounts in places quite near each other; as in the English lake districts just alluded to. It is sometimes difficult to account for these variations. In some lagoons in New Granada, South America, the writer has known three or four heavy rains to occur weekly for some months, during which not a drop fell on hills about 1000 feet high, within ten miles' distance, and within full sight. At another locality, almost a dead-level plain, fully three-quarters of the rains that fell for two years, at a spot two miles from his residence, occurred in the morning; while those which fell about three miles from it, in an opposite direction, were in the afternoon.

---

*Precipitation includes snow, hail, and sleet, melted. Unmelted snow is estimated at 10 inches snow = 1 inch rain.

The relation between precipitation and stream-flow is greatly affected by the existence of forests or crops, by the slope and character of ground on the water-shed, especially as to rate of absorption, by the season of the year, the frost in the ground, etc. The stream-flow may ordinarily be taken as varying between 0.2 and 0.8 of the rainfall. Streams in limestone regions frequently lose a very large proportion of their flow through subterranean caverns.

Assuming a fall of 2 feet in 1 year (= 76,379 cubic feet per square mile per day), that half the rainfall is available for water supply, and that a per capita consumption of 4 cubic feet (= 30 gallons) per day is sufficient, one square mile will supply 19,095 persons; or a square of 38.25 feet on a side will supply one person.

An inch of rain amounts to 3630 cubic feet; or 27155 U. S. gallons; or 101.3 tons per acre; or to 2323200 cubic feet; or 17378743 U. S. gallons; or 64821 tons per square mile at 62½ lbs per cubic foot.

The most destructive rains are usually those which fall upon snow, under which the ground is frozen, so as not to absorb water.

Table 2. Maximum intensity of rainfall for periods of 5, 10, and 60 minutes at Weather Bureau stations equipped with self-registering gauges, compiled from all available records to the end of 1896.

(From Bulletin D of U. S. Department of Agriculture.)

Stations.	Rate per hour for—			Stations.	Rate per hour for—		
	5 min.	10 mins.	60 mins.		5 min.	10 mins.	60 mins.
	Ins.	Inches.	Inches.		Ins.	Inches.	Inches.
Bismarck	9.00	6.00	2.00	Chicago	6.60	5.92	1.60
St. Paul	8.40	6.00	1.80	Galveston	6.48	5.58	2.55
New Orleans	8.16	4.86	2.18	Omaha	6.00	4.80	1.55
Milwaukee	7.80	4.20	1.25	Dodge City	6.00	4.20	1.34
Kansas City	7.80	6.60	2.40	Norfolk	5.76	5.46	1.55
Washington	7.50	5.10	1.78	Cleveland	5.64	3.66	1.12
Jacksonville	7.44	7.08	2.20	Atlanta	5.46	5.46	1.50
Detroit	7.20	6.00	2.15	Key West	5.40	4.80	2.25
New York City	7.20	4.92	1.60	Philadelphia	5.40	4.02	1.50
Boston	6.72	4.98	1.68	St. Louis	4.80	3.84	2.25
Savannah	6.60	6.00	2.21	Cincinnati	4.56	4.20	1.70
Indianapolis	6.60	3.90	1.60	Denver	3.60	3.30	1.18
Memphis	6.60	4.80	1.86	Duluth	3.60	2.40	1.35

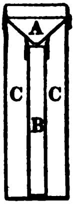
The weight of freshly fallen snow, as measured by the author, varies from about 5 to 12 lbs per cubic foot; apparently depending chiefly upon the degree of humidity of the air through which it had passed. On one occasion, when mingled snow and hail had fallen to the depth of 6 inches, he found its weight to be 31 lbs per cubic foot. It was very dry and incoherent. A cubic foot of heavy snow may, by a gentle sprinkling of water, be converted into about half a cubic foot of slush, weighing 20 lbs.; which will not **slide or run off** from a shingled roof sloping 30°, if the weather is cold. A cubic block of snow saturated with water until it weighed 45 lbs per cubic foot, just slid on a rough board inclined at 45°; on a smoothly planed one at 30°; and on slate at 18°; all approximate. A prism of snow, saturated to 52 lbs per cubic foot, one inch square, and 4 inches high, **bore a weight** of 7 lbs; which at first compressed it about one-quarter part of its length. European engineers consider 6 lbs per square foot of roof to be sufficient **allowance for the weight of snow;**

and 8 lbs for the pressure of wind; total, 14 lbs. The writer thinks that in the U. S. the allowance for snow should not be taken at *less* than 12 lbs; or the total for snow and wind, at 20 lbs. There is no danger that snow on a roof will become saturated to the extent just alluded to; because a rain that would supply the necessary quantity of water would also by its violence wash away the snow; but we entertain no doubt whatever that the united pressures from snow and wind, in our Northern States, do actually at times reach, and even surpass, 20 lbs per square foot of roof. **The limit of perpetual snow** at the equator is at the height of about 16000 feet, or say 3 miles above sea-level; in lat 45° north or south, it is about half that height; while near the poles it is about at sea-level.

Rain Gauges. Plain cylindrical vessels are ill adapted to service as rain gauges; because moderate rains, even though sufficient to yield a large run-off from a moderate area, are not of sufficient depth to be satisfactorily measured unless the depth be exaggerated. The inaccuracy of measurement, always considerable, is too great relatively to the depth.

In its simplest and most usual form, the gauge (see Fig.) consists essentially of a funnel, A, which receives the rain and leads it into a measuring tube, B, of smaller cross-section. The funnel should have a vertical and fairly sharp edge, and, in order to minimize the loss through evaporation, it should fit closely over the tube, and its lower end should be of small diameter.

The depth of water in the tube is ascertained by inserting, to the bottom of the tube, a measuring stick of some unpolished wood which will readily show to what depth it has been wet. The stick may be permanently graduated, or it may be compared with an ordinary scale at each observation. The tube is usually of such diameter that the area of its cross-section, minus that of the stick, is one-tenth of the area of the funnel mouth. The depth of rainfall is then one-tenth of the depth as measured by the stick.



DIMENSIONS OF STANDARD U. S. WEATHER BUREAU RAIN GAUGE.

				Ins.
A.	Receiver or funnel.		Diameter	8
B.	Measuring tube.	Height 20 ins.	"	2.53
C C.	Overflow attachment and snow gauge.		"	8

Such gauges, with the tubes carefully made from seamless drawn brass tubing, cost about $5.00 each; but an intelligent and careful tinsmith, given the dimensions accurately, can construct, of galvanized iron, for about $1.00 a gauge that will answer every purpose of the engineer.

The exposure has a very marked effect upon the results obtained. The funnel should be elevated about 3 ft, in order to prevent rain from splashing back into it from the ground or roof. If on a roof, the latter should be flat, and preferably 50 ft wide or wider, and the gauge should be placed as far as possible from the edges; else the air currents, produced by the wind striking the side of the building, will carry some of the rain over the gauge. No objects much higher than the gauge should be near it, as they produce variable air currents which may seriously affect its indications.

An overflow tank, C, should be provided, for cases of overfilling the tube.

Water, freezing in the gauge, may burst it, or force the bottom off, or at least so deform the gauge as to destroy its accuracy.

To measure snow, the funnel is removed, and the snow is collected in the overflow attachment or other cylindrical vessel deep enough to prevent the snow from being blown out, and the cross-sectional area of which is accurately known. The snow is then melted, either by allowing it to stand in a warm place, or, with less loss through evaporation, by adding an accurately known quantity of luke-warm water. In the latter case, the volume of the added water must of course be deducted from the measurement.

Rainfall equivalent of snow. Ten inches of snow are usually taken as equivalent to 1 in of rain; but, according to various authorities, the equivalent may vary between 2½ and 34; *i. e.*, between 25 and 1.84 lbs. per cubic foot.

Self-recording gauges, of which several forms are on the market, are quite expensive, and, even when purchased from regular makers, seldom perfectly reliable. Gauges using a small tipping bucket register inaccurately in heavy rains; those using a float are limited as to the total depth which they can register; while those which weigh the rain, if exposed, are affected by wind.

Table 3. Details of Precipitation, in the United States.

Records from 1871 or earlier, to 1891 inclusive.

(From Bulletin C of U. S. Department of Agriculture, 1894.)

State.	City.	Average per-cent. of days on which rain fell.	Average percentage days with rain. *					Heaviest in 1 day, inches.	Max. No. of con-sec. days	
			Trace to 0.25 inch.	0.26 to 0.50 in.	0.51 to 1.00 in.	1.01 to 2.00 ins.	2.01 to 3.00 ins.		With-out rain	With rain
Alabama	Mobile	40.1	24.3	5.4	5.1	3.8	0.9	7.3	43	18
Arizona	Yuma †	6.6	5.6	0.5	0.3	0.2	0.0	1.7	167	6
Arizona	Whipple Barr'ks	15.7	10.2	2.8	1.9	0.8	0.1	4.2	88	9
California	San Francisco	21.8	14.2	3.2	2.6	1.5	0.2	4.7	142	19
Colorado	Denver	28.7	24.2	2.8	1.2	0.4	0.0	6.5	39	15
Connecticut	New London	43.0	28.1	6.2	5.4	2.6	0.5	11.8	19	14
Dis of Columbia,	Washington	40.8	27.2	5.7	4.6	2.6	0.5	4.2	26	12
Florida	Jacksonville	44.7	29.8	5.3	5.1	3.2	0.8	6.2	34	32
Florida	Key West	38.8	27.9	4.3	3.9	1.9	0.6	8.2	39	22
Georgia	Augusta	37.2	22.9	5.7	4.8	3.1	0.5	4.9	26	17
Georgia	Savannah	36.4	22.3	5.5	4.5	3.0	0.7	8.6	32	17
Illinois	Cairo	38.5	25.0	5.5	4.9	2.6	0.4	4.2	28	13
Illinois	Chicago	44.0	32.5	5.3	4.2	1.7	0.2	5.6	21	17
Indiana	Indianapolis	44.2	30.3	6.0	5.2	2.2	0.4	4.3	23	14
Iowa	Davenport	40.8	29.7	5.2	3.9	1.6	0.3	5.2	24	11
Iowa	Keokuk	35.4	24.5	4.9	3.8	1.6	0.5	4.8	26	11
Kansas	Leavenworth	31.9	20.7	4.8	3.5	2.3	0.4	3.6	36	11
Louisiana	New Orleans	40.6	24.9	5.0	5.5	3.8	1.0	8.9	28	17
Louisiana	Shreveport	35.9	22.6	4.7	4.4	3.1	0.8	6.9	39	28
Maine	Portland	43.4	29.8	6.3	4.6	2.2	0.4	3.9	20	11
Maryland	Baltimore	41.2	27.5	6.1	4.7	2.3	0.5	4.5	29	14
Massachusetts	Boston	39.8	25.8	6.1	5.0	2.3	0.5	4.4	25	10
Michigan	Detroit	46.7	36.0	5.5	3.8	1.2	0.2	3.9	19	18
Michigan	Marquette	49.0	39.0	5.3	3.4	1.2	0.1	5.2	21	14
Minnesota	Duluth	44.5	34.7	5.0	3.2	1.4	0.2	3.1	26	14
Minnesota	St. Paul	38.9	30.3	4.6	2.7	1.1	0.2	3.7	30	11
Mississippi	Vicksburg	36.4	21.1	5.1	5.2	3.6	1.1	5.4	28	12
Missouri	St. Louis	38.4	26.4	5.4	4.4	1.7	0.4	4.5	28	11
Nebraska	Omaha	34.1	24.4	3.8	3.6	1.7	0.5	5.0	26	12
New Jersey	Atlantic City ‡	41.0	27.5	5.8	5.2	2.1	0.3	9.0	38	13
New Mexico	Fort Wingate	14.6	10.1	2.6	1.4	0.4	0.1	4.5	124	7
New York	Buffalo	54.2	41.3	7.0	4.4	1.3	0.2	3.2	22	22
New York	New York	39.6	25.7	6.1	4.8	2.4	0.5	6.2	26	12
New York	Oswego	58.3	46.5	6.8	3.6	1.1	0.2	3.6	18	44
North Carolina	Wilmington	36.2	20.7	6.0	5.4	3.0	0.6	8.0	32	11
Ohio	Cincinnati	46.4	32.5	6.4	5.1	2.0	0.3	3.0	19	15
Ohio	Cleveland	52.2	40.2	6.4	3.9	1.4	0.3	3.6	18	33
Ohio	Toledo	44.3	33.9	5.5	3.5	1.3	0.1	3.2	18	13
Pennsylvania	Philadelphia §	43.6	30.6	5.8	4.6	2.1	0.3	5.2	22	12
Pennsylvania	Pittsburg	51.3	38.3	7.0	4.5	1.3	0.2	3.6	17	22
South Carolina	Charleston	34.7	20.6	4.9	4.7	3.0	0.9	8.3	34	10
Tennessee	Knoxville	43.4	26.8	6.9	6.0	3.0	0.6	5.6	25	18
Tennessee	Memphis	39.8	24.4	5.9	5.2	3.3	0.7	8.9	27	12
Tennessee	Nashville	42.0	26.4	6.1	5.8	3.0	0.6	5.2	23	19
Texas	Galveston	34.6	22.8	3.7	3.6	2.9	0.9	7.9	31	13
Virginia	Lynchburg	40.3	26.2	6.2	5.1	2.3	0.4	4.7	18	17
Virginia	Norkfolk	42.0	26.2	6.1	5.7	3.3	0.5	4.6	26	15
Wisconsin	Milwaukee	44.7	34.0	5.2	3.9	1.4	0.2	3.7	21	14
Wyoming	Cheyenne	30.5	26.7	2.4	1.2	0.2	0.0	1.9	43	19

*** For instance, Alabama, Mobile, trace to 0.25 inch, 24.3, means that on 24.3 per cent. of the days embraced within the 20 years, rain fell to a depth of from a trace to 0.25 inch.**

† From October 1875 only. ‡ From January 1874 only. § From May 1872 only.

WATER.

Pure water, as boiled and distilled, **is composed of** the two gases, hydrogen and oxygen; in the proportions of 2 measures hydrogen to 1 of oxygen; or 1 weight of hydrogen to 8 of oxygen. Ordinarily, however, it contains several foreign ingredients, as carbonic and other acids; and soluble mineral, or organic substances. When it contains much lime, it is said to be *hard;* and will not make a good lather with soap. **The air** in its ordinary state **contains** about 4 grains of water per cubic foot.

The average **pressure of the air at sea level, will balance a column of water** 34 feet high; or about 30 inches of mercury. At its boiling point of 212° Fah, its bulk is about one twenty-third greater than at 70°.

Its weight per cubic foot is taken at $62\frac{1}{2}$ lbs, or 1000 ounces avoir; but $62\frac{1}{3}$ lbs would be nearer the truth, as per table below. It is about 815 times heavier than air, when both are at the temperature of 62°; and the barometer at 30 inches. With barometer at 30 inches the weight of perfectly pure water is as follows. At about 39° it has its maximum density of 62.425 lbs per cubic foot.

Temp, Fah.	Lbs per Cub Ft.	Temp, Fah.	Lbs per Cub Ft.
32°	62.417	70°	62.302
40°	62.423	80°	62.218
50°	62.409	90°	62.119
60°	62.367	212°	59.7

Weight of sea water 64.00 to 64.27 lbs per cubic foot, or say 1.6 to 1.9 lbs per cubic foot more than fresh water. See also p 328.

Water has its **maximum density** when its temperature is a little above 39° Fah; or about 7° above the freezing point. By best authorities 39.2°. From about 39° it expands either by cold, or by heat. When the temperature of 32° reduces it to **ice,** its weight is but about 57.2 lbs. per cubic foot; and its specific gravity about .9175, according to the investigations of L. Dufour. Hence, as ice, it has expanded one-twelfth of its original bulk as water; and the **sudden expansive force** exerted at the moment of freezing, is sufficiently great to split iron water-pipes; being probably not less than 30000 lbs per square inch. Instances have occurred of its splitting cast tubular posts of iron bridges, and of ordinary buildings, when full of rain water from exposure. It also loosens and throws down masses of rock, through the joints of which rain or spring water has found its way. Retaining-walls also are sometimes overthrown, or at least bulged, by the freezing of water which has settled between their backs and the earth filling which they sustain; and walls which are not founded at a sufficient depth, are often lifted upward by the same process.

It is said that **in a glass tube ¼ inch in diameter,** water will not freeze until the temperature is reduced to 23°; and in tubes of less than $\frac{1}{50}$ inch, to 3° or 4°. Neither will it freeze until considerably colder than 32° in rapid running streams. **Anchor ice,** sometimes found at depths as great as 25 feet, consists of an aggregation of small crystals or needles of ice frozen at the surface of rapid open water; and probably carried below by the force of the stream. It does not form under frozen water.

Since ice floats in water; and a floating body displaces a weight of the liquid equal to its own weight, it follows that a cubic foot of floating ice weighing 57.2 lbs, must displace 57.2 lbs of water. But 57.2 lbs of water, one foot square, is 11 inches deep: therefore, floating ice of a cubical or parallelopipedal shape, will have $\frac{11}{12}$ of its volume under water; and only $\frac{1}{12}$ above; and a square foot of ice of any thickness, will require a weight equal to $\frac{1}{11}$ of its own weight to sink it to the surface of the water. In practice, however, this must be regarded merely as a close approximation, since the weight of ice is somewhat affected by enclosed air-bubbles.

Pure water is usually assumed to **boil** at 212° Fah in the open air, at the level of the sea; the barometer being at 30 inches; and at about 1° less for every 520 feet above sea level, for heights within 1 mile. In fact, its boiling point varies like its freezing point, with its purity, the density of the air, the material of the vessel, &c. In a metallic vessel, it may boil at 210°; and in a glass one, at from 212° to 220°; and it is stated that if all air be previously extracted, it requires 275°.

It evaporates at all temperatures; **dissolves** more substances than any other agent: and has a greater capacity for heat than any other known substance.

It is compressed at the rate of about one-21740th, (or about $\frac{1}{100}$ of an inch in $18\frac{1}{10}$ feet,) by each atmosphere or pressure of 15 lbs per square inch. When the pressure is removed, its elasticity restores its original bulk

Effect on metals. The lime contained in many waters, forms deposits in metallic water-pipes, and in channels of earthenware, or of masonry; especially if the current be slow. Some other substances do the same; obstructing the flow of the water to such an extent, that it is always expedient to use pipes of diameters larger than would otherwise be necessary. The lime also forms very hard **incrustations at the bottoms of boilers**; very much impairing their efficiency; and rendering them more liable to burst. Such water is unfit for locomotives. We have seen it stated that the Southwestern R R Co, England, prevent this lime deposit, along their limestone sections, by dissolving 1 ounce of sal-ammoniac to 90 gallons of water. The salt of sea water forms similar deposits in boilers; as also does mud, and other impurities.

Water, either when very pure, as rain water; or when it contains carbonic acid, (as most water does,) **produces carbonate of lead in lead pipes**; and as this is an active poison, such pipes should not be used for such waters. Tinned lead pipes may be substituted for them. If, however, sulphate of lime also be present, as is very frequently the case, this effect is not always produced; and several other substances usually found in spring and river water, also diminish it to a greater or less degree. **Fresh water corrodes wrought iron more rapidly than cast**; but the reverse appears to be the case with **sea water**; although it also affects wrought iron very quickly; so that thick flakes may be detached from it with ease. The corrosion of iron or steel by sea water increases with the carbon. Cast-iron cannons from a vessel which had been sunk in the fresh water of the Delaware River for more than 40 years, were perfectly free from rust. Gen. Pasley, who had examined the metals found in the ships Royal George, and Edgar, the first of which had remained sunk in the sea for 62 years, and the last for 133 years, "stated that the cast iron had generally become quite soft; and in some cases resembled plumbago. Some of the shot when exposed to the air became hot; and burst into many pieces. The wrought iron was not so much injured, *except when in contact with copper, or brass gun-metal.* Neither of these last was much affected, except when in contact with iron. Some of the wrought iron was reworked by a blacksmith, and pronounced superior to modern iron." "Mr. Cottam stated that some of the guns had been carefully removed in their soft state, to the Tower of London; and in time (within 4 years) *resumed their original hardness.* Brass cannons from the Mary Rose, which had been sunk in the sea for 292 years, were considerably honeycombed in spots only; (perhaps where iron had been in contact with them.) The old cannons, of wrought-iron bars hooped together, were corroded about ¼ inch deep; but had probably been protected by mud. The cast-iron shot became redhot on exposure to the air; and fell to pieces like dry clay!"

"Unprotected parts of cast-iron sluice-valves, on the sea gates of the Caledonian canal, were converted into a soft plumbaginous substance, to a depth of ¾ of an inch, within 4 years; but where they had been coated with common Swedish tar, they were entirely uninjured. This softening effect on cast iron appears to be as rapid even when the water is but slightly brackish; and that only at intervals. It also takes place on cast iron imbedded in salt earth. Some water pipes thus laid near the Liverpool docks, at the expiration of 20 years were soft enough to be cut by a knife; while the same kind, on higher ground beyond the influence of the sea water, were as good as new at the end of 50 years."

Observation has, however, shown that **the rapidity of this action depends much on the quality of the iron**; that which is dark-colored. and contains much carbon mechanically combined with it, corrodes most rapidly: while hard white, or light-gray castings remain secure for a long time. Some cast-iron sea-piles of this character, showed no deterioration in 40 years.

Contact with brass or copper is said to induce a galvanic action which greatly hastens decay in either fresh or salt water. Some muskets were recovered from a wreck which had been submerged in sea water for 70 years near New York. The brass parts were in perfect condition; but the iron parts had entirely disappeared. **Galvanizing** (coating with zinc) acts as a preservative to the iron, but at the expense of the zinc, which soon disappears. The iron then corrodes. If iron be well heated, and then coated with **hot coal-tar**, it will resist the action of either salt or fresh water for many years. It is very important that the tar be perfectly *purified.* Such a coating, or one of paint, will not prevent **barnacles and other shells** from attaching themselves to the iron. Asphaltum, if pure, answers as well as coal-tar.

Copper and bronze are very little affected by sea water.

No galvanic action has been detected where brass ferules are inserted into the water-pipes in Philadelphia.

The most prejudicial exposure for iron, as well as for wood, is that to alternate wet and dry. At some dangerous spots in Long Island Sound, it has been the practice to drive round bars of rolled iron about 4 inches diameter, for supporting signals. These wear away most rapidly between high and low water; at the rate of about an inch in depth in 20 years; in which time the 4-inch bar becomes reduced to a 2-inch one, along that portion of it. Under fresh water especially, or under ground, a thin coating of coal-pitch varnish, carefully applied, will protect iron, such as water-pipes, &c., for a long time. See page 655. The sulphuric acid contained in the water from coal mines corrodes iron pipes rapidly. **In the fresh water of canals,** iron boats have continued in service from 20 to 40 years. **Wood remains sound** for centuries under either fresh or salt water, if not exposed to be worn away by the action of currents; or to be destroyed by marine insects.

Sea water weighs from 64 to 64.27 lbs per cubic foot, or say from 1.6 to 1.9 lbs per cubic foot more than fresh water, varying with the locality, and not appreciably with the depth. The excess, over the weight of fresh water, is chiefly common salt. At 64 lbs per cubic foot, 35 cubic feet weigh 2240 lbs. **Sea water freezes** at about 27° Fahr. The ice is fresh; but (especially at low temperatures) brine may be entrapped in the ice.

A teaspoonful of powdered alum, well stirred into a bucket of **dirty water,** will generally purify it sufficiently within a few hours to be drinkable. If a hole 3 or 4 feet deep be dug in the sand of the sea-shore, the infiltrating water will usually be sufficiently fresh for washing with soap; or even for drinking. It is also stated that water may be preserved sweet for many years by placing in the containing vessel 1 ounce of black oxide of manganese for each gallon of water.

It is said that water kept in zinc tanks; or flowing through iron tubes galvanized inside, rapidly becomes poisoned by soluble salts of zinc formed thereby; and it is recommended to coat zinc surfaces with asphalt varnish to prevent this. Yet, in the city of Hartford, Conn, service pipes of iron, galvanized inside and out, were adopted in 1855, at the recommendation of the water commissioners; and have been in use ever since. They are likewise used in Philadelphia and other cities to a considerable extent. In many hotels and other buildings in Boston, the "Seamless Drawn Brass Tube" of the American Tube Works at Boston, has for many years been in use for service pipe; and has given great satisfaction. It is stated that the softest water may be kept in brass vessels for years without any deleterious result.

The action of lead upon some waters (even pure ones) is highly poisonous. The subject, however, is a complicated one. An injurious ingredient may be attended by another which neutralizes its action. Organic matter, whether vegetable or animal, is injurious. Carbonic acid, when not in excess, is harmless.

Ice may be so impure that its water is dangerous to drink.

The popular notion that hot water freezes more quickly than cold, with air at the same temperature, is erroneous.

TIDES.

The tides are those well-known rises and falls of the surface of the sea and of some rivers, caused by the attraction of the sun and moon. There are two rises, floods, or high tides; and two falls, ebbs, or low tides, every 24 hours and 50 minutes (**a lunar day**); making the average of 6 hours 12½ minutes between high and low water. These intervals are, however, **subject to great variations;** as are also the heights of the tides; and this not only at different places, but at the same place. These irregularities are owing to the shape of the coast line, the depth of water, winds, and other causes. *Usually* at new and full moon, or rather a day or two after, (or twice in each lunar month, at intervals of two weeks,) the tides rise higher, and fall lower than at other times; and these are called **spring tides.** Also, one or two days after the moon is in her *quarters*, twice in a lunar month, they both rise and fall less than at other times; and are then called **neap tides.** From neap to spring they rise and fall more daily; and vice versa. The **time of high water** at any place, is generally two or three hours after the moon has passed over either the upper or lower meridian; and is called the **establishment** of that place; because, when this time is established, the time of high water on any other day may be found from it in most cases. The total height of spring tides is *generally* from 1¼ to 2 times as great as that of neaps. The great **tidal wave** is merely an *undulation*, unattended by any current, or progressive motion of the particles of water. Each successive high tide occurs about 24 minutes later than the preceding one; and so with the low tides.

EVAPORATION, FILTRATION, AND LEAKAGE.

The amount of evaporation from surfaces of water exposed to the natural effects of the open air, is of course greater in summer than in winter; although it is quite perceptible in even the coldest weather. It is greater in shallow water than in deep, inasmuch as the bottom also becomes heated by the sun. It is greater in running, than in standing water; on much the same principle that it is greater during winds than calms. It is probable that the average daily loss from a reservoir of moderate depth, from evaporation alone, throughout the 3 warmer months of the year, (June, July, August,) rarely exceeds about $\frac{3}{10}$ inch, in any part of the United States. Or $\frac{1}{10}$ inch during the 9 colder months; except in the Southern States. These two averages would give a daily one of .15 inch; or a total annual loss of 55 ins, or 4 ft 7 ins. It probably is 3.5 to 4 ft.

By some trials by the writer, in the tropics, ponds of pure water 8 ft deep, in a stiff retentive clay, and fully exposed to a very hot sun all day, lost during the dry season, precisely 2 ins in 16 days; or ⅛ inch per day; while the evaporation from a glass tumbler was ¼ inch per day. The air in that region is highly charged with moisture; and the dews are heavy. Every day during the trial the thermometer reached from 115° to 125° in the sun.

The total annual evaporation in several parts of England and Scotland is stated to average from 22 to 38 ins; at Paris, 34; Boston, Mass, 32; many places in the U. S., 30 to 36 ins. This last would give a daily average of $\frac{1}{10}$ inch for the whole year. Such statements, however, are of very little value, unless accompanied by memoranda of the circumstances of the case; such as the depth, exposure, size and nature of the vessel, pond, &c, which contains the water, &c. Sometimes the total annual evaporation from a district of country exceeds the rain fall; and vice versa.

On canals, reservoirs, &c, it is usual to combine the loss by evaporation, with that by filtration. The last is that which soaks into the earth; and of which some portion passes entirely through the banks, (when in embankt;) and if in very small quantity, may be dried up by the sun and air as fast as it reaches the outside; so as not to exhibit itself as water; but if in greater quantity, it becomes apparent, as *leakage*.

E. H. Gill, C E, states the average evaporation and filtration on the Sandy and Beaver canal, Ohio, (38 ft wide at water surface; 26 ft at bottom; and 4 ft deep,) to be but 13 cub ft per mile per minute, *in a dry season*. Here the exposed water surf in one mile is 200640 sq ft; and in order, with this surf, to lose 13 cub ft per min, or 18720 cub ft per day of 24 hours, the quantity lost must be $\frac{18720}{200640} = .0933$ ft, = 1⅛ inch in depth per day. Moreover, one mile of the canal contains 675840 cub ft; therefore, the number of days reqd for the combined evaporation and filtration to amount to as much as all the water in the canal, is $\frac{675840}{18720} = 36$ days. Observations in warm weather on a 22-mile reach of the Chenango canal, N York, (40; 28; and 4 ft,) gave 65½ cub ft per mile per min; or 5 times as much as in the preceding case. This rate would empty the canal in about 8 days. Besides this there was an excessive leakage at the gates of a lock, (of only 5½ ft lift,) of 479 cub ft per min, 22 cub ft per mile per min; and at aqueducts, and waste-weirs, others amounting to 19 cub ft per mile per min. The leakage at other locks with lifts of 8 ft, or less, did not *exceed* about 350 cub ft per min, at each. On other canals, it has been found to be from 50, to 500 ft per min. On the Chesapeake and Ohio canal, (where 50, 32, and 6 ft,) Mr. Fisk, C E, estimated the loss by evap and filtration in 2 weeks of warm weather, to be equal to all the water in the canal. **Professor Rankine assumes 2 ins per day, for leakage of canal bed, and evaporation, on English canals.** J. B. Jervis, C E, estimated the loss from evap, filtration, and leakage through lock-gates, on the original Erie canal, (40, 28, and 4 ft,) at 100 cub ft per mile per min; or 144000 cub ft per day. The water surf in a mile is 211200 sq ft; therefore, the daily loss would be equal to a depth of $\frac{144000}{211200} = .682$ ft, = say 8⅛ ins.

On the Delaware division of the Pennsylvania canals, when the supply is temporarily shut off from any long reach, the water falls from 4 to 8 ins per day. The filtration will of course be much greater on embankts, than in cuts. In some of our canals, the depth at high embankts becomes quite considerable; the earth, from motives of economy, not being filled in level under the bottom of the canal; but merely left to form its own natural slopes. At one spot at least, on the Ches and Ohio canal, where one side is a natural face of vertical rock, this depth is 40 ft. Such depths increase the leakage very greatly; especially when, as is frequently the case, the embankts are not puddled; and the practice is not to be commended, for other reasons also.

The total average loss from reservoirs of moderate depths, in case the earthen dams be constructed with proper care, and well settled by time, will not exceed about from ½ to 1 inch per day; but in new ones, it will usually be considerably greater.

The loss from ditches, or channels of small area, is much greater than that from navigable canals; so that long canal feeders usually deliver but a small proportion of the water which enters them at their heads.

MECHANICS. FORCE IN RIGID BODIES.

In the following pages we endeavor to make clear a few elementary principles of Mechanics. The opening articles are devoted chiefly to the subject of matter *in motion;* for, while an acquaintance with this is perhaps not absolutely required in obtaining a *working* knowledge of those principles of Statics which enter so largely into the computations of the civil engineer, yet it must be an important aid to their intelligent appreciation.

Art. 1 (a). Mechanics may be defined as that branch of science which treats of the effects of force upon matter.

This broad definition of the word "Mechanics" includes hydrostatics, hydraulics, pneumatics. etc., if not also electricity, optics, acoustics, and indeed all branches of physics; but we shall here confine ourselves chiefly to the consideration of the action of extraneous forces upon bodies supposed to be *rigid*, or incapable of change of shape.

(b) Mechanics is divided into two branches, namely:

Kinematics; or the study of the *motions* of bodies, without reference to the *causes* of motion; and

Dynamics, or the study of force and its effects.

The latter is sub-divided into

Kinetics; which treats of the relations between force and *motion;* and

Statics; which considers those special, but very numerous, cases, where *equal and opposite* forces counteract each other and thus destroy each other's motions.

Art. 2 (a). Matter, or **substance,** may be defined as whatever occupies space; as metal, stone, wood, water, air, steam, gas, etc.

(b) **A body** is any portion of matter which is either more or less completely separated in fact from all other matter, or which we take into consideration by itself and as if it were so separated. Thus, a stone is a body, whether it be falling through the air or lying detached upon the ground, or built up into a wall. Also, the wall is a body; or, if we wish, we may consider any portion of the wall, as any particular cubic foot or inch in it, as a body. The earth and the other planets are bodies, and their smallest atoms are bodies.

A train of cars may be regarded as a body; as may also each car, each wheel or axle or other part of the car, each passenger, etc., etc.

Similarly, the ocean is a body, or we may take as a body any portion of it at pleasure, such as a cubic foot, a certain bay, a drop, etc.

(c) But in what follows we shall (as already stated) consider chiefly *rigid* bodies; *i. e.*, bodies which undergo no change in *shape*, such as by being crushed or stretched or pulled apart, or penetrated by another body. All actual bodies are of course more or less subject to some such changes of shape; *i. e.*, no body is *in fact* absolutely rigid; but we may properly, for convenience, suppose such bodies to exist, because many bodies are so nearly rigid that under ordinary circumstances they undergo little or no change of shape, and because such change as does occur may be considered under the distinct head of Strength of Materials.

(d) But while bodies are thus to be regarded as incapable of change of *form*, it is equally important that we regard them as *susceptible* to change of *position* as *wholes*. Thus, they may be upset or turned around horizontally or in any other direction, or moved along in any straight or curved line, with or without turning around a point within themselves. In short they are capable of *motion*, as *wholes*.

Art. 3 (a). Motion of a body is change of its position in relation to another body or to some real or imaginary point, which (for convenience) we *regard* as fixed, or at rest. Thus, while a stone falls from a roof to the ground, its position, relatively to the roof, is constantly changing, as is also that relatively to the ground and that relatively to any given point in the wall; and we say that the stone is *in motion relatively to either of those bodies*, or to any point in them. But if two stones, A and B, fall from the roof at the same instant and reach the ground at the same (subsequent) instant, we say that although each moves, relatively to roof and ground, yet they have *no motion relatively to each other;* or, they are *at rest* relatively to each other; for their position in regard to each other does not change; *i. e.*, in whatever direction and at whatever distance stone A may be from stone B at the time of starting, it remains in that same direction, and at that same distance from B during the whole time of the fall. Similarly, the roof, the wall and the ground are at rest relatively to each other, yet they are in motion relatively to a falling stone. They are also in motion relatively to the sun, owing to the earth's daily rotation about its axis, and its annual movement around the sun.

(b) If a train-man walks toward the rear along the top of a freight train just as fast as the train moves forward, he is in *motion* relatively to the *train;* but, as a whole, he is at *rest* relatively to *buildings*, etc. near by; for a spectator, standing at a little distance from the track, sees him continually opposite the same part of such building, etc. If the man on the train now stops walking, he comes to *rest* relatively to the *train*, but at the same time comes into *motion* relatively to the surrounding *buildings*, etc., for the spectator sees him begin to move along with the train.

(c) Since we know of no absolutely fixed point in space, we cannot say, of any body, what its *absolute* motion is. Consequently, we do not know of such a thing as absolute *rest*, and are safe in saying that *all* bodies are in motion.

Art. 4 (a). The **velocity** of a moving body is its *rate* of motion. A body (as a railroad train) is said to move with **uniform velocity**, or **constant velocity**, when the *distances* moved over in *equal times* are *equal to each other*, no matter how small those times may be taken.

(b) The **velocity is expressed** by stating the *distance* passed over *during some given time*, or which *would* be passed over during that time if the uniform motion continued so long. Thus, if a railroad train, moving with constant velocity, passes over 10 miles in half an hour, we may say that its velocity, during that time, is (*i. e.*, that it moves *at the rate* of) 20 miles per hour, or 105,600 feet per hour, or 1760 feet per minute, or 29⅓ feet per second. Or, we may, if desirable, say that it moves at the rate of 10 miles in *half* an hour, or 88 feet in *three* seconds, etc.; but it is generally more convenient to state the distance passed over in a *unit* of time, as in *one* day *one* hour, *one* second, etc.

(c) If, of two trains, A and B, moving with constant velocity,

A moves 10 miles in half an hour,
B moves 10 miles in quarter of an hour,

then the velocities are,

A,	20 miles per hour,
B,	40 miles per hour.

In other words, the velocity of a body (which may be defined as the *distance* passed over in a given *time*) is *inversely* as the *time* required to pass over a given *distance*.

(d) By **unit velocity** is meant that velocity which, by common consent, is taken as equal to *unity* or *one*. Where English measures are used, the unit velocity generally adopted in the study of Mechanics is 1 foot per second.

(e) When we say that a body has a velocity of 20 miles per hour, or 10 feet per second, etc., we do not imply that it will necessarily travel 20 miles, or 10 feet, etc.; for it may not have sufficient time for that. We mean merely that it is traveling at the *rate* of 20 miles per hour, or 10 feet per second, etc.; so that *if it continued* to move at that same rate for an hour, or a second, etc., it would travel 20 miles, or 10 feet, etc.

(f) When velocity *increases*, it is said to be **accelerated.** When it *decreases*, it is said to be **retarded.** If the acceleration or retardation is in exact proportion to the time; that is, when during any and every equal interval of time, the same degree of change takes place, it is *uniformly* accelerated, or retarded. When otherwise, the words *variable* and *variably* are used.

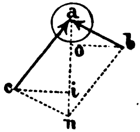
(g) A body may have, at the same time, **two or more independent velocities** requiring to be considered. For instance, a ball fired vertically upward from a gun, and then falling again to the earth, has, during the whole time of its rise and fall, (1st) the *uniform upward* velocity with which it leaves the muzzle, and (2nd) the continually *accelerated downward* velocity given to it by gravity, which acts upon it during the whole time. Its *resultant* (or apparent) velocity at any moment is the *difference* between these two.

Thus, immediately after leaving the gun, the downward velocity given by gravity is very small, and the resultant velocity is therefore upward and very nearly equal to the whole upward velocity due to the powder. But after awhile the downward velocity (by constantly increasing) becomes equal to the upward velocity; *i. e.*, their difference, or the resultant velocity, becomes *nothing;* the ball at that instant stands still; but its downward velocity continues to increase, and immediately becomes a little greater than the upward velocity; then greater and greater, until the ball strikes the ground. At that instant its resultant velocity is

$$= \left\{\begin{array}{c}\text{the downward velocity which it would}\\ \text{have acquired by falling } \textit{during the}\\ \textit{whole time of its rise and fall.}\end{array}\right\} - \left\{\begin{array}{c}\text{the uniform upward}\\ \text{velocity given by the}\\ \text{powder.}\end{array}\right.$$

We have here neglected the resistance of the air, which of course retards both the ascent and the descent of the ball.

(h) As a further illustration, regard *a b n c* as a raft drifting in the direction *c a* or *n b*. A man on the raft walks with uniform velocity from corner *n* to corner *c* while the raft drifts (with a uniform velocity a little greater than that of the man) through the distance *n b*. Therefore, when the man reaches corner *c*, that corner has moved to the point which, when he started, was occupied by *a*. The man's resultant motion, relatively to the bed of the river or to a point on shore, has therefore been *n a*. His motion at right angles to *n a*, due to his walking, is *i c*, but that due to the drifting of the raft is *o b*. These two are equal and opposite. Hence his *resultant* motion *at right angles* to *n a* is nothing; he does not move from the line *n a*. His walking moves him through a distance equal to *n i*, in the direction *n a;* and the drifting through a distance equal to *i a*, and the sum of these two is *n a*.



(i) All the motions which we see given to bodies are but *changes* in their unknown *absolute* motions. For convenience, we may confine our attention to some one or more of these changes, neglecting others.

Thus, in the case of the ball fired upward from a gun (see (*g*) above) we may neglect its uniform upward motion and consider only its constantly accelerated downward motion under the action of gravity; or, as is more usual, we may consider only the *resultant* or *apparent* motion, which is first upward and then downward. In both cases we neglect the motions of the ball caused by the several motions of the earth in space.

Art. 5 (a). Force, the cause of change of motion. Suppose a perfectly smooth ball resting upon a perfectly hard, frictionless and level surface, and suppose the resistance of the air to be removed. In order to merely *move* the ball horizontally (*i. e.*, to set it in motion—to change its state of motion) some *force* must act upon it. Or, if such a ball were already in motion, we could not retard or hasten it, or turn it from its path without exerting force upon it. For, as stated in **Newton's first law of motion, "every body continues in its state** of rest or of motion in a straight line, except in so far as it may be compelled by impressed forces to change that state." On the other hand, if a force acts upon a body, the motion of the body must undergo change.

(b) Force is an action between two bodies, tending either to separate them or to bring them closer together. For instance, when a stone falls to the ground, we explain the fact by saying that a force (the attraction of gravitation) tends to draw the earth and the stone together.

Magnetic and electric attraction, and the cohesive force between the particles of a body, are other instances of *attractive* force.

(c) **Force applied by contact.** In practice we apply force to a body (B) by causing contact between it and another body (A) which has a tendency to motion toward B. A *repulsive* force is thus called into action between the two bodies (in some way which we cannot understand), and this force pushes B forward (or in the direction of A's tendency to move) and pushes A backward, thus *diminishing* its forward tendency.*

If, for instance, a stone be laid upon the ground, it tends to move downward, but does not do so, because a repulsive force pushes it and the earth apart just as hard as the force of gravity tends to draw them together.

Similarly, when we attempt to lift a moderate weight with our hand, we do so by giving the hand a tendency to move upward. If the hand slips from the weight, this tendency moves the hand rapidly upward before our will force can check it. But otherwise, the repulsive force, generated by contact between the hand (tending upward) and the weight, moves the latter upward in spite of the force of gravity, and pushes the hand downward, depriving it of much of the upward velocity which it would otherwise have. It is perhaps chiefly from the **effort,** of which we are conscious in such cases, that we derive our notions of "**force.**"

When a moving billiard ball. A, strikes another one, B, at rest, the tendency of A to continue moving forward is resisted by a repulsive force acting between it and B. This force pushes B forward, and A backward, retarding its former velocity. As explained in Art. 23 (*a*), the repulsive force does not exist in either body until the two meet.

(**d**) The repulsive force thus generated by contact between two bodies, continues to act only so long as they remain in contact, and only so long as they tend (from some extraneous cause) to come closer together. But it is generally or always accompanied by an additional repulsive force, due to the *compression* of the particles of the bodies and their tendency to return to their original positions. This *elastic* repulsive force may continue to act after the tendency to compression has ceased.

(**e**) **Force acts either as a pull or as a push.** Thus, when a weight is suspended by a hook at the end of a rope, gravity *pulls* the weight downward, the weight *pushes* the hook, and the hook *pulls* the rope, each of these actions being accompanied, of course, by its corresponding and opposite "reaction." When two bodies collide, each pushes the other, generally for a very short time.

(**f**) **Equality of action and reaction.** A force always exerts itself *equally* upon the two bodies between which it acts. Thus, the force (or attraction) of gravitation, acting between the earth and a stone, draws the earth upward just as hard as it draws the stone downward; and the repulsive force, acting between a table and a stone resting upon it, pushes the table and the earth downward just as hard as it pushes the stone upward. This is the fact expressed by **Newton's third law of motion,** that "to every action there is always an equal and contrary reaction." For measures of force, see Arts. 11, 12, 13.

If a cannon ball in its flight cuts a leaf from a tree, we say that the *leaf* has reacted against the *ball* with precisely the same force with which the ball acted against the leaf. That degree of force was sufficient to cut off a leaf, but not to arrest the ball. A ship of war, in running against a canoe, or the fist of a pugilist striking his opponent in the face, receives as violent a blow as it gives; but the same blow that will upset or sink a canoe, will not *appreciably* affect the motion of a ship, and the blow which may seriously damage a nose, mouth, or eyes, may have no such effect upon hard knuckles.

The resistance which an abutment opposes to the pressure of an arch; or a retaining-wall to the pressure of the earth behind it, is no greater than those pressures themselves; but the abutment and the wall are, for the sake of safety, made *capable* of sustaining much greater pressures, in case accidental circumstances should produce such.

(**g**) In most practical cases **we have to consider only one** of the two bodies between which a force acts. Hence, for convenience, we commonly speak as if the force were divided into *two* equal and opposite forces one for each of the two bodies, and confine our attention to *one* of the bodies and the force acting upon it, neglecting the other. Thus we may speak of the force of steam in an engine as acting upon the *piston*, and neglect its equal and opposite pressure against the head of the cylinder.

(**h**) That point of a body to which, theoretically, a force is applied, is called the **point of application.** In practice we cannot apply force to a *point*, according to the scientific meaning of that word; but have to apply it distributed over an appreciable *area* (sometimes very large) of the surface of the body.

---

* We ordinarily express all this by saying simply that A pushes B forward, and this is sufficiently exact for practical purposes; but it is well to recognize that it is merely a convenient expression and does not fully state the facts, and that every force *necessarily* consists of *two* equal and opposite pulls or pushes exerted between two bodies.

For the present we shall assume that the line of action of the force passes through the center of gravity of the body and forms a right angle with the surface at the point of application.

Art. 7 (a). Acceleration. When an unresisted force, acting upon a body, sets it in motion (*i. e.*, gives it velocity) in the direction of the force, this velocity *increases* as the force continues to act; each equal interval of time (if the force remains constant) bringing its own equal increase of velocity.

Thus, if a stone be let fall, the force of gravity gives to it, in the first inconceivably short interval of time, a small velocity downward. In the next equal interval of time, it adds a second equal velocity, and so on, so that at the end of the second interval the velocity of the stone is twice as great, at the end of the third interval three times as great, as at the end of the first one, and so on. We may divide the time into as small equal intervals as we please. In each such interval the constant* force of gravity gives to the stone an equal increase of velocity.

Such increase of velocity is called acceleration.† When a body is thrown vertically *upward*, the downward acceleration of gravity appears as a *retardation* of the upward motion. When a force thus acts *against* the motion under consideration, its acceleration is called *negative*.

Art. 8 (a). The rate of acceleration† is the acceleration which takes place *in a given time*, as one second.

(b) The **unit rate of acceleration** is that which adds unit of velocity in a unit of time; or, where English measures are used, one foot per second, *per second*.

(c) For a *given rate* of acceleration, the *total* accelerations are of course proportional to the *times* during which the velocity increases at that rate.

Art. 9 (a). Laws of acceleration. Suppose two blocks of iron, one (which we will call A) twice as large as the other (*a*), placed each upon a perfectly frictionless and horizontal plane, so that in moving them horizontally we are opposed by no force tending to hold them still. Now apply to each block, through a spring balance, a pull such as will keep the pointer of each balance always at the same mark, as, for instance, constantly at 2 in both balances. We thus have equal forces acting upon unequal masses.‡ Here the rate of acceleration of *a* is double that of A; for **when the forces are equal the rates of acceleration are inversely as the masses.**

In other words, in one second (or in any other given time) the small block of iron, *a*, will acquire twice the increase of velocity that A (twice as large) will acquire; so that if both blocks start at the same time from a state of rest, the smaller one, *a*, will have, at the end of any given time, twice the *velocity* of A, which has twice its mass.

(b) Again, let the two masses, A and *a*, be equal, but let the *force exerted* upon *a* be twice that exerted upon A. Then the rate of acceleration of *a* will (as before) be twice that of A; for, **when the masses are equal, the rates of acceleration are directly as the forces.**

(c) We thus arrive at the principle that, in any case, **the rate of acceleration is directly proportional to the force and inversely proportional to the mass.**

---

* We here speak of the force of gravity, exerted in a given place, as constant, because it is so for all practical purposes. Strictly speaking, it increases a very little as the stone approaches the earth.

† Since the *rate* of acceleration is generally of greater consequence, in Mechanics, than the *total* acceleration, or the "acceleration" proper, scientific writers (for the sake of brevity) use the term "acceleration" to denote that *rate*, and the term "*total* acceleration" to denote the total increase or decrease of velocity occurring during any given time. Thus, the *rate* of acceleration of gravity (about 32.2 ft. per second per second) is called, simply, the "acceleration of gravity." As we shall not have to use either expression very frequently, we shall, generally, to avoid misapprehension, give to each idea its *full* name; thus, "total acceleration" for the *whole change of velocity* in a given case, and "rate of acceleration" for the *rate* of that change.

‡ The mass of a body is the quantity of matter that it contains.

(d) Hence, if we make the two forces proportional to the two masses, the rates of acceleration will be equal; or, **for a given rate of acceleration, the forces must be directly as the masses.**

(e) Hence, also, a greater force is required to impart a given velocity to a given body in a short time than to impart the same velocity in a longer time. For instance, the forward coupling links of a long train of cars would snap instantly under a pull sufficient to give to the train in two seconds a velocity of twenty miles per hour, supposing a sufficiently powerful locomotive to exist. In many such cases, therefore, we have to be contented with a slow, instead of a rapid acceleration.

A string may safely sustain a weight of one pound suspended from our hand. If we wish to impart a great upward velocity to the weight *in a very short time*, we evidently can do so only by exerting upon it a great force; in other words, by jerking the string violently upward. But if the string has not tensile strength sufficient to transmit this force from our hand to the weight, it will break. We might safely give to the weight the desired velocity by applying a *less force* during a *longer time*.

(f) When a stone falls, the *force* pulling the earth upward is (as remarked above) equal to that which pulls the stone downward, but the *mass* of the earth is so vastly greater than that of the stone that its motion is totally imperceptible to us, and would still be so, even if it were not counteracted by motions in other directions in other parts of the earth. Hence we are *practically*, though *not absolutely*, right when we say that the earth remains at rest while the stone falls.

(g) But in the case of the two billiard balls (Art. 5 *c* p. 333), we can clearly see the result of the action of the force upon each of the two bodies; for the second ball, B, which was at rest, now moves forward, while the forward velocity of the first one, A, is diminished or destroyed, its backward motion thus appearing as a *retardation* of its *forward* motion. And, (since the same force acts upon both balls)

$$\begin{matrix}\text{mass}\\ \text{of A}\end{matrix} : \begin{matrix}\text{mass}\\ \text{of B}\end{matrix} :: \begin{matrix}\text{rate of acceleration}\\ \text{of B}\end{matrix} : \begin{matrix}\text{rate of negative acceleration}\\ \text{of A}\end{matrix}$$

or (since the force acts for the same time upon both balls)

$$\begin{matrix}\text{mass}\\ \text{of A}\end{matrix} : \begin{matrix}\text{mass}\\ \text{of B}\end{matrix} :: \begin{matrix}\text{forward velocity}\\ \text{of B}\end{matrix} : \begin{matrix}\text{loss of forward velocity}\\ \text{of A}\end{matrix}$$

(h) REMARK. A man cannot *lift* a weight of 20 tons; but if it be placed upon proper friction rollers, he can *move* it horizontally, as we see in some drawbridges, turntables, &c.; and if friction and the resistance of the air could be entirely removed, he could move it by a single breath; and it would continue to move forever after the force of the breath had ceased to act upon it. It would, however, move very slowly, because the force of the single breath would have to diffuse itself among 20 tons of matter. He can move it, if it be placed in a suitable vessel in water, or if suspended from a long rope. A powerful locomotive that may move 2000 tons, cannot lift 10 tons vertically.

If we imagine two bodies, each as large and heavy as the earth, to be precisely balanced in a pair of scales without friction, a single grain of sand added to either scale-pan, would give motion to both bodies.

Art. 10 (a). The constant force of gravity is a uniformly accelerating force when it acts upon a body falling freely; for it then increases the velocity at the uniform rate of .322 of a foot per second during every hundredth part of a second, or 32.2 feet per second in every second. Also when it acts upon a body moving down an inclined plane; although in this case the increase is not so rapid, because it is caused by only a part of the gravity, while another part presses the body to the plane, and a third part overcomes the friction. It is a uniformly retarding force, upon a body thrown vertically upward; for no matter what may be the velocity of the body when projected upward, it will be diminished .322 of a foot per second in each hundredth part of a second during its rise, or 32.2 feet per second during each entire second. At least, such would be the case were it not for the varying resistance of the air at different velocities. It is a uniformly straining force when it causes a body at rest, to press upon another body; or to pull upon a string by which it is suspended. The foregoing expressions, like those of momentum, strain, push, pull, lift, work, &c., do not indicate different *kinds* of force; but merely different kinds of *effects* produced by the one grand principle, force.

(b) The above 32.2 feet per second is called the **acceleration of gravity**; and by scientific writers is conventionally denoted by a small **g**; or, more correctly speaking, since the acceleration is not precisely the same at all parts of the earth, g denotes the acceleration per second, whatever it may be, at any particular place.

Art. 11 (a). Relation between force and mass. The mass of a body is the quantity of matter which it contains. *One* cubic foot of water has *twice* as great a mass as *half* a cubic foot of water, but a *less* mass than *one* cubic foot of iron. Thus, the *size* of a body is a measure of mass between bodies of the *same* material, but not between bodies of *different* materials.

(b) When bodies are allowed to fall freely in a vacuum at a given place, they are found to acquire equal velocities in any given time, of whatever different materials they may be composed. From this we know (Art. 9 (*d*), p. 335), that the *forces* moving them downward, viz.: their respective *weights* at that place, must be proportional to their *masses*.

Thus, *in any given place*, the *weight* of a body is a perfect measure of its *mass*. But the *weight* of a given body *changes* when the body is moved from one level above the sea to another, or from one latitude to another; while the *mass* of the body of course remains *the same* in all places. Thus, a piece of iron which weighs a pound at the level of the sea, will weigh *less* than a pound by a spring balance, upon the top of a mountain close by, because the attraction between the earth and a given mass diminishes when the latter recedes from the earth's center. Or if the piece of iron weighs one pound near the North or South Pole, it will, for the same reason, weigh *less* than a pound by a spring balance if weighed nearer to the equator and at the same level above the sea.

The difference in the weight of a body in different localities is so slight as to be of no account in questions of ordinary practical Mechanics;* but scientific exactness requires a measure of mass which will give the same expression for the quantity of matter in a given body, wherever it may be; and, since weighing is a very *convenient* way of arriving at the quantity of matter in a body, it is desirable that we should still be able to express the mass in terms of the weight. Now, when a given body is carried to a higher level, or to a lower latitude, its loss of weight is simply a decrease in the *force* with which gravity draws it downward, and this same decrease also causes a decrease of the *velocity* which the body acquires in falling during any given time. The change in velocity, by Art. 9 (*b*), p. 334, is necessarily proportional to the change in weight.

Therefore, if the weight of a body at any place be divided by the velocity which gravity imparts in one second at the same place (and called g, or the *acceleration of gravity* for that place), the *quotient* will be the same at all places, and therefore serves as an invariable measure of the mass.

(c) By common consent, the **unit of mass,** in scientific Mechanics, is said to be that quantity of matter to which a *unit* of force can give unit rate of acceleration. This unit rate, in countries where English measures are used, is one foot per second, per second. It remains then to adjust the units of *force* and of *mass*. Two methods (an old and a new one) are in use for doing this. We shall refer to them here as methods A and B respectively.

(d) In method A, still generally used in questions of *statics*, the **unit of force** is fixed as that force which is equal to the *weight of one pound* in a certain place; *i. e.*, the force with which the earth at that place attracts a certain standard piece of platinum called a pound; and the unit of *mass* is not this standard piece of metal, but, as stated in (c), that mass to which this unit force of one pound gives, in one second, a velocity of one foot per second. Now the one pound attraction of the earth upon a mass of *one pound* will (Art. 1, p. 330) in one second give to that mass a velocity = g or about 32 feet per second; and (Art. 9 (*a*), p. 334), for a given force the masses are inversely as the velocities imparted in a given time. Therefore, to give in one second a velocity of only *one* foot per second (instead of g or about 32) the one pound unit of force would have to act upon a mass g times (or about 32 times) that which weighs one pound.

This could be accomplished, with an Attwood's machine, Art. 16 (*c*), p. 339, by making the two equal weights each = 15 ½ lbs. and the third weight = 1 lb.

---

* The greatest discrepancy that can occur at various heights and latitudes, by adopting weight as the measure of quantity, would not be likely to exceed 1 in 300; or, under ordinary circumstances, 1 in 1000.

By method A, therefore, the *unit* of mass is g times (or about 32 times) the mass of the standard piece of metal called a pound; *i. e.*, a body containing one such unit of mass weighs g lbs. or about 32 lbs.; or, **by method A,**

$$\text{the } weight \text{ of any given body in lbs.} = g \times \text{the } mass \text{ of the body, in units of mass.}$$

Or,

$$\text{the } mass \text{ of a body, in units of mass} = \frac{\text{the weight of the body, in pounds}}{g}$$

For instance:

in a body weighing	the mass is about
½ pound	$\frac{1}{64}$ unit of mass
1 "	$\frac{1}{32}$ " "
2 "	$\frac{1}{16}$ " "
32 "	1 " "
64 "	2 " "

It has been suggested to call this unit of mass a "Matt."

(e) **In method B,** the *mass* of the standard pound piece of platinum is taken as the **unit of mass** and is called **a pound;** and the force which will give to it in one second a velocity of one foot per second is taken as the unit of *force*. This small unit of force is called a **poundal.** In order that it may in one second give to the mass of one pound a velocity of only one foot per second, it must (by Art. 9 b), be $\frac{1}{g}$ $\left(\text{or about } \frac{1}{32}\right)$ of the *weight* of said pound mass.

Hence, **by method B,**

$$\text{the } mass \text{ of any given body, in } pounds = \frac{\text{the } weight \text{ of the body in } poundals}{g}$$

and

$$\text{the } weight \text{ of a body, in } poundals = g \times \text{the } mass \text{ of the body in } pounds.$$

For instance:

in a body weighing	the mass of the body is about
½ poundal = $\frac{1}{64}$ pound	$\frac{1}{64}$ pound
1 " = $\frac{1}{32}$ "	$\frac{1}{32}$ "
2 " = $\frac{1}{16}$ "	$\frac{1}{16}$ "
32 " = 1 "	1 "
64 " = 2 "	2 "

(f) **For convenience,** we sometimes disregard the scientific requirement that the unit of force must be that which will give unit rate of acceleration to unit mass, and take a *pound* of *matter* as our unit of *mass*, and a *pound weight* as our unit of *force*. Our unit of force will then in one second give a velocity of g (or about 32.2 feet per second) to our unit of mass. In *Statics*, we are not concerned with the *masses* of bodies, but only with the *forces* acting upon them, including their *weights*.

Art. 12 (a). Impulse. By taking, as the unit of force, that force which, in one second, will give to unit mass a velocity of one foot per second, we have (by Art. 9, p. 334), in any case of unbalanced *force* acting upon a *mass* during a given *time*:

$$\text{Velocity} = \frac{\text{force} \times \text{time}}{\text{mass}} \quad \ldots \ (1)$$

$$\text{Force} = \frac{\text{velocity} \times \text{mass}}{\text{time}} \quad \ldots \ (2)$$

$$\text{Mass} = \frac{\text{force} \times \text{time}}{\text{velocity}} \quad \ldots \ (3)$$

$$\text{Time} = \frac{\text{mass} \times \text{velocity}}{\text{force}} \quad \ldots \ (4)$$

also

$$\text{Force} \times \text{time} = \text{mass} \times \text{velocity}. \quad \ldots \ (5)$$

To the **product, force × time,** in equation (5), writers now **give the name impulse,** which was formerly given to *collision* (now called **impact**). See Art. 24 (*a*). The term *impulse*, as now used, conveys merely the idea of *force* acting through a certain length of *time*. Equation (5) tells us that an impulse (the product of a force by the time of its action) is numerically equal to the *momentum** which it produces. Equation (2) tells us that any force is numerically equal to the momentum which it can produce *in one second*. In other words, the **momentum** of a body moving with a given velocity is numerically equal to the *force* which in one second can produce or destroy that velocity in that body; or, a **force** is numerically equal to the **rate per second** at which it can produce momentum. Thus, forces are proportional to the momentums which they can produce in a given time; or, in a given time, equal forces produce equal momentums. Therefore a force must always give equal and opposite momentums to the two bodies between which it acts.

Art. 13 (a). The usual way of measuring a force is by ascertaining the amount of some other force which it can counteract. Thus we may measure the weight of a body by hanging it to a spring balance. The scale of the balance then indicates the amount of tension in the spring; and we know that the weight of the body is equal to the tension, because the weight just prevents the tension from drawing the hook upward.

Thus, **forces are conveniently expressed in weights,** as in pounds, tons, &c., and they are generally so measured in Statics, and in our following articles.

(b) A force may be constant or variable. When a stone rests upon the ground, the pull of gravity upon it (*i. e.*, its weight) remains constant, neither increasing nor decreasing. But when a stone is thrown upward its weight decreases very slightly as it recedes from the earth, and again increases as it approaches it during its fall. In this case, the force of gravity, acting upon the stone, decreases or increases *steadily*. But a force may change *suddenly*, or *irregularly*, or may be *intermittent;* as when a series of unequal blows are struck by a hammer. In what follows we shall have to do only with forces supposed to be *constant*.

Art. 14 (a). Density. The *densities* of materials are proportional to the *masses* contained in a given *volume*, as a cubic inch; or *inversely* as the *volume* required to contain a given *mass*. Or, since the weights at a given place are proportional to the masses, the densities are proportional to the weights per unit of volume (or "specific gravities") of the materials. Thus, a body weighing 100 lbs. per cubic foot is twice as dense as one weighing only 50 lbs. per cubic foot at the same place.

Art. 15 (a). Inertia. The inability of matter to set itself in motion, or to change the rate or direction of its motion, is called its inertia, or inertness. When we say that a certain body has twice the inertia (inertness) of a smaller one, we mean that twice the *force* is required to give it an equal rate of acceleration; and that, since all force (Art. 5 *f*), acts equally in both directions, we experience twice as great a *reaction* (or so-called "resistance") from the larger body as from the smaller one. The "inertia" of a body is therefore a measure of the *force* required to produce in it a given rate of acceleration; or, which is the same thing, it is a measure of the *mass* of the body. We may therefore consider "inertia" and "mass" as identical.

(b) What is called the "resistance of inertia" of a body, is simply the reaction, (*i. e.*, one of the two equal and opposite actions) of whatever force we apply to the body. Hence, its amount depends not only upon the mass of the body, but also upon the rate of acceleration which we choose to

---

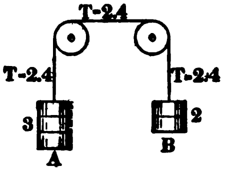
*The **momentum** of a body (sometimes called its "quantity of motion") is equal to the product obtained by multiplying its *mass* by its velocity. If we adopt the pound as the unit of mass, as in "method B," Art. 11 (*e*), the product, *weight* in *pounds* × velocity, is numerically either exactly or nearly the same as the product, *mass* in pounds × velocity, depending upon whether or not the body is in that latitude and at that level where a *mass* of one pound is said to *weigh* one pound. But the product, *weight* in *poundals* × velocity, is exactly g times (about 32.2 times) the product, *mass* in pounds × velocity; also, by "method A," *weight* in *pounds* × velocity = g × *mass* in "*matts*" × velocity.

give to it. Therefore we cannot tell, from the mass or weight of a body alone, what its "resistance of inertia" in any given case will be.

Art. 16 (a). Forces in opposite directions. When two equal and opposite forces act upon a body at the same time, and in the same straight line, we say that they destroy each other's tendencies to move the body, and it remains at rest. If two *unequal* forces thus act in opposition, the smaller force and an equal portion of the greater one are said to counteract each other in the same way, but the remainder of the greater force, acting as an unbalanced or unresisted force, moves the body in its own direction, as it would do if it were the only force acting upon it.

Thus, when we move bodies, in practice, we encounter not only the "resistance of inertia" (*i. e.*, we not only have to exert force in order to move inert matter), but we are also opposed by other *forces*, acting against us, as friction, the resistance of the air, and, often, all or a part of the *weight* of the body. By "resistances," in the following, we mean such resisting *forces*, and do not include in the term the "resistance of *inertia*."

(b) If separated, the two bodies, A and B, of 3 ℔s and 2 ℔s respectively, would fall with equal accelerations = g; each unit, $\frac{W}{g}$, of mass being acted upon by its own weight, W. But, connected as they are, A will move downward, and B upward, with an acceleration = only $\frac{g}{5}$; for now an unbalanced force of only $3 - 2 = 1$ ℔ must give acceleration to a mass of $\frac{3+2}{g} = \frac{5}{g}$. But, to give to a mass, B, of $\frac{2}{g}$, an accel of $\frac{g}{5}$, requires a force of $\frac{2}{g} \cdot \frac{g}{5} = \frac{2}{5}$ ℔ = 0.4 ℔. This, plus 2 ℔s (required to balance the *weight* of B) is the tension, 2.4 ℔s existing throughout the cord. Exerted at A, this tension balances 2.4 of the 3 ℔s weight of A. The remainder ($3 - 2.4 = 0.6$ ℔) of the weight, acting downward upon the mass, $\frac{3}{g}$, of A, gives to it the required acceleration of $\frac{g}{5}$; for here $\frac{\text{force}}{\text{mass}} = 0.6 \div \frac{3}{g} = \frac{0.6\,g}{3} = 0.2\,g = \frac{g}{5}$.



Or we may regard the total tension, 2.4 ℔s, in the cord at A, as acting upon A and giving to it a *negative* or *upward* acceleration of $2.4 \div \frac{3}{g} = 0.8\,g$, which, deducted from g (the acceleration which A would otherwise have) leaves

$$\text{Acceleration} = g - 0.8\,g = 0.2\,g = \frac{g}{5}.$$

Let W = weight of A
w = weight of B
F = net force available for acceleration = W − w
M = combined mass of both bodies = $\frac{W + w}{g}$
m = mass of B = $\frac{w}{g}$
a = acceleration
T = tension in cord.

$$\text{Then: } a = \frac{F}{M} = (W - w) \div \frac{W + w}{g} = \frac{g\,(W - w)}{W + w}$$

$$T = w + m\,a = w + \frac{w}{g}a = w + \frac{w}{g} \cdot \frac{g\,(W - w)}{W + w} = w\left(1 + \frac{W - w}{W + w}\right).$$

(c) An "**Atwood's Machine**" consists essentially of a pulley, a flexible cord passing over the pulley, two equal weights (one suspended at each end of the cord), and a third weight, generally much lighter than either of the other two. The two equal weights balance each other by means of the pulley and cord. The third weight is laid upon one of the other two weights. The force of gravity, acting upon the third weight, then sets the masses of the three weights in motion at a small but constantly increasing velocity. In order to do this it must also overcome the friction of the pulley and cord, and the rigidity of the latter; but, as these are made as slight as possible, they are, for convenience, neglected. The machine is used for illustrating the acceleration given to inert matter by unbalanced force, and forms an excellent example of the two distinct duties which a moving force generally has to perform, viz: (1st) the balancing of resistance, and (2nd) acceleration.

(d) In the case of **a locomotive, drawing a train on a level,** friction and the resistance of the air are the only resistances to be balanced; for the weight of the train here opposes no resistance. Unless the force of the steam is *more* than sufficient to balance the resistances, it cannot move the train. If it exceeds the resistances, the excess, however slight, gives motion to the inert matter of the train. If, at any moment while the train is moving, the force of the steam becomes *just equal to the resistances* (whether by an increase of the latter or by diminishing the force) the train will move on at a *uniform* velocity equal to that which it had at the moment when the force and resistance were equalized; and, if these could always be kept equal, it would so move on forever.

But so long as the excess of steam pressure over the resistances continues to act, the velocity is increased at each instant; for during each such instant the excess of force gives a small velocity *in addition* to that already existing.

On a level railroad, let

P = the total tractive force of the locomotive = say 13 tons
W = weight of locomotive = 50 tons
w = weight of train = 336 tons
R = resistance of locomotive (including internal friction, etc.) = 3 tons
r = resistance of train = 1 ton
F = net force available for acceleration $= P - R - r = 9$ tons
M = mass of engine and train $= \dfrac{W + w}{g} = \dfrac{50 + 336}{32.2} = 12$
m = mass of train $= \dfrac{w}{g} = \dfrac{336}{32.2} = 10.44$
a = acceleration
T = tension on draw-bar.

Then: Acceleration $= a = \dfrac{F}{M} = \dfrac{9}{12} = 0.75$ ft per second per second.

The tension T on the draw-bar = resistance of train + force causing acceleration a, or $T = r + m\,a = 1 + 10.44 \times 0.75 = 1 + 7.83 = 8.83$ tons.

This tension, T, pulling backward against the locomotive, causes there a *retardation*, or *negative* acceleration, of $\dfrac{T}{\text{mass of locomotive}} = \dfrac{8.83\,g}{50} = 5.69$ ft per sec per sec, and thus reduces, by that amount, the acceleration which the locomotive would otherwise have, and which would be $= \dfrac{(P - r)\,g}{50} = \dfrac{10 \times 32.2}{50} = 6.44$. This, less 5.69, = 0.75 ft per sec per sec = acceleration of train.

(e) If the tractive force of a locomotive exceeds the resistances, due to friction, grades, and air, the velocity will be accelerated; but it then becomes more difficult to maintain the excess of force, for the pistons must travel faster through the cylinders, and the boiler can no longer supply steam fast enough to maintain the original cylinder pressure. Besides, some of the resistances increase with increase of velocity. We thus reach a speed at which the engine, although exerting its utmost force, can do no more than balance the resistances. The train then moves with a uniform velocity equal to that which it had when this condition was reached.

When it becomes necessary to stop at a station some distance ahead, steam is shut off, so that the steam force of the engine shall no longer counterbalance or destroy the resisting forces; and the number of the resistances themselves is increased by adding to them the friction of the brakes. The resistances, thus increased, are now the only forces acting upon the train, and their acceleration is *negative*, or a *retardation*. Hence, the train moves more and more slowly, and must eventually stop.

(f) **Caution.** When two opposite forces are in equilibrium, an addition to one of the forces does not always form an unbalanced force; for in many cases the *other force increases equally*, up to a certain point. For instance, when we attempt to lift a weight, W, its downward *resistance*, R, remains constantly just equal to our upward pull, P, however P may vary, until P *exceeds* W. Thus, R can never *exceed* W, but may be much less than it. Indeed, when we stop pulling, R ceases, although W (the attraction between the earth and the weight) of course remains unchanged throughout. Such variation of resisting force, to meet varying demands, occurs in all those innumerable cases where structures sustain varying loads within their ultimate strength.

Art. 17 (a). Work. Force, when it moves a body,* is said to do "work" upon it. The *whole* work done by the force in moving the body through any distance is measured by multiplying the *force* by the distance; or: Work = Force × distance. If the *force* is taken in *pounds*, and the *distance* in *feet*, the product (or the *work* done) will be in *foot-pounds;* if the force is in tons and the distance in inches, the product will be in inch-tons; and so on.†

Thus, if a force of	moves a body through	we have work =
1 pound	10,000 feet	10,000 foot-pounds
100 pounds	100 "	10,000 "
10,000 "	1 foot	10,000 "

or, in any case, if the force be F pounds, the whole work done by it in moving a body through s feet, is $F\,s$ foot-pounds.

(**b**) The foot-pound, the foot-ton, the inch-pound, the inch-ton, etc., etc., are called **units of work.**†

For practical purposes, in this country, forces are most frequently stated in pounds, and the distances (through which they act) in feet. Hence **the ordinary unit of work** is the **foot-pound. The metric unit of work** is the **kilogram-meter,** *i. e.* 1 kilogram raised 1 meter = 2.2046 pounds raised 3.2809 feet, = 7.2331 foot-pounds. 1 foot-pound = 0.13825 kilogram-meter.

(**c**) In most cases, a portion at least of the **work** done by a force is **expended in overcoming resistances.** Thus, when a locomotive begins to move a train, a portion of its force works against, and balances, the resistances of friction or of an up-grade, while the remainder, acting as unbalanced force upon the inert mass of the train, increases its velocity.

An upward pull of exactly one pound will not raise a one pound weight, but will merely balance the downward force of gravity. If we increase the upward pull from one pound (= 16 ounces) to 17 ounces, the ounce so added, being unbalanced force, will give motion to the mass, and will accelerate its upward velocity as long as it continues to act. If we now reduce the upward pull to 1 pound, thus making it just equal to the downward pull of gravity, the body will move on upward with a uniform velocity; but if we reduce the upward force to 15 ounces (= $\frac{15}{16}$ pound), then there will be an unbalanced *downward* force of 1 ounce acting upon the body, and this downward force will generate in the body a downward or negative acceleration or *retardation*, and will *destroy* the upward velocity in the same time as the *upward* excess of 1 ounce required to *produce* it.

During any time, while the 17 ounces upward "force" were acting against the 16 ounces downward "resistance," the product of *total* upward force × distance must be *greater* than that of *resistance* × distance. The excess is the work done in accelerating the velocity, by virtue of which the body has acquired kinetic energy or capacity for doing work in coming to rest.

On the other hand, while the upward velocity was being *retarded*, the product of total upward force × dist was *less* than that of *resistance* × dist, the difference being the work done by the kinetic energy against the resistance of gravity.

In practice, the term "work" is usually restricted to that *portion* of the work which a force performs in balancing the *resistances* which act against it; in other words, to the work done by so much of the force as is equal to the resistance.

With this restriction, we have work = *force* × dist, = *resistance* × dist.

Thus, if the resistance be a friction of 4 lbs., overcome at every point along a distance of 3 feet; or if it be a weight of 4 lbs., lifted 3 feet high, then the work done amounts to 4 × 3 = 12 foot-lbs, provided the initial and the final velocities are equal.

(**d**) In cases **where the velocity is uniform,** as in a steadily running machine, the force is necessarily equal to the resistance; and where the velocities at the beginning and end of any work are equal (as where the machine starts from rest and comes to rest again) the *mean* force is equal to the *mean* resistance. In such cases, therefore, the two products, mean force × distance, and mean resistance × distance, are equal, and we have, as before,

Work = *force* × dist = *resistance* × dist.

---

* A man who is standing still is not considered to be working, any more than is a post or a rope when sustaining a heavy load; although he may be supporting an oppressive burden, or holding a car-brake with all his strength; for his force *moves nothing* in either case.

† These products must not be confounded with *moments*, = force × leverage.

(f) In calculating the work done by machinery, etc., allowance must be made for this expenditure of a portion of the work in overcoming resistances. Thus, in pumping water, part of the applied force is required to balance the friction of the different parts of the pump; so that a steam or water "power," exerting a force of 100 lbs., and moving 6 feet per second, cannot raise 100 lbs. of water to a height of 6 feet per second. Therefore machines, so far from *gaining power*, according to the popular idea, actually lose it in one sense of the word. In *starting* a piece of machinery, the forces employed have (1st) to balance, react against, or destroy the resisting force of friction and the cohesive forces of the material which is to be operated on; and (2d) to give motion to the unresisting matter of the machine and of the material operated on, after the resisting forces which had acted upon them have thus been rendered ineffective. But after the desired velocity has been established, the forces have *merely to balance the resistances* in order that the velocity may continue uniform.

(g) That portion of the work of a machine, etc., which is expended against friction is sometimes called "**lost work**" or "**prejudicial work,**" while only that portion is called "**useful work**" which renders visible and tangible service in the shape of output, etc. Thus, in pumping water, the work done in overcoming the friction of the pump and of the water is said to be lost or prejudicial, while the useful work would be represented by the product, weight of water delivered × height to which it is lifted.

The distinction, although artificial, and somewhat arbitrary, is often a very convenient one; but the work is of course not actually "lost," and still less is it "prejudicial;" for the water could not be delivered without first overcoming the resistances. A merchant might as well call that portion of his money lost which he expends for clerk-hire, etc.

(h) For a given force and *distance*, **the work done is independent of the time;** for the product, force × distance, then remains the same, whatever the time may be. But the distance through which a given *force* will work at a given velocity is of course proportional to the time during which it is allowed to work. Thus, in order to lift 50 pounds 100 feet, a man must do the same work, (= 5000 foot-pounds) whether he do it in one hour or in ten; but, if he exerts constantly *the same force*, he will lift 50 lbs. ten times as high in ten hours as in one, and thus will do ten times the work. Thus, for a given *force*, **the work is proportional to the time.**

Art. 18 (a), Power. The quantity of any work may evidently be considered without regard to the *time* required to perform it; but we often require to know the *rate* at which work can be done; that is, how much can be done within a certain *time*.

The rate at which a machine, etc. can work is called its *power*. Thus, in selecting a steam-engine, it is important to know how much it can do *per minute*, *hour*, or *day*. We therefore stipulate that it shall be of so many *horse-powers*; which means nothing more than that it shall be capable of overcoming resisting forces at the rate of so many times 33,000 foot-pounds per minute when running at a uniform velocity, i. e., when force × distance = resistance × distance.

(b) The **horse-power**, 33,000 foot-pounds per minute, or 550 foot-pounds per second, is the **unit of power, or of rate of work,** commonly used in connection with engines. The **metric horse-power,** called "force de cheval," "cheval-vapeur," or (German) "Pferdekraft," is 75 kilogram-meters per second = 542.48 ft.-lbs. per sec. = 32,549 ft.-lbs. per minute = 0.9863 horse-power. 1 horse-power = 1.0138 "force de cheval." In theoretical Mechanics the **foot-pound per second** is used in English measure; and the **kilogram-meter per second** in metric measure,

1 foot-pound per second = 0.13825 kilogram-meter per second.
1 kilogram-meter per second = 7.2331 foot-pounds per second.

(c) Up to the time when the velocity becomes uniform, the **power, or rate of work,** of the train, in Art. 16 (*d*), is *variable*, being gradually *accelerated*. For in each second it overcomes its resistances (and moves its point of application) *through a greater distance* than during the preceding second. Also, after the steam is shut off, the rate of work is variable, being gradually *retarded*. When the force of the steam just balances the resistances, the rate of work is uniform.

(d) **Power = force × velocity.** Since the *rate* of work is equal to the work done *in a given time*, as so many *foot-pounds per second*, we may find it by dividing the work in foot-pounds done during any given time by the number of seconds in that time. Thus

$$\text{Power} = \text{rate of work} = \frac{\text{force in pounds} \times \text{distance in feet}}{\text{time in seconds}}.$$

But this is equivalent to

$$\text{Power} = \text{rate of work} = \text{force in pounds} \times \frac{\text{distance in feet}}{\text{time in seconds}}$$

$$= \text{force in lbs.} \times \text{velocity in feet per second.}$$

Or, if we treat only of the work of *that force which overcomes resistances:* or in cases where the velocity is either uniform throughout or the same at the beginning and end of the work;

$$\underset{\text{in ft-lbs. per sec.}}{\text{Power,}} = \underset{\text{in ft-lbs. per sec.}}{\text{rate of work}} = \underset{\text{in lbs.}}{\text{resistance,}} \times \underset{\text{in ft per sec.}}{\text{velocity,}}$$

Thus, if the resistance is 3300 lbs. and is overcome through a distance of 10 feet in every minute; or if the resistance is 33 lbs. and is overcome through a distance of 1000 feet per minute, the rate of the work is in each case the same, namely, 33,000 foot-pounds per minute, or one horse-power; for

$$\overset{\text{lbs.}}{3300} \times \overset{\text{vel.}}{10} = \overset{\text{lbs.}}{33} \times \overset{\text{vel.}}{1000} = 33{,}000 \text{ foot-pounds per minute.}$$

(e) The same "power" which will overcome a given resistance through a given distance, in a given time, will also overcome any other resistance through any other distance, in that same time, provided the resistance and distance when multiplied together give the same amount as in the first case. Thus, the power that will lift 50 pounds through 10 feet in a second, will in a second lift 500 pounds, 1 foot; or 25 pounds, 20 feet; or 5000 pounds $\frac{1}{10}$ of a foot. In practice, the adjustment of the speed to suit different resistances, is usually effected by the medium of **cog-wheels, belts, or levers.** By means of these the engine, water-wheel, horse, or other motive power, exerting a given force and running at a given velocity, may be made to overcome small resistances rapidly, or great ones slowly, as desired.

Art. 19 (a). **The work which a body can do by virtue of its motion;** or (which is the same thing) **the work required to bring the body to rest. Kinetic energy, vis viva, or "living force."** As already remarked, a force equal to the weight of any body, at any place, will, in one second, give to the mass or matter of the body a velocity $= g$, or (on the earth's surface) about 32.2 feet per second. Or if a body be thrown upward with a velocity $= g$, its weight will stop it in one second.

Since, in the latter case, the velocity at the beginning and at the end of the second are, respectively, $= g$ feet per second, and $= 0$, the *mean* velocity of the body is $\frac{g}{2}$ feet per second. Therefore, during the second it will rise $\frac{g}{2}$ feet, or about 16 feet. In other words, the work which any body can do, by virtue of being thrown vertically upward with an initial velocity (velocity at the *start*) of g feet per second, is equal to the product of its weight multiplied by $\frac{g}{2}$ feet. Or,

$$\text{work in foot-pounds} = \text{weight} \times \frac{g}{2}$$

Notice that in this case (since the initial velocity v is equal to g), $\frac{v}{g} = 1$.

Suppose now that the same body be thrown upward with *double* the former velocity; *i. e.*, with an initial velocity equal to $2\,g$ (or about 64 feet per second). Since gravity requires (Art. 8 c), *two* seconds to impart or destroy this velocity, the body will now move upward during *two* seconds, or twice as long a *time* as before. But its mean *velocity* now is g, or twice as great as before. Therefore, moving for double the time and with double the velocity, it will travel *four times* as *far*, overcoming the same resistance as before (viz.: its own weight) through *four* times the distance.

Thus, by making its initial velocity $v = 2\,g$, *i. e.*, by *doubling* its $\frac{v}{g}$, making it $= 2$, we have enabled the body to do *four* times the work which it could do when its $\frac{v}{g}$ was 1; so that the work in the second case is equal to the product of that in the first case multiplied by the *square* of $\frac{v}{g}$; or.

$$\text{Work in ft-lbs.} = \text{weight} \times \frac{g}{2} \times \left(\frac{v}{g}\right)^2$$
$$= \text{weight} \times \frac{g}{2} \times \frac{v^2}{g^2}$$
$$= \text{weight} \times \frac{v^2}{2g}$$

And it is plain that this would be the case for any *other* velocity. Now the total amount of the work which the body can do, is independent of the amount of the resistance against which it is done; for if we increase the resistance we diminish the distance in the same proportion, so that their product, or the amount of work, remains the same. The above formula therefore, applies to all cases; *i. e.*, the **total amount of work,** in foot pounds, which any body will do, against any resistance, by virtue of its motion alone, in coming to rest, is

$$\textbf{Work} = \text{weight of moving body, in lbs.} \times \frac{\text{square of its velocity in ft per sec'd}}{2g}$$
$$= \text{weight of moving body, in lbs.} \times \text{fall in ft required to give the velocity}$$
$$= \frac{\text{weight of moving body, in lbs.}}{g} \times \frac{\text{square of its velocity in ft per second}}{2}$$

In these equations, the *weight* is that which the body has in any given place, and g is the acceleration of gravity at that same place.

(b) Since the $\frac{\text{weight of a body}}{g}$ is its mass (Art. 11, p. 336), the last formula becomes, by "method A," Art. 11 (*d*),

$$\frac{\textbf{work}}{}\ \text{in foot-}\textit{pounds} = \text{mass of moving body in ``}\textit{matts}\text{''} \times \frac{\text{square of its velocity in ft per second}}{2}$$

and by "method B," Art. 11 (e),

$$\textbf{work}\ \text{in foot-}\textit{poundals} = \text{mass of moving body in }\textit{pounds} \times \frac{\text{square of its velocity in ft per second}}{2}$$

(c) In the above equations the *left* hand side represents the *work* (or resistance overcome through a distance) in any given case, while the *right* hand side represents the **kinetic energy** of the body, by which it is enabled to do that work. Some writers call this energy **"vis viva,"** or "living force" a name formerly given (for convenience) to a quantity just *double* the energy, or = mass × velocity^2.

(d) As an illustration of the foregoing, take a train weighing 1,120,000 pounds, and moving at the rate of 22 feet per second. The kinetic energy of such a train is

$$\text{energy} = \text{weight} \times \frac{\text{velocity}^2}{2g}; \quad \text{or,}$$

$$1{,}120{,}000 \text{ lbs.} \times \frac{22^2}{64.4} = 8{,}400{,}000 \text{ ft.-lbs.}$$

That is, if steam be shut off, the train will perform a work of 8,400,000 ft.-lbs. in coming to rest. Thus, if the sum of all the resistances (of friction, air, grades, curves, etc.) remained constantly = 5000 lbs.,* the train would travel

$$\frac{8{,}400{,}000 \text{ ft.-lbs.}}{5000 \text{ lbs.}} = 1680 \text{ ft.}$$

(e) We thus see that the total quantity of work which a body can do by virtue of its motion alone, and without assistance from extraneous forces, is in proportion to the weight of the body and to the *square* of its velocity when it begins to do the work. For example, suppose that a train, at the moment when steam is shut off, has a velocity of 10 miles an hour and that the kinetic energy, which that velocity gives it, will by itself carry the train against the resistances of the road, etc., for a distance of *one* quarter of a mile before it stops. Then, if steam be shut off while the train is moving at 5, 20, 30 or 40 miles per hour (*i. e.* with ½, 2, 3 or 4 times 10 miles per hour) the train will travel $\frac{1}{16}$, 1, 2¼ or 4 miles (or ¼, 4, 9 or 16 times ¼ mile) before coming to rest.[*]

---

* In practice, this would not be the case.

But the *rate* of work done is proportional simply to the resistance and the *velocity* (Art. 18*d*, p. 342). Therefore, the locomotive whose steam is shut off at 20, 30 or 40 miles per hour, will require, for running its 4, 9 or 16 quarters of a mile, but 2, 3 or 4 times as many seconds as it required at 10 miles per hour.

The same principle applies to all cases of acceleration or of retardation.[†] For instance, in the case of a falling body, the *distance* through which it must fall in order to acquire any given velocity is as the *square* of that velocity, but the *time* required is simply as the *velocity*. Also, if a body is thrown vertically upward with any given velocity, the *height* to which it will rise by the time gravity destroys that velocity, will be as the *square* of the velocity, but the *time* will be simply as the velocity.

Art. 20 (a). The **momentum** of a moving body (or the product of its mass by its velocity) is the *rate*, in foot-pounds per second, at which it works against a resisting force *equal to its own weight*, as in the case of a body thrown vertically upward. At the instant when it comes to rest, its momentum, or rate of work, is of course = *nothing*. Therefore its *mean* rate of work, or *mean* momentum, is one-half of that which it has at the moment of starting.

Thus, suppose such a body to weigh 5 lbs. Then, whatever its velocity may be, 5 pounds is the resisting force, against which it must work while coming to rest. Let the initial velocity be 96 feet per second. Then its

$$\text{momentum} = \text{mass} \times \text{velocity} = 5 \times 96 = 480 \text{ foot-pounds per second:}$$

and, while coming to rest, its

$$\textit{mean} \text{ momentum} = \text{mass} \times \frac{\text{velocity}}{2} = 240 \text{ foot-pounds per second.}$$

Now, in falling, the weight of the body (5 lbs.), would *give* it a velocity of 96 feet per second in about *three* seconds. Consequently, in rising, it will *destroy* its velocity in the same time. In other words, the time $= \frac{\text{velocity}}{\text{acceleration}} = \frac{\text{velocity}}{g} = \frac{96}{32} = 3$. Three seconds, therefore, is the *time* during which it can work. Now, if the mean *rate* of work in foot-pounds per second (at which a body can work against a resistance) be multiplied by the *time* during which it can continue so to work, the *product* must be the total *work* done. Or, in this case,

$$\begin{aligned}\begin{array}{c}\text{work}\\ \text{in ft-lbs.}\end{array} &= \begin{array}{c}\text{mean rate of work}\\ \text{in ft-lbs. per sec.}\end{array} \times \begin{array}{c}\text{time,}\\ \text{or No. of secs.}\end{array} = 240 \times 3 = 720 \text{ foot-pounds.}\\ &= \text{weight} \times \frac{\text{velocity}}{2} \times \frac{\text{velocity}}{g}\\ &= \text{weight} \times \frac{\text{velocity}^2}{2\,g}\text{, as in Art. 19 }(a)\text{,} = 5 \times \frac{96^2}{64} = 720 \text{ ft-pounds.}\end{aligned}$$

(b) We may notice also that since, in the case of a falling body, or of one thrown upward, $\frac{\text{velocity}}{g}$ is the *time* during which it must fall in order to acquire a given velocity, or during which it must rise in order to lose it, therefore,

$$\frac{\text{velocity}}{2} \times \frac{\text{velocity}}{g} = \text{mean velocity} \times \text{time} = \text{distance traversed;}$$

so that

$$\begin{aligned}&\text{weight} \times \frac{\text{velocity}^2}{2\,g} = \text{weight} \times \frac{\text{velocity}}{2} \times \frac{\text{velocity}}{g} =\\ &\text{weight} \times \text{distance traversed} \quad = \text{the work.}\end{aligned}$$

---

[*] This supposes, for convenience, that the resistances remain uniform throughout, and are the same in all the cases, which, however, would not hold good in practice.

[†] Retardation is merely acceleration in a direction opposite to that of the motion which we happen to be considering.

Art. 21 (a). Energy is indestructible. Energy, expended in work, is not destroyed. It is either transferred to other bodies, or else stored up in the body itself; or part may be thus transferred, and the rest thus stored. But, although energy cannot be destroyed, it may be rendered useless to us. Thus, a moving train, in coming to rest on a level track, transfers its kinetic energy into other kinetic energy; namely, the useless heat due to friction at the rails, brakes and journals; and this heat, although none of it is *destroyed*, is dissipated in the earth and air so as to be practically beyond our recovery.

Art. 22 (a). Potential energy, or possible energy, may be defined as stored-up energy. We lift a one-pound body one-foot by expending upon it one foot-pound of energy. But this foot-pound is stored up in the "system" (composed of the earth and the body) as an addition to its stock of potential energy. For, while the stone falls through one foot, the system will acquire a kinetic energy of one foot-pound, and will part with one foot-pound of its potential energy.

(b) The *potential* energy of a "system" of bodies (such as the earth and a weight raised above it, or the atoms of a mass of powder, or those of a bent spring) depends upon the relative *positions* of those bodies, and upon their *tendencies to change* those positions. The *kinetic* energy of a system (such as the earth and a moving train of cars) depends upon the *masses* of its bodies and upon their *motion* relatively to each other.

Familiar instances of potential energy are—the weight or spring of a clock when fully or partly wound up, and whether moving or not; the pent-up water in a reservoir; the steam pressure in a boiler; and the explosive energy of powder. We have mechanical energy in the case of the weight or springs or water; heat energy in the case of the steam, and chemical energy in that of the powder.

(c) In many cases we may conveniently estimate the *total* potential energy of a system. Thus (neglecting the resistance of the air) the explosive energy of a pound of powder is = the weight of any given cannon ball × the height to which the force of that powder could throw it, = the weight of the ball × (the square of the initial velocity given to it by the explosion) ÷ $2g$. But in other cases we care to find only a certain definite *portion* of the total potential energy. Thus, the *total* potential energy of a clock-weight* would not be exhausted until the weight reached the center of the earth; but we generally deal only with that portion which was stored in it by winding-up, and which it will give out again as kinetic energy in running down. This portion is = the weight × the height which it has to run down = the weight × (the square of the velocity which it would acquire in falling *freely* through that height) ÷ $2g$.

(d) There are many cases of energy in which we may hesitate as to whether the term "kinetic" or "potential" is the more appropriate. Thus, the pressure of steam in a boiler is believed to be due to the violent *motion* of the particles of steam, which bombard the inner surface of the boiler-shell; so that, from *this* point of view, we should call the energy of steam *kinetic*. But, on the other hand, the shell itself remains stationary; and, until the steam is permitted to escape from the boiler, there is no outward evidence of energy in the shape of work. The energy remains stored up in the boiler ready for use. From *this* point of view, we may call the energy of steam *potential* energy.

(e) It seems reasonable to suppose that further knowledge as to the nature of other forms of energy, apparently potential (as is that of steam), might reveal the fact that all energy is ultimately kinetic.

Art. 23 (a). There is much **confusion of ideas** in regard to those actions to which, in Mechanics, we give the names, "**force,**" "**energy,**" "**power,**" etc. This arises from the fact that in every-day language these terms are used indiscriminately to express the same ideas.

Thus, we commonly speak of the "force" of a cannon-ball flying through the air, meaning, however, the repulsive force which *would be* exerted between the ball and a building, etc. with which it might come into contact. This force would tend to move a part of the building along in the direction of the flight of the ball, and would move the ball backward; (*i. e.*, would retard its forward motion). But this great repulsive "*force*" does not exist until the ball strikes the building. Indeed, we cannot even tell, from the velocity and weight of the ball, what the *amount* of the force will be, for this depends upon the strength, etc., of the *building*. If the building is of glass, the force may be so slight as scarcely to retard the motion of the ball perceptibly, while, if the building is an earth embankment, the force will be much greater, and may retard the motion of the ball so rapidly as to entirely stop it before it has gone a foot farther.

---

* For convenience we may thus speak of the energy of a *system* of bodies (the earth and the clock-weight) as residing in only *one* of the bodies.

The moving ball has great (kinetic) *energy;* but the only *force* that it exerts during its flight is the comparatively very slight one required to push aside the particles of air.

The energy of the ball, and therefore the total work which it can do, are independent of the nature of the obstruction which it meets; but since the work is the product of the resistance offered and the distance through which it can be overcome, the distance must be inversely as the resistance offered; or (which is the same thing) inversely as the force required of, and exerted by, the ball in balancing that resistance.

Since work, in ft.-lbs. = force, in lbs., × distance traversed, in feet, we have

$$\text{force, in lbs.} = \frac{\text{work, in ft.-lbs.}}{\text{distance traversed, in feet}} = \begin{array}{c}\text{rate of work,}\\ \text{in ft.-lbs. } \textit{per foot.}\end{array}$$

Art. 24 (**a**). **An impact,** blow, stroke or collision takes place when a moving body encounters another body. The peculiarity of such cases is that the *time of action* of the repulsive force due to the collision is so *short* that generally it is impossible to measure it, and we therefore cannot calculate the force from the momentum produced by it in either of the two bodies: but since both bodies undergo a great change of velocity (*i. e.*, a great acceleration) during this short time, we know that the repulsive force acting between them must be very great.

We shall consider only cases of **direct impact.** or impact where the centers of gravity of the two bodies approach each other in one straight line, and where the nature of the surfaces of contact is such that the repulsive force caused by the impact also acts through those centers and in their line of approach.

(**b**) This force, acting equally upon the two bodies (Art. 5 *f*), for the same length of time (namely, the time during which they are in contact), necessarily produces equal and opposite changes in their momentums (Art. 12, p. 338). Hence, the total momentum (or product, mass × velocity) of the *two* bodies is always the same after impact as it was before.

(**c**) But the relative behavior of the two bodies, after collision, depends upon their elasticity. If they could be perfectly inelastic, their velocities, after impact, would be equal. In other words, they would move on together. If they could be perfectly *elastic,* they would separate from each other, after collision, with the same velocity with which they approached each other before collision.

(**d**) Between these two extremes, neither of which is ever perfectly realized in practice, there are all possible degrees of elasticity, with corresponding differences in the behavior of the bodies. The subject, especially that of *indirect* impact, is a very complex one, but seldom comes up in practical civil engineering.

(**e**) "In some careful experiments made at Portsmouth dock-yard, England, a man of medium strength, and striking with a maul weighing 18 lbs., the handle of which was 44 inches long, barely *started* a bolt about ⅛ of an inch at each blow; and it required a quiet pressure of 107 tons to press the bolt down the same quantity; but a small additional weight pressed it completely home."

GRAVITY, FALLING BODIES.

Bodies falling vertically. A body, falling freely in vacuo from a state of rest, acquires, by the end of the first second, a velocity of about 32.2 feet per second; and, in each succeeding second, an *addition* of velocity, or acceleration, of about 32.2 feet per second. In other words, the velocity receives in each second an acceleration of about 32.2 feet per second, or is accelerated at the *rate* of about 32.2 feet per second, *per second*. This rate is generally called (for brevity, see foot-note,† p. 334), simply the **acceleration of gravity** (but see * below), and is denoted by **g**. It increases from about 32.1 feet per second, per second, at the equator, to about 32.5 at the poles. In the latitude of London it is 32.19. These are its values at sea-level; but at a height of 5 miles above that level it is diminished by only about 1 part in 400. For most practical purposes it may be taken at 32.2.

Caution. Owing to the resistance of the air none of the following rules give perfectly accurate results in practice, especially at great vels. The greater the specific gravity of the body the better will be the result. The air **resists** both **rising** and **falling** bodies.

If a body be thrown vertically upwards with a given vel, it will rise to the same height from which it must have fallen in order to acquire said vel; and its vel will be retarded in each second 32.2 ft per sec. Its average ascending velocity will be half of that with which it started; as in all other cases of uniformly retarded vel. In falling it will acquire the same vel that it started up with, and in the same time. See above Caution.

Acceleration acquired*

in a given time $= g \times \text{time}$

in a given fall from rest $= \sqrt{2\,g \times \text{fall}}$.

$$\left.\begin{matrix}\text{in a given fall from rest}\\ \text{and given time}\end{matrix}\right\} = \frac{\text{twice the fall}}{\text{time}}$$

Time required

$$\text{for a given acceleration} = \frac{\text{acceleration}}{g}$$

$$\text{for a given fall from rest} = \sqrt{\frac{\text{fall}}{\frac{1}{2}\,g}} = \frac{\text{fall}}{\frac{1}{2}\text{ final velocity}}$$

$$\left.\begin{matrix}\text{for a given fall from rest}\\ \text{or otherwise}\end{matrix}\right\} = \frac{\text{fall}}{\text{mean vel}} = \frac{\text{fall}}{\frac{1}{2}\,(\text{initial vel} + \text{final vel})}$$

Fall

$$\text{in a given time (starting from rest)} = \text{time} \times \tfrac{1}{2}\text{ final vel} = \text{time}^2 \times \tfrac{1}{2}\,g$$

$$\left.\begin{matrix}\text{in a given time (starting}\\ \text{from rest or otherwise)}\end{matrix}\right\} = \text{time} \times \text{mean vel} = \text{time} \times \frac{\text{initial vel} + \text{final vel}}{2}$$

$$\left.\begin{matrix}\text{reqd for a given acceleration}\\ \text{(starting from rest)}\end{matrix}\right\} = \frac{\text{acceleration}^2}{2\,g}.$$

during any one given second (counting from rest)

$$= g \times \left(\text{number of the second (1st, 2d, \&c)} - \tfrac{1}{2}\right)$$

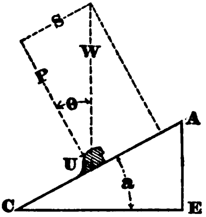
during any equal consecutive times (starting from rest) $\propto$ 1, 3, 5, 7, 9, &c.

Calling $g = 32.2$ we have	At the end of the									
	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	10th.
	seconds									
Velocity; ft per sec.	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8	322.0
Dist fallen since end of preceding sec; ft.	16.1	48.3	80.5	112.7	144.9	177.1	209.3	241.5	273.7	305.9
Total dist fallen; ft.	16.1	64.4	144.9	257.6	402.5	579.6	788.9	1030.4	1304.1	1610.0

---

* By "acceleration," *in this article*, we mean the *total* acceleration; *i. e.*, the whole change of velocity occurring in the given time or fall. For the *rate* of acceleration we use simply the letter *g*.

Descent on inclined planes. When a body, U, is placed upon an inclined plane, A C, its whole weight W is not employed in giving it velocity (as in the case of bodies falling vertically) but a portion, P, of it (= W × cosine of o = W × cosine of a*) is expended in perpendicular pressure against the plane; while only S, (= W × sine of o = W × sine of a,*) acts upon U in a direction parallel to the surface A C of the plane, and tends to slide it down that surf.



The acceleration, generated in a given body in a given time, is proportional to the force acting upon the body in the direction of the acceleration Hence if we make W to represent by scale the acceleration g (say 32.2 ft per sec) which grav would give to U in a sec if falling freely, then S will give, by the same scale, the acceleration in ft per sec which the actual sliding force S would give to U in one sec if there were no friction between U and the plane. We have therefore

theoretical acceleration down the plane = g × sine of a.

Therefore we have only to substitute "g. sin a" in place of "g;" and the *sloping* distance or "slide" A C in place of the corresponding *vertical* distance or "fall" A E in the equations, in order to obtain the accelerations etc as follows:

on an inclined plane without friction.

In the following, the slides A C are in feet, the times in seconds, and the velocities and accelerations in feet per second.†

Acceleration† of sliding velocity

$$\text{in a given time} = \left.\begin{matrix}\text{vert accel acquired in falling}\\ \text{vert during the same time}\end{matrix}\right\} \times \sin a$$

$$= g.\ \sin a \times \text{time}$$

$$\left.\begin{matrix}\text{in a given slide, as A C,}\\ \text{from rest}\end{matrix}\right\} = \frac{\text{slide}}{\frac{1}{2}\ \text{time}}$$

$$= \left\{\begin{matrix}\text{vert accel acquired in falling}\\ \text{freely thro the corresponding}\\ \text{vert ht A E}\end{matrix}\right\} = \sqrt{2\,g.\ \text{AE}}$$

$$= \sqrt{2\,g.\ \sin a \times \text{slide}}$$

Time required

$$\text{for a given sliding acceleration} = \frac{\text{sliding acceleration}}{g.\ \sin a}$$

$$\text{for a given slide, as A C, from rest} = \frac{\text{slide}}{\frac{1}{2}\ \text{final sliding velocity}} = \sqrt{\frac{\text{slide}}{\frac{1}{2}\,g.\ \sin a}}$$

$$= \frac{\text{time reqd to fall freely thro the corresponding vert ht A E}}{\sin a}$$

$$\left.\begin{matrix}\text{for a given slide, from}\\ \text{rest or otherwise}\end{matrix}\right\} = \frac{\text{slide}}{\text{mean sliding vel}} = \frac{\text{slide}}{\frac{1}{2}\,(\text{initial} + \text{final sliding vels})}$$

$$\textbf{Cosine a} = \frac{\text{base E C}}{\text{length A C}} = \frac{\text{horizontal stretch, as E C, of any length, as A C}}{\text{that length}} = \frac{\sqrt{\text{AC}^2 - \text{AE}^2}}{\text{AC}}$$

$$\textbf{Sine a} = \frac{\text{height A E}}{\text{length A C}} = \frac{\text{fall, A E, in any given length, A C}}{\text{that length}} = \frac{\sqrt{\text{AC}^2 - \text{EC}^2}}{\text{AC}}$$

---

[*] Because o and a are equal.

[†] By acceleration, *in this article*, we mean the total acceleration, *i. e.*, the whole change in velocity occurring in the given time or slide. For the *rate* of acceleration we use simply the letter g.

Slide, as A C

in a given time, starting from rest = time × ½ final sliding vel

= time ^{2} × ½ g. sin a.

in a given time, starting from rest or otherwise = time × mean sliding vel

= time × ½ (initial + final, sliding vels)

required for a given sliding acceleration (starting from rest) $= \dfrac{\text{sliding acceleration}^2}{2\,g.\ \sin a}$

But in practice the sliding on the plane is always opposed by friction. To include the effect of friction, we have only to substitute

"$g \times \left[\sin a - (\cos a.\ \text{coeff fric})\right]$" in place of "$g.\ \sin a$" in the above equations. Because

Friction = Perpendicular pressure P × coefficient of friction

= weight W × cosine a × coefficient of friction

and

retardation of friction = g × cosine a × coefficient of friction.

Resultant sliding acceleration

= theoretical sliding accel (due to the sliding force, S) — retardation of fric

= (g. sin a) — (g. cosine a. coeff fric)

$= g \times \left[\sin a - (\text{cosine } a.\ \text{coeff fric})\right]$

If the retardation of friction (= g. cos a × coeff fric) is *not less* than the total or theoretical accel ("g. sin a") the body cannot slide down the plane.

PENDULUMS.

The numbers of vibrations which diff pendulums will make in any given place in a given time, are *inversely* as the square roots of their lengths; thus, if one of them is 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; but its number of vibrations will be but ½, ⅓, or ¼ as great. The times in which diff pendulums will make a vibration, are *directly* as the sq rts of their lengths. Thus, if one be 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; and so also will be the time occupied in one of its vibrations.

The length of a pendulum vibrating seconds at the level of the sea, in a vacuum, in the lat of London (51½° North) is 39.1393 ins; and in the lat of N. York (40¾° North) 39.1013 ins. At the equator about $\frac{1}{10}$ inch shorter; and at the poles, about $\frac{1}{10}$ inch longer. Approximately enough for experiments which occupy but a few sec, we may at any place call the length of a seconds pendulum in the open air, 39 ins; half sec, 9¾ ins; and may assume that long and short vibrations of the same pendulum are made in the same time; which they actually are, *very* nearly. For measuring depths, or dists by sound, a sufficiently good sec pendulum may be made of a pebble (a small piece of metal is better) and a piece of thread, suspended from a common pin. The length of 39 ins should be measured from the centre of the pebble.

In starting the vibrations, the pebble, or *bob*, must not be *thrown* into motion, but merely *let drop*, after extending the string at the proper height.

To find the length of a pendulum reqd to make a given number of vibrations in a min, divide 375 by said reqd number. The square of the quot will be the length in ins, near enough for such temporary purposes as the foregoing. Thus, for a pendulum to make 100 vibrations per min, we have $\frac{375}{100} = 3.75$; and the square of $3.75 = 14.06$ ins, the reqd length.

To find the number of vibrations per min for a pendulum of given length, in ins, take the sq rt of said length, and div 375 by said sq rt. Thus, for a pendulum 14.06 ins long, the sq rt is 3.75; and $\frac{375}{3.75} = 100$, the reqd number.

Rem. 1. By practising before the sec pendulum of a clock, or one prepared as just stated, a person will soon learn to count 5 in a sec, for a few sec in succession; and will thus be able to divide a sec into 5 equal parts; and this may at times be useful for very rough estimating when he has no pendulum.

Centre of Oscillation and Percussion.

Rem. 2. When a pendulum, or any other suspended body, is vibrating or oscillating backward and forward, it is plain that those particles of it which are far from the point of suspension move faster than those which are near it. But there is always a certain point in the body, such that if *all* the particles were concentrated at it, so that all should move with the same actual vel, neither the number of oscillations, nor their angular vel, would be changed. This point is called the *center of oscillation*. It is not the same as the cen of grav, and is always farther than it from the point of suspension. It is also the *centre of percussion* of the suspended vibrating body. The dist of this point from the point of susp is found thus: Suppose the body to be divided into many (the more the better) small parts; the smaller the better. Find the weight of each part. Also find the cen of grav of each part; also the dist from each such cen of grav to the point of susp. Square each of these dists, and mult each square by the wt of the corresponding small part of the body. Add the products together, and call their sum *p*. Next mult the weight of the entire body by the dist of its cen of grav from the point of susp. Call the prod *g*. Divide *p* by *g*. This *p* is the **moment of inertia** of the body, and if divided by the wt of the body the sq rt of the quotient will be the **Radius of Gyration.**

Angular Velocity.

When a body revolves around any axis, the parts which are farther from that axis move faster than those nearer to it. Therefore we cannot assign a stated linear velocity in feet per second, or miles per hour etc, that shall apply to *every part* of it. But every part of the body revolves around an entire circle, or through an angle of 360°, in the same time. Hence, all the parts have the same velocity in *degrees* per second, or in *revolutions* per second. This is called the angular velocity. Scientific writers measure it by the length of the arc described by any point in the body in a given time, as a second, the length of the arc being measured by the number of times the length *of its own radius* is contained in it. When so measured,

$$\begin{array}{c}\text{Angular velocity}\\ \text{in radii per second}\end{array} = \frac{\text{linear velocity (in feet etc) per sec}}{\text{length of radius (in feet etc)}}$$

Here, as before, the angular velocity is the same for all the points in the body, because the velocities of the several points are directly as their radii or distances from the axis of revolution.

In each revolution, each point describes the circumference of the circle in which it revolves $= 2\pi r$ ($\pi = 3.1416$ etc; r = radius of said circle). Consequently, if the body makes n revolutions per second, the length of the arc described by each point in one second is $2\pi r n$; and the angular velocity of the body, or *linear* velocity of any point, *measured in its own radii*, is

$$a = \frac{2\pi r n}{r} = 2\pi n = \text{say } 6.2832 \times \text{revs per } \textit{second} = \text{say } .1047 \times \text{revs per } \textit{minute}.$$

Moment of Inertia.

Suppose a body revolving around an axis, as a grindstone; or oscillating, like a pendulum. Suppose that the distance from the axis of revolution (which, in the pendulum, is the point of suspension) to each individual particle of the body, has been measured; and that the square of each such distance has been multiplied by the weight of that particle to which said distance was measured.

The sum of all these products is the moment of inertia of the body. Or

$$\textbf{Moment of inertia} = \left\{\begin{array}{c}\text{the sum,}\\ \text{for all the particles}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{weight}\\ \text{of}\\ \text{particle}\end{array}\right. \times \begin{array}{c}\text{square of dist}\\ \text{of particle from}\\ \text{axis of revolution}\end{array}$$

or, $I = \Sigma\, d^2 w.$

Scientific writers frequently use the *mass* of each particle; i e, $\dfrac{\text{its weight}}{\text{acceleration } (g) \text{ of gravity, or about } 32.2}$ instead of its weight, in calculating the moment of inertia.

In practice we may suppose the body to be divided into portions measuring a cubic inch (or some other small size) each; and use these instead of the theoretical infinitely small particles. The smaller these portions are taken, the more nearly correct will be the result.

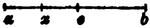
When the moment of inertia of a mere *surface* is wanted (instead of that of a *body*), we suppose the surface to be divided into a number of small *areas*, and use them instead of the *weights* of the small portions of the body.

$$\textbf{Moment of inertia} = \begin{array}{c}\text{weight of body,}\\ \text{or}\\ \text{area of surface}\end{array} \times \begin{array}{c}\text{square of}\\ \text{radius of gyration}\end{array}$$

Table of Radii of Gyration.

Body	Revolving around	Radius of Gyration
Any body or figure	any given axis	$\sqrt{\dfrac{\text{moment of inertia around the given axis}}{\text{weight of body, or area of surface}}}$
Solid cylinder	its longitudinal axis	$\text{radius of cylinder} \times \sqrt{\dfrac{1}{2}}$ $= \text{radius of cylinder} \times \text{about } .7071$
ditto	a diam, midway between its ends	$\sqrt{\dfrac{\text{length}^2}{12} + \dfrac{\text{radius}^2 \text{ of cylinder}}{4}}$
ditto, infinitely short (circular surface)	a diameter	$\dfrac{\text{radius of cylinder}}{2}$
Hollow cylinder	its longitudinal axis	$\sqrt{\dfrac{\text{inner rad}^2 + \text{outer rad}^2}{2}}$
ditto, infinitely thin	ditto	radius of cylinder
ditto, of any thickness	a diam midway between its ends	$\sqrt{\dfrac{\text{inner rad}^2 + \text{outer rad}^2}{4} + \dfrac{\text{length}^2}{12}}$
ditto, infinitely thin	ditto	$\sqrt{\dfrac{\text{radius}^2 \text{ of cylinder}}{2} + \dfrac{\text{length}^2}{12}}$
ditto, infinitely thin and infinitely short (circumference of a circle)	a diameter	$\text{radius of cylinder} \times \sqrt{\dfrac{1}{2}}$ $= \text{radius of cylinder} \times \text{about } .7071$
Solid sphere	a diameter	$\sqrt{\dfrac{\text{radius}^2 \text{ of sphere}}{2.5}}$ $= \text{radius of sphere} \times \sqrt{.4}$ $= \text{radius of sphere} \times \text{about } .63246$

Table of Radii of Gyration.—CONTINUED.

Body	Revolving around	Radius of Gyration
Hollow sphere of any thickness	a diameter	$\sqrt{\frac{2\,(\text{outer rad}^5 - \text{inner rad}^5)}{5\,(\text{outer rad}^3 - \text{inner rad}^3)}}$
ditto, thin	ditto	approx (outer rad + inner rad) × .4085
ditto, infinitely thin (spherical surface)	ditto	radius of sphere × $\sqrt{\frac{2}{3}}$ = radius of sphere × about .8165
Straight line, *a b*	any point, *x*, in its length	$\sqrt{\frac{a x^3 + x b^3}{3\,a b}}$
	either end, *a* or *b*	length *a b* × $\sqrt{\frac{1}{3}}$ = length *a b* × about .5775
	its center, *c*	*a c* × $\sqrt{\frac{1}{3}}$ = length *a b* × about .2887
Solid cone	its axis	radius of base of cone × $\sqrt{.3}$ = radius of base of cone × .5477
Circular plate, of rectangular cross section	See Solid cylinder	For the *thickness* of plate or ring, measured perpendicularly to the plane of the circumference, take the *length* of the cylinder.
Circular ring, of rectangular cross section	See Hollow cylinder	
Square, rectangle and other surfaces	For *least* radius of gyration, or that around the *longest* axis, see p 496 and 497.	

CENTRIFUGAL FORCE.

When a body *a*, Fig. 1, moves in a circular path *abd*, it tends. at each point, as *a* or *b*, to move in a tangent *at* or *bt′* to the circle at that point. But at each point, as *a*, etc., in the path, it is *deflected* from the tangent by a force acting toward the center, *c*, of the circle. This force may be the tension of a string, *ca*, or the attraction between a planet at *c* and its moon *a*, or the inward pressure of the rails, *a b*, on a curve, etc., etc. Like all force, it is an action between *two* bodies, tending either to separate them or to draw them closer together, and acting equally upon both. (See Art. 5 (*b*), p. 332). In the case of the string, it *pulls* the body *a*, *toward* the center, *c*, and the nail or hand, etc., at *c*, toward the body at *a* or *b*, etc.; *i. e.*, *from* the center. In the case of a car on a curve it *pushes* the car *toward* the center, and the rails *from* the center. The pull or push on the *revolving* body *toward* the center is called the **centripetal force**; while the pull or push tending to move the *deflecting* body *from* the center is called the **centrifugal force.** These two "forces," being merely the two "sides" (as it were) of the same stress, are necessarily equal and opposite, and can only exist together. The moment the stress or tension exceeds the strength (or inherent cohesive force) of the string, etc., the latter breaks. The centripetal and centrifugal forces therefore instantly cease; and the body, no longer disturbed by a deflecting force, moves on, at a uniform velocity,[*] in a tangent, *at* or *bt′*, etc., to its circular path; *i. e.*, at right angles to the direction which the centrifugal force had at the moment it ceased.

(*a*). **A single revolving body,** *a*, Fig. 1. Let

f = the centrifugal or centripetal force, in pounds.
W = the weight of the body *a*, in pounds,
R = the radius *ca* of the path of the *center of gravity* of the body *a*, in feet,
v = the uniform velocity of the body *a* in its circular path *abd*, in feet per second,
n = the number of revolutions per minute,
g = the acceleration of gravity = say 32.2 feet per second per second, $900\,g$ = about 28980.
π = circumference ÷ diameter = say 3.1416. π^2 = about 9.8696.

Then, for the **centrifugal force,** f:

If we have the velocity v in feet per second: $f = W\dfrac{v^2}{R\,g}$ [†] . . . (1)

If we have the number n of revolutions per minute: $f = W\dfrac{\pi^2 R n^2}{900\,g}$ [‡] . . . (2)

f = about $.0003406\,W R n^2$ [§] . . . (3).

---

[*] Neglecting friction, gravity, the resistance of the air, etc.

[†] For let *at*, Fig. 1, represent the amount and direction of the velocity v of the body at *a* in feet per second. Then at the end of one second the body will have reached the point *b* (the arc *ab* being made = *at*), and the amount and direction of its velocity at *b* will then be represented by the line *bt′* = *at* in length, but differing in direction. Drawing *cu* and *cu′* at the center, equal and parallel respectively to *at* and *bt′*, we find that the *change in the direction* of the motion (*i. e.*, the acceleration toward the center) during the second is represented by the arc *uu′*; and, since angle *acb* = angle *ucu′*, we have the proportion, radius R or *ac* : *ab* or *at* : : *cu* or *at* : arc *uu′*. In other words, the acceleration *uu′* in one second, or rate of acceleration, is = $\dfrac{at^2}{R} = \dfrac{v^2}{R}$; and, for the force causing that acceleration, we have

$$f = \text{mass of body} \times \text{rate of acceleration} = \text{mass of body} \times \frac{v^2}{R} = W\frac{v^2}{R\,g}.$$

[‡] By formula (1), $f = W\dfrac{v^2}{R\,g}$. But $v = \dfrac{2\,\pi\,R\,n}{60}$; and $v^2 = \dfrac{4\pi^2\,R^2\,n^2}{3600} = \dfrac{\pi^2\,R^2\,n^2}{900}$.

Hence, $f = W\dfrac{\pi^2\,R^2\,n^2}{900\,R\,g} = W\dfrac{\pi^2\,R\,n^2}{900\,g}$.

[§] Formula (3) is obtained from (2) by substituting the values 9.8696 and 28980 for π^2 and $900\,g$ respectively.

(*b*) **Wheels and discs.** Suppose the rim of a wheel to be cut into very short slices, as shown (much exaggerated) at *a*, Fig. 2. Then for each slice, as *a*, by formula (1): $f =$ weight W of slice $\times \frac{v^2}{R\,g}$;* and if each slice were connected

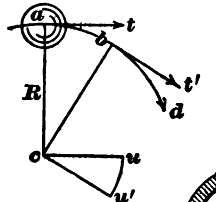


Fig. 1

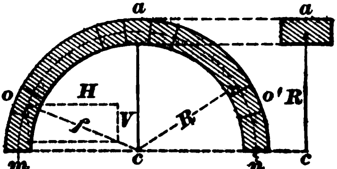


Fig. 2

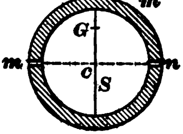


Fig. 3

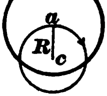


Fig. 4

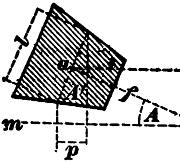


Fig. 5

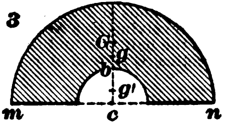


Fig. 6

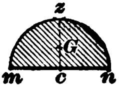


Fig. 7

with the center by a separate string, the sum of the stresses in all the strings (neglecting friction between adjacent slices) would be:

F = sum of centrifugal forces of all the slices† = weight of *rim* $\times \frac{v^2}{R\,g}$ . . . (4).

But the stress with which we are usually concerned in such cases (viz.: **the tension in the rim itself** in the direction of a tangent to its own circumference) is *much less* than the theoretical quantity F obtained from formula (4), being in fact only $\frac{1}{6.2832}$ of it. For suppose first that the same thin rim is cut only at two opposite points *m* and *n*, Fig. 3, and that its two halves are held together only by the string S.

---

* If the rim is very thin in proportion to its diameter *mn*, we may take the center of gravity of each slice as being in a circle *mn* midway between the inner and outer edges of the rim, so that $R = \frac{\text{inner radius} + \text{outer radius}}{2}$. In a rim of appreciable thickness, this is not the case, because each slice is a little thicker at its outer than at its inner end. See Fig. 5. Hence its center of gravity is a little outside of the curved line *mn*, Fig. 2.

† In a perfectly balanced rim (*i. e.*, a rim whose center of gravity coincides with its center of rotation, as in Fig. 3) the centrifugal forces of the particles on one side of *c* counterbalance those on the opposite side. Here, too, R = 0 Hence, *as a whole*, such a rim has *no centrifugal force; i. e.*, no tendency to leave the center in any one direction by virtue of its rotation. But if the two centers do *not* coincide (Fig. 4), then the rim is a single revolving body, and its centrifugal force is: $f =$ weight of entire rim $\times \frac{v^2}{R\,g}$; where R is the distance between the two centers, and *v* the velocity of the center of gravity *a*. The force *f* acts in the line joining the two centers.

Then:*

$$\text{semi-circumference } mzn : \text{diameter } mn :: \frac{F}{2} : \text{pull on the string S};$$

so that

$$\textbf{pull on string } \text{S} = \frac{\text{half weight}}{\text{of rim}} \times \frac{v^2}{\text{R}\,g} \times \frac{2}{\pi} = \frac{\text{weight}}{\text{of rim}} \times \frac{v^2}{\text{R}\,g\,\pi} = \frac{\text{F}}{\pi} = \frac{\text{F}}{3.1416} \quad \ldots (5);$$

and if the rim is now made complete by joining the ends at m and n, and if the string S is removed, then the pull on the string by formula (5) will be equally divided between m and n. Hence each cross-section, as m or n, of the rim, will sustain a tensile stress equal to half the pull on the string; or

$$\textbf{tension in rim} = \frac{\text{F}}{2\,\pi} = \frac{\text{F}}{6.2832} = \frac{\text{weight of rim} \times v^2}{6.2832\ \text{R}\,g} \quad \ldots\ldots\ldots (6).$$

The centripetal force, f, Fig. 2, holding any part o of the rim to its circular path, is the resultant of the two equal tensions at the ends of that part.

For the *stress per square inch* of cross-section of rim, we have:

$$\textbf{unit stress} = \frac{\text{tension in rim}}{\text{area A of cross-section of rim, in square inches}}$$

$$= \frac{\text{F}}{6.2832\ \text{A}} = \frac{\text{weight of rim} \times v^2}{6.2832\ \text{A R}\,g} \quad \ldots\ldots (7).$$

We shall arrive at the same result if we reflect that the pull in the string S, or the *sum* of the two tensions at m and n, is equal to the centrifugal force f of either *half* of the rim, revolving, as a single body, about the center c. Find the center of gravity G of the half rim, and then, in formula (1), use the velocity of that point, and the radius cG instead of velocity at z and radius cz respectively; thus:

$$\text{pull in string} = f = \frac{\text{centrifugal force}}{\text{of half-rim}} = \text{weight of half-rim} \times \frac{(\text{velocity at G})^2}{c\text{G} \times g};$$

and half of this is the tension in each cross-section of the rim.†

If the rim were infinitely thin, cG, Fig. 3, would be 0.6366 cz.

If its thickness must be taken into consideration, and if it is of rectangular cross-section, find the centers of gravity g and g', Fig. 6, of the whole semicircular segment cz and of the small segment cb respectively ($cg = 0.4244\ cz$, and $cg' = 0.4244\ cb$. Then

$$g'\text{G} = gg' \times \frac{\text{area of entire segment } cz}{\text{area of half rim}}.$$

For rims of other than rectangular cross-section, use formulæ (4), (5) and (6).

In a disc, such as a grindstone, the tension in each full cross-section mn, Fig. 7, is equal to the centrifugal force f of *half* the disc. Let W = weight of half disc. The distance cG from the center c to the center of gravity G of the half disc, is $c\text{G} = 0.4244\ cz$; and the

---

* In Fig. 2, let the centrifugal force of any slice, o, be represented by the diagonal, f, of a rectangle, whose sides, H and V, are respectively parallel and perpendicular to the given diameter mn. Then H and V represent the components of f in those two directions. The equal and opposite horizontal components H, of o and of the corresponding slice o', being parallel to mn, have no tendency to pull the rim apart at m or n. Hence, the pull on a string S, Fig. 3, perpendicular to mn, is the sum of the components V of all the slices. For each very thin slice, Fig. 5 (greatly exaggerated) we have (since angle A = angle A′):

$$\frac{\text{Length } l}{\text{of slice}} : \frac{\text{its horizontal}}{\text{projection, } p} :: \frac{\text{centrifugal force}}{f\text{, of slice}} : \frac{\text{its vertical}}{\text{component V.}}$$

Hence, for the entire half-rim mn, Fig. 3 (made up of such slices), we have:

$$\frac{\text{Length } mzn}{\text{of half-rim}} : \frac{\text{its horizontal}}{\text{projection } mn} :: \begin{matrix}\text{the sum of the centrif-}\\ \text{ugal forces of all the}\\ \text{slices of the half-rim,}\end{matrix} : \begin{matrix}\text{the sum of the}\\ \text{vertical compo-}\\ \text{nents V for all}\\ \text{those slices;}\end{matrix}$$

which is identical with the proportion at top of page.

† The *rims* of revolving wheels are usually made strong enough to resist the tension due to the centrifugal force, without aid from the *spokes*, which thus have merely to support the *weight* of the wheel. But if the rim *breaks*, the centrifugal forces of its fragments come entirely upon the spokes; and, since the breakage is always irregu- some of the spokes will always receive more than their share.

$$\textbf{tension in } mn = f = \mathrm{W}\,\frac{(\text{vel. at G})^2}{\text{rad. } c\mathrm{G} \times g} = \mathrm{W}\,\frac{0.4244^2\,(\text{vel. at } z)^2}{0.4244\; cz \times g}$$

$$= \mathrm{W}\,\frac{0.4244\,(\text{vel. at } z)^2}{cz \times g} \quad \ldots\ldots\ (8).$$

$$= \mathrm{W}\,\frac{0.4244\,\pi^2\,n^2\,cz}{900\,g} \quad \ldots\ldots\ (9).$$

The stress ***per square inch*** **in any full section** ***mn*** **is**

$$\textbf{unit stress} = \frac{\text{tension in } mn}{\text{area of cross-section in square inches}},$$

$$= \mathrm{W}\,\frac{0.4244\,(\text{velocity at } z)^2}{\text{diam. } mn\text{, ins.} \times \text{thickness, ins.} \times cz \times g} \quad \ldots (10).$$

$$= \mathrm{W}\,\frac{0.4244\,\pi^2\,n^2\,cz}{\text{diam. } mn\text{, ins.} \times \text{thickness, ins.} \times 900\,g} \quad \ldots (11).$$

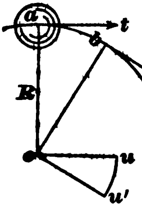


Fig. 1

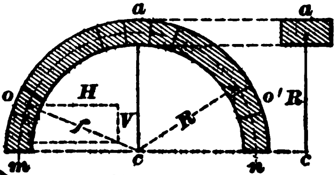


Fig. 2

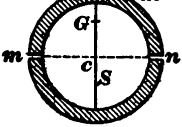


Fig. 3

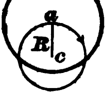


Fig. 4

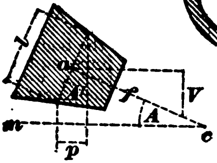


Fig. 5

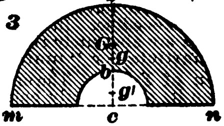


Fig. 6

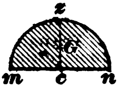


Fig. 7

f = the centripetal force, in pounds, acting upon a single revolving body, *a*, Figs. 1, 2, 4 and 5, or upon the half-rim or half-disc, Figs. 3, 6 and 7 = the centrifugal force exerted by such body.

F = the sum of the centrifugal forces *f*, of all the particles of a rim, Fig. 3.

W = the weight of the body, in pounds.

R = the radius *ca*, Figs. 1, 4 and 5, of the path of the *center of gravity* of the body.

v = the uniform velocity of the body in its circular path, in feet per second.

n = the number of revolutions per minute.

g = the acceleration of gravity = say 32.2 feet per second. 900 *g* = about 28980.

$\pi = \dfrac{\text{circumference}}{\text{diameter}}$ = say 3.1416. π^2 = about 9.8696.

In a rolling wheel, each point in the rim, during the moment when it touches the ground, is stationary *with respect to the earth;* but each particle has the same velocity *about the center* as if the latter were stationary, and hence the centrifugal force has no effect upon the weight.

STATICS.

FORCES.

1. **Statics Defined.** The science of statics, or of equilibrium of forces, takes account of those very numerous cases where the forces under consideration are in equilibrium, or balanced. It embraces, therefore, all cases of bodies which are said to be "at rest."*

2. **In the problems usually presented** in civil engineering, a certain given force, or certain given forces, applied to a stationary* body (as a bridge or building) tend to produce motion, either in the structure as a whole or in one or more of its members; and it is required to find and to apply another force or other forces which will balance the tendency to motion, and thus permit the structure and its members to remain at rest. See ¶ 33, below.

3. **Equilibrium.** Suppose a body to be acted upon by certain forces. Then those forces are said to be in equilibrium, when, as a whole, they produce no change in the body's state of rest or of motion, either as regards its motion as a whole along any particular line (motion of translation), or as regards its rotation about any point, either within or without the body. In such cases the body also is said to be in equilibrium. See ¶ 84, below.

4. A body may be in equilibrium as regards the forces under consideration, even though not in equilibrium as regards other forces. Thus, a stone, held between the thumb and finger, is in equilibrium as regards their two equal pressures, even though it may be lifted upward by the excess of the muscular force of the arm over the attraction between the earth and the stone. Similarly, on a level railroad, a car is in equilibrium as regards gravity and the upward resistance of the rails, although the horizontal pull of the locomotive may exceed the resistance to traction.

5. **Molecular Action.** Any force, applied to a body, is in fact made up of a system of forces, often parallel or nearly so, applied to the several particles of the body. Thus, the attraction exerted by the earth upon a grain of sand or upon the moon is, strictly speaking, a cluster of nearly parallel forces exerted upon the several particles of those bodies; but, for convenience, and so far only as concerns their tendency to move the body as a whole, we conceive of such forces as replaced by a single force, equal to their sum and acting in one line. In thus considering the forces, we assume that the bodies are absolutely rigid, so that each of them acts as a single "**material particle**" or "**material point.**"

6. **Transmission of Force.** The upward pressure of the ground, upon a stone resting upon it, acts directly only upon those particles which are nearest to the ground. These, in turn, exert a (practically) equal upward force upon those immediately above them, and so on; and the force is thus transmitted throughout the stone.

7. **Rigid Bodies.** In treating of bodies as rigid, we assume that the intermolecular forces hold the several particles absolutely in their original relative positions.

It is not the *material* that resists being broken, but the *forces* which hold its particles in their places. Thus, a cake of ice may sustain a great pressure; but its particles yield readily when its cohesive forces are destroyed by a melting temperature.

8. **Force Units.** The force units generally used in statics are those of weight, as the pound and the kilogram. See Conversion Tables, p. 235.

In statics we have no occasion to consider the masses of bodies (except in so far as these determine their weights, or the force of gravity exerted upon them), bodies being regarded merely as the media upon and through which the forces under consideration are exerted. Hence we require, in statics, no units of mass; and, as the bodies are regarded as being "at rest," no units of time, velocity, acceleration, momentum, or energy.

---

* Strictly speaking, *absolute* rest is scarcely conceivable, since all bodies are actually in motion (see Art. 3, p. 331), so that unbalanced forces produce merely *changes in the states of motion* of bodies. Yet, for a body to be at rest, *relative to other bodies*, is a very common condition, and, in practical statics, we usually regard the body under consideration as being at rest relatively to the earth or to some other large body, so that the change of state of motion, due to the action of unbalanced force upon it, consists in a change from relative rest to relative motion. See ¶ 33, below.

9. Forces, how Determined. A force is fully determined when we know (1) its amount (as in pounds, or in some other weight unit), (2) its direction, (3) its sense (see ¶ 10), and (4) its position or its point of application.

10. When a force is represented by a line, the length of the line may be made to represent by scale the amount of the force, and its direction and position may often be made to indicate those of the force, while the sense of the force may be shown by arrows or letters affixed to the lines, or by the signs, + and —.

Thus, the *directions* of the forces represented by lines *a* and *b*, Fig. 1, are vertical, and those of *c* and *d* are horizontal. The *sense* of *a* is upward, of *b* downward, of *c* right-handed, of *d* left-handed. Thus, *a* and *b* are of like direction, but of opposite sense; and so with *c* and *d*. In treating of vertical or horizontal forces, we usually call upward or right-handed forces **positive,** and downward or left-handed forces **negative,** as indicated by the signs, + and —, in Fig. 1. When a force is designated by two letters, attached to the line representing it, one at each end of the line, the sense of the force may be indicated by the order in which the letters are taken. Thus, in Fig. 1, having regard to the directions of the arrows, we have forces, *e f*, *h g*, *k l*, and *n m*.

11. Line of Action, etc. The point (see ¶ 5) at which a force P, Fig. 2, is supposed to be applied, as *a*, is called its point of application. But the force is transmitted, by the particles, throughout the body (see ¶ 6), and the effect of the force, as regards the body as a whole, is not changed if it be regarded as acting at any other point, as *b*, in its line of action. We may therefore regard any point in that line as a point of application of the force. For instance, the tendency to move the stone, Fig. 2, as a whole, will not be changed if, instead of *pushing* it, at *a*, we apply a *pull* (in the same direction and in the same sense) at *b*; and if a weight, P, be laid upon the top of the hook, at *b*, Fig. 3, it will have the same tendency, to move the hook as a whole, as it has when suspended from the hook as in the Fig.

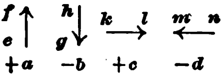


Fig. 1.

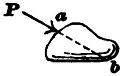


Fig. 2.

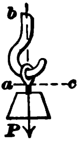


Fig. 3.

A force cannot actually be applied to a body at a point outside of the substance of the body, as between the upper and lower portions of the hook in Fig. 3, yet this portion also of the line *a b* is a part of the line of action of the force. The vertical force, exerted by the weight, P, is transmitted to *b* by means of bending moments in the bent portion of the hook.

12. Stress. (See Art. 1, Strength of Materials, p. 454.) Opposing forces, applied to a body by contact (see Art. 5 c, p. 332), cause stress, or the exertion of intermolecular force, within it, or between its particles, tending to pull them apart (tension) or to press them closer together (compression). **The stress, due to two equal opposing forces,** is equal to *one* of them.

Tension and Compression. Ties, Struts, etc. If the action of the forces tends to pull farther apart the particles of the body upon which they act, the stress is called a tension or pull, or a tensile stress. If it tends to press them closer together, the stress is called a pressure, compression or push, or a compressive stress. A long slender piece sustaining tension is called a tie. One sustaining compression is called a strut or post. One capable of sustaining either tension or compression is called a tie-strut or strut-tie.

MOMENTS.

13. Moments. If, from any point, o, or o', Fig. 4, a line, $o\ c$ or $o'\ s$, be drawn normally to the line of action, $n\ m$, of a force, P_1, whether the point, o or o', be within or outside of the body upon which the force, P_1, is acting, said line, $o\ c$ or $o'\ s$, is called the **arm** or **leverage** of the force about such point; and if the amount of the force, in lbs., etc., be multiplied by the length of the arm, in ft., etc., then the product, in ft.-lbs., etc., is called the **moment** of the force about that point.* The moment represents the total tendency of the force to produce rotation about the given point. A force has evidently no moment about any point in its line of action.

14. Sense of Moments. Since the moment of P_1 about o, Fig. 4, tends to cause rotation (about that point) in the direction of the motion of the hands of a clock, as we look at the clock and at the figure, or from left to right, as indicated by the arrow on the circle around o, it is called a **clockwise** or **right-hand** moment; but the moment of the same force about o' tends to produce rotation from right to left. Hence it is called a **counter-clockwise** or **left-hand** moment, as is also that of P_2 about o. Right-hand or clockwise moments are conventionally considered as **positive**, or +, and left-hand or counter-clockwise moments as **negative**, or —.

15. The plane of a moment is that plane in which lie both the line of action and the arm of the force.

16. The resultant or combined tendency of two or more moments in the same plane is equal to the algebraic sum of the several moments. Thus, Fig. 4, if the forces, P_1, P_2, and P_3, are respectively 6, 5, and 3 lbs., and if the arms, $o\ c$, $o\ y$, and $o\ e$, of their moments about o are respectively 7, 6, and 3 ft., we have

$$\begin{aligned} & P_1 . o\ c - P_2 . o\ y + P_3 . o\ e \\ = {} & 6 \times 7 - 5 \times 6 + 3 \times 3 \\ = {} & 42 - 30 + 9 = 21 \text{ ft.-lbs.} \end{aligned}$$

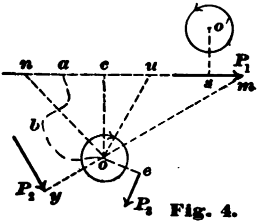


Fig. 4.

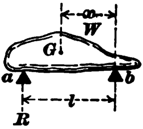


Fig. 5.

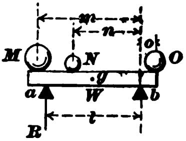


Fig. 6.

17. If the algebraic sum of the moments is zero, they are in **equilibrium** and tend to cause no rotation of the body about the given point.

Thus, in Fig. 5, where W is the weight, and G the center of gravity of the body, and R the upward reaction of the left support, a, taking moments about the right support, b, we have $R\ l - W\ x =$ zero; or $R\ l = W\ x$. Hence, having W, x and l, to find R, we have $R = \frac{W\ x}{l}$.

Similarly, in Fig. 6, where $W =$ weight of beam alone, and g, the center of gravity of W, is at the center of the span l, so that the leverage $b\ g$ of the weight of the beam about b, is $= \frac{l}{2}$, we take moments about b, thus:

$$R\ l + O\ o - W \frac{l}{2} - M\ m - N\ n = \text{zero; or}$$

$$R = \frac{M\ m + N\ n + W\ \frac{l}{2} - O\ o}{l}.$$

---

*Note that a very small force may have a great moment about a point, while a much greater force, passing nearer to the same point, may have a smaller moment about it; or, passing through the point, no moment at all.

In Fig. 7, where W is the weight of the beam itself, and w its leverage, taking moments about b, we have

$$+ \mathrm{R}\, l + \mathrm{O}\, o - \mathrm{N}\, n - \mathrm{W}\, w + \mathrm{M}\, m = 0;$$

$$\text{Hence,} \qquad \text{Reaction at } a = \mathrm{R} = \frac{\mathrm{W}\, w + \mathrm{N}\, n - \mathrm{M}\, m - \mathrm{O}\, o}{l}.$$

In any case, if W be the combined weight and G the common center of gravity, of the beam and its several loads, and x the horizontal distance of that center from the right support, b; and if l be the span, R the reaction of the left support, a, and R′ that of the right support, b, we have

$$\mathrm{R} = \frac{\mathrm{W}\, x}{l}; \text{ and } \mathrm{R}' = \mathrm{W} - \mathrm{R}.$$

If $x = \frac{l}{2}$, R is $= \frac{\mathrm{W}}{2} = \mathrm{R}'$.

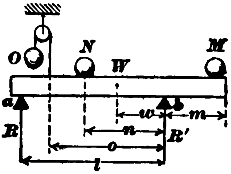


Fig. 7.

Note that the moments of two or more forces, about a given point, may be in equilibrium, while the forces themselves are not in equilibrium. See ¶ 84, below.

18. Center of Moments. So far as concerns equilibrium of moments, it is immaterial what point is selected as a center of moments; but it is generally convenient to take the center of moments in the line of action of one (or more, if there be concurrent forces, see ¶ 19) of the unknown forces, for we thus eliminate that force or those forces from the equation.

CLASSIFICATION OF FORCES.

19. Classification of Forces. Concurrent, Colinear, Coplanar, and Parallel Forces. Forces are called concurrent when their lines of action meet at one point, as a, b, c, d, e and f, or f and g, Fig. 8; non-concurrent when they do not so meet, as c and g; colinear when their lines of action coincide, as a and b, or c and d; non-colinear when they do not coincide, as b and f; coplanar when their lines of action lie in one plane,* as a, b, c, d and e, or b, f and g, etc.; non-coplanar, as c and g, or b, f and d, when they do not lie in one plane; parallel when their lines of action are parallel, as b and g; non-parallel when those lines are not parallel, as b and f.

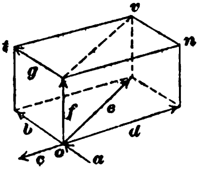


Fig. 8.

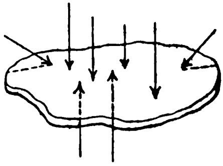


Fig. 9.

---

*Acting *upon* a plane, as in Fig. 9, must not be confounded with acting *in* that plane, as in Figs. 70, etc.

Any two parallel forces must be coplanar. Three or more parallel forces may or may not be coplanar. Any two concurrent forces must be coplanar. Three or more concurrent forces may or may not be coplanar. Any two coplanar forces must be either parallel or concurrent.

COMPOSITION AND RESOLUTION OF FORCES.

20. Resultant. A single force, which can produce, upon a body considered as a whole, the same effect as two or more given forces combined, is called the resultant of those forces. Thus, in Fig. 10 (*b*), a downward pressure, G, $= w + W$, is the resultant of the downward pressures w and W; and, in Fig. 11 (*b*), a downward pressure, $= W - w$, is the resultant of the downward pressure W and the upward pull w of the left-hand string.*

21. Component. Any two or more forces which, together, produce, upon a body considered as a whole, the same effect as one given force, are called the components of that force, which thus becomes their resultant. Thus, in Fig. 10 (*b*), w and W are the components of the total force, G, $= w + W$. In Fig. 11 (*b*), $+ W$ ($= 5$) and w ($= -3$) are the components of G.*

22. If we take into account the resultant of any given forces, those forces (components) themselves must of course be left out of account, as regards their action upon the body as a whole; although we may still have to consider their effect upon its particles. Vice versa, if the forces (components) are considered, their resultant must be neglected.

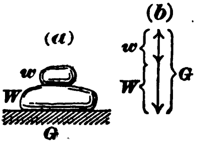


Fig. 10.

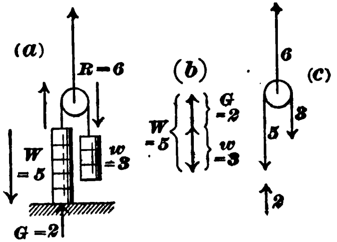


Fig. 11.

23. Anti-resultant. The anti-resultant of one or more forces is a single force which, acting upon any body or system of bodies considered as a whole, produces an effect equal, but opposite, to that of their resultant. In other words, the anti-resultant is the force required to hold the given force or forces in equilibrium. Thus, in Fig. 10 (*b*), the upward reaction, G, of the ground, is the anti-resultant of the two downward forces, w and W; and the downward resultant, $W + w$, of W and w, is the anti-resultant of G. In Fig. 11 (*b*), G (upward) is the anti-resultant of W (downward) and w (acting upward through the left-hand string). Similarly, this upward pull of w is the anti-resultant of W and G.

24. In any system of balanced forces (forces in equilibrium), any one of the forces is the anti-resultant of all the rest; and any two or more of them have, for their resultant, the anti-resultant of all the rest. In such a system, the resultant (and the anti-resultant) of all the (balanced) forces is zero.

25. Anti-component. The anti-components of a given force, or of a given system of forces, are any two or more forces whose resultant is the anti-resultant of the given force or of the given system of forces.

26. Composition and Resolution of Forces. The operation of finding the resultant of any given system of forces is called the composition of forces; while that of finding any desired components of a given force is called the resolution of the force.

---

* For convenience, we here reverse the convention of ¶ 10.

Colinear Forces.

27. Let the vertical line, w, Fig. 10 (b), represent, by any convenient scale, the weight of the upper stone in Fig. 10 (a), and W that of the lower stone. Then, $w + W$, = G, = the combined length of the two lines, gives, by the same scale, the combined weight of the two stones, and a vertical line G, coincident with them, equal to their sum, and pointing upward, would represent their anti-resultant, or the reaction of the ground.

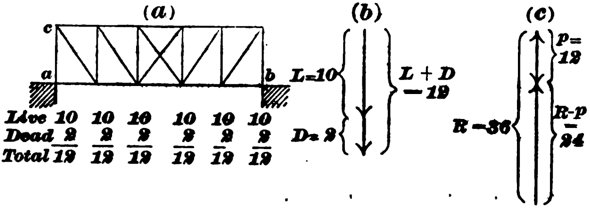


Fig. 12.

28. Similarly, if, at each panel point of the lower chord in the bridge truss in Fig. 12 (a), we have 2 tons dead load (weight of bridge and floor, etc.*) and 10 tons live load (train, vehicles, cattle, passengers, etc.), the combined length of the two lines in Fig. 12 (b), L = 10, and D = 2, gives the total panel load of 12 tons.

29. In Fig. 11 the pressure, 5 lbs., of W upon the ground, is diminished by the 3 lbs. upward pull of the cord, transmitted from the smaller weight w, leaving 2 lbs. upward pressure to be exerted by the ground in order to maintain equilibrium. The upward reaction, R, of the pulley is $= w + W - G = 3 + 5 - 2 = 6$. This is represented graphically in Fig. 11 (c).

30. In the truss shown in Fig. 12 (a), the total dead and live load is $= 6 \times 12 = 72$ tons, and half this total load, or 36 tons, rests upon each abutment. Hence, to preserve equilibrium, each abutment must exert an upward reaction of 36 tons; but, in order to ascertain how much of these 36 tons is *transmitted through the end-post*, $a\ c$, we must deduct from it the 12 tons which we assume to be originally concentrated, as dead and live load, at the panel point a; for this portion is evidently not transmitted through $a\ c$. Accordingly, in Fig. 12 (c), we draw R upward, and equal by scale to 36 tons; and, from its upper end, draw p downward and = 12 tons. The remainder of R, $= R - p = 36 - 12 = 24$ tons, is then the pressure transmitted through $a\ c$.

31. Colinear forces are called **similar** when they are of like sense, and **opposite** when of opposite sense. The same distinction applies to resultants.

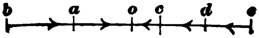


Fig. 13.

32. For equilibrium, under the action of colinear forces, it is, of course, necessary that the sum of the forces acting in one sense be equal to the sum of those acting in the opposite sense, or, in other words, that the algebraic sum of all the forces be zero. Thus, in Fig. 13, if the forces are in equilibrium, the sum, $b\,a + a\,o$, of the two right-handed forces must be equal to the sum, $e\,d + d\,c + c\,o$, of the three left-handed forces. Or, considering the right-handed forces, $b\,a$ and $a\,o$, as positive, and the left-handed forces, $e\,d$, $d\,c$ and $c\,o$, as negative, as in ¶ 10, we have, as the condition of equilibrium of colinear forces:

$$b\,a + a\,o - o\,c - c\,d - d\,e = 0.$$

---

* The dead load is, of course, never actually concentrated upon one chord, as here indicated; but it is often assumed, for convenience, that it is so concentrated.

In other words, the algebraic sum of all the forces must be zero; or, more briefly,

$$\Sigma \text{ forces} = 0,$$

where the Greek letter Σ (sigma), or sign of summation, is to be read "The sum of—."

33. Two equal and opposite forces, acting upon a body, are commonly said to keep it at rest; but, strictly speaking, they merely *prevent each other* from moving the body, and thus *permit* it to remain at rest, so far as they are concerned; for they cannot keep it at rest against the action of any third force, however slight and in whatever direction it may act; and the body itself has no tendency to move.

34. Unequal Opposite Forces. If two opposite forces, acting upon a body, are unequal, the smaller one, and an equal portion of the greater one, act against each other, producing no effect upon the body as a whole; while the remainder, the resultant, moves the body in its own direction.

Concurrent Coplanar Forces. The Force Parallelogram.

35. Composition. Let the two lines, $a\,o$, $b\,o$, in any of the diagrams of Fig. 14, represent, in magnitude, direction and sense, concurrent forces whose lines of action meet at the point o. Then, in the parallelogram, $a\,c\,b\,o$, formed upon the lines $a\,o$, $b\,o$, the resultant of those two forces is represented, in magnitude and in direction, by that diagonal, R, which passes through the point, o, of concurrence. The parallelogram, $a\,c\,b\,o$, is called a **force parallelogram.**

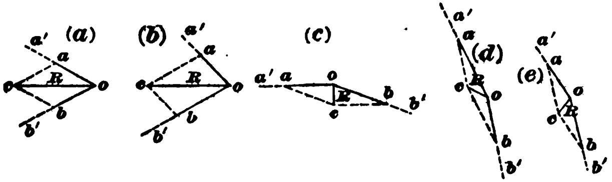


Fig. 14.

36. Resolution. Conversely, to find the components of a given force, $o\,c$, Fig. 14, when it is resolved in any two given directions, $o\,a$, $o\,b$, draw the lines, $o\,a'$, $o\,b'$, in those directions and of indefinite length, and upon these lines, with the diagonal $R = o\,c$, construct the force parallelogram $a\,c\,b\,o$. The sides, $o\,a$, $o\,b$, of the parallelogram then represent the required components in amount and in direction.

37. Caution. The two forces, $a\,o$ and $b\,o$, Fig. 14, may act either toward or from the point o; or, in other words, they may act either as pulls or as pushes; but the lines representing them in the parallelogram, and meeting at the point, o, must be drawn, either both as pushes or both as pulls; and the resultant, R, as represented by the diagonal of the parallelogram, will be a pull or a push, according as the two forces are represented as pulls or as pushes.

38. Thus, in Fig. 15 (*a*), the inclined end-post of the truss pushes obliquely downward toward o, with a force represented by $a'\,o$, while the lower chord pulls away from o, toward the right, with a force represented by $o\,b'$. If, now, we were to construct, in Fig. 15 (*a*), the parallelogram $o\,a'\,c'\,b'$, we should obtain the diagonal $o\,c'$ or $c'\,o$, which does not represent the true resultant. In fact, as one of the two forces acts toward, and the other from, the point, o, we could not tell (even if R' were the *direction* of the resultant) in which *sense* its arrow should point.

We must first either suppose the push, $a'\,o$, in the end-post, toward o, to be carried on beyond o, so as to act as a pull, $o\,a$, Fig. 15 (*b*) (of course, in the same direction and sense as before), thus treating both forces as pulls; or else we must similarly suppose the pull, *o b′*, in the chord, to be transformed into the push, *b o*, of Fig. 15 (*c*), thus treating both forces as pushes. In either case we obtain the true resultant, R (= *a′ b′*, Fig. 15 *a*), which, in this case, represents the vertical downward pressure of the end of the truss upon the abutment.

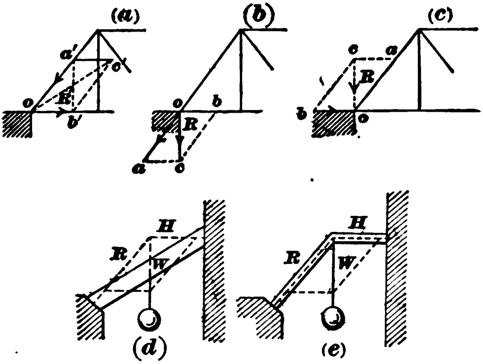


Fig. 15.

Caution. The tensile force, exerted at the end of a flexible tie, necessarily acts in the line of the tie; but, in general, the pressure, exerted at the end of a strut, acts in the line of the axis of the strut only when all the forces producing it are applied at the other end of the strut. Thus, in Fig. 15 (*d*), the components, R and H, of the weight, W, do not coincide with the axis of the beam which supports the load; but in Fig. 15 (*e*), where the weight acts at the intersection of the two struts, its components, R and H, do coincide with the axes of the struts. See also Figs. 143 and 145 (*b*).

39. Demonstration. The rational demonstration of the principle of the force parallelogram is given in treatises on Mechanics. (See Bibliography.) It may be established experimentally as indicated in Fig. 16, where *c o* represents by scale the pull shown by the spring balance C, while *o a* and *o b* represent those shown by A and B respectively.

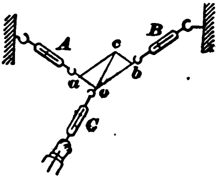


Fig. 16.

40. Equations for Components and Resultant. Given the amounts of the forces, *a* and *c*, or of the resultant, R, and the angles formed between them, Fig. 17 (*a*), we have*:

---

* See dotted lines, Fig. 17 (*a*), noting that $c' = c$; $c . \sin (x + y) = R . \sin x$, and $a . \sin (x + y) = R . \sin y$.

$$R = c\,\frac{\sin (x + y)}{\sin x} = a\,\frac{\sin (x + y)}{\sin y}$$

$$c = R\,\frac{\sin x}{\sin (x + y)}; \quad a = R\,\frac{\sin y}{\sin (x + y)}.$$

If the angle between the two forces is 90°, Fig. 17 (*b*), these formulas become:

$$R = \frac{c}{\cos y} = \frac{a}{\cos x}$$

$$c = R \cos y; \quad a = R \cos x.$$

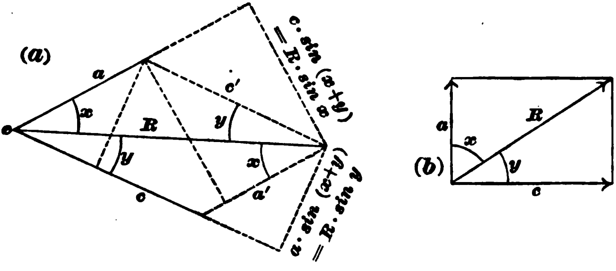


Fig. 17.

41. Position and Sense of Resultant. Figs. 18. If the lines representing the components be drawn in accordance with ¶¶ 37 and 38, and if a straight line, *m n* or *m′ n′*, be drawn through the point, *o*, of concurrence, in such a way that both forces are on one side of that line, then the line representing the resultant will be found upon the same side of that line with the components, and between them; and it will act toward the line, *m n* or *m′ n′*, if the components act toward it, and vice versa. The resultant is necessarily in the same plane with its two components.

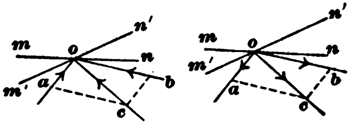


Fig. 18.

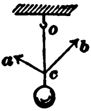


Fig. 19.

42. If one of the components is colinear with the force, it is the force itself, and the other component is zero. In other words, a force cannot be resolved into two non-colinear components, one of which is in the line of action of the force. Thus the rope, *o c*, Fig. 19, may receive assistance from *two* additional ropes, pulling in the directions *a c*, and *c b*; for the resultant of their pulls may coincide with *o c*; but, so long as *o c* remains vertical, no *single* force, as *c a* or *c b*, can relieve it, unless acting in its own direction *c o*.

43. In Fig. 20, the load, P, placed at C, is suspended entirely by the vertical member B C, and exerts directly no pull along the horizontal member, C E. Neither does a pull in the latter exert any effect upon the force acting in B C, so long as B C remains vertical. But the tension in B C, acting at B, does exert a thrust *o a* along B D, although that member is at right angles to B C; for B C meets there also the inclined member A B; and the tension *o d* is thus resolved into *o a* and *o b*, along B D and B A respectively. The horizontal thrust, *o a*, in B D, is really the anti-resultant of the horizontal component, *d b*, of the oblique thrust in the end-post B A, at its head, B, which thrust is = the pull in A E, due to P.

44. In Fig. 21, the tension, *o c*, in the inclined tie, D G, is resolved, at D, into *o a* and *o b*, acting at right angles to each other along D F and D E respectively.

45. A resultant may be either greater or less than either one of its two oblique components, but it is always less than their sum. If the components are equal, and if the angle between them = 120°, the resultant is equal to one of them. Therefore the same weight which would break a single vertical rope or post, would break two such ropes or posts, each inclined 60° to the vertical.

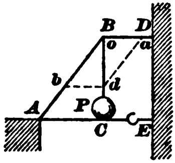


Fig. 20.

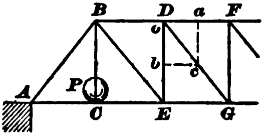


Fig. 21.

The Force Triangle.

46. The Force Triangle. Inasmuch as the two triangles, into which a parallelogram is divided by its diagonal, are similar and equal, it is sufficient to draw either one of these *triangles*, *a o c* or *b o c*, Figs. 14, 16, 18, instead of the entire parallelogram.

47. If three concurrent coplanar forces are in equilibrium, the lines representing them form a triangle; and the arrows, indicating their senses, follow each other around the triangle. Thus, in Fig. 22 (*a*), we have, acting at *o* and balancing each other there, three forces: viz., (1) the vertical downward force *o c* of the weight, acting as a pull through the rope *o c*, (2) the horizontal thrust *a o* through the beam *a o*, and (3) the upward inclined thrust *b o* of the strut *o b*, all acting in the senses (*o c*, *a o*, *b o*) in which the letters are taken, and as indicated by the arrows.

48. Each of the forces in Fig. 22 (*b*) and (*c*) is the anti-resultant of the other two in the same triangle; and, if its sense be reversed, it becomes their resultant. Thus, *o c*, Fig. 22 (*b*), is the anti-resultant, and *c o* the resultant, of *c a* and *a o*; and *o c*, Fig. 22 (*c*), is the anti-resultant, and *c o* the resultant of *c b* and *b o*, *c b* being parallel to *a o*, Fig (*b*), and representing the thrust exerted by the horizontal beam against the joint *o*, Fig. (*a*).*

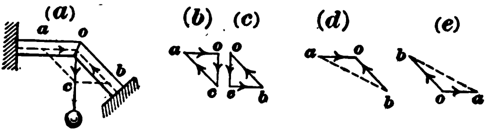


Fig. 22.

---

*Fig. 22 (*d*) and (*e*), representing the same two forces, *a o*, *b o*, of Fig. 22 (*a*), show the erroneous resultant (*a b*) obtained if the lines are drawn with their arrows pointing both toward or both from the meeting-point of the lines. See ¶¶ 37, 38. A comparison of any force parallelogram, as that in Fig. 18, with either of the two force triangles composing it, will show that this, while apparently contradicting ¶¶ 37 and 38, is merely another statement of the same fact. The apparent contradiction is due to the fact that, in the force triangle, the lines representing the forces do not meet at the point, *o*, of concurrence of the forces.

49. Conversely, if the three sides of a triangle be taken as representing, in direction and in amount, three concurrent forces whose senses are such that arrows, representing them and affixed to their respective sides in the triangle, follow each other around it, then those forces are in equilibrium.

50. The three forces, Fig. 23, are proportional, respectively, to the sines of their opposite angles. Thus:

$$\text{Force } a : \text{force } b : \text{force } c = \text{Sin A} : \sin \text{B} : \sin \text{C}.$$

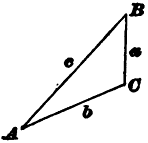


Fig. 23.

51. Example. In Fig. 24, the half arch and its spandrel, acting as a single rigid body, are assumed to be held in equilibrium by their combined weight, W, the horizontal pressure h at the crown, and the reaction R of the skewback, which is assumed to act through the center of the skewback. In the force triangle $c\ s\ t$, $c\ s$, acting through the center of gravity of the half arch and spandrel, represents the known weight W, and $s\ t$ is drawn horizontal, or parallel to h. From c, where h, produced, meets the line of action of W, draw $c\ t$ through the center of the skewback. Then $s\ t$ and $c\ t$ give us the amounts of h and R respectively.

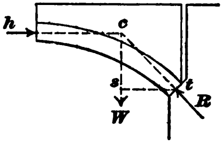


Fig. 24.

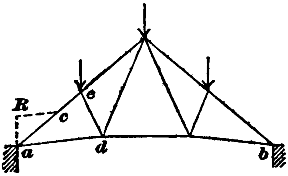


Fig. 25.

52. Example. Let Fig. 25 represent a roof truss, resting upon its abutments and carrying three loads, as shown by the arrows. Draw a R vertically, to represent the proportion of the loads carried by the left abutment, a, or, which is the same thing, the vertical upward reaction of that abutment. Then, drawing R c, parallel to the chord member, $a\ d$, to intersect $a\ e$ in c, we have, for the stresses in $a\ e$ and $a\ d$, due to the three loads:

Stress in $a\ e = a\ c$
“ “ $a\ d = \text{R}\ c$

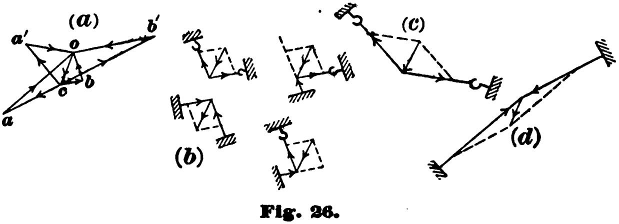


Fig. 26.

53. While any two or more given forces, as $o\ b$ and $b\ c$, Fig. 26 (a) (arrows reversed), or $o\ b'$ and $b'\ c$, or $o\ a$ and $a\ c$, or $o\ a'$ and $a'\ c$, can have but one resultant $o\ c$; a single force, as $o\ c$, may be resolved into two or more concurrent components in any desired directions. In other words, there is an infinite number of possible systems of concurrent forces which have $o\ c$ for their resultant.

Rectangular Components.

54. Resolutes, or Rectangular Components. A very common case of resolution of forces is that where a force, as the pressure, *c n*, of the post, Fig. 27, is to be resolved into components at right angles to each other, as are the vertical and horizontal components *c t* and *t n* in Fig. 27 (*a*). Two such components, taken together, are called the resolutes or rectangular components of the force. The joint, *o d*, in Fig. 27 (*a*), is properly placed at right angles to *c n*; but the joint *c i b*, Fig. 27 (*b*), provides also against accidental changes in the direction of *c n*. In Fig. 27 (*b*), the surfaces, *c i* and *i b*, are preferably proportioned as the components, *c t* and *t n*, Fig. 27 (*a*), respectively, by similarity of triangles, *c i b*, *c t n*.

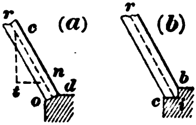


Fig. 27.

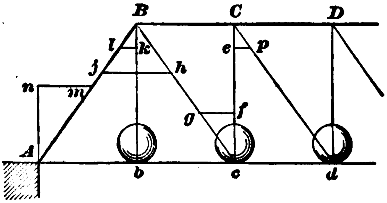


Fig. 28.

55. Example. In bridge and roof trusses it is often required to find the vertical and horizontal resolutes of the stress in an inclined member, or to find the stress brought upon an inclined member by a given vertical or horizontal stress applied at one of its ends, in conjunction with another stress (whose amount may or may not be given) at right angles to it.

Thus, in Fig. 28, the tension C *p* in the diagonal C *d* is resolved into a compression *e p* along the upper chord member C D* and a compression C *e* in the post C *c*.* Adding to C *e* the load at *c*, and representing their sum by *f c*, we have tension *f g* in chord member *c d*, and tension *c g* in the diagonal B *c*. Making B *h* = *c g*, we have *j h*, compression in B C, and B *j*, compression in the end-post or batter post B A. But the load at *b* also sends to B, through the hip vertical B *b*, a load (tension) equal to itself. Representing this by B *k*, we have *l k* as its component along the chord member B C, and B *l* as its component along the end-post B A. Now, making A *m* = the sum of B *j* and B *l*, we find the vertical resolute A *n* = so much of the vertical reaction of the abutment as is due to the three loads only, and the horizontal resolute *m n* = the corresponding stress in the chord member, A *c*.

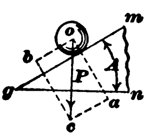


Fig. 29.

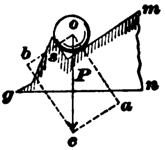


Fig. 30.

56. Example. Inclined Plane. Again, in Fig. 29, let it be required to find the two resolutes of P (the weight of the ball) respectively parallel and perpendicular to the inclined plane. The former is the tendency of the ball to move down the plane, and is called the **tangential component.** The

---

*The stress thus found is not necessarily the *total* stress in the member. The compression in C *c* (neglecting its own weight and that of the top chord) is due entirely to the tension C *p* in C *d*, acting at its top, and hence C *e* represents the total compression in C *c*; but *e p* is only a *portion* of the compression sustained by C D; for B C also contributes its share toward this.

latter is the pressure of the ball against the plane, and is called the **normal component.**

Here we have only to draw the triangle of forces *o a c*,* drawing *o c* = P to represent the weight of the ball, and *o a* and *a c* in the required directions. Then *o a* and *a c* give respectively the normal and the tangential components of the force, P.†

57. If we suppose the inclined plane *g m*, Fig. 29, to be frictionless, and if the body *o* is to be prevented from sliding down the plane, by means of a force applied in a direction parallel to the plane, that force must be = *c a*.

Thus, in Fig. 30, supposing the plane *o m* to be frictionless, we have *a c* = pressure against the stop, *s*.

58. Table of normal and tangential components for different angles of inclination:

Inclination or Slope of the Plane. The sloping length is = vertical height / sine, col 6.					Pres. on Plane, in parts of the wt. Or, nat. cos. of angle of Plane.	Pres. on Plane, in lbs per ton.	Tendency down the Plane, in parts of the wt. Or, nat. sine of angle of Plane.	Tendency down the Plane, in lbs per ton.
Vert.	Hor.	Ft. per mile.	Deg.	Min.				
1 in	3.	1760.00	18	26	.9487	2125	.3162	708.
1 in	4.	1320.00	14	2	.9702	2173	.2425	543.
1 in	5.	1056.00	11	19	.9806	2196	.1962	439.
1 in	6.	880.00	9	28	.9864	2210	.1645	368.
1 in	8.	660.00	7	8	.9923	2223	.1242	278.
1 in	9.	588.66	6	20	.9939	2226	.1108	247.
1 in	10.	528.00	5	43	.9950	2229	.0996	223.
1 in	11.4	461.94	5	00	.9962	2231	.0872	195.
1 in	12.	440.00	4	46	.9965	2232	.0831	186.
1 in	14.3	369.23	4	00	.9976	2232	.0698	156.
1 in	15.	352.00	3	49	.9978	2233	.0666	149.
1 in	19.1	276.73	3	00	.9986	2237	.0523	117.
1 in	20.	264.00	2	52	.9987	“	.0500	112.
1 in	23.1	229.04	2	30	.9990	“	.0436	97.7
1 in	25.	211.20	2	17	.9992	2238	.0398	89.2
1 in	28.6	184.36	2	00	.9994	“	.0349	78.2
1 in	30.	176.00	1	55	“	“	.0334	74.8
1 in	32.7	161.47	1	45	.9995	2239	.0305	68.4
1 in	35.	150.86	1	38	.9996	“	.0285	63.8
1 in	38.2	138.22	1	30	.9997	2240	.0262	58.6
1 in	40.	132.00	1	26	“	“	.0250	56.0
1 in	45.8	115.29	1	15	“	“	.0218	48 8
1 in	50.	105.60	1	9	.9998	“	.0201	45.0
1 in	57.3	92.16	1	0	“	“	.0175	39.1
1 in	60.	88.00	0	57½	.9999	“	.0167	37.4
1 in	70.	75.43	0	49	“	“	.0143	32.0
1 in	76.4	69.12	0	45	“	“	.0131	29.3
1 in	80.	66.00	0	43	“	“	.0125	28.0
1 in	90.	58.67	0	38	“	“	.0111	24.9
1 in	100.	52.80	0	34	1.0000	“	.0100	22.4
1 in	114.6	46.07	0	30	“	“	.0087	19.6
1 in	125.	42.24	0	27½	“	“	.0080	17.9
1 in	150.	35.20	0	23	“	“	.0067	15.0
1 in	175.	30.17	0	19⅝	“	“	.0057	12.8
1 in	200.	26.40	0	17	“	“	.0050	11.2
1 in	229.2	23.04	0	15	“	“	.0044	9.77
1 in	250.	21.12	0	14	“	“	.0041	9.18
1 in	300.	17.60	0	11½	“	“	.0033	7.39
1 in	343.9	15.35	0	10	“	“	.0029	6.52
1 in	400.	13.20	0	8⅝	“	“	.0025	5.60
1 in	500.	10.56	0	7	“	“	.0020	4.48
1 in	600.	8.80	0	6	“	“	.0017	3.81
1 in	800.	6.60	0	4⅓	“	“	.0013	2.91
1 in	1000.	5.28	0	3⅜	“	“	.0010	2.24
1 in	3437.	1.54	0	1	“	“	.0003	0.65
Level.		0.00	0	0	“	“	.0000	0.00

---

* Or *o b c*. If both triangles are drawn, we have the force parallelogram, *o a c b*.

†The line *a c* (or *c a*) is called the **projection** of *o c* upon the inclined plane; and *o a* (or *a o*) is the projection of *o c* upon a *normal* to the inclined plane.

59. Equations. In Fig. 29,

$$oa = P\,.\cos coa$$
$$ac = P\,.\sin coa$$

and, since the angle coa between the vertical oc and the normal component oa is equal to the angle A of inclination between the plane gm and the horizontal gn, we have:

$$\text{Normal component,}\quad oa = P\,.\cos A.$$
$$\text{Tangential component,}\quad ac = P\,.\sin A.$$

60. When a force is resolved into *rectangular* components, as in Figs. 29 and 30, each of these components represents the *total effort or tendency* which *that force alone* can exert *in that direction.*

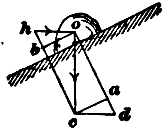


Fig. 31.

Thus, in Fig. 31, the utmost force which the weight oc alone can exert *perpendicularly against the plane* is that represented by the component oa. True, if, in order to prevent the body from sliding down the plane, we apply a force in some other direction, such as the horizontal one, ho, instead of the tangential one bo, and find the components of oc in the directions ho and oa, we shall find the normal component od greater than before; but the increase ad is due entirely to the normal component, hb, of the horizontal force ho. Thus, the only effect upon the body o, and upon the plane, of substituting ho for bo, is to add the normal component, hb, of the former, to that (oa) of oc.

Stress Components.

61. Stress Components. In Fig. 32, let ao and bo be any two forces, and co their resultant. From a and b draw aa' and bb' at right angles to the diagonal oc of the force parallelogram $aobc$, and construct the sub-parallelograms (rectangles), $oa'aa''$ and $ob'bb''$. Each of the original components, oa, ob, is thus resolved into two sub-components, perpendicular to each other, one of which is perpendicular also to the resultant, oc, while the other coincides with oc in position and in sense. Now, perpendiculars, let fall from the opposite angles of a parallelogram upon its diagonal, are equal.

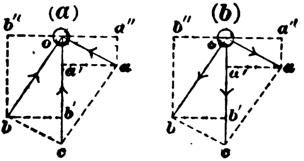


Fig. 32.

Hence the two colinear forces, oa'' and ob'', acting upon the body at o, are equal and opposite (although the lines, $a'a$ and $b'b$, representing them, are not opposite). Hence also they are in equilibrium, and their only effect upon the body is a stress of compression in Fig. 32 (*a*), and of tension in Fig. 32 (*b*). They may therefore be called the stress components. The other two sub-components (oa' of oa, and ob' of ob) combine to form the resultant oc, which is equal to their sum, and which tends to move the body o in its own direction.

62. The two great forces, $o a$, $o b$, in Fig. 33 (*b*) have the same resultant, $o c, = o c'$, as the two small forces, $o a'$ $o b'$, in Fig. 33 (*a*), although their stress components, $a'' a, = b'' b$, are much greater.

63. It often happens that one of the components is itself normal to the resultant. Thus, in Fig. 22, where $o c$ is vertical, its component, $o a$, is horizontal, and the perpendicular, let fall from a upon $o c$, represents its horizontal anti-component, $a o$. Here the horizontal and the inclined beam sustain equal horizontal pressures; but the vertical pressure, $o c, =$ the weight, W, is borne entirely by the inclined beam.

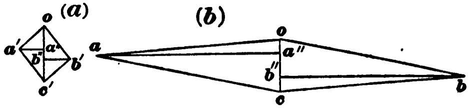


Fig. 33.

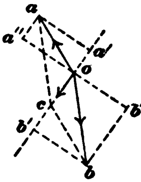


Fig. 34.

64. When, as in Fig. 34, the resultant, $o c$, forms, with one of the original components, $o a$ and $o b$, an angle, $a o c$, greater than 90°, the perpendiculars, $a a'$, $b b'$, from a and b, must be let fall upon the line of the resultant *produced*. Here, however, as before, the two equal and opposite sub-components, $o a''$ and $o b''$, are in equilibrium at o, while the other two sub-components, $o b'$ and $o a'$, go to make up the resultant $o c$; which, however (since $o b'$ and $o a'$ here act in *opposite* senses) is equal to their *difference*, and not to their sum, as in Fig. 32.

Fig. 34 shows that a *downward* force, $o c$, may be so resolved that one of its components is an *upward* force, $o a$, *greater* than the original *downward* force, and that the pressure, $o b$, has a component, $o b'$ or $b'' b$, parallel to $o c$, and greater than $o c$ itself; for $b'' b = o b' = o c + c b'$.

Applied and Imparted Forces.

65. Applied and Imparted Forces. In Fig. 29, the ball is free to roll down the inclined plane. Hence, although the entire weight P of the ball is *applied* to the body $g m n$, only the *normal component* $o a$ is *imparted* to it or exerts any pressure upon it, and this pressure is in the direction $o a$.

But in Fig. 30, the body $g m n$ receives and resists not only the normal component $o a$, but also (by means of the stop s) the *tangential* component $o b$; and the entire force P, or $o c$, is thus *imparted* to the body $g m n$, pressing it in the direction $o c$.

Composition and Resolution of Concurrent Forces by Means of Co-ordinates.

66. In Fig. 35 (*a*) let the three coplanar forces E, F and G act through the point x. Draw two lines, H H, and V V, Fig. 35 (*b*), crossing each other at right angles, as at o.* These lines are called rectangular co-ordinates. From o, draw lines E o, F o, G o, parallel to E x, F x, G x, Fig. 35 (*a*), and equal respectively to the forces E, F, and G by any convenient scale. Resolve each of these forces, Fig. 35 (*b*), into two components, parallel to H H and V V respectively. Thus, E o is resolved into $t o$ and $n o$, F o into $u o$ and $e o$, G o into $i o$ and $m o$. Then, summing up the resolutes, we have:

Sum of horizontal resolutes $= u o - i o - t o = - s o$, and

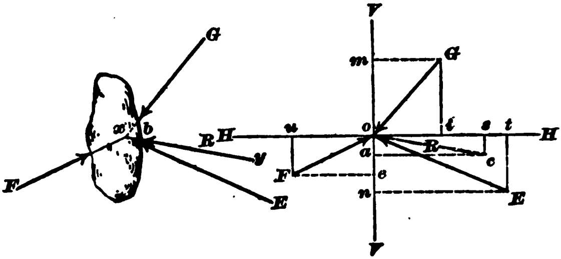
Sum of vertical resolutes $= n o + e o - m o = a o$;

---

*It is only for convenience that the co-ordinates are usually drawn (as in Fig. 35) at right angles. They may be drawn at any other angle (see Fig. 36); but, in any case, the forces must of course be resolved into components *parallel to the co-ordinates*, whatever the directions of those co-ordinates may be.

and — *s o* and *a o* are the resolutes of the resultant, R, of the three forces E, F and G.

67. When a system of (concurrent) forces is in equilibrium, the algebraic* sum of the components of all the forces, along either of the two co-ordinates, is zero. Thus, in Fig. 35 (*b*) or 36, if the sense of R be such that it shall act as the anti-resultant of the other three forces E, F and G, its component, *o s* or *o a*, along either co-ordinate, will be found to balance those of the other forces along the same co-ordinate.



(*a*) (*b*)

Fig. 35.

Hence we have the very important proposition that: When a system of concurrent coplanar forces is in equilibrium, the algebraic sums of their components, in any two directions, are each equal to zero.

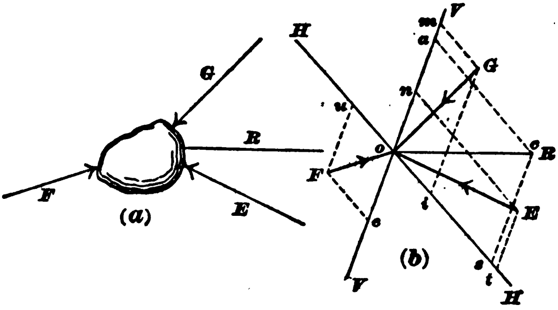


Fig. 36.

68. Conversely, in a system of concurrent forces, if the algebraic sums of the components in any two directions are each equal to zero, the forces are in equilibrium.

If the sum of the components in one of any two directions is not equal to zero, the forces cannot be in equilibrium. Thus, in Fig. 35 (*b*) or 36 (*b*), the sum of the components, along either one (as VV) of the two co-ordinates, may be zero; and yet, if the sum of those along the other co-ordinate is not zero, their resultant, or algebraic sum, will move the body, on which they act, in the direction of that resultant.

---

*The components being taken as + or ÷, according to the sense of each.

69. With vertical and horizontal co-ordinates, the condition of equilibrium[*] becomes:

The sum of the horizontal resolutes must be equal to zero;
The sum of the vertical resolutes must be equal to zero;

or, more briefly:

$$\Sigma \text{ horizontal resolutes} = 0$$
$$\Sigma \text{ vertical resolutes} = 0$$

Conversely, if these conditions are fulfilled, the forces are in equilibrium.

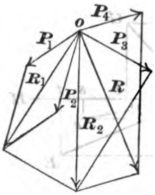


Fig. 37.

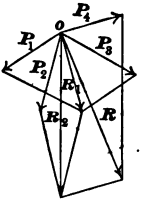


Fig. 38.

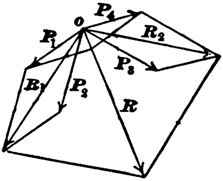


Fig. 39.

70. Resultant of More than Two Coplanar Forces. Where it is required to find the resultant of more than two concurrent and coplanar forces, as in Fig. 37, we may first find the resultant R_1 of any two of them, as of P_1 and P_2; then the resultant, R_2, of R_1 and a third force, as P_3; and so on, until we finally obtain the resultant R of all the forces. This resultant is evidently concurrent and coplanar with the given forces.

71. It is quite immaterial **in what order the forces are taken.** Thus, we may, as in Fig. 38, first combine P_1 and P_3; then their resultant R_1 with P_2, obtaining R_2; and, finally, R_2 with P_4, obtaining R; or, as in Fig. 39, we may first combine any two of the forces, as P_1 and P_2, obtaining their resultant R_1; then proceed to any other two forces, as P_3 and P_4, and obtain their resultant R_2; and finally combine the two resultants, R_1 and R_2, obtaining the resultant R.

The Force Polygon.

72. The Force Polygon. Comparing Figs. 37 and 38 with Figs. 40 and 41, respectively, we see that we may arrive at the same resultant R by simply drawing, as in Fig. 41, lines representing the several forces in any order, but following each other according to their senses. It will be noticed that this is merely an abbreviation of the process of drawing the several force parallelograms.

73. Resultant and Anti-resultant. The line,—R, required to complete the polygon, represents the *anti*-resultant of the other forces if its sense is such that it *follows* them around the polygon, as in Fig. 40. If its sense is *opposed* to theirs, as in Fig. 41, it is their *resultant*, R.

74. In other words, if any number of concurrent forces, as P_1, P_2, P_3, P_4 and R, Figs. 37 and 38,[†] are in equilibrium, the lines representing them, if drawn in any order, but so that their senses follow each other, will form a closed polygon, as in Fig. 40 (or in Fig. 41 if the sense of R be reversed).

75. Conversely, if the lines representing any system of concurrent coplanar forces, when drawn with their senses following each other, form a closed polygon, as in Fig. 40, those forces are in equilibrium.

---

[*] With non-concurrent forces, another condition must be satisfied. See ¶ 83.

[†] R is here regarded as tending *upward*, so as to form the *anti*-resultant of the other forces.

It will be noticed that the force triangle, and the straight line representing a system of colinear forces, Figs. 10 and 11, ¶¶ 20, etc., or a system of parallel forces, Figs. 55, etc., ¶¶ 111, etc., are merely special cases of the force polygon.

76. In a force polygon, Fig. 42, any one of the forces is the anti-resultant of all the rest. Any two or more of the forces balance all the rest; or, their resultant is the anti-resultant of all the rest.

If a line *a c* or *b d*, Fig. 42, be drawn, connecting any two corners of a force polygon, that line represents the resultant, or the anti-resultant (according as its arrow is drawn) of all the forces on either side of it. Thus:

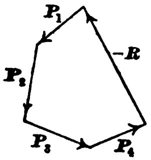


Fig. 40.

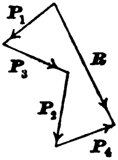


Fig. 41.

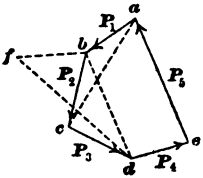


Fig. 42.

a c	is the resultant	of	P_1 P_2	and the anti-resultant		of	P_3 P_4 P_5
c a	"	"	"	P_3 P_4 P_5	"	"	"	P_1 P_2
b d	"	"	"	P_2 P_3	"	"	"	P_4 P_5 P_1
d b	"	"	"	P_4 P_5 P_1	"	"	"	P_2 P_3

77. Knowing the *directions* of *all* the forces of a system, as $P_1 \ldots\ldots P_5$, Fig. 42, and the *amounts* of *all but two* of them, as P_2 and P_3, we may find the amounts of those two by first drawing the others, P_4, P_5 and P_1, as in the figure. Then two lines *b c* and *c d*, drawn in the directions of the other two and closing the polygon, will necessarily give their amounts.

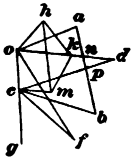


Fig. 43.

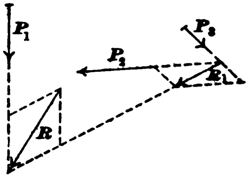


Fig. 44.

78. If any two points, as *o* and *c*, Fig 43, be taken, then the force or forces represented by any line or system of lines joining those two points will be equivalent to *o c*. Thus: *o c* = *o a b c* = *o d c* = *o n p c* = *o h k m c* = *o h m c* = *o f c* = *o g c*, etc., etc.

Similarly, in Fig. 42, the force polygon *a b c d e a* is equivalent to the force polygon *a b f d e a*, and to the force triangle, *a b c a*, each being = zero.

Non-concurrent Coplanar Forces.

79. Non-concurrent Coplanar Forces. Fig. 44. The process of finding the resultant of three or more coplanar but non-concurrent forces is the same as if they were concurrent. Thus, let P_1, P_2 and P_3 represent three such forces.* We may first find the resultant R_1 of any two of them, as P_2

---

*Any *two* coplanar non-parallel forces, as P_1 and P_2, or P_2 and P_3 are necessarily concurrent (see ¶ 19); but there is no single point in which the *three* forces meet.

and P_3; and then, by combining R_1 with the remaining force P_1, we find the resultant R of the three forces. Here the line R represents the resultant, not only in amount and in direction, but also in position. That is, the line of action of the resultant coincides with R.

80. The resultant R is the same, in amount and in direction, as if the forces were concurrent, and its position is the same as it would have been if their point of concurrence were in the line of R. If there are more than three forces, we proceed in the same way.

81. Conversely, the resultant R, or any other force, may be resolved into a system of any number of concurrent or nonconcurrent coplanar forces, in any directions, at pleasure. Thus, we may first resolve R into P_1 and R_1; then either of these into two other forces, as R_1 into P_2 and P_3, and so on.

82. If a system of non-concurrent coplanar forces is in equilibrium, the forces will still be in equilibrium if they are so placed as to be concurrent; provided, of course, that their directions, senses and amounts remain unchanged; but it does not follow that a system of forces, which is in equilibrium when concurrent, will remain in equilibrium when so placed as to be non-concurrent.

Thus, the five forces, P_1 . . . . . P_5, Fig. 45 (*a*), may be so placed, as in Fig. 45 (*b*), that the resultant *a c*, of P_1 and P_2, does not coincide with the resultant *c a* of P_3, P_4 and P_5, but is parallel to it. These two resultants then form a couple. (See ¶¶ 155, etc.)

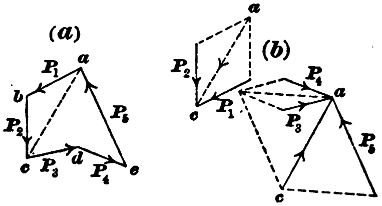


Fig. 45.

83. Third Condition of Equilibrium. Hence, the conditions of equilibrium for concurrent forces, stated in ¶ 69,

$$\Sigma \text{ vertical components} = 0$$
$$\Sigma \text{ horizontal components} = 0$$

do not suffice for non-concurrent forces, and a third condition must be added, viz.:—

$$\Sigma \text{ moments} = 0;$$

i. e., the moments of the forces, taken about any point, must be in equilibrium.

A system of forces in equilibrium has no resultant; hence it has no moment about any point. In other words, the moments of the forces, as well as the forces themselves, are in equilibrium.

84. The resultant of a system of unbalanced non-concurrent forces, acting upon a body, may be either

(1) a single force, acting through the center of gravity of the body; or

(2) a couple; *i. e.*, two equal and parallel forces of opposite sense (see ¶¶ 155, etc.); or

(3) either (a) a single force, acting through the center of gravity of the body, and a couple; or (b) a single force, acting elsewhere than through the center of gravity of the body.

In Case (3), the two alternative resultants are interchangeable; *i. e.*, a single force, acting elsewhere than through the center of gravity of the body, may always be replaced by an equivalent combination consisting of an equal parallel force, acting through the center of gravity of the body, and a couple, and vice versa. See ¶¶ 161, etc.

The resultant gives to the body, in Case (1), motion of translation in a straight line, without rotation; in Case (2), rotation without translation; and in Case (3), both translation and rotation. See foot-note (*), ¶ 1.

85. The force polygon, ¶ 72, Figs. 40, etc., and the method by coordinates, ¶ 66, Fig. 35, therefore, give us only the amount, direction and sense of the resultant of non-concurrent forces, and *not* its *position*. To find the position of the resultant of non-concurrent forces, we may have recourse to a figure, like Fig. 44, where the forces are represented in their actual positions, or to the cord polygon, ¶¶ 86, etc., Fig. 46.

The Cord Polygon.

86. In the force triangle any two of the three lines may be regarded as representing, by their directions, the positions of two members (two struts or two ties, or one strut and one tie) of indefinite length, resisting the third force; while their lengths give the amounts of the forces which those members must exert in order to maintain equilibrium.

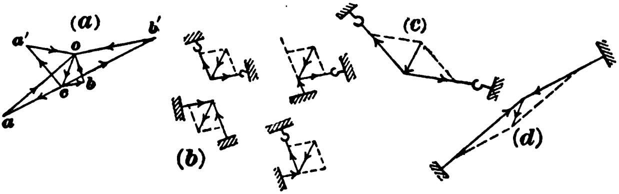


Fig. 26 (repeated).

87. Thus, in Fig. 26 (*b*), are shown four different systems, of two members each, inclined respectively like the forces *c b* and *b o* in Fig. 26 (*a*) and balancing the third force *o c*. The stresses in these two members are given by the lengths of the lines *c b* and *b o* in Fig. 26 (*a*).

The members acting as struts are represented, in Fig. 26 (*b*), as abutting against flat surfaces, while those acting as ties are represented as attached to hooks, against which they pull.

In Fig. 26 (*c*) and (*d*) are indicated systems of members, inclined like the forces *c a′* and *a′ o*, *c a* and *a o*, respectively, of Fig. 26 (*a*), by which the third force *o c* might be supported.

88. In the force polygon *a b c d e a*, Fig. 46 (*b*), representing the four forces, P_1, P_2, P_3, P_4, of Fig. 46 (*a*), if we select, at pleasure, any point *o* (called the **pole**) and draw from it a series of straight lines *o a*, *o b*, etc. (called **rays**), radiating to the ends, *a*, *b*, *c*, etc., of the lines P_1, P_2, etc., representing the forces, we shall form a series of force triangles, *a o b*, *b o c*, etc.

Thus, in the triangle *a b o* we have the force P_1, or *a b*, balanced by the two forces *o a* and *b o*; in the triangle *b c o*, the force P_2, or *b c*, balanced by the two forces *o b* and *c o*; and so on.

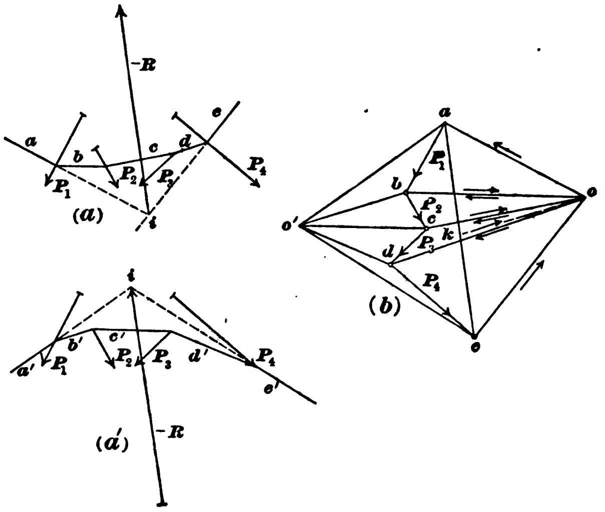
89. The Cord Polygon. If, now, in Fig. 46 (*a*), we draw the lines *a* and *b*, parallel respectively to the rays *o a* and *o b* of Fig. 46 (*b*) and meeting in the line representing the force P_1, they will represent the positions of two tension members of indefinite length, which will balance the force P_1 by exerting forces represented, in amount as well as in direction, by the rays *o a* and *b o*, Fig. 46 (*b*). Again, taking pole *o′*, Fig. 46 (*b*), instead of *o*, we have *a′* and *b′*, Fig. 46 (*a′*), parallel respectively to the rays, *o′ a* and *o′ b*, and representing a pair of struts performing the same duty.

90. Similarly, the lines *b* and *c*, Fig. 46 (*a*), parallel respectively to rays *o b* and *o c*, represent two tension members, which, with stresses equal respectively to *o b* and *c o*, Fig. 46 (*b*), balance the force P_2.

91. We thus obtain, finally, a system of five tension members, *a b c d e*, Fig. 46 (*a*), which, if properly fastened at the ends *a* and *e* respectively, will, by exerting forces represented respectively by the rays, *o a*, *o b*, *o c*, etc., Fig. 46 (*b*), balance the four given forces P_1, P_2, P_3 and P_4.

92. The figure *a b c d e*, Fig. 46 (*a*), is called a cord polygon, funicular polygon, or equilibrium polygon.

93. Resultant, Anti-resultant. Amount and Direction. In the force polygon, Fig. 46 (*b*) or (*d*), the line *e a*, joining the end of the last force-line *d e* with the beginning of the first one *a b*, represents the anti-resultant of the given system of four forces, and *a e* their resultant. Evidently, therefore, the rays *a o* and *o e*, which represent two components of *a e*, represent also, in direction and in amount, two forces which would balance *e a*, or which would be equivalent to the given system of (four) forces.



Figs. 46 (*a*), (*a'*) and (*b*).

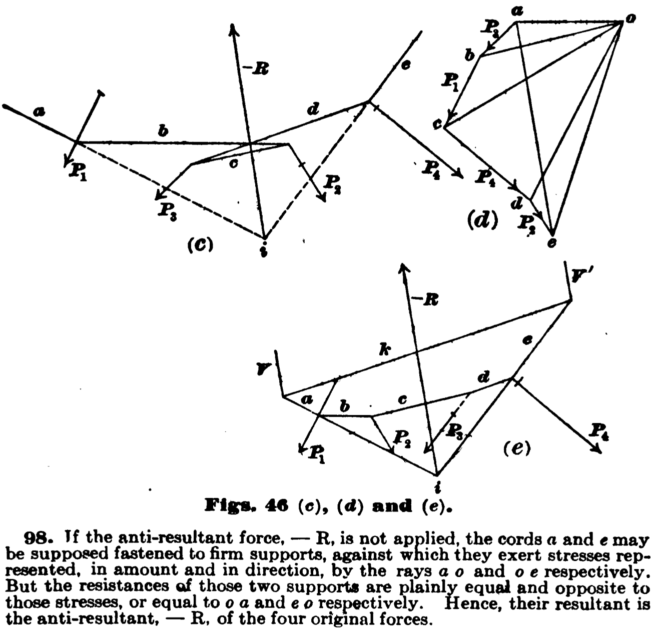
94. Position of Resultant. Hence, in the cord polygon, Fig. 46 (*a*), the intersection, *i*, of the cords *a* and *e*, parallel respectively to the rays *o a* and *e o*, is a point in the line of action of the resultant R; and, if we imagine *a i* and *e i* to be rigid rods, and apply, at *i*, a force, — R, equal and parallel to *a e*, but of opposite sense, that force will be the anti-resultant of the (four) given forces, and we shall have a frame-work *b c d i* of cords and rods, kept in equilibrium by the action of the five forces, P_1, P_2, P_3, P_4 and — R.

95. By choosing other positions of the pole, as *o'*, Fig. 46 (*b*), or by differently arranging the given forces, as in Fig. 46 (*c*), we merely change the shape of the cord polygon, and (in some cases) reverse the sense of the stresses in the members. Thus, in Fig. 46 (*a*), all the stresses are tensions, or pulls; while in Fig. 46 (*c*) *a*, *b*, *d* and *e* are tensions or pulls, and *c* is a compression or push.

96. In constructing the cord polygon, Fig. 46 (*a*), (*a'*), (*c*), and (*e*), care must be taken to draw the cords in their proper places; and for this it is necessary to remember, simply, that the two rays pertaining to any particular force line in the force polygon, Fig. 46 (*b*), represent those members which, in the cord polygon, Fig. 46 (*a*), take the components of that force.

Thus, *o a* and *b o*, Fig. 46 (*b*), pertain to the force P_1: *o b* and *c o* to the force P_2. Hence, in Fig. 46 (*a*) or (*c*) we draw *a* and *b* (parallel respectively to *o a* and *b o*) meeting in the line of action of P_1; *b* and *c* (parallel respectively to *o b* and *c o*) meeting in the line of action of P_2, etc., etc.

97. Each ray in the force polygon, Fig. 46 (*b*), including the outside ones, is thus seen to pertain to two forces, and each force has two rays. The two cords, parallel respectively to the two rays of any force, must be drawn to meet in the line of action of that force; and each cord must join the lines of action of the two forces to which its parallel ray pertains. The lines, *a*, *b*, *c*, etc., in the *cord* polygon, Fig. 46 (*a*) and (*c*), give merely the *inclinations* of members which, as there arranged, would sustain the given forces. The lengths of these lines have nothing to do with the amounts of the *stresses*. These are given by the lengths of the corresponding *rays* in the *force* polygon, Fig. 46 (*b*).



Figs. 46 (*c*), (*d*) **and** (*e*).

98. If the anti-resultant force, — R, is not applied, the cords *a* and *e* may be supposed fastened to firm supports, against which they exert stresses represented, in amount and in direction, by the rays *a o* and *o e* respectively. But the resistances of those two supports are plainly equal and opposite to those stresses, or equal to *o a* and *e o* respectively. Hence, their resultant is the anti-resultant, — R, of the four original forces.

99. If, Fig. 46 (*e*), the two end members *a* and *e* were attached merely to two ties, V and V′, parallel to the anti-resultant, —R, they would evidently draw the ends of those ties inward toward each other. To prevent this, let the strut *k* be inserted, making it of such length that the ties V and V′ may remain parallel to — R, and draw *o k*, Fig. 46 (*b*), parallel to *k*. Then *a k* and *k e* give the stresses in V and V′ respectively.

100. If the anti-resultant, — R, found by means of the force polygon, be applied in a line passing through the intersection of the outer (initial and final) members in the cord polygon, all the forces, including of course the anti-resultant, will be in equilibrium. In other words, coplanar forces are in equilibrium if they may be so drawn as to form a closed force polygon, and if a closed cord polygon may be drawn between them. But if the anti-resultant be applied elsewhere, we shall have a couple, composed of the anti-resultant, — R, and the resultant R of the forces.

Concurrent Non-coplanar Forces.

101. Any two of the concurrent forces, as *o a* and *o c*, Fig. 47 (*a*) or (*b*), are necessarily coplanar. Find their resultant, *o r*, which must be coplanar with them and with a third force *o b*. Then the resultant, R, of *o r* and *o b* is the resultant of the three forces. If there are other forces, proceed in the same way.

102. No three non-coplanar forces, whether concurrent or not, can be in equilibrium.

103. Force Parallelopiped. The resultant of any three concurrent non-coplanar forces, *o a*, *o b*, *o c*, Figs. 47, will be represented by the diagonal *o* R, of a parallelopiped, of which three converging edges represent the three forces.

104. Methods by Models. **(a) For three forces.** Construct a box, Fig. 47 (*a*) or (*b*), with three convergent edges representing the three forces in position and amount. Then a string *o* R, joining the proper corners, will represent the resultant.

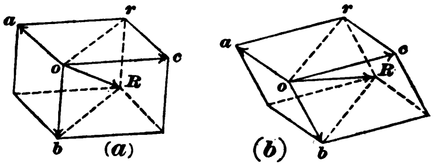


Fig. 47.

Or, let *a o*, *b o*, *c o*, Fig. 48 (*a*), be three forces, meeting at *o*. Draw on pasteboard the three forces *a o*, *b o*, *c o*, as in Fig. 48 (*b*), with their *actual* angles *a o b*, *b o c*, *c o a*, and find the resultant *w o* of the middle pair, *b o* and *c o*. Cut out neatly the whole figure, *a o a c w b a*. Make deep knife-scratches along *o b*, *o c*, so that the two outer triangles may be more readily turned at angles to the middle one. Turn them until the two edges *o a*, *o a* meet, and then paste a piece of thin paper along the meeting joint to keep them in place. Stand the model upon its side *o b w c* as a base, and we shall have the *slipper* shape *a o b w*, Fig. 48 (*c*); *o w* being the sole, and *a o b* the hollow foot. In the model, the force *a o* and the resultant *w o* of the other two forces, are now in their actual relative positions. To find their resultant, cut out a separate piece of pasteboard, R *a o w*, with R *a* and R *w* parallel respectively to *w o* and *a o*. Draw upon each side of it the diagonal R *o*. Paste this piece inside the model, with its lower edge *w o* on the line *w o*, Fig. 48 (*b*), and its edge *a o* in the corner *a o*. This done, R *o* represents the resultant of *a o*, *b o*, *c o*, Fig. 48 (*a*), in its actual position relative to them.

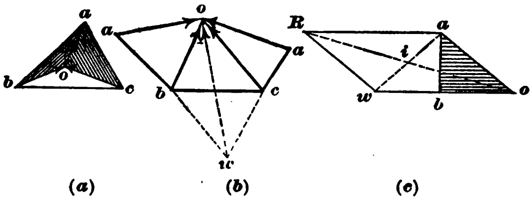
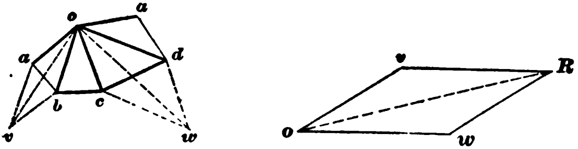


Fig. 48.

105. **(b) For four forces,** as *a o*, *b o*, *c o*, *d o*, in Fig. 49. Draw them as in Fig. 49 (*a*), with their angles *a o b*, *b o c*, etc. Draw also the resultants *v o*, of *a o* and *b o*; and *w o*, of *c o* and *d o*. Then cut out the entire figure, as before, and paste together the two edges *a o*, *a o*. Hold the model in such a way that two of its *planes* (as *a o b* and *b o c*) form the same angle with each other as do the two corresponding planes between the forces. Then we have the two resultants *v o*, *w o*, Fig. 49 (*b*), in their *actual relative positions*. Cut out a separate piece of pasteboard R *v o w*, Fig. 49 (*b*), draw the diagonal R *o* on each side of it, and paste it inside the model, with *o v* and *o w* on the corresponding lines of the model. . Then R *o* will represent the resultant of the four forces, *a o*, *b o*, *c o*, *d o*, in its actual position relative to them.

The model may be made of wood, the triangles *a o b*, *b o c*, etc., being cut out separately, the joining edges bevelled, and then glued together.

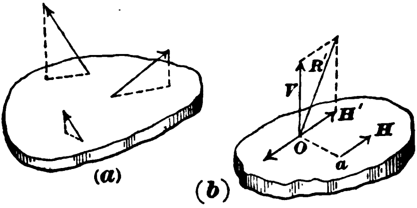


(*a*) **Fig. 49.** (*b*)

Non-concurrent Non-coplanar Forces.

106. Non-concurrent Non-coplanar Forces. Fig. 50 (*a*). (For parallel non-coplanar forces, see ¶¶ 110, etc.) Resolve each force into two rectangular components, one normal to an assumed plane, the other coinciding with the plane.* Find the resultant of the (coplanar) components coinciding with the plane, by methods already given, and that of the normal (parallel) components, by ¶¶ 110, etc. If these two resultants are coplanar, they are also concurrent, and their resultant (which is the resultant of the system) is readily found.

107. If not, let V, Fig. 50 (*b*), be the resultant normal to the plane, and H the resultant lying in the plane. By ¶ 162, substitute, for H, the equal and parallel force H′, meeting V at O, and the couple H . O *a*, and find the resultant, R′, of V and H′. The system of forces is thus reduced to the single force R′ and the couple H . O *a*. For Couples, see ¶ 155.



(*a*) (*b*)

Fig. 50.

108. Moments of Non-coplanar Forces. The action of the weight W of the wall, Fig. 51 (*a*), and of the non-coplanar forces P_1 and P_2, may be represented as in Fig. 51 (*b*), where the axle *a′ c′* represents the edge *a c* about which the wall tends to turn, while the bars or levers represent the leverages of the forces. So far as regards the overturning stability of the wall, regarded as a rigid body and as capable of turning only about the edge *a c*, it is immaterial whether an extraneous force, as P_1, is applied at *p* or at *q*; but it is plainly not immaterial as regards a tendency to swing the wall around horizontally, or to fracture it; or as regards pressures (and consequent friction) between the axle *a′ c′* and its bearings. For equilibrium, $P_1 m = P_2 h + W. \frac{b}{2}$. Here a torsional or twisting stress is exerted in the axle, and the pressures of its ends in the bearings are more or less modified; but, so far as merely the equilibrium of the moments is concerned, we may suppose all of the forces and their moments to be shifted into one and the same plane, as in Fig. 51 (*c*).

---

*Wires, stuck in a board representing the plane, will facilitate this.

109. In cases like that represented in Fig. 51, it is usual, for convenience, to restrict ourselves to a supposed vertical *slice*, *s*, 1 foot thick, and to the forces acting upon such slice; supposing the weight of the slice to be concentrated at its center of gravity, and the extraneous forces to be applied in the same vertical plane with gravity. In effect, we are then dealing with a slice indefinitely thin, but having the weight of the 1-ft. slice.

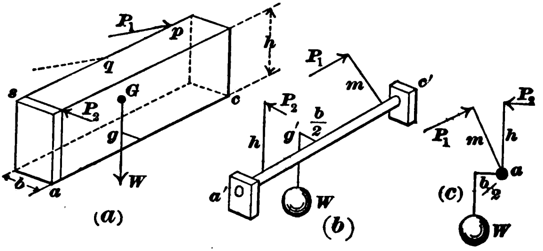


Fig. 51.

PARALLEL FORCES.

110. The resultant of any number of parallel forces, whether they are in the same plane or not, and whether in the same direction or not, is parallel to them and = their algebraic sum.

Coplanar Parallel Forces.

111. The resultant of any number of coplanar parallel forces is in the same plane with them, whether the forces are of the same or of opposite sense; and the leverages, or arms, of such forces, and of their resultant, about any given point in the same plane, are in one straight line. Thus, in Fig. 56 (*a*), where the five forces, *a*, *b*, *c*, *d* and *e* are in one plane, their resultant, R, is in that same plane; and the leverages of the forces, and of R, about any point, as *b* or *v*, in the same plane, are in the straight line R *v*.

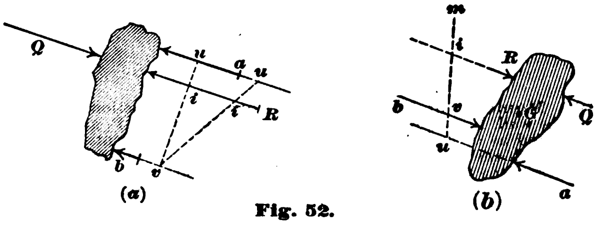


Fig. 52.

112. The resultant, R, or anti-resultant, Q, Fig. 52, **of two parallel forces,** *a* and *b*, intersects any straight line, *u v*, joining the directions of the two forces. Hence, if three parallel forces are in equilibrium, they are in the same plane. In Fig. 52 (*a*), the two forces, *a* and *b*, are of like sense. R is then between *a* and *b*, and R = *b* + *a*. In Fig. 52 (*b*), *a* and *b* are of opposite sense. R is then not between *a* and *b*, and R = *b* — *a*.

113. To find the position of the resultant, draw and measure any straight line, *u v*, joining the lines of action of the forces. It is immaterial whether *u v* is perpendicular to said directions, or not. The line representing the resultant cuts *u v*, and its position is found thus:

$$u\,i = u\,v \times \frac{b}{R}; \quad \text{and } v\,i = u\,v \times \frac{a}{R}.$$

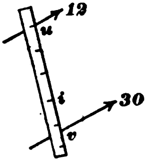


Fig. 53.

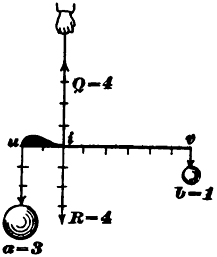
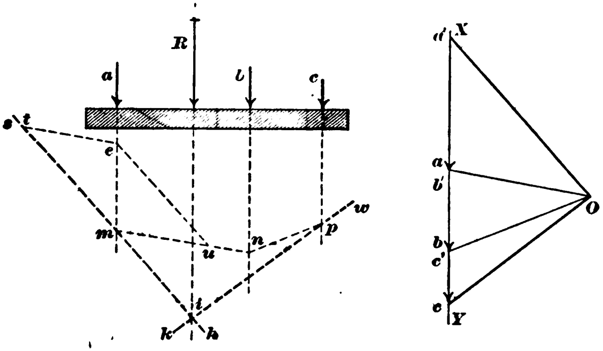


Fig. 54.

114. This may be conveniently done by making *u v* equal, by any convenient scale, to the sum of the forces, as in Fig. 53, where $u\,v = 42$. Then make *u i equal*, by the same scale, to the force at *v*, or *v i* equal to the force at *u*. Then a line, R, Fig. 52 (*a*), drawn through *i* parallel to *a* and *b*, gives the position and direction of their resultant; and its *amount* is equal to the *sum* of *a* and *b;* or $R = a + b$. In other words, if a force, Q, parallel to *a* and *b*, and equal to their sum, but of opposite sense, be applied to the body anywhere in a line passing through *i*, it will balance *a* and *b*, or will be their anti-resultant.



(*a*) Fig. 55. (*b*)

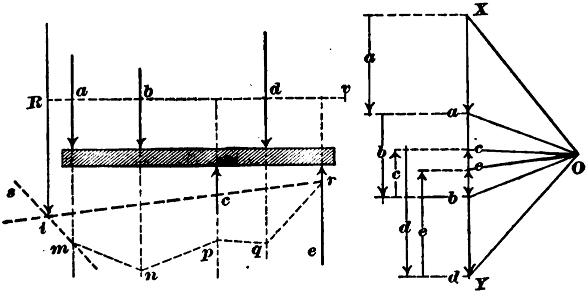
115. The position of the resultant, so found, satisfies the condition of equilibrium of moments: thus, $b.vi - a.ui = \text{zero}$.

If the two forces are equal, their resultant R is evidently midway between them.

116. In the common steelyard, Fig. 54, the two forces *a* and *b*, of Fig. 52 (*a*), are represented by the two weights, $a = 3$ pounds at *u*, and $b = 1$ pound at *v*, with leverages *ui* and *vi* respectively, as 2 : 6, or as 1 : 3.

It will be noticed that in Fig. 56 (*a*) the resultant, R, owing to the positions and amounts of the several forces, falls outside of the system of given forces.

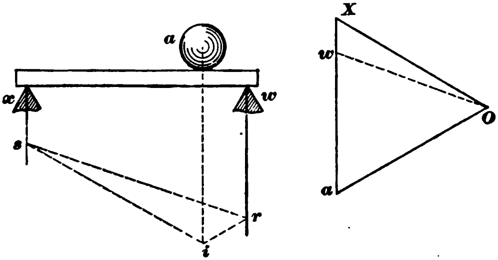
117. Figs. 55 to 58 illustrate the application of the cord polygon (¶¶ 86 to 100) to coplanar parallel forces. Here the force polygon is necessarily a straight line.



(*a*) (*b*)

Fig. 56.

118. Resolution. Let Fig. 57 (*a*) represent **a beam bearing a single concentrated load,** *a*, elsewhere than at its center; and let it be required to find the pressure on each of the two supports, *w* and *x*.

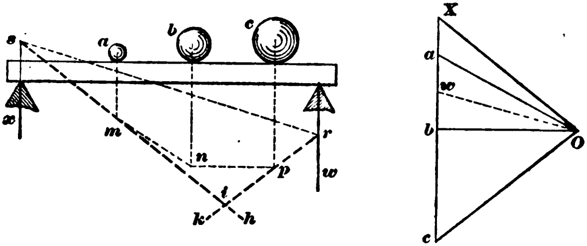


(*a*) (*b*)

Fig. 57.

Draw X *a*, Fig. 57 (*b*), to represent the load *a* by scale, and rays X O, *a* O, to any point O not in the line X *a*. In Fig. (*a*), from any point, *i*, in the vertical through the point, *a*, where the load is applied, draw *i s* and *i r*, parallel respectively to O X and O *a*. Join *r s*, and in Fig. (*b*) draw O *w* parallel to *r s*. Then the two segments, *w a* and X *w*, of X *a*, give by scale the pressures upon the two supports, *w* and *x* respectively. The greater pressure will of course be upon the support nearest to the load; but we may be guided also by remembering that the segment X *w*, adjoining the radial line O X in Fig. (*b*) represents the pressure on that support, *x*, Fig. (*a*), which pertains to the line *i s* parallel to O X; and vice versa.

119. Fig. 58 represents a case where there are several loads on the beam. Here the intersection, *i*, of the lines *h s* and *k r*, Fig. (*a*), drawn parallel respectively to O X and *c* O, Fig. (*b*) shows the position of the resultant of the three loads. Here, as in Fig. 57, we join *r s*, Fig. (*a*), and draw O *w*, Fig. (*b*), parallel to *r s*. Then X *w*, Fig. (*b*), gives the pressure upon *x*, and *w c* that upon *w*.



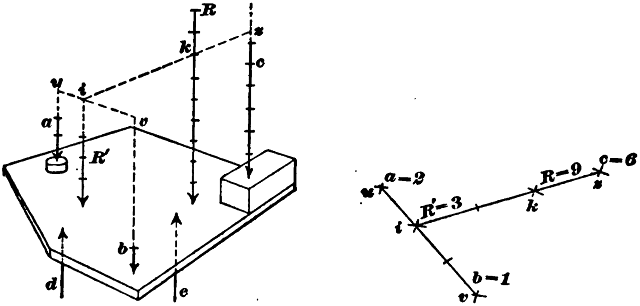
(*a*) **Fig. 58.** (*b*)

Non-coplanar Parallel Forces.

120. Non-coplanar Parallel Forces. Fig. 59 (*a*). Between the lines of action of any two of the forces, as *a* and *b*, draw any straight line, *u v*, and make

$$ui = uv \times \frac{b}{a+b}; \quad \text{or} \quad vi = uv \times \frac{a}{a+b}.$$

Through *i* draw R′, parallel to *a* and *b*, and equal to their sum. Then is R′ the resultant of *a* and *b*. Then, from any point, *i*, in the line of action of R′, draw *i z* to any point, *z*, in the line of action of *c*, and make $ik = iz \times \frac{c}{c+R'}$; or $zk = iz \times \frac{R'}{c+R'}$. Through *k* draw R parallel to *a*, *b* and *c*, and equal by scale to their sum. Then is R the resultant of the three forces, *a*, *b* and *c*. If there are other forces, proceed in the same way with them.



(*a*) **Fig. 59.** (*b*)

121. In Fig. 59 (*a*) we have shown the forces, *a* and *c*, acting upon surfaces raised above the general plane, merely in order to illustrate the fact that it is not at all necessary that the forces be supposed to act upon or against a plane surface.

122. Although Fig. 59 (*a*) illustrates the method of finding the resultant of non-coplanar parallel forces, yet it plainly does not give the actual relative positions of the forces and their resultant; because it is necessarily drawn in a kind of perspective, and therefore all the parts cannot be measured by a scale. The true relative positions may of course be represented in plan, as by the five stars, *a*, *b*, *c*, *i* and *k*, Fig. 59 (*b*), corresponding to the points where the forces and resultants intersect some one chosen plane. But it is now impossible to represent the forces themselves by lines. They must therefore be stated in figures, as is here done. It is then easy to find the positions of the resultants, as before.

123. If there are also **forces acting in the opposite direction,** as *d* and *e*, Fig. 59 (*a*), find their resultant separately. We thus obtain, finally, two resultants of opposite sense. These resultants may be equal or unequal, and colinear or non-colinear. If they are non-colinear, see ¶ 84, and Couples, ¶¶ 155, etc.

124. Method by projections, Fig. 60. First find the projections, a', b' and c' of the forces, a, b and c, upon any plane, as $x\ y$, parallel to them; and then their projections, a'', b'', and c'', upon a second plane, $x\ v$, parallel to them and normal to the first. Find the position, R′, of the resultant of a', b' and c', in plane $x\ y$, and that R″, of a'', b'' and c'', in plane $x\ v$. Now, as the lines, a', b', c', and a'', b'', c'', are projections of the forces, a, b and c, so R′, R″, are projections of the resultant, R, of the forces. The position of R is therefore at the intersection of two planes, R R′ and R R″, perpendicular to the planes; $x\ y$ and $x\ v$, and standing upon the projections R′ and R″, of the resultant, R. $R = a + b + c$.

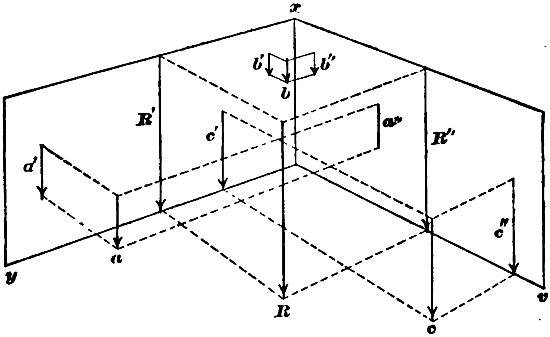


Fig. 60.

CENTER OF GRAVITY.

125. If a body, Fig. 1,* or a system of bodies, Fig. 2, be held successively in different positions, (*a*), *b*), etc., the resultant of the parallel forces of gravity, acting upon its particles and indicated by the arrows in the figures, will occupy different positions, relatively to the figure of the body or system. That point, where all these positions, or lines of gravity, meet, is called the center of gravity of the body or system. Thus, if a homogeneous cylinder be stood vertically upon either end, the line of gravity will coincide with the axis of the cylinder; but if the cylinder be then laid upon its side, the line of gravity will intersect the axis at right angles and will bisect it Hence, in the cylinder, the center of gravity is at the center of the axis.

126. About the center of gravity the moments of all the forces of gravity are in equilibrium, in whatever position the body or system may be. Hence, the body, or system, if suspended by this point, and acted upon by gravity alone, will balance itself; *i. e.*, if at rest it will remain at rest; or, if set in motion revolving about its center of gravity, and then left to itself, it will continue to revolve about that center indefinitely and with uniform angular velocity. Or, if suspended freely from any point, it will oscillate until the center of gravity comes to rest vertically under such point.

---

* Figs. 1 to 45, relating to Center of Gravity, are numbered independently of the rest of the series of figures relating to Statics.

127. In some bodies, such as the cube, or other parallelopiped, the sphere, etc., the center of *gravity* is also the center of the *weight* of the body; but very frequently this is not the case. Thus, in a body *a b*, Fig. 2, with its center of gravity at G, there is more weight on the side *a* G, than on the side G *b*.

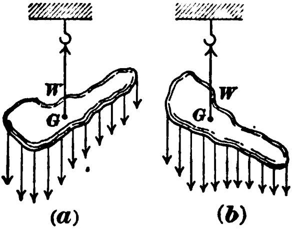


Fig. 1.

Stable, Unstable, and Indifferent Equilibrium.

128. A body is said to be in stable equilibrium when, as in the pendulum, it is so suspended that, if swung a little to either side, it tends to oscillate until it comes to rest again, with its center of gravity vertically under the point of suspension.

129. It is said to be in unstable equilibrium when, as in the case of an egg, stood upon its point, it is so supported that, if swung a little to either side, and left to itself, it swings farther out from the vertical and eventually falls.

130. It is said to be in indifferent equilibrium when, as in the case of a grindstone, supported by its horizontal axis, or of a sphere resting upon a horizontal table, it is so suspended or supported that, if made to rotate about its center of gravity and then left to itself, it will continue in that state of rest or of angular motion in which it is left.

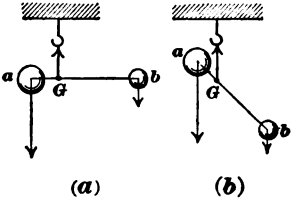


Fig. 2.

General Rules.

131. The following general rules (1) to (6), form the basis of the special rules, (7) to (39).

In speaking of the center of gravity of one or more **bodies,** we shall assume, for simplicity, that they are *homogeneous* (*i. e.*, of uniform density throughout) and of the same density with each other. The center of *gravity* is then the same as the center of *volume*, and we may use the *volumes* of the bodies (as in *cubic feet*, etc.) in the rules, instead of their *weights* (as in *pounds*, etc.).

In applying these general rules to **surfaces,** use the *areas* of the surfaces, and in applying them to **lines,** use the *lengths* of the lines, in place of the weights or volumes of the bodies.

In all of the rules and figures, pp. 388 to 398, **G represents** the center of gravity, except where otherwise stated.

(1). **Any two bodies,** Fig. 3. Having found the center of gravity, g, g', of each body, by means of the rules given below: then **G** is in the line joining g and g'; and

$$g\,G = g\,g' \times \frac{\text{weight of } g'}{\text{sum of weights of } g \text{ and } g'}$$

$$g'\,G = g\,g' \times \frac{\text{weight of } g}{\text{sum of weights of } g \text{ and } g'}$$

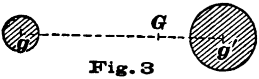


Fig. 3

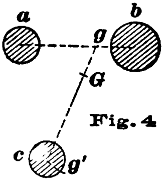


Fig. 4

(2). **Any number of bodies,** as a, b and c, Fig. 4, whether their centers of gravity are in the same plane or not.

First, by means of rule (1) find the center of gravity, g, of any two of the bodies, as a and b. Then the center of gravity, **G**, of the three bodies, a, b and c, is in the line $g\,g'$ joining g with the center of gravity, g' of c; and

$$g\,G = g\,g' \times \frac{\text{weight of } c}{\text{sum of weights of } a,\, b \text{ and } c};$$

$$g'\,G = g\,g' \times \frac{\text{sum of weights of } a \text{ and } b}{\text{sum of weights of } a,\, b \text{ and } c};$$

and so on, if there are other bodies.

(3). In many cases, a **single complex body** may be supposed to be divided into parts whose several centers of gravity can be readily found. Then the center of gravity of the whole may be found by the foregoing and following rules. Thus, in Fig. 5, we may find separately the centers of gravity of the two parallelopipeds and of the cylinder between them (each in the center of its respective portion of the whole solid); and in Fig. 6 the centers of gravity of the square prism and the square pyramid (the latter by rule (36), and then, knowing in either case the weights of the several parts, find their common center of gravity as directed in rules (1) and (2).

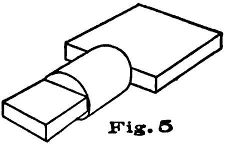


Fig. 5

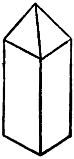


Fig. 6

(4). **Any hollow body,** or body containing one or more openings, Fig. 7 Find the common center of gravity, g', of the openings by rule (1) or (2), and

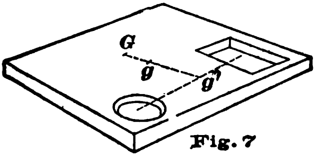


Fig. 7

the center of gravity, g, of the entire figure, as though it had no openings Then G is in the line $g\,g'$, extended, and

$$g\,G = g\,g' \times \frac{\text{sum of volumes of openings}}{\text{volume of entire body} - \text{volumes of openings}}$$

$$g'\,G = g\,g' \times \frac{\text{volume of entire body}}{\text{volume of entire body} - \text{volumes of openings}}$$

REMARK. For convenience, we have shown the several centers of gravity, g, g', G, upon the *surface* of the figure. In the real solid (supposed to be of uniform thickness) they would of course be in the middle of its thickness and immediately under the positions shown in the figure.

(5). In any line, figure or body, or in any system of lines, figures or bodies, any plane passing through the center of gravity is called a "**plane of gravity**" for said line, etc., or system of lines, etc. The intersection of two such planes of gravity is called a "**line of gravity.**" The center of gravity is (1st) the intersection of two lines of gravity; (2nd) the intersection of three planes of gravity, or (3rd) the intersection of a plane of gravity with a line of gravity not lying in said plane.

If a figure or body has an axis or plane of **symmetry** (*i. e.*, a line or plane dividing it into two equal and similar portions) said axis or plane is a line or plane of gravity. If a figure or body has a central point, said point is the center of gravity.

In Fig. 1, the string represents a line of gravity; and any plane with which the string coincides is a plane of gravity. Thus G may often be conveniently found, especially in the case of a flat body, by allowing it to hang freely from a string attached alternately at different corners of it, or by balancing it in two or more positions over a knife-edge, etc., and finding G in either case by the intersection of the lines or planes of gravity thus found.

(6). **The graphic method** of finding the resultant of parallel forces may often be advantageously used for finding the center of gravity of a compound body or figure, or of a system of bodies or figures, when the centers of gravity of the several parts are known.

Thus, in Fig. 8, let a, b and c represent three figures or bodies whose centers of gravity are in one plane. Draw vertical lines through said centers, and construct the polygon of forces, $x\,a\,b\,c$, Fig. 9, making the lines $x\,a$, $a\,b$, etc., proportional to the weights of a, b and c; and from any convenient point O draw radial lines O x, O a, etc. In Fig. 8, draw $m\,h$, $m\,n$, $n\,p$, and $p\,k$, parallel respectively to O x, O a, O b, O c. Then a vertical line, i G, drawn through the intersection, i, of $m\,h$ and $p\,k$, is a line of gravity of the system or figure. If the body or figure is *symmetrical*, as in the cross section of a T rail, I beam or deck beam, etc., the axis of symmetry, dividing the figure, etc. into two similar and equal parts, is also a line of gravity, and its intersection with the line i G already found is the required center of gravity G. In such cases it is generally most convenient to draw the lines through the several centers of gravity *perpendicular* to the axis of symmetry, so that the line of gravity found will also be perpendicular to it.

But if, as in Fig. 8, the body or figure, etc., is not symmetrical, we must find a second line of gravity, the intersection of which with the first will give the center of gravity, G. To do this, repeat the process, drawing another set of parallel lines through the several centers of gravity, Fig. 8. It will be most convenient to draw them horizontally, or *at right angles* to those already drawn, and in the following instructions we suppose this to be done.

Then draw a second funicular polygon, $m' n' p' i'$, Fig. 8, making the lines, $m' n'$, etc., *perpendicular* (instead of parallel) to the radial lines O x, etc., Fig. 9; and draw the second line of gravity, i' G, through i', perpendicular to the first. Then G is at the intersection of the two lines of gravity.

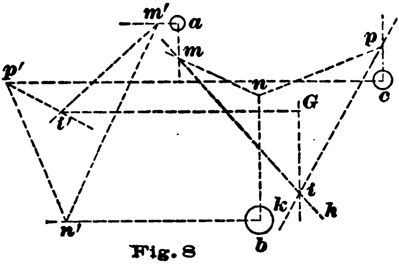


Fig. 8

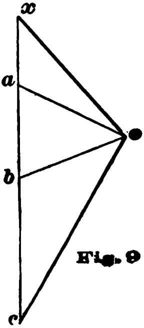


Fig. 9

The drawing of the second funicular polygon is often less simple than that of the first, because in the second the parallel lines through the several centers of gravity do not necessarily follow each other in the same order as in the first. Bear in mind that the two lines (as $n' p'$, $n' m'$) meeting in the parallel line (as $b n'$) pertaining to any given part, b, of the figure, must be perpendicular respectively to those radial lines (O a, O b) which meet the ends of the line, $a b$, that represents that same part.

Figs. 10 and 11 show the application of the same process to an irregular figure composed of three rectangles, a, b and c. The lettering is the same as in Figs. 8 and 9; but in Fig. 10 it happens that i' and p' of the second funicular polygon fall upon the same point.

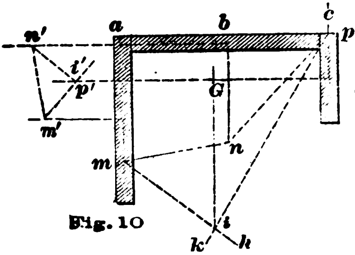


Fig. 10

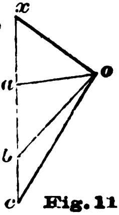


Fig. 11

If the centers of gravity of the several bodies, or of the several parts of the body, etc., are in more than one plane, we must find their projections upon certain planes, and apply the process to those projections.

Special Rules.

132. Special Rules, derived from the general rules, (1) to (6).

Lines.

(7). Straight line. G is in the line, and at the middle of its length.

(8). Circular arc,* aob, Figs. 12 and 13 (center of circle at c). G is in the line co joining the center of the circle with the middle of the arc, and

$$cG = \text{radius } ac \times \frac{\text{chord } ab}{\text{length of arc } aob}.$$

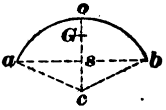


Fig. 12

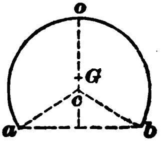


Fig. 13

(8a). If the arc is a semi-circle,*

$$cG = \text{radius } ac \times \frac{2}{\pi} = \text{radius } ac \times 0.6366.$$

(8b). Approximate rules for distance sG, Fig. 12, from *chord* to center of gravity.

If rise so = .01 chord ab; sG = .666 so	If rise so = .30 chord ab; sG = .653 so
" " " = .10 " "; " = .665 so	" " " = .35 " "; " = .649 so
" " " = .15 " "; " = .663 so	" " " = .40 " "; " = .645 so
" " " = .20 " "; " = .660 so	" " " = .45 " "; " = .641 so
" " " = .25 " "; " = .657 so	" " " = .50 " "; " = .637 so

(9). Triangle, abc, Fig. 14. The center of gravity, G, of its three sides* is the center of the circle inscribed by a triangle, def, whose corners are in the centers of the sides of the given triangle.

(10). Parallelogram (square, rectangle, rhombus or rhomboid). The center of gravity of the four sides* is at the intersection of the diagonals.

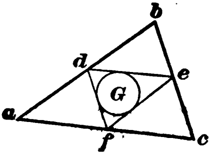


Fig. 14

(11). Circle, ellipse, or **regular polygon.** The center of gravity of the outline or circumference* is the center of the figure.

(12). Regular prism, right or oblique, and **right regular pyramid,** or **frustum.** The center of gravity of the edges* is the center of the axis. In the *prism*, the position of G is not affected by either including or excluding the sides of *both* of the polygons forming the ends.

(12a). Cycloid.* See p. 194.

Surfaces.

A. Plane surfaces.

We now treat of the centers of gravity of plane *surfaces*, which may be regarded as infinitely thin flat bodies. The rules for surfaces may be used also for actual flat bodies, in which, however, the center of gravity is in the middle of the thickness, immediately under the points found by the rules.

(13) Parallelogram (square, rectangle, rhombus or rhomboid), **circle, ellipse** or **regular polygon.** G is the center of the figure; or the intersection of any two diameters, or the middle of any diameter. In a **Parallelogram,** G is the intersection of the two diagonals.

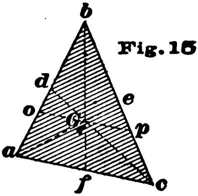
(14). Triangle, Fig. 15. G is at the intersection of lines (as ae and cd) drawn from any two angles, a and c, to the centers, e and d, of the sides, bc

---

* We are now treating of *lines* only; not of the surfaces bounded by the For surfaces, see rules (13), etc., etc.

and $a\,b$, respectively opposite to said angles. Such lines are called "medial lines."

$e\,G = \frac{1}{3}\,a\,e$; $d\,G = \frac{1}{3}\,c\,d$; $f\,G = \frac{1}{3}\,b\,f$ (f being the middle of $a\,c$).



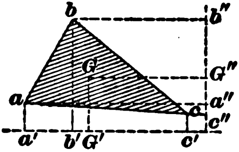


Fig. 16

(14a). Fig. 15. Or, on either one of the sides (as $a\,b$), meeting at any angle, a make $a\,o = \frac{1}{3}\,a\,b$. Draw $o\,p$ parallel to the other side, $a\,c$. Then $o\,G = \frac{1}{3}\,o\,p$, and G is at the intersection of $o\,p$ with any medial line, as $a\,e$, etc.

(14b). Fig. 16. If $a\,a'$, $b\,b'$, $c\,c'$ and G G′ are the distances of the three corners and of G from any straight line or plane $a''\,c'$; then

$$G\,G' = \frac{1}{3}\,(a\,a' + b\,b' + c\,c').$$

This gives us the position of the line of gravity G G″. In the same way we find the distance G G″ of G from any second line or plane, $b''\,c''$. This gives us the position of a second line of gravity G G′. G is at the intersecton of G G′ and G G″.

(14c). Fig. 17. The distance $G\,n$ of G in any direction from any side, as $a\,c$ (extended if necessary) is $= \frac{1}{3}$ the distance $n'\,b$ measured in a parallel direction from the same side to the opposite angle, b.

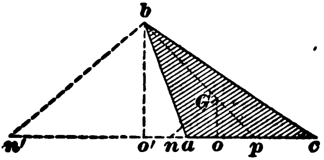


Fig. 17

It follows from this that the *shortest* distance, $G\,o$, of G from any side (as $a\,c$) is $= \frac{1}{3}$ the shortest distance, $o'\,b$, from the same side to its opposite angle b. It follows also that $p\,G = \frac{1}{3}\,p\,b$, as in Rule (14).

(15). Trapezium or trapezoid, Fig. 18. For trapezoids, see also Rule (16). Draw the two diagonals, $a\,c$ and $b\,d$. Divide either of them, as $a\,c$, into two equal parts, $a\,m$ and $c\,m$. From b, on $b\,d$, lay off $b\,n = d\,s$ (or from d lay off $d\,n = b\,s$). Join $m\,n$. G is in $m\,n$, and

$$m\,G = \frac{1}{3}\,m\,n.$$

(G is the center of gravity of the triangle $a\,c\,n$).

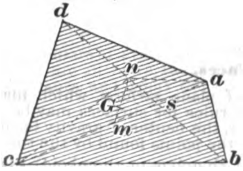


Fig. 18

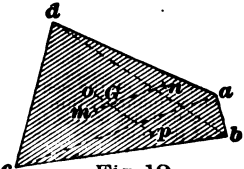


Fig. 19

(15a). Or, Fig. 19, find first the centers of gravity, m and n, of the two triangles, $c\,b\,d$ and $a\,b\,d$, into which the trapezium is divided by one of its diagonals, $b\,d$. Join $m\,n$. Then find the centers of gravity, o and p, of the two triangles, $d\,a\,c$ and $b\,a\,c$, into which the trapezium is divided by its *other* diagonal, $a\,c$. Join $o\,p$. Then G is the intersection of $m\,n$ and $o\,p$.

(16). **Trapezoid only,** Fig. 20. G is in the line *e f* joining the centers, *e* and *f*, of the two parallel sides, *a b* and *c d*. To find its position in said line, prolong either parallel side, as *a b*, in either direction, say toward *i*; and make *b i* equal to the opposite side, *c d*. Then prolong said opposite side, *c d*, in the opposite direction, making *d h* = *a b*. Join *h i*. Then G is the intersection of *h i* and *e f*. Or

$$f\,G = \frac{e f}{3} \times \frac{2\,a b + c d}{a b + c d}; \qquad \text{or} \quad o\,G = \frac{e n}{3} \times \frac{2\,a b + c d}{a b + c d}$$

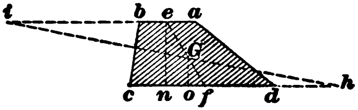


Fig. 20

(17). **Regular polygon.** G is the center of the figure.

(17 *a*). **Irregular polygon.** If the polygon be divided into any two portions, as by any diagonal, G must be in the line (of gravity) joining the centers of gravity of those two portions. If we again divide the whole polygon into two *other* parts by *another* diagonal, and join the centers of gravity of those two parts, G is the intersection of the two lines of gravity.

(17 *b*). Or we may divide the polygon into triangles, find the center of gravity of each triangle, by Rules (14), etc., and then find G by general Rule (1), (2) or (6).

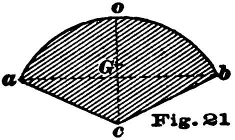


Fig. 21

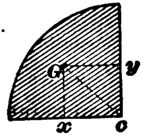


Fig. 22

(18). **Circular sector,** *a o b c*, Fig. 21. (Center of circle at *c*).

$$c\,G = \frac{2}{3}\,\text{radius } a c \times \frac{\text{chord } a b}{\text{arc } a o b} = \frac{\text{radius}^2 \times \text{chord}}{3 \times \text{area}}.$$

For length of arc, see p. 141.

(18 *a*). If the sector is a **sextant,**

$$c\,G = \text{radius} \times \frac{2}{\pi} = \text{radius} \times 0.6366.$$

(18 *b*). If the sector is a **quadrant,** Fig. 22,

$$c\,G = \frac{4}{3}\,\text{radius} \times \frac{\sqrt{2}}{\pi} = \text{radius} \times 0.6002.$$

$$c\,x = x\,G = \frac{4}{3}\,\text{radius} \times \frac{1}{\pi}.$$

(18 *c*). If the sector is a **semi-circle,**

$$c\,G = \frac{4}{3}\,\text{radius} \times \frac{1}{\pi} = \text{radius} \times 0.4244$$

$$= (\text{approximately})\ \text{radius} \times \frac{14}{33}.$$

(19). Circular segment, *a o b s*, Fig. 23. (Center of circle at *c*).

$$c\,G = \frac{\text{cube of chord } a\,b}{12 \times \text{area of segment}}.$$

(19*a*). If the segment is a **semi-circle,**

$$c\,G = \frac{4}{3}\,\text{radius} \times \frac{1}{\pi} = \text{radius} \times 0.4244$$

$$= \text{(approximately) radius} \times \frac{14}{33}.$$

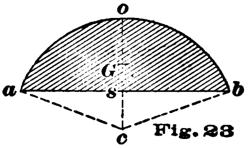


Fig. 23

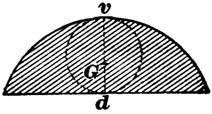


Fig. 24

(20). Cycloid, Fig. 24. (Vertex at *v*).

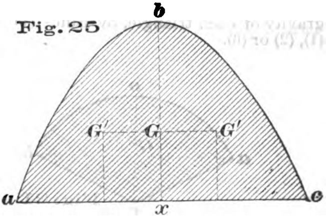
$$v\,G = \frac{7}{12}\,v\,d.$$

(21). Parabola, *a b c*, Fig. 25. *a c* is the base; *a x* and *c x*, ordinates; and the height or axis, *b x*, an abscissa. Center of gravity at **G,** in the axis *x b*, and

$$x\,G = \frac{2}{5}\,x\,b.$$

(21 *a*). Semi-parabola, *a b x* or *c b x*. Center of gravity at G′, and

Fig. 25



$$x\,G = \frac{2}{5}\,x\,b; \qquad G\,G' = \frac{3}{8}\,a\,x = \frac{3}{8}\,c\,x.$$

(22). Ellipse, *m n o p*, Fig. 26. The center of gravity, *c*, of the whole ellipse is at the center of the figure.

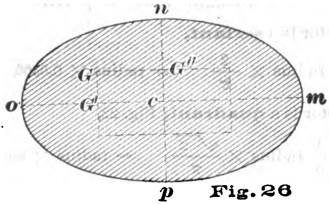


Fig. 26

G is the center of gravity of the quarter ellipse, *o n c*.
G′ " " " " " half " *n o p*.
G″ " " " " " " " *m n o*.

$$c\,G' = \frac{4}{3}\,o\,c \times \frac{1}{\pi} = 0.4244\,o\,c = \text{(approximately)}\ \frac{14}{33}\,o\,c.$$

$$c\,G'' = G'\,G = \frac{4}{3}\,c\,n \times \frac{1}{\pi} = 0.4244\,c\,n = \text{(approximately)}\ \frac{14}{33}\,c\,n.$$

(23). Any plane figure. Draw the figure to scale on stout card-board. Cut it out and balance it in two or more positions over the edge of a table or on a knife-edge; and mark on it the several positions of the supporting edge. Where these intersect is the center of gravity. Considerable care is of course necessary to obtain very close results by this method. Before balancing the card, its upper edges should be marked off into small equal spaces. Otherwise it will be difficult to locate the positions of the supporting edge. The paper on which the figure is prepared must of course be so stiff that the figure will not bend when balanced on the knife-edge. See Rule (5).

B. Surfaces of Solids.*

(24). Curved surface* of **sphere** or **spheroid (ellipsoid). G** is the center of the figure.

(25). Curved surface* of any **spherical zone,** as a **spherical segment, hemisphere.** etc., Figs. 27. **G** is the center of the **axis** or height, *a o*.†

In the **hemisphere,** $o\,G = \frac{1}{2}$ radius.†

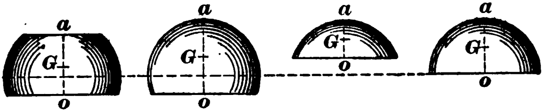


Fig. 27

(26). Right or oblique **prism,** whose ends are either regular figures or parallelograms (this includes the **cube** and other **parallelopipeds**); and right or oblique **cylinder** (circular or elliptic). *Surface** (either including both or excluding both of the two parallel ends). **G** is the center of the **axis,** or line joining the centers of the two parallel ends.

(27). Curved surface*† of right **cone,** Fig. 28 (circular or elliptic), or slanting surfaces*† of right regular **pyramid,** Fig. 29. **G** is in the axis *o a* (the line joining the apex and the center of the base); and

$$o\,G = \frac{1}{3}\,o\,a.$$

In an *oblique* cone or pyramid, the perpendicular distance of **G*** from the base is one-third of the perpendicular height, as in the right cone and pyramid: *but does not lie in the axis.*

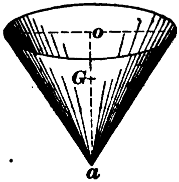


Fig. 28

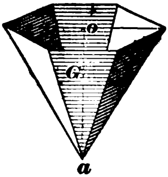


Fig. 29

(28). Frustums with top and base parallel, Figs. 30 and 31. Curved surface*† of frustum of right cone (circular or elliptic); or slanting surfaces*† of frustum of right regular pyramid. **G** is in the axis *o a* (the line joining the centers of the two parallel ends); and

$$o\,G = \frac{1}{3}\,o\,a \times \frac{\text{circumference of } o + 2 \text{ circumference of } a.}{\text{circumference of } o + \text{circumference of } a.}$$

---

* We treat now of the *surfaces* of solids, not of their contents or volumes or weights. For these, see Rules (29), etc.

† If the **top** or **base** is to be included, see Rules (1) and (2).

In the **conic frustum,** Fig. 30, we may use the *radii* of the two ends; and in the **frustum of a regular pyramid,** Fig. 31, any *side* of each end (as *b c* and *d e*) instead of the circumferences.

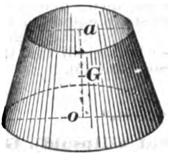


Fig. 30

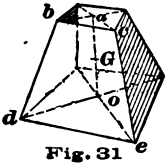


Fig. 31

Solids.

In the following rules for center of gravity of solids, the solid is supposed to be *homogeneous; i. e.,* of uniform density throughout; so that the center of *gravity* is the center of *magnitude* or of *volume.*

(29). **Sphere** and **spheroid (ellipsoid).** G is the center of the body.

(30). **Hemisphere,** Fig. 32. (Center of sphere at *c*). Height *c* T = radius *c b*. G is in the axis, *c* T, and

$$cG = \frac{3}{8}\,cT = \frac{3}{8}\text{ radius } cb.$$

(31). **Spherical sector,** Fig. 33. (Center of sphere at *c*).

$$cG = \frac{3}{4}\left(\text{radius } cb - \frac{h}{2}\right).$$

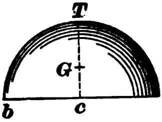


Fig. 32

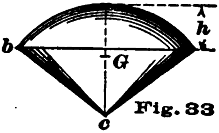


Fig. 33

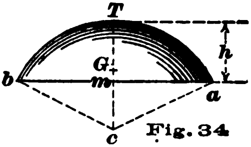


Fig. 34

(32). **Spherical segment,** *a m b* T, Fig. 34. Center of *sphere* at *c*. Center of *base* at *m*. Rise or height of segment = *m* T = *h*. G is in the axis *m* T; and

$$cG = \frac{3}{4} \times \frac{(2\text{ radius } cb \text{ of sphere} - \text{height } h)^2}{3\text{ radius } cb \text{ of sphere} - \text{height } h}$$

$$mG = \frac{\text{height, } h}{2} \times \frac{2\,(\text{radius } mb \text{ of base})^2 + (\text{height, } h)^2}{3\,(\text{radius } mb \text{ of base})^2 + (\text{height, } h)^2}$$

$$= \frac{\text{height, } h}{4} \times \frac{4 \times \text{radius } cb \text{ of sphere} - \text{height, } h}{3 \times \text{radius } cb \text{ of sphere} - \text{height, } h}.$$

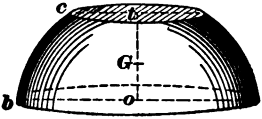


Fig. 35

(33). **Spherical zone,** Fig. 35.

$$oG = \frac{ot}{2} \times \frac{2\,(\text{radius } ob \text{ of base})^2 + 4\,(\text{radius } tc \text{ of top})^2 + (\text{height } ot)^2}{3\,(\text{radius } ob \text{ of base})^2 + 3\,(\text{radius } tc \text{ of top})^2 + (\text{height } ot)^2}.$$

(34). **Prism,** regular or irregular, right or oblique (including the **cube** and other **parallelopipeds**), and **cylinder,** circular or elliptic, etc., regular or irregular, right or oblique. G is the center of the axis joining the centers of gravity of the two ends.

(34 *a*). **A flat body,** such as an iron plate, etc, may be treated as a very short cylinder or prism. See (34)

(35). **Ungula of a cylinder,** circular, or elliptic (provided one of the axes of the ellipse coincides with the oblique cutting plane); right or oblique. Figs. 36 and 37.

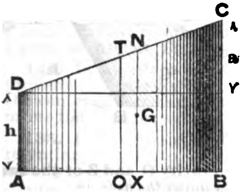


Fig. 36

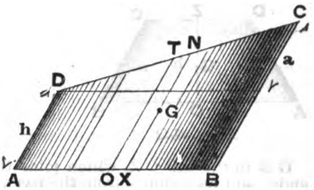


Fig. 37

Let O T be the axis (joining the centers of gravity of the ends), and X N a line drawn parallel to the axis, in the plane, A B C D, passing through the axis and through the uppermost and lowermost points C and D of the oblique cutting plane. Then the position of G in the plane A B C D, is found thus:

$$OX = \frac{OB}{4} \times \frac{a}{2h + a};$$

$$XG = \frac{XN}{2} = \frac{1}{4}\left(2h + a + \frac{1}{4}\,\frac{a^2}{2h + a}\right).$$

(35 *a*), Figs 38 and 39. If the oblique plane C D *meets* the base, A B, at A, so that $h = 0$, while C D remains a complete ellipse or circle, this becomes

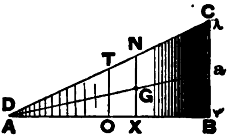


Fig. 38

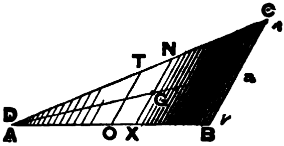


Fig. 39

$$OX = \frac{OB}{4};$$

$$XG = \frac{XN}{2} = \frac{5}{16}a.$$

(36). **Cone,** Figs. 40 and 41, circular, elliptic, etc., right or oblique; or **pyramid,** regular or irregular, right or oblique. The center of gravity G is in the axis O T, drawn from the apex, or top, T, to the center of gravity O of the base; and

$$OG = \frac{OT}{4}.$$

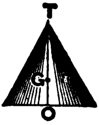


Fig. 40

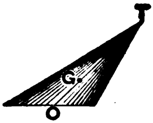


Fig. 41

(37). **Frustum of a cone,** Figs. 42 and 43, circular or elliptic, right or oblique; or **of a pyramid,** regular or irregular, right or oblique; provided the two ends A B and C D are parallel.

Call the area of the large end A, and that of the small end *a*; and let *h* be the height O Z of the frustum, *measured along its axis.* Then

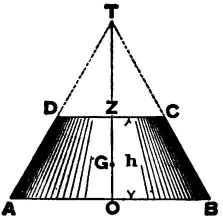


Fig. 42

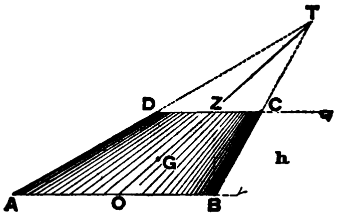


Fig. 43

G is in the axis O Z, which joins the centers of gravity O and Z of the two ends; and its distance from the base, A B, *measured along the axis*, is

$$OG = \frac{h}{4} \times \frac{A + 2\sqrt{Aa} + 3a}{A + \sqrt{Aa} + a}.$$

(37*a*). In a frustum of a circular cone, right or oblique, with parallel ends, this becomes

$$OG = \frac{h}{4} \times \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2},$$

where R and *r* are the radii of the large and small ends of the frustum respectively.

(38). Figs. 44 and 45. **Frustum,** A B C D, **of a cone,** circular, elliptic, etc., right or oblique; or **of a pyramid,** regular or irregular, right or oblique; whether the ends are parallel or not. By rule (36) find the center of gravity N of the *entire* pyramid (or cone, as the case may be) A B T, of which the frustum forms the lower part; and the center of gravity S of the smaller pyramid or cone D C T (= entire pyramid or cone, minus the frustum). Also find the *volume* of each: thus,

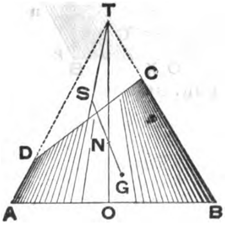


Fig. 44

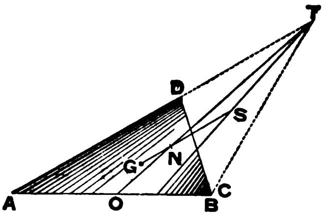


Fig. 45

$$\text{Volume of pyramid or cone} = \frac{\text{area of base} \times \text{perpendicular height}}{3},$$

and

$$\begin{array}{c}\text{Volume of}\\ \text{the frustum}\\ \text{A B C D}\end{array} = \begin{array}{c}\text{volume of}\\ \text{entire pyramid}\\ \text{or cone, A B T}\end{array} - \begin{array}{c}\text{volume of}\\ \text{smaller}\\ \text{one, D C T}\end{array}$$

Then the center of gravity G of the frustum A B C D is in the extension of the line S N; and

$$NG = SN \times \frac{\text{volume of smaller pyramid or cone, D C T}}{\text{volume of frustum, A B C D}}.$$

(39). **Paraboloid.** G is in the axis, and at one-third of its length from the base.

LINE OF PRESSURE.
CENTER OF FORCE OR OF PRESSURE.

Position of Resultant.

133. In ¶¶ 133 to 154 we discuss the position of the resultant, or line of pressure, of a system of parallel forces acting against a surface. For the changes in that position within a structure, due to the action of non-parallel forces, see Arches, Dams, etc., ¶¶ 251, etc.

134. In a system of parallel forces, acting against a surface, the line of pressure, or pressure line, is the position of the resultant of the forces; and the center of force or center of pressure is the point where the pressure line meets that surface against which the forces act.

135. If the lengths of the lines which represent the forces be taken as representing weights, to scale, then the position of the pressure line is the line of gravity (see (5), ¶ 131) corresponding to those weights.

136. Thus, in Fig. 55 (*a*), ¶ 117, if the three forces, *a*, *b* and *c*, be taken as weights, represented to scale by the arrows, *a*, *b* and *c*, respectively, then the resultant R of the three forces occupies the position of the line of gravity of the three weights.

137. Again, in a mass of sand, Fig. 61,* with an irregular surface, we may suppose the mass to consist of innumerable vertical columns of sand, of different heights, and exerting pressures proportional to those heights. Here, also, the pressure line is the vertical line of gravity of the mass, and the center of pressure against the base of the containing box is the point where said pressure line meets that base.

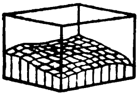


Fig. 61.

138. Although we are usually concerned with forces acting against *surfaces*, so that the lines representing the forces form a solid and not merely a surface, yet, in a majority of the cases which occur in civil engineering, we may, for convenience, regard the forces as concentrated in a single plane, and therefore as acting against a mere line.

139. Thus, in the case of an arch, pressing against its skewback, the pressure is ordinarily distributed over all or a considerable part of the bearing surface of the skewback; but we may, for convenience, regard it as concentrated in a single plane, midway between, and parallel to, the two faces of the arch.

140. Similarly, in the case of the water pressure against the back of a dam (or against a small strip of the back, extending from the water surface to the bottom, or to any other depth), the water, of course, presses upon the entire surface of such strip; but we may, for convenience, regard the pressure as concentrated in a vertical plane normal to the back of the dam and meeting it in the vertical axis of the assumed strip.

141. We have just seen (¶¶ 138 to 140) that, when a system of parallel pressures acts against a surface, they may often be assumed to act, in one plane, against a single line—viz., the intersection of that plane with the surface. It also frequently happens that such forces are so distributed along that line that the lines representing the forces are either of equal length or of lengths increasing uniformly from one end of the line to the other.

---

*Following Fig. 60, of Parallel Forces, ¶ 124. Figs. 1 to 45, illustrating Center of Gravity, are numbered independently of the rest of the series of figures relating to Statics.

142. Thus, in the case of water resting upon a horizontal surface, Fig. 62, the pressure is uniformly distributed, and the diagram, Fig. (*b*), representing the pressures, is a rectangle bounded by a horizontal line, and its center of gravity, G, is at the center of the figure. Hence, the center of pressure, *c*, is at the center of the line *a b*, or *l*.

Here the unit pressure, p, is uniform, and $R = p\,l$.

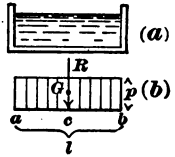


Fig. 62.

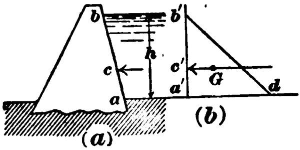


Fig. 63.

143. But when the water presses horizontally against a vertical or inclined surface, *a b*, Fig. 63, the unit pressure increases uniformly from zero, at the water surface, *b*, to a max at the bottom, *a*; and the hor pressures are represented, in Fig. (*b*), by the ordinates of the triangle *b' a' d*. Since the resultant passes through the center of gravity, G, of the triangle, the center of pressure, *c*, is at such a depth that $c\,a = \frac{1}{3}\,a\,b$, and $c'\,a' = \frac{1}{3}\,h$. See Rule (14 *c*) under Center of Gravity.

Here the mean horizontal unit pressure, p, is half the maximum horizontal pressure at *a*, and the total horizontal pressure is $= p\,h$.

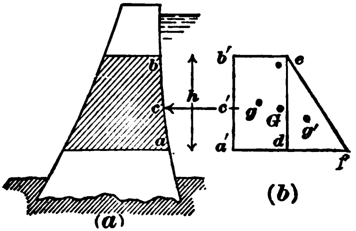


Fig. 64.

144. Again, if we consider only the water pressures against a certain *part*, *a b*, Fig. 64, of the depth of the back of a dam, the diagram, Fig. (*b*), representing the horizontal unit pressures, becomes a trapezoid, composed of a rectangle *b' d*, and a triangle *e d f*, with their centers of gravity at *g* and *g'* respectively; and the center of pressure, *c*, on *a b*, is opposite their common center of gravity (center of gravity of trapezoid), G. If *h* be the vertical depth of the portion considered, then

$$a'\,c' = \frac{h}{3} \times \frac{2\,b'\,e + a'\,f}{b'\,e + a'\,f}.$$

See Rule (16) under Center of Gravity. See also Center of Pressure, under Hydrostatics.

Distribution of Pressure.

145. Conversely, if two surfaces, as those of a masonry joint, are in such contact that the pressure is, or may be regarded as, regularly distributed, and if the position of the resultant is known, the rectilinear figure, representing the distribution of pressure, may be drawn by means of the principles just stated.

146. In Figs. 65 to 68 inclusive, let

o = the center of the joint $a\ b$ between the two surfaces;
R = the total pressure = resultant of all the pressures;
c = point of application of resultant, R;
l = $a\ b$ = the length of the joint;
x = $o\ c$ = the distance of the center of pressure from the center of the joint;
y = $\frac{l}{2} - x = a\ c$ = distance of center of pressure from nearest end of joint;
p = the mean unit pressure = $\frac{R}{l}$;
p_a = the maximum unit pressure;
p_b = the minimum unit pressure.

¶¶ 147 to 154 apply equally whether the surface is horizontal, vertical or inclined, and whether the forces are normal or inclined to it.

147. If x is not greater than $\frac{l}{6}$, or, in any case, if the joint is capable of sustaining tension, as well as compression, we have:

$$\text{Maximum unit pressure} = p_a = p\ (1 + \frac{6\ x}{l});$$

$$\text{Minimum unit pressure} = p_b = p\ (1 - \frac{6\ x}{l}).$$

If x exceeds $\frac{l}{6}$, and if the joint is incapable of resisting tension, see ¶¶ 151, 152, 154.

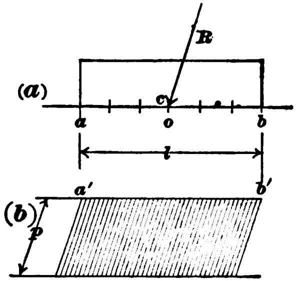


Fig. 65.

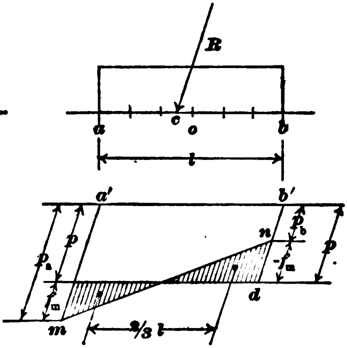


Fig. 66.

148. Demonstration. In Fig. 66, where the parallelogram $a'\ d$ represents the total pressure R as it would be if uniformly distributed along l, we see that the moment of R, about o, which changes the parallelogram $a'\ d$ into the trapezoid $a'\ b'\ n\ m$, is equivalent to a couple (see Couples, ¶ 155. etc.) composed of two forces—viz., a pressure, f (not shown) distributed over $o\ a$ and represented by the shaded triangle on the left, and a tension, $-f$, or diminution of pressure, distributed over $o\ b$ and represented by the triangle on the right. The forces, f and $-f$, act through the centers of gravity of these two triangles respectively; and the distance of each of these centers of gravity from the center, o, of the joint, measured parallel to the joint, is $= \frac{2}{3} \cdot \frac{l}{2}$. Hence the distance between the two centers of gravity, measured parallel to the joint, is $= \frac{2\,l}{3}$. Let x be the eccentricity, $c\,o$, of R, measured along the joint, and let A_R and A_C (not shown) be the lever arms of R and of the couple, respectively, about the center, o, of the joint. Then, since R is parallel to f and $-f$, A_R to A_C, and x to l, we have:

$$A_R : A_C = x : \frac{2\,l}{3}.$$

If R is normal to the joint, we have: $A_R = x$; and $A_C = \frac{2\,l}{3}$.

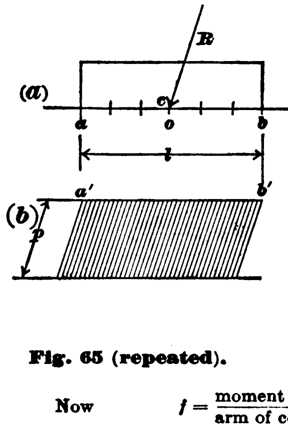


Fig. 65 (repeated).

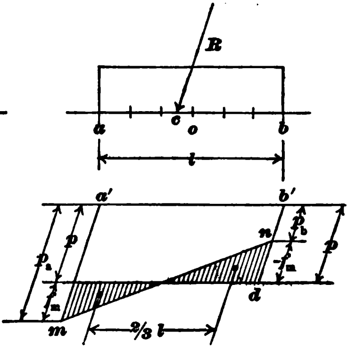


Fig. 66 (repeated).

Now $\qquad f = \frac{\text{moment of R}}{\text{arm of couple}} = \frac{R\,.\,A_R}{A_C}.$

Hence, $\qquad f = \frac{R\,.\,x}{\frac{2\,l}{3}} = \frac{R}{l}\,.\,\frac{3\,x}{2} = p\,\frac{3\,x}{2}.$

The *mean* additional pressure on $o\,a$ (or mean tension on $o\,b$) is $= \frac{f}{\frac{1}{2}\,l}$ and the corresponding maximum additional pressure is

$$f_m = 2\,\frac{f}{\frac{1}{2}\,l} = \frac{4}{l}\,f = \frac{4}{l}\,p\,\frac{3\,x}{2} = p\,\frac{6\,x}{l}.$$

$$\text{Now } p_a = p + f_m = p + p\,\frac{6\,x}{l} = p\,(1 + \frac{6\,x}{l})$$

$$\text{and } p_b = p - f_m = p - p\,\frac{6\,x}{l} = p\,(1 - \frac{6\,x}{l}).$$

149. If, as in Fig. 65, the center of pressure, c, is at the center, o, of the surface, we have $x = o\,c =$ zero, and the pressure, R, is uniformly distributed over the surface.

150. "The Middle Third." If, as in Fig. 67, $x = \frac{l}{6}$,—*i. e.*, if the resultant, R, of all the forces, meets the surface at the edge of the middle third of that surface, then $p_a = 2\,p$; and $p_b = 0$. See ¶¶ 143 and 148.

151. When, as in Fig. 68 (*a*), x exceeds $\frac{l}{6}$,—*i. e.*, when the center of pressure, c, falls beyond the middle third of the surface of pressure, a portion, $s\,b$, of the surface, is in tension, the maximum tension, p_b, Fig. 68 (*b*), being $= p\,(1 - \frac{6\,x}{l})$ as above; maximum pressure $= p\,(1 + \frac{6\,x}{l})$, and total pressure on $a\,s = \frac{p_a\,.\,a\,s}{2} =$ R *plus* the tension in $s\,b$; but if, as usually

happens in masonry, the surfaces are incapable of resisting tension, the total pressure, R, is simply concentrated upon a portion $a\,v$, Fig. 68 (*c*), of the surface, $a\,v$ being $= 3\,y$.

$$\text{Then, mean unit pressure on } a\,v = \frac{R}{a\,v} = \frac{R}{3\,y};$$

$$p_a = 2\,\frac{R}{3\,y} = 2\,\frac{p\,l}{3\,y}.$$

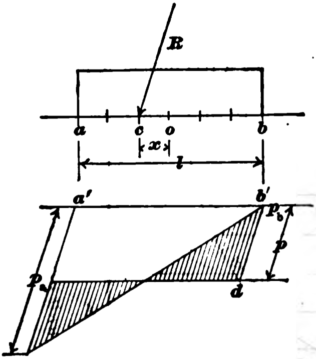


Fig. 67.

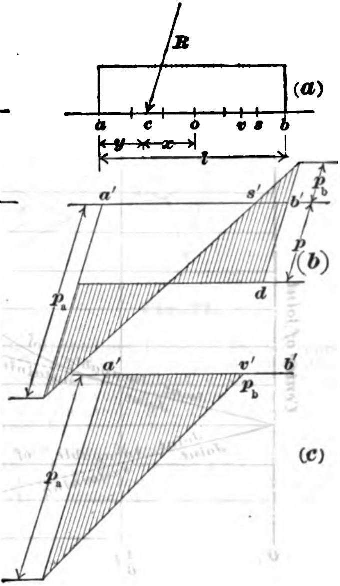


Fig. 68.

152. Hence, in a joint incapable of sustaining tension, Fig. 68 (*c*), if $p_a = 2\,\frac{R}{3\,y}$ is the maximum permissible unit stress, the distance $a\,c$, from the center of pressure, c, to the nearest end of the joint, must not be less than $y = \frac{2\,R}{3\,p_a}$.

153. If the joint can resist tension, Fig. 68 (*b*), we substitute, in the equation, $p_a = p\,(1 + \frac{6\,x}{l})$, the value of $x = \frac{l}{2} - y$, and, solving for y, we have

$$y = \frac{2}{3}\,l - \frac{p_a\,l}{6\,p}.$$

154. The influence diagrams, Fig. 69 (see ¶¶ 339, etc., and Trusses, ¶¶ 79, etc.), show the changes in the maximum and minimum unit pressures, p_a and p_b, as the center of pressure, c, recedes from the center, o, of the joint. The diagrams are constructed for a mean unit pressure, p, of 1. If the surfaces of the joint are capable of sustaining tension, every part of the joint always sustains either pressure or tension; and (see dotted lines

Fig. 69) the maximum unit pressure, $p_a = p\,(1 + \frac{6\,x}{l})$, see ¶ 146, increases proportionally with x; becoming $= 4\,p = \frac{4\,R}{l}$ when c reaches the end, a, of the joint, and $\frac{x}{l} = \frac{1}{2}$. The maximum tension, p_b, is then $= 2\,p = \frac{2\,R}{l}$. But if the surfaces are incapable of sustaining tension (see solid lines, Fig. 69), the increase of p_a is proportional to x only so long as $x < \frac{l}{6}$;—*i. e.*, so long as the resultant of all the pressures falls within the middle third of the base $a\,b$. When that limit is exceeded, the maximum unit pressure, p_a, begins to increase more rapidly than does the distance, x, of c, from the center, o, of the joint, the diagram becoming a rectangular hyperbola; so that, if the resultant could be actually applied at the very edge of the joint, the unit pressure there would become infinite.

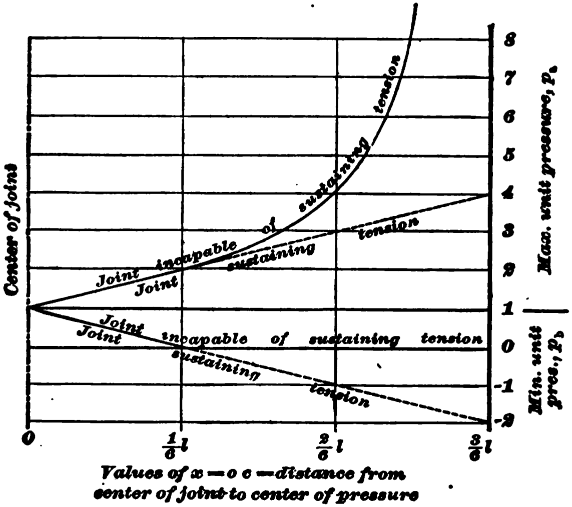


Fig. 69.

COUPLES.

155. Couples. Two equal parallel forces, p and q, or p' and q', Fig. 70,* of opposite sense, are called a couple. A couple has no tendency to move the body † as a whole in any straight line. In other words, the two forces, forming a couple, can have no resultant. Their only tendency is to make the body revolve about its center of gravity, G, and in the **plane of the couple**—*i. e.*, the plane in which the two forces lie. A body with a fixed axis can revolve only in a plane normal to that axis. The actual plane of rotation of a free body depends upon the distribution of mass in the body, and is not necessarily the plane of the couple.

---

* Figs. 70 to 75 are supposed to be seen in perspective, and the forces are supposed to act in the planes shown.

† See foot-note (*), ¶ 1.

156. The moment of a couple is equal to the product of one of the two forces, p or q, into the perpendicular distance, d, between the two forces. Or, in our figures,

$$\text{moment of couple} = p \,.\, d = q \,.\, d.$$

157. Graphic Representation of Couples. A couple, M or N, Fig. 70, is indicated, in amount, in direction and in sense, by a line, L or L′, normal to the plane of the couple, so placed that, looking along it toward that plane, the couple appears positive or right-handed, and of such length as to represent, by scale, the moment of the couple. In Fig. 70, the two couples M and N are of opposite sense. Hence the lines L and L′, representing them, project in opposite directions from their respective planes.

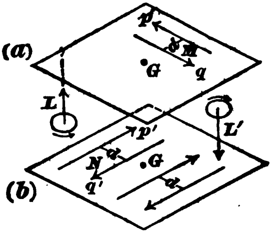


Fig. 70.

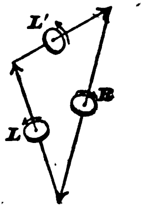


Fig. 71.

158. Composition of Couples. If the lines, L and L′, Fig. 71, represent two couples, in accordance with ¶ 157, then the line R, completing the triangle, will, in the same way, represent their resultant or anti-resultant. As drawn, with its arrow *following* those of the other two sides, it represents their *anti*-resultant. For their resultant, the arrow on R, and that indicating the direction of rotation, must be reversed.

159. Equality of Couples. Two couples, M and N, in the same plane, Fig. 72 or Fig. 73, or in parallel planes, Fig. 70, are equal if their moments are equal, whether or not the forces of one of the couples be equal or parallel to those of the other. In Fig. 73, the two couples, M and N, are of like sense; in Figs. 70 and 72, of opposite sense.

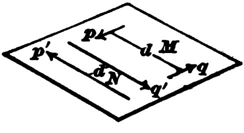


Fig. 72.

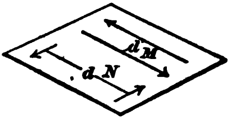


Fig. 73.

160. Since a couple has no resultant (¶ 155), it can have no anti-resultant; *i. e.*, no single force can balance a couple and thus preserve equilibrium. (But see ¶ 168.) To do this requires an equal and opposite couple. Thus, in Fig. 72 the couple M is balanced by the equal and opposite couple N. If, as in Fig. 72, the two couples are in the same plane, and if we find first the resultant of either pair of non-parallel forces, as p and p', and then those of the other pair, q and q', we shall find these resultants equal and opposite, maintaining equilibrium.

161. Any couple, as M, Fig. 73, **may be replaced** by any other equal couple, N, in the same plane or in a parallel plane, and of like sense.

162. If, to a force, P, Fig. 74 (*a*), **we add a couple,** M, Fig. 74 (*b*), in the same plane with the force, we may replace the couple, M, by an equal and like couple, N, Fig. (*c*), composed of the forces, —P and P′, each = P, placing —P opposite P, as shown. Then P and —P counteract each other, and we have left only P′, equal and parallel to P; and, since Pd = M, we have

$d = \frac{M}{P}$. In other words, the effect of the addition of the couple, M, Fig. (*b*), to the force, P, is simply to shift the line of action of P, parallel with itself, through the distance, *d*. If the couple M is left-handed, as in the figure, P will be shifted to the right (looking in its own direction), and vice versa.

163. **Conversely**, the force, P′, Fig. (*c*), is equivalent to the combination of force P and couple M, Fig. (*b*).

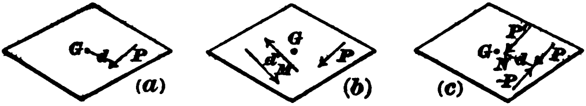


Fig. 74.

164. Again, having only the force P′, Fig. (*c*), if we apply, at a distance, *d*, from P′, the two opposite forces, P and —P, each equal and parallel to P′, we shall thus substitute, for P′, the equal and parallel force, P, and a couple = P*d* = M.

165. Hence, also, the combination of the force P and the couple M, Fig. (*b*), is equivalent to the combination of the force P and the couple N, Fig. (*c*).

166. If the moment of the couple, M, Fig. (*b*), or N, Fig. (*c*), be equal and opposite to the moment of the force P about the center of gravity, G, of a body, we have $d = \frac{M}{P}$ = distance from P to G. In other words, the effect of such a couple is to shift the force, P, parallel with itself, to a line passing through the center of gravity, G.

167. Hence, the effect of a force, P, Fig. (*a*), applied to a body at a distance, *d*, from its center of gravity, G, is equivalent to the combined effect of an equal and parallel force, P′, Fig. (*c*), applied at the center of gravity, and a couple (as M, Fig. *b*) = P*d*, and of like sense, applied to any part of the body in a plane parallel to P and P′.

168. It will be seen that, although (¶ 160) no single force can balance a couple and establish equilibrium, yet, if a force, P, be so applied that its moment, P*d*, about the center of gravity, G, of the body, is equal and opposite to the moment of the couple, it will counteract the tendency to rotation, due to the couple, and substitute for it a motion of translation only.

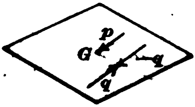


Fig. 75.

169. Thus, in Fig. 75, where the force, *p*, acts through the center of gravity, G, of the body, let a force, —*q*, equal and opposite to *q*, be applied in the same line with it. Then rotation will be prevented, and the body will move * under the action of *p* (= the resultant of the three forces), which acts through the center of gravity, G, of the body. The rotation will similarly be prevented if a force *less* than *q* be applied *farther* from G than *q* is; or if a force *greater* than *q* be applied *nearer* G than *q* is; provided always that the moment of said third force, about G, be equal and opposite to that of the couple *p q*. But in the first case the resultant of the three forces (being always equal to the third force) will be less, and in the second case greater, than *p*.

170. If, to a couple, be added a third force, colinear with one of the forces of the couple, we have the case of two unequal parallel forces of opposite sense. See ¶ 112, under Parallel Forces.

---

* See foot-note (*), ¶ 1.

FRICTION.*

171. When one rough body rests upon another, the projections and depressions, forming **the roughnesses** of their surfaces of contact, **interlock** to a greater or less extent; and, in order to slide one over the other, we must expend a portion of the sliding force, either in *separating* the bodies (as by lifting the upper one) sufficiently to clear the projections, or in breaking off some of the projections and clearing the others.

172. Even the most highly polished flat surface, as *x y*, Fig. 76, is *not* (as it appears to the eye) a *plane*, but is, in fact, a more or less jagged surface, as would appear under a sufficiently powerful microscope; so that the force, *a b*, instead of forming the *apparent* angle, *a b x*, with *one* smooth surface, *x y*, of application, really becomes a series of parallel forces, as *c*, *d* and *e*, which form *other* angles with a number of surfaces, *m m*, *n n*, etc., of application, *inclined* (often in different directions) to the general surface, *x y*, as shown. Among these surfaces may be some, as *m m*, at right angles to the applied force; and, the force *c* will be imparted to them in its original direction, although applied *obliquely* to the *apparent* surface, *x y*. In the case of the two forces, *d* and *e*, applied to the surfaces, *n n* and *s s*, if the sliding tendencies along the two surfaces are *equal* and act in *opposition* to each other, the *combined* resistance of the two surfaces, *n n* and *s s*, is directly opposite to the forces, as would be that of a single surface at right angles to those forces.

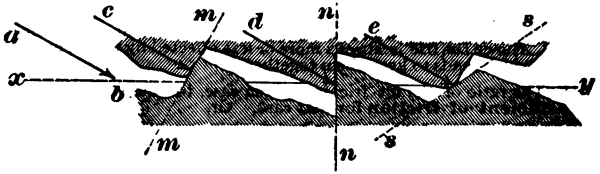


Fig. 76.

173. It is of course entirely out of the question to ascertain the exact resistance of each such microscopic projection in any given case. Instead of this, we find by experiment the *combined* resistance which *all* of the projections, in a given case, offer to the sliding force, and give to this resistance the name of **friction.**

174. Friction always tends to *prevent relative motion of the two bodies between which it acts; i. e.*, motion of one of the bodies relatively to the other. In doing so, however, it tends equally to *cause* relative motion † between each of those two and *a third, or outside body*. Thus, the fric between a belt and the pulley *driven by it* tends to prevent slipping between *them;* but thus tends to make the belt slip on the *driving* pulley, and sets the *driven* pulley and its shaft in motion *relatively to the bearing* in which the shaft revolves. *This* motion is resisted by the fric *between journal and bearing*; and *this* fric, in turn, tends equally to make the *bearing* revolve with the journal, and to make the belt slip on the driven pulley.

175. The fric between two bodies *at rest* relatively to each other is called **static friction,** or fric of rest. That between two bodies *in relative motion* is called **kinetic friction** or fric of motion.

176. The ultimate or maximum static fric between two bodies, as U and L, Fig. 77 (or the greatest fric resistance which they are capable of opposing to any sliding force when at rest), is equal to a force (as that of the wt F) which is just upon the point of making U begin to slide upon L.[*] Thus fric, like other forces, may be expressed in *weights*, as in lbs.

---

* "Friction" (meaning rubbing) is a misnomer in so far as it implies that rubbing must take place in order to produce the resistance. For we meet this resistance, not only during rubbing, but also before motion (or rubbing) takes place. "Resistance of roughness" would better express its nature.

† See foot-note (*), ¶ 1.

177. A resistce cannot exceed the force which it resists.[†] Therefore if F is less than the ult static fric between U and L, the *frictional resistce actually exerted* by them is also less. When F is = the ult fric (and U is therefore on the point of sliding) the actual resistce is = the ult stat fric. If F exceeds the ult stat fric, the excess gives motion to U.

178. If, when a body is in motion, all extraneous forces and resistces are removed or kept in equilib, it moves at a uniform vel. Hence, if the force, F, Fig. 77, is just = the ultimate kinetic fric between U and L, their vel is uniform. If F exceeds this, the excess *accelerates* the vel. If the ult kinetic fric exceeds F, the excess *retards* the vel. Thus **the actual frictional resistce** exerted by two bodies *in relative motion* is = **their ult kinetic fric** = that force (as F) which can just maintain their relative vel uniform.

179. Hence, if the hor surf S upon which L rests, could be made perfectly frictionless, the pres of L against the lug *m* (which would then always be = the *actual* fric resistce between U and L) would also be = their *ult* fric so long as U continued in motion over L, and might therefore be greater or less than or = F; but when U was at rest the pres against *m* would be = F, and less than (or at most just =) the ult fric.

Coefficient of Friction.

180. Since no surface can be made *absolutely* smooth, some separation of the two bodies must in all cases take place in order to clear such projections as exist. Hence the fric is always more or less affected by the amount of the perp pres which tends to keep them together.

181. The ratio of the ult fric, in a given case, to the perp pres, is called the **coefficient of friction** for that case. Or,

$$\text{Coefficient of friction} = \frac{\text{ultimate friction}}{\text{perpendicular pressure}}$$

and

$$\text{Ultimate friction} = \text{perp pres} \times \text{coeff of fric.}$$

Thus, if a force F, Fig. 77, of 10 lbs, just balances the ult fric between U and L, and if the wt of U (the perp pres in this case since the surf between U and L is *hor*) is 50 lbs, then the coeff of fric between U and L is $= \frac{10 \text{ lbs}}{50 \text{ lbs}} = 0.2.$

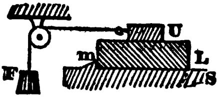


Fig. 77.

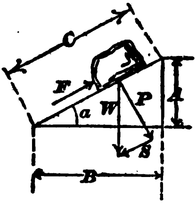


Fig. 78.

182. The coeff is usually expressed decimally, or by a common fraction; but sometimes, as in the case of railroad cars and engines, in lbs (of fric) per ton (of perp pres). Or by the "angle of fric" in degs and mins.

---

[*] We here neglect the fric of the string and pulley, and assume that *all* the force of the wt F is transmitted by the string to U.

[†] If a resisting *force* exceeds the force resisted, the excess is not resistce, but motive force.

183. Angle of Friction. In Fig. 78, let W = the weight of the body, P = its pressure normal to the plane, and S = the component tending to slide the body down the plane.

When the angle *a* is such that the body is just on the point of sliding down the plane, it is called the angle of friction, or angle of repose. The friction F and the sliding force S are then equal.

But $\frac{S}{P} = \frac{F}{P} = \frac{A}{B}$ = coefficient of friction = tan *a*. Hence F = P tan *a* = W cosin *a* . tan *a*.

184. Frictional Stability. Let R, Fig. 79, be the resultant of all the forces pressing a body against a plane, and N a normal to the plane. If the angle *i* between R and N exceeds the angle of friction (*a*, Fig. 78) between the two surfaces in contact, the body will slide on the plane, but not otherwise. If *i* does not exceed the angle of friction, the entire resultant R will be imparted to the plane and in its own direction, and not merely its normal component V, as would be the case if the surfaces were frictionless.

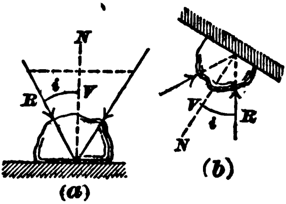


Fig. 79.

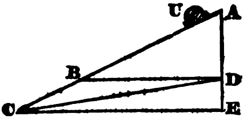


Fig. 80.

185. To find the coeff of kinetic fric, allow one of the bodies, U, Fig. 80, to slide down an inclined plane A C formed of the other one and having any convenient known steepness A C E greater than the angle of fric (¶ 183). Note the *vert* dist A E through which U descends in sliding any dist as A C (A E = A C × sine of A C E); also its actual *sliding* vel in ft per sec on reaching C. Calculate the *vert* dist A D through which it would have to descend *along the plane* (from A to B) to acquire that vel *if there were no fric.*

$$\left(\text{A D} = \frac{\text{velocity}^2 \text{ in ft per sec}}{\text{twice the accel } g \text{ of grav}^*}\right).$$

Find D E (= A E — A D), and the hor dist E C corresponding to A C (E C = A C × cosine A C E = $\sqrt{\text{A C}^2 - \text{A E}^2}$). Then

$$\text{Coeff of the average fric in sliding from A to C} = \frac{\text{D E}}{\text{E C}},$$

because, if we let A E represent the *total* sliding force expended (in acceleration and in overcoming the fric), then A D represents the portion of A E expended on *vel*, and D E that expended on *fric*, and, since C E represents the perpendicular pressure (¶ 183),

$$\frac{\text{D E}}{\text{E C}} = \frac{\text{friction}}{\text{prep pres}} = \text{coeff.}$$

186. Or, find sine and tangent of A C E; and the dist A C (= time² in secs × ½ *g* * × sine of A C E) through which U *would* slide in a given time *if there were no fric*. Measure the dist A B through which it *actually slides* in that time; and find B C = A C — A B. Then

$$\left.\begin{array}{l}\text{coeff of the average}\\ \text{fric in sliding from A to B}\end{array}\right\} = \text{tan D C E} = \text{tan A C E} \times \frac{\text{B C}}{\text{A C}}$$

because

---

* *g* = about 32.2 ft per second per second.

(1st.) A C : A B : B C :: A E : A D : D E

:: the theoretical velocity due to total sliding force : the actual velocity : the frictional retardation

:: the total sliding force : sliding force employed in giving the actual velocity : the friction, or the sliding force required to balance the friction.

And, if A E is = the total sliding force, then E C is = the perpendicular pressure, and

$$\frac{DE}{EC} = \text{the coefficient of friction} = \text{tangent of } DCE.$$

(2nd) Owing to the similarity of the two triangles, A B D and A C E, we have

$$AC : BC :: AE : DE :: \frac{AE}{EC} : \frac{DE}{EC} :: \text{tangent } ACE : \text{tangent } DCE.$$

187. In 1831 to 1834, **Gen'l Arthur Morin*** experimented with pressures not exceeding about 30 lbs per sq in; and arrived at the following conclusions in regard to sliding fric where the perp pres is considerably less than would be necessary to abrade the surfs appreciably. These were for a long time generally regarded as constituting the **three fundamental laws of fric.**

1st. The ult fric between two bodies is proportional to the total perp force which presses them together; *i e*, the **coeff is independent of the perp pres** and of its *intensity* (pres *per unit of surf*). Hence

2d. For any given *total* perp pres, **the coeff is independent of the area of surf in contact.**

If upon a hor support we lay a brick, measuring 8 × 4 × 2 ins, first upon its long edge (8 × 2 ins) and then upon its side (8 × 4 ins), we double the area of contact, while the total pres (the wt of the brick) remains the same, and *thus reduce the pres per sq in* by one-half. Consequently (the coeff remaining practically the same) we have only half the *fric per sq in*. But we have twice as many sq ins of contact, and therefore the same *total* fric.

But if we can increase or diminish the area of contact *without affecting the pres per sq in*, the *total* pres will of course *vary as the area*, and the total fric will vary in the same proportion, for the coeff remains the same. Thus, if we place two similar sheets of paper between the leaves of a book (taking care not to place both sheets between the same two leaves) and then squeeze the book in a letter-copying press, it will require about twice as much force to pull out both sheets as to pull out only one of them.

3d. Although the coeff of *static* fric between two bodies is often much greater than their coeff of kinetic fric; yet **the coeff of kinetic fric is independent of the vel.**

This applies also (approx) to the *fric*, and hence to the *work* (in *foot*-pounds etc) of overcoming fric through a given *dist*; for then the work (= resistce × dist) is independent of the vel. But in a given *time*, the dist (and consequently the work also) of course varies as the vel.

188. (a) Some kinds of surfaces appear to interlock their projections much more perfectly when at rest relatively to each other, than when in even very slow motion; and in some cases the degree of interlocking seems to increase with time of contact. Hence there is often a great diff in amount between fric of rest and fric of motion. Thus, Gen'l Morin found that with oak upon oak, fibres of the two pieces at right angles, the resistce to sliding while still at rest, and after being for "some time in contact," was about one eighth greater than when the pieces had a relative vel of from 1 to 5 ft per sec.

(b) But experience shows that even very slight jarring suffices to remove this diff; and since all structures, even the heaviest, are subject to occasional jarring, (as a bridge, or a neighboring building, or even a hill, during the passage of a train; or a large factory by the motion of its machinery; or in numberless cases, by the action of the wind) it is expedient, in construction, not to rely on fric for *stability* any further than the coeff for *moving* fric will justify. When it is to be regarded as a resistce, which we must provide force for overcoming, it should be taken at considerably *more* than our tabular statement.

---

* See his "Fundamental Ideas of Mechanics", translated by Jos. Bennett; D. Appleton & Co., New York, 1860.

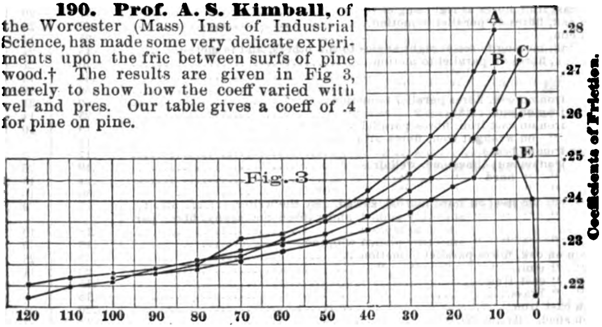
Table of moving friction, of perfectly smooth, clean, and dry, plane surfaces, chiefly from Morin.

Materials Experimented with.	Coeff of Fric; or Proportion of Fric to the Pres.	Angle of Fric.	
		Deg.	Min.
Oak on oak; all the fibers parallel to the motion	.48	25	38
" " moving fibres at right angles to the others; and to the motion	.32	17	45
" " all the fibres at right angles to the motion	.34	18	47
" " moving fibres on end; resting fibres parallel to the motion	.19	10	46
" cast iron, fibres at right angles to motion	.37	20	19
Elm on oak, fibres all parallel to motion	.43	23	17
Oak on elm, " " "	.25	14	3
Elm on oak, moving fibres at right angles to the others, and to motion	.45	24	16
Ash on oak, fibres all parallel to motion	.40	21	49
Fir on oak, " " " "	.36	19	48
Beech on oak " " " "	.36	19	48
Wrought iron on oak, fibres parallel to motion	.62	31	47
Wrought iron on elm, " " " "	.25	14	3
Wrought iron on cast iron, fibres parallel to motion	.19	10	46
" " on wrought iron, fibres all parallel to motion	.14	7	58
Wrought iron on brass	.17	9	39
Wrought iron on soft limestone, well dressed	.49	26	6
" " " hard " " "	.24	18	30
" " " " " " " wet	.30	16	42
" " or steel on hard marble, sawed. By the writer .... about	.17	9	39
" " " " " smoothly planed, and rubbed mahogany, fibres parallel to motion	.18	10	12
" " " " " smoothly planed wh pine	.16	9	6
Cast iron on oak, fibres parallel to motion	.49	26	6
" " " elm, " " " "	.20	11	19
" " " cast iron	.15	8	32
" " " brass	.15	8	32
Steel on cast iron	.20	11	19
Steel on steel. By the writer	.14	7	59
Steel on brass	.15	8	32
Steel on polished glass. By the writer .... about	.11	6	17
" quite smooth, but not polished; on perfectly dry planed wh pine, fibres parallel to motion .... about	.16	9	6
" quite smooth, but not polished; on perfectly dry planed and smoothed mahogany, fibres parallel to motion .... about	.18	10	12
Yellow copper on cast iron	.19	10	46
" " on oak	.62	31	48
Brass on cast iron	.22	12	25
" on wrought iron, fibres parallel to motion	.16	9	6
" on brass	.20	11	19
" on perfectly dry planed wh pine, fibres parallel to motion .... about	.19	10	46
" " " " " and smoothed mahogany, fibres parallel to motion .... about	.24	13	30
Polished marble on polished marble. By the writer .... average	.16	9	6
" " on common brick .... "	.44	23	45
Common brick on common brick .... "	.64	32	38
Soft limestone well dressed, on the same	.64	32	38
Common brick, on well-dressed soft limestone	.65	33	2
" " " " " hard "	.60	31	00
Oak across the grain, on soft limestone, well dressed	.38	20	48
" " " " " hard " " "	.38	20	48
Hard limestone on hard limestone, both " "	.38	20	48
" " " soft " " " "	.67	33	50
Soft " " hard " " " "	.65	33	2
Wood on metal, generally, .2 to .62 .... mean	.41	22	18
Wood, *very smooth*, on the same, generally, .25 to .5 .... "	.38	20	48
Wood, " " on metal, " .2 to .62 .... "	.41	22	18
Metal on metal, very smooth, dry " .15 to .22 .... "	.18	10	12
Masonry and brickwork, dry " .6 to .7 .... "	.65	33	2
" " " with wet mortar .... about	.47	25	30
" " " " slightly damp mortar .... "	.74	36	30
" on dry clay .... "	.51	27	00
" " moist " .... "	.33	18	15
Marble, sawed; on the same; both dry. By the writer.● ...... average "	.4	21	49
" " " " " both damp .... " ..● ...... " "	.55	28	49
" " on perfectly dry planed wh pine. " ..● ...... " "	.45	24	14
" " on damp planed wh pine .... " ..● .... "	.6	31	00
" polished, on perfectly dry planed wh pine " .... "	.26	14	35
White pine, perfectly dry; planed; on the same; all the fibres parallel to motion .... about	.4	21	48
" " damp, planed; on the same .... "	.6	31	00

● But after a few trials the surfaces become so much smoother as to reduce the angles as much as from 2° to 5°; the sliding blocks weighing about 30 ℔s each

189. Recent experiments, with much greater variations of pres and of vel, and with more delicate apparatus for detecting slight changes in the coeff, although giving conflicting results,[*] show that **the three laws** in ¶ 187 are far from correct for surfs moving at high vels, and under great pres; and that they **are only approximately correct** for ordinary vels and pressures; for the coeff is found to vary both with the intensity of the pres and with the vel, as also with the temperature.[*] But in the cases with which the civil engineer has mostly to deal, slight diffs in the character of the surfs, or even in the dampness of the air, will often cause much greater changes of coeff than those due to any probable changes of pres, vel and temp: so that, within the limits of abrasion, we may generally take Morin's rules as sufficiently correct for such cases.

190. Prof. A. S. Kimball, of the Worcester (Mass) Inst of Industrial Science, has made some very delicate experiments upon the fric between surfs of pine wood.[†] The results are given in Fig 3, merely to show how the coeff varied with vel and pres. Our table gives a coeff of .4 for pine on pine.



Line A shows coeffs at diff vels under a pres of 1.58 lbs per sq in.
" B " " " 1.59 " "
" C " " " 1.60 " "
" D " " " 1.61 " "
" E " " " 4.17 " "

It will be seen that at low vels the coeff decreased when the pres per sq in was almost imperceptibly increased; but this diff disappeared as the vel increased. At vels from 4 to 120 ins per sec, the coeff generally decreased as the vel increased; rapidly at first, but more slowly as the vel became greater. This agrees with other recent expts. But at very low vels (.08 to 5 ins per sec) Prof. Kimball found the coeff (line E) *increasing very rapidly with the vel.*

We have made the scale of coeffs large in order to show their variations, which are so slight that they would otherwise be scarcely perceptible. Less delicate expts would have failed to show them at all.

191. (a) In 1878 **Capt. Douglas Galton** and **Mr. George Westinghouse, Jr.,** made careful experiments in England to ascertain the effect of friction in connection with **railway brakes.**[‡] The friction and pressure were automatically recorded by means of hydraulic gauges. With cast iron brake-blocks and steel-tired wooden wheels, 43½ inches in diameter, they found coefficients about as shown in Fig. 4.

---

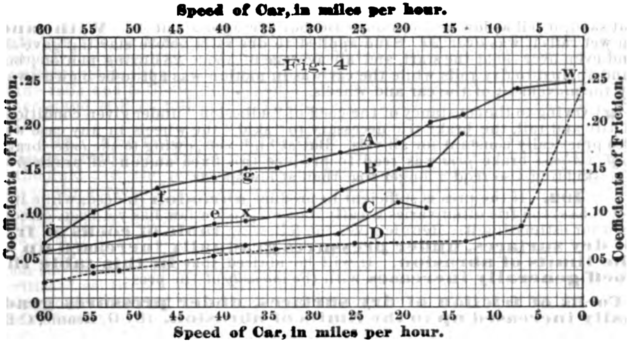
[*] This is not surprising in view of the extent to which the coeff is affected by the nature of the surf. If the shape of the minute projections is such that they fit into each other as perfectly under small pressures as under great ones, and if they are too strong to be broken by the pressures applied, the coeff, as stated in the 1st law, should be independent of the pres. But if high pres wedges the projections of one body more closely between those of the other, the coeff should increase under such pres. On the other hand, if the higher pres breaks down the projections while the lower ones are unable to do so, the coeff should decrease under the higher pres. The particles thus broken off may either act as a lubricant and thus still further *reduce* the fric and its coeff, or (if angular and hard) may *increase* it. Change of area of contact, under a given total pres, may, by affecting the *intensity* of the pres, make changes in the coeff similar to those just mentioned.

At high vels the roughnesses have not time to interlock as perfectly as at low vels. Hence we should expect a less coeff at high vels. But high vel generally increases the number of projections broken away; and these may either increase or diminish the coeff, as explained above. High vel often *indirectly* affects it by increasing the *temperature*.

[†] Silliman's Journal (American Journal of Science) March 1876 and May 1877.

[‡] See Proc, Instn of Mechl Engrs, London, June and Oct 1878 and April 1879; and "Engineering," London, 1878; vol. 25, pp 452, 469, 490; vol. 26, pp 153, 386, 395.

The points in lines A, B and C show the average *brake* coeffs, or coeffs of sliding fric between the tread of a *rolling* wheel and the *brake-block*.



Line A shows brake coeffs obtained immed'y after application of brake
" B " " 5 secs " "
" C " " 15 " " "
" D shows *rail* coeffs or coeffs of sliding fric between the tread of a *sliding* or "*skidding*" wheel (held fast by the brake) and the *rail*.

(b) From lines A B and C it appears that the **brake coeff** obtained at a given length of time after the application of the brake was generally **greater at low than at high vels.** But where the vel was maintained uniform the **brake coeff diminished as block and wheel remained longer in contact.** Thus, lines A and B show that at 37½ miles per hour the brake coeff was .154 when the brake was first applied (point *g*), but fell to .096 in 5 secs (*x*). Line A (immed'y after application) shows a higher brake coeff (.132 at *f*) at 47½ miles than line B (5 secs after application) shows at 37½ miles (.096 at *x*).

The diminution of the *rail* coeff with length of time of application of brake, was scarcely noticeable.

(c) When the brake fric (owing to the reduction of vel and consequent increase of coeff) becomes = the "adhesion" or static fric between the rail and the tire of the rolling wheel, the vel of rotation rapidly falls below that due to the vel of the car; *i e*, **the wheel begins to "skid"** or slide along the rail; and in from .75 to 3 secs the rotation of the wheel ceases entirely.

(d) **The rail coeff,** line D, **is generally much less than the brake coeff,** lines A, B and C. The pres on the rail (= the wt on a wheel) was about 5000 lbs per sq in, or greatly in excess of the limit of abrasion. That at the brake was about 200 lbs per sq in. A few expts were made with brake blocks having but ⅓ of the usual area of contact, and therefore 3 times the pres per sq in under a given total pres. They failed to show conclusively that this caused any marked change in the coeff.

(e) **The rail coeff,** line D, like the brake coeff, **increases as the vel diminishes;** slowly at first, but much more rapidly as the speed becomes less; until, at the moment of stopping, it is generally even greater than the brake coeff just before skidding. With steel tires on *iron* rails at high vels it was somewhat greater than on steel rails, but this diff disappeared as the vel diminished.

(f) **Locomotives** overcome resistces = from ¼ to ⅓ or more of the wt on all the drivers; *i e*, they have a coeff of .33 or more, although the experimental coeff for steel on steel in motion at low pres, is only about .15. But the cases are so diff that a similarity in their coeffs could hardly be expected. The great wt, say from 2 to 6 or even 7 tons, on a driver, is concentrated on a surf (where the wheel touches the rail) about 2 ins long × about ½ inch wide, or = say 1 sq in. The pres per sq in thus greatly exceeds not only that upon which the tables are based, but also the limit of abrasion. Besides, any point in the tread, during the instant when it is acting as the fulcrum for the steam pres in the cyl, is *stationary* upon the rail. Its fric (miscalled "adhesion") is therefore *static*.

Capt. Galton found that **the coeff of "adhesion"** was independent of the vel, and depended only on the character of the surfs in contact. With a four-wheeled car having about 5000 lbs load on each wheel, it was generally over .20 on dry rails; in some cases .25 or even higher. On wet or greasy rails, without sand, it fell as low as .15 in one case, but averaged about .18. **With sand** on wet rails it was over .20. Sand applied to dry rails before starting gave .35 and even over .40 at the start, and an average of about .28 during motion; but sand applied to dry rails while the car was in motion was apt to be blown away by the movement of the car and wheels.

(g) Owing to the constancy of the coeff of "adhesion" under given conditions of tire and rail, the brake fric necessary to "skid" the wheels in any case was also practically constant for all vels. But at high vels, owing to the *lower* brake *coeff*, a *higher* brake *pres* was reqd to produce this fixed amount of brake *fric* The skidding also reqd a longer *time* than at low speeds.

192. If the pres is sufficient to produce **abrasion** (indeed, while it is much less) the fric often varies greatly, but no precise law has yet been discovered for estimating it. Rennie gives the following **table of coeffs of fric of dry surfaces, under pressures gradually increased up to the limits of abrasion.** It will be noticed that **in this table the coeff generally increases** with the *intensity* of the pres:

Coeffs of friction of dry surfaces, under pressures gradually increased up to the limits of abrasion. (By G. Rennie, C E.)

Pres. in Lbs. per Square Inch.	Wrought Iron on Wrought Iron.	Wrought Iron on Cast Iron.	Steel on Cast Iron.	Brass on Cast Iron.
32.5	.140	.174	.166	.157
186	.250	.275	.300	.225
224	.271	.292	.333	.219
336	.312	.333	.347	.215
448	.376	.365	.354	.208
560	.409	.367	.358	.233
672	......	.376	.403	.233
709	......	.434	......	.234
784	......	......	......	.232
821	......	......	......	.273

193. (a) Rolling friction, or that between the circumf of a rolling body and the surf upon which it rolls, is somewhat similar to that of a pinion rolling upon a rack. In disengaging the interlocking projections, or in lifting the wheel over an obstacle *o*, Figs 5 and 6, the motive force F, instead of *dragging* one over the other, as in Fig 76, p. 407, acts at the end of a bent lever F R W Figs 5 and 6, the other end W of which acts in a direction *perp* to the contact surf; and in practical cases of rolling fric proper the leverage R W of the resisting wt of the wheel and its load is very much less, in proportion to that (F R) of the force F, than in our exaggerated figs. Hence the force F reqd to *roll* a wheel etc is usually very much less than would be necessary to *slide* it.

(b) There are usually **two ways of applying the force** in overcoming rolling fric: 1st (Fig 5) at the *axis* of the rolling body; as the force of a horse is applied at the axle of a wagon-wheel; or that of a man at the axle of a wheel-barrow: 2d (Fig 6) at the *circumf;* as when workmen push along a heavy timber laid on top of two or more rollers; or as the ends of an iron bridge-truss play backward and forward by contraction and expansion, on top of metallic rollers or balls (p725). In Fig 5 we have, in addition to the rolling fric of the circumf of the wheel on its support, the sliding fric of the axle in its bearing. In Fig 6 we have only rolling fric, but at both top and bottom of the wheel.

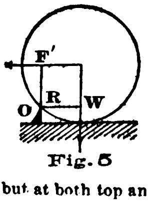


Fig. 5

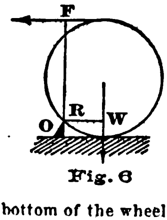


Fig. 6

(c) When the obstacles *o* are very small, as in the case of cart-wheels on smooth hard roads, or of car-wheels on iron or steel rails, the leverage

(F R) of F becomes, practically, in Fig 5 the *radius*, and in Fig 6 the *diam*, of the wheel; while that (R W) of the resistce is very small. Hence, neglecting axle fric in Fig 5, the force F reqd to overcome rolling fric in such cases is directly as the wt W of and on the wheel, and inversely as the diam of the wheel.

The few expts that have been made upon the coeffs of rolling fric, apart from axle fric, are too incomplete to serve as a basis for practical rules.

(d) The fric (or "**adhesion**") between wheel and rail, which enables a locomotive to move itself and train, or which tends to make a car-wheel revolve notwithstanding the pres of the brake, is a resistce to the *sliding* of the wheel on the rail; and is therefore not rolling but *sliding* fric; *static* when the wheels either stand still or roll perfectly on the rails; and *kinetic* when they slip or "skid".

194. The friction of liquids moving in contact with solid bodies **is independent of the pressure,** because the "lifting" of the particles of the fluid over the projections on the surf of the solid body, is aided by the pres of the surrounding particles of the liquid, which tend to occupy the places of those lifted. Hence we have, for liquids, no coeff of fric corresponding with that (= resistce ÷ pres) of solids. The resistce is believed to be directly as the area of surf of contact. Recent researches indicate that Resistce = a coeff × area of surf × vel^n, in which both n and the coeff depend upon the vel and upon the character of the surf: and that at low vels $n = 1$, but that at a certain "critical" vel (which varies with the circumstances) n suddenly becomes $= 2$, owing to the breaking up of the stream into marked counter currents or eddies. The resistance of fluid fric arises principally from the counter currents thus set in motion, and which must be brought into compliance with the direction of the force which is urging the stream forward.

195. Table of coefficients of moving friction of smooth plane surfaces, when kept perfectly lubricated. (Morin.)

Substances.	Dry Soap.	Olive Oil.	Tallow.	Lard.	Lard & Plumbago.
Oak on oak, fibres parallel to motion	.164	....	.075	.067	
" " " fibres perpendicular to motion	....	....	.083	.072	
" on elm, fibres parallel to motion	.136	....	.073	.066	
" on cast iron, fibres parallel to motion	....	....	.080		
" on wrought iron, " " "	....	....	.098		
Beech on oak, fibres " " "	....	....	.055		
Elm on oak, " " " "	.137	....	.070	.060	
" on elm, " " " "	.139				
" cast iron, " " " "	....	....	.066		
Wrought iron on oak, fibres parallel, greased and wet, .256.					
" " " " fibres parallel to motion	.214	....	.085		
" " on elm, " " " "	....	.055	.078	.076	
" " on cast iron, " " "	....	.066	.103	.076	
" " on wrought iron, " " "	....	.070	.082	.081	
" " on brass, fibres " " "	....	.078	.103	.075	
Cast iron on oak, fibres parallel to motion	.189				
" " " " " " " " greased and wet, .218	....	.075	.078	.075	
" " on elm, " " " "	....	.061	.077	....	.091
" " on cast iron, with water, .314	.197	.064	.100	.070	.055
" " on brass	....	.078	.103	.075	
Copper on oak, fibres parallel to motion	....	....	.069		
Yellow copper on cast iron	....	.066	.072	.068	
Brass on cast iron	....	.077	.086		
" on wrought iron	....	.072	.081	....	.089
" on brass	....	.058			
Steel on cast iron	....	.079	.105	.081	
" on wrought iron	....	....	.093	.076	
" on brass	....	.053	.056	....	.067
Tanned oxhide on cast iron, greased and very wet, .365	....	.133	.159		
" " on brass	....	.191	.241		
" " on oak, with water, .29					

The launching friction of the wooden frigate Princeton was found by a committee of the Franklin Institute in 1844, to average about .067 or one-fifteenth of the pressure during the first .75 of a second and .022 or one forty-fifth for the next 4 seconds of her motion. The slope of the ways was 1 in 13, or 4 degrees 24 minutes. They were heavily coated with tallow. Pressure on them = 15.84 lbs. per square inch, or 2280 lbs. per square foot. In the first .75 of a second the vessel slid 2.5 inches; in the next 4 seconds 15 feet 6.5 inches; total for 4.75 seconds 15.75 feet.

196. The friction of lubricated surfaces varies greatly with the character of the surfs and with that of the lubricant and the manner of its application. If the lubricant is of poor quality, and scantily and unevenly applied under great pres, it may wear away in places and leave portions of the dry surfs in contact. The conditions then approximate to those of unlubricated surfaces. But if the best lubricants for the purpose are used, and supplied regularly and in proper quantity, so as to keep the surfs always perfectly separated, the case becomes practically one of liquid friction, and the resistce is very small. Between these two extremes there is a wide range of variations (see table, ¶ 197 (*d*)), the coeff being affected by the smallest change in the conditions. Where any degree of accuracy is reqd, we would refer the reader to the experimental results given in Prof. Thurston's very exhaustive work,* devoted exclusively to this intricate subject.

197. (**a**) Expts by Mr. Arthur M, Wellington upon the **fric of lubricated journals**† gave a gradual and continuous increase of coeff as the vel of revolution diminished from 18 ft per sec (= a car speed of 12 miles per hour) to a stop. This increase was very slight at high vels, but much more rapid at low ones; as in Figs 3 and 4. At vels from 2 to 18 ft per sec the coeff was much less under high pressures than under low ones; but at starting there was little diff in this respect. The coeff increased rapidly as the **temperature** rose from 100° to 120° and 150° Fahr.

(**b**) Prof. Thurston, also experimenting with **lubricated journals,**‡ found that at starting, the coeff increased with increase of pres, as it did also when in motion, if the pres greatly exceeded the max (say 500 to 600 lbs per sq in) allowable in machinery. He also found that at high vels the coeff *increased* very slowly (instead of continuing to decrease) as the vel increased.

(**c**) Prof. Thurston gives the following **approx formulæ for journal friction** at ordinary temperatures, pressures and speeds, with journal and bearing in good condition and well lubricated:

$$\textbf{Coeff for starting} = (.015 \text{ to } .02) \times \sqrt[3]{\text{pres in lbs per sq in.}}$$

$$\textbf{Coeff} \text{ when the shaft is } \textbf{revolving} = (.02 \text{ to } .03) \times \frac{\sqrt[5]{\text{vel in ft per } min}}{\sqrt[2]{\text{pres in lbs per sq in.}}}$$

At pressures of about 200 lbs per sq in:

$$\textbf{Temperature of minimum fric}\text{; in Fahr degs} = 15 \times \sqrt[3]{\text{vel in ft per } min}$$

Caution. The leverage, with which journal fric resists motion, **increases with the diam** of the journal.

(**d**) The following figures, selected from a table of experimental results given by Prof. Thurston, merely show **the extent to which the coeff of journal fric is affected by pres, vel and temperature;** and hence the risk incurred in rigidly applying general rules to such cases. In these expts the character of journal and bearing, the lubricant and its method of application, remained the same throughout. Where these vary, still further, and much greater, variations in the coeff may occur.

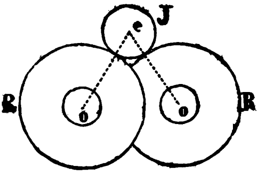
Steel journal in bronze bearing, lubricated with standard sperm oil.

Temperature. Fahrenheit.	**Speed of revolution**									
	30 feet per *minute*			100 feet per *minute*			500 ft per *min*		1200 ft per *min*	
	Pressures.									
	200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	200 lbs per sq in	100 lbs per sq in
	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff
130°	.0160	.0044	.125	.0087	.0019	.0630	.0053	.0037	.0065	.0075
90°	.0056	.0031	.094	.0040	.0019	.0630	.0075	.0061	.0100	.0150

---

* Friction and Lost Work in Machinery and Mill Work. John Wiley & Sons, New York.
Traus Amer Soc of Civil Engrs, New York, Dec. 1884.
Journal of the Franklin Institute. Nov. 1878.

(e) **Where the force is applied first on one side of the journal and then on the opposite side,** as in crank pins, the fric is less than where the resultant pres is always upon one side, as in fly-wheel shafts; because in the former case the oil has time to spread itself alternately upon both sides of the journal.



(f) **Friction rollers.** If a journal J, instead of revolving on ordinary bearings, be supported on friction rollers R, R, the force required to make J revolve will be reduced in nearly the same proportion that the diam of the axle *o* or *o* of the rollers, is less than the diam of the rollers themselves.

Mr. Wellington experimented with a patent bearing on this principle, invented by Mr. A. Higley. Diam of rollers R R, 8 ins; of their axles *o o* 1¾ ins; of the journal *c*, 3½ ins. Here, theoretically,

$$\text{fric of patent journal} = \text{fric of } 3\tfrac{1}{2} \text{ in journal} \times \frac{\text{diam of axles } o\,o}{\text{diam of rollers R R}} = \frac{1\tfrac{3}{4} \text{ ins}}{8 \text{ ins}}$$

or as 1 to 4.6. Under a load of 279 lbs per sq in, Mr. Wellington found it about as 1 to 4 when starting from rest; and about as 1 to 2 at a car speed of 10 miles per hour.

198. (a) **Resistance of railroad rolling stock.** This consists of rolling fric between the treads of the wheels and the rails (the treads also sometimes *slide* on the rails, as in going around curves); of sliding fric between the journals and their bearings, and between the wheel flanges and the rail heads; of the resistce of the air; and of oscillations and concussions, which consume motive power by their lateral and vert motions, and also increase the wheel and journal frics.

Its amount depends greatly upon the condition of the road-bed and rails (as to ballast, alignment, surf, spaces at the joints, dryness etc); upon that of the rolling stock (as to wt carried, kind of springs used, kind and quantity of lubricant, condition and dimensions of wheels and axles etc); upon grades and curvature; upon the direction and force of the wind; and upon many minor considerations. Experiments give very conflicting results.

(b) During the summer of 1878, **Mr. Wellington** experimented with loaded and empty box and flat freight cars, passenger and sleeping cars, and at speeds varying from 0 to 35 miles per hour. The cars were started rolling (by grav) down a nearly uniform grade of .7 foot per 100 feet, or 36.5 feet per mile, and 6400 ft long. Their resistces were calculated as in ¶ 185. "The rails were of iron, 60 lbs per yd, and the track was well ballasted and in good line and surf, but not strictly first class." The following approx figures are deduced from Mr. Wellington's expts upon cars fitted with ordinary journals:*

Car Resistance in pounds per ton (2240 lbs) of weight of train, on straight and level track in good condition.

Speed of train in miles per hour	Empty cars				Loaded cars			
	Axle, tire and flange	Oscillation and concuss'n	Air	Total	Axle, tire and flange	Oscillation and concuss'n	Air	Total
0	14	0	0	14	18	0	0	18
10	6	.6	.4	7	4	.6	.4	5
20	6	2.7	1.3	10	4	2.	1.	7
30	6	5.3	2.7	14	4	4.7	2.3	11

(c) With the Higley patent anti-fric roller journal, the resistce to *starting* was but about 4 lbs per ton.

(d) About midway in the track experimented upon, was a **curve** of 1° deflection angle (5730 ft rad) 3000 ft long, with its outer rail elevated 3 to 4 ins above the inner one. The rise of the outer rail was begun on the tangent, about 500 ft before reaching the curve. In the first 500 ft of the curve the resistce was greater than that encountered just before reaching the curve, by from .6 to 2.1 (average 1.1) lbs per ton. In the last 500 ft of the curve this excess had diminished to from .2 to .9 (average .6) lbs per ton. Owing to the continuance of the down grade on the curve, the vel increased as the train traversed the curve; but it does not clearly appear whether the decrease in curve resistce was due to the increase in vel, or to the fact that the oscillations caused by entering the curve gradually ceased as the train went on.

---

* Transactions, American Society of Civil Engineers, Feb 1879.

(e) **Mr. P. H. Dudley,** experimenting with his "**dynagraph**"[*] obtained results from which the following are deduced:

Train Resistance in pounds per ton (2240 lbs) of weight of train, including grades.

Description of train			Trip	Average **speed.** Miles per hour	Average **resistance.**
Loaded cars	Empty cars	Weight tons (2240 lbs)			
29	2	526	Toledo to Cleveland. 95 miles	20	8.34
37	0	638	Cleveland to Erie 95.5 miles	20	7.67
25	2	458	Erie to Buffalo. 88 miles	20	8.89

"With the long and heavy trains of the L. S. & M. S. Ry, of 600 to 650 tons, it reqd less fuel with the same engine to run trains at 18 to 20 miles per hour than it did at 10 to 12 miles per hour", owing to the fact that at the higher speeds steam was used expansively to a greater extent, and hence more economically.

199. The work, in ft-lbs, reqd to overcome fric through any dist, is = the fric in lbs × the dist in ft. In order that a body, started sliding or rolling freely on a hor plane and then left to itself, may do this work; *i e,* may slide or roll through the given dist, its **kinetic energy** (= its wt in lbs × its vel² in ft per sec ÷ 2g[†]) must = the first-named prod. Conversely, **the dist in ft** through which such a body will slide or roll on a hor plane, is

$$= \frac{\text{its kinetic energy in ft-lbs, at start}}{\text{fric in lbs}}$$

$$= \frac{\text{wt of body in lbs} \times \text{initial vel}^2 \text{ in ft per sec}}{\text{wt of body in lbs} \times \text{coeff of fric} \times 2g^\dagger} = \frac{\text{initial vel}^2 \text{ in ft per sec}}{\text{coeff of fric} \times 2g^\dagger}$$

The time reqd, in secs, is $= \dfrac{\text{dist in ft, so found}}{\text{mean vel, in ft per sec}} = \dfrac{\text{dist in ft}}{\frac{1}{2}\text{ initial vel in ft per sec}}$

Suppose two similar locomotives, A and B, each drawing a train on a level straight track; A at 10 miles, and B at 20 miles, per hour. The total resistce of each eng and train (which, for convenience, we suppose to be independent of vel) is 1000 lbs. Hence the force, or total steam pres in the two cyls reqd to balance the fric and thus maintain the vel, is the same in each eng. In traveling *ten miles* this force does the same amount of work (1000 lbs × 10 miles = 10000 pound-miles) in each eng, and with the same expenditure of steam in each; although B must supply steam to its cyls *twice as fast* as A, in order to *maintain* in them the same pres. In *one hour* the force in A does 10000 lb-miles as before, but that in B does (1000 lbs × 20 miles =) 20000 lb-miles, and with twice A's expenditure of steam.

But in fact the resistce of a given train is much greater at higher vels. See table, ¶ 198 (*b*) And even if we still assumed the resistce to be the same at both vels, B must exert more force than A in order to *acquire* a vel of 20 miles per hour while A is acquiring 10 miles per hour.

---

[*] An inst for measuring the strain on the draw-bar of a locomotive, or the force which the latter exerts upon the train.

[†] g = acceleration of gravity = say 32.2; $2g$ = say 64.4.

200. Natural Slope. When granular materials, as sand, earth, grain, etc., are deposited loosely, as when they are shoveled from a cutting or dumped from a cart, the angle, formed between a level plane and the sloping surface of the pile of material, is called the natural slope. This angle depends upon the friction and adhesion between the separate particles of the material, and often varies, in one and the same material, from time to time, with changes in weather conditions, etc., especially with dampness.

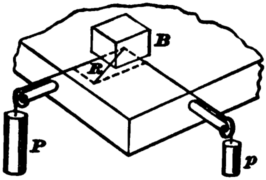


Fig. 85.

201. Any force, p, Fig. 85, acting upon a body, B, will suffice to move the body (see foot-note (*), ¶ 1), provided it exceeds the sum, S, of all resistances, including friction between B and the surface upon which B rests, or if it forms, with any other force or forces, P, a resultant, R, greater than S.

If, before the application of p, the body is already in uniform motion, P is = S; and any force, p, however small, will suffice to change the direction of motion. This accounts for the ease with which a *revolving* shaft may be slid longitudinally in its bearings, and for the fact that a cork may be more easily drawn if we first give it a twisting motion in the neck of the bottle.

LEVERS.

202. Classes of Levers. Figs. 86. Levers are classed according to the relative positions of "power," * "weight" * and fulcrum as follows:

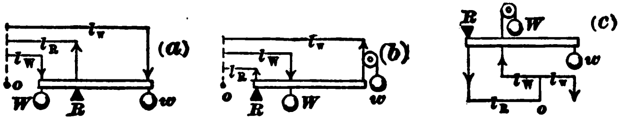


Fig. 86.

Fig. (*a*), Class 1. Fulcrum R between power w and weight W;
" (*b*), " 2. Weight W between power w and fulcrum R;
" (*c*), " 3. Power W between weight w and fulcrum R.

In class 2, the leverage of the power is necessarily greater than that of the weight. In class 3, vice versa.

203. In Fig. 86, taking the moments of the forces about any point at pleasure, as o, we have, for equilibrium:

$$\begin{aligned}&\text{Fig. } (a),\ W \,.\, l_W - R\, l_R + w \,.\, l_w = 0;\\&\text{Fig. } (b),\ W \,.\, l_W - R\, l_R - w \,.\, l_w = 0;\\&\text{Fig. } (c),\ W \,.\, l_W - R\, l_R + w \,.\, l_w = 0.\end{aligned}$$

---

* When levers are used for lifting weights or for overcoming other resistances, the force applied is called the "power," and the resistance to be overcome is called the "weight."

204. Compound levers, Fig. 87, may be used where there is not room for the arms of a single lever of sufficient length. In a compound lever, neglecting friction,

$$\frac{\text{weight}}{\text{power}} = \frac{\text{product of lengths of power arms}}{\text{product of lengths of weight arms}} = \frac{8 \times 10 \times 2}{2 \times 1 \times 4} = \frac{160}{8} = 20.$$

The three levers of Fig. 87, taken separately and beginning at the power end, give:

$$\frac{\text{weight}}{\text{power}} = \frac{8}{2} = 4; \ \frac{10}{1} = 10; \ \frac{2}{4} = \frac{1}{2};$$

and $4 \times 10 \times \frac{1}{2} = 20$, as before.

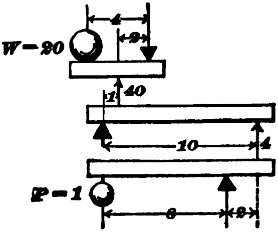


Fig. 87.

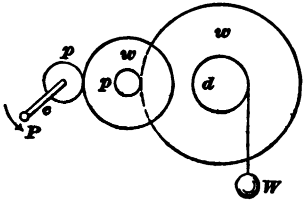


Fig. 88.

205. Toothed or Cog Gearing. Wheels and Pinions. Fig. 88. These are a series of continuous compound levers. The power is usually applied to a crank, *c*, and the weight is attached to a drum, *d*. The larger wheel, *w*, on a given shaft, is called the wheel; the smaller one, *p*, the pinion.

Let c = the radius of the crank, d = that of the drum, m = the product of the radii of the pinions, and n = the product of the radii of the wheels. Then, neglecting friction,

$$\frac{\text{weight}}{\text{power}} = \frac{c \,.\, n}{m \,.\, d}.$$

Instead of the several radii, we may of course use the corresponding diameters or circumferences; and, as the teeth are necessarily of equal "pitch" (length, measured along the circumference), the number of teeth on a wheel or pinion is usually taken instead of the radius.

When the ratio, $\frac{\text{weight}}{\text{power}}$, is great, the system is said to be of **high gear.** When that ratio is small, we have **low gear.**

Compound levers and gearing are used for converting low into high velocity, as well as for lifting great weights by means of small powers. When used for increasing the velocity, the positions of power and of weight are the reverse of those shown in Figs. 87 and 88.

206. Whenever the power and the weight balance each other, either in a single lever, or in a connected system of levers or leverages, of any kind whatever, then if we suppose them to be put into motion about the fulcrum, their respective velocities will be in the same proportion or ratio as their leverages; that is, if the leverage of the power is 2, 5, or 50 times as great as that of the weight, the power will move 2, 5, or 50 times as fast as the weight. Therefore, by observing these velocities, we may determine the ratio of the leverages. The weight and the power are to each other, therefore, *inversely* as their velocities, as well as inversely as their leverages.

207. No mechanical advantage is gained by merely increasing the *length* of a lever, as by curving it, as at *abo*, Fig. 4, ¶ 13, or by giving it an inclination to the line of action of the power, P, as at *o m, o u* or *o n.*

208. Thus, in Fig. 89, representing a bent lever, $a\,f\,b$, the *length* of the *lever*, or of any of its members, as $f\,b$, must not be confounded with the *arm* or *leverage* of the *force* acting upon the lever. These may or may not be equal. Thus, the member $f\,b$ is much longer than the member $f\,a$; yet, if the arms, $f\,a$ and $f\,c$, of the forces or weights are equal, the weights n and m must also be equal in order to insure equilibrium.

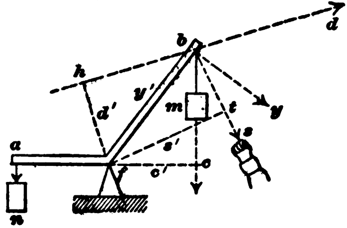


Fig. 89.

209. If the weight m be removed, a force c, or s, or y, or d, with leverage $= c', s', y', d'$, respectively, may be applied at any point, as b, to balance the moment of n. In any case this force must be such that

$$\text{force} \times \text{its leverage} = n \,.\, a\,f.$$

Hence,

$$\text{force} = \frac{n \,.\, a\,f}{\text{leverage of force}}.$$

210. Hence also the force required is *least* when, as at y, it is *perpendicular* to the length of the member $f\,b$; for the leverage (which evidently *cannot exceed* $f\,b$) is then greatest. The force required increases as it deviates in either direction from the line $b\,y$ (perpendicular to $f\,b$) and approaches more nearly to the direction of $f\,b$ itself; for its leverage then constantly decreases. No force, however great, could balance the moment of n about f, if applied in the direction $f\,b$, or $b\,f$; for such a force would have no moment about f.

211. Similarly, in Fig. 90, the moment, about a, of a load W, placed at b, is $=$ W . $a\,c$, or the same as if it were placed at c, and not $=$ W . $a\,b$.

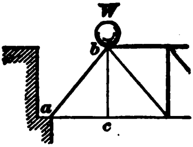


Fig. 90.

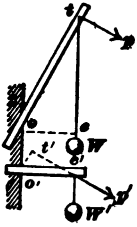


Fig. 91.

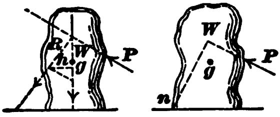
212. In Figs. 91, also, the moments W. $o\,e$ and W'. $o'\,e'$, of the equal weights, W and W', are equal. But if forces, p and p', be applied in directions perpendicular to the longer beam, $o\,t$, the leverage $o\,t$ of p becomes about 6 times that ($o'\,t'$) of p'. Hence a force, p, applied at t, has about the same bending moment as a parallel force $= 6\,p$, applied at e'.

STABILITY.

213. Stability. Figs. 92. If the resultant, R, Fig. (*a*), of the force P and weight W, falls beyond the base, as shown, then the overturning moment of P, Fig. 92 (*b*), about the toe *n*, will exceed the moment of stability of the weight W about the same point, and the body will overturn about *n*. If not, it will stand.

214. Assuming stability against overturning, the body will slide if the horizontal component, *h*, of R, Fig. 92 (*a*), exceeds the frictional and other resistances.

215. In practice, the toe, *n*, or the ground beneath it, might yield if the stone revolved upon it, or if R fell near *n* (see ¶¶ 145, etc.); but this is a question of strength of materials. Cement, clamps, etc., between the base and the ground, would add a third force, and thus change the problem.



(*a*) (*b*)

Fig. 92.

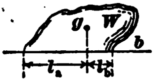


Fig. 93.

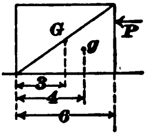
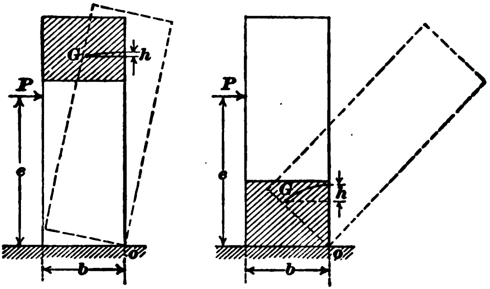


Fig. 94.

216. Owing to the greater leverage, l_a, Fig. 93, of W about *a*, the moment of stability is much greater about *a* than about *b*.

217. In Fig. 94, let G = 2 lbs.; *g* = 1 lb.; leverages = 3, 4 and 6 ft., as shown. Then the moment of stability of the rectangular body, G, against a horizontal force, P, is = 3 G = 3 × 2 = 6 ft.-lbs.; and the moment of the lower triangular body, *g*, is = 4 *g* = 4 × 1 = 4 ft.-lbs.; so that, although the larger body weighs twice as much as the smaller one, yet its moment of stability is only 1.5 times as great.



(*a*) (*b*)

Fig. 95.

218. Work of Overturning. In Figs. 95 (*a*) and (*b*), let the shaded portion of each figure be of lead, and the remainder of wood, and let the center of gravity of the entire body, in each case, be at G. Then, since the weight, W, is the same in both cases, as is also its leverage of stability, about *o*, $= \frac{b}{2}$, the moment of stability, $= \frac{1}{2} b.\mathrm{W}$, is the same in both cases, as is also the force, P, required to balance that moment when applied at a given elevation, *e*. As overturning proceeds, the weight, W, remaining unchanged, the leverage and moment of stability, and the overturning moment required, decrease, becoming = 0 when the bodies reach the positions shown by the dotted lines. If the elevation, e, remains constant, the force, P, required for overturning, decreases in the same proportion as the leverage, etc.

219. But in order that the bodies may be overturned by the force of gravity alone, they must be brought into the positions shown by the dotted lines. This requires that the weights of the bodies be lifted through a height = the distance, h, through which their centers of gravity, G, are raised. Hence

$$\text{work of overturning} = W.h.$$

Since h is greater in Fig. (b), the work of overturning is greater in that case. In civil engineering we are generally concerned with the amount of the *force* which will *begin* overturning, rather than with the amount of *work* required to *complete* the overthrow.

220. Stability against overturning is of course affected, and may be increased, by forces other than the weight of the body itself. Thus, the stability of a bridge pier is ordinarily increased by the weight of the bridge itself if this be brought upon the pier symmetrically. Otherwise the weight of the bridge may either increase or diminish the stability of the pier, according to circumstances.

221. The coefficient of stability, in any given case, is the ratio of the moment of stability to the overturning moment. Or,

$$\text{Coefficient of stability} = \frac{\text{moment of stability}}{\text{overturning moment}}.$$

222. Let the weight, W, of the stone in Fig. 96 be 10 lbs., G its center of gravity, and $o\,g$ = 2 feet. Then the moment of the weight about o, or the moment of stability about o, is 10 × 2 = 20 ft.-lbs.; and, if $o\,n$ = 5 feet, a force $P = \frac{20}{5} = 4$ lbs., will just hold in equilibrium the moment of the weight; so that, except at the corner, o, no pressure will be exerted upon the base $o\,m$, although the stone remains in contact with the base. If the force P exceeds 4 lbs., the stone will begin to turn about o. If P is less than 4 lbs., the stone will exert a pressure upon the base $o\,m$.

Let the stone be supported at o and at m only. The leverage of the supporting force R, at m, is = the length $o\,m$ of the base, = l. Let P = 1 and base $o\,m$ = 4.5 ft. Then, for equilibrium,

$$W\,.\,o\,g - P\,.\,o\,n - R\,.\,o\,m = 0;$$

or,

$$20 \text{ ft.-lbs.} - 1 \times 5 = 4.5 \times R;$$

or.

$$R = \frac{20 - 5}{4.5} = 3.33\,.\,.\,.\,.\ \text{lbs.}$$

In other words, a vertical upward force, R, of 3.33 . . . lbs., at m, will maintain equilibrium.

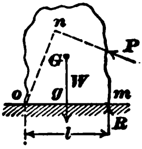


Fig. 96.

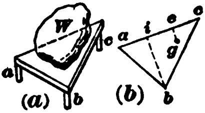


Fig. 97.

223. In Fig. 97 (b), let g be the center of gravity of the load W and the table, combined, Fig. 97 (a). Then, upward reaction of $b = \frac{W\,.\,e\,g}{i\,b}$. Those of a and c may be similarly found.

224. In Fig. 98, let h be the horizontal force exerted at the crown by the left-hand half of the arch, against the half-arch shown, and e its leverage about o. Let W be the weight of the half-arch with its spandrel, acting as a single rigid body, and l its leverage about o. Then, for equilibrium, we have

$$h \cdot e = W \cdot l; \text{ or } h = \frac{W \cdot l}{e}.$$

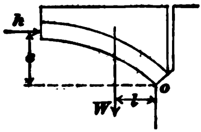


Fig. 98.

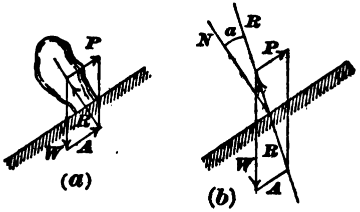


Fig. 99.

Stability on Inclined Planes.

225. Stability on Inclined Planes. Fig. 99. Here, as in ¶ 213, if the resultant, R, of the force P and weight W, falls beyond the base,—*i. e.*, if the overturning moment exceeds the moment of stability,—the body will overturn. If not, it will stand.

The force, P, in any given direction, required to prevent overturning, is = the anti-resultant, A, of weight W and reaction R; and reaction R = anti-resultant of force P and weight W.

226. Neglecting friction, as in Fig. 99 (a), R will be normal to the plane. Taking friction into account, Fig. 99 (b), R may form, with a normal, N, to the plane, an angle, a, not exceeding the angle of friction between the body and the plane. R may be either uphill or downhill from N.

227. In Fig. 100, the body B has less stability against overturning about its toe, a, than has the similar body, A, when the force, n, tends to upset it downhill; but a greater stability than A against overturning about c under the action of a force tending to upset it uphill.

228. The body C, which would upset if upon a horizontal base, would be stable against overturning if placed upon an inclined plane, as at D. Assuming $a\,o = t\,c$, a given upward vertical force would have the same overturning moment, whether applied at a or at c. But a given horizontal force, applied at any given height, as at g, has a greater leverage, $g\,o$, when pushing downhill than when pushing uphill. In the latter case its leverage is only $g\,t$.

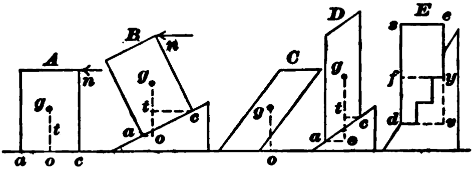


Fig. 100.

229. Structures built upon slopes are liable to slide. This may be obviated by cutting the slope into horizontal steps, as at $d\,y$, Fig. E; but the vertical faces of such steps break the bond of the masonry; and, moreover, the joints being more numerous, and the mortar therefore in greater quantity, on the deeper side, $s\,d$, than on the shallower uphill side, $e\,y$, the structure is liable to unequal settlement, the downhill side settling most and tending to split away from the uphill portion, as might be the case with a foundation firm in some parts and compressible in others. Hence, when circumstances permit, it is preferable to level off the foundation, as at *d v;* or, if the structure has to withstand downhillward pressures, *v* should be lower than *d*, and the courses of masonry laid with a corresponding inclination.

THE CORD.

230. The Cord. Figs. 101 (*a*) and (*b*) and 102 (*a*) and (*b*). In ¶¶ 230 to 239 we deal with cords supposed to be perfectly flexible, inextensible, frictionless, weightless and infinitely thin.

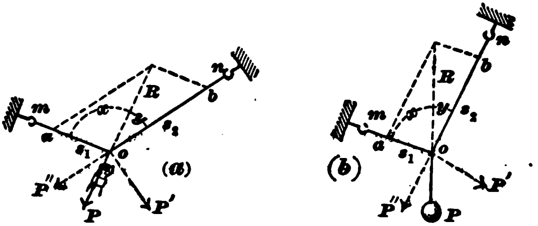


Fig. 101.

231. Let P be the external force applied to the cord at the knot or pin, *o*, and let R be the resultant of the stresses, s_1 and s_2, or *o a* and *o b*, in the two segments, *o m* and *o n*, of the cord. Then, for equilibrium, R must be equal to and colinear with P.

232. Knowing the amount of P (= R), the tensions s_1 and s_2 may be found by means of ¶ 36; and, vice versa, given s_1 and s_2, we may find R (= P) by ¶ 35. Or see ¶ 40.

233. If, as in Figs. 101 (*a*) and (*b*), the force P be applied to the cord, at *o*, by means of a fixed knot, incapable of sliding along the cord, so that the segments, *o m* and *o n*, of the cord, are of fixed lengths, and the angle, $x + y$, between them, of fixed magnitude, then the force may be applied in any direction, as P or P′, passing between the two segments of the cord; and the components, s_1 and s_2, will be equal only when R (P produced) forms equal angles, *x* and *y*, with the two segments of the cord. If the direction of the force, as P″, coincides with either segment, as *o n*, of the cord, that segment transmits the entire force, P″, and the other segment none.

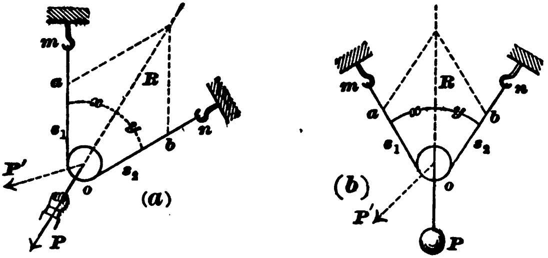


Fig. 102.

234. But if, as in Figs. 102 and 103, the force P be applied to the cord by means of a frictionless ring, slip-knot, pin or pulley, etc., then, for equilibrium, the two stresses, s_1 and s_2, must be equal, as must also the two angles, *x* and *y;* and, if we suppose the direction of the force P to be changed, as to P′, the pin and the cord will readjust themselves, as indicated by the dotted lines in Fig. 103, until the pin finally comes to rest at that point, *o*′, where the angles, x' and y', are equal, and also the stresses, s_1' and s_2'.

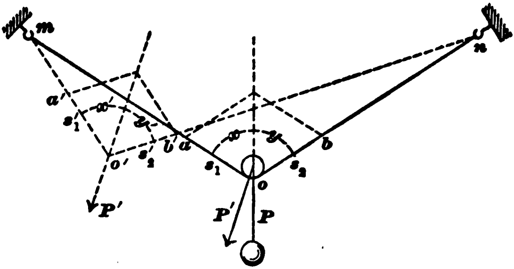


Fig. 103.

235. Even though the pin or pulley be rigidly fixed to some external object, as at *o*, Fig. 104, yet, if there is no friction at its axle, or between it and the cord, the components, s_1 and s_2, will still be equal, and their resultant, R, will bisect the angle, $x + y$, between them. In other words, the angles, x and y, will be equal.

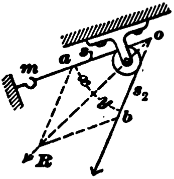


Fig. 104.

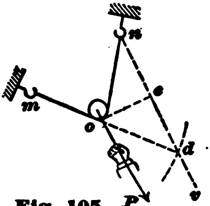


Fig. 105.

236. When the pin is movable, Figs. 102 and 103, to find the position, *o*, Fig. 105, which it will assume. From the end, *n*, of one of the segments, *o n*, of the cord, draw *n v* parallel to P. From the end, *m*, of the other segment, with radius $= m\,o + o\,n$, = length of cord, describe an arc, cutting *n v* in *d*. Bisect *n d* in *e*. Draw *e o* normal to *n v*, intersecting *m d* in *o*. Then *o* is the required point.

237. Whether *o* be a fixed knot or a movable pin or pulley, it is always in the circumference of an ellipse whose foci are at the ends, *m* and *n*, of the cord.

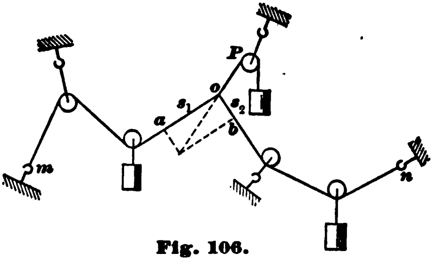
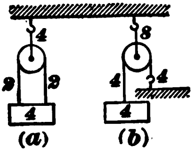


Fig. 106.

238. From the foregoing it follows that, if *o*, Fig. 106, be a fixed knot, and if the other pins or pulleys, etc., are frictionless, the stress *a o*, or s_1, will be transmitted uniformly throughout the left segment of the cord, from *o* to its end at *m*; and *b o*, or s_2, throughout the right segment, from *o* to *n*.

239. Caution. Note that, in Fig. 107 (*b*), the stresses in all the cords are twice as great as the stresses in the corresponding cords in Fig. 107 (*a*), although each Fig. shows a load = 4 suspended from the pulley. Thus, if the weight be that of a man, hanging by the rope, and if the rope, in Fig. (*a*), be just sufficiently strong to hold, it will break if he gives one end of the rope to another man to hold, or makes it fast, as in Fig. (*b*).



(*a*) (*b*)

Fig. 107.

The Funicular Machine.

240. When the angles, x and y, Figs. 101, etc., are very great, a very small force, P, will balance a very great stress, s_1 or s_2, in the cord. When $x = y = 90°$, we have $\cos x = \cos y = 0$, and $s_1 = s_2 =$ infinity, however small P may be. If a line, *m n*, joining the ends of the cord, is horizontal or inclined, the weight of the cord itself acts as a force P. Hence

"There is no force, however great, can stretch a cord, however fine, into a horizontal line that shall be absolutely straight."

241. The funicular machine takes advantage of the fact that, when the total angle, $x + y$, between the two segments of the cord, approaches 180°, a small force, P, may balance great stresses, s_1 and s_2. Thus, in Fig. 108, let W represent a heavy boat (seen in plan) which is to be hauled ashore. One end of a rope being made fast to the bow of the boat, the rope is passed around one smooth post, *n*, to another, *m*, around which it is given one or more whole turns; and a man stands at the end, *e*, to take in the slack; while others, taking hold of the rope between *m* and *n*, pull it, in the direction of P, into a position *m o n*. If the two angles, x and y, are equal, the component in the segment *o n* exceeds P, so long as the angle x exceeds 60°, and a pull, equal to this component (except in so far as it is reduced by the rigidity of the rope and by its friction against the post *n*), is exerted upon the boat at W, drawing it a short distance up the beach. The rope is then straightened again, from *m* to *n*, by taking in the slack at *e*, and the operation is repeated as often as may be necessary.

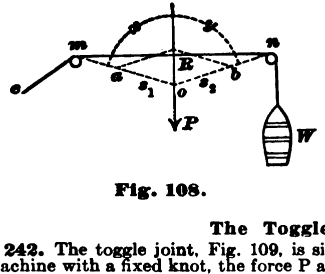


Fig. 108.

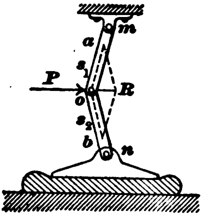


Fig. 109.

The Toggle Joint.

242. The toggle joint, Fig. 109, is simply an inversion of the funicular machine with a fixed knot, the force P and the components, s_1 and s_2, being pushes or compressions, instead of pulls or tensions. The joint being unable to move along the arms, the force P may be applied in any direction at pleasure, but it is usually exerted in a direction forming approximately equal angles with the two arms.

The Pulley.

243. Figs. 110 show the relations of stresses and weights in several arrangements of fixed and movable pulleys. Thus, in (*a*), 1 ℔. balances 1 ℔., in (*b*) 2 ℔s., in (*c*) and in (*d*) 4 ℔s. In each case, if the bodies or weights be set in motion, their velocities are inversely as their masses. See ¶ 206.

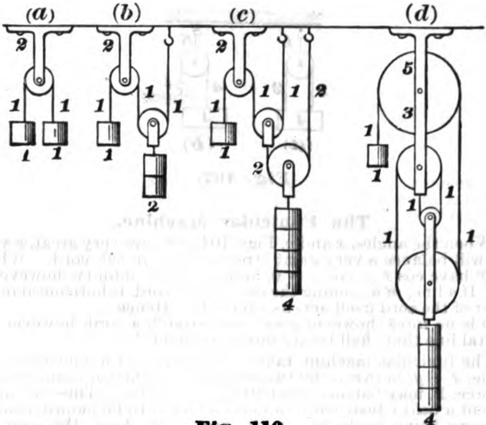


Fig. 110.

244. The simple pulley, Fig. 110 (*a*), is used simply for convenience of changing direction of stress, for the forces at the two ends of the cord are equal; but in the compound pulley, Figs. 110 (*b*), (*c*), (*d*), a small force (the "power"), moving rapidly, at one part of the rope, balances a greater force (the "weight"), moving slowly, at another part. Hence, the compound pulley is used for the purpose of overcoming great resistances slowly, by means of small forces, moving rapidly.

245. To set such a system in motion * (*i. e.*, to raise the "weight") requires that the equilibrium be disturbed by making the "power" exceed the stress in the cord due to the "weight." But the motion, once generated, will continue indefinitely if the "power" is made sufficiently greater than the "weight" to balance the resistances of friction, etc.

The Loaded Cord or Chain.

246. In Figs. 111 the principle of the cord polygon, ¶¶ 86, etc., is applied to the case of a flexible cord or chain, sustaining four loads, $p_1 \ldots p_4$, at fixed points, and exerting a horizontal † pull, H, at its lower end, and an inclined pull, R, at its upper end. The loads, $p_1 \ldots p_4$, are represented by the vertical line, 0–4, Fig. 111 (*a*); the horizontal pull, H, by 0–*c*; the amount and direction of the inclined pull, R, at the upper end of the cord, by 4–*c*. and the tensions in the segments, 1–2, 2–3 and 3–4, by the rays, 1–*c*, 2–*c* and 3–*c*, respectively.

247. The horizontal tension, H (= the horizontal component of the tension in each segment), is uniform throughout the cord; but the vertical component of the tension in any segment is equal to the sum of the loads between that segment and the pulley, *m*. Thus, the vertical component

---

* See foot-note (*), ¶ 1.

† See foot-note (*), ¶ 249.

(0–2, Fig. *a*) of the tension, *c*–2, in segment 2–3, is = $p_1 + p_2$; that in segment 3–4 is 0–3 = $p_1 + p_2 + p_3$, etc.

248. If all the loads (including W) be increased in the same proportion, as indicated by the dotted lines in Fig. 111 (*a*), or diminished in the same proportion, the new triangles, *c*′ 4′ 0, etc., Fig. (*a*), will be similar to the old, and the profile of the cord, Fig. (*b*), will remain unchanged, although the stresses in its segments will of course be increased or diminished in the same proportion.

249. In Fig. 111 we make the weight, W, which is necessarily equal to the horizontal* pull, H (see The Cord, ¶¶ 230, etc.), equal also to the sum of the loads, $p_1 \ldots p_4$. When this is the case, the cord segment, *a*–4, next to the support, *a*, and the corresponding line, *c*–4, Fig. (*a*), will be inclined 45° to the vertical.

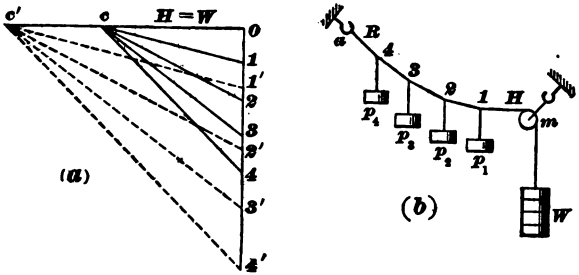


Fig. 111.

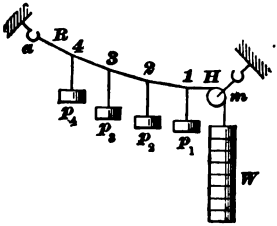


Fig. 112.

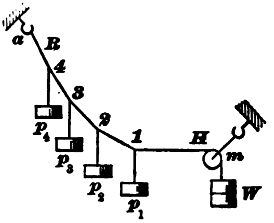


Fig. 113.

250. But if, while the loads, $p_1 \ldots p_4$, remain unchanged, we raise the pulley *m*, so as to keep H horizontal,* we shall obtain a flatter curve, as in Fig. 112; and, for equilibrium, H (= W) must be made *greater* than the sum of $p_1 \ldots p_4$. On the other hand, if we place the pulley, *m*, lower than in Fig. 111 (still keeping H horizontal), we obtain a deeper curve, as in Fig. 113; and H (= W) must be made *less* than the sum of $p_1 \ldots p_4$.

---

* In Figs. 111, 112 and 113 we suppose the weight, W, and the position of the pulley, *m*, to be so adjusted, relatively to the support, *a*, that the pull, H, shall remain horizontal.

ARCHES, DAMS, ETC. THRUST AND RESISTANCE LINES.

The Arch.

In ¶¶ 251 to 257 are given the elements of the commonly accepted theory of the arch. For practical considerations, see ¶¶ 258 to 266, and Stone Bridges.

251. If Figs. 111, 112 and 113 be inverted, the cord segments will represent struts, sustaining compression, as do the stones of a masonry arch, Fig. 114.

Thrust Line. Resistance Line.

252. In the case of an arch, Fig. 114, assuming * that the horizontal thrust H, at the crown, *m*, and the reaction, R, of the skewback, *a*, act at the center (or at some other definite point) of crown and of skewback, respectively, their amounts, and the direction of the reaction, R, may be found by means of the Force Triangle, ¶ 51, or by Moments, ¶ 224. (See ¶ 257.) We then suppose the half-arch and its spandrel to be divided, by vertical planes,* Fig. 114 (*b*), into a number of segments, as shown; and, finding the weight and the center of gravity of each such segment (see ¶¶ 257 and 266), we treat these segments as we treated the loads, $p_1 \ldots p_4$, of Figs. 111 to 113, laying them off from 0 to 6, Fig. 114 (*a*), and laying off 0–*c* horizontal and = H. The rays, *c*–1, *c*–2, etc., then give, theoretically,* the directions and amounts of the pressures exerted by the segments, 1, 2, etc., respectively.

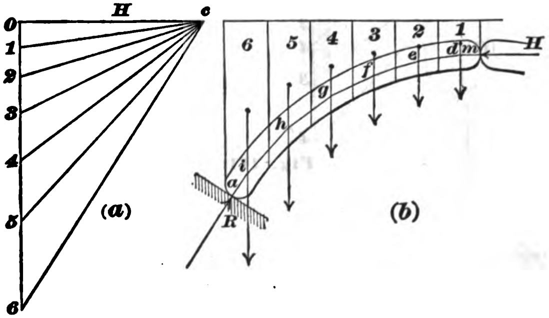


Fig. 114.

The broken line, *m d e f . . . . a*, Fig. 114 (*b*), thus formed, is called the **thrust line,** or line of resultants. It corresponds with the cord polygons of Figs. 111 (*b*), etc.*

253. **The resistance line** is a broken line joining the points where the several resultants, forming the thrust line, cut the respective joints between the arch stones.

254. When the planes, by which the arch is supposed to be divided into segments, are vertical,* as in Fig. 114 (*b*), and, indeed, in most actual arches, the thrust and resistance lines, Fig. 115, practically coincide; but if these planes are far from vertical, as in Fig. 115, the two lines separate, the resistance line being always the outer one.

Thus, in Fig. 115 (where the thrust line is shown solid, and the resistance line dotted), noticing where resultant *a* cuts joint A, where resultant *b* cuts joint B, etc., it will be seen that the two lines practically coincide as far as to joint C, where they begin to diverge.

---

* See Practical Considerations, ¶¶ 258, etc.

255. In ¶ 252 we assumed that the arch and its spandrel are divided into vertical segments, incapable (except in the arch ring) of exerting other than vertical pressures. The theoretical resistance line, thus obtained, may, especially in deep arches, pass from the thickness of the arch ring in places; so that, if no other forces were acting, the arch would open at such places; on the intrados when the resistance line cuts the extrados, and vice versa; but such opening is usually prevented by other forces, such as the horizontal or inclined pressures of the spandrels. The actual resistance line is thus confined within the thickness of the arch ring. In general, the actual resistance line, Fig. 116, approaches the extrados at the crown, and the intrados at the haunches, so that the arch tends to sink at the crown (opening there on the intrados), and to rise at the haunches (opening there on the extrados), as shown.

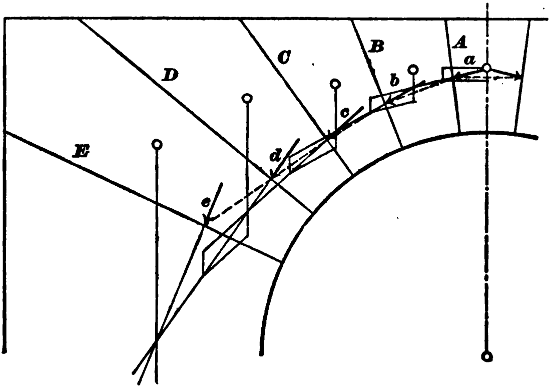


Fig. 115.

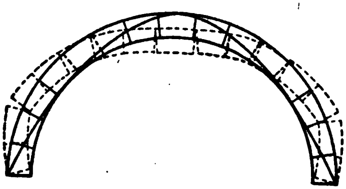


Fig. 116.

256. In order to avoid any tendency of the joints to open at either side, the arch should be so designed that the actual resistance line shall everywhere be within the middle third (see ¶¶ 145, etc.) of the depth of the arch ring.

257. In general, the design of an arch is reached by a series of approximations. Thus, a form of arch and spandrel must be assumed in advance, in order to find their common center of gravity for the purpose of determining the horizontal thrust, H, and the skewback reaction, R, as in ¶ 252; and, if it is afterward found necessary to modify the form first assumed, in order to satisfy the requirements of ¶ 256, or for other reasons, we may have to recompute H and R, again modifying the design, and so on.

Practical Considerations.

258. While the theoretical thrust and resistance lines, based upon the foregoing assumptions, are easily found, much uncertainty exists as to the positions of the actual thrust and resistance lines in a masonry arch.

259. In the first place, we do not know through what points in the crown and skewback, respectively, the resultants, H and R, pass.

260. Again, we have assumed that the loads on the arch, like those on the cord, Figs. 111 to 113, are incapable of acting otherwise than vertically; whereas the spandrel walls and filling, which form a large portion of the load on a masonry arch, may offer resistances acting in other directions. If the loading were a liquid, like water, its pressures upon the arch ring would be radial, like those of the particles of steam, in a boiler, upon the boiler tubes; and this condition is probably more or less closely approximated in the case of a loading of clean dry sand; and, less closely, in the case of earth filling. Hence, although the determination of the theoretical thrust and resistance lines in an arch is facilitated by the assumption that the arch is correctly represented by the inverted cord, the distinction between the two cases must be borne in mind when drawing practial conclusions from the lines so found.

261. Thus, in many cases, the theoretical thrust and resistance lines cut the intrados or the extrados in places, thus passing entirely out of the arch ring; so that this would inevitably fall (see ¶ 255), were it not for horizontal or inclined resistances exerted by the upper parts of the abutments through the spandrel walls and filling.

262. Hence, in order to determine the actual resistance line, we should not only have to know through what points, in crown and in skewback respectively, the resultants, H and R, pass, but we should also have to ascertain and take into account the possible horizontal and inclined resistances of the spandrel walls and filling. But, as this is ordinarily impracticable, we content ourselves either with determining the theoretical thrust and resistance lines, as directed above, and then estimating, as well as may be, the resistances of the spandrels, or with reasoning by analogy from the behavior of actual structures. See Stone Bridges.

263. If the inverted cord correctly represented the actual thrust line in a masonry arch, the arch stones, in elliptic or in deep segmental arches, would have to be made inordinately deep, in order that the resistance line should nowhere leave the middle third of their depth (see ¶¶ 145, etc.); and it might therefore appear rational to make the profile of the arch correspond approximately with the thrust line, which usually approaches a parabola. But, owing to the spandrel resistances, the actual thrust line, even in semicircular arches, probably seldom greatly oversteps the middle third.

264. With a wall or a deep continuous filling, over an arch, if the arch were to settle, or were to be removed, the wall and the filling above it would form an arch, as indicated by the broken lines in Fig. 117; and only that portion below this arch would fall out. Hence, only this portion can properly be regarded as pressing upon the arch.

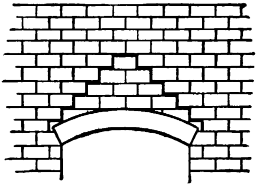


Fig. 117.

265. Neglecting the strength of the mortar, the inclination of each joint between two arch stones must of course be such that the angle, between the thrust, at any joint, and a normal to that joint, shall be less than the angle of friction. See ¶¶ 183, 184.

266. It is often the case that the spandrels or the spandrel filling are of less specific gravity than the arch ring. In such cases, in order to facilitate the finding of the lines of gravity of the segments, we may, before dividing the half-arch and its spandrels into vertical segments (¶ 252), consider the lighter structure of the spandrels as being reduced to an equivalent depth of material having equal specific gravity with the arch. The areas of the several segments, as seen in profile, and as thus reduced, may then be taken as representing their weights. Thus, in Fig. 118, where *t t t* represents the top of the spandrels, the curved line *e e e* represents the top of a filling of equal weight per foot run with the spandrels, but of equal specific gravity with the arch ring. When, as in Fig. 119, the spandrels consist of a series of transverse arches, we may assume that the main arch carries a series of loads concentrated at the piers of these transverse arches.

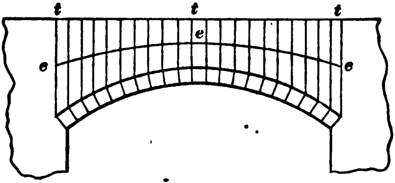


Fig. 118.

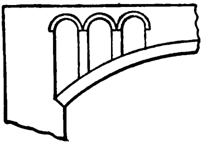


Fig. 119.

The Masonry Dam.

267. A dam must be secure against sliding, on its base or on any plane within the body of the dam, against overturning, and against crushing of the material at any point and consequent opening of a seam at either face of the dam.

268. The dam will be secure against sliding if the resultant of all the pressures, upon any surface, forms, with a normal to that surface, an angle less than the angle of friction of the surface. See ¶¶ 183, etc. In practice, the base of the dam is let well down into the rock foundation, as indicated in Fig. 122 (*a*), and continuity of joints is avoided by making all the stones break joints. The angle of friction thus becomes, in effect, 90°, and sliding cannot occur without shearing the stones themselves.

269. If the material is sufficiently strong to resist crushing, under the maximum unit stresses brought upon it, and if the resultant of all the forces acting upon any section falls within the body of the dam, the dam will be secure against overturning. But see ¶ 270.

270. For a given total pressure upon any section, the maximum unit pressure in the section would be least when the resultant cut the middle point of the section. See Center of Pressure, ¶¶ 133, etc. It is generally impracticable to secure this; but the dam must be so designed that, under the maximum unit pressure, the given material shall not be taxed beyond its safe crushing strength. If this is done, and if, under all conditions, the center of pressure is kept within the middle third (see ¶ 150) of each horizontal section throughout the dam, there will be no tendency to open on either face of the dam.

271. Let Fig. 120 represent a stone block, resting upon a solid foundation and intended to sustain the pressure, p, of quiet water on one side. Through the center of gravity, g, of the block, draw o' N vertically, to represent the weight, W, of the block. Then the point, s, where o' N meets the foundation, is the center of pressure for the block alone, *i. e.*, when the water is removed.

272. Let h be the depth of water back of the block, and let the block be one foot in length, measured normally to the paper. Then the amount, in pounds, of the water pressure, against the vertical back, $a\,b$, is $p = 62.5\,h \times \frac{1}{2}\,h$, and its center of pressure is at a depth, $d = \frac{2}{3}\,h$, below the water surface.

273. Combining p with W (¶ 35) we obtain R as their resultant, and r as the center of pressure upon the foundation when the block is sustaining the water pressure.

274. Let Fig. 121 represent several such blocks superposed. Let

g_1 = cen of grav, p_1 = cen of water pres, for block 1;
g_2 = " " " p_2 = " " " " " blocks 1 and 2 combined;
g_3 = " " " p_3 = " " " " " " 1, 2, and 3 combined,
etc.

Then, finding r_1 and s_1, r_2 and s_2, r_3 and s_3, etc., for joints 1–2, 2–3, 3–4, etc., as before, we have the points r_1, r_2, r_3, etc., in the resistance line for full dam, and the points s_1, s_2, s_3, etc., in the resistance line for empty dam.

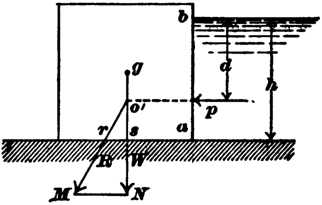


Fig. 120.

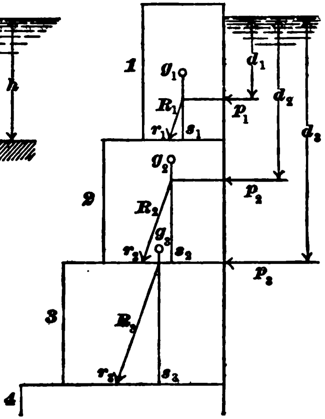


Fig. 121.

275. In Fig. 122 (*a*) the curves, $u\,u$ and $d\,d$, indicate the up-stream and down-stream limits, respectively, of the middle third of the plane separating each two blocks, assuming the profile with vertical back; and the points $r_1 \ldots . r_7$ and $s_1 \ldots . . s_7$ are points in the corresponding resistance lines for full dam and for empty dam respectively.

276. While theory would require the cross-section of the dam to terminate in a sharp angle at the top, it is, of course, always made heavier in practice, as indicated in Fig. 122 (*a*).

277. For joint 2–3 we may suppose that block 2 had at first been designed rectangular, as shown by dotted line $c\,a$ (Fig. 122 (*c*), showing blocks 1 and 2 enlarged); but this makes the center of pressure, r', for the full dam, fall beyond the middle third of the narrow base, $a\,b$. We therefore try the trapezoidal shape $c\,i\,b$, with its wider base, $i\,b$, and find that, with this, the center of pressure, r_2, although further down-stream than before, falls within the middle third of said wider base. The remainder of the profile is determined by similar trials.

278. Graphic Method. Suppose the cross-section to be divided, by horizontal sections, into numerous blocks, 1, 2, 3, 4, etc., of a depth approximately = the top width of the dam. In Fig. 122 (*b*), draw 0–1, 1–2, 2–3, 3–4, etc., vertically, to represent, by scale, the weights of the several blocks, respectively, and 0–1′, 1′–2′, 2′–3′, 3′–4′, etc., horizontally, to represent the water pressures against said blocks respectively; and draw 1′–1, 2′–2, 3′–3, 4′–4, etc., representing, in amount and in direction, the total inclined pressures upon joints 1–2, 2–3, 3–4, etc., and upon the base, respectively. Thus, 2′–2 represents the resultant of the water pressure (see Hydrostatics) upon blocks 1 and 2, and the combined weight of those blocks. From the points, o_1, o_2, etc., Fig. 122 (*c*), where these two forces meet in each case, draw $o_1 r_1$, $o_2 r_2$, etc., parallel respectively to 1′–1, 2′–2, etc., Fig. 122 (*b*), to the corresponding joint. We thus obtain points, $r_1 \dots r_6$, in the resistance line for the case where the dam is filled to the assumed depth.

The foregoing refers to the diagram for a dam already completed or designed. In designing *de novo*, we of course begin at the top, and lay off the lines 0–1, 0–1′ and 1′–1 in Fig. (*b*) for the first block; then lines 1–2, 1′–2′ and 2′–2, for the second block, and so on; making necessary changes, as in ¶ 277.

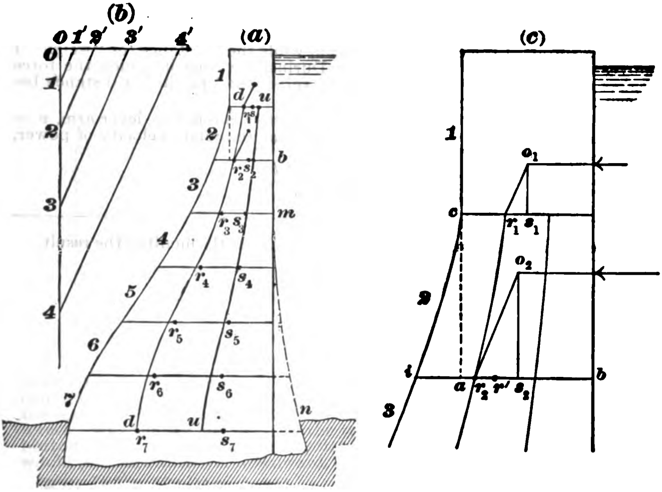


Fig. 122.

279. In order that the resistance line, $r_1 \dots r_6$, for the full dam, may be brought well within the middle third, it may sometimes be necessary to adopt a somewhat unwieldy cross-section; but, in view of the imminent danger involved in the smallest opening on the up-stream side of the dam, (see ¶ 281), it is well here to err on the safe side.

280. As this process is carried further down, the angle formed between the down-stream face and the vertical becomes considerable; and the middle third, in each of the lower joints, is thus brought further down-stream. The centers of pressure, $s_1 \dots s_6$, for empty dam, may then fall beyond the middle third, on the up-stream side, as indicated at joints 5–6 and 6–7. To obviate this, the up-stream face is sometimes given a curved profile, as at *m n*.

Practical Considerations.

281. The assumption of ideal conditions is particularly dangerous in the case of masonry dams. Thus, any compression of the material at the down-stream face may open seams on the up-stream face; and water, entering these seams, will exert a wedge-like action, shifting the resistance line further down-stream, thus still further increasing the tendency to crushing on the down-stream face and to opening on the up-stream face. Again, if any relatively smooth joints have been left, the water, thus penetrating into or under the dam, increases the tendency to slide, not only by diminishing the effective weight of the upper portions, but also by acting as a lubricant upon the seam where it penetrates.

It has been suggested that failures of dams may have been occasioned, in part at least, by vacuum, formed in front of the down-stream face, by the action of the sheet of water falling in front of that face.

282. Theoretically, the deflections of arches, dams and other structures composed of blocks, may be found by means of the formulas in ¶¶ 162–167 of Trusses; but, owing to uncertainty as to the values of the moduli of elasticity, E, of building stones and of mortar, and to the relative inaccuracy of finish in masonry work, the formulas are of but little practical value in such cases.

THE SCREW.

283. The screw is a spiral inclined plane. The force (or "power") describes a spiral, at the end of a lever arm, while the resistance (or "weight") moves along the axis of the screw. During the time in which the force makes one revolution, the resistance traverses the "pitch," or distance between the centers of two adjacent threads.

284. Hence, if P = power, w = weight, d = pitch, l = lever arm, v = rectilinear velocity of weight, and V = linear (circular) velocity of power, we have, theoretically:*

$$\frac{w}{P} = \frac{V}{v} = \frac{2\pi l}{d}.$$

---

* Neglecting friction, which, however, very greatly modifies the result.

FORCES ACTING UPON BEAMS AND TRUSSES.

Conditions of Equilibrium.

285. In beams and trusses, for equilibrium, it is necessary and sufficient that the resisting forces, exerted by the material of the structure, and the moments of those forces, shall balance the external or destructive forces and their moments. We here discuss chiefly the destructive forces. For the resisting forces, see Stresses, under Trusses, and Beams or Transverse Strength, under Strength of Materials.

286. The destructive forces are (1) the loads upon the structure, including its own weight, "live" or moving loads, wind, etc., and (2) the reactions of the supports. We shall here discuss the action of vertical loads only, including (*a*) the dead load, or the weight of the structure itself, together with the roadway, etc., and (*b*) the live, moving or extraneous load of vehicles, trains, persons, etc. The action of horizontal loads (wind, centrifugal force, etc.) is governed by similar laws, and is discussed under Stresses, in Trusses.

287. Let Fig. 123 (*a*) represent a cantilever, resting upon a support, *b*, and bearing a load, W, at its outer end, *a*. The cantilever is prevented from turning about *b*, by the tension, T, of a horizontal chain, and by the compression, C, in a horizontal strut.[*] Neglecting the weight of the cantilever itself, the cantilever is acted upon by four external forces, forming two couples; one couple consisting of two vertical forces—viz., the load, W, and the reaction, R′, of the support; the other couple consisting of two horizontal forces—viz., the tension, T, near the top, and the compression, C, near the bottom. Were it not for the reaction, R′, of the support, *b*, the load, W, would pull the cantilever downward, as indicated in Fig. 123 (*b*).

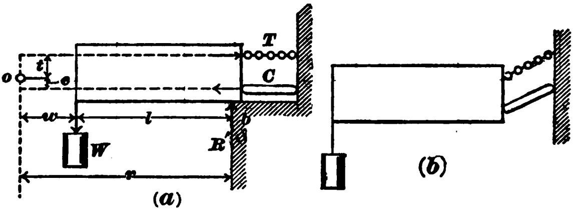


Fig. 123.

288. In Fig. 123 (*a*) we have:

Algebraic sum of vertical forces $= R' - W = 0$;

" " " horizontal " $= T - C = 0$;

" " " moments, about any point, as *o*,

$$W.w - R'.r + T.t + C.c = 0.$$

---

[*] In Figs. 123 to 127, inclusive, and Figs. 132 and 133, showing cantilevers, beams and part beams, acted upon by loads, by reactions, by pulls of chains and by pushes of struts, the arrows denote forces acting *upon the cantilever or beam or upon its segments*, and not forces acting upon the load, the supports, or the connecting chains or struts. Thus, the tension in a chain tends to *draw together* the two bodies which it connects. Hence, in these cases, the corresponding arrows point *toward each other*. On the other hand, the compression in a strut tends to *separate* the two bodies between which it acts. Hence its two arrows point *away from each other*.

289. If, as in Fig. 124, the horizontal forces are exerted at the end farthest from the support, and at the same distance apart as before, their amounts and senses must remain respectively the same as before; but we now have compression, C, at the top, and tension, T, below. Or, if Fig. 123 be inverted, R′ acts as the load, and W as the upward reaction; and we have, as in Fig. 124, compression, C, at top, and tension, T, below. Thus, Fig. 124 is practically Fig. 123 inverted.

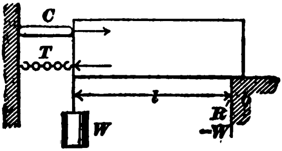


Fig. 124.

290. The condition described in ¶ 289, Fig. 124, represents also the condition in each segment, A, B, of a beam, Figs. 125 (*a*) and (*b*) or Figs. 126 (*a*) and (*b*), supported at both ends and bearing a concentrated load, W + W, Fig. 125, or W + *w*, Fig. 126.

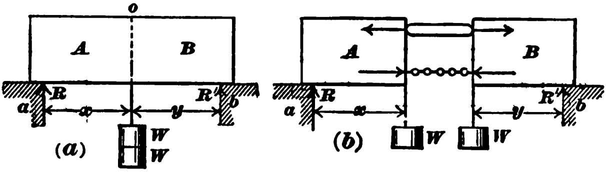


Fig. 125.

291. Suppose the beam, Fig. 125 (*a*) or Fig. 126 (*a*), to be divided into two cantilevers, or part beams, as in Fig. 125 (*b*) or Fig. 126 (*b*); each part sustaining, at its end, a part of the original load. (See ¶ 292.) The stresses in the strut and chain, Figs. (*b*), take the place of stresses in the material (situated in the dotted line) of the truss or beam, Figs. (*a*). In a truss, these forces are exerted by the chords: in a beam, by the particles or fibers throughout the section.

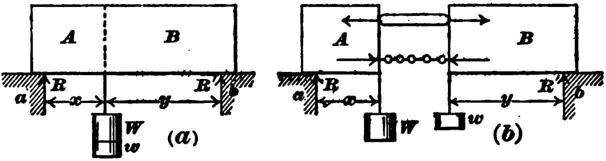


Fig 126.

292. If, as in Fig. 125 (*a*), the load is at the center of the span, the spans, *x* and *y*, of the cantilevers, Fig. 125 (*b*), are equal, as are also the loads, W = W, carried by them. But if, as in Fig. 126 (*a*), the load, W + *w*, on the beam, is not at the center of the span, the partial loads, W and *w*, supposed to be supported at the ends of the two cantilevers, or part beams, respectively, Fig. 126 (*b*), are unequal, and inversely proportional to their leverages about their respective supports. Hence, the moments of the two opposite couples are equal. The reaction of each support is equal to the weight carried by the cantilever resting upon it.

End Reactions.

293. In a cantilever, Fig. 127, there is but one vertical support; the reaction, R′, of that support, is = the sum of all the loads, including the weight of the cantilever itself; and the reaction due to each partial load is = such partial load. Thus, if B = weight of cantilever,

$$R' = W + w + B.$$

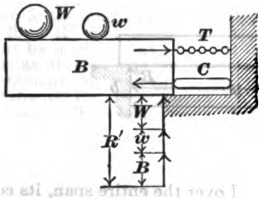


Fig. 127.

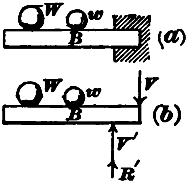


Fig. 128.

294. The reaction, R′, must not be confounded with other vertical forces. Thus, a cantilever is often supported as in Fig. 128 (*a*). The couple, composed of two horizontal forces, T and C, Fig. 127, is then replaced by a couple composed of two vertical forces, V and V′, Fig. 128 (*b*); one of which, V′, coincides with the reaction, R′. Here, R′ + V′, acting upward, is the anti-resultant of W, *w*, B and V, acting downward.

295. In a beam, Fig. 129, the sum of the two end reactions is = the sum of all the loads, including the weight of the beam itself.

296. The reaction, R, of the left support, *a*, Fig. 129, due to the load, W, alone, is $R = W \cdot \frac{y}{l}$ (see ¶ 17), and the reaction, R′, of the right support, *b*, is $= W - R = W \cdot \frac{x}{l}$. If the load is central, $\frac{x}{l} = \frac{y}{l} = \frac{1}{2}$, and $R = R' = \frac{W}{2}$.

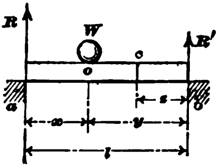


Fig. 129.

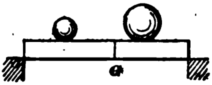


Fig. 130.

297. Graphically, Fig. 156, suppose a concentrated load, W (not shown), to be placed on the beam at any point, as *c*. Draw *a′ a″* and *b′ b″*, vertical and each = W. Join *a″ b′*; also join *a′ b″*, and draw *g h* vertically through *c′*. Then the ordinate, *c′ g*, to the upper line, *a″ b′*, and the ordinate, *c′ h*, to the lower line, *a′ b″*, give the left and the right end reactions, R and R′, respectively.

298. Where, as in Fig. 130, there are two or more loads (in which the weight of the beam may or may not be included), the reactions due to each load may be separately obtained, the sums of these reactions giving the total reactions; or, the common center of gravity, G, of all the loads may first be found (see ¶¶ 125, etc.), and then the reactions found as for a single load, W, Fig. 129; the combined weight of the loads, whose center of gravity is at G, being supposed concentrated there.

299. In a beam, Fig. 131, under a load, W, uniformly distributed over any part of the span, let G be the center of gravity of the load, and let x and y be the segments of the span, l, to the left and right of G respectively. Then, neglecting the weight of the beam, $R = W\frac{y}{l}$; and $R' = W - R = W\frac{x}{l}$.

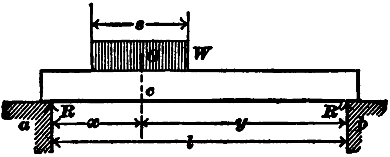


Fig. 131.

300. If the load is uniformly distributed over the entire span, its center of gravity is at the center of the span, and we have:

$$\frac{x}{l} = \frac{y}{l} = \frac{1}{2}; \text{ and } R = R' = \frac{W}{2}.$$

Moments and Shears.

301. In order to determine what internal stresses are required, at any point in the span, to maintain equilibrium, we may suppose the cantilever or beam to be cut in two by a section, $c\,c$, Fig. 132 or Fig. 133, at such point, and inquire what forces must be applied, in the section, in order to maintain equilibrium and hold in position the two segments, E and F, into which the section, $c\,c$, divides the span, Fig. 132, or that part of the span between the load and a support, Fig. 133. The forces, so ascertained, are evidently equivalent to those actually exerted, for the same purpose, by the material of the beam itself.

302. In Figs. 132 and 133, moments of loads and of reactions, or *external* or bending moments, are indicated by arrows *below* the cantilever and beam respectively; while the resisting moments of the *internal* forces are indicated by arrows *within* the body of the cantilever or beam respectively.

303. In the cantilever, Fig. 132, the load, w, = 4 lbs., distant 6 ft. from the section, $c\,c$, produces there a left-hand or negative moment of $6\,w = 6 \times 4 = 24$ ft.-lbs. Hence, for equilibrium, the horizontal strut and chain, at $c\,c$, must exert a right-hand or positive resisting moment of 24 ft.-lbs.; and, being 2 ft. apart, they must exert a tension, T, and compression, C, of $\frac{24}{2} = 12$ lbs. each. At the support, moment of load $= 9\,w = 9 \times 4 = 36$ ft.-lbs.; and $T' = C' = \frac{36}{2} = 18$ lbs.

304. But, considering only the forces thus far discussed, we should find the right segment, F, acted upon, at $c\,c$, by a left-hand couple, $= d \times T = d \times C = 2 \times 12 = 24$ ft.-lbs.; and, at the support, by a right-hand couple, $= d \times T' = d \times C' = 2 \times 18 = 36$ ft.-lbs. In other words, there would be an unbalanced excess of right-hand moment, $= 36 - 24 = 3\,R' = 3 \times 4 = 12$ ft.-lbs., acting upon F. F also receives, at the support, the upward reaction, R', = 4 lbs., of that support. Similarly, the couple, $d \times T = d \times C$, at $c\,c$, exerts, upon the left segment, E, an apparently unbalanced right-hand moment of $2 \times 12 = 6\,w = 6 \times 4 = 24$ ft.-lbs., and E receives, from the load, w, a downward pull = 4 lbs.

305. For equilibrium, therefore, the vertical chain at $c\,c$ must exert a tension $= S = w = R' = 4$ lbs., pulling F downward, and E upward. The downward tension, $-S$, acting on F at $c\,c$, forms, with the reaction, R', of the support, a left-hand couple $= 3\,R' = 3 \times 4 = 12$ ft.-lbs., balancing the excess of right-hand moment acting upon F; while the upward tension, $+S$, acting on E at $c\,c$, forms, with the weight, w, a left-hand couple, $= 6\,w = 6 \times 4 = 24$ ft.-lbs., balancing the excess of right-hand mom. acting on E.

306. Similarly, if we suppose the cantilever cut through by a section at any other point, we shall find that a vertical force, $= S = w = R'$, acting upward upon the left segment and downward upon the right segment, is required in order to maintain equilibrium and to transmit the load, *w*, to the support, so that the two segments may act unitedly as a single cantilever. This force, S, is called a shear. See ¶¶ 325, etc. Without it, section E would fall, as in Fig. 123 (*b*).

307. In the beam, Fig. 133, the total load is 16 lbs.; and, its distances, 3 ft. and 9 ft., from the left and from the right support respectively, being as 1 to 3, the end reactions (¶¶ 293, etc.) are as 3 to 1; or $R = 16 \times \frac{3}{4} = 12$ lbs.; $R' = 16 \times \frac{1}{4} = 4$ lbs. We therefore regard the beam as being cut by a section at the load (as well as at *c c*), and the total load of 16 lbs. as divided into two portions; one, $W = R = 12$ lbs., attached to the end segment, M; and the other, $w = R' = 4$ lbs., supported by the middle segment, E. Here, as in Fig. 132, segments E and F together form a cantilever, 9 ft. long, loaded with a weight, *w*, of 4 lbs., at its end; but, in Fig. 133, the horizontal resisting forces, T′ and C′, by which the entire cantilever (E + F) is upheld, are exerted, not at the support, as in Fig. 132, but at the end farthest from the support.

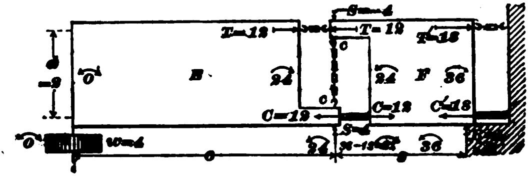


Fig. 132.

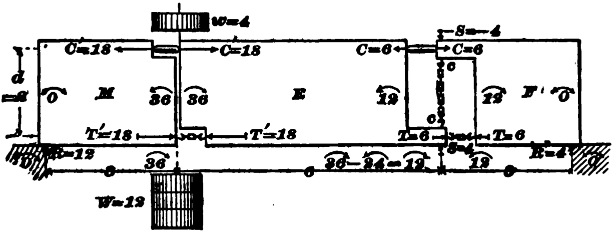


Fig. 133.

308. We have, therefore, in Fig. 133 :—at *c c*,
Bending moment, positive,
Acting on $E = 2\,T' - 6\,w = 2 \times 18 - 6 \times 4 = 36 - 24 = 12$ ft.-lbs.
Acting on $F = 3\,R' = 3 \times 4 = 12$ ft.-lbs.
Resisting moment, negative, $= 2\,T = 2\,C = 2 \times 6 = 12$ ft.-lbs. Hence, $T = C = 6$ lbs.; and shear, $S = w = R' = 4$ lbs.

309. In Fig. 132 or in Fig. 133, considering the segment extending from the load to either support (in Fig. 132 there is but one such segment), it will be seen that, at the free end of any such segment, the horizontal stresses are zero, and that they increase uniformly to a maximum at the other end of the segment. Thus, in Fig. 132, they increase uniformly from 0, at the loaded or free end, to 18 lbs., at the support; while, in Fig. 133, they increase uniformly from 0, at each support, or free end, to 18 lbs., at the load.

Moments in Cantilevers.

310. In a cantilever, Fig. 134, each load exerts, about any point between itself and the support, a moment = its weight × the horizontal distance of its center of gravity from such point; and the total moment, at any point, is the sum of the moments of the several loads about that point. Thus, neglecting the weight of the cantilever itself, we have:

about b, moment $= \mathrm{A} . x + \mathrm{B} . y$;
" c, " $= \mathrm{A} . m + \mathrm{B} . n$;
" d, " $= \mathrm{A} . z$;
" A, or any point beyond A, moment = 0.

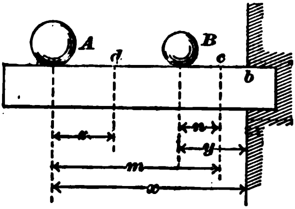


Fig. 134.

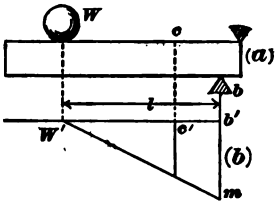


Fig. 135.

311. In a cantilever, Fig. 135, the maximum leverage of any load, W, is evidently its distance, l, from the support, b. Hence, the maximum bending moment of any load upon a cantilever is at the support, and is $= \mathrm{W} . l$. From this maximum, the moment diminishes uniformly to zero, at the load. See ¶ 309.

312. Draw $b'\,m$, Fig. 135 (b), to represent the maximum moment by scale, and join m W′. Then, for any point, c,
moment = ordinate at c' to line m W′.

313. In a cantilever, Fig. 136 (a), with two or more concentrated loads, W and w, let
$b'\,m$, Fig. 136 (b), = moment of W at the support;
$b'\,m'$, Fig. 136 (b), = moment of w, at the support.

Then, for both loads, W and w, neglecting the weight of the beam,
at d, moment = moment of W alone, = ordinate at d';
at c, moment = sum of moments of W and w,
= sum of two ordinates, $c'\,n$ and $c'\,n'$, at c'.

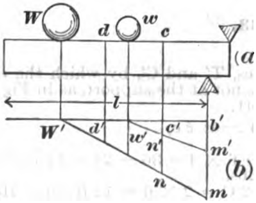


Fig. 136.

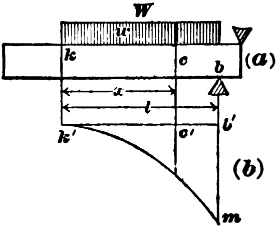


Fig. 137.

314. In a cantilever, Fig. 137 (a), under a load, W, uniformly distributed over a length, l, beginning at the support, b, the maximum moment, at the support, b, is $= \mathrm{W} . \frac{l}{2}$.

In Fig. 137 (*b*), make $b'\,m$ = said max moment, and draw a semi-parabola $m\,k'$, with apex at k'. Then, at any other section, *c*, the moment is represented by the ordinate, c', of said parabola, and is $= w \,.\, \frac{x}{2}$, where w = the weight of that portion of W beyond *c*, and x = the length of that portion.

At *k*, or at any point beyond *k*, moment = 0.

315. In Fig. 138, neglecting the weight of the cantilever itself, let W represent the weight of the whole load, and *w*, that of the shaded portion, concentrated at their respective centers of gravity, G and *g*. Then,

about *b*, moment = W . *x*;
" *c*, " = W . *y*;
" *d*, " = *w* . *v*;
" *k*, or any point beyond *k*, moment = 0.

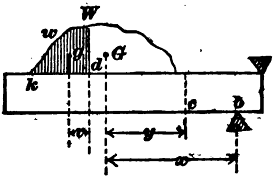


Fig. 138.

Moments in Beams.

316. In a beam, Fig. 129, the upward reaction of each abutment exerts, about any point, a moment = reaction × distance of support from such point; but any load, between such point and the support, exerts a contrary moment = load × distance of load from such point. Thus,

about *c*, moment $= R' \,.\, z = R\,(l - z) - W\,(y - z)$.

At each support, the moment is 0.

317. In a beam, Fig. 129, carrying a single concentrated load, W, the moment, $R'.z$, at any point, *c*, is $= R'z = W\frac{x}{l} \,.\, z = R\,(l - z) - W\,(y - z) = W \,.\, \frac{y}{l}\,(l - z) - W\,(y - z)$. At a point, as *c*, not under the load, the moment, $R'z$, is evidently less than the moment, $R'.y$, about the point, *o*, under the load. In other words, the maximum moment is at the point, *o*, under the load.

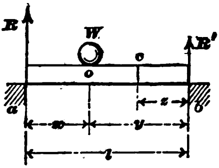


Fig. 129 (repeated).

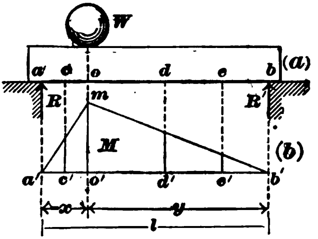


Fig. 139.

318. From the point, o', Fig. 139 (*b*), corresponding to the point, *o*, Fig. (*a*), where the load is applied, erect an ordinate, $o'\,m$, equal by scale to the (maximum) moment, $= R' \,.\, y = R \,.\, x$, at that point. Join $a'm\,b'$. Then the ordinate to $a'm$, or to $m\,b'$, at any point, c', d', e', etc., represents by scale the moment at the corresponding point, *c*, *d*, *e*, etc., in the span.

319. When the load, W, is at the center of the span, *l*, Fig. 140, each end reaction is $=\frac{W}{2}$. Hence, the moment, *s'*, at any point, *s*, distant *y* from a support, as *b*, is

$$\text{moment} = \frac{W}{2}.y.$$

At the center of the span (*i. e.*, at the point under the central load, W) we have:

$$\text{maximum moment, M,} = \frac{W}{2}\cdot\frac{l}{2} = \frac{W.l}{4}.$$

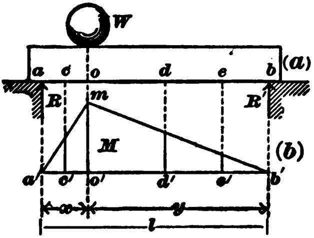


Fig. 139 (repeated).

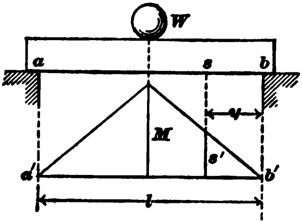


Fig. 140.

In order that the maximum moment (at *o*, Fig. 139) due to an eccentric load, W, may be equal to the maximum moment (at center of span, *l*) due to a given center load, C, we must have

$$W\frac{x}{l}.y = C.\frac{l}{4};\ \text{or}\ W = C\frac{l}{4}.\frac{l}{xy} = C\frac{\left(\frac{l}{2}\right)^2}{xy}.$$

320. When there are two or more concentrated loads, *c*, *d*, *e*, Fig. 141, treat each load as in Fig. 139, making each short ordinate, *m*, *m'*, *m''*, represent the maximum moment of its single load, *c*, *d* or *e*, alone. Make the long ordinates, M, M' and M'' = the sums of the separate moments, as measured at *c'*, at *d'*, and at *e'*, respectively. Then the ordinate to *a'* M M' M'' *b'*, at any point, represents the total moment at that point, due to the several loads combined.

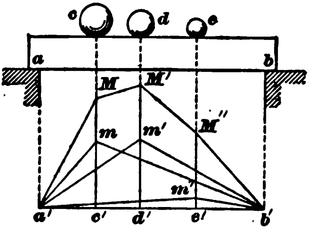


Fig. 141.

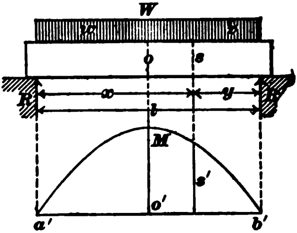


Fig. 142.

321. In a beam, Fig. 142, under a uniform load, W, covering the span, *l*, the maximum moment is at the center of the span, and is

$$\text{moment} = R.\frac{l}{2} - \frac{W}{2}.\frac{l}{4} = \frac{W}{2}.\frac{l}{2} - \frac{W}{2}.\frac{l}{4} = \frac{W}{2}.\frac{l}{4} = \frac{W.l}{8}.$$

Make o'M, by scale, = the maximum moment; and draw the parabola, a' M b', with vertex at M. Then the moment, at any section, as s, is represented by the corresponding ordinate, s', to that parabola.

Let w and v = the portions of W to the left and right of s, respectively. Then, moment at $s = \frac{w}{2} y$[*] $= \frac{v}{2} x$[*] $= \frac{1}{2}$ moment due to whole load, W, concentrated at s.[*]

At either support, moment = 0.

In Fig. 131, at a point, c, under the center of gravity of a load, W, uniformly distributed over a portion, s, of the span, neglecting the weight of the beam,

$$\text{moment} = R.x - \frac{W}{2} . \frac{s}{4} = R.x - \frac{W.s}{8} = R'.y - \frac{W.s}{8}.$$

322. Let W = the total load, whether concentrated or uniform, and let l = the span. Then the maximum moment, M, is as given below:

Cantilever.	Load, W, at end.	M at support	$M = W\,l;$
"	" " uniform.	" " "	$M = \frac{W\,l}{2};$
Supported beam.†	" " at center.	" at center	$M = \frac{W\,l}{4};$
" "	" " uniform.	" " "	$M = \frac{W\,l}{8};$
Fixed beam.‡	" " at center.	" " " or support	$M = \frac{W\,l}{8};$
" "	" " uniform.	" " support	$M = \frac{W\,l}{12}.$

323. In the inclined beam, Fig. 143, the inclined distances may be used, instead of the horizontal distances, in finding the reactions. Thus,

$$\text{reaction } R' = W . \frac{x}{l} = W . \frac{x'}{l'}.$$

But, in finding moments of vertical forces, we must of course use the horizontal, not the inclined, distances. Thus, at c, moment R'c; not R'c'.

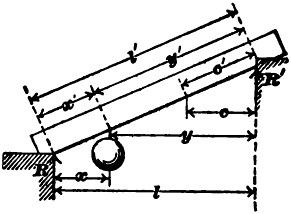


Fig. 143.

---

[*] Moment at $s = R'\,y - v\,\frac{y}{2} = \frac{W}{2}\,y - \frac{v}{2}\,y = \frac{W - v}{2}\,y = \frac{w}{2}\,y;$
$= R\,x - w\,\frac{x}{2} = \frac{W}{2}\,x - \frac{w}{2}\,x = \frac{W - w}{2}\,x = \frac{v}{2}\,x.$

With W concentrated at s,

$$\text{moment at } s = W\,\frac{x}{l}\,y = w\,y = W\,\frac{y}{l}\,x = v\,x.$$

† Beam supported at each end, but not fixed.

‡ Beam fixed at each end.

324. In curved beams, the same principles apply as in straight beams. Thus, Fig. 144, at s, moment $= W . l$. Again, in Fig. 145 (*a*), reaction $R' = \frac{W . x}{l}$, and at s, moment $= R' . y$. Or, as in Fig. 145 (*b*), from o, where the load is applied, draw $o\,a$ and $o\,b$, to the two supports respectively, and, by means of the force parallelogram, find the components, p and q, of W. Then, at s,

$$\text{moment} = p . n.$$

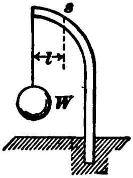


Fig. 144.

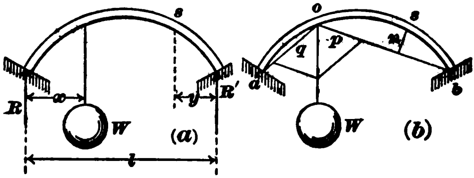


Fig. 145.

Shear.

325. In the beam, *a b*, Fig. 146 (*a*), consider the segments, *a c* and *c b*, to the left and to the right respectively of the plane *n n*. Besides the horizontal forces acting across the plane *n n*, we have seen (¶ 305) that we require also, for equilibrium, a vertical force, = the left end reaction, R, acting downward upon the left segment, *a c*, and forming a couple with R; and, at the same time, acting upward on the right segment, *c b*, being = the load, W, minus the right end reaction, R′. This force is called the shear, S, in the section *n n*. It may be regarded as the transmission of the vertical forces from loads to supports or vice versa.

326. The two segments, *a c* and *c b*, thus tend to slide vertically past each other, the right segment, *c b*, tending downward, owing to the preponderance of the load, W, over the right end reaction, R′; and this tendency is resisted by the shear, S, which is = the left end reaction, R. The same tendency exists uniformly between W and *a*, and is resisted throughout by a shear = S = R.

327. Between the load, W, and the right support, *b*, also, a uniform shear exists; but here the shear, S′, is = the right end reaction, R′, = R — W; and, whereas the shear, S, to the *left* of the load was *right*-handed or *clockwise* (the portion to the *right* of any section, *n n*, receiving the *downward* force), and is called *positive*, or +, the shear on the *right* of the load is *left*-handed or *counterclockwise* (the portion to the *left* of any section receiving the *downward* force), and is called *negative*, or —.

328. The shears, S and S′, to the left and to the right of the load, W, are represented by the diagrams in Fig. 146 (*b*); that, S, on the left of the load being drawn *above* the zero line, *a′ b′*, to indicate a *positive* shear, and vice versa.

329. Comparing Figs. 146, 147 and 148, notice that, between the left support, *a*, and the load, W, Fig. 146, we have positive shears, S = 90, Fig. 146, and $s = 15$, Fig. 147; so that, in Fig. 148, where *both* loads, W and *w*, are placed upon the same beam, we have, between *a* and W, a total positive shear of $S + s = 90 + 15 = 105$. Between the right support, *b*, and the load, *w*, Fig. 147, we have negative shears, $S' = -30$, Fig. 146, and $s' = -45$, Fig. 147; so that, in Fig. 148, between *b* and *w*, we have a total negative shear $= S' + s' = -30 - 45 = -75$. But, between the points of application of W and of *w*, we have $S' = -30$, Fig. 146, and $s = +15$, Fig. 147; leaving, between W and *w*, Fig. 148, $s + S' = 15 - 30 = -15$. If the total right end reaction, $R' + r'$, exceeds *w*, as we here suppose, the shear, at any point between the two loads, W and *w*, Fig. 148, is negative, as indicated; and vice versa.

330. In any section, the shear is = the reaction at either end, minus any loads between that end and the given section.

331. If, as in Fig. 149, the right end reaction, R′ + r', is = the load, w, then the left end reaction, R + r, is = the load, W; and there is no shear at any point between the two loads. In other words, if the beam be cut by a section at any point between W and w, horizontal forces alone will preserve equilibrium, no vertical forces being required, since the two segments have no tendency to slide vertically past each other.

332. A similar condition exists in any section where the sign of the shear changes from + to — or vice versa. Thus, if the beam be cut by a section immediately under W, Fig. 146 or 148, or under w, Fig. 147, horizontal forces equivalent to the fiber stresses in the beam, will suffice to preserve equilibrium, without a vertical force, or shear; there being no tendency of the two segments to slide past each other. Also, when, as in Fig. 149, under W and under w, the shear changes, in amount, from any value, on one side of a section, to 0, on the other side, the shear in the section itself is = 0.

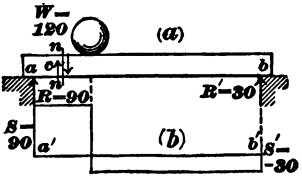


Fig. 146.

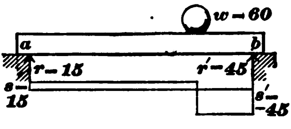


Fig. 147.

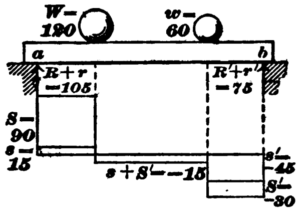


Fig. 148.

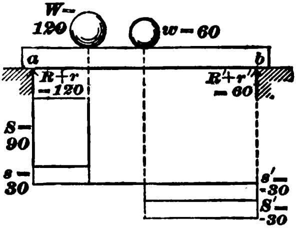


Fig. 149.

333. But in the section under w, Fig. 148, where the shear changes in amount, although not changing sign from + to — or vice versa, there is a shear = the *less* of the two shears on the opposite sides of the section, for this is the amount of the shear transferred through the section, or is the tendency of either segment to slide past the other.

334. With any number of loads, if that portion of the total load to the left of any section be called X, and that portion to the right of the same section be called Y, it will be found that the shear in the section is equal to the difference between that part of X which goes to the right support, b, and that part of Y which goes to the left support, a.

335. With a load, W, Fig. 150 (a), uniformly distributed over the entire span, the maximum shear, $= R = R' = \frac{W}{2}$, is at each support, a and b. The minimum shear, = 0, is at the center, c, of the span, which is also the point of maximum bending moment, see ¶ 321 and Fig. 142. At any point, *d*; the shear is given by the corresponding ordinate, *d′*, Fig. 150 (*b*). See Relation between Moment and Shear, ¶¶ 359, etc.

336. With a load, W, uniformly distributed over any part, *y*, of the span, Fig. 151 (*a*), find the end reactions, R and R′, as in ¶ 299. Then

between a and d, shear $= S = R$;
" e and b, " $= S' = R'$;
at c, " $= 0$.

$$x = dc = y \cdot \frac{R}{W}; \quad z = y - x = ce = y \cdot \frac{R'}{W}.$$

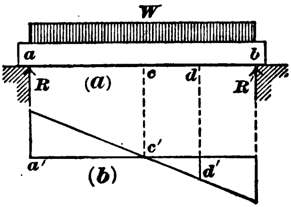


Fig. 150.

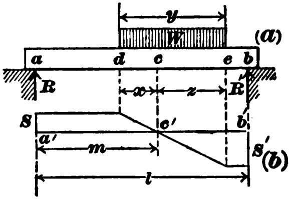


Fig. 151.

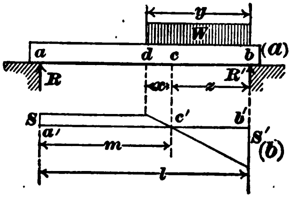


Fig. 152.

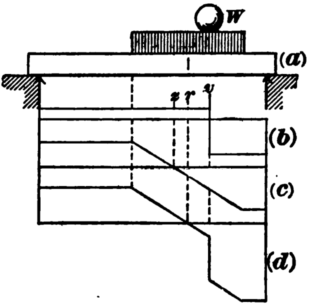


Fig. 153.

337. When the loaded portion, *y*, of the span, begins at one of the supports, *b*, Fig. 152 (*a*), then since $R = W\frac{\frac{1}{2}y}{l} = W\frac{y}{2l}$, we have

$$x = dc = y \cdot \frac{R}{W} = y\frac{W\frac{y}{2l}}{W} = y\frac{y}{2l} = \frac{y^2}{2l}.$$

338. When a concentrated load, W, Fig. 153, is added to a load uniformly distributed over the entire span, or over a part of it, each load produces the same shears as if it alone were upon the span. Those due to W are represented in Fig. 153 (*b*), while those due to the uniform load are represented in Fig. 153 (*c*). The resultant shear, due to both loads combined, is represented in Fig. 153 (*d*). Note that, between *v* and *r*, the addition of W, with its *positive* shear, *reduces* the *negative* shear due to the uniform load, and that, between *r* and *z*, the addition of W reverses the negative shear; also that it shifts the zero point from *z* to *r*.

For Continuous Beams, see Beams, under Strength of Materials.

Influence Diagrams.

339. The end reactions, due to a given load, and consequently the moments, shears and stresses, produced, at any given point in a span, by such load, vary as the position of the load, relatively to the supports, is changed. A diagram, Figs. 154 (*b*), 155 (*b*), 156 (*b*), showing the changes thus produced at any given point, is called an influence diagram, or influence line.*

Influence Diagram for Moments.

340. Thus, in Fig. 154 (*b*), *a′Mb′* is the moment influence diagram for the point, *c*, under a single concentrated load, W.†

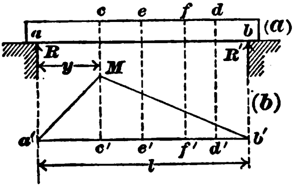


Fig. 154.

341. In Fig. 154, let l be the span, x the variable distance of the load, W,† from the right support, *b*, and y the constant distance, *a c*, of a given point, *c*, from the left support, *a*. Then, for any position of W, the left end reaction, R, is = W . $\frac{x}{l}$; and the moment of that reaction about *c*, = R . y, = W . $\frac{x}{l}$. y. The right end reaction is R′ = W $\frac{l-x}{l}$, and its moment, about *c*, is R′ $(l - y)$ = W $\frac{l-x}{l}$ $(l - y)$.

So long as W is between *b* and *c*, the moment at *c* is = R . y = W. $\frac{x}{l}$. y.

342. Since W, y and l are constant, the moment, at *c*, while W is between *b* and *c*, is proportional to the variable distance, x, of the load from *b*. It therefore increases uniformly, from 0, when W is at *b*, to its maximum value, M, when W is at *c*. See ¶ 317. Hence, if the ordinate, *c′* M, be made equal, by scale, to the maximum moment, M, then the moment, at *c*, for any position, *d*, *e* or *f*, of W, between *c* and *b*, is given by the corresponding ordinate, *d′*, *e′* or *f′*, to the line *b′* M. Similarly, the moments, at *c*, for any positions of W between *c* and *a*, are given by the ordinates to the line *a′* M.

343. For the moment, at *c*, for any number of loads, in any positions, find the moment, at *c*, for each load separately, as above, and take their sum.

344. It is customary to construct the moment influence diagram for the moments of a load, W, = unity (1 ton, 1 pound, 1 thousand kilograms, etc.). Each ordinate must then be multiplied by its corresponding load, measured in the corresponding unit, in order to obtain the required moment.

345. When W is at the point, *c*, we have $x = l - y$. Hence, ordinate, *c′* M, = maximum moment, = W . $\frac{l-y}{l}$. y; or, if W = 1, *c′* M = $\frac{l-y}{l}$. y. The area of the diagram, *a′Mb′*, is = $\frac{l}{2}$. *c′* M = $\frac{l}{2}$. $\frac{l-y}{l}$. y = $\frac{(l-y)\,y}{2}$.

---

* See "Calculation of the Stresses in Bridges for Actual Concentrated Loads," by Prof. Geo. F. Swain, "Trans. Am. Soc. C. E.," vol. XVII, July, 1887.

† Inasmuch as the load, in this discussion, occupies different positions at different times, it is not shown in Fig. 154.

346. If a load, = 1, be distributed over a length, = 1, at *c*, Fig. 155 (*a*), the resulting moment, at *c*, may be represented by the area of the rectangle standing on *c*′, Fig. (*b*), the height of said rectangle being the ordinate, *c*′ M, and its length = 1. Similarly, the moment, at *c*, due to a uniformly distributed load, *e f*, of 1 per unit length, Fig. (*a*), may be represented by the sum of the areas of the rectangles between *e*′ and *f*′, Fig. (*b*); and, if we suppose the load, *e f*, Fig. (*a*), of 1 per unit length, to be divided into a very large number of very narrow vertical strips, the resulting moment, at *c*, may be taken as represented by the area of the shaded trapezoid over *e*′ *f*′, Fig. 155 (*b*). The moment, at *c*, due to a load of *p* (lbs., tons, etc.) per unit length, and occupying the same length, *e f*, is = *p* × area of trapezoid over *e*′ *f*′, Fig. 155 (*b*).

347. Hence, the maximum moment, at *c*, due to a uniform load of *p* (lbs., tons, etc.) per unit of length, occurs when that load covers the entire span. This maximum moment is = *p* × area *a*′M*b*′, $= p\,\frac{(l-y)\,y}{2}$. See ¶ 345.

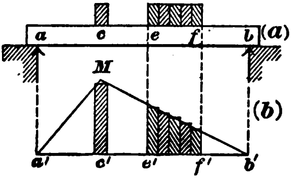


Fig. 155.

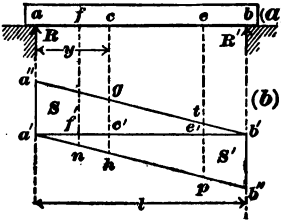


Fig. 156.

Influence Diagram for Shear.

348. Under a single concentrated load, the shear, at any point between the load and either support, is = the reaction of that support. See ¶¶ 326 and 327.

349. In the shear influence diagram, Fig. 156, as in the moment influence diagram, Fig. 154, let *l* be the span; *x* the variable distance of the load, W, from the right support, *b*, and *y* the constant distance, *a c*, of a given point, *c*, from the left support, *a*. Then, for any position of W, the left end reaction, R, or the shear, S, at any point between the load and the left support, is $= \mathrm{W}\,.\,\frac{x}{l}$; and the right reaction, R′, or the shear, S′, at any point between the load and the right support, is $= \mathrm{W}\,.\,\frac{l-x}{l}$.

350. The influence line for shear, like that for moments, ¶ 344, is usually constructed for a load = unity, so that $\mathrm{S} = \mathrm{R} = \frac{x}{l}$; and $\mathrm{S}' = \mathrm{R}' = \frac{l-x}{l}$. Each ordinate of the shear diagram must then be multiplied by W, in order to obtain the required shear.

351. Since W (= 1) and *l* are constant, R and S vary directly (and R′ and S′ inversely) with *x*. Thus, when W (= 1) is at *b*, we have *x* = 0; S = R = 0, and S′ = R′ = W = 1. When W (= 1) is at *a*, we have *x* = *l*; S = R = 1, and S′ = R′ = 0. Draw *a*′ *a*″ and *b*′ *b*″, each = W (= 1), and join *a*″ *b*′ and *a*′ *b*″. Then, with W at *c*, the (positive) shear, S, at each point, as *f*, between *c* and *a*, is given by the ordinate, *c*′ *g*, to the line *a*″ *b*′; while the ordinate, *c*′ *h*, to the line, *a*′ *b*″, gives the (negative) shear at each point, as *e*, between *c* and *b*.

352. Similarly, with W at *e*, the ordinate, *e*′ *t*, gives the (positive) shear at each point, as *c*, between *e* and *a*; while *e*′ *p* gives the (negative) shear at each point between *e* and *b*.

353. It will be noticed that, as the load, W, passes from one side to the other of any point, as *c*, the shear at that point is reversed, the total change in shear being $= h\,c' + c'\,g = h\,g =$ the load, W.

354. With a load, W ($= 1$), at *b*, the shear at *c* is $= 0$. See ¶ 351. As the load advances from *b* toward *a*, the positive shear, $S = R = \frac{x}{l}$, at *c*, increases in proportion to the ordinates to the line $b'\,g$, becoming $= c'\,g = \frac{l - y}{l}$, when W is just to the right of *c*. With W just to the left of *c*, we have, negative shear at $c = S' = R' = c'\,h = \frac{y}{l}$. But as W proceeds from *c* to *a*, this negative shear, at *c*, decreases in proportion to the ordinates to the line $h\,a'$, becoming 0 when W reaches *a*. Thus, $a'hgb'$ is the shear influence diagram for the point, *c*. Similarly, $a'ptb'$ is the shear influence diagram for the point, *e*, etc.

355. If a series of nearly uniform and equidistant concentrated loads, such as the wheel loads of a locomotive and train, come upon the span, at the support, *b*, and advance toward *a*, the shear at *c* evidently increases until the first load reaches *c*. It is then suddenly diminished, by an amount = the first load, as that load passes *c*. It then continues to diminish, as each wheel passes over *c*, but more slowly, until the first load reaches *a*. See ¶ 358.

356. With a *uniformly distributed* load, of unity per unit length, moving as in ¶ 355, the shears at *c* (see ¶ 346) are represented by the *areas* of those portions of the diagram, $a'hgb'$, successively covered by the load, portions of the diagram below the zero line, $a'\,b'$, being taken as negative. Thus, when the head of the load reaches *e*, the (positive) shear at *c* is given by the area of the triangle, $b'\,e'\,t$. With head of load at *c*, the shear at *c* reaches its maximum, and is given by the area of triangle, $b'\,c'\,g$. With head of load at *f*, the shear at *c* is = area $b'\,c'\,g$ — area $f'\,c'\,h\,n$.

357. Similarly, the shears at *e* are given by the areas of portions of the diagram, $a'\,p\,t\,b'$.

358. Fig. 157* shows the influence diagram, 0 *d z* 14 16, for the shears at point 6, for a given uniformly distributed load, at least as long as the span, coming upon the span at point 0, passing across it, and leaving it at point 8; also the corresponding diagram, 0 *e* 16, for the left support, 8; and that, 0 *g* 16, for the right support, 0.

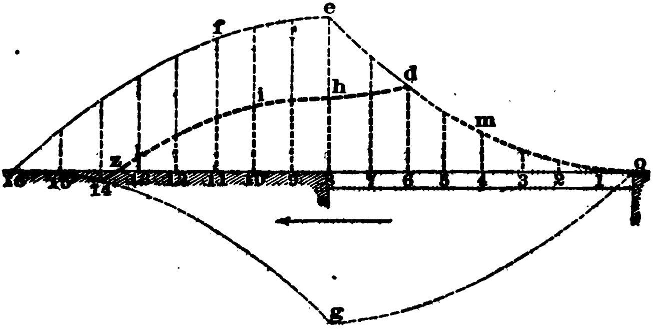


Fig. 157.

For the action of internal resisting forces in beams and trusses, see Transverse Strength, under Strength of Materials, and Stresses, under Trusses.

---

* First published in our 9th Edition, 1885.

Relation between Moment and Shear.

359. The shear, at any point in the span, is simply the **rate at which the bending moment is changing** at that point.

360. Thus, in Fig. 158, the moment, M, Fig. (*b*), at the support, *b*, due to the concentrated load, W, of 6 lbs., is $= W\,l = 6 \times 4 = 24$ ft.-lbs.; but, between the support and the load, the moment is decreasing at the *uniform rate* of 6 ft.-lbs. for each foot of x, or 6 ft.-lbs. per foot $=$ 6 lbs.; and this 6 lbs. is the *uniform shear*, V, Fig. (*c*), throughout the beam. Hence the shear diagram, Fig. (*c*), is a *horizontal* line; *i. e.*, its ordinates are of *equal* length.

361. Again, in Fig. 159, the shear diagram ordinates between a'' and o'', Fig. (*c*), are *positive*; showing the (algebraic) *increase* of the bending moment, M, Fig. (*b*), as we proceed from the left support, *a*, toward the center, *o*, of the span; while the *negative* shear diagram ordinates, between o'' and b'', show the (algebraic) *decrease* of the bending moment as we proceed from the center, *o*, to the right support, *b*. At the center, *o*, the rate of change of bending moment is zero, as is also the vertical shear.

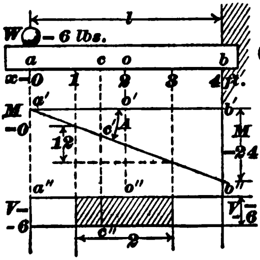


Fig. 158.

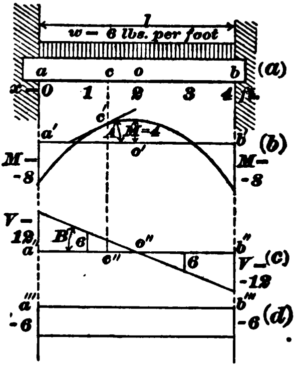


Fig. 159.

362. Both in Figs. 158 and 159, the bending moment, M, is constantly changing; but in Fig. 158 its *rate of change* ($=$ 6 ft.-lbs. per ft. of span) is *constant*. Hence, the *moment* diagram is a *straight inclined* line, and the *shear* diagram is a *horizontal* line; whereas in Fig. 159 the rate of change of bending moment is constantly *varying*, being $=$ 12 ft.-lbs. per foot of span (shear $=$ 12 lbs.) at the support, and diminishing to zero at the center, *o*, of the span. Hence, in Fig. 159, the *moment* diagram, Fig. (*b*), is no longer straight, but *curved;* and the *shear* diagram, Fig. (*c*), is no longer horizontal, but *inclined.*

363. But, in Fig. 159 (*c*), the shear, V, or the *rate* of change of the bending moment (although no longer *constant*, as it was in Fig. 158 (*c*)), nevertheless *diminishes uniformly*, as we proceed from *a* toward *b*. Thus, at the point, 1, Fig. 159, midway between *a* and *o*, the bending moment is changing at the rate of 6 ft.-lbs. per foot, or *half* as fast as at *a*. Hence, the shear diagram, although no longer a *horizontal* line, is still a *straight* line; and the *uniform* decrease ($=$ 6 ft.-lbs. per foot per foot) in the rate of change of the bending moment, or the *uniform* decrease ($=$ 6 lbs. per foot) in shear, is indicated by the *horizontal* diagram in Fig. 159 (*d*).

364. In either Fig., let a straight line be drawn, tangential to the moment diagram (*b*), at any point, c', and forming, with the horizontal zero line, $a'\,b'$, an angle, A. (In Fig. 158, this line coincides with the moment diagram.) Then the tangent of A is given by the shear diagram ordinate, c'', corresponding to the point, *c;* or, for any point, $V = \tan A$.

365. In Fig. 158, where this angle, A, Fig. (*b*), is *constant*, the shear ordinates, Fig. (*c*), are of *constant* length. In other words, the shear diagram, Fig. 158, is a *horizontal* line.

366. Since the shear diagram ordinates represent *forces* (as in lbs., etc.) and the abscissas represent *distances* (as in ft., etc.) the product of the distance between any two shear ordinates, multiplied by the mean of those ordinates, is an *area* representing a *moment* (in ft.-lbs., etc.). This moment is = the *difference* between the two moments represented by the corresponding ordinates in the moment diagram.

367. Thus, in Fig. 158 (*b*), the increase in (negative) bending moment, between points 1 and 3, is = 18 — 6 = 12 ft.-lbs.; and, in Fig. 158 (*c*), the moment represented by the (shaded) area, between the same two points, is = 2 ft. × 6 lbs. = 12 ft.-lbs. In Fig. 159, the (algebraic) increase in bending moment, Fig. (*b*), between the left support, *a*, and the center, *o*, of the span, is = 8 + 4 = 12 ft.-lbs.; and the moment, represented by the shear diagram area (triangle) between the same two points, Fig. (*c*), is

$$= \frac{1}{2} V \frac{l}{2} = \frac{12 \times 2}{2} = 12 \text{ ft.-lbs.}$$

368. Again, in Fig. 159, at any two points equally distant from the center, *o*, of the span, the moments are equal; or difference of moments = zero; and, since shear ordinates *below* the zero line, $a'' \; b''$, Fig. (*c*), are considered as *negative*, the algebraic sum of the two corresponding shear triangles, Fig. (*c*), is also = zero.

Similarly, areas in Fig. 159 (*d*) correspond to differences of ordinates in Fig. 159 (*c*).

STRENGTH OF MATERIALS.

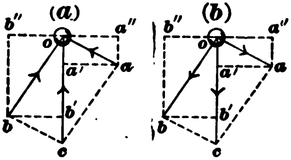
GENERAL PRINCIPLES.

Art. 1 (a) Stress or **Strain**[*] occurs when force acts upon a body in such a way that its particles tend to move (at the same time) with different velocities or in different directions; to do which they must either separate from each other or come closer together. This occurs, for instance, when a body is so placed as to oppose the relative motion of two other bodies; as when a block is placed between a weight and a horizontal table. In this case, each of the two bodies (the weight and the table) imparts a force to the opposing body (the block); and the stress is the opposition of these equal forces. The tendency of the particles of the block to separate or to come closer together calls into action the **inherent forces** of its material, and these act between the particles and tend to keep them in their original relative positions.

(b) **Compression and Tension.** If two opposite forces are simultaneously imparted to a body in the same straight line, the stress is either compressive (when the forces act *toward* each other) or tensile (when they act *from* each other).

Compressive stress tends to *push* the particles *closer together*. *Tensile* stress tends to *pull* them *farther apart*.[†]

(c) **If two imparted forces,** as $a\,o$, $b\,o$, **meet at an angle,** as at o; then two equal and opposite components, $a''\,o$ and $b''\,o$, will cause compressive or tensile stress in the body, while the other two, $a'\,o$ and $b'\,o$, unite to form the resultant, $c\,o$, which, unless balanced by other forces, *moves* the body in its own direction, and, in doing so, produces another stress among the particles of the body. See (a), above.



(d) **If the two forces are parallel,** forming a "couple," as in a punch and die, the stress is a **shear** (tending to *slide* some of the particles over the others, see p. 499), and is accompanied also by a **transverse stress** (causing a tensile stress in some of the particles and a compressive stress in others) as in the case of a beam. The *transverse* stress is proportional to the *distance* between the two forces (*i. e.*, to the arm or leverage of the couple), so that, when they are very close together, as in a pair of shears, the transverse stress is very small and is neglected, and the shearing stress alone is considered.

(e) **If two contrary couples, in different planes,** act upon a body, the stress is called **torsion** or twisting. See p. 499. Thus, torsion takes place in a brake axle when we try to turn it while its lower end is held fast by the brake chain.

(f) But the ultimate tendency of any of these forms of stress is either to *separate* certain particles or to *drive them closer together*, as in cases (tensile or compressive stress) where the two forces are in one line.[†] We shall, therefore, in these introductory articles, consider only this simple and fundamental form of stress, assuming that it is caused by the action of two opposite imparted forces, acting in the same straight line so that they are *entirely* employed in causing the stress.

(g) **A stress may be stated** in any unit of weight, as in pounds, and is equal to *one* of the two opposite forces.

---

[*] For another use of the word "strain," see Art. 2 (d).

[†] Indeed, even in cases of *compressive* stress, it is only by the *separation* of the particles that the structure of the body and its inherent forces can be destroyed.

Art. 2 (a) Stretch and Rupture. It appears from experiment that the inherent cohesive forces called into action by the first application of any stress are always less than that stress, however small it may be. In other words; any stress, however slight, is believed to produce *some* derangement of the particles. But the inherent forces *increase* with this derangement (up to a certain point) and thus, in many cases, they become equal to the stress and so prevent further derangement. When the stress exceeds the greatest inherent force which the body can exert, the particles separate to such an extent that the inherent forces cease to act. The body is then said to be broken, or ruptured.

(b) **Different materials behave very differently** when under stress. Brittle ones seem to resist almost perfectly up to a certain point, allowing no perceptible derangement of the particles; and then yield suddenly and entirely. In ductile materials, on the contrary, considerable derangement takes place before the inherent resisting forces finally yield.

(c) The **ultimate stress** of a body is that which is just sufficient to break it or crush it, or, in short, to destroy its structure so that it can no longer resist. In other words, a stress just less than the ultimate is the greatest stress to which the body can be subjected.

Caution. In brittle materials, such as brick, stone, cement, glass, cast-iron, etc., especially when subjected to *tension*, the point of rupture is clearly marked, and hence the ultimate strength may in such cases be stated with precision. But with ductile or malleable materials, such as copper, lead and wrought iron, especially when under *compression*, it is often difficult or impossible to state the ultimate strength definitely. For instance, a cube of lead may be gradually crushed into a thin flat sheet without rupture. In other words, there is practically no load which can break it by crushing. In such cases, we may arbitrarily assume some given amount of distortion as marking the point of ultimate stress. Thus, by the "ultimate" load of a rolled iron beam we mean "that one which so cripples the beam that it continues to yield indefinitely without increase of load." Such assumptions, however, necessarily give rise to some ambiguity, and care should therefore always be taken to define or to ascertain clearly in what sense the term "ultimate stress" is employed.

The **ultimate strength** of a material (or, more briefly, its **strength**) is the greatest inherent force which its particles can exert in opposition to a stress. In other words, it is that inherent resistance which is just equal to the ultimate stress. Hence strength, like stress, is stated in units of weight, and we may use the terms "ultimate strength" and "ultimate stress" indifferently, as denoting practically the same thing.

(d) For want of a convenient and appropriate name for the **change of shape** caused by stress; modern writers have, rather unfortunately, given to it the name of **strain**,* which, in ordinary language, is used to signify *stress*, as above defined. We prefer to use the word "**stretch**" for change of shape, in inches, etc. (regarding *compression* as *negative* "stretch"), and "strain" or "stress" for the action of the two opposing forces, in pounds, etc.

(e) By the "**length**" of a body, we mean its dimension measured *in the line of the stress*; and, by "**area**," the area of the resisting cross section at right angles to that line. Thus, if a slab of iron, 2 inches thick and 10 inches square, be laid flat upon a smooth and horizontal surface, and if a load be placed upon it so as to be uniformly distributed over its upper flat surface, the "length" is 2 inches, and the "area," 10 x 10 = 100 square inches.†

(f) **Units adopted.** Unless otherwise stated, we shall understand the stress in any case to be given in pounds, the stretch and the length in inches, and the area in square inches.

---

* The word "strain" is not thus defined, even as a scientific term, in either Webster's or Worcester's dictionary.

† Under stresses approaching the ultimate stress, the area of the cross section generally increases under compression, and diminishes under tension, to different extents in different materials; but we are here concerned only with cases within the limit of elasticity, (Art. 4 a, p. 458) and in such cases the change of area is generally very slight and may be neglected.

Art. 3 (a). If the total stress (in lbs., etc.) upon a body be divided by the area of the resisting surface (in square inches, etc.) the quotient, $\frac{\text{stress}}{\text{area}}$, is the *mean* **stress per unit of area,** or (as it is sometimes called) the *intensity* of the stress. Or,

$$\textbf{Stress per unit of area} = \frac{\text{total stress}}{\text{area}}.$$

Thus, if a bar of iron, 2 inches wide by 1 inch thick, (having therefore 2 square inches of area of cross section) and 10 feet long, be subjected to a total tensile stress of 20,000 lbs. in the direction of its (10 ft.) length, we have

$$\left.\begin{matrix}\text{mean stress}\\ \text{per unit of area}\end{matrix}\right\} = \frac{\text{total stress}}{\text{area}} = \frac{20{,}000 \text{ lbs.}}{2 \text{ square in.}} = 10{,}000 \text{ lbs per square inch.}$$

Caution. Strictly speaking, the stress on a surface is seldom distributed uniformly over it. Thus, in the case of the bar just referred to, if the stress is applied by means of grips, clamping the sides and edges of the bar, the stress per square inch near those sides and edges is probably greater than that near the center of the bar, because the stress is not perfectly and uniformly transmitted from the outer to the inner fibres. And, in cases of compression, the load, instead of being uniformly distributed over the surface, as it appears to be, is often in fact supported by a few projecting portions of it. In practice, these considerations are often of the greatest importance, but in studying the *principles* of resistance, we may, for convenience, temporarily neglect them, and assume the stresses to be uniformly distributed over their respective areas.

(b) If the total stretch of a body (in inches, etc.), under any given stress, be divided by the original length of the body (in the same measure), the quotient is the **stretch per unit of length.** Or,

$$\textbf{Stretch per unit of length} = \frac{\text{total stretch}}{\text{original length}}.$$

Thus: if the foregoing bar, 10 feet (or 120 inches) long, is found to stretch .04 inch, under its load of 20,000 lbs. total, or 10,000 lbs. per square inch, we have

$$\begin{matrix}\text{stretch per unit of length}\\ \text{under said load}\end{matrix} = \frac{\text{total stretch}}{\text{original length}} = \frac{.04}{120} = .00033 \text{ inch per inch.}$$

(c) **The Modulus of Elasticity.** In materials which undergo a perceptible stretch before rupture, it has been found by experiment that up to a certain degree of stress, called the limit of elasticity (Art. 4 *a*, p. 458), the ratio, $\frac{\text{total stress}}{\text{total stretch}}$, in any given body, remains very nearly constant. In other words, within the limit mentioned, equal additions of stress cause practically equal additional stretches.

In order to compare bodies of different dimensions, we state the same fact by saying that, within the elastic limit the quotient, $\frac{\text{stress } \textit{per unit of area}}{\text{stretch } \textit{per unit of length}}$ remains practically constant.

This quotient, as found by experiment with any given material, is called the **Modulus of Elasticity** of that material, and is usually denoted by the capital letter **E.** It is of course expressed in the same unit as the stress per unit of area, as, for instance, in pounds per square inch; but it is usually stated simply in *pounds*, the words "per square inch" being understood.

(d) It will be noticed that the greater the stress required to produce a given stretch in a body, the greater is its modulus of elasticity. Hence the modulus E is a measure of the resistance which the body can make against a change in shape. This resistance we call the "**elasticity**" of the body, although in every-day language (and, indeed, often in a scientific sense also) we apply the term "elasticity" rather to the ability of a body to sustain considerable distortion without losing its power of returning to its original shape.

(e) Since

$$\textbf{Stress per unit of area} = \frac{\text{total stress}}{\text{area}},$$

and since

$$\textbf{stretch per unit of length} = \frac{\text{total stretch}}{\text{original length}},$$

we may find the modulus of elasticity of any material, from experiment upon any specimen of it, thus:

$$\textbf{Modulus of elasticity} = \frac{\text{total stress} \times \text{original length}}{\text{total stretch} \times \text{area}}. \quad \ldots (1)$$

From this we have the following equations:

$$\begin{array}{l}\textbf{Total stress,}\text{ in lbs.}\\ \text{required for a given}\\ \text{total stretch, in inches}\end{array} = \begin{array}{c}\text{modulus of}\\ \text{elasticity in lbs}\\ \text{per square inch}\end{array} \times \frac{\begin{array}{c}\text{total stretch}\\ \text{in inches}\end{array} \times \begin{array}{c}\text{area}\\ \text{in square ins}\end{array}}{\text{original length in inches}} \quad \ldots (2)$$

$$\begin{array}{c}\textbf{Stress per unit of area,}\\ \text{in lbs. per square inch, required}\\ \text{for a given stretch, in inches}\end{array} = \begin{array}{c}\text{modulus of}\\ \text{elasticity in lbs.}\\ \text{per square inch}\end{array} \times \begin{array}{c}\text{stretch per}\\ \text{unit of length}\end{array} \quad \ldots (3)$$

$$\begin{array}{c}\textbf{Total stretch,}\text{ in}\\ \text{inches, under any stress}\end{array} = \begin{array}{c}\text{total stress}\\ \text{in lbs.}\end{array} \times \frac{\text{original length, in inches}}{\begin{array}{c}\text{modulus of elasticity,}\\ \text{in lbs. per square inch}\end{array} \times \begin{array}{c}\text{area, in}\\ \text{sq. ins}\end{array}} \quad \ldots (4)$$

$$= \begin{array}{c}\text{original length}\\ \text{in inches}\end{array} \times \frac{\text{stress, in lbs. per square inch}}{\text{modulus of elasticity in lbs. per sq. inch}} \quad \ldots (5)$$

$$\textbf{Stretch per unit of length} = \frac{\text{stress, in lbs. per square inch}}{\text{modulus of elasticity, in lbs per sq. in.}} \quad \ldots (6)$$

The modulus of elasticity is used chiefly in connection with the stiffness of beams. In any beam, supported at both ends and loaded at the center:

$$\begin{array}{c}\textbf{Modulus of}\\ \textbf{elasticity,}\text{ in}\\ \text{lbs. per sq. inch}\end{array} = \frac{\left(\begin{array}{c}\text{load}\\ \text{in lbs.}\end{array} + \begin{array}{c}\frac{5}{8}\text{ weight of clear}\\ \text{span of beam, in lbs.}\end{array}\right) \times \begin{array}{c}\text{cube of span}\\ \text{in inches}\end{array}}{48 \times \begin{array}{c}\text{deflection,}\\ \text{in inches}\end{array} \times \text{moment of inertia}} \quad \ldots (7)$$

or,

$$E = \frac{(W + \frac{5}{8} w)\, l^3}{48\, \Delta\, I} \quad \ldots (8)$$

If the beam is *rectangular*, this becomes

$$\begin{array}{c}\textbf{Modulus of}\\ \textbf{elasticity,}\text{ in}\\ \text{lbs per square inch}\end{array} = \frac{\left(\begin{array}{c}\text{load}\\ \text{in lbs.}\end{array} + \begin{array}{c}\frac{5}{8}\text{ weight of clear}\\ \text{span of beam, in lbs}\end{array}\right) \times \begin{array}{c}\text{cube of span}\\ \text{in inches}\end{array}}{4 \times \begin{array}{c}\text{deflection,}\\ \text{in inches}\end{array} \times \begin{array}{c}\text{breadth,}\\ \text{inches}\end{array} \times \begin{array}{c}\text{cube of depth}\\ \text{in inches}\end{array}} \quad \ldots (9)$$

or,

$$E = \frac{(W + \frac{5}{8} w)\, l^3}{4\, \Delta\, b\, d^3} \quad \ldots (10)$$

Corresponding formulæ for modulus of elasticity in beams otherwise supported and loaded, may be readily deduced from those for deflection

(f) If equal additions of stress could produce equal additional stretches in a body to an indefinite extent, both within and beyond the elastic limit, then a stress equal to the Modulus of Elasticity would *double* the length of a bar when applied to it in *tension*, or would *shorten* it to *zero* when applied in *compression*. In other words, if equation (5),

$$\text{total stretch} = \begin{array}{c}\text{original length}\\ \text{in inches}\end{array} \times \frac{\text{stress per square inch}}{\text{modulus of elasticity}},$$

held good *beyond* the elastic limit, as it does (approximately) within that limit, and if we could make the stress per square inch *equal* to the modulus of elasticity, we should have total stretch = original length.

For example, a one-inch square bar of wrought iron will, within the limit of elasticity, stretch or shorten, on an average, about $\frac{1}{12000}$ of its length under each additional load of 2240 lbs. If it could continue to stretch or shorten indefinitely at this rate, it is evident that 12000 times 2240 lbs., or 26 880 000 lbs., (which is about the average modulus of elasticity for such bars) could either stretch the bar to double its length or reduce it to zero.

If equal additional stresses applied to a bar could indefinitely produce stretches, each bearing a *constant proportion to the increased length of the bar*, if in *tension*; or to the *diminished* length, if in *compression*; then the same load which would double the length of the bar if applied in tension, would reduce it to *half* its length, if applied in compression.

(*g*) We give below a **table of average Moduli of Elasticity**, in round numbers, for a few materials; remarking, by way of caution, that, even in the case of ductile materials, the stretches produced by stresses within the elastic limit are so small, and (owing to differences in the character of the material) so irregular, that a satisfactory average can be arrived at only by comparing many experiments; while, in the case of materials, such as stone, brick, etc., where almost no perceptible stretch takes place before rupture, it is scarcely worth while to give any values as representing the actual moduli. Thus, eighteen experiments upon a single brand of neat cement for the St. Louis bridge, indicated a Modulus varying from 800 000 to 6 930 000 (!) pounds per square inch in *tension*, and from 500 000 to 1 500 000 in *compression*.

(*h*) Owing to the fact that the stretches within the elastic limit are seldom, if ever, *exactly* proportional to the stresses, but only approximately so, the modulus of elasticity, as found by experiment for a given material, will generally vary somewhat with the stress at which the stretch is taken.

Art. 4 (*a*) The stress beyond which the stretches in any body increase perceptibly faster than the stresses, is called the **limit of elasticity** of that body. Owing to the irregularity in the behavior of different specimens of the same material, and to the extreme smallness of the distortions caused in most materials by moderate loads, and because we often cannot decide just when the stretch begins to increase faster than the load, the elastic limit is seldom, if ever, determinable with exactness and certainty.* But by means of a large number of experiments upon a given material we may obtain useful average or minimum values for it, and should in all cases of practice keep the stresses well within such values; since, if the elastic limit be exceeded (through miscalculation, or through subsequent increase in the stress or decrease in the strength of the material) the structure rapidly fails. The table, below, gives approximate average elastic limits for a few materials.

(*b*) Brittle materials, such as stones, cements, bricks, etc., can scarcely be said to have an elastic limit; or, if they have, it is almost impossible to determine it; since rupture, in such bodies, takes place before any stretch can be satisfactorily measured. Thus, in the 18 specimens of one brand of cement, referred to in Art. 3 *g* above, the experiments indicated an elastic limit varying between 16 and 104 (!) pounds per square inch in *tension*, and from 424 to 1502 in *compression*.

(*c*) Experiments show that a small permanent "set" (stretch) probably takes place in all cases of stress even under very moderate loads; but ordinarily it first becomes noticeable at about the time when the elastic limit is exceeded. Many writers define the elastic limit as that stress at which the first marked permanent set appears.

(*d*) The **elastic ratio** of a material is the quotient, $\frac{\text{elastic limit}}{\text{ultimate strength}}$. It is usually expressed as a decimal fraction.

---

* The U. S. Board appointed to test Iron, Steel, &c., found a variation of nearly 4000 lbs. per square inch in the elastic limit of bars of one make of rolled iron, prepared with great care and having very uniform tensile strength; and, in another very carefully made iron, a difference of over 30 per cent. between two bars of the same size. Report, 1881, Vol. 1, p. 31.

(e) Table of Moduli of Elasticity and of Elastic Limits for different materials.

The values here given are approximate averages compiled from many sources. Authorities differ considerably in their data on this subject. See Art. 3 (*g*) and (*h*), and Art. 4 (*a*) and (*b*).

MATERIAL.	Modulus or Coeff of Elasticity.	Stretch or Compression in a length of 10 ft. under a load of 1000 lbs per sq in.	Stretch or Compression in a length of 10 ft. under a load of 1 ton per sq in.	Approx elas limit.
	lbs per sq in.	Ins.	Ins.	lbs per sq in.
Ash	1 600 000	.075	.168	4500
Beech	1 300 000	.092	.207	4000
Birch	1 400 000	.086	.192	5000
Brass, cast	9 200 000	.013	.029	6000
" wire	14 200 000	.009	.019	16000
Chestnut	1 000 000	.120	.269	4500
Copper, cast	18 000 000	.007	.015	6300
" wire	18 000 000	.007	.015	10000
Elm	1, 000 000	.120	.269	2000
Glass	8 000 000	.015	.034	3200
Iron, cast	12 000 000	.010	.022	4500
	to	to	to	to
	23 000 000	.005	.012	8000
" " average	17 500 000	.007	.015	6250
" wrought, in either bars, sheets or plates	18 000 000	.006	.015	20000
	to	to	to	to
	40 000 000	.003	.007	40000
" " " average	29 000 000	.004	.009	30000
" wire, hard	26 000 000	.005	.010	27000
" wire ropes	15 000 000	.008	.018	18000
Larch	1 100 000	.109	.244	2300
Lead, sheet	720 000	.167	....	1100
" wire	1 000 000	.120	....	1100
Mahogany	1 400 000	.086	.192	2700
Oak	1 000 000	.120	.269	....
	to	to	to	
	2 000 000	.060	.134	....
" average	1 500 000	.080	.179	3200
Pine, white or yellow	1 600 000	.075	.168	3300
Slate	14 500 000	.008	.018	3700
Spruce	1 600 000	.075	.168	3300
Steel bars	29 000 000	.004	.009	34000
	to	to	to	to
	42 000 000	.003	.006	44000
" " average	35 500 000	.003	.007	39000
Sycamore	1 000 000	.120	.269	4000
Teak	2 000 000	.060	.134	6000
Tin, cast	4 600 000	.026	....	1500

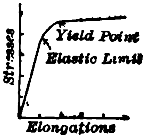


Fig. D.

(f) Yield point. In testing specimens of iron and steel, it is commonly found that, after passing the elastic limit, see (Art 4 *a*), a point is reached where the ratio between the stretch and the stress suddenly begins to increase more rapidly. This is commonly called the yield point. See Fig. D.

Art. 5 (a) Resilience is a name given to the *work* (as in inch-pounds) which must be done in order to produce a certain stretch in a given body. This work is equal to

$$\text{resilience in inch-pounds} = \text{said stretch in inches} \times \text{mean stress in pounds employed in producing the stretch.}$$

The *total* resilience is the work done in causing *rupture*. The *elastic* resilience (frequently called, simply, *the resilience*) is that done in causing the greatest stretch possible *within the elastic limit.*

(b) Suddenly applied loads. Place a weight of 4 lbs. in a spring balance, but let it be upheld by a string fastened to a firm support in such a way that the scale of the balance shall show only 1 lb. By now cutting this string with a pair of scissors, we *suddenly* apply 4 — 1 = 3 lbs.; and the weight will descend rapidly, until, for an instant, the scale shows about 1 + *twice* 3 = 7 lbs. In other words, the load of 3 lbs. applied *suddenly* (but without jar or shock) has produced nearly *twice* the stretch that it could produce if added grain by grain, as in the shape of sand.

For, when the load is first applied, the *inherent* forces, as noticed in Art. 2 (*a*), are insufficient to counteract its stress. Hence the load begins to stretch the spring. The work thus done is equal to the product, suddenly applied weight of 3 lbs. × the stretch of the spring; and it has been expended (except a small portion required to counteract friction) in bringing the resisting forces into action, thus storing in the spring potential energy nearly sufficient to do the same work; *i. e.*, to lift the weight (3 lbs.) to the point (1 lb. on the scale) from which it started. But a portion of this energy has to work against friction and the resistance of the air. Therefore the weight does not rise quite to its original height.

The shortening of the spring nearly to its original length has now reduced its inherent forces almost to zero; and the weight again falls, but not so far as before. It thus vibrates through a less and less distance each time, and finally comes to rest at a point (4 lbs. on the scale) midway between its highest and lowest positions (1 lb. and 7 lbs.) Thus, within the limit of elasticity, **a load applied suddenly** (though without shock) **produces temporarily a stretch nearly equal to twice that which it could produce if applied gradually;** *i. e.*, twice that which it can *maintain* after it comes to rest.

Remark. If the load is added in small instalments, each applied suddenly, then each instalment produces a small temporary stretch and afterward maintains a stretch half as great. Thus, under the last small instalment, the bar stretches temporarily to a length greater than that which the total load can maintain, by an amount equal to half the small temporary stretch produced by the sudden application of the last small instalment.

(c) The Modulus of Elastic Resilience (often called, simply, the Modulus of Resilience) of a material, is the work done upon *one cubic inch of it* by a gradually applied load equal to the elastic limit. Or,

$$\text{Modulus of resilience} = \text{stretch in inches } \textit{per inch of length} \text{ at the elastic limit} \times \text{mean stress } \textit{in lbs. per square inch} \text{ causing that stretch}$$

If, as is usually done, we assume this mean stress to be ½ the elastic limit, then, by formula (6)

$$\text{Modulus of resilience} = \frac{\text{elastic limit}}{\text{modulus of elasticity}} \times \tfrac{1}{2} \text{ elastic limit}$$

$$= \tfrac{1}{2}\,\frac{\text{square of elastic limit}}{\text{modulus of elasticity}}.$$

The elastic resilience of any piece is then

$$\text{Resilience} = \text{modulus of resilience} \times \text{volume of piece in cubic inches.}$$

The modulus of resilience of a material is a measure of its capacity for resisting shocks or blows.

Elastic Ratio. The elastic ratio of a material is the ratio between its elastic limit (Art. 4*a*, and its ultimate strength (Art. 2*c*.

Thus, if the ultimate tensile strength of a steel bar be 70,000 pounds per square inch, and its elastic limit in tension 39,900 pounds per square inch, its elastic ratio is

$$\frac{39{,}900}{70{,}000} = 0.57.$$

Inasmuch as it is now generally conceded that the permissible working load of a material should be determined by its elastic limit rather than by its ultimate strength, it follows that, other things being equal, a high elastic ratio is in general a desirable qualification; but, on the other hand, it is possible, by modifying the process of manufacture, to obtain material of high elastic ratio, but deficient in "body" or in resilience—*i. e.*, in capacity to resist the effect of blows or shocks, or of sudden application or fluctuation of stress.

In the manufacture of steel it is found that the elastic ratio is increased by increasing the reduction of area in hammering or rolling, and that the rate of increase of elastic ratio with reduction of area increases rapidly as the reduction becomes very great. This is indicated by the following experiments by Kirkaldy on steel plates:[*]

Plates 1 inch thick, mean elastic ratio 0.53
" $\frac{3}{4}$ " " " " 0.53
" $\frac{1}{2}$ " " " " 0.54
" $\frac{1}{4}$ " " " " 0.61

---

[*] Annual Report of the Secretary of the Navy, Washington, 1885, Vol. I. p. 499; and Merchant Shipping Experiments on Steel, Parliamentary Paper, C. 2897, London, 1881.

Art. 6. A section may be weakened by increasing its width. On pp 400, etc., we considered the case where the width of the base is fixed and where the point of application of the resultant of the forces acting upon it is shifted to different positions along the base. We will now notice the case where the resultant is applied at a constant distance from one end of the base, but where the base is of varying width, so that this constant distance may be equal to, or greater or less than, the half width of the base.

Let Fig. E represent a side view of a bar of uniform thickness = 1,* but (as

Fig. E.

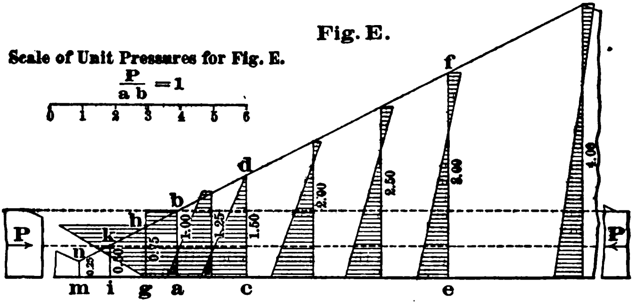
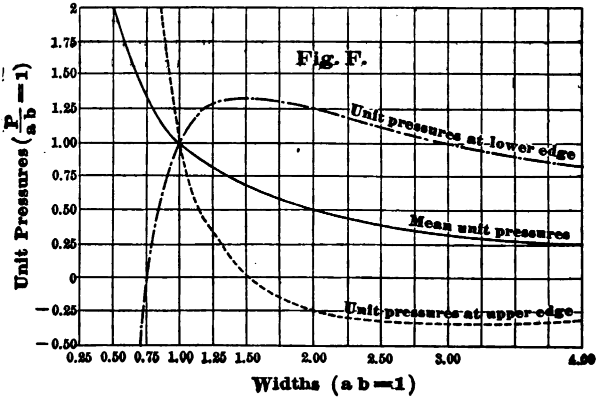


Fig. F.



shown) of varying width, and subjected to pressures, the resultant P of which is = 1,* and passes through the center of that section *a b* whose width is 1.* †

---

* We adopt the value 1 for the pressure P, the width *a b*, and the thickness merely in order to facilitate the explanation. It is not essential to the application of the principle.

† We here suppose ourselves dealing with a perfectly rigid and homogeneous material. In practice, these values would be more or less modified by yielding of the particles under stress, by unevennesses in the surfaces of the supposed cross sections, etc. Nevertheless, the general principles here laid down hold good.

The pressure per unit of area of cross section (or "unit stress") in the section ab is then $\frac{P}{ab \times 1} = \frac{P}{1} = P = 1$[*], and may be assumed to be uniformly distributed over it.

But at other sections of the bar the resultant is nearer to one edge than to the other, and the unit stress can then no longer be assumed to be uniformly distributed over the cross section, but, as explained on pp. 400 to 403 is a maximum at the edge nearest to the resultant, and diminishes gradually and uniformly to a minimum at the farther edge.

This is indicated by the shaded triangles, etc., in Fig. E and by the curves in Fig. F, which show, for the several sections in Fig. E, the mean unit stress[*] and the unit stresses at the upper and lower edges respectively, calculated by the rules on pp. 231 *a* and 231 *b*.

These stresses are also given in the following table:

Unit Stresses in Fig. E; the unit stress $\frac{P}{ab}$ in section ab being taken as 1.[†]

Section.	Width.	Stress per unit of area of cross section.		
		Mean.	At lower edge *m e*.	At upper edge *n f*.
	4.00	0.25	0.8125	− 0.3125 [‡]
e f	3.00	⅓	1.00	− ⅓
	2.50	0.40	1.12	− 0.32
	2.00	0.50	1.25	− 0.25
c d	1.50	⅔	1⅓	0
	1.25	0.80	1.28	0.32
a b	1.00	1.00	1.00	1.00
g h	0.75	1⅓	0	2⅔
i k [¶]	0.50	2.00	− 4 [‡]	8.00
m n [¶]	0.25	4.00	− 32 [‡]	40.00

It is important to notice that for a given force P, and for widths less than 3 *a b*, the strongest section of this bar is *not the widest one*, but that (*a b*) at which the resultant P passes through the center of the section. In other words, **a bar may be weakened by additions to its cross section** if those additions are such as to cause the resultant of the pressures to pass elsewhere than through the center of any cross section. This fact is entirely independent of the *weight* of the added portion.

Among the sections wider than *a b*, the weakest is that (*c d*) whose width is = 1.5 *a b*. At that section the lower edge *m e* has its maximum unit stress $\left(= 1\tfrac{1}{3} \times \frac{P}{ab}\right)$ while at *d* in the upper edge there is no pressure. Beyond *c d* the upper edge *n f* is in *tension*[‡] and the unit pressure along *m e* decreases, becoming again $= \frac{P}{ab}$ at *e f*, where the width *e f* is = 3 *a b*, and decreasing still further with further increase in width.

---

[*] In the case discussed on pp. 400 to 403, the mean pressure $u\,n = \frac{P}{u\,v}$, remained constant so long as the entire surface *u v* was called into play. Here, on the contrary, the area of the section varies. Hence the mean unit pressure varies also, and inversely as the area.

[†] See foot-note *, p. 462.

[‡] In the present discussion, as well as in that on pp. 400 to 403, we have assumed cases of *compression* for illustration, but the principle involved applies equally to cases where the force applied is *tensile*. In such cases, however, the terms "pressure" and "tension" are of course reversed.

[¶] The unit stresses at the edges in section *i k* are too great to be shown conveniently in either figure; while those in section *m n* (as the table shows) far exceed the limits of the figures. The pressure at *k* would be $\frac{P}{o} = \infty$ (infinity) were it not for the tensions in the lower part of the section.

When the width becomes *less* than that at $a\,b$, as at $g\,h$, etc., the *upper* edge of the bar comes nearer to the resultant than the lower edge, and hence receives the maximum pressure.

When the width $= \frac{3}{4}\,a\,b$, as at $g\,h$, the distance of the resultant from the upper edge is $\frac{1}{3}$ the width of the section. The pressure at the lower edge is then $= 0$; the mean pressure in $g\,h$ is $\frac{P}{a\,b} \times \frac{1}{0.75} = 1\frac{1}{3} \times \frac{P}{a\,b}$, and the pressure at the upper edge is twice the mean pressure in $g\,h$, or $2\frac{2}{3} \times \frac{P}{a\,b}$.

When the width becomes less than $\frac{3}{4}\,a\,b$, as at $i\,k$ and $m\,n$, the pressure at the lower edge $m\,e$ becomes negative or tensile.* Thus, when, as at $i\,k$, the width is $= \frac{1}{2}\,a\,b$, and the resultant passes through the upper edge, the unit pressure at that edge is $= 8 \times \frac{P}{a\,b}$, while the lower edge sustains a *tension* of $4 \times \frac{P}{a\,b}$; and, as the section is further reduced, these stresses are still further and very rapidly increased.

The condition of those sections (such as $m\,n$) where the line of the resultant passes outside the section, is similar to that of the section $m\,n$ of a bent hook sustaining a load, as in Fig. G.

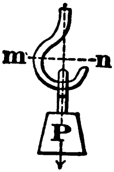


Fig. G.

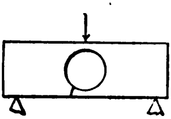


Fig. H.

Messrs. William Sellers & Co., of Philadelphia, had occasion to test a number of cast-iron beams, each having a large circular opening, as in the annexed figure. These beams broke, not at the smallest section directly under the center of the opening, but a little to one side, where the section was deeper, as indicated in Fig. H.

Fatigue of Materials. In the following articles on Strength of Materials, the ultimate or breaking load is that which will, during its first application, rupture the given piece within a *short* time. But Wöhler's and Spangenberg's experiments show that a piece may be ruptured by **repeated applications** of a load much *less* than this; and that the oftener the load is applied the less it needs to be in order to produce rupture. Thus, wrought iron which required a tension of 58000 lbs per sq inch to break it in 800 applications, broke with 35000 lbs per sq inch applied about 10 million times; the stress, after each application, returning to zero in both cases.

The *diff* between the maximum and minimum tension in a piece subjected to tension only, or between the max and min compression in a piece subjected to comp only; or the *sum* of the max tension and max comp in a piece subjected alternately to tension and comp; is called the **range of stress** in the piece. Stresses alternating between 0 and any point within the elastic limit may be repeated many million times without producing rupture.*

For a given number of applications, the load required for rupture is least when the range of stress is greatest. If the stress is alternately comp and tension, rupture takes place more readily than if it is always comp or always tension. That is, it takes place with a less range of stress applied a given number of times, or with a less number of applications of a given range of stress. For a given range of stress and given number of applications, the most unfavorable condition is where the tension and comp are equal.

The above facts are now generally taken into consideration in designing members of important structures subject to moving loads. For instance, Mr. Jos. M. Wilson, C. E., Mem. Inst. C. E. (London Eng.), Mem. Am. Soc. C. E., uses the following formulæ for determining the "permissible stress" in iron bridges, in lbs per sq inch; in order to provide the proper area of cross section for each member.

For pieces subject to *one kind of stress only* (all comp or all tension)

$$\mathbf{a} = u\dagger\left(1 + \frac{\text{min stress in the piece}}{\text{max stress in the piece}}\right)$$

For a piece subject *alternately to comp and tension*, find the max comp and the max tension in the piece. Call the *lesser* of these two maxima "max lesser", and the other or *greater* one, "max greater". Then

$$\mathbf{a} = u\dagger\left(1 - \frac{\text{max lesser}}{2\,\text{max greater}}\right)$$

For a piece whose *max comp and max tension are equal*, this becomes

$$\mathbf{a} = u\dagger\left(1 - \frac{1}{2}\right) = \frac{u}{2}$$

The above **a** is the permissible *tensile* stress in lbs per sq inch on any member; but the permissible *compressive* stress is found by "Gordon's formula" for pillars, p 495, using **a** (found as above) as the numerator, instead of *f*. For *a* in the *divisor* or *denominator* of Gordon's formula (which must not be confounded with the **a** of the *foregoing* formulæ) Mr. Wilson uses for wrought iron:

when both ends are fixed	36000
when one end is fixed and one hinged	24000
when both ends are hinged	18000

Experiments show that materials may fail under a **long continued stress** of much less intensity than that produced by the ult or bkg load.

---

[*] This does not always hold in cases where the elastic limit has been artificially raised by process of manufacture, etc. Oft-repeated alternations between tension and compression below such a limit reduce it to the natural one. A slight flaw may cause rupture under comparatively few applications of a range of stress but little greater, or even less, than the elastic limit. Rest between stresses increases the resisting power of a piece. In many cases, stresses a little beyond the elastic limit, even if oft-repeated, raise that limit and the strength, but render the piece brittle and thus more liable to rupture from shocks; and a little further increase of stress rapidly lessens, or may entirely destroy, the elasticity. A *tensile* stress above the elastic limit greatly lowers, or may even destroy, the *compressive* elasticity, and vice versa. If a tensile stress, by stretching a piece, reduces its resisting area, it may thus reduce its *total* strength, even though the strength *per sq in* has increased. Mr. B. Baker finds that hard steel fatigues much faster under repeated loads than soft steel or iron.

[†] u = 6500 lbs per sq inch for rolled iron in compression
= 7000 lbs " " " tension (plates or shapes).
= 7500 lbs " for double rolled iron in tension (links or rods).

TRANSVERSE STRENGTH.

1. In Statics, ¶¶ 285, etc., we discuss the action of external or destructive forces upon cantilevers, beams and trusses. We here discuss the reaction of the internal or resisting forces (stresses) in solid cantilevers and beams, in order to determine their loads. See also Trusses.

2. Unless otherwise stated or apparent, we assume that the stresses in all parts of the cantilever or beam are within the elastic limit.

Conditions of Equilibrium.

3. For equilibrium, the internal forces, and their moments, must balance the external forces and their moments. In other words, if the cantilever or beam be supposed cut by a section at any point, we must have

$$\begin{array}{lll}(1) & \Sigma \text{ vertical forces} & = 0 \\ (2) & \Sigma \text{ horizontal forces} & = 0 \\ (3) & \Sigma \text{ moments} & = 0\end{array}$$

Or:

(1) Algebraic sum of the internal vertical *stresses* = algebraic sum of the external vertical *forces* on either side of the section;

(2) Sum of horizontal *tensile* stresses = sum of horizontal *compressive* stresses; and

(3) Algebraic sum of the moments of the internal *stresses* = algebraic sum of moments of external *forces* on either side of the section.

4. Cantilevers and beams of uniform cross-section have usually a superabundance of strength against shearing, and fail (if at all) near the middle, where the bending moment is greatest. Hence the discussion of their resistance turns principally upon equilibrium of *moments*. For their resistance to vertical shear, see Statics, ¶¶ 325, etc., and p. 499. See also Horizontal Shear, ¶¶ 51 to 53, below.

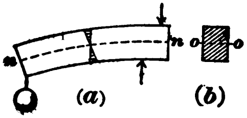


Fig. 1.

5. For equilibrium, therefore, the resisting moment, R (= the sum of the resisting moments, r, of all the particles in any cross-section of the cantilever or beam, Fig. 1 or 2), must be equal to the bending moment, M, or algebraic sum of the moments of all the external forces on either side of the section.

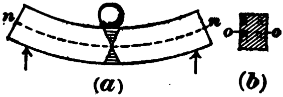


Fig. 2.

Reactions of Fibers.

6. In a *truss* or *framed* beam (see Trusses) the resistance of each of its two chords is regarded as acting in a line passing through the centers of gravity of the cross-sections of the chord; but, in a solid cantilever, Fig. 1, or beam, Fig. 2, the total resisting moment is the sum of the separate resisting moments of the several fibers throughout the cross-section.

Neutral Surface. Neutral Axis.

7. When a cantilever (or beam) bends, the fibers in the upper (or lower) part of each cross-section are extended, while those in the lower (or upper) part are compressed (see Figs. 1 and 2); the extension and compression being greatest at the top and bottom of the section, and thence decreasing uniformly inward toward a surface, *n n*, Figs. (*a*), near the center of the cross-section. In this surface, which is called the **neutral surface,** the fibers are neither extended nor compressed. The line, *o o*, Figs. (*b*), formed by the intersection of the neutral surface with any cross-section of the cantilever or beam, is called the **neutral axis** of that section.

8. In order that the algebraic sum of all the horizontal stresses in the cross-section may be zero, as required for equilibrium, the neutral axis must pass through the center of gravity of the section. Hence, the neutral surface passes through the centers of gravity of all the cross-sections.

9. The neutral axis may be found by balancing the section (cut out of cardboard) over a knife-edge. Or see Center of Gravity, under Statics, ¶¶ 125, etc. Every section has an indefinite number of neutral axes, all passing through its center of gravity in as many different directions. The axis required, in any given case, is that one which is normal to the plane of the bending moment under consideration.

In the following discussion, we assume that the neutral axis of the section is normal to the line of action (usually vertical) of the load, as it generally is. For other cases, as, for instance, the case of roof purlins, see "The Determination of Unit Stresses in the General Case of Flexure," by Prof. L. J. Johnson, Boston Soc. of Civil Engineers, in Jour. Ass'n of Eng'ng Socs., vol. XXVIII, No. 5, May, 1902.

Resisting Moment. Unit Stress.

10. It is assumed that the extension or compression of each fiber, and therefore the resisting force actually exerted by it, is proportional to its vertical distance, *t*, above or below the neutral axis.

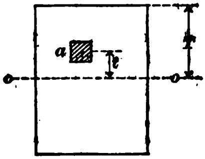


Fig. 3.

In Fig. 3, let

T = the distance from the neutral axis, *o o*, to the fiber farthest from that axis, either above or below the axis;
S = the unit stress in said farthest fiber;
t = the distance from the neutral axis to any given fiber;
s = the unit stress in said given fiber;
a = the area of said given fiber;
F = the total stress in said given fiber;
r = the resisting moment of said given fiber about the neutral axis;
M = bending moment at the cross-section under consideration;
R = the resisting moment of the entire cross-section;
= Σr = the sum of the resisting moments of all the fibers;
I = the moment of inertia of the cross-section. See ¶¶ 14, etc.;
= $\Sigma t^2 a$ = the sum, for all the fibers, of $t^2 a$;
X = the section modulus, $= \frac{I}{T} = \frac{R}{S}$. See ¶¶ 25, etc.

Then the unit stress, in any given fiber, is $= s = S\frac{t}{T}$; its total stress, F, is $= a\,s = S\,a\frac{t}{T}$; and its resisting moment, r, is $= F\,t = S\,a\frac{t^2}{T}$. Hence, the resisting moment, R, of the entire section, is

$$R = M = \Sigma r = \Sigma S\,a\frac{t^2}{T} = \frac{S}{T}\Sigma t^2 a = \frac{S}{T}\,.\,I.$$

Hence, also,

$$S = \frac{MT}{I}; \quad I = \frac{MT}{S}; \quad \text{and } T = \frac{SI}{M}.$$

Since,

$$\frac{S}{s} = \frac{T}{t}, \text{ it follows that } \frac{S}{T} = \frac{s}{t}; \text{ and}$$

$$R = M = \frac{S}{T} . I = \frac{s}{t} . I.$$

The resisting moment, R, is = S X, and the moment of inertia, I, = T X.

When beams are tested to destruction, the value attained by S is called the **Modulus of Rupture.**

11. It will be noticed that the strengths of similar beams of any shape, and those of rectangular beams, whether similar or not, are directly proportional to the product, width × square of depth. See ¶ 63.

12. When the stress, S, upon the extreme fibers, is = the elastic limit of the material, failure is imminent. The permissible unit stress is usually taken as not more than half the elastic limit, and the safe load is that under which S does not exceed the permissible unit stress.

13. The same quantity of material that composes a solid beam, Fig. 2, would present greater resistance to bending or breaking if it were cut in two lengthwise along the neutral surface, *n n*, and converted into top and bottom chords of a truss; because, first, the *leverage* with which the resistance acts is thus greatly increased; and, second, the depths of the chords are so small, compared with their distances from the neutral axis, that their fibers may be assumed to act *unitedly* and *equally*. Hence, practically, *all* the fibers in the upper chord must be crushed, or *all* those in the lower pulled apart, *at the same instant*, before the truss can give way; whereas, in the solid beam, the extreme upper or lower fibers yield first; then those next to them, and so on, one after the other.

Moment of Inertia.

14. Unlike the moment of a force, which is the product of a force and a distance, the moment of inertia, being the sum of the products of areas of fibers by the squares of their distances from the neutral axis, is a purely *geometrical* quantity. Thus, the moment of inertia of a given section depends solely upon the dimensions and shape of that section, and is independent of the material and the span of the beam and of the manner in which it is supported or loaded.

Unit of Moment of Inertia. The moment of inertia of a figure being the product of an area by the square of a distance, its unit is the fourth power of a unit of length. Thus, in a rectangle 3 ins. wide and 4 ins. deep, $I = \frac{b\ d^3}{12} = \frac{3 \times 64}{12} = \frac{192}{12} = 16$ biquadratic inches = 16 inch^4.

In a rectangle 1 inch wide and 6 ins. deep, $I = \frac{1 \times 6^3}{12} = 18$ inch^4.

15. Comparing similar sections of any shape, their moments of inertia (~~like their strengths, see ¶ 11~~) are proportional to the product, breadth × cube of depth.

16. The following illustrated table, pp. 469–471, gives, for several figures of frequent occurrence,

(1) I = the moment of inertia = $\Sigma\, t^2\, a$;

(2) T = the distance from the neutral axis to the farthest fiber;

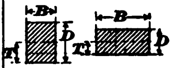
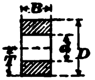
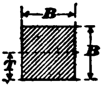
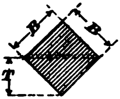
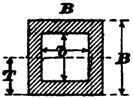
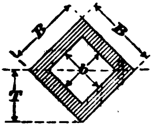
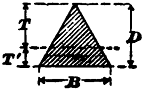
(3) X = the section modulus = $\frac{I}{T} = \frac{\Sigma\, t^2\, a}{T} = \frac{R}{S} = \frac{M}{S}$;

(4) A = the area of the cross-section.

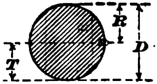
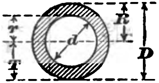
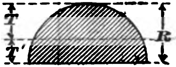
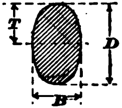
17. In sections where the distance from the neutral axis to the lowermost fiber, and the corresponding section modulus, differ from those (T and X) pertaining to the uppermost fiber, those corresponding to the lowermost fiber are distinguished as T′ and X′ respectively.

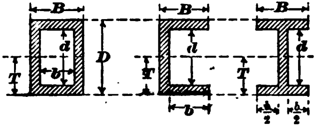
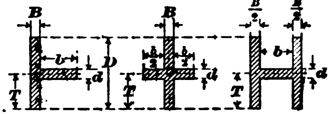
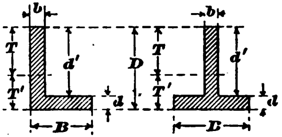
18. In each figure the neutral axis is indicated by a horizontal line crossing the section.

Moments of Inertia, etc.

		I Moment of inertia.	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus.	A Area of Section.
1		$\frac{BD^3}{12}$	$\frac{D}{2}$	$\frac{BD^2}{6}$	BD
2		$\frac{B(D^3 - d^3)}{12}$	$\frac{D}{2}$	$\frac{B(D^3 - d^3)}{6D}$	B (D - d)
3		$\frac{B^4}{12}$	$\frac{B}{2}$	$\frac{B^3}{6}$	B^2
4		$\frac{B^4}{12}$	$\frac{B}{\sqrt{2}}$	$\frac{\sqrt{2}}{12} B^3$ $= 0.118 B^3$	B^2
5		$\frac{B^4 - b^4}{12}$	$\frac{B}{2}$	$\frac{B^4 - b^4}{6B}$	$B^2 - b^2$
6		$\frac{B^4 - b^4}{12}$	$\frac{B}{\sqrt{2}}$	$\frac{\sqrt{2}}{12} \cdot \frac{B^4 - b^4}{B}$ $= 0.118 \frac{B^4 - b^4}{B}$	$B^2 - b^2$
7		$\frac{BD^3}{36}$	$T = \frac{2}{3} D$ $T' = \frac{1}{3} D$	$X = \frac{BD^2}{24}$ $X' = \frac{BD^2}{12}$	$\frac{BD}{2}$

Moments of Inertia, etc. continued. ($\pi = 3.14159\ldots$)

		I Moment of inertia.	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus.	A Area of Section.
8	 Circle	$\frac{\pi R^4}{4} = 0.7854\ R^4$ $\frac{\pi D^4}{64} = 0.0491\ D^4$	$R = \frac{D}{2}$	$\frac{\pi R^3}{4} = \frac{\pi D^3}{32}$	$\pi R^2 = \frac{\pi D^2}{4}$
9	 Hollow Circle	$\frac{\pi(R^4 - r^4)}{4} = \frac{\pi(D^4 - d^4)}{64}$	$R = \frac{D}{2}$	$\frac{\pi(R^4 - r^4)}{4R} = \frac{\pi(D^4 - d^4)}{32D}$	$\pi(R^2 - r^2) = \frac{\pi(D^2 - d^2)}{4}$
10	 Semicircle	$\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4 = 0.1098\ R^4$	$T = 0.5756\ R$ $T' = 0.4244\ R$	$X = 0.1908\ R^3$ $X' = 0.2587\ R^3$	$\frac{\pi R^2}{2} = \frac{\pi D^2}{8}$
11	 Ellipse	$\frac{\pi B D^3}{64}$	$\frac{D}{2}$	$\frac{\pi B D^2}{32}$	$\frac{\pi B D}{4}$

		I Moment of inertia	T Distance from neutral axis to farthest fiber	$X = \frac{I}{T}$ Section modulus	A Area of Section
12	 B B B d d d D T T T b b $\frac{b}{2}$ $\frac{b}{2}$	$\frac{BD^3-bd^3}{12}$	$\frac{D}{2}$	$\frac{BD^3-bd^3}{6D}$	$BD-bd$
13	 B B $\frac{B}{2}$ $\frac{B}{2}$ b $\frac{b}{2}$ $\frac{b}{2}$ b D d d d T T T	$\frac{BD^3+bd^3}{12}$	$\frac{D}{2}$	$\frac{BD^3+bd^3}{6D}$	$BD+bd$
14	 b b T T T' T' d' d' D d d B B	$I = \frac{B[T'^3-(T'-d)^3]+b[T^3+(T-d)^3]}{3}$	$T' = \frac{Bd^2+bd'(D+d)}{2[BD-(B-b)d']}$; $T = D - T'$;	$X' = \frac{I}{T'}$; $X = \frac{I}{T}$;	$Bd+bd'$

19. The moment of inertia of any figure, about its neutral axis, is the sum of the moments of inertia of its several parts, about that same axis.

20. Let I = the moment of inertia of the entire figure about its neutral axis, *o o*;

i = the moment of inertia of any part, about the neutral axis, *o o*, of the entire figure;

m = the moment of inertia of that part, about its own neutral axis;

a = the area of that part;

t = the distance of its center of gravity from the neutral axis, *o o*, of the entire figure.

Then $I = \Sigma i$; and $i = m + a t^2$.

21. Thus, in Fig. 4,

$$i_1 = \frac{B D^3}{12} + B D t_1^2;$$

$$i_2 = \frac{b d^3}{12} + b d t_2^2;$$

and $I = i_1 + i_2$.

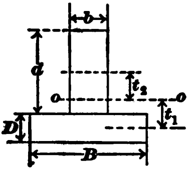


Fig. 4.

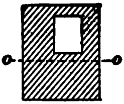


Fig. 5.

22. Hence, in any hollow section, as in the hollow rectangle, Fig. 5, let I' = the moment of inertia of the whole figure (including both the shaded and the unshaded rectangles), i = that of the missing or unshaded rectangle, and I = that of the shaded portion; all referred to the neutral axis, *o o*, of the *shaded portion*. Then $I = I' - i$.

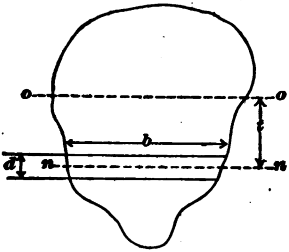


Fig. 6.

23. In the case of an irregular section, as Fig. 6, let the section be divided into numerous strips, parallel to the neutral axis and narrow enough to be considered as rectangular; and proceed as in ¶¶ 19 to 21.

. The narrower the strips are taken, the less m becomes. If the strips ken so narrow (relatively to the depth of the section) that m may be cted, then $I = \Sigma t^2 a$, as in ¶ 10. The strips need not be of uniform .

The Section Modulus.

. **Definition.** If the resisting moment, $R = \frac{S}{T} . \Sigma t^2 a$, be divided e unit stress, S, in the extreme fibers, the quotient, $X = \frac{R}{S} = \frac{\Sigma t^2 a}{T}$, is called the Section Modulus. This, like the moment of inertia, ¶¶ tc., is a purely *geometrical* quantity, depending solely upon the dimen- and shape of the section, and being independent of the material, of the , and of the manner of loading.

. Having the section modulus, X, we have only to multiply it by the stress, S, in the extreme fibers, in order to obtain the resisting moment, r $R = S X$.

. Multiplying the section modulus, X, by the distance, T, from the al axis to the farthest fibers, we obtain the moment of inertia, I; or, T X.

. The section modulus is usually given in tables of rolled beams, chan- nd shapes. See tables of Carnegie Beams, etc.

Loading. Strength.

. The following illustrated table gives the maximum moment, M, sponding to a given load, W, and the load, W,[*] corresponding to a unit stress, S, for different conditions of support and of loading. In table,

M = maximum bending moment;
W = the total extraneous load [*] on the beam, whether concentrated at one point (as shown) or uniformly distributed over the span;
l = the span;
S = the unit stress, in the fibers farthest from the neutral axis, due to the extraneous load, W;[*]
T = the distance from the neutral axis to the farthest fibers;
I = the moment of inertia.

rectangular beams,

b = breadth;
d = depth;
I = moment of inertia $= \frac{b d^3}{12}$;
$n = \frac{W l}{S b d^2}$.

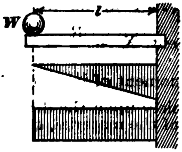
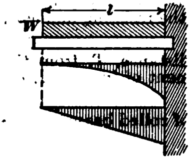
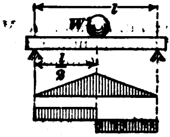
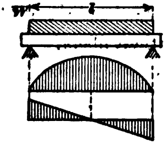
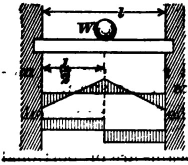
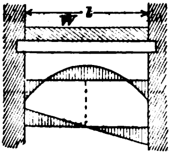
the two diagrams under each loading, the first represents the mo- s, and the second the shears, in the several parts of the span.

. If S = the permissible unit fiber stress, then, in the foregoing formulas, the permissible extraneous load.[*]

. It will be noticed that the strengths of similar beams are proportional eir values of $\frac{b d^2}{l}$; *i. e.*, the strengths of beams of similar cross-sections rectly proportional to their breadths and to the squares of their depths, nversely proportional to their spans.

---

[*] The beam is here supposed to be without weight. See ¶¶ 42, etc.

For symbols, see opposite page.	M Maximum bending moment.		Load		n
			General	In rectangular beams	
	At Support.	$M = W\,l$	$W = \frac{R}{l}$ $= S\cdot\frac{I}{T\,l}$	$W = \frac{S}{12}\cdot\frac{b\,d^3}{T\,l}$ $= \frac{S}{6}\cdot\frac{b\,d^2}{l}$	$\frac{1}{6}$
	At Support.	$M = \frac{W\,l}{2}$	$W = 2\,\frac{R}{l}$ $= 2\,S\cdot\frac{I}{T\,l}$	$W = \frac{S}{6}\cdot\frac{b\,d^3}{T\,l}$ $= \frac{S}{3}\cdot\frac{b\,d^2}{l}$	$\frac{1}{3}$
	At Center.	$M = \frac{W\,l}{4}$	$W = 4\,\frac{R}{l}$ $= 4\,S\cdot\frac{I}{T\,l}$	$W = \frac{S}{3}\cdot\frac{b\,d^3}{T\,l}$ $= \frac{2\,S}{3}\cdot\frac{b\,d^2}{l}$	$\frac{2}{3}$
	At Center.	$M = \frac{W\,l}{8}$	$W = 8\,\frac{R}{l}$ $= 8\,S\cdot\frac{I}{T\,l}$	$W = \frac{2\,S}{3}\cdot\frac{b\,d^3}{T\,l}$ $= \frac{4\,S}{3}\cdot\frac{b\,d^2}{l}$	$\frac{4}{3}$
	At Center and at Support.				
	At Support.	$M = \frac{W\,l}{12}$	$W = 12\,\frac{R}{l}$ $= 12\,S\cdot\frac{I}{T\,l}$	$W = S\cdot\frac{b\,d^3}{T\,l}$ $= 2\,S\cdot\frac{b\,d^2}{l}$	2

Symbols in Table Opposite.

S = unit stress, due to W, in the extreme fibers;
T = distance from the neutral axis to the extreme fibers;
I = moment of inertia; M = maximum bending moment;
W = load; l = span.

In rectangular beams,

$$b = \text{breadth}, \quad d = \text{depth}, \qquad n = \frac{W\,l}{S\,b\,d^2}; \qquad W = n\,S\,\frac{b\,d^2}{l}.$$

Beam 1 Inch Square, 1 Foot Span.

32. In a beam, 1 inch square and 1 foot (12 inches) span, supported at th ends, we have, for the extraneous center load:*

$$' = S \cdot \frac{2\,b\,d^2}{3\,l} = \frac{n\,S\,b\,d^2}{l} = S \cdot \frac{2}{3 \times 12} = \frac{S}{18}; \text{ and } S = 18\,W'.$$

33. For any other rectangular beam, let L = the span in feet. Then the traneous center load,* W, required to produce the same unit stress, S, in e extreme fibers, is

$$W = W'\,\frac{b\,d^2}{L}.$$

34. Thus, for yellow pine, let S = the permissible unit stress = $\frac{1}{2}$ the stic limit $= \frac{3240}{2} = 1620$ lbs. per sq. in. Then, for a beam 1 inch square, foot span, supported at both ends and loaded at center; the permissible d,* W′, is

$$W' = \frac{n\,S}{12} = \frac{S}{18} = \frac{1620}{18} = 90 \text{ lbs.},$$

d, for a joist, 3 × 12 ins., 20 ft. span; the permissible extraneous * center d is

$$W = W'\,\frac{b\,d^2}{L} = 90 \times \frac{3 \times 144}{20} = 1944 \text{ lbs.};$$

d the permissible extraneous uniform load is = 2 W = 2 × 1944 = 3888 s.

35. If the load, W, the span, L, and the coefficient, W′, are given, we ve. $b\,d^2 = \frac{W\,L}{W'}$. Thus, in the case of the yellow pine joist, mentioned ¶ 34, of 20 ft span, with a uniform extraneous* load, where 2 W = × 1944 = 3888 lbs., we have $b\,d^2 = \frac{W\,L}{W'} = \frac{1944 \times 20}{90} = 432$.

36. Then, if either b or d is given, the other is easily found. If not, sign, to either of them, an arbitrary value. Thus, if $b = 6$, we have $d^2 = \frac{2}{} = 72$; and $d = \sqrt{72}$ = say $8\frac{1}{2}$. With $b = 3$, $d^2 = 144$, and $d = 12$.

37. With the slide rule, in the foregoing example, place the runner at 432; d, assuming $b = 6$, place 1 (or 10) on the slide, opposite 6 on the rule. en, in the scale of square roots, on the slide, opposite 432 on the rule, will found 838 and 265. The former of these represents the desired root, and take 8.5 as a sufficient approximation.

38. If the relation, S = 18 W, held beyond the elastic limit, and if W′ = e center *breaking* load, in lbs., on a beam of any material, 1 inch square, t. span, supported at each end; then, for any other beam of the same material, and of breadth b ins., depth d ins., and span L ft., the center *breaking* d would be $W = W'\,\frac{b\,d^2}{L}$.

39. Notwithstanding the defective basis of this method, as applied to ds beyond the elastic limit, its simplicity renders it very convenient, and is much in use. See the following table of values of W′, and example, ¶ 40.

---

* The beam is here supposed to be without weight. For the weight of the am itself, see ¶¶ 42, etc.

Center Breaking Loads, W′, in Pounds, for Beams 1 Inch Square, 1 Foot Span, Supported at Each End.

In wooden beams in practice deduct one-third part, to allow for knots, crooked grain, &c.

	W′
WOODS.	
Ash, English	650
" Amer White (Author).	650
" Swamp	400
" Black	600
Arbor Vitæ, Amer	250
Balsam, Canada	350
Beech,	
" Amer	850
Birch, Amer Black	550
" Amer Yellow	850
Cedar, Bermuda	400
" Guadaloupe	600
" Amer White, or Arbor Vitæ	250
Chestnut	450
Elm, Amer White	650
" Rock, Canada	800
Hemlock	500
Hickory, Amer	800
" " Bitter nut	800
Iron Wood, Canada	600
Locust	700
Lignum Vitæ	650
Larch	400
Mahogany	750
Mangrove, White	680
" Black	550
Maple, Black	750
" Soft	750
Oak, English	550
" Amer White (by Author).	600
" " Red, Black, Basket	850
" Live	600
Pine, Amer White..(by Author).	450
" " Yellow " "	500
" " Pitch " "	550
" Georgia	850
Poplar	550
Poon	700
Spruce (by Author).	450
" Black	550
Sycamore	500
Tamarack	400
Teak	750
Walnut	550
Willow	350
METALS.	
Brass	850
Iron, cast, 1500 to 2700....average	2100
" " common pig	2000
" " castings from pig	2300
" " employed in our tables	2025
" " for castings 2½ or 3 ins thick	1800?
Iron, wrought, 1900 to 2600.....av	2250
Wrought iron does not *break*; but at about the average of 2250 lbs its elas limit is reached.	
Steel, hammered or rolled; elas destroyed by 3000 to 7000..	5000
Under heavy loads hard steel snaps like cast iron, and soft steel bends like wrought iron.	
STONES, ETC.	
Blue stone flagging, Hudson River	125
Brick, common, 10 to 30..average	20
" good Amer pressed, 30 to 50 average	40
Caen Stone	25
Concrete, see article on Concrete.	
Granite, 50 to 150 average	100
" Quincy	100
Glass, Millville, N. Jersey, thick flooring ....(by Author).	170
Mortar, of lime alone, 60 days old	10
" 1 measure of slacked lime in powder, 1 sand	8
" 1 measure of slacked lime in powder, 2 sand	7
Marble, Italian, White (Author)	116
" Manchester, Vt, " "	95
" East Dorset, Vt, " "	111
" Lee, Mass, " "	86
" Montg'y Co, Pa, Gray "	103
" " " Clouded "	142
" Rutland, Vt, Gray "	70
" Glenn's Falls, N.Y. Black "	155
" Baltimore, Md, white, coarse "	102
Oolites, 20 to 50	35
Sandstones, 20 to 70........average	45
" Red of Connecticut and New Jersey	45
Slate, laid on its bed, 200 to 450, av	325

40. Example. In the yellow pine joist of ¶¶ 34 and 36, 3 × 12 ins., ft. span, we have, from the table, W′ = say 500 lbs. Hence

Center breaking load * $W = W' \frac{bd^2}{L} = \frac{500 \times 3 \times 144}{20} = 10{,}800$ lbs., or out 5.5 times the permissible load, found, by means of the permissible unit ress, in ¶ 34.

Dimensions.

41. Since $W = n\,S\,\frac{bd^2}{l} = W' \frac{bd^2}{L}$, and $W' = \frac{n\,S}{12}$, we have *

Breadth $= b = \frac{W\,l}{n\,S\,d^2} = \frac{W\,L}{W'\,d^2}$;

Depth $= d = \sqrt{\frac{W\,l}{n\,S\,b}} = \sqrt{\frac{W\,L}{W'\,b}}$;

ere W = extraneous load * required;
W' = extraneous load on beam 1 inch square, 1 ft. span;
n = coefficient from last column of table $= \frac{W\,l}{S\,b\,d^2}$;
S = unit fiber stress;
l = span in inches;
L = span in feet.

Weight of Beam Considered as Load.

12. For simplicity we have hitherto regarded our cantilevers and beams having no weight of their own; and, in beams of the moderate dimensions ially employed in buildings, their own weight, *w*, is so small, in comparison h their loads, W, that it may often be safely neglected; but in larger ms it must generally be taken into account. The loads, found as above, h S = greatest permissible unit stress, must then be regarded as including only the extraneous load, W, but also the weight, *w*, of the beam itself, a length = span.

13. If the beam is prismatic,—*i. e.*, of uniform cross-section,—its weight, acts as a uniformly distributed load, and we have, for the extraneous load, in the case of a concentrated center load on a beam, or of a concentrated d at the end of the span, *l*, in cantilevers,

$$W = \text{whole load} - \frac{w}{2};$$

he case of a uniform load,

$$W = \text{whole load} - w.$$

4. In finding the breadth or the depth of a rectangular beam; uired to carry a given load with a given span and given unit stress, we y provide for the weight by successive approximations. Thus,

5. To find the breadth, *b*, required for a beam of given depth, *d*. glecting the weight, *w*, of the beam, find the first approximate breadth, *b*, the formulas in ¶ 41, for the extraneous load, W. Next, calculate the ght, *w*, of a beam with width, *b*, treat said weight as a uniform load; and, the same formulas, find the additional breadth, *b′*, required to carry this itional load, *w*. Then $b + b' =$ a second approximate breadth. If nec- ry, find the weight, *w′*, of a beam of breadth, *b′*, and, from this, a second itional breadth, *b″*, required to carry it. Then $b + b' + b'' =$ a third roximate breadth, and so on.

6. To find the depth, *d*, required for a beam of given breadth, *b*; a first approximate depth, *d*, by the formula, ¶ 41, for the extraneous l, W. Find the weight, *w*, of a beam of that depth; and again apply the nula, using (in place of W) $W + w$ if W is a uniform load, or $W + \frac{w}{2}$ if W concentrated load. The depth, *d′*, so found, is a second approximation, may again apply the formula, as before, using the weight, *w′*, of beam of th *d′*; or, more simply, increase the breadth, as in ¶ 45.

The beam is here supposed to be without weight. See ¶¶ 42, etc.

47. In practice, beams of rectangular section are almost always of timber; and such beams are economically obtainable only in certain commercial sizes. Hence, the second approximation will usually be all that is required.

Strengths and Weights of Similar Beams of Different Dimensions. Comparison between Models and Actual Structures.

48. In any given beam, let $W_1 =$ the load causing any given unit stress, S. Then, $W_1 = \frac{n\,S\,b\,d^2}{l}$ (for n, see table, p. 474); and, in any similar beam, of a times the breadth, depth and span, the corresponding load, $W = \frac{n\,S\,a\,b\,a^2\,d^2}{a\,l}$. Hence, the ratio of their *loads* is $\frac{W}{W_1} = a^2$; or $W = a^2\,W_1$. but the ratio of their *weights* is $\frac{w}{w_1} = \frac{ab\;ad\;al}{b\;d\;l} = a^3$; or $w = a^3\,w_1$.

49. In other words, comparing one beam with another, of a times its breadth, depth and span, their *strengths* are as the *squares* of their respective dimensions; but their *weights* are as the *cubes* of those dimensions.

50. Hence, if a model of a beam will just break under a uniform load (including its own weight, w) = 2, 3 or 4, etc., times its own weight, then a beam of similar cross-section, but of 2, 3 or 4, etc., times its breadth, depth and span, will just break under its own weight alone.

Horizontal Shear.

51. When (Figs. 7 and 8) deflection occurs in a cantilever or beam composed of separate horizontal layers, like a pile of loose boards, the several layers slide upon each other; but, if they are firmly joined together, or otherwise prevented from sliding, they exert, upon each other, a horizontal shearing force. In any section, this force diminishes uniformly from a maximum, at the neutral surface, $n\,n$, to zero, at the top and bottom of the section.

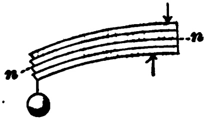


Fig. 7.

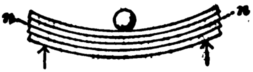


Fig. 8.

52. In any section of a rectangular beam, the maximum horizontal shear, per unit of neutral surface, is

$$H = \frac{3\,V}{2\,b\,d};$$

where $V =$ the vertical shear in the section, and b and $d =$ the breadth and the depth of the section, respectively.

In words, the unit horizontal shear, at any point, is directly proportional to the vertical shear at that point. Hence, the horizontal shear diagram is similar, in character, to the vertical shear diagram; but is opposite in sense, positive vertical shear corresponding to negative horizontal shear.

53. If the horizontal shear is resisted by a fastening applied at only one point, said fastening must be made sufficiently strong to resist the *sum of all* the horizontal shears between such point and that where the shear is = 0.

54. In Fig. 9, diagrams (b) and (c) show respectively the moments and the vertical shears due to concentrated and distributed loads on a beam as shown; and, in Fig. (d), each ordinate represents the force which must be applied, at the corresponding point, in order to resist the sum of all the horizontal shears between that point and the point of zero shear. Ordinates above a zero line indicate positive moments or shears, and vice versa. In *positive* moments, the segment to the *left* of a ion tends to turn *clockwise*. In *positive* shears, the *left*-hand segment ls to slide *upward* or the *upper* segment to slide toward the *right*. ween *a* and *c*, between *c* and *d*, between *g* and *h*, and between *h* and ll the diagrams are straight lines, Figs. (*b*) and (*d*) being inclined, Fig. (*c*) horizontal. At *c* and at *h*, Figs. (*b*) and (*d*) change their ination, and Fig. (*c*) shifts its position. Between *d* and *f*, and ween *f* and *g* (*i. e.*, under the distributed load), Figs. (*b*) and (*d*) are abolic curves, and Fig. (*c*) shows inclined straight lines. At *f*, Figs. and (*d*) change curvature, and Fig. (*c*) shifts its position. At *e*, the it of maximum moment, Figs. (*c*) and (*d*) change signs. See Relation ween Moment and Shear, Statics, ¶¶ 359 to 368.

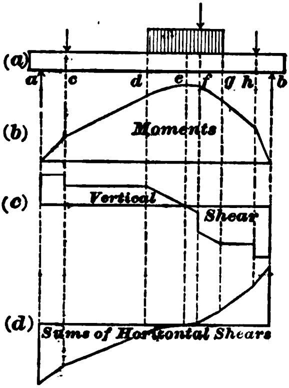


Fig. 9.

5. Inasmuch as the horizontal shear is a resistance to bending, its neglect, ie common theory of beams, as heretofore explained, is in general on the of safety. But, in beams composed of horizontal layers, means must rovided for its transmission from one layer to the next.

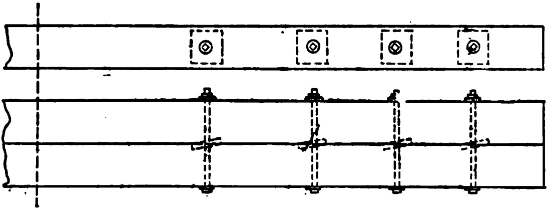
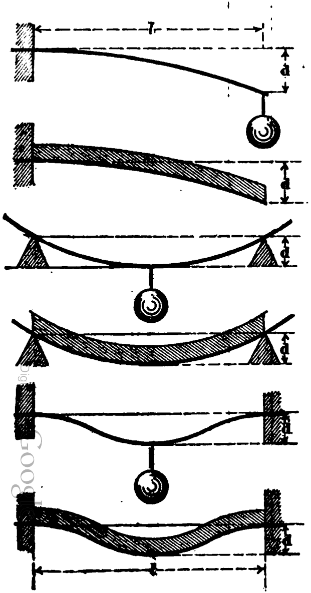


Fig. 10.

3. Thus, deep wooden beams, Fig. 10, are frequently built up of two or e timbers, one above the other. In order to prevent deflection, due to sliding of these timbers upon each other, blocks are inserted between n at intervals, as shown, or the adjacent sides of the timbers are so hed as to interlock. In either case, the timbers are tightly bound ther. The blocks or notches then serve to transmit the horizontal shear n one timber to the other. In Fig. 10 the blocks are more numerous near ends of the span, as required by the diagram of horizontal shear, Fig. 9 (*d*).

DEFLECTIONS OF BEAMS OF UNIFORM CROSS SECTION THROUGHOUT. (For signification of letters, see opposite.)



If the beam is	Deflection d, in inches, caused by extraneous load.	by weight of beam.	by weight of beam and load.	Extraneous load for a given deflection (weight of beam neglected.)
Fixed at one end, loaded at the other	$\frac{1}{3}\ \frac{l^3\ W}{E\ I}$	$\frac{1}{8}\ \frac{l^3\ w}{E\ I}$	$\frac{1}{3}\ \frac{l^3\ (W + \frac{3}{8}\ w)}{E\ I}$	$3\ \frac{d\ E\ I}{l^3}$
" " " uniformly	$\frac{1}{8}$ "	$\frac{1}{8}$ "	$\frac{1}{8}\ \frac{l^3\ (W + w)}{E\ I}$	8 "
Supported at both ends, loaded at the center	$\frac{1}{48}$ "	$\frac{5}{384}$ "	$\frac{1}{48}\ \frac{l^3\ (W + \frac{5}{8}\ w)}{E\ I}$	48 "
" " " uniformly	$\frac{5}{384}$ "	$\frac{5}{384}$ "	$\frac{5}{384}\ \frac{l^3\ (W + w)}{E\ I}$	$\frac{384}{5}$ "
Fixed " " at the center	$\frac{1}{192}$ "	$\frac{1}{384}$ "	$\frac{1}{192}\ \frac{l^3\ (W + \frac{1}{2}\ w)}{E\ I}$	192 "
" " " uniformly	$\frac{1}{384}$ "	$\frac{1}{384}$ "	$\frac{1}{384}\ \frac{l^3\ (W + w)}{E\ I}$	384 "

Deflections.

57. The opposite table gives the **deflections** within the elastic limit, of any *prismatic* beam (beam of uniform cross section throughout) **under different arrangements of support and of load;** also (in the last column) the *extraneous* **load which will produce a given deflection,** without assistance from the weight of the beam itself. All the formulæ are based upon the assumption that the increase of deflection is proportional to increase of load.

The letters signify as follows:

d = deflection of beam, in inches (see Figs).
W = weight of extraneous load, in pounds.
w = " " clear span of beam, in pounds.
l = clear span of beam, in inches (see Figs).
E = modulus of elasticity of the material of the beam, in lbs per sq inch.
I = moment of inertia of the cross section of the beam, in inches.

From the principles embodied in the opposite table, we find that in beams of similar cross section and of the same material, and within the elastic limit, the load, and deflections (neglecting the weight of the beam itself) are as follows:

With the same	The deflections under a given extraneous load are
span	inversely as the breadths and as the cubes of the depths
" and breadth	" " " " "
" " depth	" " breadths
breadth " "	directly " cubes of the spans

With the same	The extraneous loads for a given deflection are
span	directly as the breadths and as the cubes of the depths
" and breadth	" " " " "
" " depth	" " breadths
breadth " "	inversely " cubes of the spans

Deflection in Terms of Extreme Fiber Stress. In table, p. 474, the load $W = k S \frac{I}{T\,l}$; where k = a coefficient, as below; S = unit stress in extreme fibers; I = moment of inertia; T = distance from neutral axis to extreme fiber, and l = span. From the table opposite, we have: $W = m \frac{d\,E\,I}{l^3}$; where m = a coefficient, as below; d = deflection, and E = modulus of elasticity. Hence, $k S \frac{I}{T\,l} = m \frac{d\,E\,I}{l^3}$; and $d = \frac{k}{m} \cdot \frac{l^2 S}{E T} = \frac{l^2 S}{E\,T\,c}$; where $c = \frac{m}{k}$.

In a cantilever, loaded at end, $m = 3$; $k = 1$; $c = 3$.
" " " uniformly, $m = 8$; $k = 2$; $c = 4$.
In a beam, supported, and loaded at center, $m = 48$; $k = 4$; $c = 12$.
" " " " " uniformly, $m = 76.8$; $k = 8$; $c = 9.6$.
" " fixed, " " at center, $m = 192$; $k = 12$; $c = 16$.
" " " " " uniformly, $m = 384$; $k = 12$; $c = 32$.

Elastic Limit.

58. Under moderate loads, the deflections are practically proportional to the load. When they begin to increase perceptibly faster than the load, the latter is said to have reached the elastic limit, or limit of elasticity. It is generally at this point that the "**permanent set**" first becomes noticeable; i. e., after removal of the load, the beam fails to return to its original unstrained condition, and remains more or less bent. The deflections then also begin to increase *irregularly;* and to continue indefinitely without further increase of load. In short, the beam is in danger. Hence, the actual load *must never exceed* the elastic limit; and should not exceed from one-third to two-thirds of it, according to circumstances.

The limit of elasticity of a beam of any particular form, or material, is **determined by experiment** with a similar beam, as in the case of constants for breaking loads, &c. Thus, load a beam at the center, by the careful gradual addition of small equal loads; carefully note down the deflection that takes place within some minutes (the more the better) after each load has been applied; in order to ascertain when the deflections begin to increase more rapidly than the loads; for when this takes place, the load for elastic limit has been reached.*

It is not the deflections of the *whole* beam that are to be noted, but those of its clear span only. Several beams should be tried, in order to get an *average* constant, for even in rolled iron beams of the same pattern, and same iron, there is a very appreciable difference of strengths and deflections.

Then, to get the constant, using the *total load* applied during the equal deflections, including half the weight of the beam itself,

$$\textbf{Constant for elastic limit} = \frac{\text{Span in feet} \times \text{Total load in lbs.}}{\text{Breadth in inches} \times \text{Square of depth in inches}}$$

The constant, for wooden beams, may be had, near enough for common practice, by taking *one third* of the *breaking* constants in the table, page 476.

Said constant, thus calculated, is the elastic limit of a beam of the given shape and material, 1 inch broad, 1 inch deep, and of 1 foot span, supported at both ends and loaded at the center. To obtain from it the elastic limit of any other beam of the same design ‡ and the same material, similarly supported and loaded, but of other dimensions,

$$\textbf{Elastic limit} = \text{constant} \times \frac{\text{breadth in inches} \times \text{square of depth in inches}}{\text{span in feet}}.$$

If the beam is	Multiply the result by
supported at both ends and loaded at center,	1
" " " " " " uniformly,	2
fixed † " " " " " at center,	2
" " " " " " uniformly,	3
" " one end " " at other end,	¼
" " " " " " uniformly,	½

---

*Of course, in practice, it is frequently difficult to ascertain with precision, when, or under what load, the deflections actually do begin to increase more rapidly than the successive loads. For although *by theory* the deflections are practically equal for equal loads, until the elastic limit is reached, yet in *fact* they are subject to more or less irregularity; for no material composing a beam is perfectly uniform throughout in texture and strength. Hence, instead of regular increase of deflection, we shall have an alternation of larger and smaller ones. Therefore, some judgment is required to determine the final point; in doing which, it is better, in case of doubt, to lean to the side of *safety*. It is assumed always that the load is not subject to jars or vibrations. These would increase the deflections.

† A beam is said to be "fixed" at either end when the tangent to the longitudinal axis of the deflected beam at that end remains always horizontal.

‡ The *shapes* of the two beams need not be *similar*. For instance, the constant deduced from experiments upon any rectangular beam is applicable to any other rectangular beam, whether square or oblong.

The Elastic Curve.

59. When a cantilever, Fig. 1, or a beam, Fig. 2, supported or fixed in any manner, bends, under the action of any load, the neutral surface, *n n*, forms a curve, such that, at any section,

$$R = \frac{E\,I}{M} = \frac{I\,S}{M\,k},$$

where

R = the radius of curvature, at the section;
M = the bending moment, at the section;
I = the moment of inertia of the section;
E = the elasticity coefficient of the material, $= \frac{S}{k}$;
S = any unit stress within the elastic limit;
k = the unit "stretch" (elongation or compression) produced by S in the given material.

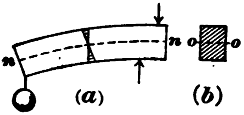


Fig. 1 (repeated).

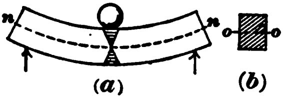


Fig. 2 (repeated).

The Deflection Coefficient.

60. Definition. The deflection coefficient, for any given material, is the deflection, in inches, of a beam, of that material, 1 inch square and of 1 foot span, supported at each end, and carrying, at its center, an extraneous load of 1 lb. — ⅝ *w'*, where *w'* = weight of clear span of beam alone, in lbs.

61. Let *y* = the deflection coefficient for any given material. Then, in any rectangular beam, of the same material, with center load or uniform load, let

b = the breadth, in inches;
d = the depth, in inches;
L = the span, in feet;
w = the weight, in lbs., of the clear span of the beam itself;
W = center load + ⅝ *w*;
= ⅝ (uniform load + *w*).

Then, in the given beam,

Deflection $= Y = y\,\frac{W\,.\,L^3}{b\,.\,d^3}$; **Breadth*** $= b = W\,.\,\frac{L^3\,.\,y}{d^3\,.\,Y}$;

Load $= W = Y\,.\,\frac{b\,.\,d^3}{L^3\,.\,y}$; **Depth*** $= d = L\sqrt[3]{\frac{W\,y}{b\,Y}}$.

62. The deflection coefficient, *y*, for any given material, is obtained by experiment, thus: At the center of any rectangular beam, of the given material, placed horizontally upon two supports, at any convenient and known distance apart, place any load that is within the elastic limit, and measure the resulting deflection, Y. Let W = the extraneous center load + ⅝ *w*, where *w* = the weight, in lbs., of the clear span of the beam itself. Then the deflection coefficient is

$$y = Y\,.\,\frac{b\,.\,d^3}{W\,.\,L^3};$$

where *b* and *d* = the breadth and depth, in inches, and L = the span, in feet, of the experimental beam.

---

* In calculating the breadth or the depth, if it is necessary to provide for the weight of the beam itself, we first let W = the extraneous load only, and then proceed by successive approximations, as in ¶¶ 45 and 46, remembering, however, that in the case of deflections, 5–8 of the weight of each additional section is to be taken as equivalent center load, and not 1–2 as in the case of strengths.

63. The ratio between any two homologous lines, in any two similar figures, is constant. Hence, in determining or using coefficients, whether for strength or for deflection, by comparing beams of similar sections but of different sizes, we may use any two homologous lines in place of the two breadths, or in place of the two depths, or the same line may be taken in place of both breadth and depth. Thus, in Figs. 11,

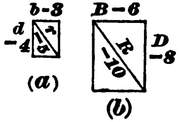


Fig. 11.

$$\frac{B}{b} = \frac{D}{d} = \frac{R}{r} = 2.$$

Hence,

$$\frac{B\,D^2}{b\,d^2} = \frac{384}{48} = 8 = 2^3 = \frac{R\,R^2}{r\,r^2}; = \frac{1000}{125} = 8 = 2^3;$$

Also,

$$\frac{B^3\,D}{b_3\,d} = \frac{1728}{108} = 16 = 2^4 = \frac{R^3\,R}{r^3\,r} = \frac{10{,}000}{625} = 16 = 2^4.$$

64. Deflection coefficients, being the deflections, y, in inches, at the centers, of beams 1 inch square and of 1 foot span, supported at each end and loaded at center with extraneous loads of 1 lb. — 5–8 the weight of the clear span of the beam itself.

		y Average
Cast iron,	0.000018 to 0.000036,	0.000027
Rolled bar iron,	0.000012 to 0.000024,	0.000018
Rolled tool steel,	0.000010 to 0.000020	0.000015
White oak, well seasoned,		0.00023
Best Southern pitch pine, well seasoned, White ash, well seasoned,		0.00027
Hickory, well seasoned,		0.00016
White pine, well seasoned, Ordinary yellow pine, well seasoned, Spruce, well seasoned, Good, straight-grained hemlock, well seasoned, Ordinary oaks, well seasoned,		0.00032

65. Caution. The deflections of timber of the same kind vary greatly with the degree of seasoning, the age of the tree, the part from which the beam is cut, etc. The coefficients given above are averages deduced from our own experiments on good pieces, well seasoned, on which the loads were allowed to remain for months. In all kinds, less than 2 per cent. of the breaking load produced a permanent set in a few months. Several of the sticks bore their breaking loads for months before actually giving way. The vibrations and jars, to which all structures are exposed, in time increase the deflections.

66. Eccentric Concentrated Loads. Let Y, Fig. 12 (*a*), be the deflection, at the center of the span (*i. e.*, at the point of application of the load) of a beam supported at each end, due to a load, W, within the elastic limit, at the center of the span. Then, if the same load, W, be placed eccentrically upon the same beam, as in Fig. 12 (*b*), the deflection, Y', at the point, *c*, of application of the load, and due to the load, W, is

$$Y' = Y\,\frac{16\,m^2\,n^2}{l^4};$$

where

l = the span;
m and n = the segments into which the load divides the span.

67. Uniform Loads. Let Y be the deflection, due to any central extraneous load (within the elastic limit), on a beam supported at each end. Then the deflection, Y′, of the same beam, due to the same load uniformly distributed over the span, is

$$Y' = \frac{5}{8} Y.$$

68. Inclined Beams. If the beam is inclined, use the horizontal projection of its span, in place of the span, *l*, in determining its deflections.

69. Cylindrical Beams. Let Y be the deflection of a square beam under any given load. Then, for a cylindrical beam whose diameter = side of the square, the deflection, under the same load, is = 1.698 Y.

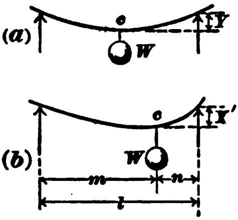
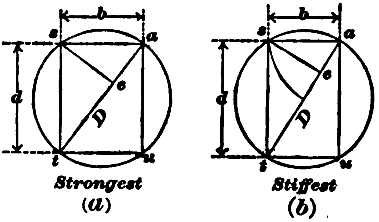


Fig. 12.



Strongest (a)

Stiffest (b)

Fig. 13.

70. Figs. 13 (*a*) and (*b*) show, respectively, the strongest and the stiffest rectangular sections which can be cut from a given cylindrical log, of diameter, D. In the strongest section Fig. (*a*), $ac = \frac{D}{3}$, and $b = \sqrt{\frac{1}{3} D^2}$. In the stiffest section Fig. (*b*), $ac = \frac{D}{4}$, and $d = \sqrt{\frac{3}{4} D^2}$.

71. Maximum Permissible Deflection. Under even a perfectly safe load, a beam may bend too much for certain purposes. Thus, to prevent the cracking of the plaster of ceilings, it is usual to limit the deflection of beams to $\frac{\text{span}}{360} = \frac{1}{30}$ inch per foot of span = 3⅓ ins. per 100 ft. In long lines of shafting, for machinery, the deflection is usually limited to $\frac{\text{span}}{1200} =$ 1 inch per 100 feet of span; in highway bridges to $\frac{\text{span}}{640} = \frac{3}{16}$ inch in 10 ft.; in railroad bridges to $\frac{\text{span}}{1600} = \frac{3}{4}$ inch per 100 feet.

72. Let Y = the maximum permissible deflection, in inches per foot of span, in any given case;
y = the deflection coefficient, ¶¶ 60, etc.
L = the clear span of the beam, in feet;
w = the weight of the clear span of the beam, in lbs.;
W = the center load + ⅝ w;
= ⅝ (uniform load + w).

Then, Y L = the deflection, in inches, for the whole span, L, and we have, for the permissible load, W, and the required breadth, *b*, and depth, *d*, for a rectangular beam (see ¶ 61):

$$\textbf{Load} = W = \frac{Y L b d^3}{L^3 y} = \frac{Y b d^3}{L^2 y};$$

$$\textbf{Breadth}^* = b = W \cdot \frac{L^2 y}{d^3 Y};$$

$$\textbf{Depth}^* = d = W \cdot \frac{L^2 y}{b Y}.$$

---

* See foot-note to ¶ 61.

Suddenly Applied Loads.

73. Suppose a load to be applied to a flexible beam suddenly, though without falling or jarring; as, for instance, if it be supported by a cord which allows it just to touch the beam without bearing upon it, and the cord be then suddenly cut in two. The deflection of the beam, in such a case, is theoretically twice as great as when the same load is applied gradually, as by very slowly relaxing the cord, or by dividing the weight into small fragments and applying them at intervals, one by one. See Art. 5 (*b*), under Strength of Materials. Hence the strength of the beam (within the elastic limit) is much more severely taxed in the former than in the latter case. A heavy train, coming very rapidly upon a bridge, presents a condition intermediate between the two.

Cantilevers and Beams of Uniform Strength.

74. For equilibrium, the resisting moment, R, of any section, must balance the bending moment, M, at that section. Or,

$$\frac{S}{T} . I = M; \text{ or, } S = M . \frac{T}{I};$$

where S = unit stress in extreme fibers;
T = distance from neutral axis to extreme fibers;
I = moment of inertia of section.

75. In a beam of uniform cross-section, therefore, since T and I are uniform throughout the span, the unit stress, S, on the extreme fibers, varies with the bending moment, M. For uniform strength against bending moments, the cross-section must so vary that $\frac{T}{I}$ shall be inversely proportional to M, in order that S may remain constant.

76. The following table shows, in elevation and in plan, the theoretical shapes of rectangular cantilevers and beams of uniform strength against bending moments, under concentrated and uniform loads. In practice, some of these shapes would of course have to be made stronger near their ends, in order to provide a sufficient section to resist shear.

77. Notwithstanding the reduction in material which would be effected, by using beams of uniform strength, their use is seldom economical, except in the case of cast iron. In timber, the material removed would not be saved; and, in steel, the saving in material would often be offset by the cost of additional labor.

Moreover, it will be noticed that the **deflections** of beams of uniform strength, under a given loading, are considerably greater than those of beams of uniform cross-section.

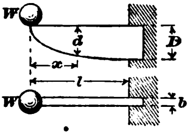
In the table,

W = concentrated load;
w = uniform load per unit of span;
l = span;
x = distance from a support to any given section;
d = depth of beam at that section;
b = breadth of beam at that section;
D = maximum depth of beam;
B = maximum breadth of beam;
S = unit stress in extreme fibers;
E = elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;
Y′ = deflection, due to extraneous load, in beam of uniform strength;
Y = deflection, due to extraneous load, in beam of uniform cross-section = maximum cross-section of beam of uniform strength.

Cantilevers of Rectangular Cross-section and of Uniform Strength. Profiles, Plans and Deflections.

For symbols, see ¶ 77.

Concentrated Load, – *W*, at end.

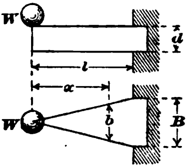


Breadth, *b*, constant.

Profile, parabola, with vertex at load.

$$d = \sqrt{\frac{6\,W\,x}{S\,b}}\,; \quad Y' = \frac{8\,W\,l^3}{E\,b\,D^3} = 2\,Y;$$

D = Maximum depth.

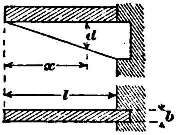


Depth, *d*, constant.

Plan, triangle.

$$b = \frac{6\,W\,x}{8\,d^2}\,; \quad Y' = \frac{6\,W\,l^3}{E\,B\,d^3} = \frac{3}{2}\,Y.$$

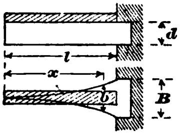
Uniform Load, = *w* per unit of span.



Breadth, *b*, constant.

Profile, triangle.

$$d = x\sqrt{\frac{3\,w}{S\,b}}$$



Depth, *d*, constant.

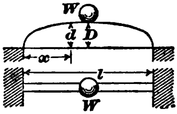
Plan, two parabolic curves, with vertices at free end.

$$b = \frac{3\,w\,x^2}{8\,d^2}\,, \quad Y' = \frac{3\,W\,l^3}{E\,B\,d^3} = 2\,Y.$$

Beams of Rectangular Cross-section and of Uniform Strength. Profiles, Plans and Deflections.

For symbols, see opposite page.

Concentrated Load, = *W*, at center.

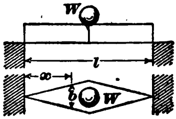


Breadth, *b*, constant.

Profile, two parabolic curves, with vertices at supports.

$d = \sqrt{\frac{3Wx}{Sb}}$ D at center of span.

$Y' = \frac{Wl^3}{2EbD^3} = 2Y.$

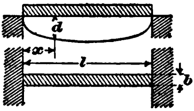


Depth, *d*, constant. B = maximum width

Plan, two triangles

$b = \frac{3Wx}{Sd^2}$; $Y' = \frac{3Wl^3}{8EBd^3} = \frac{3}{2}Y.$

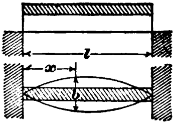
Uniform Load, = *w* per unit of span.



Breadth, *b*, constant.

Profile, ellipse or semi-ellipse.

$d = \sqrt{\frac{3w}{Sb}(lx - x^2)}.$



Depth, *d*, constant

Plan, parabolas with vertices at center of span

$b = \frac{3w}{Sd^2}(lx - x^2)$

Symbols in table opposite:

W = concentrated load;
l = span;
d = depth of beam at that sec;
D = maximum depth of beam;
w = uniform load per unit of span;
x = dist from a support to given sec;
b = breadth of beam at that sec;
B = maximum breadth of beam;
S = unit stress in extreme fibers;
E = elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;
Y' = deflection due to extraneous load in beam of uniform strength;
Y = deflection, due to extraneous load, in beam of uniform cross-section = max cross-section of beam of uniform strength.

Continuous Beams.

78. A continuous beam is one which rests upon more than two supports.

79. The resistances and deflections of continuous beams, like those of beams with fixed ends, are determined by means of the elastic curve, using the calculus. The more important facts, thus deduced, are indicated in Fig. 14 and illustrated table, ¶ 89.

80. Fig. 14 represents the general character of the deflections, and the variations of the moments and of the shears, in uniformly loaded continuous beams.

81. Moments. Fig. 14 (b). Ordinates drawn *above* the zero line, a' b', represent *positive* moments, or those where the segment of the beam, to the left of any section, tends to revolve *clockwise;* and vice versa.

82. At each end of the beam, at one point, i (called the inflection point, or point of contrary flexure) in each end span, and at two such points in each remaining span, the moment is zero.

83. At another point, m, in each span, the positive moment reaches a maximum for that span; while the negative moments reach their maxima at the supports. Both the positive and the negative moments vary in the different spans; but, if the spans are equal, then the moments, at any two points equidistant from the center of the whole beam, are equal.

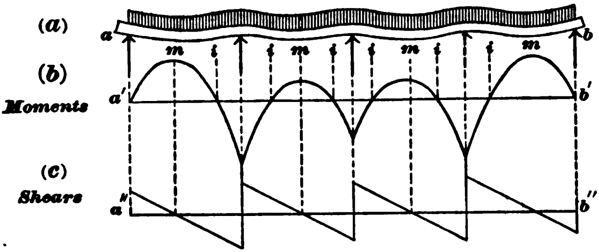


Fig. 14.

84. The moment diagram, between each support and the point, m, of maximum positive moment on either side of it, is a semi-parabola, with its apex at m.

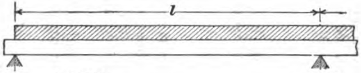
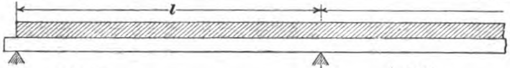
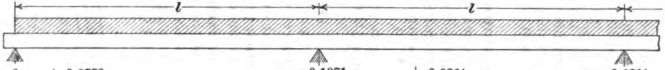
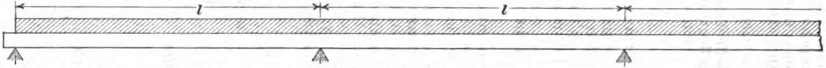
85. Shears. Fig. 14 (c). Ordinates drawn *above* the zero line, a'' b'', represent *positive* shears, or those in which the *left*-hand segment, at any section, tends to slide *upward* past the *right*-hand segment; and vice versa.

86. At the point, m, of maximum moment, in each span, the shear is zero. Between each such point and the next support on the left, the shear is positive, and vice versa.

87. At each support the shear suddenly changes, by an amount = the reaction of the support.

88. The shear diagram is a series of straight lines.

Continuous Beams.

Number of		
Spans	Supports	
2	3	$m\,w\,l^2$ = Moment. $v\,w\,l$ = Shear. $a\,l$ = Distance from left support of any span to point of maximum positive moment in that span. $x\,l$ = Distance from any left support to inflection point. w = Load per unit of span. l = Span.  $m =$ 0 + 0.0703 −0.125 $a =$ 0.375 $x =$ 0.75 $v = 0$ + 0.375 − 0.625 + 0.625
3	4	 $m =$ 0 + 0.08 − 0.10 + 0.025 $a =$ 0.4 0.5 $x =$ 0.8 { 0.276 0.724 $v = 0$ + 0.4 − 0.6 + 0.5
4	5	 $m =$ 0 + 0.0772 − 0.1071 + 0.0364 − 0.0714 $a =$ 0.3930 0.5357 $x =$ 0.7860 { 0.2659 0.8055 $v = 0$ + 0.3929 − 0.6071 + 0.5357 − 0.4643 + 0.4643
5	6	 $m =$ 0 + 0.0779 − 0.1053 + 0.0332 − 0.07895 + 0.0461 $a =$ 0.3947 0.5264 0.5 $x =$ 0.7894 { 0.2680 0.7830 { 0.1964 0.8036 $v = 0$ + 0.3947 − 0.6053 + 0.5263 − 0.4737 + 0.5000

89. The illustrated table opposite represents the conditions theoretically existing in uniformly loaded continuous beams of from two to five equal spans. Only the left half of each such beam is shown, the right half being symmetrical with it.

90. The Figs. show the amount of the maximum positive moment in each span, that of the negative moment at each support, and the shear on each side of each support.

91. The Figs. show, also, the coefficient, a, for the distance, $a\,l$, from the left support of each span to the point of maximum moment in that span; and the coefficient, x, for the distance or distances, $x\,l$, from the same support to the inflection point or points in that span. In each central span, the sum of the two values of x is $= 1$. In each end span, $x = 2\,a$.

92. In each central span, the point of maximum positive moment is at the center of the span. In other words, the deflection in that span is symmetrical, or $a = 0.5$.

93. The numerical sum of the two shears, one on each side of a support, is = the reaction of that support. At each central support, the shears, on its two sides, are equal.

In the Figs.,

w = load per unit of span;
l = span;
m = the coefficient for moment;
$m\,w\,l^2$ = moment;
v = the coefficient for shear;
$v\,w\,l$ = shear;
a = the coefficient for distance to point of maximum moment;
$a\,l$ = distance from left support of any span to point of maximum positive moment in that span;
x = the coefficient for distance to inflection point;
$x\,l$ = distance from left support of any span to either inflection point in that span.

94. Fig. 15 shows the values of m and of v in a uniformly loaded non-continuous beam. Comparing these with the corresponding values in continuous beams, as shown in the illustrated table, opposite, we see that the continuous beam has considerable theoretical advantage. But see ¶ 95.

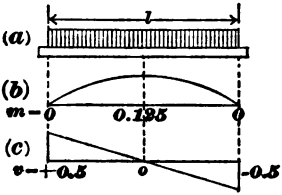


Fig. 15.

95. Certain practical considerations, however, materially reduce these advantages in many cases. Thus, in a continuous railroad bridge of 100 ft. spans, so designed that the maximum deflection shall not exceed $\frac{3}{4}$ inch, a settlement of $\frac{3}{4}$ inch, in an intermediate pier, would deprive the bridge of the support of such pier, and thus practically throw two adjacent spans into one, bringing upon their members stresses far in excess of those for which they were designed. Again, with moving loads, the theoretical advantage may at times be much less than that due to a stationary load and indicated in the illustrated table.

Cross-shaped Beam.*

96. In a cross-shaped beam, Fig. 16, of homogeneous material, loaded at center, let

$$\begin{array}{rcl} W & = & \text{the load;} \\ E & = & \text{the elasticity coefficient} = \dfrac{\text{unit stress}}{\text{unit stretch}}; \\ Y & = & \text{the deflection at center;} \end{array}$$

$$\left.\begin{array}{rcl} L, l & = & \text{the spans of} \\ D, d & = & \text{the depths of} \\ T, t & = & \text{the half depths of} \\ I, i & = & \text{the moments of inertia of} \\ S, s & = & \text{the unit stresses in the extreme fibers of} \\ P, p & = & \text{the portions of W borne by} \end{array}\right\} \text{the two branches respectively.}$$

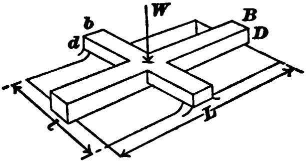


Fig. 16.

Then (see illustrated table, p. 480), since the deflection is necessarily the same for both branches,

$$\frac{L^3 P}{E I} = \frac{l^3 p}{E i}; \text{ or}$$

$$\frac{P}{p} = \frac{I . l^3}{i . L^3};$$

and, since $P = 4 . \frac{S . I}{T . L}$, and $p = 4 . \frac{s . i}{t . l}$ (see table, p. 474), we have

$$\frac{S}{s} = \frac{D}{d}\left(\frac{l}{L}\right)^2.$$

97. In other words, in order that both branches may be equally strong, their depths (independently of their breadths) must be inversely as the squares of their spans, or their spans inversely as the square roots of their depths.

Resistance of Plates.

98. The laws governing the resistance of plates, to pressures normal to their surfaces, are but imperfectly understood; and formulas respecting them must be used with caution and as probable approximations.

99. Rectangular plate, with central load, W.

t = the thickness of the plate;
L = its longer span;
l = its shorter span;
S = the maximum unit fiber stress;
C = a coefficient. See p. 493.

$$S = \frac{3}{2} C . \frac{L . l}{L^2 + l^2} \frac{W}{t^2}; \qquad W = \frac{2}{3} S . \frac{(L^2 + l^2) t^2}{C L l}$$

For a square plate, L = l, and

$$S = \frac{3}{4} C \frac{W}{t^2}; \qquad W = \frac{4}{3} S \frac{t^2}{c}.$$

---

* See foot-note (†), p. 493.

100. Rectangular plate, uniformly loaded. Let

w = the uniform load per unit of surface; other letters as in ¶ 99.

Then, according to Grashof,*

$$S = \frac{1}{2} C . \frac{L^2 l^2}{L^2 + l^2} . \frac{w}{t^2}; \qquad w = 2 S . \frac{(L^2 + l^2)\, t^2}{C L^2 l^2}.$$

For a square plate, L = l. Hence,

$$S = \frac{1}{4} C . L^2 . \frac{w}{t^2}; \qquad w = 4 S . \frac{t^2}{C L^2}.$$

101. Value of C.

If the plate is	For uniform load.	For central load.
merely supported along its four edges,	C = 1.125	C = 2.00
firmly secured along its four edges,	C = 0.75	C = 1.75

102. Circular plate, uniformly loaded. Let

w = the load per unit of surface;
S = the unit fiber stress in the material;
E = its elasticity coefficient = $\frac{\text{unit stress}}{\text{unit stretch}}$;
r = the radius of the unsupported portion of the plate;
t = the thickness of the plate;
Y = the deflection at the center.

Then, according to F. Reuleaux,† if the plate is merely *supported,*

$$w = S \left(\frac{t}{r}\right)^2; \qquad t = r \sqrt{\frac{w}{S}}; \qquad Y = \frac{5\, w\, r^4}{6\, E\, t^3}.$$

If the plate is firmly *secured,*

$$w = \frac{3}{2} S \left(\frac{t}{r}\right)^2; \qquad t = r \sqrt{\frac{2}{3} . \frac{w}{S}}; \qquad Y = \frac{w\, r^4}{6\, E\, t^3}.$$

For strengths of cylinders, pipes, etc., see Hydrostatics, Art. 17.

TRANSVERSE AND LONGITUDINAL STRESS COMBINED.

103. Although the combination of longitudinal and transverse stress in the same piece is objectionable, it is often unavoidable. Thus, in a timber roof, the rafters generally act both as columns and as beams.

In such cases, the total unit stress, S, in the extreme fibers, is the sum of the uniform stress, S_c, due to direct compression or tension, and the extreme fiber stress, S_b, due to bending moments only, under the action of the transverse and longitudinal loads combined. Or $S = S_c + S_b$.

Let M_b = the bending moment due to the transverse load; M_c = the bending moment due to longitudinal load, P; and M = the total resultant bending moment, $= M_b - M_c$ when the longitudinal load is tensile; $= M_b + M_c$ when the longitudinal load is compressive.

But $M_c = P\, d$, where P = the longitudinal load, and d = its leverage, = the deflection of the beam, due to all causes; and (see ¶ 57) $d = \frac{l^2 S_b}{E\, T\, c}$; where l = span, S_b = unit stress in extreme fibers, due to bending; E = modulus of elasticity; T = distance from neutral axis to extreme fibers, and c = a coefficient, whose values, for different cases, are given in ¶ 57.

Hence, $M_c = P \frac{l^2 S_b}{E\, T\, c}$; and resultant moment $M = M_b \pm P \frac{l^2 S_b}{E\, T\, c}$. The resisting moment, R (see ¶ 10), is $= S_b \frac{I}{T}$; and, for equilibrium, R = M.

Hence, $S_b . \frac{I}{T} = M_b \pm P \frac{l^2 S_b}{E\, T\, c}$; whence we derive, for the extreme fiber stress, S_b, due to bending only, under the action of the transverse and longitudinal loads combined,

---

* "Theorie der Elasticität und Festigkeit," Berlin, 1878.

† "Der Konstrukteur," Braunschweig, 1889. "The Constructor," translated by H. H. Suplee, Philadelphia, 1893.

$$S_b = \frac{M_b T}{I + \frac{P l^2}{E c}}$$ where the longitudinal stress is tensile

$$S_b = \frac{M_b T}{I - \frac{P l^2}{E c}}$$ where the longitudinal stress is compressive

Besides this we have the unit stress, S_o, due directly to the longitudinal load, P, and $= \frac{P}{A}$, where A is the area of cross-section of the beam. Hence, for the total unit stress, S, in the extreme fibers, we have

$$S = S_o + S_b = \frac{P}{A} + \frac{M_b T}{I \pm \frac{P l^2}{E c}}.$$

When the deflection, d, is negligible, $M = 0$; $S_b = \frac{M T}{I}$, as in ¶ 10; and $S = \frac{P}{A} + \frac{M T}{I}$.

It is often assumed that the resultant unit stress, S, in the extreme fibers, is equal to the sum of the longitudinal and transverse unit stresses, and the piece is then so designed that the resultant unit stress, so obtained, shall not exceed the permissible unit stress.

STRENGTH OF PILLARS.

The foregoing remarks on crushing or compressive strength refer to that of ieces so short as to be incapable of yielding except by crushing proper. Pieces onger in proportion to their diameter of cross section are liable to yield by ending sideways. They are called pillars or columns.

The law governing the strength of pillars is but imperfectly understood; and he best formulæ are rendered only approximate by slight unavoidable and unuspected defects in the material, straightness and setting of the column. A ery slight obliquity between the axis of a pillar and the line of pressure may educe the strength as much as 50 per cent; and differences of 10 per cent or nore in the bkg load may occur between two pillars which to all appearances re precisely similar and tested under the same conditions. Hence a liberal actor of safety should be employed in using any formulæ or tables for pillars.

In our following remarks on this subject, the pillars are supposed to sustain *constant* load; and the ultimate or breaking load referred to is that one which rould, during its first application, cripple or rupture the pillar in a short time. ut struts in bridges etc often have to endure stresses which vary greatly in mount from time to time. Their ultimate load is then less.

Long pillars with **rounded ends,** as in Fig 1, have less strength than hose with **flat ends,** whether free or firmly fixed.

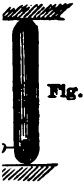


Fig. 1.

In steel bridges and roofs, the ends of the struts are frequently sustained by means of pins or bolts passing through (across) them, at either one or both ends. These we will call **hinged ends.** Pillars so fixed are about intermediate in strength between those with flat and those with round ends. There is much uncertainty about this and all such matters. The strength of a given hinged-end pillar is increased to an important extent by increasing the diameter of the pin.

The **formula** in most general use for the strength of pillars, is that attributed to Prof. **Lewis Gordon** of Glasgow, and alled by his name. With the use of the proper coefficients for the given case, t gives results agreeing approximately with averages obtained in practice with illars of such lengths (say from 10 to 40 diams) as are commonly used.

It is as follows

$$\text{Breaking load in lbs per sq inch of area of cross section of pillar} = \frac{f}{1 + \frac{l^2}{r^2\,a}}$$

n which

f is a coefficient depending upon the nature of the material and (to ome extent) upon the shape of cross section of the pillar. It is often taken, pproximately enough, as being the ult crushing strength of short blocks of the iven material. For good American wrought iron, such as is used for pillars, 0000 is generally used; for cast iron 80000. Mr. Cleeman* found for mild steel 15 per cent carbon) 52000; and for hard steel (.36 per cent carbon) 83000 lbs. r. C. Shaler Smith gives 5000 for Pine.

a, for wrought iron, is usually taken as follows:

	a =
when both ends of the pillar are flat or fixed	36000 to 40000
when both ends of the pillar are hinged	18000 to 20000
when one end is flat or fixed, and the other hinged	24000 to 30000

For cast iron about one eighth of these figures is generally used; and for pine bout one twelfth.

l is the length of the pillar. If the pillar has, between its ends, supports which prevent it from yielding side-ways, the length is to be measured between such supports.

r is the least radius of gyration of the cross section of the pillar. *l* and *r* must be in the same unit; as both in feet, or both in inches.

---

* Proceedings Engineers' Club of Phila, Nov 1884.

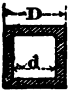
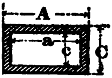
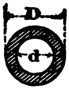
Radius of gyration. Suppose a body free to revolve around an axis which passes through it in any direction; or to oscillate like a pendulum hung from a point of suspension. Then suppose in either case, a certain given amount of force to be applied to the body, at a certain given dist from the axis, or from the point of suspension, so as to impart to the body an angular vel; or in other words, to cause it to describe a number of *degrees* per sec. Now, there will be a certain point in the body, such that if the entire wt of the body were there concentrated, then the same force as before, applied at the same dist from the axis, or from the point of suspension as before, would impart to the body the same angular motion as before. This point is the center of gyration; and its dist from the axis, or from the point of suspension, is the *Radius of gyration*, of the body. In the case of *areas*, as of cross-sections of pillars or beams, the *surface* is supposed to revolve about an imaginary axis; and, unless otherwise stated, this axis is the neutral axis of the area, which passes through its center of gravity. Then

Radius of gyration $= \sqrt{\text{Moment of inertia} \div \text{Area}}$

Square of radius of gyration $=$ Moment of inertia $\div$ Area

In a circle, the radius of gyration remains the same, no matter in what direction the neutral axis may be drawn. In other figures its length is different for the different neutral axes about which the figure may be supposed to be capable of revolving. Thus, in the I beam, page 898, the radius of gyration about the neutral axis X Y is much greater than that about the longer neutral axis A B. In rules for pillars the *least* radius of gyration must be used.

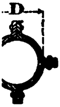
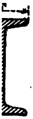
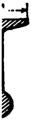
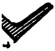
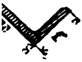
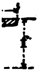
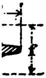
The following formulæ enable us to find the least radius of gyration, and the square of the least radius of gyration, for such shapes as are commonly used for pillars.

Shape of cross section of pillar	**Least radius of gyration**	**Square of least radius of gyration**
Solid square	$\frac{\text{side}}{\sqrt{12}}$*	$\frac{\text{side}^2}{12}$
 Hollow square of uniform thickness	$\sqrt{\frac{D^2 + d^2}{12}}$	$\frac{D^2 + d^2}{12}$
Solid rectangle	$\frac{\text{least side}}{\sqrt{12}}$*	$\frac{\text{least side}^2}{12}$
 Hollow rectangle of uniform thickness	$\sqrt{\frac{C^3 A - c^3 a}{12\,(C A - c a)}}$	$\frac{C^3 A - c^3 a}{12\,(C A - c a)}$
Solid circle	$\frac{\text{diameter}}{4}$	$\frac{\text{diameter}^2}{16}$
 Hollow circle of uniform thickness	$\sqrt{\frac{D^2 + d^2}{16}}$	$\frac{D^2 + d^2}{16}$

---

* $\sqrt{12}$ = about 3.4641.

The following are only approximate:

Shape of cross section of pillar		Least radius of gyration	Square of least radius of gyration
	Phœnix column.	D × 0.3636	D² × 0.1322
	Carnegie Z-bar column.	B × 0.590	B² × 0.348
	I beam.	$\frac{F}{4.58}$	$\frac{F^2}{21}$
	Channel.	$\frac{F}{3.54}$	$\frac{F^2}{12.5}$
	Deck beam.	$\frac{F}{6}$	$\frac{F^2}{36.5}$
	Angle, with equal legs	$\frac{F}{5}$	$\frac{F^2}{25}$
	Angle, with unequal legs	$\frac{Ff}{2.6\,(F+f)}$	$\frac{F^2f^2}{13\,(F^2+f^2)}$
	T, with F = f	$\frac{F}{4.74}$	$\frac{F^2}{22.5}$
	Cross, with F = f		

The young engineer must bear in mind that the breakg and the safe loads per sq inch, of pillars of any given material, are not constant quantities: but diminish as the piece becomes longer in proportion to its diam. If a very long piece or pillar be so braced at intervals as to prevent its bending at those points, then its length becomes virtually diminished, and its strength increased. Thus, if a pillar 100 ft long be sufficiently braced at intervals of 20 ft, then the load sustained may be that due to a pillar only 20 ft long. Therefore, very long pillars used as bridge piers, &c, are thus braced; as are also long horizontal or inclined pieces, exposed to compression in the form of upper chords of bridges; or as struts of any kind in bridges, roofs, or other structures.

Mistakes are sometimes made by assuming, say 5 or 6 tons per sq inch, as the safe compressing load for cast iron; 4 tons for wrought; 1000 pounds for timber; without any regard to the length of the piece.

But although the final crushing loads, as given in tables of strengths of materials, are usually those for pieces not more than about 2 diams high, they will not be much less for pieces not exceeding 4 or 5 diams.

Cautions. Remember a heavily loaded cast-iron pillar is easily broken by a side blow. Cast-iron ones are subject to hidden voids. All are subject to jars and vibrations from moving loads. It very rarely happens that the pres is equally distributed over the whole area of the pillar; or that the top and bottom ends have perfect bearing at every part, as they had in the experimental pillars † Cast pillars are seldom perfectly straight, and hence are weakened.

Hollow pillars intended to bear heavy loads should not be cast with such mouldings as *a a*; or with very projecting bases or caps such as *g*, Fig 19. It is plain that these are weak, and would break off under a much less load than would injure the shaft of the pillar. When such projecting ornaments are required, they should be cast separately, and be attached to a prolongation of the shaft, as *cd*, by iron pins or rivets.

Ordinarily, it is better to adopt a more simple style of base and cap, which, as at *b*, can be cast in one piece with the pillar, without weakening it.

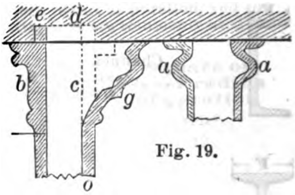


Fig. 19.

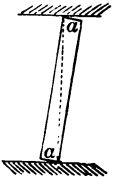


Fig. 20.

When a flat-ended pillar, Fig. 20, is so irregularly fixed, that the pressure upon it passes along its diagonal *a a*, it loses much of its strength. Hence the necessity for equalizing, as far as possible, the pressure over every part of the top and bottom of the pillar; a point very difficult to secure in practice.

---

† In important cases both ends should be planed perfectly true.

SHEARING STRENGTH.

Shearing or detrusion occurs when a body is acted upon by two opposite forces in parallel and closely adjacent planes, tending to slide some of the particles over the others. In Fig 1, the two forces are (1) the downward pressure of the weight, W, and (2) the upward reaction of the support, A.

In **single shear,** Fig 1, the shearing area, a, = the section $g\,g$. In **double shear,** Fig 2, $a = g\,g + o\,o = 2 \times g\,g$. In Fig 3, $a = 6 \times$ cross section of piece. In Fig 4 (single shear), a = section $c\,c$. In punching rivet holes, a = circumference of hole × thickness of plate.

In any case, if S = the ultimate unit shearing stress, Shearing strength = S a.

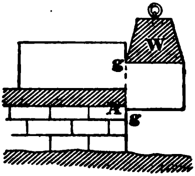


Fig. 1

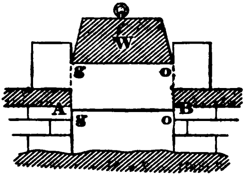


Fig. 2

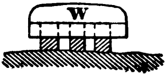


Fig. 3

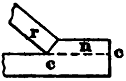


Fig. 4

Ultimate unit shearing stress, S, in lbs per sq inch. The following figures indicate the range of values of S in metals and in timber.

Metals. Wrought iron, 35,000 to 55,000; cast iron, 20,000 to 30,000; steel, 45,000 to 75,000; copper, 33,000.

		With the grain, Fig 4.	Across the grain.
Timber: From our experiments:	Spruce	250 to 500	3,250
	White pine	"	2,500
	Hemlock	"	"
	Yellow pine	......	4,300 to 5,600
	Oak	400 to 700	....
	White oak	......	4,400

TORSIONAL STRENGTH.

Torsion occurs when a body is acted upon by two couples or moments of contrary sense and in different planes. Thus, torsion takes place in a brake axle when we try to turn it while its lower end is held fast by the brake chain; and in shafting, when it transmits the motive power of an engine to tools. Suppose such a body to be divided, by cross sections, into layers. Then each layer tends to shear across from those next to it. Hence, in order to maintain equilibrium, each two adjacent layers must exert, in the cross section between them, an internal resisting moment equal to one of the two external and contrary torsional moments.

Resisting moment in a circular cross section of a cylindrical shaft. Let

P = the torsional force of one of the two external moments..... ....in pounds;
l = its leverage, = its distance from the axis of the shaft.................in inches;
M = P l = external or torsional moment..in inch-pounds;
T = distance from axis to farthest fibers, = radius of shaft ....in inches;
D = diameter of shaft, = 2 T...in inches;
S = unit shearing stress in farthest fibers......................in pounds per sq inch;
t = distance from axis to any given fiber...in inches;
s = unit stress in said given fiber ....in pounds per square inch;
a = area of said given fiber..in square inches;
F = total stress in said fiber ....in pounds;
r = resisting moment of said fiber about the axis...................in inch-pounds;
R = internal resisting moment of the entire cross section
= $\Sigma\, r$ = sum of resisting moments of all the fibers.................in inch-pounds;
I_p = polar moment of inertia* of the cross section
= moment of inertia of cross section about the axis of the shaft
= $\Sigma\, t^2 a$, = the sum, for all the fibers, of $t^2 a$.................................in inches.

---

* In any figure, the polar moment of inertia, I_p, is = the sum of the greatest and the least moments of inertia of the same figure, about two axes lying in the figure and intersecting in its center. In a solid circle, each of these is a moment of inertia about a diameter, and is = $\pi\, T^4 \div 4$. Hence, in such a circle, $I_p = \pi\, T^4 \div 2$.

Then the unit stress, in any given fiber, is $s = S\,t \div T$; its total stress, F, is $= a\,s = S\,a\,t \div T$; and its resisting moment, r, is $= F\,t = S\,a\,t^2 \div T$. For equilibrium, the internal resisting moment, R, of the entire section, must be = the external torsional moment, M.

Hence, for the **Internal Resisting moment,** R, we have:

$R = M = \Sigma\, r = \Sigma\, S\,a\,t^2 \div T = (S \div T)\, \Sigma\, t^2 a = (S \div T)\, I_p$.

Hence, also, $S = M\,T \div I_p$; $M = S\,I_p \div T$; and $P = M \div l = S\,I_p \div (T\,l)$.

In a solid circle, $I_p = \pi\,T^4 \div 2$. Hence, $S = 2\,M\,T \div (\pi\,T^4) = 2\,M \div (\pi\,T^3)$; $M = S\,\pi\,T^3 \div 2$; $P = S\,\pi\,T^3 \div (2\,l)$; and

$$\textbf{Diameter,}\ D, = 2\,T = 2 \times \sqrt[3]{\frac{2\,P\,l}{\pi\,S}} = \sqrt[3]{\frac{5.1\ M}{S}} = 1.72\ \sqrt[3]{\frac{M}{S}}.$$

For approximate ultimate values of S, for torsion, use the values for shearing, p 499, with **safety factors** from 5 to 10.

Horse power of shafting. In one revolution, the force, P lbs, describing a circle with radius = l ins, does a work = $2\,\pi\,l\,P$ inch-lbs, and, in n revolutions, work = $2\,\pi\,l\,P\,n$ inch-lbs. If n be the number of revolutions *per minute*, the horse power is:

$H = 2\,\pi\,l\,P\,n \div (12 \times 33{,}000) = 2\,\pi\,M\,n \div (12 \times 33{,}000)$;

or, since $P\,l = M = R = S\,I_p \div T$, we have:

$H = S\,\pi\,n\,I_p \div (12 \times 16{,}500\ T)$; and $S = 12 \times 16{,}500\ T\,H \div (\pi\,n\,I_p)$.

In a solid cylindrical shaft, $I_p = \pi\,T^4 \div 2$. Hence,

$H = S\,\pi\,n\,\pi\,T^4 \div (12 \times 33{,}000\ T) = S\,\pi^2\,n\,T^3 \div (12 \times 33{,}000)$;

$S = 12 \times 33{,}000\ H \div (\pi^2\,n\,T^3) = 12 \times 3{,}343\ H \div (n\,T^3)$;

$n = 321{,}000\ H \div (D^3\,S)$: and

$$\textbf{Diameter,}\ D = 2\,T = 2 \times \sqrt[3]{\frac{12 \times 3{,}343\ H}{S\,n}} = \sqrt[3]{\frac{321{,}000\ H}{S\,n}} = 68\ \sqrt[3]{\frac{H}{S\,n}}.$$

The higher the speed, the less is the force, and hence the less is the strength of shaft, required in order to transmit a *given horse-power;* but if the speed is increased by increasing the torsional force, the horse-power transmitted is thereby increased also.

Example. Given, a wrought iron shaft; let S = 6,000 lbs per sq inch; P = 7,500 lbs; l = 10 ins; M = 75,000 inch-lbs. Required the diameter, D. Here, $D = 1.72 \times \sqrt[3]{M \div S} = 1.72 \times \sqrt[3]{75{,}000 \div 6{,}000} = 1.72 \times \sqrt[3]{12.5} = 1.72 \times 2.32 = 4$ ins. Let the horse-power, H, = 25. Then $n = 321{,}000\ H \div (D^3\,S) = 321{,}000 \times 25 \div (4^3 \times 6{,}000) = 21$ revolutions per minute. Checking, $D = 68 \times \sqrt[3]{H \div (Sn)} = 68 \times \sqrt[3]{25 \div (6{,}000 \times 21)} = 68 \times 0.058 =$ say 4 inches.

Rectangular Sections. The foregoing equations are based upon the assumption that the stress increases uniformly from the axis of the shaft outward. It has been shown (notably by St. Venant*) that this assumption is not applicable to square and rectangular sections. In a rectangle, let B = the longer, b = the shorter side, and $c = b \div B$. Then $S = M\,(3 + 1.8\,c) \div (B\,b^2)$; and $P = S\,B\,b^2 \div [(3 + 1.8\,c)\,l]$. In a square, with side = b, this becomes: $S = 4.8\,M \div b^3$: $M = S\,b^3 \div 4.8$. $P = S\,b^3 \div (4.8\,l)$.

The angle of torsion is that described by one of the external torsional moments, relatively to the other. Within the elastic limit, this angle is proportional to the torsional moment, M, and (assuming l constant) proportional to the force, P. Other things being equal, the angle is proportional to the distance between the planes of the two contrary external moments, and, in a solid cylindrical shaft, is inversely proportional to D^4. It is inadvisable to allow the angle of torsion to exceed 1° in a length = 20 diameters, in shafts revolving in one direction. In reciprocating shafts allow still less. See Fatigue, p 465.

Practical Considerations. In many cases the diameter of the shaft must be made greater than that required by the foregoing formulas; as in a long shaft, in order to keep the angle of torsion within permissible limits; in fly-wheel and other shafts, carrying considerable bending loads in addition to the torsion; and, in most cases, to allow for additional moments due to alternate acceleration and retardation.

---

*See Treatise on Natural Philosophy, by Sir William Thomson and Peter Guthrie Tait, Part II, New Edition, Cambridge, 1890, pp 236, etc.

HYDROSTATICS.

Art. 1. Hydrostatics treats of the pressure of water and of other fluids at rest.

At any given point within a fluid **the pressure is equal in all directions;** and the pressure against any point of any surface, whether plane or curved, is **normal to the pressed surface** (or to a plane tangent to that surface) at that point.

The intensity of the pressure is proportional to the depth of the point below the water surface.

Pressure against any plane surface.

Let

a = the area of the pressed surface;

h = the vertical depth of the center of gravity of the pressed surface below the free surface of the fluid;

H = the total depth of the fluid;

w = the weight of a unit volume of the fluid;*

p = the mean unit pressure on the pressed surface;

P = the total pressure on the pressed surface;

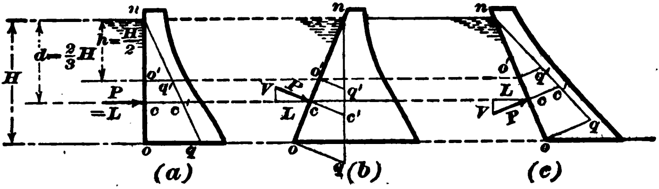


Fig. 1.

Then the **mean unit pressure,** p, is equal to the weight of a prism of the fluid, whose base is = 1, and whose length is = h; or

$$p = h\,w;$$

and the **total pressure,** P, is equal to the weight of a prism of the fluid, whose base is = a, and whose length is = h. Or

$$P = a\,h\,w = a\,p.$$

In the diagrams of Figs. 1 and 2, the *ordinates* (supposed to be drawn from, and normally to, the pressed surfaces respectively) represent the *unit* pressures (as in lbs. per sq. inch, kilograms per sq. centimeter, etc.), and the *area,* opposite any given surface, represents the *total* pressure on that surface. Thus, in Fig. 1 (a), the unit pressures at n, at o', at c and at o, are represented by n (= 0), by $o'\,q'$, by $c\,c'$ and by $o\,q$ respectively; and the total pressure on $n\,o$ is represented by the area $n\,o\,q$, that on $n\,o'$ by the area $n\,o'\,q'$, that on $o'\,o$ by the area $o\,q'$, and that on $o'\,c$ by the area $c\,q'$.

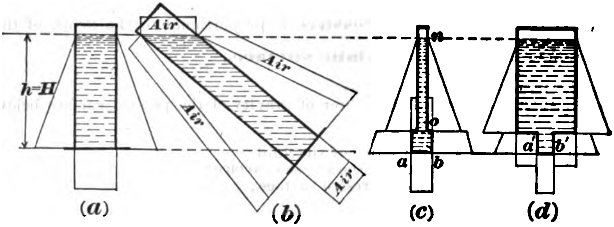
The center of pressure upon any surface is opposite the center of gravity of the area representing the total pressure upon that surface. Thus, in Figs. 1, the center of pressure on $n\,o$, is opposite the center of gravity of the triangle, $n\,o\,q$, or at a depth, d, $= \frac{2}{3}$ H, below the water surface. See The Center of Pressure, Arts 8, etc., also ¶¶ 133 etc., of Statics.

The hydrostatic paradox. For a given depth, h, both the mean unit pressure, p, and (for a given area, a) the total pressure, P, are independent of the quantity of water. Thus, in Figs. 1, the walls sustain as great a pressure from a vertical film of water only an inch thick, as if the water extended back for miles. In Fig. 2 (*b*), the excess of weight of water, over that in Fig. 2 (*a*), is carried by the lower sloping *wall* of the vessel. The total pressures, P, upon the equal bases, *a b* and *a′ b′*, Figs. 2 (*c*) and (*d*), are equal. In Fig. 2 (*c*), the total pressure upon the base is greater, and in Fig. 2 (*d*) less, than the weight of the water; but, in either case, the algebraic sum of all the vertical pressures in the vessel (*upward* pressures taken as *negative*) is = the weight of all the water in the vessel; and the algebraic sum of all the horizontal pressures is = 0.

---

* For water, w = about 62.5 lbs. per cub. ft., = about 0.0362 lbs. per cub. inch.

Thus, let the lower part of Fig. 2 (*c*) represent a cubical box, 8 feet on a side, and filled with water. Now let the tube, *n o*, 36 ft. high and of 0.287 inch bore, be filled with water. The water in the tube alone, although weighing only about 1 pound, will cause an additional bursting pressure of 2250 ℔s. per square foot, or say 884 tons total, to be exerted upon the top, bottom and sides of the box.



(*a*) (*b*) (*c*) (*d*)

Fig. 2.

Air pressure on water surface. In addition to the pressure of the water itself, the free surface of any body of water sustains also the pressure of the air, = about 14.7 ℔s. per sq. inch. This pressure (transmitted through the water to the walls of the vessel) is indicated by the diagrams (parallelograms) marked "air" in Fig. 2 (*b*). In most cases, the pressure against a surface, due to the air pressure on the water surface, is counter-balanced by an equal pressure of the air in the opposite direction, as against the outer sides of the walls of the vessels in Figs. 2, and against the down-stream sides of the dams in Figs. 1.

Strictly speaking, the air pressure, exerted directly against the walls, Fig. 2, or against the dams, Figs. 1, being exerted practically at the centers of gravity of those surfaces, and therefore at lower elevations than the opposite pressure due to the air pressure on the water surface, is a very little greater than the latter. In a dam 100 ft deep, this difference would amount to about 0.027 ℔ per sq inch, = 0.0018 atmosphere.

Braces for dams. The water pressure being normal to the pressed surface, the posts, Fig. 3, must also be normal to the surface, D, if they are to receive that pressure longitudinally and thus avoid bending moments; but other considerations may forbid their being so placed. Thus, if the face, D, of the dam, is nearly vertical, the posts would have to be made inordinately long; and their consequent weakness, as pillars (unless made inconveniently thick) might more than offset the advantage due to directness of pressure. Moreover, the feet of the down-stream posts would project beyond the crest of the dam, and would thus be liable to injury by ice or logs, etc., tumbling over the dam.

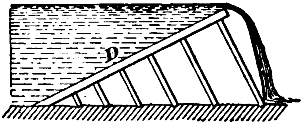


Fig. 3.

Inasmuch as the pressure increases uniformly from the water surface downward, the posts are usually placed closer together near the bottom than near the top, although the shortness of the lower ones renders them stronger as pillars. Similarly, the hoops on tanks, if of uniform strength, are placed closer together near the bottom.

Horizontal and vertical components. In Figs. 1 (*b*) and (*c*), the force triangle (Statics, ¶¶ 46, etc.) gives us the horizontal and vertical components, L and V, of the total normal pressure, P. Or, if *n o* be taken, in each case, as representing the total normal pressure, P, by scale, then H = the total horizontal pressure. In Fig. 1 (*a*), with pressure against a vertical surface, L = P, and V = 0.

In Fig. (*b*), the vertical component, V, presses the wall downward against its base; but in Fig. (*c*) it tends to uplift and overturn it.

The depth, H, being the same in each of the three figures, Figs. 1 *a*, *b* and *c*, the *vertical* projections of the three submerged surfaces are equal, and hence the total *horizontal* pressures are equal in the three cases; but the *horizontal* projection, and consequently the total *vertical* pressure, vary with the inclination of the surface. Thus, in Fig. 1 (*a*), the horizontal projection and the vertical pressure are each = 0.

Pressures in cubical and other vessels, full of water. Let F = the weight of water contained in a prismatic vessel; and $f = \frac{F}{3}$ = the weight of that in a conical or pyramidal vessel of the same base and height.

In a cubical vessel, we have
pressure on base . . . . = F;
" " one side . . . . = $\frac{F}{2}$;
" " base and four sides together = $F + 4\,\frac{F}{2} = 3\ F$.

In a conical or pyramidal vessel, we have
pressure on base . . . . = $3f = F$.

In a spherical vessel, we have
total pressure . . . . = 3 × weight of water.

Art. 2. Unequal pressures in opposite directions. In Figs. 4, let *a* = the area of that portion, *n′ o*, which is subjected to pressure on both sides; and let H and *h* = the vertical depths of the center of gravity of *n′o* below the two water surfaces, *n* and *n′* respectively. Then, in each Fig., the large triangle, *n o q*, represents the sum of the pressures of the deeper water on the left against the entire wall, *n o;* the trapezoid, *m o*, represents the sum = *a* H *w*, of the pressures of the water on the left against the portion, *n′ o;* and the smaller triangle, *n′ o q′*, represents the sum = *a h w*, of the pressures of the shallower water on the right against the portion, *n′ o*. Then the parallelogram, *n′ q*, represents the *excess* of pressure, from the left, against the portion, *n′ o*. This excess, due to the difference, H–*h*, between the two levels, is *uniformly* distributed over *n′ o*, the uniform excess unit pressure being represented by the ordinate *n′ m* = *q″ q*.

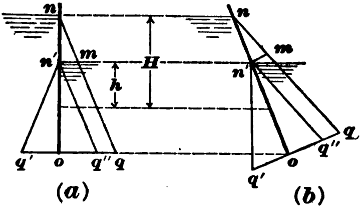


Fig. 4.

The pressure coming from the right against the portion *n′ o*, and represented by the triangle *n′ o q′*, is balanced by an equal portion (represented by the triangle *n′ o q″*) of the total pressure, *m o*, from the left against the portion *n′ o*; and the centers of these two pressures, each being at a depth = $\frac{2}{3}$ *n′ o* below *n′*, are opposite. Hence these two pressures are in equilibrium. But the center of the excess pressure, *n′ q*, from the left, is opposite the center of gravity of the parallelogram, *n′ q*, or at the center of gravity of *n′ o*. Hence the portion *n′ o*, considered independently of *n′ n*, is acted upon by the unbalanced force *n′ q*, coming from the left, acting through its center of gravity and therefore tending to move it bodily toward the right, without rotation.

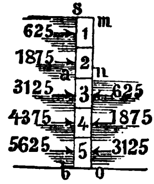


Fig 5

This will be understood by means of Fig 5, which may represent five planks, 1, 2, 3, 4, and 5, forming a dam, and seen endwise; each one 1 ft in depth, and say 20 ft long hor; making the area of each surf pressed, equal to 20 sq ft. The pres in lbs against each separate 20 sq ft of area, calculated by the rule in Art 1, is shown in the fig. Now, the outward pres against the upper *immersed* 20 ft area, or that of plank 3, is 2125 lbs; while the counter-pres against it from the other side is 625 lbs; making the *excess* of outward pres equal to 3125 — 625 = 2500 lbs. Again, at the lowest plank, number 5, the outward pres exceeds the inward one by 5625 — 3125 = 2500 lbs, the same as in the upper one. And so of any other equal area of surf, at any depth whatever; the excess depending upon the vert height of *m n*, will be equally distributed over *a b*. It only remains to show that the total excess of outward pres against *a b*, is equal in amount to the wt of a uniform column of water with a base equal in area to *a b*, and with a height equal to *m n*. Thus, we have seen that in the instance before us, the excess amounts to 3 times 2500 lbs, or to 7500 lbs. Now, the wt of the column of water will be 60 (or area of *a b*) × *m n* (or 2 ft) × 62.5 lbs = 7500 lbs; or the same as the *excess* pres on *a b*.

The excess of pres against the entire side *s b*, over that against *n o*, is evidently the diff between those two pressures calculated respectively by the rule in Art 1.

Art. 3. Surfaces, vert, as *b m c s, a n o t,* Fig 6, **or otherwise, of equal widths,** *b m, a n;* **commencing at the level,** *b a n m,* **of the water, but extending to diff depths,** *m c, n o,* **measured vert; and having the same inclination to the surf of the water; sustain total pressures proportional to the squares of those depths.**

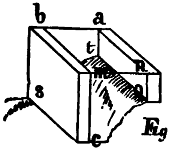


Fig 6

In Fig 6, let the two vert sides, *a n o t*, and *b m c s*, of a vessel, have the same width *a n*, and *b m*; then if the depth *m c*, be 2, 3, 4, 5, &c, times greater than the depth *n o*, the pres against the surf *b m c s*, will be 4, 9, 16, 25, &c, times greater than that against *a n o t*. This will be seen by referring to the pressures figured on the left side of Fig 5, where, as stated in Art 2, the surf of plank 1, exposed to the pres on the left side, is 20 sq ft; that of planks 1 and 2, 40 sq ft; that of planks 1, 2, and 3, 60 sq ft, &c. All these surfs commence at the level of the water; and all of them being vert, are of course at the same inclination with the water surf; but their depths are respectively 1, 2, and 3 ft. The pres against the surf of 1, is 625 lbs; that against the surf of 1, 2, is 625 + 1875 = 2500; and that against the surf of 1, 2, 3, is 625 + 1875 + 3125 = 5625. But 2500 is *four* times 625; and 5625 is *nine* times 625. And the pres against the entire surf *s b*, (which is 5 times as deep as plank 1,) is 25 times as great as that against plank 1; or 625 × 25 = 15625 lbs = the sum of all the pressures marked on the left side of Fig 5.

This follows, from the Rule in Art 1; for twice the area of surf, mult by twice the vert depth of the cen of grav below the surf, must give 4 times the pres; three times the area, by three times the depth, must give 9 times the pres, &c.

It follows, also, that at any particular *point*, or against any *given* area placed at various depths, the pres will increase simply as the vert depth: thus, if there be three areas, each one sq ft, placed in the same positions, but with their centers of grav respectively 8, 16, and 24 ft below the surf, the pres against them will be respectively *as* 8, 16, and 24; or *as* 1, 2, and 3.

Art. 4. The pressure of quiet water, in any one given direction, against any given plane surface, whether vertical, horizontal, or inclined, is equal to the weight of a prismatic column of water, the area of whose section, parallel to its base, is equal to the area of the projection of the given surface taken at right angles to the given direction, and whose height is equal to the vertical depth of the center of gravity of the given surface below the upper surface of the water. Hence the

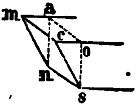


Fig 7

RULE. **To find the pres in lbs,** mult together the area in sq ft of the projection taken at right angles to the given direction; the vert depth in ft of the cen of grav of the pressed surf below the upper surf of the water; and the constant 62.5 lbs wt of a cub ft of water.

Ex. Let *m c s n*, Fig 7, be an inclined surf, sustaining the pres of water which is level with its top *m c*. Then the *total* pres against *m c s n*, and at right angles to it, as found by the rule in Art 1, is an illustration of the present rule; because the projection of *m c s n*, taken at right angles to the given direction, or parallel to *m c s n*, is in fact *m c s n* itself, or equal to it. Hence the rule in Art 1 is merely a simple modification of the present one, applicable to the case of *total* pres against any surf.

But if it be reqd to find only the vert or downward pres against ***m c s n,*** in pounds, mult together the area of the hor projection *a o c m* in sq ft; the vert depth in ft of the cen of grav of *m c s n* below the surf; and 62.5. Or if only the *hor* pres against *m c s n* be sought, mult together the area of the vert projection *a o s n*; the vert depth of the cen of grav of *m c s n*; and 62.5.

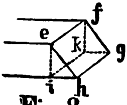


Fig 8

In Fig 8 also, the *total* pres against *e f g h* is found by rule in Art 1; while the *hor* and vert pressures against it are found as in Fig 7, by using the projections *e f k i*, and *k i g h*. In Fig 7 the vert pres is downward; while in Fig 8 it is upward; but this circumstance in no respect affects the rule.

REM. 1. At any given depth, the pres, perp to any given surf, is the same in all directions; but Figs 7 and 8 show that the total pres *oblique* to a given surf will be less than the perp one at the same depth; because an oblique projection of a surf must be less than the surf itself, which last is the projection when the pres is perp to it. Thus, in a reservoir, the total pres perp to a sloping side, as *m n s c*, Fig 7, is greater than either the vert or the hor pres upon it.

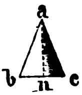


Fig 9

Again, let Fig 9 represent **a conical vessel full of water;** its base $b\,c$, 2 ft diam; its vert height $a\,n$, 3 ft; then the circumf of the base will be 6.2832 ft; the area of the base 3.1416 sq ft; the length of its slant side $a\,b$ or $a\,c$, 3.16 ft; the area of its curved slanting sides will be $\frac{6.2832 \times 3.16}{2} = 9.93$ sq ft; and the vert depth of the cen of grav of the slanting sides will be at two-thirds of the vert height $a\,n$ from the apex a, or 2 ft.

Here, to find the total pres against the base, we have by rule in Art 1, $3.1416 \times 3 \times 62.5 = 589.05$ lbs. For the total pres against the slant sides, by the same rule, $9.93 \times 2 \times 62.5 = 1241.25$ lbs. For the vert pres upward against the entire area of the slant sides, we have given the area of the base (which is here the hor projection of the slant sides) $= 3.1416$; and the vert depth of the cen of grav of the slant sides, 2 ft. Therefore, $3.1416 \times 2 \times 62.5 = 392.7$ lbs, the upward vert pres.

Finally, for the hor pres in any given direction against the slant sides of *one half* of the cone, we have the vert projection of that half, represented by the triangle $a\,b\,c$, with its base 2 ft, and its perp height 3 ft; and consequently, with an area of 3 sq ft. The depth of its cen of grav is 2 ft; therefore, $3 \times 2 \times 62.5 = 375$ lbs, the reqd hor pres.*

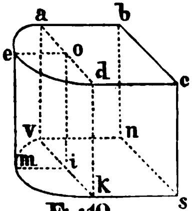


Fig 10

In Fig 10, which represents a vessel full of water, the *total* pres against the semi-cylindrical surf $a\,v\,e\,m\,d\,k$, and perp to it, must be also hor, because the surf is vert; but inasmuch as the surf is *curved*, this total pres, as found by rule in Art 1, acts against it in many directions, which might be represented by an infinite number of radii drawn from o as a center. But let it be reqd to find the hor pres in lbs, in one direction only, say parallel to $o\,e$, or perp to $a\,d$; which would be the force tending to tear the curved surf away from the flat sides $a\,b\,n\,v$, and $d\,c\,s\,k$, by producing fractures along the lines $a\,v$ and $d\,k$; or which would tend to burst a pipe or other cylinder. In this case, mult together the area of the vert projection $a\,d\,k\,v$ in sq ft; the depth of the cen of grav of the curved surf in ft; (which, in the semi-cylinder would be half of $e\,m$, or of $o\,i$;) and 62.5. Since the resulting pres is resisted equally by the strength of the vessel along the two lines $a\,v$ and $d\,k$, it is plain that each single thickness along those lines need only be sufficient to resist safely *one half* of it; and so in the case of pipes, or other cylinders, such as hooped cisterns or tanks. See Art 17.

Should the pres against only one half of the curved surf, as $e\,d\,m\,k$ be sought, and in a direction parallel to $o\,d$. tending to produce fractures along the lines $e\,m$, and $d\,k$, then use the vert projection $o\,e\,m\,i$; with the same depth; and 62.5 as before.

It follows, that if the face of a metallic piston be made concave or convex, no more pres will be reqd to force the piston through any dist, than if it were flat; for the pres against the face of the piston, in the direction in which it moves, must be measured by the area of a projection of that face, taken at right angles to said direction; and the area of said projection will be the same in all three cases.

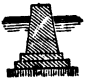


Fig $10\frac{1}{2}$

REM. 2. **If a bridge pier, or other construction, Fig 10½, be founded on sand or gravel,** or on any kind of foundation through which water may find its way underneath, even in a very thin sheet, then the upward pres of the water will take effect upon the pier; and will tend to lift it, with a force equal to the wt of the water displaced by the pier; (Arts 18, 19. In other words, the effective wt of the *submerged* portion of the pier, will be reduced 62½ lbs per cub ft; or nearly the half of the ordinary wt of masonry.

But if the foundation be on rock, covered with a layer of cement to prevent the infiltration of water beneath the masonry, no such effect will be produced; but on the contrary, the vert pres downward, afforded by the battering sides of the pier, and by its offsets, will tend to hold it down, and thus increase its stability; which, in *quiet* water, will then actually be greater than on land.

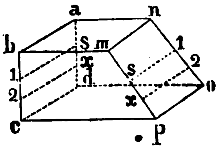


Fig 11

Art. 5. To divide a rectangular surf, whether vert as *a b c d*, or inclined as *m n o p*, Fig 11, whose top *a b* or *m n* is level with the surf of the water, by a hor line *x* 2, such that the total pres against the part above said hor line, shall equal that against the part below it.

RULE. Mult one half of the length of $b\,c$, or $m\,p$, as the case may be, by the constant number 1.4142; the prod will be b 2, or $m\,x$.

Ex. Let $b\,c = 12$ ft. Then $6 \times 1.4142 = 8.4852$ ft; or b 2. Let $m\,p = 16$ ft. Then $8 \times 1.4142 = 11.3136$ ft, or $m\,x$.

REM. The line x 2, thus found, must not be confounded with the *cen of pres*, which is entirely diff. See Art 8.

Art. 6. In a rectangular surf, whether vert as *a b c d*, or inclined as *m n o p*, Fig 11, whose top *a b* or *m n* coincides with the surf of the water, to find any number of points, as 1, 2, &c, through which if hor lines, as 1 *s*, 2 *x*, &c, be drawn, they will divide the given surf into smaller rectangles, all of which shall sustain equal pressures.

RULE. First fix on the number of small rectangles reqd. Then for point 1 from the top, mult the number 1, by this number of rectangles. Take the sq rt of the prod. Mult this sq rt by the entire length

---

* **In a sphere** filled with a fluid the total inside pres = 3 times wt of fluid.

b c or m p, as the case may be. Div the prod by the number of rectangles. The quot will be the dist b 1, or n 1, as the case may be.

For the dist b 2, or n 2, proceed in precisely the same way; only instead of the number 1, use the number 2 to be mult by the number of rectangles: and so use successively the numbers 3, 4, 5, &c, if it be reqd to find that number of points.

Ex. Let b c = 10 ft; and let it be reqd to find 2 points, 1 and 2, for dividing the rectangular surf a b c d into 3 rectangular parts, which shall sustain equal pressures. Here we have for point 1,

1 × 3 = 3. The sq rt of 3 = 1.732. And 1.732 × 10 (or b c) = 17.32. And $\frac{17.32}{3 \text{ rectangles}} = 5.773$ ft = b 1.

For point 2, we have

2 × 3 = 6. The sq rt of 6 = 2.449. And 2.449 × 10 (or b c) = 24.49. And $\frac{24.49}{3 \text{ rectangles}} = 8.163$ ft = b 2.

And so for any number of points.

REM. 1. **This rule will be found useful in spacing the cross-bars of lock-gates; the hoops around cylindrical cisterns; and the props to a structure, like Fig 3.**

REM. 2. **For dividing any surf, as *o b c d*, Fig 12, which is not rectangular, in the same manner,** with an accuracy sufficient for most practical purposes, perhaps the following method is as convenient as any.

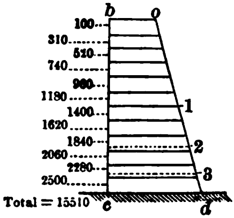


Fig. 12.

RULE. First div the surf, as in Fig 12, into several small hor parts, equal or not, at pleasure. Then by Rule in Art 1, find the pres on each part separately, as is supposed to be done in the numbers on the left hand of the fig. The sum of these (in this case 15510) is the total pres against the entire surf *o b c d*. Now suppose we wish to div this surf in 4 parts bearing equal pres; first div 15510 by 4 = 3878. Then beginning at the top, add together a number of the separate pressures sufficient to amount to 3878; by this means find point 1. Then proceed with the addition until the sum amounts to twice 3878, or 7756, which will indicate point 2; and in the same manner find point 3, by adding up to three times 3878, or 11634. Then the hor dotted lines ruled through points 1, 2, and 3, will give the reqd divisions approximately. In this manner the hoops of conical, and other shaped vessels, may be spaced nearly enough for practical purposes.

Art. 7. The transmission of pressure through water. Water, in common with other fluids, possesses the important property of transmitting pres equally in all directions. Thus, suppose the vessel, Fig 13, to be entirely closed, and filled with water; and suppose the transverse area of T, C, D, and E, to be each equal to one sq inch. Then, if by means of a piston, or otherwise, a pres of 1 ℔, 1 ton, or any other amount, be applied to the one sq inch of area of T, C, D, or E, every sq inch of the inner surf of the vessel, and of the pipe a, will instantly receive, at right angles to itself, an equal pres of 1 ℔, or 1 ton, &c: in addition to the pres which it before sustained from the water itself; and this will occur if the vessel consist of parts even miles asunder; as, for instance, if T were miles distant from E; and united to it by a long series of tubes. If the vessel were a strong steam boiler full of water, a single pres of a few hundred pounds at T, C, &c, would burst it. See also Fig 2 (c) and paragraph above it.

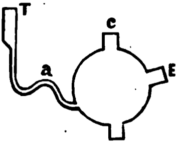


Fig. 13.

The hydrostatic press acts on this principle. Any body, within the vessel, would also receive an equal additional pres on each sq inch of its surf.

If the top of T be open, the air will press upon the sq inch of the exposed surf of water to the extent of nearly 15 ℔s; and the same degree of pres will also be transmitted to every sq inch of the interior surf of the vessel, and its connecting tubes; but no danger of bursting will result from this atmospheric pres, because the air also presses every sq inch of the outside of the vessel to the same extent.

Air, and other gaseous fluids, transmit pres equally in all directions, like liquids; but not as rapidly.

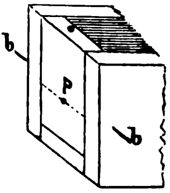


Fig. 14.

Art. 8. The center of pressure. Let Fig 14 represent a vessel full of water, and suppose the side P to be perfectly loose, so as to be thrown outward by the slightest pres of the water from within. Now, there is but one single point, P, in every surf so pressed, no matter what its shape may be, to which if we apply a force equal to the pres of the water, and in a direction opposite to said pres, the side P will be thereby prevented from yielding. Such point is called the *center of pressure.* It must not be understood by this that the actual amount of pres of the water against that part of the surface which is above the hor dotted line passing through P, is equal to that of the water below said line; but that the sum of the products of the several pressures above it, mult by their several leverages, or vert dists from P, is equal to the sum of the products of the pressures below, mult by their leverages; or, in other words, that the sum of the *moments* around the point P, of the pressures above the line, is equal to the sum of the *moments* of those below it; so that if a hor iron rod *b b* were passed entirely through the side P, at the same level as the dotted line, as shown in the fig, so as to serve as a hinge for the side P to turn on, the side would have no tendency to turn.

Art. 9. To find the cen of pres of a quiet fluid, against a plane surface. Fig 15.

1. The center of pressure of a quiet fluid against any plane surface whose width is uniform throughout its depth, whether said surface be vertical, as *e o*, or inclined, as c *a*, (or inclined in the opposite direction:) and whose top *c*, or *e*, coincides with the hor water surf; is distant *vert* below the water surf, two-thirds of the vert depth, *s x*, from said water surf to the bottom of the plane; as at *n* and *i*. Inasmuch as a hor line at ⅔ of the depth of *s x*, intersects both c *a* and *e o* at ⅔ of their lengths respectively, we might say at once that the center of pres against a plane of uniform width is at two-thirds of its length below the water surface.

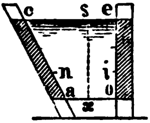


Fig. 15.

Throughout Art 9 any measure, as yard, foot, or inch &c, may be used.

2. But if the hor top *a*, or *o*, Fig 16, of the rectangular plane *a g*, or *o h*, be covered to some depth with water, then the *vert* depth *s m*, of the cen of pres *d*, or *e*, below the surf of the water, will be equal to

$$\tfrac{2}{3} \text{ of } \frac{\text{cube of } sc - \text{cube of } sw}{\text{square of } sc - \text{square of } sw}$$

where *s c* is the vert depth of the bottom, and *s w* the vert depth of the top, of the pressed surf, below the water surf. Or, in words: From the cube of *s c*, take the cube of *s w*; and call the rem *a*. Then, from the square of *s c*, take the square of *s w*; and call the rem *b*. Div *a* by *b*, and take two-thirds of the quot for *s m*.

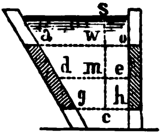


Fig. 16.

3. When a plane surf of any shape whatever, whether rectangular, triangular, or circular, &c; whether vert as *op*, Fig 17, or inclined as *m n*, is *entirely immersed*, so as to be pressed over the entire area of *both* sides· but by *diff depths* of water on its two sides; then the cen of pres coincides with the *cen of grav* of the pressed surf.

In the 3 foregoing figures the supposed surfaces are shown edgewise, so that their widths do not appear.

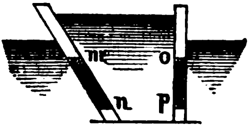


Fig. 17.

4. In any triangular plane surf, whether right-angled, or otherwise, as *a b c*, Fig 18; whether vert. or inclined; the *base a b* of which coincides with the hor surf of the water; the cen of pres o, will be in the center of the line *c r*, which bisects the base *a b*.

5. But if the triangle, as *a s c*, vert, or inclined, have its apex, *a*, at the surf of the water; and its base *s c*, *hor;* then the cen of pres *x*, will also be in the line *a m* which bisects the base; but *a x* will be ¾ of *a m*.

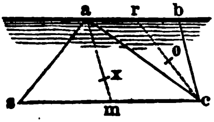


Fig. 18.

6. If any plane triangle *a b c*, Fig 19, base up, and hor: have its base *a b* covered to some depth *n d*, with water; then the cen of pres *o*, will be in the line *c s* which bisects the base; and *n o* will be equal to

$$\frac{mx^2 + (2mx \times ma) + 3ma^2}{(mx + 2ma) \times 2.}$$

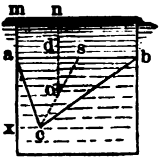


Fig. 19.

7. The center of pres against any plane *rectangular* surface, Fig 20, whether vert as *m n*, or inclined as *p o*, or *w x*; having its top coinciding with the surf of the water; and pressed by diff depths of water on its opposite sides, as shown in the fig; will be *vert* below the *upper* water surf, a dist equal to

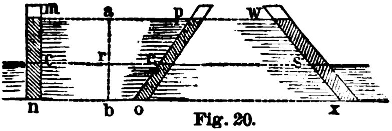


Fig. 20.

$$\frac{\left(\begin{matrix}\textit{area of surf } \mathrm{m\,n},\\ \textit{or } \mathrm{p\,o}, \textit{ or } \mathrm{w\,x}\end{matrix} \times \frac{\textit{sq of vert depth } \mathrm{a\,b}}{3}\right) - \left(\begin{matrix}\textit{area of surf}\\ \mathrm{c\,n}, \textit{ or } \mathrm{e\,o}, \textit{ or } \mathrm{s\,x}\end{matrix} \times \frac{\textit{sq of vert depth } \mathrm{r\,b}}{3}\right) - \left(\begin{matrix}\textit{vert}\\ \textit{depth}\\ \mathrm{a\,r}\end{matrix} \times \begin{matrix}\textit{area of}\\ \textit{surf } \mathrm{c\,n},\\ \textit{or } \mathrm{e\,o}, \textit{or } \mathrm{s\,x}\end{matrix} \times \frac{\textit{vert depth } \mathrm{r\,b}}{2}\right)}{\left(\begin{matrix}\textit{area of surf}\\ \mathrm{m\,n}, \textit{ or } \mathrm{p\,o}, \textit{ or } \mathrm{w\,x}\end{matrix} \times \textit{half of } \mathrm{a\,b}\right) - \left(\begin{matrix}\textit{area of surf}\\ \mathrm{c\,n}, \textit{ or } \mathrm{e\,o}, \textit{ or } \mathrm{s\,x}\end{matrix} \times \textit{half of } \mathrm{r\,b}\right).}$$

8. To find the center of pressure against either a circular or an elliptic surface, pressed on one side only; whether vertical or inclined; and having its top either coinciding with the surface of the water or below it.

Let h = the vertical depth of the center of pressure below the water surface.

r = the vertical or inclined *semi*-diameter of the surface.

d = the *vertical* distance of the *center* of the pressed surface, below the water surface.

Then

$$h = \frac{r^2}{4d} + d.$$

In a **vertical circle,** with top at water surface, $h = 1\frac{1}{4} \times$ radius.

Art. 10. Walls for resisting the pressure of quiet water. A study of our remarks on retaining-walls for earth, pp. 603, etc., will be of use in this connection. It is of course assumed that the water does not find its way under the wall; and that the wall cannot slide. In making calculations for walls to resist the pressure of either earth or water, it is convenient to assume the wall to be but one foot in *length* (not height, or thickness); for then the number of cubic feet contained in it, is equal to that of the square feet of area of its cross-section, or profile; so that these square feet, when multiplied by the weight of a cubic foot of the masonry, give the weight of the wall. In ordinary cases, it is well, for safety, to assume that the water extends down to the very bottom line of the wall.

Now, by Art 1, the total pressure of quiet water, against the rectilineal back of a wall, whether vertical or sloping, is found in lbs, by multiplying together the area in square feet of the part actually pressed, (or in contact with the water;) half the *vertical* depth of the water, in feet, (being the *vertical* depth of the center of gravity of a rectilineal back, below the surface); and the constant 62.5 lbs; and this total pressure is always *perpendicular to the pressed area.*

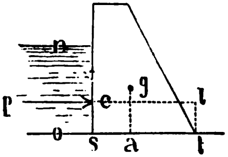


Fig 20½.

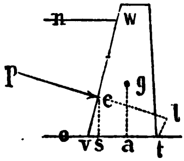


Fig 21.

When the back of the wall is vertical, as in Fig 20½, this pressure p is of course less than when it is battered; and is also *horizontal;* and it tends to overthrow the wall, by making it revolve around its outer toe, or edge t. The center of pressure is at c; $c\,s$ being ⅓ the *vertical* depth $o\,n$; in other words, the entire pressure of the water, so far as regards overthrowing the wall as one mass, may be considered as concentrated at the point c; where it acts with an overthrowing leverage $t\,l$. The pressure in lbs, multiplied by this leverage in feet, gives the *moment* in foot-lbs of the overturning force. The wall, on the other hand, resists in a vertical direction $g\,a$, with a moment equal to its weight (supposed to be concentrated at its center of gravity g), multiplied by the horizontal distance $a\,t$, which constitutes the leverage of the weight with respect to the point t as a fulcrum. If the moment of the water is greater than that of the wall, the latter will be overthrown; but if less, it will stand.

In Fig 21 the overturning moment of the water is equal to its calculated pressure $p \times$ its leverage $t\,l$; while the moment of stability of the wall is equal to its weight $\times$ its leverage $a\,t$. By aid of a drawing to a scale, we may on this principle ascertain whether any proposed wall will stand. For we have only to calculate the pressure p, then apply it at c, and at right angles to the back; prolong it to l; measure $t\,l$ by the same scale. Then calculate the weight of wall; find its center of gravity g; draw $g\,a$ vertical, and measure the leverage $a\,t$. We then have the data for calculating the two moments.

If the water, instead of being quiet, is liable to waves, the wall should be made thicker.

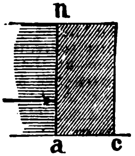


Fig. 22.

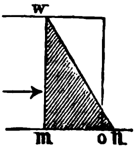


Fig. 23.

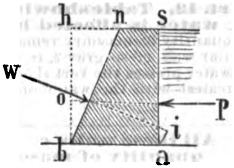


Fig. 24.

Art. 11. To find the thickness at base of a wall required to be safe against *overturning* under the pres of quiet water level with its top, and pressing against its entire vert back. **Caution.** See Art. 18.

(1st) Vertical wall, Fig 22.

$$\text{Thickness in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety}^{*}}{3 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

(2d) Right angled triangular wall, Fig 23.

$$\text{Thickness at base in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety}^{*}}{2 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

$$= \text{thickness, } m\,o\text{, of vertical wall} \times 1.225.$$

Notwithstanding their greater thickness at base, such triangular walls contain, as seen by the fig, not much more than half the quantity of masonry reqd for vert ones of equal stability. This is owing to the fact that their cent of grav is thrown farther back; thus increasing the leverage by which the wt of the wall resists overthrow.

(3d) Wall with vertical back and sloping face, Fig 24.

$$\text{Thickness at base in feet} = \sqrt{\frac{(\text{Ht}^2\text{, ft} \times \text{factor of safety}^{*}) + (\text{batter } h\,n^2\text{, ft} \times \text{sp grav of wall})}{3 \times \text{specific gravity of wall}}}$$

$$= \text{Height in feet} \times \text{the proper decimal in the following table.}$$

Fig. 22.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.	Resist = 2 pres.	Resist = 3 pres.
Dressed Granite...	2.5	156	.447	.516	.633
Dressed Sandstone	2.2	137	.477	.550	.674
Mortar Rubble.....	2.	125	.500	.578	.707
Brickwork	1.8	112	.527	.609	.746
Fig. 23.					
Dressed Granite...	2.5	156	.548	.633	.775
Dressed Sandstone	2.2	137	.584	.675	.826
Mortar Rubble.....	2.	125	.613	.707	.866
Brickwork	1.8	112	.646	.746	.913

Fig. 24.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.				Resist = 2 pres.			
			Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.	Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.
Dressed Granite...	2.5	156	.449	.458	.487	.532	.519	.526	.551	.593
Dressed Sandstone	2.2	137	.480	.488	.515	.558	.552	.560	.583	.622
Mortar Rubble.....	2.	125	.502	.510	.536	.578	.571	.586	.609	.646
Brickwork	1.8	112	.530	.539	.562	.602	.610	.618	.640	.674

$$^{*}\text{Factor of safety} = \frac{\text{Required moment of stability of wall}}{\text{overturning moment of water}}.$$

Art. 12. Table showing how the stability of a wall sustaining water is affected by a change in the form of the wall; the quantity of masonry remaining the same. **Rem.** When the base of a triangular wall, of sp grav 2, is less than ½ the ht, the stability is greatest when the water presses the vert side; but if the base exceeds ½ the ht, the stability is greatest with the water on the battered side. **Caution.** See Art. 13.

	All these walls contain precisely the same quantity of masonry. The masonry is supposed to be mortar rubble, weighing 125 lbs per cubic foot; or twice as much as water; or about the same as ordinary rough mortar rubble. If the sp gr of the masonry is actually greater or less than this, the safety also will be greater or less, in precisely the same proportion.	**Base in parts of height.**	**Approx resist of wall.**
1	Vertical wall	.5	1.5
2	Face vertical; back batters one-tenth height	.55	1.8
3	" " " " one-fifth "	.6	2.2
4	" " " " one-fourth "	.625	2.6
5	" " " " one-third "	.667	3.5
6	" " " " four-tenths "	.7	4.9
7	" " " " one-half "	.75	14.0
8	Back vertical; face batters one-tenth height	.55	1.8
9	" " " " one-fifth "	.6	2.1
10	" " " " one-fourth "	.625	2.2
11	" " " " one-third "	.667	2.4
12	" " " " four-tenths "	.7	2.6
13	" " " " one-half "	.75	2.9
14	Back and face, each batter one-tenth height	.6	2.2
15	" " " " one-fifth "	.7	3.4
16	" " " " one-fourth "	.75	4.6
17	" " " " one-third "	.833	9.0
18	" " " " four-tenths "	.9	36.0

Art. 13. Liability of wall or foundation to crush under unequal distribution of pressure. Arts 11 and 12 apply only to the *stability* of a *rigid* wall resting upon a *rigid* base, and therefore incapable of failure *except by overturning as a whole.* They show that the *stability* is greatest when the water presses against the *sloping* side. But in practice the point where the resultant of all the pressures on the base of the wall cuts the base, must not be so near to either toe as to endanger a *crushing* of wall or of foundation. This consideration often makes it best to let the water press against the *vert* back, notwithstanding the consequent loss in stability.

Art. 14. Fig. 25 shows, to scale, a dam wall at Poona, India, designed by Mr. Fife, C. E., of England. It is of mortar rubble, of 150 lbs per cub ft. Its total vert height is 100 ft; thickness *uv* at base, 60 ft 9 ins; at top, *rx*, 13 ft 9 ins. The front *ru* slopes 42 ft in 100 ft; and the back *xv*, 5 ft in 100 ft. Its foundation is 7 feet deep; but we here assume that the water presses against its *entire* back *xv*. Through the cen of grav G draw G*s* vert. From *c*, where the direction of the pres P of the water strikes G*s*, lay off *cn* by scale = 139.6 tons (of 2240 lbs) water pres against 1 ft in length of *xv;* and *ct* = 249.4 tons wt of 1 ft length of wall. Complete the parallelogram *cnmt* of forces. Its diag *cm* represents the resultant of all the pressures upon the base *uv*, and cuts the base at *a*, 20 ft back from the toe *u*. Doing the same with the 151.4 tons pres *p* against *ru*, we get the resultant *oy*, which is greater than *cm*, and cuts the base (at *i*) only 12.7 ft from the toe *v*, or 7.3 ft less than *a* is from *u*.

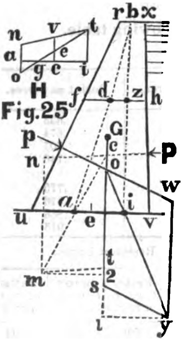


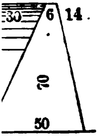
Fig.25

Hence, when the water presses against *xv* the wall is less liable to fracture or crushing, and the earth foundation *uv* is more evenly loaded, and hence less liable to yield unequally so as to cause cracks in the wall. On this account *xv* is made the back of the wall, although the moment of stability of the wall is then only 2.2 (calling the overturning moment of the water 1), while if the water pressed against *ru* it would be 3, or 36 per cent greater.

ırt. 15. The points *a* and *i*, Fig 25, are called **centers of pressure** ›n the base, or **centers of resistance** of the base. If similar points, as nd *z*, be found in the same way for other lines, as *f h*, by treating a part (as *h f*) of the wall as if it were an entire wall; a slightly curved line joining ›se points is called the **line of pressure.** Thus, *b a* is the line of pres- 'e when the water presses against *x v*. Each point, as *d*, in *b a*, shows where y joint, as *f h*, drawn through that point, is cut by the resultant of all the ces acting upon said joint. *b i* is the line of pres when the water presses ıinst *r u*. These lines do not show the *direction* of the resultants. Thus, at *a*, : latter is *c m*, not *b a*. The angle between the direction of the resultant and a ɔ at right angles to the bed or joint, must be less than the angle of friction the materials forming the joint.

f from the end *m* or *y* of the resultant of the pressures upon any joint, we .w *m* 2 or *y l* hor, then *c* 2 or *o l* (as the case may be) measures the entire *vert* s on that joint; and *m* 2 or *y l* measures the *hor* pres against the back of the ll, which tends to cause sliding at the same joint. If the direction of the re- :ant comes within the limit stated in the preceding paragraph, *m* 2 or *y l* will less than the frictional resistance to sliding, which last is = *c* 2 (or *o l*) × the ff of friction for the surfaces forming the joint. Hence sliding cannot take :e. Sliding never occurs in the *masonry* of walls of ordinary forms. Good rtar, well set, aids to prevent sliding, but it is better not to rely upon it. But ire walls have slidden on slippery foundations.

ırt. 16. In California is this dam of a mining reservoir, built of ıgh stone without mortar, founded on rock. Height, 70 feet; base, 50; top, 6; water-slope, 30 feet; outer-slope, 14. To prevent leaking the water-slope is only covered with 3-inch plank bolted horizontally to 12 by 12 inch strings, built into the stone-work. All laid with some care by hand, except a core of about one-fifth of the mass, which was roughly thrown in. Cost about $3 per cubic yard. It has been in use since 1860.



Rem. If a dam is compactly backed with earth at its natural slope, and in sufficient quantity to prevent the water from reaching the dam, the pressure against the dam will not be increased.

ırt. 17. To find the thickness of a cylinder to resist safely the ssure of water, steam, &c, against its interior. If riveted, see next page.

Vhere the thickness is less than one-thirtieth of the dius, as it is in most cases, the usual formula

$$) \quad \textbf{Thickness in inches} = \frac{\text{pressure}}{\text{safe strength}} \times \text{radius}^{*}$$

mployed. It regards the material as being subjected only to a direct tensile ıin, which is sufficiently correct in such thin shells.

'or somewhat greater pressures and thicknesses, Professor Reuleaux (Der Konstrukteur, p 52) gives

$$) \quad \textbf{Thickness in inches} = \frac{\text{pressure}}{\text{safe strength}} \left(1 + \frac{\text{pressure}}{2 \times \text{safe strength}}\right) \times \text{radius.}^{*}$$

'or very great pressures and thicknesses, as in hydraulic sses, cannons, &c, Professor Reuleaux (Konstrukteur, p 53) gives Lamé's nula:

$$) \quad \textbf{Thickness in inches} = \left(\sqrt{\frac{\text{safe strength} + \text{pressure}}{\text{safe strength} - \text{pressure}}} - 1\right) \times \text{radius.}^{*}$$

he three formulæ give results as follows, pressures and strengths in lbs per are inch:

meter.	Radius.	Pressure.	Safe tensile strength.	Thickness, inches.		
				Formula (1).	Formula (2).	Formula (3).
nches.	10 inches.	50	10000	**.05**	.050125	.05
"	"	500	"	.50	**.5125**	.513
"	"	5000	"	5.00	6.25	**7.32**

he thicknesses given by the formulæ appropriate to the several pressures are ıted in **heavy type.** It will be seen that in these cases the results differ slightly, except for *very* great pressures.

In all three formulæ take the *radius in inches*, and the pressure and strength *›ounds per square inch*.

Rem. 2. Want of uniformity in the cooling of thick castings makes them proportionally weaker than thin ones, so that in order to reduce thickness in important cases we should use only best iron remelted 3 or 4 times, by which means an ult cohesion of about 30000 lbs per sq inch may be secured. But even with this precaution **no rule will apply safely** in practice to cast cylinders whose thickness exceeds either about 8 to 10 ins, or the inner rad however small.

Under a pres of 8000 lbs per sq inch, **water will ooze through cast iron 8 or 10 ins** thick; and under but 250 lbs per sq inch, through .5 inch.

Table of thicknesses of single-riveted wrought iron pipes, tanks, standpipes, &c, by the above rule, to bear with a safety of 6 a quiet pressure of 1000 ft head of water, or 434 lbs per sq inch; the ult coh of fair quality plate iron being taken at 48000 lbs per sq inch, or at 8000 lbs for a safety of 6; which is farther reduced to 8000 × .56 = 4480 lbs, to allow for weakening by rivet holes; for **single-riveted** cyls have but about .56 of the strength of the solid sheet; and **double-riveted** ones about .7. With the above pres and other data, the rule here leads to thickness = .1016 × inner rad in ins.

Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Di. Ft.	Ths. Ins.
.5	.025	5	.254	16	.813	30	1.52	60	3.05	120	10	6.09
1.0	.051	6	.305	18	.914	33	1.68	66	3.35	132	11	6.70
1.5	.076	8	.406	20	1.016	36	1.83	72	3.66	144	12	7.31
2.0	.102	10	.508	22	1.117	42	2.13	84	4.27	192	16	9.75
3.0	.152	12	.609	24	1.219	48	2.44	96	4.88	240	20	12.19
4.0	.203	14	.711	27	1.371	54	2.74	108	5.49	288	24	14.63

For a less head or pressure, or for any safety less than 6, it is safe and near enough in practice, to reduce the thickness of wrought iron cyls in the same proportion as said head, pres, or safety is less than the tabular one.

Double-riveted cylinders, Fairbairn says, are about 1.25 times as strong as single-riveted. Hence they may be one-fifth part thinner. **Lap-welded ones** are nearly 1.8 times as strong as single-riveted; and hence may be only .56 as thick.

Many continuous miles of **double-riveted pipes in California** have been in use for years with safetys of but 2 to 2.6. In one case the head is 1720 ft, with a pres of 746 lbs per sq inch; diam 11.5 ins; thickness, .34 inch.

Cast iron city water pipes must be thicker than required by formula (1), in order to endure rough handling and the effects of "water-ram" (due to sudden stoppage of flow, see second Rem, p 513), and to provide against irregularity of casting and the air bubbles or voids to which all castings are more or less liable. In the following table the ultimate tensile strength of cast iron is taken at 18,000 lbs per square inch. Column A gives thicknesses by Mr. J. T. Fanning's formula (Hydraulic Engineering, p 454).

$$\left.\begin{matrix}\text{Thickness}\\ \text{in inches}\end{matrix}\right\} = \frac{(\text{pres, lbs per sq in} + 100) \times \text{bore, ins}}{.4 \times \text{ultimate tensile strength}} + .333\left(1 - \frac{\text{bore, ins}}{100}\right).$$

These correspond with average practice. The addition of 100 lbs to the pres is made in order to allow for water-ram. Column B gives thicknesses by formula (1), taking coefficient of safety = 8 (thus making safe tensile strain = 2250 lbs per square inch) and adding three-tenths of an inch to each thickness given by the formula:

Head in feet	50		100		200		300		500		1000	
Pressure, lbs per sq in.	21.7		43.4		86.8		130		217		434	
Bore, ins.	Thickness of pipe, in Inches.											
	A	B	A	B	A	B	A	B	A	B	A	B
2	.36	.31	.37	.32	.38	.34	.39	.36	.42	.40	.48	.51
3	.37	.31	.38	.33	.40	.35	.42	.40	.45	.45	.54	.60
4	.39	.32	.40	.34	.42	.38	.45	.42	.50	.50	.61	.72
6	.41	.33	.43	.36	.47	.42	.50	.48	.57	.60	.75	.94
8	.45	.34	.47	.38	.52	.47	.57	.55	.66	.70	.90	1.15
10	.47	.35	.50	.40	.56	.50	.62	.60	.74	.81	1.04	1.35
12	.49	.36	.53	.42	.60	.54	.67	.66	.82	.91	1.18	1.57
16	.55	.38	.60	.46	.70	.62	.79	.77	.98	1.10	1.46	2.00
18	.57	.39	.63	.48	.74	.65	.85	.84	1.06	1.21	1.60	2.20
20	.61	.40	.67	.50	.79	.68	.91	.90	1.15	1.31	1.75	2.50
24	.66	.42	.73	.53	.87	.77	1.02	1.01	1.30	1.51	2.03	2.84
30	.74	.45	.83	.59	1.01	.89	1.19	1.19	1.55	1.82	2.46	3.47
36	.82	.47	.93	.65	1.15	1.01	1.36	1.37	1.80	2.12	2.88	4.11
48	.98	.53	1.13	.77	1.42	1.24	1.70	1.73	2.28	2.78	3.73	5.38

Table of thickness of lead pipe to bear internal pressures with a safety of 6; taking the ultimate cohesion of lead at 1400 lbs per sq inch.

Rem. Although these thicknesses are safe against quiet pressures, they might not resist shocks caused by too sudden closing of stop-cocks against running water.

Bore in Inches.	Heads in Feet.					Bore in Inches.	Heads in Feet.				
	100	200	300	400	500		100	200	300	400	500
	Pres in lbs per sq inch.						Pres in lbs per sq inch.				
	43.4	86.8	130	174	217		43.4	86.8	130	174	217
	Thickness in Inches.						Thickness in Inches.				
¼	.026	.055	.089	.128	.171	1	.102	.221	.357	.511	.682
⅜	.038	.083	.134	.192	.256	1¼	.127	.276	.447	.639	.853
½	.051	.111	.179	.256	.341	1½	.153	.332	.536	.767	1.02
⅝	.064	.138	.223	.320	.427	1¾	.178	.387	.626	.895	1.20
¾	.076	.166	.268	.383	.512	2	.204	.442	.714	1.02	1.36
⅞	.089	.193	.313	.447	.597						

Rem. The valves of water-pipes must be closed slowly, and the necessity for this precaution increases with their diams. Otherwise the sudden arresting of the momentum of the running water will create a great pressure against the pipes in all directions, and throughout their entire length behind the gate, even if it be many miles; thus endangering their bursting at any point. Hence stop-gates are shut by screws.

Art. 18. Buoyancy. When a body is placed in a liquid, whether it float or sink, it evidently displaces a bulk of the liquid equal to the bulk of the immersed portion of the body; and the body, in both cases and at any depth, and in any position whatever, is buoyed up by the liquid with a force equal to the weight of the liquid so displaced. Thus, if we immerse entirely in water a piece of cork, *c*, *c*, Fig 26, or any body of less specific gravity than water, the cork will, by its weight, or force of gravity, tend to descend still deeper; but the upward buoyant force of the water, being greater than the downward force of gravity of the cork, will compel the latter to rise. In this case, the cork receives a total downward pressure equal to the weight of the vertical column of water above it, shown by the vertical lines in vessel 1; and a total upward pressure equal to the weight of the column shown in vessel 2. The difference between these two columns is evidently (from the figs) equal to the bulk of the cork itself; therefore the difference between their weights or pressures (or, in other words, the buoyancy of the water) is equal to the weight or pressure of the water which would have occupied the place of the cork; or, in other words, of the water which is displaced by the cork. This *difference*, or buoyancy, will plainly be very nearly the same at any depth whatever of *entire* immersion. It increases slightly with the depth, owing to increase in the density of the water; but, on the other hand it is diminished by compression of the cork. Now the cork, if left to itself, will continue to rise until a portion of it reaches above the surface, as in vessel 3. The downward pressing column then ceases to exist; and the cork is pressed downward only by its own weight. But, as it now remains stationary, the upward pressure of the water must be equal to the weight of the cork. But the upward pressure of the water arises only from the shaded column shown in vessel 3; and this column is (as in the case of total immersion) equal to the bulk of water displaced. Therefore, *in all cases*, the buoyancy is equal to the weight of water displaced; and when the body *floats* on the surface, the buoyancy, or the weight of water displaced, is also equal to the weight of the body itself.

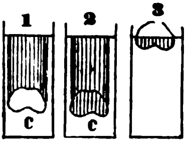


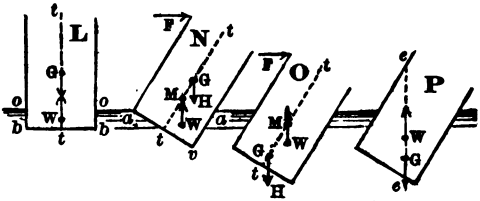
Fig. 26.

If the body be of a substance **heavier than water,** its weight is greater than the buoyancy of the displaced water, and the body therefore sinks, with a force equal to the difference between the two. Thus, a cubic foot of cast iron weighs 450 lbs., and a cubic foot of water 62.5 lbs., so that the iron sinks with a force of 450 — 62.5 = 387.5 lbs.

The same principle applies to **other fluids.** Thus, light bodies, such as smoke, a balloon, etc., in air, all tend, like a cork in water, to fall; but the air, being heavier, crowds them out of the lower positions which they tend to assume, and pushes them upward.

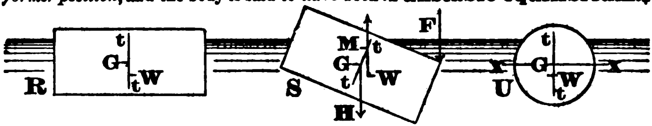
Although a pound of lead and a pound of feathers, weighed in the air, balance each other, yet in a vacuum the feathers will outweigh the lead, by as much as the bulk of the air displaced by them outweighs that displaced by the iron.

The downwd force of grav may be regarded as concentrated at the cen of grav G of a floating body. The upwd pres, or buoyancy,[†] of the water may similarly be regarded as acting at the cen of gr W of the displaced water.[*] W is also called the **center of pressure,** or **of buoyancy,** of the water; and a vert line drawn through it is called the **axis,** or **vertical, of buoyancy,** or **of flotation.** Ordinarily,[‡] W shifts its position with every change in that of the body. Thus in L it is at the cen of gr of the rectangle *o o b b*; and in N at that of the triangle *a a v*.



When a floating body, L, P or R, is at rest, and undisturbed by any third force, as F, it is said to be in **equilibrium,** and G and W are then in the same vert line *t t* Figs L and R, or *e e* Fig P; which line is called the **axis, or vertical, of equilibrium.**[§]

When a third force, as F, causes the axis of equilib to lean, as in Figs N, O and S, then if a vert line be drawn upwd from the cen W of buoy, the point M where said line cuts said axis, is called the **metacenter** of the body.[‖] G and W are then no longer in the same vert line;[‡] and the two opp and vert forces, grav and buoy, acting upon those points respectively, form a "couple" and, when the third force F is removed, they no longer hold the body in equilib, but cause it to rotate. If (as in Figs O and S) the positions of G and W are then such that the metacenter M is *above* the cen of gr G, this rotation will tend to *restore the body to its former position*, and the body is said to have been (before the application of the third force F) in **stable equilibrium.**[¶] But if (as in N) M is *below* G the direction of rotation is such as to *upset* the body, by causing it to *depart further from its former position*, and the body is said to have been in **unstable equilibrium.**[‡]



The tendency or moment in ft-lbs of a floating body either to upset or to right itself, is,

$$= \text{the wt of the body (or the equal upwd pres of the water) in lbs} \times \text{the hor dist between W M and G H, Figs N, O and S, in ft.}$$

The third force F may of course be so great as to overpower the tendency of the body to right itself. Thus, a ship may upset in a hurricane, although judiciously loaded and ballasted for ordinary winds. A hor section of a body at water-line is called its **plane of flotation.**

---

[*] **The body is in fact acted upon by other forces,** such as the hor pressures of the water against its immersed portions; but as all of these in any one given direction are balanced by equal ones in the opposite direction, they have no effect upon the forces G and W. It is also acted upon by the air, which presses it downwards with a force of 14.73 lbs per sq inch; but this is balanced by an equal pres of the surrounding air upon the surface of the water.

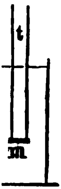
[†] **This buoyancy is made up** of the parallel upward pressures of the innumerable vert filaments of the displaced water as shown by Fig 26, and the **axis of flotation** is their resultant, as in the case of parallel forces.

[‡] The shape of a body (as that of a sphere or cylinder U) may be such that the position of its cen of buoy W, relatively to that of its cen of gr G, is not changed by the rotation of the body about a given axis (as any axis of the sphere or the *longitudinal* axis of the cyl), but remains constantly in the same vert line with G, so that the body, in rotating, remains in equilib. Such a body is said to be in **indifferent equilibrium** about said axis. But if a cyl U be made to rotate about its *transverse* axis *x x*, it plainly comes under the remarks on Figs R and S, and may (before rotating) be in either stable or unstable equilib about that axis according to the way in which its wt is distributed.

[‖] This metacenter shifts its position on the line *t t* according to the inclination of the latter.

[§] **Uneven loading,** instead of a third force, may cause a vessel at rest to lean as at P; and yet the vessel so leaning may be in equilib: for its axis *e e* of equilib may be vert, although not coinciding with the **axis of symmetry** of the vessel, as it does at *t t* in L.

[¶] In *floating* bodies, this may sometimes (as in Figs R and S) be the case even when the cen of *buoy* W (*not* the *metacenter*) is *below* the cen of gr G; because, when the body is forced to lean, W moves to another point in it, and this point may be such as to bring M above G. W is always below G in bodies of uniform density, floating at rest, if any part of the body is above water. When such bodies are entirely submerged, W and G coincide.



ig 27

Art. 19. A body lighter than water, if placed at the bottom of a vessel containing water, will not rise unless the water can get under it, to buoy it, or press it upward, as the air presses a balloon or smoke upward. Thus, if one side of a block of light wood, *perfectly* flat and smooth, be placed upon the similarly flat and smooth bottom of a vessel, and held there until the vessel is filled with water, the downward pres will keep it in its place, until water insinuates itself beneath through the pores of the wood. But if the wood be smoothly varnished, to exclude water from its pores, it will remain at the bottom.

On the other hand, a piece of metal may be prevented from sinking in water, by subjecting it to a sufficient *upward* pres only, while the downward pres is excluded. Thus, if the bottom of an open glass tube, *t*, Fig 27, and a plate of iron *m*, be made smooth enough to be water-tight when placed as in the fig; and if in this position they be placed in a vessel of water to a depth greater than about 8 times the thickness of the iron, the upward pres of the water will hold the iron in its place, and prevent its sinking; use it is pressed upward by a column of water heavier than both the column of air, and its own ht, which press it downward. On this principle iron ships float.

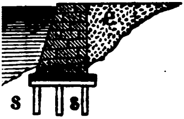


Fig 28

Rem. 1. **A retaining-wall, as in Fig 28, founded on piles,** may be strong enough to resist the pres of the earth *e* behind it, in case water does not find its way underneath; and yet may be overthrown if it does; or even if the earth *s s* around the heads of the piles becomes saturated with water so as to form a fluid mud. In either case, the upward pres of the water against the bottom of the wall will virtually reduce the wt of all such parts as are below the water surf, to the extent of 62½ lbs per cub ft; or nearly one-half of the ordinary wt of rubble masonry in mortar.

Rem. 2. Although the piles under a wall, as in Fig 28, may be abundantly sufficient to sustain the wt of the wall; and the wall equally strong *in itself* to resist the pres of the backing *e*; yet if oil *s s* around the piles be soft, both they and the wall may be pushed outward, and the latter thrown by the pres of the backing *e*. From this cause the wing-walls of bridges, when built iles in very soft soil, are frequently bulged outward and disfigured. In such cases, the piling, the wooden platform on top of it, should extend over the whole space between the walls; or else other remedy be applied.

rt. 20. Draught of vessels. Since a *floating* body displaces a wt of liquid l to the wt of the body, we may determine the wt of a vessel and its cargo, by ascertaining how r cub ft of water they displace. The cub ft, mult by 62½, will give the reqd wt in lbs. Suppose, stance, a flat-boat, with vert sides, 60 ft long, 15 ft wide, and drawing unloaded 6 ins, or .5 of In this case it displaces 60 × 15 × .5 = 450 cub ft of water; which weighs 450 × 62½ = 28125 which consequently is the wt of the boat also. If the cargo then be put in, and found to sink oat 2 ft *more*, we have for the wt of water displaced by the cargo alone, 60 × 15 × 2 × 62½ = 0 lbs; which is also the wt of the cargo. So also, knowing beforehand the wt of the boat and , and the dimensions of the boat, we can find what the draught will be. Thus, if the wt as before 0625 lbs, and the boat 60 × 15, we have 60 × 15 × 62½ = 56250; and $\frac{140625}{56250}$ = 2.5 ft the required ht. In vessels of more complex shapes, as in ordinary sailing vessels, the calculation of the nt of displacement becomes more tedious; but the principle remains the same.

HYDRAULICS.

Flow of Water through Pipes.

Much of the theory of hydraulics is still matter of dispute. This, and the unavoidable imperfections of actual work, render it advisable to use liberal safety factors in applying hydraulic formulas. Even new pipes are liable to tuberculations, which materially diminish the flow, and these sometimes greatly increase under the action of chemicals in the water. Air in the pipes also diminishes the flow.

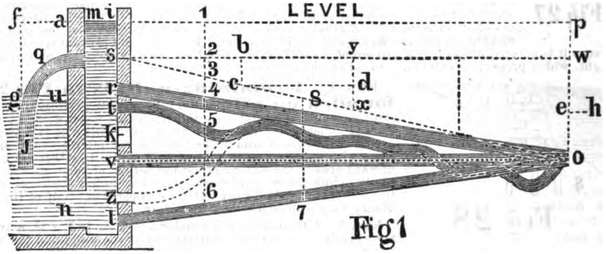


Fig 1

The term **HEAD** or **TOTAL HEAD** of water, as applied to the flowage of water through canals, pipes, or openings in reservoirs, &c, means the *vert* dist iv or po, Fig 1, from the level surf, mi, of the water in the reservoir, or source of supply, to the center (or more properly to the cen of grav) o, of the orifice (whether the end of a pipe, ro, to, vo, zo, lo; or any other kind of opening) through which the disch takes place freely, *into the air*; or the vert dist au, or fg, from the same surf, mi, to the level *surf*, gu, of the water in the lower reservoir; when the disch takes place *under water*. Thus, in the case of disch *into the air*, the vert dist iv or po, is the total head for either of the pipes ro, to, vo, zo, or lo; and ik is the head for the orifice, k, in the side of the reservoir. And for disch *under water*, au, or fg, is the head for either the pipe j, or the opening n; without any regard whatever to their depths below the surf of the lower water; which, according to the older authorities, do not at all affect their disch.

A portion of a pipe may have a head greater than the total head of the entire pipe. Thus the point 6 in the pipe lo, has a head 6 1; while the entire pipe has only the head po.

Both in theory and in practice it is immaterial as regards the vel, and the quantity of water discharged, whether the pipe is inclined downward, as ro, Fig 1; or hor, as vo; or inclined upward, as lo; provided the total head po, and also the length of the pipe, remain unchanged. If one pipe is longer than another, its sides will evidently present more friction against the water, and thus diminish the vel and the quantity of disch. The inclined pipes, ro, lo, being of course a little longer than the hor one vo, will therefore each disch a trifle less water; but if the hor one were extended slightly beyond o, so as to give it the same length as the others, then each of the three would disch the same quantity in the same time.

Art. 1 *a*. Divisions of the Total Head. In any pipe, as so, ro, to, vo, zo, or lo, Fig 1, the total head has three distinct duties to perform; 1st, to overcome the resistance to *entry* at s, r, t, v, z, or l; 2d, to overcome the resistances *within* the pipe; and, 3d, to give to the water, entering the pipe, the uniform *velocity* with which it actually flows.

For convenience, we regard the total head as divided into three portions, corresponding to these duties; namely, 1st, the *entry* head; 2d, the *resistance*, or *friction*, head; and, 3d. the *velocity* head.

Art. 1 *b*. The **velocity head** is the height through which a body must fall, in vacuo, to acquire the vel with which the water actually flows into the pipe. It is therefore $= \frac{v^2}{2g}$, in which v is the vel in ft per sec; and g is the acceleration of gravity, or 32.2

Art. 1 *c*. Experiment shows that, with the usual sharp-edged entry, the **entry head** is, near enough for practice, = half the vel head. If the entry is shaped like Fig 7, scarcely any entry head will be required. But, in pipes longer than about 1000 diameters, the entry head bears so slight a proportion to the total head, that this advantage is of but little importance. It becomes more apparent in shorter pipes.

Art. 1 *d*. In Fig 1 we will assume that for any of the pipes, is represents the sum of the vel and entry heads. Then the remainder sv, or wo, of the total head, is the **friction head**; or the head which is just sufficient to *balance* the friction and other resistances *within* the pipe; and, since the *entry* head balances the resistance at the *entrance* to the pipe, the *velocity* head has only to give velocity to the water in the vessel, causing it to enter pipe as rapidly as it flows through it, and thus keeping the pipe supplied. If, by shortening the , or by smoothing its inner surf, we diminish the total friction, then a less friction head will be iired; but the vel will, at the same time, be increased, and this will require a greater vel head, entry head, so that the three together make up the total head, as before. Since the *friction* is il to the force or head reqd to overcome it, it also is represented by *w o*.

Art. 1 *e*. The friction head may as in *v o*, *s o*, and *l o*, Fig 1, be all *above* the entrance ie pipe, and therefore *outside* of the pipe; or, as in a pipe laid from *s* to *o*, it may be all *below* entrance, and *within* the pipe; or, as in *r o* and *l o*, it may be partly above, and partly below, the ance; and therefore partly within, and partly without, the pipe. The vel and disch, *after the is filled*, are not affected by this difference in position of the entry end; but the *pressures* in the , and the vels *while the water is filling an empty pipe*, are affected by it, as explained in Arts 1 *l* 1 *o*.

Art. 1 *f*. But it is necessary that the entry end of the pipe ould be placed so far below the surf *m i*, that there shall be left, re the cen of grav of the entry end, at least a head, *i s*, sufficient to perform the duties of the entry vel heads. If the entry end of any of the pipes be raised above *s*, a portion of the *vel* head will n the pipe. In other words, the head *in* the pipe will be more than sufficient to overcome the stances in the pipe; and the surplus will act as vel head, and will give greater vel to the water ie pipe. The *reduced* head thus left above the entry end will plainly be insufficient to maintain supply for the *greater* vel, and the pipe will run only partly full.

ordinary cases of pipes of considerable length, the sum of the entry and vel heads theoretically iired, is but a small portion of the total head, and rarely exceeds a foot. Indeed, in a pipe of iderable diameter, the upper half of its cross section at the entry end may often be more than igh to provide sufficient entry and vel heads above the cen of grav of said cross section; so that top of the entry end might, so far as these considerations alone are concerned, project above the of the water in the reservoir. But the end of the pipe should in practice always be entirely be- the surf; otherwise air and floating impurities will be drawn into it, and cause obstructions. eover, the water surf of reservoirs is always liable to considerable changes of height; and the y end of the pipe must be placed at such a depth that the water can flow into it with sufficient when at its *lowest* stages. As before stated, this will cause no diminution or increase of disch.

Art. 1 *g*. To find the friction head reqd for any part of pipe; knowing the fric head reqd for the whole pipe. Since the friction, in a of uniform diam, is (other things being equal) in proportion to its length; and since *w o*, Fig 1, esents the total friction, or reqd friction head, we have

Total length of the pipe : Length of the given portion : : *w o* : The friction head reqd for that portion.

having drawn *w o* by scale, *s w* hor, and *s o*;

Total length of the pipe : Length of the given portion : : *s o* : A dist, as *s c*, to be laid off from *s* on *s o*.

: : *s w* : A dist, as *s b*, to be laid off from *s* on *s w*.

n a vert line, as *b c*, drawn from *b* or *c*, and joining *s w* and *s o*, gives by scale the friction head

Art. 1 *h*. If the pipe is straight, as *r o*, *v o*, *l o*, the friction in any part *begin- at the reservoir*, as *l* 6 in the pipe *l o*, may be found at once by drawing a line $\overline{6\ \ 1}$ vert upward the axis of the pipe at 6. The line $\overline{2\ \ 3}$ will then give the friction in *l* 6. It also gives the fric- in *r* 4, or in that part of *v o* which lies between *v* and the dotted line $\overline{1\ \ 6}$. It must be remembered that all the pipes in Fig 1 are supposed to be of the same actual length. They would thus end at different points *o*, and strictly, a separate diagram must be drawn for each pipe. In a part of the pipe *not* beginning at the reservoir, as in *r o*, *v o*, or *l o*, between points vertically under *c* and *x*, the amount of friction is given by the line *d x*, for it is plainly = *y x* — *b c*.

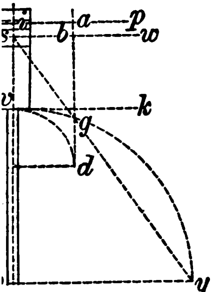


Fig.1 A

Art. 1 *j*. If the pipe is vert, as *v o*, Fig 1 A; let *t s* (on its axis *t o*) represent, as before, the sum of the vel and entry heads. From *s*, *v*, and *o*, respectively, draw hor lines *s w*, *v k*, and *o y*, making *o y* = *v o*. Draw the oblique line *s y*. Then, to find the friction in any part, as *v q*, beginning at the reservoir; from *q* lay off *q d* hor, and equal to *v q*, and draw the vert line *a d*, crossing *s y* at *g*. Then *b g* will give the friction in *v q*.

Art. 1 *k*. If the pipe is curved, and if the curvature is uniformly distributed along its length, or so slight that it may be neglected; the friction heads reqd for the several portions of the pipe, may be found in the same way as for straight pipes, as in Art 1 H. Otherwise they must be found by proportion, as in Art 1 G.

Art. 1 *l*. While water is filling an empty pipe, the excess of the total head above the requirements of friction, &c, gives to the water a **greater vel** than it has after the pipe is filled; but this gradually decreases as the advancing water encoun- the friction along the increased lengths of pipe filled; and finally becomes least when the water the whole length, and begins to flow from the disch end, *o*. But if only the vel and entry s are left above the entry end, as in a pipe laid from *s* to *o*, there will plainly be no such excess tal head, and, consequently, no such change of vel during the filling of the pipe.

Art. 1 *m-r*.* Relation between discharge, area, velocity and pressure. In Fig 1 *B-D*, where the pipe, *b F*, running full, receives water from an unlimited reservoir, *R*, at *b*, and discharges through an orifice, *F*; the volume of water, passing any given cross section of *b F*, in a given time, is constant and equal to the rate of discharge at *F*. Thus;—if the rate of discharge, at *F*, be *Q* cubic feet per second, then *Q* cubic feet will pass each cross section of the pipe, *b F*, per second.

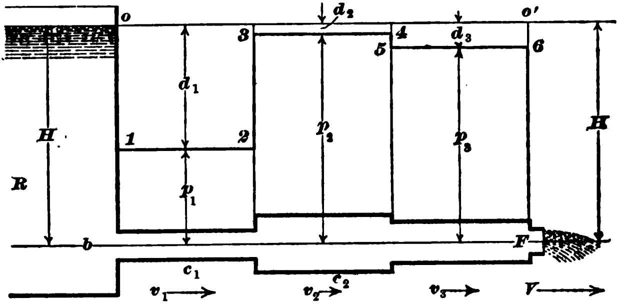


Fig. 1 B-D.

Let $a =$ the area of cross section, and $V =$ the velocity, of the stream issuing through the short pipe beyond *F*. *V* is called the velocity of efflux.

Let A_1, A_2, etc., be the different areas of cross section of *b F*, and let v_1, v_2, etc., be the velocities at those cross sections respectively. Then $Q = a\,V = A_1\,v_1 = A_2\,v_2$, etc.; or $V = \frac{Q}{a}$, $v_1 = \frac{Q}{A_1}$, $v_2 = \frac{Q}{A_2}$, etc. In other words, the velocities are inversely as the areas of cross section. Also, $a = \frac{Q}{V}$, $A_1 = \frac{Q}{v_1}$, $A_2 = \frac{Q}{v_2}$, etc.

The losses of pressure, due to the velocities, respectively, are $d_1 = \frac{v_1^2}{2g}$, $d_2 = \frac{v_2^2}{2g}$, etc.; as represented by the ordinates between the line *o o'*, of static pressure, and the diagram, *o* 1 2 3 4 5 6 *F*, of actual pressures. The difference, due to velocity, between the pres heads at any two points, as c_1 and c_2, where the velocities are v_1 and v_2 respectively, is $p_2 - p_1 = d_1 - d_2 = \frac{v_1^2}{2\,g} - \frac{v_2^2}{2\,g} = \frac{v_1^2 - v_2^2}{2\,g}$.*

The remaining pressure head, p_1, p_2, etc., at any point, is = static head in reservoir — velocity head at the point, $= H - d_1$, $H - d_2$, etc.

The loss of pressure head, at *F*, is $(6\ F) = p_3 = H - d_3$; and the pressure drops to zero; i.e., to the atmospheric pressure.

Art. 1 *s*. Open piezometers. If the lower ends of vertical or inclined tubes, open at both ends, be inserted into a pipe, *b F*, Fig. 1 *B D*, as at c_1, c_2, etc., the water surface, in these tubes, will stand at heights, p_1, p_2, etc., corresponding to the pressure heads at the points where the tubes are inserted. Such tubes are called open piezometers. In order that the water level may be observed, they are of glass, at least in those portions where that level is likely to be found. An obstruction, in the pipe, between c_2 and *F*, would *raise* the level in a piezometer at c_2; while an obstruction between *b* and c_2 would *lower* it.

---

* In Art. 1 *m-r*, for simplicity, we neglect all resistances, including those due to the abrupt enlargements and contractions of the pipe.

Art. 1 t. If we imagine any pipe, full of water, to be supplied with a number of piezometers, then a line, joining the tops of the columns of water in the several piezometers, is called the **hydraulic grade line.**

Art. 1 u. In a straight tube of uniform diam throughout, as $r o$, $v o$, or $l o$, Fig 1, running full and discharging freely into the air, the hyd grade line is a straight line drawn from its disch end o to a point s immediately over the entry end of the pipe, and at a depth below the surf equal to the sum of the vel and entry heads.

If the orifice at *o* be contracted, the hyd grade line must be drawn from s to some point, as e, immediately over o, and depending, for its height, upon the amount of contraction at o. But in this case the point s will also be higher than before, because the vel in the pipe is reduced by the contraction; and the sum $i s$ of the vel and entry heads will be less.

If the disch at *o* is under water, the effect upon the position of the grade line will be the same as that of a contraction of the orifice at o. The point e will be on the surf of the lower water, and immediately over o.

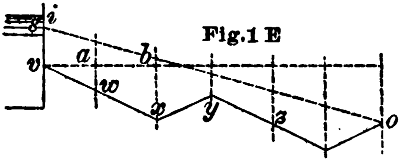


Fig.1 E

Art. 1 v. If the pipe, of uniform diam, (whether discharging freely or through a contracted opening at o, whether into the air or under water), **is bent or curved,** the hyd grade line will still be straight, **provided** the resistances are equal in each equal division of the *hor* length of the pipe, as in Fig 1 E, where equal divisions $v w$, $w x$, &c, of the *total* length, correspond with equal divisions $v a$, $a b$, &c, of the *hor* length.

But in Fig 1 F, the hyd grade line will take the shape $s a o$. For if, in accordance with Art. 1 G, we divide $s o$ into two equal parts, $s m$, $m o$, corresponding with the two equal parts $v r$, $r o$, of the length of the pipe, we obtain $m c = a e$ for the head consumed in the resistances in $v r$, leaving only $r a$ for the pres head at r.

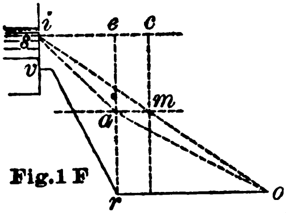


Fig.1 F

Art 1 w. In a very large vessel, the total head upon any point at the level of the entrance l to a pipe $l o o'$ Fig 1 G, is represented by $i l$, as already explained but of this total head a portion, as $i s$, is required to act as velocity head and entry head for the entrance at l, leaving only $s l$ as the pressure head upon a point in the *pipe*, immediately to the right of l. Thus while the pressure, in pounds per square inch, in the *vessel* at l, is

$$p = i l \times 0.434 \quad .$$

that in the *pipe* at l is

$$p = s l \times 0.434.$$

But now a portion, as $s v$, of $s l$, is expended in $l o$ in balancing or "overcoming" the resistances throughout that portion of the pipe; and, in doing this work, it gradually diminishes from $s v$ (at l) to nothing (at o) as indicated by the dotted line $s e$. Thus, at the point 6, a portion $= b c$ has already been expended in overcoming the resistances in the pipe between l and 6, leaving $c 6$ as the pressure head at 6, of which $c m$ must still be expended against resistances in the wide pipe between 6 and o, leaving $m 6 = v l = e o$ as the pressure head for a point just to the left of the contraction at o. The pressure in $l o$ is thus gradually diminished from $s l$ (at l) to $e o = v l$ (at o).

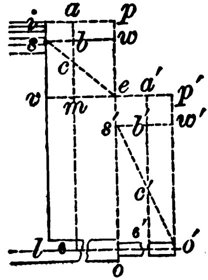


Fig.1 G

Now a portion $e s'$ of $e o$ is required to act as velocity and entry head for the entrance o to the narrower portion $o o'$ of the pipe; because we need at o not only an additional *entry* head to overcome the resistance due to the square shoulder formed by the contraction, but also an additional *velocity* head to give the increase of velocity which must take place as the water passes from the wide pipe $l o$ to the narrower one $o o'$; for, so long as a pipe runs *full* and the discharge remains constant, the *velocity* in each part of the pipe must be *inversely as the area* of cross section of that part; because in each second the same quantity of water passes each point; and this constant quantity is = area × velocity. Hence, as the area diminishes, the velocity increases.

There remains, therefore, $s' o$ as the pressure head upon a point in the narrow part just to the right of o; and this in turn gradually diminishes to nothing at the end *o′* of the pipe, as indicated by the dotted line *s o′*, being all expended in overcoming the resistances in *o o′*. We thus have, for the hydraulic gradient in Fig 1 G, the broken line *i s e s′ o′*.

When the pressure is thus diminished by overcoming resistances, or by accelerating velocity, the diminution is called **loss of head.** Thus we say that *i s* is *lost* at the entrance *l*, *s v* as friction head in *l o*, *e s′* at the contraction *o*, and *s′ o* as friction head in *o o′*,

At *o′*, *all* the available head, *i l*, has been used up. The water flowing out at *o′*, therefore exerts no lateral pressure, so that the stream flows out in parallel lines, and its capacity for forward pressure is due entirely to its kinetic energy (energy of motion) and to the pressure of the air upon the surface in the reservoir at *i*; but this last is of course balanced by the air pressure from without against the opening *o′*. Where a great reduction of cross sectional area in a pipe is followed (down-stream) by a re-enlargement, the increase of velocity may (under certain circumstances) consume not only the entire head of *water*, but also a portion or all of the *atmospheric* pressure on the surface in the reservoir, thus causing a partial or complete vacuum at the constriction. See the Venturi Meter.

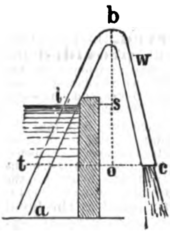


Fig. M.

The syphon, or siphon. If one leg *a b* of a bent tube or pipe *a b c*, Fig M, of any diam, filled with water, and with both its ends stopped, be placed in a reservoir of water, as in the fig; and if the stoppers be then removed, the water in the reservoir will begin to flow out at *c*, and will continue to do so until its level is reduced to *t*, which is the same as that of the *highest* end *c* of the pipe or syphon. The flow will then stop. The parts *a b* and *b c* are called the legs of the syphon, *b* being its highest point; and this is correct so far as relates to it merely as a piece of tube; but considering it purely with regard to its character as a hydraulic machine, the part *t a* below the level of the highest end *c*, may be entirely neglected; for the water in the reservoir will not be drawn down below the level of the highest end, whether that be the inner or the outer one. Therefore, if the disch end be above the water in the reservoir, as, for instance, at *w*, no flow will take place. The *vert* height *b o*, from the highest part of the syphon, to the lowest level *t*, to which the reservoir is to be drawn down, must not, theoretically, exceed about 33 or 34 ft; or that at which the pres of the air will sustain a column of water. Practically it must be less, to allow for the friction of the flowing water, and for air which forces its way in. And still less at places far above sea level; for at such the reduced weight of the atmospheric column will not balance so great a height of water. In order readily to understand, or at any time to recall the principle on which the syphon acts, bear in mind that we may theoretically consider the end of the inner leg to be not actually immersed *below* the water surf, but only to be kept precisely *at* it, as the surf descends while the water is flowing out: but may regard the vert dist *b o* as the length of the outer leg; and a varying dist, which at first is *b s*, and finally *b o* (as the surf of the reservoir descends) as the length of the inner leg; and that the flow continues only while *this* outer leg is longer than *this* inner one. The books are wrong in saying that the outer leg *b c* *must* be longer than the inner one *b a*, in order that the water may run at all. The principle then is simply this: that both these legs *b c*, and *b i*, being first filled with water, (the part *i a* being considered at first as a portion of the *reservoir*, and not of the syphon,) it follows that when the stoppers are removed from the ends *c* and *a*, the air presses equally against these ends; but the great vert head of water *b o* in the outer leg *b c*, presses against the air at *c*, with more force than the small head of water *b s* in the inner leg *b i*, does against the air at *a* or *i*.* Consequently, the water in *b c* will tend to fall out more rapidly than that in *b i*; and as it commences to fall, would produce a vacuum at *b*, were it not that the pres of the air against the other end *a* or *i*, forces the water up *i b*, to supply the place of that which flows out at *c*. In this manner the flow continues until the surf of the water in the reservoir descends to *t*, on the same level as *c*. The pressures of the vert heads *b o*, *b o*, in the two legs *b c*, *b t*, being then equal, it ceases.

The syphon principle may be employed for draining ponds into lower ground at a considerable dist, even though an elevation of several feet (in practice perhaps not *exceeding* about 28 ft above the level to which the pond is to be reduced) may intervene. In such a case an **escape** must be provided at the summit (or summits, if there are more than one) of the bends, for the disch of free air, which will inevitably enter, and soon stop the flow, unless this precaution be taken. The air-valve will not answer for this, because as soon as the valve *v* opens, the syphon becomes in effect two separate tubes open at top; and the water will fall in both. An orifice at the escape will be needed for filling the syphon at the start; and to prevent the water thus introduced, from running out, stopcocks must be provided at the ends, and kept closed until the filling is completed.

The greatest pains must be taken to make all the joints perfectly air-tight.

The motive power or head which causes the flow in a syphon, is the vert dist *s o*, from the surf of the reservoir, to the disch end *c*; or in other words, it is the diff, *s o*, between the theoretical lengths *b s* and *b o*, of the two legs. Coniently, the farther *c* is below *s* the more rapid will be the flow; and it is plain ; as the surf gradually sinks below *s*, the less rapid will the flow become. Havthis head, the entire length *a b c* of the syphon, and its diam, all in ft, the h may be found approximately by either of the rules given in Art 2 for straight 2s. These rules give 55⅓ galls per min, instead of the 43⅓ galls actually dischd Col Crozet's syphon, with a head of 20 ft. See below.

---

* Said pressure of the air is of course not exerted directly at *a* or *i*; but is transmitted to *a* through the water in the vessel; and thence upward to *i* through the water in the siphon.

n a true syphon, *a g n y o* Fig 1½, *free from air* inside, and *running full*, 3 **total head** *p o* is measured vertically from the surface *m i* in the reser- to the center of gravity of the outlet *o*, as in Fig 1; **the hydraulic adient** (with the restriction named in Art 1 *v*) is, as before, a straight line *s r o* drawn from the foot *s* of the combined entry and velocity heads to the end *o*; and the velocity and discharge are the same as they would be if all parts of the pipe were brought below *s r o*. But see cautions 1 and 2, below.

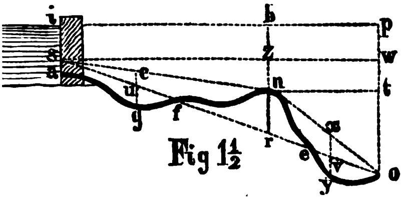


Fig 1½

The pressure at any point, *g*, *n* or *y*, is then given by a vertical line, *g u*, *n r* or *y v*, drawn m the point in question to *s r o*: but for points, as *n*, situated *above s r o*, this ssure is *negative* or *inward;* while at points where *s r o* and the pipe are at the 1e level, as at *f* and *e*, there is neither pressure nor vacuum.

'aution 1. But if the water be admitted to the empty pipe at *a*, while the l *o* is open, the pipe will not form a true syphon. The part *a g n* will then run l, and will have *s c n* as its hydraulic gradient; but upon reaching, at *n*, a tion *n o* of the pipe with a much steeper grade, the water will run off, in *n o*, h a velocity greater than that with which it arrives from *a n*. Hence the eam in *n o* will have a less area of cross section than in *a n*, and therefore can- fill *n o*, but will run off in it as in an open gutter.

'aution 2. The tendency to vacuum at points above *s r o* causes an accuilation, at *n*, of particles of **air** that have been carried into the syphon by the ter or have found their way in through imperfect joints, etc.; and these ng about a condition approaching that described in Caution 1; for their pansive force, by reducing the negative pressure or vacuum *n r* at *n*, diminishes 3 total head *h r* of the part *a g n*, while, by practically reducing the cross-secn of the syphon at *n*, they require that a portion of the remaining head be 2d at *n*, as entry head to overcome the resistance caused by the contraction, d as velocity head to give the increase of velocity needed for passing the narwed section at *n*. Now since the friction head required for the part *a g n* reiins about the same, the velocity head in the reservoir is considerably diminied, and the water arrives at *n* too slowly to keep *n o* filled. The accumulation air at *n* thus retards the flow and disturbs the distribution of the pressures, that these are no longer correctly indicated by vertical lines drawn to *s r o*.

At Blue Ridge Tunnel, Virginia, Col. C. Crozet constructed a drainage phon 1792 ft long of cast iron faucet pipes 3 ins bore, 9 ft long. Its summit was t above the surface of the water to be drained; and its discharge end was 20 ft low said surface, thus giving it a head of 20 ft. At the summit 570 ft from the let, was an ordinary cast iron **air-vessel** with a chamber 3 ft high and 15 ins ner diam. In the stem connecting it with the syphon was a **cut-off stopock**; and at its top was an opening 6 ins diam, closed by an air tight screw lid. each end of the syphon was a stopcock. **To start the flow** these end cks are closed, and the entire syphon and air-vessel are filled with water through e opening at top of air-vessel. This opening is then closed airtight, and the two d cocks afterwards opened; the cut-off cock remaining open. The flow then gins, and **theoretically** it should continue without diminution, except so r as the head diminishes by the lowering of the surface level of the pond. **But practice** with very long syphons this is not the case, for air begins at once disengage itself from the water, and to travel up the syphon to the summit, here it enters the air-vessel, and rising to the top of the chamber gradually ives out the water. If this is allowed to continue the air would first fill the enre chamber, and then the summit of the syphon itself, where it would act as a ad completely stopping the flow. **The water-level in the air chamber** in be detected by the sound made by tapping against the outside with a hammer.

To prevent this stoppage, the cut-off at the foot of the chamber is closed before the water is all driven out; and the lid on top being removed the chamber is refilled with water, the lid replaced, and the cut-off again opened. The flow in the meantime continues uninterrupted, but still gradually diminishing notwithstanding the refilling of the chamber; and after a number of refillings it will cease altogether, and the whole operation must then be repeated by filling the whole syphon and air chamber with water as at the start.

At Col. Crozet's syphon at first owing to the porosity of the joint-caulking, which was nothing but oakum and pitch, air entered the pipes so rapidly as to drive all the water from the chamber and thus require it to be refilled every 5 or 10 minutes; but still in two hours the syphon would run dry. The joints were then thoroughly recaulked with lead, and protected by a covering of white and red lead made into a putty with Japan varnish and boiled linseed oil. But even then the chamber had to be refilled with water about every two hours; and after six hours the syphon ran dry, and the whole had to be refilled. In this way it continued to work.

Care in making the joints air-tight, and an outside and inside coating of the pipes and air-vessel with coal pitch varnish are important precautions.

Art. 2. Approximate formulæ for the velocity of water in straight, smooth, cylindrical iron pipes, as *r o*, *v o*, *l o*, Fig. 1. Having the total head *p o*, and the length and diameter of the pipe.

$$\left.\begin{matrix}\textbf{Approx}\\ \textbf{mean vel}\\ \text{in ft per sec}\end{matrix}\right\} = \begin{matrix}\text{coefficient}\\ \textbf{m}\\ \text{as below}\end{matrix} \times \sqrt{\frac{\text{diam in ft} \times \text{total head in ft}}{\text{total length in ft} + 54 \text{ diams in ft}}}$$

Table of coefficients "m".

Diam of pipe,		m	Diam of pipe,		m
feet	inches		feet	inches	
0.1	1.2	23	1.5	18	53
0.2	2.4	30	2.0	24	57
0.3	3.6	34	2.5	30	60
0.4	4.8	37	3.0	36	62
0.5	6.0	39	3.5	42	64
0.6	7.2	42	4.0	48	66
0.7	8.4	44	5.0	60	68
0.8	9.6	46	6.0	72	70
0.9	10.8	47	7.0	84	72
1.0	12.0	48	10.0	120	77

For heads not less than 4 feet per mile, this formula gives results practically corresponding with those by Kutter's formula (p. 523) with coefficient *n* of roughness = 0.012. But slight differences, as to roughness, etc., may cause considerable variations of velocity, especially in small pipes; for, in such pipes, a given roughness of surface bears a greater proportion to the whole area of surface than in a pipe of large diameter. Extreme accuracy is not to be expected in such matters.

As in a river the velocity half way across it, and at the surface, is usually greater than at the bottom and sides, so in a pipe the velocity is greater at the center of its cross section than at its circumf. The **mean velocity** referred to in our rules is an assumed uniform one which would give the *same discharge* that the actual ununiform one does.

Hence

$$\begin{matrix}\textbf{Discharge}\\ \text{in cub ft per sec}\end{matrix} = \begin{matrix}\textbf{Mean velocity}\\ \text{in ft per sec}\end{matrix} \times \begin{matrix}\textbf{Area of cross section}\\ \text{of pipe in sq ft.}\end{matrix}$$

1 cubic foot = 7.48052 U.S. gallons
1 U. S. gallon = .13368 cubic foot = 231 cubic inches.

---

* For intermediate diameters, etc, take intermediate coefficients from the table by simple proportion.

In the case of long pipes with low heads, the sum of the velocity and entry heads is frequently so small that it may be neglected. Where this is the case, or where their amount can be approximately ascertained, **Kutter's formula**, although designed for open channels, may be used. This formula is the joint production of two eminent Swiss engineers, E. Ganguillet and W. R. Kutter, but for convenience it is usually called by the name of the latter.*

It is, properly speaking, a formula for finding the coefficient c in the well known formula,

$$\textbf{Mean velocity} = c\sqrt{\text{mean radius} \times \text{slope}}$$

$$= c\sqrt{\frac{\text{diameter}}{4} \times \text{slope}}$$

According to Kutter,

For English measure. **For metric measure.**

$$\mathbf{c} = \frac{41.6 + \frac{.00281}{\text{slope}} + \frac{1.811}{n}}{1 + \frac{\left(41.6 + \frac{.00281}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } \textit{feet}}}} = \frac{23 + \frac{.00155}{\text{slope}} + \frac{1}{n}}{1 + \frac{\left(23 + \frac{.00155}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } \textit{metres}}}}$$

See also **tables of c**, pp 566 etc.

The mean radius is the quotient, in feet or in metres, obtained by dividing the area of wet cross section, in square feet or in square metres, by the wet perimeter (see below) in feet or in metres. In pipes running full, or exactly half full, and in semicircular open channels running full, it is equal to one-fourth of the inner diameter.

The wet perimeter is the sum, *a b c o* Figs 28, 29, 30, of the lengths, *a b*, *b c*, *c o*, in feet or in metres, found by measuring (at right angles to the length of the channel) such parts of its sides and bottom as are in contact with the water. In pipes running full, it is of course equal to the inner circumference.

$$\textbf{The slope is} = \frac{\textit{friction} \text{ head } wo \text{ Fig 1,}}{\text{length of pipe measured in a straight line from end to end.}}$$

$$= \text{sine of angle } wso\text{, Fig 1.}$$

In open channels, this becomes

$$\textbf{slope} = \frac{\text{fall of water surface in any portion of the length of the channel}}{\text{length of that portion}}$$

$$= \text{fall of water surface per unit of length of channel}$$

$$= \text{sine of the angle formed between the sloping surface and the horizon.}$$

The number indicating the slope in any given case is plainly the same for English, metric and all other measures.

"**n**" is a "**coefficient of roughness**" of wet perimeter, and of course depends chiefly upon the character of the inner surface of the pipe. For iron pipes in good order and from 1 inch to 4 feet diameter, n may be taken at from .010 to .012; the lower figures being used where the pipe is in exceptionally good condition.

If the diameter, or the mean radius, is in feet, metres etc, the velocity will be in feet, metres etc, per second.

---

* See "Flow of Water," translated from Ganguillet and Kutter, by Rudolph Hering and John C. Trautwine, Jr., New York, John Wiley & Sons, 1889. $4.00.

The diameter or the slope, required for a given velocity, may be found by trial as follows: assume a diameter, or a slope, as the case may be; take the corresponding c from tables, pp 566, etc. Then say

$$\textbf{Approx Diam required}\ \text{for the given vel} = \begin{matrix}\text{mean}\\\text{radius}\end{matrix} \times 4 = \left(\frac{\text{velocity}}{c\sqrt{\text{slope}}}\right)^2 \times 4$$

$$\textbf{Approx Slope required}\ \text{for the given vel} = \left(\frac{\text{velocity}}{c\sqrt{\text{mean radius}}}\right)^2 = \left(\frac{\text{velocity}}{c\sqrt{\tfrac{1}{4}\ \text{diam}}}\right)^2.$$

With the approximate diameter (or slope) and c, thus obtained, say $v' = c\sqrt{\text{mean radius} \times \text{slope}}$. If v' is near enough to the given velocity, the assumed diameter (or slope) is the proper one. If not, try again, assuming a *greater* diameter or slope than before if v' is *less* than the required velocity, and *vice versa.*

Curves and bends do not greatly affect the discharge, so long as the total heads, and total actual lengths of the pipes remain the same; provided the tops of all the curves be kept below the hydraulic grade line; and provision be made for the escape of air accumulating at the tops of the curves.

Relation between area, velocity, and discharge.

Let q = rate of discharge (as in cubic feet per second),
v = mean velocity (as in feet per second),
a = area of cross section (as in square feet).

Then: $q = a\,v$; $v = \frac{q}{a}$; $a = \frac{q}{v}$.

Relation of discharge to diameter* and slope. If we assume velocity $= c\sqrt{\text{mean radius} \times \text{slope}}$, or $v = c\sqrt{r\,s}$ (page 523); and if the pipe be of circular cross section, we have, for the rate, Q, of discharge through a pipe of diameter, d, and area, A, of cross section, running full:—

$$Q = A\,v = \frac{\pi\,d^2}{4} \cdot c\,\frac{d^{1/2}}{2}\,s^{1/2} = \frac{c\,\pi\,d^{5/2}\,s^{1/2}}{8};$$

or: Q is proportional to the $^5/_2$ power (square root of fifth power) of the diameter, and to the $\frac{1}{2}$ power (square root) of the slope. For tables of fifth powers, and of square roots of fifth powers, see pp 67–69.

Effect of resistances.

The pressure head of running water, upon any point in a pipe between the orifice and the reservoir, is:

$$= \left\{\begin{matrix}\text{the total}\\\text{head on}\\\text{that point}\end{matrix}\right\} \text{minus} \left\{\begin{matrix}\text{the head}\\\text{due to the}\\\text{vel at}\\\text{that point}\end{matrix} + \begin{matrix}\text{the}\\\text{entry}\\\text{head}\end{matrix} + \begin{matrix}\text{the head consumed}\\\text{in overcoming re-}\\\text{sistances in the pipe}\\\text{between the reservoir}\\\text{and the point.}\end{matrix}\right\}$$

Thus, at the point 6, in the pipe, $l\,o$, Fig 1, the pressure head is $h = (8\,6) = (1\,6) - [(1\,2) + (2\,3)]$; where $(1\,2) = i\,s$ = the sum of the velocity and entry heads. At 4, in the pipe $r\,o$, $h = (3\,4) = (1\,4) - [(1\,2) + (2\,3)]$.

In Fig 1, let the straight line, $s\,o$, represent the actual length of the pipe, whether straight, bent or curved, etc.; and $s\,v$ the sum of the resistances (supposed to be uniformly distributed) within the pipe. Then, the angle, $s\,o\,v$, is called the hydraulic gradient, and sine $s\,o\,v = s\,v \div s\,o$.

In the vertical pipe, $v\,o$, Fig 1 A, the pressure, at q, is $= g\,d$.

---

*Diameter = 4 × mean radius, or $d = 4\,r$ (p 523).

ABLE OF WEIGHT OF WATER CONTAINED IN ONE FOOT LENGTH OF PIPES OF DIFFERENT BORES.

(Original.)

ater at maximum density, 62.425 lbs. per cubic foot = 1 gram per cubic centi-er; corresponding to a temperature of 4° Centigrade = 39.2° Fahrenheit.

Weight = 0.340475558 × square of bore in inches.

re. s.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.
	0.005320	6	12.25712	23	180.1116	62	1308.788
	0.021280	6¼	13.29983	24	196.1139	63	1351.347
	0.047879	6½	14.38509	25	212.7972	64	1394.588
	0.085119	6¾	15.51292	26	230.1615	65	1438.509
	0.132998	7	16.68330	27	248.2067	66	1483.112
	0.191518	7¼	17.89625	28	266.9328	67	1528.395
	0.260677	7½	19.15175	29	286.3399	68	1574.359
	0.340476	7¾	20.44981	30	306.4280	69	1621.004
	0.430914	8	21.79044	31	327.1970	70	1668.330
	0.531993	8¼	23.17362	32	348.6470	71	1716.337
	0.643712	8½	24.59936	33	370.7779	72	1765.025
	0.766070	8¾	26.06766	34	393.5897	73	1814.394
	0.899068	9	27.57852	35	417.0826	74	1864.444
	1.042706	9¼	29.13194	36	441.2563	75	1915.175
	1.196984	9½	30.72792	37	466.1110	76	1966.587
	1.361902	9¾	32.36646	38	491.6467	77	2018.680
	1.537460	10	34.04756	39	517.8633	78	2071.453
	1.723658	10½	37.53748	40	544.7609	79	2124.908
	1.920495	11	41.19754	41	572.3394	80	2179.044
	2.127972	11½	45.02789	42	600.5989	81	2233.860
	2.346089	12	49.02848	43	629.5393	82	2289.358
	2.574846	12½	53.19931	44	659.1607	83	2345.536
	2.814243	13	57.54037	45	689.4630	84	2402.396
	3.064280	13½	62.05167	46	720.4463	85	2459.936
	3.324957	14	66.73321	47	752.1105	86	2518.157
	3.596273	14½	71.58499	48	784.4557	87	2577.060
	3.878229	15	76.60700	49	817.4818	88	2636.643
	4.170826	15½	81.79925	50	851.1889	89	2696.907
	4.474062	16	87.16174	51	885.5769	90	2757.852
	4.787938	16½	92.69447	52	920.6459	91	2819.478
	5.112453	17	98.39744	53	956.3958	92	2881.785
	5.447609	17½	104.27064	54	992.8267	93	2944.773
	6.149840	18	110.31408	55	1029.9386	94	3008.442
	6.894630	18½	116.52776	56	1067.7314	95	3072.792
	7.681980	19	122.91168	57	1106.2051	96	3137.823
	8.511889	19½	129.46583	58	1145.3598	97	3203.535
	9.384358	20	136.19022	59	1185.1954	98	3269.927
	10.299386	21	150.14972	60	1225.7120	99	3337.001
	11.256973	22	164.79017	61	1266.9096	100	3404.756

The weight of water in a given length (as one foot) of any pipe or other cular cylinder **is in proportion to the square of the bore** or ner diameter. Hence the weight of water in 1 foot length of any cylinder of her diameter than those in the table can be found by multiplying that for a 1 ch pipe, 0.340475558, by the square of the inner diameter of the given cylinder in ches. Thus, for a cylinder 120 inches diameter: diameter^2 = 120^2 = 14400, and eight of water in 1 foot depth = 0.340475558 × 14400 = 4902.848 lbs. Or, weight r 120 ins. diam. = 100 × weight for 12 ins. diam. = 100 × 49.02848 = 4902.848 lbs. milarly, $\left(\frac{7}{16}\right)^2 = \frac{49}{256}$ = 0.191406, and 0.340475558 × 0.191406 = 0.065169 lb. = eight in 1 foot of $\frac{7}{16}$ inch pipe. Here, also, $\frac{7}{16}$ = *half* of $\frac{7}{8}$; hence, weight for inch = one-*fourth* of weight for $\frac{7}{8}$ inch = one-fourth of 0.260677 = 0.065169.

Weight of one square inch of water 1 foot high, at 62.425 lbs. er cubic foot = 62.425 ÷ 144 = 0.433507 lb.

---

* Actual. See nominal and actual diameters, foot note, p 526.

TABLE 3. Areas and Contents of Pipes; and square roots of Diams. (Original.) **Correct.**

* Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	* Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	* Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.
¼	.0208	.0003	.145	4.	.3333	.0873	.579	15.	1.250	1.227	1.118
5-16	.0260	.0005	.161	⅛	.3438	.0928	.586	¼	1.271	1.268	1.127
⅜	.0313	.0008	.177	¼	.3542	.0985	.596	½	1.292	1.310	1.136
7-16	.0365	.0010	.191	⅜	.3646	.1040	.604	¾	1.313	1.353	1.146
½	.0417	.0014	.204	½	.3750	.1104	.612	16.	1.333	1.396	1.155
9-16	.0469	.0017	.217	⅝	.3854	.1167	.621	¼	1.354	1.440	1.163
⅝	.0521	.0021	.228	¾	.3958	.1231	.629	½	1.375	1.485	1.172
11-16	.0573	.0026	.239	⅞	.4063	.1296	.637	¾	1.396	1.530	1.181
¾	.0625	.0031	.250	5.	.4167	.1363	.645	17.	1.417	1.576	1.190
13-16	.0677	.0036	.260	⅛	.4271	.1433	.653	¼	1.437	1.623	1.199
⅞	.0729	.0042	.270	¼	.4375	.1503	.660	½	1.458	1.670	1.207
15-16	.0781	.0048	.280	⅜	.4479	.1576	.669	¾	1.479	1.718	1.216
1.	.0833	.0055	.289	½	.4583	.1650	.677	18.	1.5	1.767	1.224
1-16	.0885	.0062	.297	⅝	.4688	.1726	.685	½	1.542	1.867	1.241
⅛	.0938	.0069	.306	¾	.4792	.1803	.693	19.	1.583	1.969	1.258
3-16	.0990	.0077	.314	⅞	.4896	.1878	.700	½	1.625	2.074	1.274
¼	.1042	.0085	.322	6.	.5	.1964	.707	20.	1.667	2.182	1.291
5-16	.1094	.0094	.330	¼	.5208	.2131	.722	½	1.708	2.292	1.307
⅜	.1146	.0103	.338	½	.5417	.2304	.736	21.	1.750	2.405	1.323
7-16	.1198	.0113	.346	¾	.5625	.2485	.750	½	1.791	2.521	1.338
½	.1250	.0123	.354	7.	.5833	.2673	.764	22.	1.833	2.640	1.354
9-16	.1302	.0133	.361	¼	.6042	.2867	.777	½	1.875	2.761	1.369
⅝	.1354	.0144	.368	½	.6250	.3068	.791	23.	1.917	2.885	1.384
11-16	.1406	.0155	.375	¾	.6458	.3276	.803	½	1.958	3.012	1.399
¾	.1458	.0167	.382	8.	.6667	.3491	.817	24.	2.000	3.142	1.414
13-16	.1510	.0179	.389	¼	.6875	.3712	.829	25.	2.083	3.409	1.443
⅞	.1563	.0192	.395	½	.7083	.3941	.841	26.	2.166	3.687	1.472
15-16	.1615	.0205	.402	¾	.7292	.4176	.854	27.	2.250	3.976	1.500
2.	.1667	.0218	.408	9.	.75	.4418	.866	28.	2 333	4.276	1.528
1-16	.1719	.0232	.414	¼	.7708	.4667	.879	29.	2 416	4.587	1.555
⅛	.1771	.0246	.420	½	.7917	.4922	.890	30.	2.500	4.909	1.581
3-16	.1823	.0260	.427	¾	.8125	.5185	.902	31.	2.584	5.241	1.607
¼	.1875	.0276	.433	10.	.8333	.5454	.913	32.	2.666	5.585	1.633
5-16	.1927	.0291	.440	¼	.8542	.5730	.924	33.	2.750	5.940	1.658
⅜	.1979	.0308	.445	½	.8750	.6013	.935	34.	2.834	6.305	1.683
7-16	.2031	.0324	.451	¾	.8958	.6303	.946	35.	2.916	6.681	1.708
½	.2083	.0341	.457	11.	.9167	.6600	.957	36.	3.000	7.069	1.732
9-16	.2135	.0358	.462	¼	.9375	.6903	.968	38.	3.166	7.876	1.779
⅝	.2188	.0375	.467	½	.9583	.7213	.979	40.	3.333	8.727	1.826
11-16	.2240	.0394	.473	¾	.9792	.7530	.990	42.	3.500	9.621	1.871
¾	.2292	.0412	.478	12.	1.	.7854	1.000	44.	3.666	10.56	1.914
13-16	.2344	.0432	.484	¼	1.021	.8184	1.010	48.	4.000	12.57	2.000
⅞	.2396	.0451	.489	½	1.042	.8522	1.020	54.	4.500	15.90	2.121
15-16	.2448	.0471	.495	¾	1.063	.8866	1.031	60.	5.000	19 63	2.236
3.	.2500	.0491	.500	13.	1.083	.9218	1.041	66.	5.500	23.76	2.345
⅛	.2604	.0532	.510	¼	1.104	.9576	1.051	72.	6.000	28.27	2.449
¼	.2708	.0576	.520	½	1.125	.9940	1.060	78.	6.500	33.18	2.550
⅜	.2813	.0621	.530	¾	1.146	1.031	1.070	84.	7.000	38.48	2.646
½	.2917	.0668	.540	14.	1.167	1.069	1.080	90.	7.500	44.18	2.739
⅝	.3021	.0716	.550	¼	1.187	1.108	1.090	96.	8.000	50.27	2.828
¾	.3125	.0767	.560	½	1.208	1.147	1.099				
⅞	.3229	.0819	.570	¾	1.229	1.187	1.110				

* **Caution.** In the tables on pages 525 and 526, the diameters or bores are the *actual* ones, as measured in inches. Wrought-iron steam, gas, and water pipes are commonly designated by fictitious or "nominal" diameters, which are mere arbitrary names for the pipes. In the smaller sizes especially, the use of these nominal diameters tends to mislead. Thus, the pipe whose "*nominal*" inner diameter is *one-eighth* inch, has an *actual* inner diameter of *full quarter* inch.

Art. 3. To find the total head required for a **given velocity,** or **given discharge,** through a straight, smooth, cylindrical iron pipe of known diam and length.

If the *discharge* is given, first find

$$\text{mean velocity in feet per second} = \frac{\text{discharge in cubic feet per second}}{\text{area of cross section of pipe in square feet}}$$

Then

$$\text{approx} \sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 54 \text{ diams}}} = \frac{\text{mean velocity in feet per second}}{\text{the proper divisor as follows}}$$

diam of pipe in ft	.05	.10	.50	1	1.5	2	3	4
divisor	40	43	46	48	51	54	58	61

(for intermediate diams, take intermediate divisors by guess.)

From table Art 2, take the coefficient m corresponding to this value of $\sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 54 \text{ diams}}}$, and to the given diam. Then

$$\textbf{Total head} \text{ in feet} = \frac{\text{mean velocity}^2 \text{ in feet per sec} \times (\text{length in ft} + 54 \text{ diams in ft})}{m^2 \times \text{diam in feet}}$$

To find the Friction head. Weisbach's formula.

$$\text{Friction head in feet} = \left(.0144 + \frac{.01716}{\sqrt{\text{vel in ft per sec}}} \right) \times \frac{\text{Length in feet}}{\text{Diam in feet}} \times \frac{\text{Vel}^2 \text{ in ft per sec}}{64.4}$$

For the total head, we have only to add together, the **friction head** so found, the **velocity head,** taken from the next table, or from Table 10, opposite the given velocity, and the **entry head** (= say half the velocity head). The sum of the velocity head and entry head rarely amounts to a foot.

TABLE 4 Of the vel, and discharge of water through straight, smooth, cylindrical cast-iron pipes; with the friction head required for each 100 feet in length; and also the velocity head. Calculated by means of Weisbach's formula, by James Thompson, A M; and George Fuller, C E, Belfast, Ireland. The vel head remains the same for any *length* of pipe; being dependent only on the *velocity* of the water in the pipe.

The entry head is equal to about half the vel head.

TABLE 4

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 3		3½		4		4½		5	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.659	5.89	.565	8.02	.494	10.4	.439	13.2	.395	16.3
2.2	.075	.780	6.48	.669	8.82	.585	11.5	.520	14.6	.468	18.0
2.4	.090	.911	7.07	.781	9.62	.683	12.5	.607	15.9	.547	19.6
2.6	.105	1.05	7.65	.901	10.4	.788	13.6	.701	17.2	.631	21.3
2.8	.122	1.20	8.24	1.03	11.2	.900	14.6	.800	18.5	.720	22.9
3.0	.140	1.35	8.83	1.16	12.0	1.02	15.7	.905	19.8	.815	24.5
3.2	.160	1.52	9.42	1.31	12.8	1.14	16.7	1.02	21.2	.915	26.2
3.4	.180	1.70	10.0	1.46	13.6	1.27	17.8	1.13	22.5	1.02	27.8
3.6	.202	1.89	10.6	1.62	14.4	1.41	18.8	1.26	23.8	1.13	29.4
3.8	.225	2.08	11.2	1.78	15.2	1.56	19.9	1.39	25.2	1.25	31.0
4.0	.250	2.28	11.8	1.96	16.0	1.71	20.9	1.52	26.5	1.37	32.7
4.2	.275	2.49	12.3	2.14	16.8	1.87	22.0	1.66	27.8	1.50	34.3
4.4	.302	2.71	12.9	2.33	17.6	2.03	23.0	1.81	29.1	1.63	36.0
4.6	.330	2.94	13.5	2.52	18.4	2.21	24.0	1.96	30.4	1.76	37.6
4.8	.360	3.18	14.1	2.72	19.2	2.38	25.1	2.12	31.8	1.91	39.2
5.0	.390	3.43	14.7	2.94	20.0	2.57	26.2	2.28	33.1	2.05	40.9
5.2	.422	3.68	15.3	3.15	20.8	2.76	27.2	2.45	34.4	2.21	42.5
5.4	.455	3.94	15.9	3.38	21.6	2.96	28.2	2.63	35.8	2.37	44.2
5.6	.490	4.22	16.5	3.61	22.4	3.16	29.3	2.81	37.1	2.53	45.8
5.8	.525	4.50	17.1	3.85	23.2	3.37	30.3	3.00	38.4	2.70	47.4
6.0	.562	4.78	17.7	4.10	24.0	3.59	31.4	3.19	39.7	2.87	49.1
6.2	.600	5.08	18.2	4.36	24.8	3.81	32.4	3.39	41.0	3.05	50.7
6.4	.640	5.39	18.8	4.62	25.6	4.04	33.5	3.59	42.4	3.23	52.3
6.6	.680	5.70	19.4	4.89	26.4	4.28	34.5	3.80	43.7	3.42	54.0
6.8	.722	6.02	20.0	5.16	27.3	4.52	35.6	4.01	45.0	3.61	55.6
7.0	.765	6.35	20.6	5.45	28.0	4.77	36.6	4.24	46.4	3.81	57.2

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 6		7		8		9		10	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.329	23.5	.282	32.0	.247	41.9	.220	53.0	.198	65.4
2.2	.075	.390	25.9	.334	35.3	.293	46.1	.260	58.3	.234	72.0
2.4	.090	.456	28.2	.390	38.5	.342	50.2	.304	63.6	.273	78.5
2.6	.105	.526	30.6	.450	41.7	.394	54.4	.350	68.9	.315	85.1
2.8	.122	.600	32.9	.514	44.9	.450	58.6	.400	74.2	.360	91.6
3.0	.140	.679	35.3	.582	48.1	.509	62.8	.453	79.5	.407	98.2
3.2	.160	.763	37.7	.654	51.3	.572	67.0	.508	84.8	.458	105
3.4	.180	.851	40 0	.729	54.5	.638	71.2	.567	90.1	.510	111
3.6	.202	.943	42.4	.808	57.7	.707	75.4	.629	95.4	.566	118
3.8	.225	1.04	44.7	.892	60.9	.780	79.6	.693	101	.624	124
4.0	.250	1.14	47.1	.979	64.1	.856	83.7	.761	106	.685	131
4.2	.275	1.25	49.5	1.07	67.3	.935	87.9	.832	111	.748	137
4.4	.302	1.35	51.8	1.16	70.5	1.02	92.1	.905	116	.814	144
4.6	.330	1.47	54.1	1.26	73.7	1.10	96.3	.981	122	.883	150
4.8	.360	1.59	56.5	1.36	76.9	1.19	100	1.06	127	.954	157
5.0	.390	1.71	58.9	1.47	80.2	1.28	105	1.14	132	1.03	163
5.2	.422	1.84	61.2	1.58	83.3	1.38	109	1.23	138	1.10	170
5.4	.455	1.97	63.6	1.69	86.6	1.48	113	1.31	143	1.18	177
5.6	.490	2.11	65.9	1.81	89.8	1.58	117	1.40	148	1.26	183
5.8	.525	2.25	68.3	1.93	93.0	1.68	121	1.50	154	1.35	190
6.0	.562	2.39	70.7	2.05	96.2	1.79	125	1.59	159	1.43	196
6.2	.600	2.54	73.0	2.18	99.4	1.90	130	1.69	164	1.52	203
6.4	.640	2.69	75.4	2.31	102	2.02	134	1.79	169	1.61	209
6.6	.680	2.85	77.7	2.44	106	2.14	138	1.90	175	1.71	216
6.8	.722	3.01	80.1	2.58	109	2.26	142	2.01	180	1.81	222
7.0	.765	3.18	82.4	2.72	112	2.38	146	2.12	185	1.90	229

TABLE 4 —(Continued.)

Vel-head in Feet.	Diam. in Inches.									
	11		12		13		14		15	
	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
.062	.180	79.2	.165	94.2	.152	110	.141	128	.132	147
.075	.213	87.1	.195	103	.180	121	.167	141	.156	162
.090	.248	95.0	.228	113	.210	133	.195	154	.182	176
.105	.287	103	.263	122	.242	144	.225	167	.210	191
.122	.327	111	.300	132	.277	156	.257	179	.240	206
.140	.370	119	.339	141	.813	166	.291	192	.271	221
.160	.416	127	.381	151	.352	177	.327	205	.305	235
.180	.464	134	.425	160	.393	188	.365	218	.340	250
.202	.514	142	.472	169	.435	199	.404	231	.377	265
.225	.567	150	.520	179	.480	210	.446	243	.416	280
.250	.623	158	.571	188	.527	221	.489	256	.457	294
.275	.680	166	.624	198	.576	232	.534	269	.499	309
.302	.740	174	.679	207	.626	243	.582	282	.543	324
.330	.803	182	.736	217	.679	254	.631	295	.589	339
.360	.867	190	.795	226	.734	265	.682	308	.636	353
.390	.935	198	.857	235	.791	276	.734	321	.685	368
.422	1.00	206	.920	245	.850	287	.789	338	.736	383
.455	1.07	214	.986	254	.910	298	.845	346	.789	397
.490	1.15	222	1.05	264	.973	309	.903	359	.843	412
.525	1.22	229	1.12	273	1.04	321	.964	372	.899	427
.562	1.30	237	1.19	283	1.10	332	1.02	385	.957	442
.600	1.38	245	1.27	292	1.17	343	1.09	397	1.01	456
.640	1.47	253	1.35	301	1.24	354	1.15	410	1.08	471
.680	1.55	261	1.42	311	1.31	365	1.22	423	1.14	486
.722	1.64	269	1.50	320	1 39	376	1.29	436	1.20	500
.765	1.73	277	1.59	330	1 46	387	1.36	449	1.27	515

Vel-head in Feet.	Diam. in Inches.									
	16		17		18		19		20	
	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
.062	.123	167	.116	189	.110	212	.104	236	.099	262
.075	.146	184	.138	208	.130	233	.123	260	.117	288
.090	.171	201	.161	227	.152	254	.144	283	.137	314
.105	.197	218	.185	246	.175	275	.166	307	.158	340
.122	.225	234	.212	265	.200	297	.189	331	.180	366
.140	.255	251	.240	284	.226	318	.214	354	.204	393
.160	.286	268	.269	302	.254	339	.241	378	.229	419
.180	.319	284	.300	321	.283	360	.269	401	.255	445
.202	.354	301	.333	340	.314	382	.298	425	.283	471
.225	.390	318	.367	359	.347	403	.328	449	.312	497
.250	.428	335	.403	378	.380	424	.360	472	.342	523
.275	.468	352	.440	397	.416	445	.394	496	.374	550
.302	.509	368	.479	416	.452	466	.429	519	.407	576
.330	.552	385	.519	435	.490	488	.465	543	.441	602
.360	.596	402	.561	454	.530	509	.502	567	.477	628
.390	.642	419	.605	473	.571	530	.541	590	.514	654
.422	.690	435	.650	492	.614	551	.581	614	.552	680
.455	.740	452	.696	511	.657	572	.623	638	.592	707
.490	.791	469	.744	529	.703	594	.666	661	.632	733
.525	.843	486	.793	548	.749	615	.710	685	.674	759
.562	.897	502	.844	567	.798	636	.755	709	.718	785
.600	.953	519	.897	586	.847	657	.802	732	.762	811
.640	1.01	536	.951	605	.898	678	.851	756	.808	838
.680	1.07	553	1.01	624	.950	700	.900	780	.855	864
.722	1.13	569	1.06	643	1.00	721	.951	803	.904	896
.765	1.19	586	1.12	662	1.06	742	1.00	827	.953	916

TABLE 4 —(Continued.)

Vel. in Feet perSec.	Vel-head in Feet.	Diam. in Inches.									
		22		24		26		28		30	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.090	316	.082	377	.076	442	.070	513	.066	589
2.2	.075	.106	348	.097	414	.090	486	.083	564	.078	648
2.4	.090	.124	380	.114	452	.105	531	.097	616	.091	707
2.6	.105	.143	412	.131	490	.121	575	.112	667	.105	766
2.8	.122	.164	443	.150	528	.138	619	.128	718	.120	824
3.0	.140	.185	475	.170	565	.157	663	.145	770	.136	883
3.2	.160	.208	507	.191	603	.176	708	.163	821	.152	942
3.4	.180	.232	538	.213	641	.196	752	.182	872	.170	1001
3.6	.202	.257	570	.236	678	.218	796	.202	923	.189	1060
3.8	.225	.284	601	.260	716	.240	840	.223	974	.208	1119
4.0	.250	.311	633	.285	754	.263	885	.244	1026	.228	1178
4.2	.275	.340	665	.312	791	.288	929	.267	1077	.249	1237
4.4	.302	.370	697	.339	829	.313	973	.290	1129	.271	1296
4.6	.330	.401	728	.368	867	.339	1017	.315	1180	.294	1355
4.8	.360	.434	760	.397	905	.367	1062	.341	1231	.318	1414
5.0	.390	.467	792	.428	942	.395	1106	.367	1283	.343	1472
5.2	.422	.502	823	.460	980	.425	1150	.394	1334	.368	1531
5.4	.455	.538	855	.493	1018	.455	1194	.423	1385	.394	1590
5.6	.490	.575	887	.527	1055	.486	1239	.452	1437	.422	1649
5.8	.525	.613	918	.562	1093	.519	1283	.482	1488	.450	1708
6.0	.562	.652	950	.598	1131	.552	1327	.513	1539	.478	1767
6.2	.600	.693	982	.635	1168	.586	1371	.544	1590	.508	1826
6.4	.640	.735	1013	.673	1206	.622	1416	.577	1642	.539	1885
6.6	.680	.778	1045	.713	1244	.658	1460	.611	1693	.570	1948
6.8	.722	.821	1077	.753	1282	.695	1504	.645	1744	.602	2008
7.0	.765	.867	1109	.794	1319	.733	1548	.681	1796	.635	2061

Art. 4 To find the discharge through a compound pipe *v o*, Fig 1 H, composed of any number of pipes, *a*, *b*, *c*, *z*, of **different diameters,** which decrease from the reservoir toward the outflow *o*.

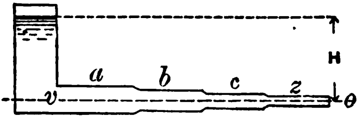


Fig.1 H

First find what part of the total head H is employed in forcing the water *through the last pipe z alone*, thus:

Let L *a*, L *b*, L *c*, and L *z*, be the lengths in ft of the pipes *a*, *b*, *c*, and *z*, respectively; D *a*, D *b*, D *c*, and D *z* their diameters in ft; and A *a*, A *b*, A *c*, and A *z* the areas of their cross sections in sq ft. Then

$$\left.\begin{array}{r}\text{The head in}\\ \text{ft employed}\\ \text{in forcing the}\\ \text{water through}\\ \text{the last pipe}\\ z\text{ alone}\end{array}\right\} = \frac{\text{The } \textit{total} \text{ head H in feet}}{1+\left[\dfrac{\mathrm{A}\,z^2\times \mathrm{D}\,z}{\mathrm{L}\,z+(54\times \mathrm{D}\,z)}\times\left(\dfrac{\mathrm{L}a+(54\times \mathrm{D}a)}{\mathrm{A}\,a^2\times \mathrm{D}\,a}+\dfrac{\mathrm{L}\,b+(54\times \mathrm{D}\,b)}{\mathrm{A}\,b^2\times \mathrm{D}\,b}+\dfrac{\mathrm{L}\,c+(54\times \mathrm{D}\,c)}{\mathrm{A}\,c^2\times \mathrm{D}c}\right)\right]}$$

However many divisions the pipe may have, proceed in the same way as above, using $\dfrac{\text{Area}^2\times\text{Diam}}{\text{Length}+54\text{ Diams}}$ for the *last* or narrowest division; and $\dfrac{\text{Length}+54\text{ Diams}}{\text{Area}^2\times\text{Diam}}$ for each of the *others*.

Then, by the formulæ, Art. 2, find the velocity in ft per second, and discharge in cub ft per second, of the last pipe *z*, using its actual diameter, length, and cross sectional area, and the head just found. Said *discharge* is evidently the discharge for the compound pipe.

For the *velocity in any portion*, as *b*, say

$$\begin{array}{c}\text{area of cross section}\\ \text{of the given portion}\end{array} : \begin{array}{c}\text{area of cross}\\ \text{section of } z\end{array} :: \begin{array}{c}\text{velocity}\\ \text{in } z\end{array} : \begin{array}{c}\text{velocity in the}\\ \text{given portion.}\end{array}$$

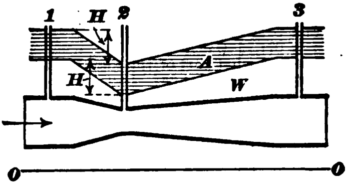
For the above rule and formula, we are indebted to Mr. Howard Murphy, C E, of Phila; and for the opportunity of testing it experimentally, to Messrs Morris Tasker & Co, Limited, Pascal Iron Works, Philadelphia.

Art. 4 a.[*] **The Venturi Meter** is designed for the measurement of the flow of liquids in pipes of large dimensions, running full.

The meter proper, patented by Clemens Herschel, consists essentially of a mere constriction in the area of cross-section of the pipe, with openings in the pipe opposite its normal and its constricted diameters, for measuring, by piezometers or pressure-gauges, the pressures at those points; while **the register**, patented by Messrs. Frederick N. Connet and Walter W. Jackson, is an elaborate mechanism, provided with clock-work and dials.

Theory.[†] Let Figs. 1 to 3 represent a Venturi meter tube, with three piezometers in place, viz.: No. 1, over the tube up-stream from the constriction; No. 2, over the constriction itself; and No. 3, over the tube down-stream from the constriction. Let the unshaded area *W* in Figs. 1 to 3, represent the depths at which the water stands above any assumed horizontal datum plane *O-O;* and let the shaded area *A* represent the uniform pressure of the atmosphere, which, for convenience, we may suppose to be converted into some liquid of the specific gravity of water, but distinguishable, by its appearance, from the water.

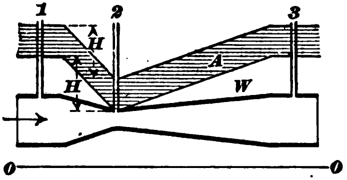
FIG. 1.



The vertical distance, between the upper boundary of this latter area and any given point in the tube, represents the combined pressure of air and water at such point.

The velocities in the meter tube, at any instant, are of necessity inversely proportional to the areas of cross section; and, as the heads corresponding to the several velocities are proportional to the squares of those velocities, the remaining or pressure heads must vary also, the smallest or lowest pressure head standing over the throat, where the velocity is greatest.

FIG. 2.



The increase of velocity, acquired by the fluid in passing from section 1 to section 2, is again given up in passing from section 2 to section 3; and, in the case of a perfect fluid, the pressure lost between sections 1 and 2 would be perfectly restored in passing from section 2 to section 3. In practice, a small total loss occurs. This loss is greater with high than with low velocities.

For a given head in piezometer No. 1 and given diameter of pipe at section 1, the expenditure of head in velocity between sections 1 and 2 increases as the area of the throat is diminished and as the throat velocity is thereby increased.[‡] In Fig. 2 is shown the case where all of the water head above the top of the throat is required to maintain the velocity through the throat.

In Figs. 1 and 2 the head, *H*, expended in the increase of velocity between sections 1 and 2 is represented by the difference in level between the tops of the two water columns 1 and 2, or between the tops of the two corresponding air columns. In Fig. 2 this difference is equal to the *total* vertical height of the water column at section 1 above the top of the throat at section 2.

---

[*] Abridged from a description prepared by the writer as Chairman of a Committee of the Franklin Institute. *Journal of the Franklin Institute*, February, 1899.

[†] The Venturi meter, apart from its merits as a measuring device, embodies important hydraulic principles. Hence its theory is here stated more fully than would otherwise be necessary.

[‡] In a given Venturi tube the pressure and velocity at the throat may be varied also by modifying those at sectons 1 and 3, as by regulating the openings of the valves of influx to and of efflux from the meter tube, by changing the total head on the system, etc.

now (Fig. 3) the throat sec- be still further reduced other conditions remain- s before), the throat veloc- ill thereby be still further ased; for the *total* pressure able for increase of veloc- etween sections 1 and 2 con- not merely in the depth of above the tube, but also e *atmospheric pressure*, rep- ted by the shaded area *A* the water *W*.

FIG. 3.

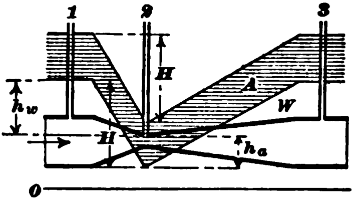
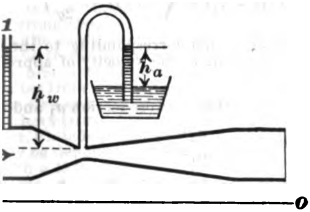


Fig. 3 all the water has dis- red from piezometer 2, and a portion of the liquid senting the air has also peared, leaving only a portion of the latter to represent such pressure as remains in the throat. In other words, the pressure within the throat is less than the atmospheric pressure.

Fig. 3, the loss of head, due to increase of velocity between sections 1 and 2, = $h_w + h_a$ = the entire available head of water, h_w, plus a portion, h_a, of the spheric pressure. The latter portion, h_a, is frequently called "the m."

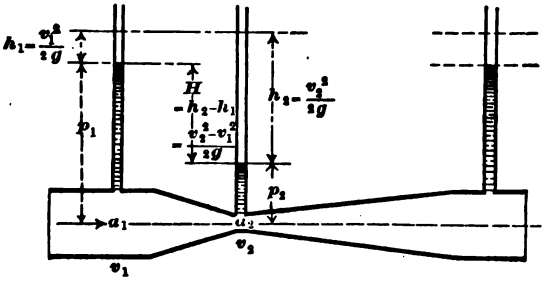
top of the water column having now disappeared below the top of the t, it is no longer feasible to ascertain the loss of head by taking the differ- between the levels of the water surfaces in piezometers 1 and 2. The degree of "vacuum" may be found, as shown in Fig. 4, by using, in place of the piezometers, a glass tube bent over and led *downward* into an open vessel containing water or mercury. The height to which the water (or the mercury, converted into feet of water) rises in this tube, shows the extent of the vacuum, or the portion, h_a, of the air pressure which has been called into service in producing the high velocity through the throat. By adding this to h_w, we obtain, as above, the total loss of head *H* between sections 1 and 2.

FIG. 4.



en the reduction of area at the throat has proceeded so far that the entire ble pressure of water and air at section 1 is required, in order to main-

FIG. 5.



e corresponding velocity through the throat (*i. e.*, when the line repre- g the upper surface of the air falls to the level of the top of the throat), no further increase of throat velocity can be secured (with a given total head over section 1) by still further narrowing the throat. If the throat is further narrowed, the velocity through it will remain the same; and, the rate of discharge being thus diminished, the velocity through section 1 will be necessarily reduced. In other words, throttling begins.

Let v_1 be the velocity in section 1, above the throat, and v_2 the "throat velocity," or velocity in the throat or section 2.

Referring to Fig. 5, the velocity head at section 1, measured from an assumed datum represented by the upper horizontal lines, is

$$h_1 = \frac{v_1^2}{2g},$$

and that at section 2 is

$$h_2 = \frac{v_2^2}{2g}.$$

Neglecting resistances to flow, the loss of head, between sections 1 and 2, or "the head on the Venturi," is equal to the increase in the velocity head, or to the loss in pressure, between a_1, and a_2, or

$$H = h_2 - h_1 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{v_2^2 - v_1^2}{2g} = p_1 - p_2 *$$

Hence,

$$h_2 = \frac{v_2^2}{2g} = \mathrm{H} + h_1$$

and **throat velocity** $= v_2 = \sqrt{2g\,(\mathrm{H} + h_1)} = \sqrt{2g\left(H + \frac{v_1^2}{2g}\right)}.$

In other words, the velocity at the throat is that corresponding to the "head H on the Venturi," plus the head corresponding to the velocity of approach v_1 in section 1.

But, since the velocities are inversely as the areas of cross-section a_1 and a_2,

$$v_1 = \frac{a_2}{a_1} v_2, \qquad \text{and } v_1^2 = \frac{a_2^2}{a_1^2} v_2^2,$$

$$H = \frac{v_2^2 - v_1^2}{2g} = \frac{v_2^2 - \frac{a_2^2}{a_1^2} v_2^2}{2g} = \frac{\left(1 - \frac{a_2^2}{a_1^2}\right) v_2^2}{2g} = \frac{\frac{a_1^2 - a_2^2}{a_1^2} v_2^2}{2g},$$

$$v_2^2 = \frac{2gHa_1^2}{a_1^2 - a_2^2},$$

and **throat velocity** $= v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gH}.$

The ratio

$$\frac{a_2}{a_1},$$

between the area a_2 of cross-section at the throat, and that, a_1, at the upper end of the up-stream cone, is called the **throat ratio.** For a ratio of 1 : 9 we have

$$\frac{a_1}{\sqrt{a_1^2 - a_2^2}} = \frac{9}{\sqrt{9^2 - 1^2}} = \frac{9}{\sqrt{80}} = \sqrt{\frac{81}{80}} = 1.0062,$$

or

$$v_2 = 1.0062\sqrt{2gH}.$$

The Venturi tube, for pipes not over 60 inches in diameter, is formed of several short sections of cast iron pipe, having the required taper, and furned with flanges, by means of which the sections are bolted together to form two truncated cones required.

---

* By Bernouilli's theorem, $p_1 + h_1 = p_2 + h_2$.

ı the smaller sizes, the shorter cone is generally in one section and the ger cone in two or more sections.

he throat section is generally made in a separate piece, and is either made ›ronze or lined with that metal.

he ends of the Venturi tube are furnished with either bell, spigot, or flanged s, according to the character of the pipe in which the tube is to be used.

'or still larger streams, such as those in masonry conduits or riveted nes, the Venturi tube may be made of wooden staves, sheet steel, cement crete, brick or other suitable material, metal being used for the throat piece where required by the pressure.

'he throat piece is surrounded by an annular chamber called the **pressure chamber,** which communicates with the interior of the throat by means ›everal holes drilled radially through the walls of the latter at equal or rly equal distances around the circumference.

similar pressure chamber is provided at the larger end of the short cone for erving the pressure in the normal section up-stream from the throat; and, t is desired to ascertain the final loss of head due to the passage of the water ough the Venturi, a similar chamber must be provided at the larger end of longer or down-stream cone.

n designating the size of the meter, the diameter of the pipe of which ›rms a part is used, and not the throat diameter. Thus, a meter for use in a ıch pipe is called a 6-inch meter.

'he register gives periodic registrations, usually every ten minutes, in ch the head $H = h_2 - h_1$, existing at the instant of registry, is recorded in ns of the total discharge in cubic feet since the last registry and as an inse in the total number of cubic feet registered. In other words, the registry olves the assumption that the average velocity, during the period between two istrations, is equal to the velocity at the instant of the following registration.

he register may be placed at a considerable distance (not exceeding, say, 500) from the Venturi tube. It must be placed at such a depth below the raulic grade line that the pressures existing in the Venturi tube shall at all es be transmitted to the register.

he pipe lines, connecting the Venturi with the register, must be covered, and ıelter from weather and frost must be provided for the register.

'he size and cost of the register are independent of the size of the Venturi.

ehavior. From experiments by Mr. Herschel,[*] [†] [¶] by the Bureau of ter, Philadelphia,[†] and by others,[†] it appears that the Venturi meter may ordiily be depended upon to give results within 3 per cent. of the true discharge.

'ith a 48 inch Venturi, Mr. Herschel [¶] found a total **loss of head,** due to passage of the water through the Venturi tube, of about 10.6 per cent. of the d H on the Venturi. With two 54 inch Venturis, Professors Marx, Wing, and kins [§] [†] found a loss of 14.9 per cent., part of which, no doubt, was due to presence of a 42 inch gate valve in the down-stream cone. This last result ıld add about 1.12 feet to the head required in pumping 20,000,000 gallons y through a 48 inch main and a Venturi having a throat ratio of 1:9.

he Venturi meter has been found to give perfectly satisfactory results in ısuring the flow of brine and very hot water.

enturi tubes are made with throat ratios ranging from 1:4½ (or 2:9) to 6. The former are adapted to high, and the latter to low velocities; for, ›re the velocity in the pipe is low, it is necessary to accelerate it greatly in throat in order to obtain sufficient loss of pressure to secure reliable initions in the register. These cannot be obtained where the throat velocity ess than about 3 feet per second. With a throat ratio of 1:16, this would › a pipe velocity of $\frac{3}{16}$ foot per second. On the other hand, a meter with gh throat ratio, adapted to low velocities, would, with high velocities, exceed upper limit of the register.

wing to its unobstructed channel, free from moving parts, the Venturi er is far less liable to clogging than the forms of meter in common use.

he **prices** of the principal sizes of the Venturi meter are as follows:—on rd cars at Providence, R. I.

6 inch	$600.00	24 inch	$1,130.00	48 inch	$3,060.00
12 inch	770.00	36 inch	1,680.00	60 inch	4,890 00

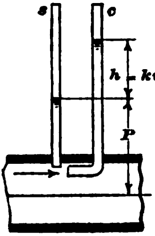
hese prices include the register, which, in the smaller sizes, constitutes the ıcipal item of cost. Discount, 1901, 10 per cent.

Trans. Am. Soc. Civil Engrs., Nov., 1887, Vol. XVII., page 228.
Journal of the Franklin Institute, Feb., 1899.
Journal New England Waterworks Assn., Vol. VIII., No. 1, Sep., 1893.
Trans. Am. Soc. Civil Engrs., Vol. XL., Dec., 1898, pp. 471, etc.

Art. 4 b. The Ferris-Pitot meter, invented and patented by Mr. Walter Ferris, of Philadelphia, is designed to measure the flow of liquids in pipes running full. It consists of a device for the registration of the results obtained by the Pitot tube, described on pages 561 and 562, and of special devices to prevent the clogging of the tubes and to permit their examination while in use.

In Fig. 6 let *P* represent the level at which the water stands in the straight Pitot tube, *s*. Then $h = kv^2$, or the difference in level between the columns in the two tubes, is the head $\left(\text{theoretically} = \frac{v^2}{2g}\right)$ due to the velocity of the water in the pipe as it impinges against the open up-stream end of the bent tube, *c*. For a given velocity, *v*, this difference, *h*, is constant, and is independent of the pressure represented by *P*.

Fig. 6.



The Ferris register, like that of the Venturi meter, records the velocity (existing at the instant of registration) in terms of the total discharge since the last registry and as an increase in the total number of cubic feet registered. The registry thus involves the assumption that the average velocity, during the period between registrations, is equal to the velocity at the end of that period. In the Ferris meter the registration is made every two minutes.

Evidently the instrument measures the velocity at only one point in the cross-section of the pipe, and it may thus be used to determine successively the velocities at any number of such points, but the velocity at such a point may or may not be equal to the mean velocity in the entire cross-section. The instrument is therefore usually calibrated by reference to some accepted standard, and the coefficient or coefficients thus obtained are used in subsequent observations.

The recording mechanism is operated by a small hydraulic motor, driven by means of the flow of the water in the pipe itself. For this purpose a second pair of Pitot tubes, is inserted into the pipe; and the current, flowing through these tubes, drives the motor without loss of water, the water used for power being returned to the pipe. If the velocity in the pipe is less than 3 feet per second it must be increased by means of a "reducer."

Experiments made by Mr. Ferris and by the Bureau of Water, Philadelphia, indicate that the Ferris-Pitot meter will ordinarily register within 3 per cent. of the true discharge.

In general, the size and cost of the registering apparatus are independent of the size of the pipe.

rt. 5. Resistance of curves and bends in water pipes.

uch uncertainty exists respecting these matters. **Weisbach's form-** a,* for the resistance due to a circular curve, Figs. 2 and 3, is

$$h = C \frac{A}{180} \cdot \frac{v^2}{2g} = \left[0.131 + 1.847 \left(\frac{r}{R} \right)^{\frac{7}{2}} \right] \frac{A}{180} \cdot \frac{v^2}{2g}, \text{ where}$$

h = head in feet required to overcome resistance due to curve or bend,
C = experimental coefficient,
A = angle of deflection, in degrees,
v = mean velocity of flow in pipe, in feet per second,
g = acceleration of gravity = 32.2 ft per sec per sec,
$\frac{v^2}{2g}$ = head theoretically due to velocity v,
D = inside diameter of pipe, in feet,
r = inside radius of pipe, in feet,
R = radius of axis of curve, in feet.

$r \div$ R =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
then C =	0.131	0.138	0.158	0.206	0.294	0.440	0.661	0.977	1.408	1.978

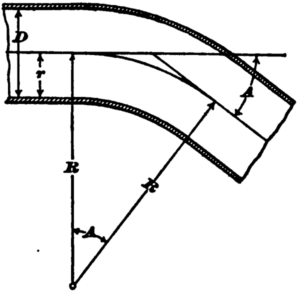


Fig. 2.

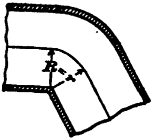


Fig. 3.

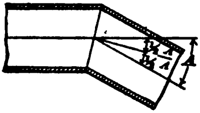


Fig. 4.
(See next page.)

ccording to this formula, the resistance due to curvature decreases rapidly increases from ½ D to 2 D; aud but little further decrease occurs beyond 5 D; but, from very careful and elaborate experiments on city water mains, 12 to 30 ins diameter, in Detroit, Mich.,† the investigators conclude that a of pipe with a curve of *short* radius R (down to a limit of R = 2½ D) causes resistance than does a line of equal length and equal total angle A, with a re of longer radius R. Their results were approximately as follows, where

H = resistance due to a section of 80 diameters in length, with a curve of A = 90° at mid-length,
h = resistance in a tangent of length = 80 diameters.

R ÷ D =	1	2	2.5	3	4	5	10	15	20	25
H ÷ h =	1.35	1.14	1.13	1.14	1.18	1.24	1.50	1.66	1.80	1.93

hey found also that the **loss of head,** due to a curve, occurs **not only the curve itself,** but that head continues to be lost in the following gent, for some distance **down stream** from the curve.

heir experiments led to the inference that even very **slight deflections,** n the line, **cause material losses of head,** and that care in securing raight alignment is therefore highly advisable. For **bends,** see next page.

Der Ingenieur, pp. 444, 445.

Paper by Gardner S. Williams, Clarence W. Hubbell, and George H. Fenkell, nsactions, American Society of Civil Engineers, 1901.

For abrupt angles, Fig. 4, Weisbach gives: **Resistance, in feet of head** =

$$h = c\,\frac{v^2}{2\,g} = (0.95\,\sin^2 \tfrac{1}{2}\,A + 2.05\,\sin^4 \tfrac{1}{2}\,A)\,\frac{v^2}{2\,g}$$

If ½ A =	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
then c =	0.03	0.14	0.36	0.74	0.98	1.26	1.56	1.86	2.16	2.43

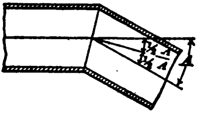


Fig. 4.

In addition to the resistance offered to flow, curves and bends involve additional labor and expense in manufacture and in laying; and vertical bends and curves lead to the formation of pockets of sediment at the feet of slopes, and of air cushions at their summits.

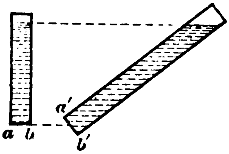


Fig. 5.

Art. 6. Although, in Fig. 5, the static pressures upon the equal bases, *a b* and *a' b'*, of the two pipes are equal (see Hydrostatics, Art. 1); yet, in order to pump water through either pipe, at a given velocity, an *additional* force is required, in order to overcome resistances to flow; and these resistances and the additional force required in order to overcome them, will be greater in the longer than in the shorter pipe.

rt. 7. Flow through orifices. Theoretically the velocity, v, of a d, flowing through a small orifice in the side or bottom of a very large vessel, qual to that acquired by a body falling freely in vacuo through a height al to the head, h, or depth, measured vertically from the level surface of the d in the vessel, to the center of gravity of the orifice; or,

$$v = \sqrt{2gh} = \sqrt{64.4h} = 8.03\sqrt{h};$$

$$h = \frac{v^2}{2g} = \frac{v^2}{64.4} = 0.0155\,v^2.$$

his law applies equally to all fluids. Thus, theoretically, mery, water, air, etc., all flow with equal velocities from a given orifice under a en head.

or deviations in practice from this theoretical law, see Art. 9, etc.

Table 10.
Velocities theoretically due to given heads.

ead	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel	Head	Vel.
eet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.
005	.57	.29	4.32	.77	7.04	1.50	9.83	7.	21.2	28	42.5	76	69.9
010	.80	.30	4.39	.78	7.09	1.52	9.90	.2	21.5	29	43.2	77	70.4
015	.98	.31	4.47	.79	7.13	1.54	9.96	.4	21.8	30	43.9	78	70.9
020	1.13	.32	4.54	.80	7.18	1.56	10.0	.6	22.1	31	44.7	79	71.3
025	1.27	.33	4.61	.81	7.22	1.58	10.1	.8	22.4	32	45.4	80	71.8
030	1.39	.34	4.68	.82	7.26	1.60	10.2	8.	22.7	33	46.1	81	72.2
035	1.50	.35	4.75	.83	7.31	1.65	10.3	.2	23.0	34	46.7	82	72.6
040	1.60	.36	4.81	.84	7.35	1.70	10.5	.4	23.3	35	47.4	83	73.1
045	1.70	.37	4.87	.85	7.40	1.75	10.6	.6	23.5	36	48.1	84	73.5
050	1.79	.38	4.94	.86	7.44	1.80	10.8	.8	23.8	37	48.8	85	74.0
055	1.88	.39	5.01	.87	7.48	1.85	10.9	9.	24.1	38	49.5	86	74.4
060	1.97	.40	5.07	.88	7.53	1.90	11.1	.2	24.3	39	50.1	87	74.8
065	2.04	.41	5.14	.89	7.57	1.95	11.2	.4	24.6	40	50.7	88	75.3
070	2.12	.42	5.20	.90	7.61	2.	11.4	.6	24 8	41	51.3	89	75.7
075	2.20	.43	5.26	.91	7.65	2.1	11.7	.8	25.1	42	52.0	90	76.1
080	2.27	.44	5.32	.92	7.70	2.2	11.9	10.	25.4	43	52.6	91	76.5
085	2.34	.45	5 38	.93	7.74	2.3	12.2	.5	26.0	44	53.2	92	76.9
090	2.41	.46	5.44	.94	7.78	2.4	12.4	11.	26.6	45	53.8	93	77.4
095	2.47	.47	5.50	.95	7.82	2.5	12.6	.5	27.2	46	54.4	94	77.8
100	2.54	.48	5.56	.96	7.86	2.6	12.9	12.	27.8	47	55.0	95	78.2
105	2.60	.49	5.62	.97	7.90	2.7	13.2	.5	28.4	48	55.6	96	78.6
110	2.66	.50	5.67	.98	7.94	2.8	13.4	13.	28.9	49	56.2	97	79.0
115	2.72	.51	5.73	.99	7.98	2.9	13.7	.5	29.5	50	56.7	98	79.4
120	2.78	.52	5.79	1 Ft.	8.03	3.	13.9	14.	30.0	51	57.3	99	79 8
125	2.84	.53	5.85	1.02	8.10	3.1	14.1	.5	30.5	52	57.8	100	80.3
130	2.89	.54	5.90	1 04	8.18	3 2	14.3	15.	31.1	53	58.4	125	89.7
135	2.95	.55	5.95	1 06	8.26	3.3	14.5	.5	31.6	54	59.0	150	98.3
140	3.00	.56	6.00	1.08	8.34	3.4	14.8	16.	32.1	55	59.5	175	106
145	3.05	.57	6.06	1.10	8.41	3.5	15.	.5	32.6	56	60.0	200	114
150	3.11	.58	6.11	1.12	8.49	3.6	15.2	17.	33.1	57	60.6	225	120
155	3.16	.59	6.17	1 14	8.57	3.7	15.4	.5	33.6	58	61.1	250	126
160	3.21	.60	6.22	1.16	8.64	3.8	15.6	18.	34.0	59	61.6	275	133
165	3.26	.61	6.28	1.18	8.72	3.9	15.8	.5	34.5	60	62.1	300	139
170	3.31	.62	6.32	1 20	8.79	4.	16.0	19.	35.0	61	62.7	350	150
175	3.36	.63	6 37	1.22	8.87	.2	16.4	.5	35.4	62	63.2	400	160
180	3.40	.64	6.42	1.24	8.94	.4	16.8	20.	35.9	63	63.7	450	170
185	3.45	.65	6.47	1.26	9.01	.6	17.2	.5	36.3	64	64.2	500	179
190	3.50	.66	6.52	1.28	9.08	.8	17.6	21.	36.8	65	64.7	550	188
195	3.55	.67	6 57	1.30	9.15	5.	17.9	.5	37.2	66	65.2	600	197
200	3.59	.68	6.61	1.32	9.21	.2	18.3	22.	37.6	67	65.7	700	212
21	3.68	.69	6.66	1.34	9.29	.4	18.7	.5	38.1	68	66.2	800	227
22	3.76	.70	6.71	1.36	9.36	.6	19.	23.	38.5	69	66.7	900	241
23	3.85	.71	6.76	1.38	9.43	.8	19.3	.5	38.9	70	67.1	1000	254
24	3.93	.72	6.81	1.40	9.49	6.	19.7	24.	39.3	71	67.6		
25	4.01	.73	6.86	1.42	9.57	.2	20.0	.5	39.7	72	68.1		
26	4.09	.74	6.91	1.44	9.63	.4	20.3	25	40.1	73	68.5		
27	4.17	.75	6.95	1.46	9.70	.6	20.6	26	40.9	74	69.0		
28	4.25	.76	6.99	1.48	9 77	.8	20.9	27	41.7	75	69.5		

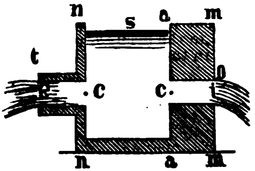


Fig. 6.

Art. 8. On the flow of water through vertical openings furnished with short tubes. When water flows from a reservoir, Fig 6, through a vert partition *m m a a*, the thickness *a m* of which is about 2½ or 3 times the least transverse dimension of the opening, (whether that dimension be its breadth, or its height;) or when, if the partition be very thin, as *n n*, the water flows through a tube, as at *t*, the length of which is about 2 or 3 times its least transverse dimension, then the effluent stream will entirely fill the opening, or the tube, as shown in Fig 6; or, in technical language, will *run with a full flow; or a full bore;* and will disch more water in a given time, than if the tube were either materially longer or shorter. For if longer than 3 times the least transverse dimension, the flow will be impeded by the increased friction against the sides of the tube; and if shorter than about twice the least transverse dimension, the water will not flow in a full stream, but in a contracted one, as shown by Fig 11. This will be the case whether the tube be circular, or rectilinear, in its cross-section.

To find approximately the actual vel, and disch into the air, through a tube, or opening, either circular or rectilinear in its outline, or cross-section; and whose length *c t*, or *c e*, in the direction of the flow, is about 2½ or 3 times its least transverse dimension; when the surface-level, *s*, Fig 6, remains constantly at the same height; and which height must not be below the upper edge of the tube, or opening.

Rule 1. Take out the theoretical vel from Table 10, corresponding to the head measured vert from the center (or more properly, the cen of grav) *c*, of the opening, to the level water surf *s*. Mult it by the coeff of disch .81. The prod will be the reqd vel, in ft per sec. Mult this actual vel by the transverse area of the opening, in sq ft. If circular, knowing its diam, this area will be found in Table 3. The prod will be the quantity of water dischd, in cub ft per sec; within, probably, 3 or 4 per cent.

Rule 2. Find the sq rt of the head in ft. Mult this sq rt by 6.5. The prod will be the actual vel in ft per sec.

Ex. An opening *c e*; or box-shaped tube *c t*, Fig 6, is 3 feet wide, by .25 of a ft high; and its length in the direction *c t* or *c e* in which the water flows is about .62 of a ft, or about 2½ times its least transverse dimension, or its height. The head from the cen of grav *c*, of the opening, to the constant surf-level *s*, is 4 feet. What will be the vel of the water; and how much will be dischd per sec?

By Rule 1. The theoretical vel (Table 10,) corresponding to a head of 4 ft is 16 ft per sec. And 16 × .81 = 12.96 ft per sec, the actual vel reqd. Again, the transverse area of the opening, or of the tube, is 3 ft × .25 ft = .75 sq ft. And .75 × 12.96 = 9.72 cub ft; the quantity dischd per sec.

By Rule 2. The sq rt of 4 is 2. And 2 × 6.5 = 13 ft per sec, the reqd vel, as before; the very slight diff being owing to the omission of small decimals in the coeffs.

Rem. 1. **If the short tube *t* projects partly inside of the vert partition *n n*,** the disch will be diminished about ⅛ part. In that case, use .71 or .7 instead of the .81 of Rule 1; or 5.7 instead of the 6.5 of Rule 2.

Rem. 2. When the thickness *a m* of the vert partition *m m a a*; or the length *c e* of the tube *t*, Fig 6, is increased to about 4 times the least transverse dimension of the opening; or of the diam, when circular; then the additional friction against its sides begins appreciably to lessen the vel and disch. In that case, or for still greater lengths, up to 100 diams, they may be found approximately, by using instead of the coeff of disch .81 in Rule 1, the following coeffs, by which to mult the theoretical vels of Table 10.

TABLE 11.

Length of Pipe in Diams.	Coeff.	Length of Pipe in Diams.	Coeff.
4	.80	40	.62
6	.76	50	.60
10	.74	60	.57
15	.71	70	.55
20	.69	80	.52
25	.67	90	.50
30	.65	100	.48

Rem. 3. When the length of the opening or tube, in the direction in which the water flows, becomes *less* than about twice its least transverse dimension, the disch is diminished; so that for lengths from 1½ times, down to openings in a very thin plate, we may use .61, instead of the .81 of Rule 1. For such openings, however, see Arts 9 and 10.

Rem. 4. But on the other hand, the disch through such short openings and tubes as are shown in Fig 6, may be increased to nearly the theoretical ones of Table 10, by merely rounding off neatly the edges of the entrance end or mouth, as in Fig 7; which is the shape, and half actual size of one with which Weisbach obtained .975 of the theoretical vel and discharge, when the head was 10 ft; and .958

with a head of one foot; so that in similar cases, .975, and .958 may be used instead of the coeff .81 in Rule 1.

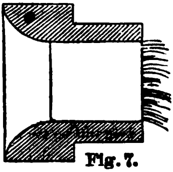


Fig. 7.

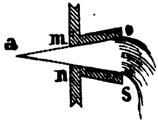


Fig. 8.

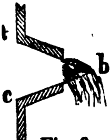


Fig. 9.

As much as .92 to .94 may be obtained by widening the opening, *m n*, toward its outer mouth, *o s*, Fig, 8, making the divergence, or angle *a*, about 5°; or by widening it toward its inner mouth, as at *t c*, Fig 9; but increasing the angle of divergence, at *b*, to from 11° to 16°. In all cases, we consider the small end as being the opening whose area must be multiplied by the vel to get the discharge.

In some experiments made with large pyramidal wooden troughs 9.5 ft long, with an inner mouth of 3.2 × 2.4 ft, and a discharging one of .62 × .44 ft; and under a head of 9½ feet, the discharge was .98 of the theoretical one, due to the smaller end. Therefore, .98 may be used in such cases, instead of the .81 of Rule 1.

Rem. 5. By using an adjutage shaped as in Fig 10, the discharge may be increased to several times that due to the head above the *center of gravity s* of the orifice *m n*: because in such cases, as explained in Art 1 *w*, the *true* head at *s*, or the head causing the rapid flow through the narrowest portion *m n*, may be much greater than the head above *s*.

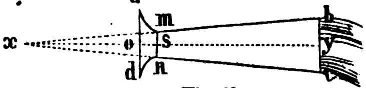


Fig. 10.

See the Venturi Meter, Art 4 a.

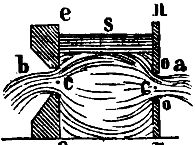


Fig. 11.

Art. 9. On the disch of water through openings in thin vert partitions, with plane or flat faces, *e e*, or *n n*, Fig 11.[*] If the face *e e*, or *n n*, instead of being plane, and vert, should be curved, or inclining in diff directions toward the opening, then the disch will be altered. When water flows from a reservoir, Fig 11, through a vert plane plate or partition *n n*, which is not thicker than about the least transverse dimension of the opening, whether that dimension be its breadth, or its height *o o*;[†] or when, if the partition *e e* itself is much thicker, we give the opening the shape shown at *b*, (which evidently amounts to the same thing,) then the effluent stream will not pass out with a *full flow*, as in Fig 6, but will assume the shape shown in Fig 11; forming, just outside of the opening, what is called the *vena contracta*, or *contracted vein*. In order that this contraction may take place to its fullest extent, or become *complete*, the inner sharp edges of the opening must not approach either the surf of the water, or the bottom or sides of the reservoir, nearer than about 1½ times the least transverse dimension of the opening. The contracted vein occurs at a dist of about half the smallest dimension of the orifice, from the orifice itself. In a circular orifice, at about half the diam dist; and ordinarily its area is about .62, or nearly ⅝ that of the orifice itself. At this point the actual mean *vel* of the stream is very nearly (about .97) the theoretical vel given by Table 10, and hence the actual *dischs* are but .62, or nearly ⅝ of the theoretical ones.

Case 1. **To find the actual disch into air,[‡] through either a circular or rectilinear[§] opening in a thin vert plane parti-**

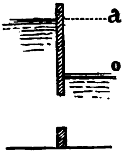


Fig. 12.

[*] We believe that these rules for thin plate are also sufficiently approximate for most practical purposes, if the opening be in the *bottom* of the reservoir; or in an *inclined*, instead of a *vert* side.

[†] When the side of a reservoir, or the edge of a plank, &c, over which water flows, has no greater thickness than this, the water is said to flow through, or over, **thin plate**, or **thin partition.**

[‡] Should the disch take place *under water*, as in Fig 12, *both surf-levels remaining constant*, then the head to be used is the *vert* diff *a o*, of the two levels. After making the calculation with this head, we should, according to Weisbach, deduct the $\frac{1}{75}$ part; inasmuch as he states that the disch is that much less when under water, than when it takes place freely into the air. Other experimenters, however, assert that it is precisely the same in both cases.

[§] If the shape of the opening is oval, triangular, or irregular, the head must be measured vert from its cen of grav.

tion, when the contraction is complete; and when the surf-level, *s*, remains constantly at the same height; water being supplied to the reservoir as fast as it runs out at the opening.*

RULE 1. When the head, measured vert from the center (or rather from the cen of grav) *c*, of the opening, to the surf-level *s* of the reservoir, is not less than 1 ft, nor more than 10 ft; and when the least transverse dimension of the opening is not less than an inch, mult the theoretical vel in ft per sec due to the head, (Table 10,) by the coefficient of disch .62. The prod will be the actual mean vel of the water through the opening. Mult this vel by the area of the opening in sq ft; the prod will be the disch in cub ft per sec, approximately.

When the head is greater than 10 ft, use .6, instead of .62.

RULE 2. Find the sq rt of the head in ft. Mult this sq rt by 5; the prod will be the vel in ft per sec; which mult by the area as before for the disch.

Ex. What will be the disch through an opening in complete contraction, whose dimensions are 6 ins, or .5 ft vert; and 4 ft hor; the vert head above the cen of grav of the opening being constantly 6 feet?

By Rule 1. The theoretical vel (Table 10,) corresponding to 6 ft head, is 19.7 ft per sec. And 19.7 × .62 = 12.214 ft, the reqd vel. Again, the area of the opening = .5 × 4 = 2 sq ft; and 12.214 × 2 = 24.428 cub ft per sec; the disch.

By Rule 2. The sq rt of 6 = 2.45; and 2.45 × 5 = 12.25 ft per sec, the reqd vel; and 12.25 × 2 = 24.5 cub ft per sec, the disch.

Both very approx even if the orifice reaches to the surface of the issuing water.

Rem. 1. The coef .62 is a mean of results of many old experimenters In 1874 Genl. T. G. Ellis of Massachusetts conducted an elaborate series (Trans Am Soc C E, Feb 1876) on a large scale, the general results of which, within less than 1 per ct, are given in the following table. See also Rem 3. The sharp edged orifices were in iron plates .25 to .5 inch thick.

Orifice.	Head above Center.	Coef.
2 ft sq.	2. to 3.5 ft.	.60 to .61
2 " long, 1 ft high	1.8 to 11.3 "	.60 to .61
2 " long, .5 high	1.4 to 17.0 "	.61 to .60
2 " diam.	1.8 to 9.6 "	.59 to .61

Rem. 2. Extreme care is reqd to obtain correct results; but for many purposes of the engineer an error of 5 to 10 per ct is unimportant.

It will rarely happen that greater accuracy is required than may be obtained by the foregoing rules; but when such does occur, aid may be derived from the following **table deduced from the experiments of Lesbros and Poncelet**, on openings 8 ins wide, of diff heights, and with diff heads. Use that coeff in the table which applies to the case, instead of the .62 of Rule 1. In some of the cases in this table, the upper edge of the opening is nearer the surf-level of the reservoir than 1½ times its least transverse dimension.

TABLE 12. Coefficients for rectangular openings in thin vertical partitions in full contraction.*

Head above cen. of grav. of opening in Feet.	Head above cen. of grav. of opening in Inches.	The breadth in all the openings = 8 inches. HEIGHT OF OPENING. Ins. 8	Ins. 6	Ins. 4	Ins. 3	Ins. 2	Ins. 1	Ins. .4
.033	.4	....	....	....	....	....	....	.70
.0666	.8	....	....	....	....	....	.65	.69
.0833	1	....	....	....	....	....	.64	.68
.125	1½	....	....	....	....	.61	.64	.68
.1666	2	....	....	....	.60	.62	.64	.68
.2083	2½	....	....	.59	.61	.62	.64	.67
.250	3	....	....	.60	.61	.62	.64	.67
.2917	3½	....	.57	.60	.61	.62	.64	.66
.3333	4	....	.58	.60	.61	.63	.64	.66
.3750	4½	.56	.59	.60	.61	.63	.64	.66
.4167	5	.57	.59	.61	.62	.63	.64	.66
.6666	8	.59	.60	.61	.62	.63	.64	.65
1	12	.60	.60	.61	.62	.63	.63	.64
3	36	.60	.60	.61	.62	.62	.63	.63
5	60	.60	.60	.61	.61	.62	.62	.62
10	120	.60	.60	.60	.60	.60	.61	.61

REM. 3. **Careful experiments on openings 4¼ ft wide, and 1 ins high**, under heads of from 6 to 15 ft, show that the coeff .62 will give results correct within $\frac{1}{30}$ part, for openings of that size also, under large heads; although the thickness of the partition varied on its diff sides, from 12 to 20 ins. It must be recollected, however, that nothing more than close *approximations* are to be attained in such matters.

REM. 4. It has been asserted by some writers, that **when two or more *contiguous* openings** are discharging at the same time from the same reservoir, they disch less in proportion than when only one of them is open. Other experiments, however, seem to show that this is not the case; it is therefore probable, at least, that the diff, if any, is but trifling.

---

* See first footnote on preceding page.

Case 2. **The discharge through thin vert partitions in com- ete contraction, when the surface-level, *m*, Fig 13, descends the water flows out into the air.** In this case, if the reservoir is smatic, that is, if its hor sections are everywhere equal; and if no water is flowing into the reser- r, to supply the place of that which flows out, then, to find the time reqd to disch the reservoir.

ULE. Inasmuch as the time in which such a reservoir entirely discharges itself, is twice that in ich the same quantity would flow out under a constant head, as in Case 1, therefore, calculate the disch in cub ft per sec by Rule 1, Art 9; div the number of cub ft contained in the reservoir, above the level *g* of the bottom of the opening, Fig 13, by this disch; the quot will be the number of sec in which a volume equal to that in the reservoir, to the depth *g*, would run out in Case 1, of a *constant* head. And *twice* this number will be the seconds reqd to empty the reservoir in Case 2, of a *varying* head.

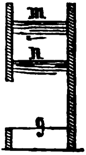


Fig. 13.

REM. If it should be reqd to find the time in which such a prismatic reservoir would *partly* empty itself, as, for instance, from *m* to *n*, Fig 13, first calculate, by the above rule, the secs necessary to empty it if it had only been filled to *n*: and afterward calculate as if it had been filled to *m*. The diff between the two times will evidently be the time reqd to empty it from *m* to *n*. If the opening is not in complete contraction, see Arts 11, &c.

If the disch is into a lower reservoir, whose surf-level remains constant, proceed in the same manner; only use the diff of level of the two surfs as the head, and afterward (according to Weisbach) increase the time $\frac{1}{75}$ part.

Art. 10. Disch from a reservoir R, Fig 14, the surf-level, *s*, which remains constantly at the same height; through opening, *o*, in thin vert partition; and in complete con- action; but entirely under water; and into a prismatic servoir, *m*.

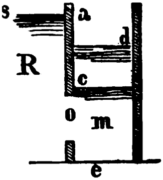


Fig. 14.

Seconds required to discharge a quantity = *c d a, the level c remaining constant.*

$$= \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft}}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required *to raise level* in *m* from *c* to *a*

$$= \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required *to raise level* in *m* from *c* to *any other level, d.*

$$= \frac{\left(\sqrt{a\,c} - \sqrt{a\,d} \text{ both in ft}\right) \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

REM. 1. **If it should be reqd to find the time of filling *m*, from s bottom *e*, up to *d*,** we may do so very approximately by calculating by first rule in Art 9, the time reqd from *e* to the center of the opening *o*, as if all that portion of disch took place into air; and afterward, from the center of the opening to *d*, by the rule just en. This case is similar to that of filling a lock from the canal reach above, in which the surf- el may be considered constant.

REM. 2. **If the bottom of the opening *o*, should coincide with e bottom of the reservoir,** then the coeff will become greater than .62. Art 11, for obtaining coeffs for imperfect contraction.

REM. 3. **If the opening, instead of being in complete con- action,** is of any of the shapes Figs 6 to 9, then a reference to Art 8 will show at coeff must be substituted for .62.

Case 3. **Disch from one prismatic reservoir, Fig 15, W, into other, X, of any comparative sizes whatever, through an pening *o*, in a plane thin vert partition, and in complete ntraction; when the water rises in X, while it falls in W.**

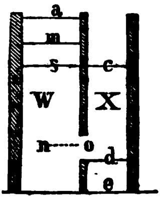


Fig. 15.

To find the time in which the water, flowing from W *into* X, *through o, will fall through the dist a s, so as to stand at the same level s c, in both reservoirs.*

In this case, the water reqd to fill X from *e* to *d*, (*d* being the bottom of the opening *o*,) flows out into the air; and the time necessary for it to do so, must be calculated separately from that reqd above *d*, which flows into water.

RULE. First from *e* to *d*. Find the hor area of each reservoir, in sq ft. Mult the hor area of X, by the vert depth *d e* in ft, for the cub ft contained in that portion. Div these cub ft by the hor area of W. The quot will be the dist *a m*, in feet, through which the water in W must descend, in order to fill X to *d*.

Seconds required to lower from *a* to *m*, and raise from *e* to *d*.

$$= \frac{\text{Twice the hor area of W in sq ft} \times \left(\sqrt{\text{head } a\,n \text{ in ft}} - \sqrt{\text{head } m\,n \text{ in ft}}\right)}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required to lower from *m* to *s*, and raise from *d* to *c*. (Very approx)

$$= \frac{\text{Hor area of X in sq ft} \times \text{twice the hor area of W in sq ft} \times \sqrt{\text{head } m\,n \text{ in ft}}}{\text{Area of opening } o \text{ in sq ft} \times \left(\text{hor area of W in sq ft} + \text{hor area of X in sq ft}\right) \times .62 \times 8.03}$$

Ex. Let the hor area of W be 100 sq ft: and that of X, 60 sq ft. Let *a n* be 20 ft; and *m n* 16 ft; and the area of the opening *o*, 3 sq ft. In what time will the water descend from *a* to *s*, and rise from *e* to *c*?

Inasmuch as the method of finding the time for filling from *e* to *d*, by the water falling from *a* to *m*, requires no further exemplification, we will confine ourselves to the *additional* time necessary for filling from *d* to *c*, by the water falling from *m* to *s*. To find this, we have, the sq rt of the head $m\,n = 4$ ft; and the sum of the 2 areas $= 100 + 60 = 160$. Hence, $\frac{4 \times 100 \times 60 \times 2}{160 \times 8.03 \times 3 \times .62} = \frac{48000}{2389.78} =$ 20.1 sec; the additional time reqd, very approximately.

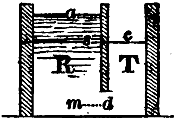


Fig. 16.

NOTE 1. **If the opening, as *d*, Fig 16, reaches to the very bottom of the reservoirs,** we may consider all the water flowing from R into T, as flowing into water. Therefore, using the head *a m*, we at once calculate the time necessary for the water in the two reservoirs to arrive at the same level *s c*, by the last process of the preceding rule; or, in other words, by the process given in the preceding example. But in this case it must be borne in mind that the opening *o* is no longer in *complete* contraction, inasmuch as the contraction along its lower edge is suppressed.

The disch will consequently be somewhat increased; and a coeff greater than .62 becomes necessary. The method of finding this, is given in the following Case 4. A reference to Art 8 will give the coeff in case the opening is shaped as Figs 6 to 9.

Art. 11. *Case* 4. The discharge through openings in plane thin vert partitions; but in incomplete contraction.

The opening may be such that contraction will take place along one portion of its perimeter, or at the top of the opening *a*, Fig 17; while it is suppressed on another portion; as at the bottom and two ends of the opening *a*; where suppression is caused by the addition of *short* side and bottom pieces c, c, c. Or it may be caused by the bottom, or ends, or both, coinciding with the bottom and sides of the reservoir. In such cases the disch will be greater than in those of complete contraction; but less than in those of full flow; inasmuch as the opening now partakes somewhat of the character of the *short* tubes of Art 8; and the coeff will rise from .62, or that which *usually* pertains to openings in full contraction; and will approach .8, or that of full flow, in proportion to the extent of perimeter along which contraction is suppressed: or even to .9 or .98 by the use of such openings as are shown by Figs 7, 8, 9.

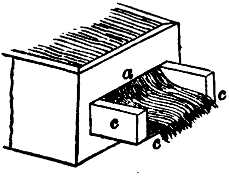


Fig. 17.

To find approximately a new coeff of disch; and the disch itself, in cases of incomplete contraction.

RULE. First find by the foregoing rules, what would be the disch in the particular case that may be under consideration, supposing the contraction to be complete. Then div that portion of the perimeter of the opening on which contraction is suppressed, by the entire perimeter. Mult the quot by the dec .152 if the opening is rectangular, or by .128 if circular. To the prod add unity, or 1. Call the sum, *g*. Then say, as unity, or 1, is to *g*, so is the coeff for complete contraction in ordinary cases (usually .62) to the reqd new coeff. *Finally*, repeat the original calculation, only substituting this new coeff in the place of .62.

According to this rule, we have the following coeff of discharge for rectangular openings within probably 3 or 4 per cent, when contraction is not suppressed on more than ¾ of the perimeter. The theoretical discharge multiplied by the corresponding coeff will give the actual discharge. When the contraction is carried farther, the coeff becomes extremely irregular, and is probably indeterminable.

For complete contraction (ordinarily)	.62
When contraction is suppressed on ¼ *the perimeter*	.64
" " " " " ½ " "	.67
" " " " " ¾ " "	.69
" " " " *entirely around the orifice*	.80

Intermediate ones can be estimated nearly enough, mentally.

REM. 1. **When, instead of a short spout, as in Fig 17, the opening is provided with an indefinitely long hor trough,** similarly attached, and open at top, there will be no practically appreciable diminution of disch below that through the simple opening as at *a*, Fig 11; provided the head measured above the cen of grav of the opening be at least as great as 2 or 2½ times the height of the opening itself. Therefore, under such circumstances the disch may be calculated by the rules in Art 9. But with smaller heads the disch diminishes considerably; so when the head above the center becomes but as great as the height of the opening, it will be but about $\frac{4}{5}$ of the calculated one. With still smaller heads, the flow becomes less much more rapidly; but has not been reduced to any rule.

REM. 2. **If, instead of being hor, the trough is INCLINED**

as much as 1 in 10, the disch will be *increased* very slightly, (some 3 or 4 per cent) over that calculated by the rules in Art 9, for the plain opening. These results were obtained by experiments on a very small scale; and should be considered as mere approximations.

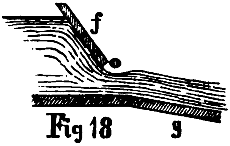


Fig 18

Art. 12. In a case like Fig 18, where contraction is supposed to be suppressed at the bottom, and at both vert sides of the opening *o*, in consequence of their coinciding with the bottom and sides of the reservoir; but where the front of the reservoir, instead of being vert, is sloped as at *f*; and when the water, after leaving the opening, flows away over a slightly sloping apron, *g*, then the disch in cub ft per sec may be approximately found by Rule 1, Case 1, Art 9, only substituting .8 in place of .62, when *f* slopes back 45°, or 1 to 1; or .74 when *f* slopes back 63°, or with a base of 1 to a rise of 2. In such cases of inclined fronts, the height of the opening must be measured *vert*, or rather *at right angles to the floor of the reservoir;* and not in a line with the sloping front.

Rem. **When the front, *f*, of the reservoir is vert, and a sloping apron or trough, *g*, is used,** having its upper edge level with the bottom of the opening, the disch is not appreciably diminished below that which takes place freely into the air, provided the head above the cen of grav of the opening is not less than from

18 to 24 ins, for an opening 6 to 9 ins high.
12 to 16 " " " 4 ins high.
9 " " " 2 ins or less, high.

Art. 13. To find, approximately, the time reqd for the emptying of a pond, or any other reservoir, as Fig 19, which is not of a prismatic shape; through an opening, *n*, near the bottom.

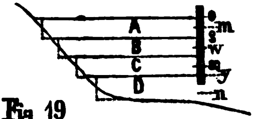


Fig 19

Rule. First ascertain the exact shape and dimensions of the reservoir. If large, and irregular, it must be carefully surveyed; and soundings taken, and figured upon a correct plan and cross-sections. Next, consider the entire body of water to be divided into a series of thin hor strata, A, B, C, D; the top line of the lower one being at least a few ins above the top of the opening *n*. It is not necessary that these strata should be of equal thickness; although the thinner they are, the more correct will the result be. The depth of the lower one, D, will vary to some extent with the height of the opening; those next above it should not exceed about a foot in thickness, until a depth of 6 or 8 feet is reached; then they may conveniently, and with sufficient accuracy, be increased to about 2 ft, for 6 or 8 ft more; and so on; becoming thicker as they approach the surf. By aid of the drawings, calculate the content of each stratum in cub ft. Now, since the strata are thin, we may, without serious error, assume each of them to be prismatic, as shown by the dotted lines; and may assume that the head under which each stratum (except the lowest) empties itself through *n*, is equal to the vert height from the center of the opening to the center of the stratum. Thus, *m n* will be the head of A; *w n*, the head of B; *x n*, the head of C. Then, for the stratum A, by Rule 1, Art 9, (only using *m n* as the head instead of *o n*,) and instead of the coeff .62 of that rule (which can only be used if *n* is in complete contraction) using .64, or whatever other coeff near the end of Art 11 applies to the case, calculate the disch in cub ft per sec. Div the content of the stratum A by this disch, and the quot will be the number of sec reqd for discharging A. Using the head *w n*, proceed in precisely the same way with the stratum B; and using the head *x n*, do the same with C. Finally, for the lower stratum D, find by Rule 1, Art 9, (with the same caution as before respecting the proper coeff,) in what time it would empty itself under a *constant* head equal to *y n*, measured from its *surf* to the center of the opening. *Double* this time will be that reqd to empty itself in the case before us, under its *varying* head. Finally, add together all these separate times; and their sum will be the entire time reqd to empty the pond, or reservoir, approximately enough for practical purposes.

The Miner's Inch.

The miner's inch is a unit of rate of discharge of water, largely used in the far western mining States. It is expressed in terms of a standard orifice, usually 1 inch square and vertical, and a standard head. The head varies from about 3 to 9 ins, but 6 ins is a commonly accepted standard. Under that head, with coefficient = 0.62, the discharge, through an orifice 1 inch square, would be: per sec, 0.0244 cu ft = 0.183 U. S. gals (of 231 cu ins); per min, 1.466 cu ft = 10.96 gals; per day, 2,111 cu ft = 15,790 gals. Usually the orifice is of fixed depth and adjustable length (see Fig.), and the standard head is obtained and maintained by such adjustment. The relation between area and shape of orifice and the discharge per sq in of area is indicated by the following table:*

Discharge, in cu ft per min, per square inch of opening, under 6 ins head.

Length of opening, ins.	4	6	8	12	24	100
Opening 2 ins high, . . .	1.473	1.480	1.484	1.487	1.490	1.494
" 4 " " . .	1.450	1.470	1.481	1.491	1.501	1.509

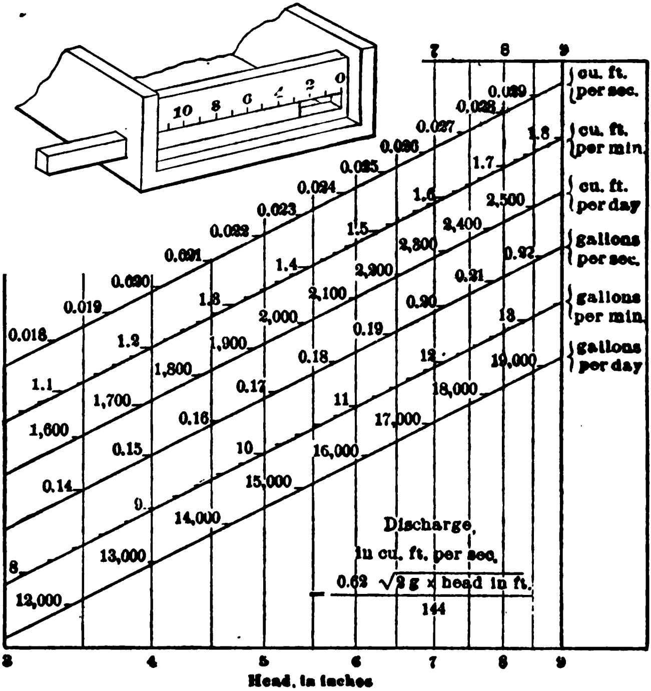


Diagram of discharge through an orifice 1 inch square.

The thickness of plank, in which the orifice is cut, varies from 1 to 3 ins, and the lower edge of the orifice is often chamfered, flaring outwardly. The bottom of the orifice is sometimes flush with the bottom of the box, but is usually raised 2 or 3 ins above it, as in our Fig.

---

* Condensed from circular of Pelton Water Wheel Co.

Art. 14 (*a*). On the discharge of water over weirs or overfalls. The weir affords a very convenient means for gauging the flow of small streams, for measuring the quantity of water supplied to water-wheels, etc.

(*b*) A measuring weir is always arranged with its back, or up-stream side, *a b*, Fig. 20, vertical, and as nearly as may be at right angles to the direction of flow of the stream. The ends, *a h*, *a h*, Figs. 21 and 22, are vertical, and the crest *a a* is horizontal.

(*c*) **End contractions.** When the weir *a a* extends entirely across the channel of approach, as in Fig. 21, so that its ends *a h*, *a h* coincide with, or form portions of, the sides *s s* of the channel, contraction (Art. 9, p. 541) takes place only on the *top* and *bottom* of the sheet of water passing over the weir, as at *m c* and at *a*, Fig. 20, and is entirely "suppressed" at the *ends*, so that the water flows out as shown in Fig. 21 *a*. Such a weir is called a **suppressed weir,** or a **weir without end contraction.** But when, as in Figs. 22 and 22 *a*,

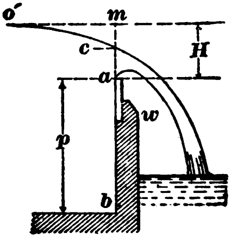


Fig.20

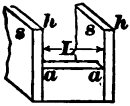


Fig. 21

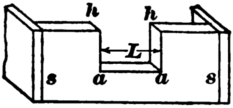


Fig. 22

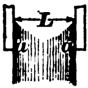


Fig. 21a

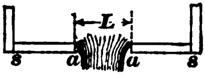


Fig. 22a

the ends *a h*, *a h* are at a distance from the sides *s s* of the channel or reservoir, contraction takes place at the *ends* of the weir, as shown at *a* and *a*, as well as over the crest. Such contraction diminishes the discharge. A weir of this kind is called a **weir with end contractions.**

Other things being equal, the extent of the contraction, and its effect upon the discharge, increase with the head H. When the length *a a* or L of the weir exceeds about 10 times the head H, the effect of the end contractions upon the discharge is nearly imperceptible; but as the length diminishes in proportion to the head, the effect of the contraction increases rapidly. Mr. Francis (Art. 14 *m*) found that when L = only 4 × H, the discharge was reduced 6 per cent. by complete end contractions. In view of the uncertainty as to the effect of end contractions, it is better to avoid them and to use weirs, like Fig. 21, where the contraction is suppressed; but if end contraction is permitted at all, it must be made *complete*;* for the coefficients given do not apply to cases of **incomplete contraction,** *i. e.*, with contraction only *partly* suppressed.

(*d*) **In a weir without end contraction,** care must be taken that the *air has free access* to the space (*w*, Fig. 20, or 22 *b*) behind the falling sheet of water. Otherwise a partial vacuum forms there, the sheet is drawn inward toward the weir, and the discharge is greatly modified. At the same time, the sheet should be prevented from *expanding laterally* as it leaves the crest. Both of these objects may be attained by prolonging the *upper portion only* of both sides of the channel a little way down-stream beyond the crest and the upper part of the falling sheet, as in Fig. 22 *b*. Mr. Francis found that such projections, by confining the sheet laterally, diminished the discharge about 0.4 per cent.

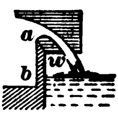


Fig.22b

(*e*) Ordinarily **the crest** is "in thin plate" or "in thin partition" (see footnote †, p. 541), so that the sheet passing over the weir touches it only at the very corner, *a*, Fig. 20. A *rounded* corner increases the discharge, as does the rounding of the edges of an orifice (Art. 8, p. 541), and a crest sufficiently wide to deflect the falling sheet diminishes the discharge (see coefficients for this case in Table 15, p. 554), but both forms introduce much uncertainty, and should therefore be avoided.

---

* The contraction is said to be "complete" when it is practically as great as it could be made by any further increase of the distance *a s*, Figs. 22 and 22 *a*; and this is believed to be attained when *a s* is made equal to the head H.

(*f*) **The length L of the crest,** Figs. 21 to 22 *a*, should be at least three times the head H, in order to reduce the effect of friction of the sides *s s* and that of end contractions where such exist. **The height** *p*, Fig. 20, of the vertical back *a b* in contact with the water should be not less than twice the head H; for, in order to reduce the velocity of approach (see Art. 14 *u*), the cross-section of **the channel** leading to the weir should be large in proportion to that of the stream *a c*. The cross-section of the channel of approach should be as *regular* as possible.

(*g*) The weir should be stoutly built, as **vibrations** of the structure may seriously modify the discharge.

(*h*) *Theoretically*, **the head** is the vertical distance H′, Fig. 24, from the crest *a* to a point *o′* where the water is *perfectly still*, and the surface therefore *horizontal*. But in fact the head is usually measured from the crest *a* to a point *o* a few feet back from the weir, where the water is only *comparatively* still, the velocity of approach being perceptible. (See Art. 14, *u*.) The difference between the head H actually measured and the head H′ to *still* water is usually very slight. It is greatly exaggerated in the figure.

The correct **measurement of the head** is a delicate matter, the discharge being increased or diminished about 1½ per cent. by 1 per cent. of increase or diminution of the head. Waves or ripples and other disturbances of the surface, and capillary attraction, are the chief sources of error.

(*i*) To avoid the latter difficulty, the **hook-gauge** is used for measuring the height of the water surface in important cases. This consists of a long graduated rod, provided at its foot with an upturned hook or point, and sliding vertically (by means of a screw motion) in a fixed support, to which is attached a vernier indicating on the scale the height of the point. The sliding rod is first run down until the point is well below the surface, and then gradually raised by means of the screw until the point just reaches the surface, which is indicated by the first appearance of a "pimple" in the water surface immediately over the hook. Under favorable circumstances a good hook-gauge may be read within from .0002 to .0005 foot.

(*j*) To avoid inaccuracies due to the **disturbance of the surface** by the current, by wind, etc., the level is sometimes taken (with the hook-gauge or otherwise) in a side chamber which communicates with the main channel of approach. The surface in the chamber maintains the same level as that in the channel itself, but is comparatively free from disturbance. Or a bucket communicating with the channel by means of a pipe, can be made to serve in the same way. Either may of course be sheltered from the wind. **Caution.** Messrs. Fteley and Stearns found that when the bucket or chamber communicated with the water *near the bottom and close behind the weir*, the head thus obtained was generally somewhat greater than that found by measurement near the surface and 6 feet back from the weir. But Mr. Francis found the difference scarcely perceptible.

(*k*) Great care is necessary in **adjusting the hook-gauge for the height of the crest;** for any error in this affects all the subsequent experiments. The hook is usually adjusted to the height of the surface when the latter just reaches the level of the crest; but this method is rendered inaccurate by capillary attraction at the crest. A more accurate method is to have, in addition to the hook-gauge, a stout *fixed* hook, pointing upward, the level of which, relatively to that of the crest, may be ascertained by means of an engineer's level, holding the rod on the crest and also on the point of the fixed hook. The water surface is then allowed to fall slowly until a "pimple" just appears over the fixed hook. It is then kept at that level and the hook-gauge adjusted accordingly. Or if the gauge-hook is a stout one, the levelling rod may be set at once upon its point without having recourse to a fixed hook. It is better to adjust the hook-gauge so as to read *zero* for the crest level, which is thus made the datum; for the reading of the hook-gauge for the water surface then gives the head H at once, and without subtracting the height of the crest.

(1) Formulæ for weir discharge.

Let

Q = the actual discharge over the weir, in cubic feet per second ; *

Q′ = the theoretical discharge over the weir, in cubic feet per second ,

H = **H′** † = the vertical distance or head *a m*, Fig. 24, p. 556, in feet,* measured from the crest *a* to the *horizontal* surface *o′* of *still* water up-stream from the weir;

L = the length *a a* of the weir, in feet,* Figs. 21 to 22 *a*;

g = the acceleration of gravity = say 32.2 feet * per second per second;

$$\left.\begin{array}{ll} c & = \text{coefficient of discharge} = \dfrac{\text{actual discharge}}{\text{theoretical discharge}} = \dfrac{Q}{Q'}; \\ m & = \dfrac{2}{3}\,c, \\ x & = \dfrac{2}{3}\,c\sqrt{2g} = m\sqrt{2g} = \text{say } 5.35\,c = \text{say } 8.025\,m. \end{array}\right\}‡$$

Then, for the **theoretical discharge,** we have

$$Q' = \frac{2}{3}\,L\,H\sqrt{2gH};\ § \quad \ldots\ldots\ldots \quad (1)$$

and for the **actual discharge,**

$$\left.\begin{array}{ll} Q & = c\,Q' \\ & = \dfrac{2}{3}\,c\,L\,H\sqrt{2gH} \quad \ldots\ldots\ldots\ldots\ldots \quad (2) \\ & = m\,L\,H\sqrt{2gH} \quad \ldots\ldots\ldots\ldots\ldots \quad (3) \\ & = x\,L\,H\sqrt{H} = x\,L\sqrt{H^3} = x\,L\,H^{\frac{3}{2}} \quad \ldots \quad (4) \end{array}\right\}\text{See foot-notes } * \dagger ‡$$

For the value of the coefficient (*c*, *m*, or *x* ‡) we have recourse to experiment, measuring the actual discharge and comparing it with the theoretical one, as in the following articles.

---

* The formulæ apply equally to any system of measures, as the English, the metric, etc. It is requisite merely that the units, of length, of time, etc., used, be the same throughout. In metric measure, g = 9.81 meters per second per second.

† *For the present* we suppose the head to be measured to *still* water, so that H = H′. When this is not the case, see "Velocity of Approach," Art. 14 (*u*), etc.

‡ It will be noticed that the formulæ (2), (3) and (4), with their corresponding coefficients, *c*, *m*, and *x*, are really identical, differing only in form. The last is the most convenient in practice, but all are met with in works on hydraulics.

§ When water issues, under a head H, from a *horizontal* orifice in the *bottom* of a vessel, the theoretical velocity (Art. 7, is $= \sqrt{2gH}$; and this may be regarded as true also for *vertical* orifices in the *sides* of vessels, provided the head H to the center of gravity of the orifice is at least two or three times the vertical dimension of the orifice; for in both cases the theoretical velocities through the several parts of the orifice may be taken as *equal*. But when a vertical orifice is nearer to the surface, or when it *reaches* to the surface as in the case of a weir, we must take into consideration the differences in the velocities with which the water issues from points at different depths.

Theoretically, the particles pass the oblique plane *a o′*, Fig. 23, in horizontal lines, with velocities ($= \sqrt{2gh} = 8.025\sqrt{h}$) proportional to the square roots of their several vertical depths *h* (not indicated in fig.) below still water surface at *o′*. Therefore if from *a m* we imagine horizontal lines *a a′*, *d d′*, *v v′*, *c c′*, etc., etc., to be drawn, representing all these velocities to any scale, then the outer ends *a′*, *d′*, *v′*, *c′*, etc., etc., of these lines will form, with *a m* and *a a′*, a parabolic segment *a m c′ a′*, the area of which is:

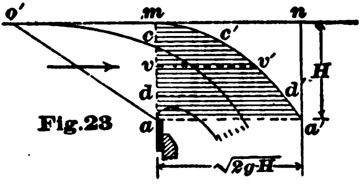


Fig.23

area $= \frac{2}{3}$ area of rectangle *a n* (see Parabola, p. 192) $= \frac{2}{3}\,a\,m \times a\,a' = \frac{2}{3}\,H\sqrt{2gH}$; and this area in square feet, multiplied by the thickness of the escaping sheet of (*m*) **Mr. James B. Francis*** experimented at "the lower locks," Lowell, Mass., in 1852, with weirs 10 feet long, 5 feet and 2 feet high, under heads from 7 to 19 inches. To apply his results, the following conditions must exist:

The head H, Fig. 20, must be between 6 and 24 inches. The height *p* of the vertical back of the weir above the bottom *b* of the channel must be at least twice the head H. The crest *a* must be "in thin partition" (foot-note † p. 541), and its length L, Figs. 21 to 22 *a*, must be at least 3 times the head H. The ends *a h*, *a h* must be vertical, and, when there is end contraction, "in thin partition."

When there is end contraction, Mr. Francis first deducts from the actual length L of the weir *one tenth*† *of the head* H for *each* end where contraction occurs. Thus, if n = the number of end contractions (two in Fig. 22),

$$Q = x \left(L - n\frac{H}{10}\right) H \sqrt{H} = x \left(L - n\frac{H}{10}\right) H^{\frac{3}{2}} \ddagger \quad \dots\dots \quad (5)$$

$$\text{In Fig. 22, } Q = x \left(L - \frac{H}{5}\right) H \sqrt{H}.\dagger = x \left(L - \frac{H}{5}\right) H^{\frac{3}{2}}$$

But within the limits specified above, the formula is very approximate *without* correction for end contraction, provided the length L of the weir is at least 10 times the head H; and within 6 per cent. of the truth when L is = 4 H. When there is no end contraction, of course no such correction is required, and the formula remains $Q = x\, L\, H \sqrt{H} \ddagger = x\, L\, H^{\frac{3}{2}}$.

Mr. Francis gives x = 3.33 for *feet*; § or

$$\textbf{Discharge} = 3.33 \times \left(\text{length} - \frac{\text{number of}}{\text{end contractions}} \times \frac{\text{head } H}{10}\right) \times H^{\frac{3}{2}}.\ddagger$$

the mean of his 88 experiments being 3.3318. The least value of x obtained by him was 3.3002, or 1 in 112 less than 3.33; and the greatest was 3.3617, or 1 in 105 more than 3.33. Hence, with $x = 3.33$, the formula will give the discharge for each of his experiments within 1 per cent. In 67 out of the 88 experiments x ranged between 3.32 and 3.35, and in 53 between 3.32 and 3.34. When x is 3.33, **m** is = 0.415, and **c** is = 0.622.

The height of the surface was measured six feet back from the weir by two hook-gauges, one on each side of the channel; and the mean of their readings was used in calculating the coefficient x.

---

water, or length L of weir, in feet, gives the theoretical discharge in cubic feet per second. Or

$$Q' = L \times \text{area } a\,m\,c'\,a' = L \times \frac{2}{3} H \sqrt{2\,g\,H}.$$

Hence, area *a m c′ a′* in sq. ft. represents the theoretical disch. in cub. ft. per sec. over 1 ft. length of weir, under head H. The theoretical *mean vel.* through the section *a o′* is

$$V = v\,v' = \frac{\text{theoretical discharge } Q'}{\text{area } a\,m} = \frac{2}{3} L\, H \sqrt{2\,g\,H} + L\, H = \frac{2}{3} \sqrt{2\,g\,H},$$

or two thirds of the theoretical horizontal velocity *a a′* of the particles passing immediately over the weir. As in the case of orifices (Art. 9, p. 541), the actual vel. *at the smallest section* of the sheet after passing the weir (corresponding to the "vena contracta") is probably very nearly equal to this theoretical velocity.

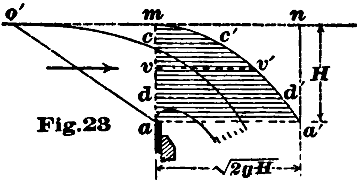


Fig.23

* "Lowell Hydraulic Experiments," Van Nostrand, New York, 1883.

† In Messrs. Fteley and Stearns' experiments this figure was not constant at 0.10, but varied between 0.061 and 0.124, generally increasing as the head decreased.

‡ We here suppose the head to be measured to the surface of *still* water, so that H and H′ (see Art. 14 *h*, p. 548) are the same. See Velocity of Approach, Art. 14 (*u*).

§ Since 1 meter = 3.2808 ft., the value of x for metric measure corresponding to Mr. Francis' 3.33, is $= 3.33 + \sqrt{3.2808} = 3.33 + 1.8113 = 1.838$.

'able 13.* Discharge in cubic feet per second for each foot length of weir in thin plate and without end contraction, by the Francis nula: **Discharge,** $Q = 3.33\ L\ H^{\frac{3}{2}} = 3.33\ L\ H\sqrt{H}$.

ery approximate also when there is end contraction, provided that L is at t = 10 H; and but about 6 per cent. in excess of the truth if L = 4 H. Mr. ncis limits the formula to heads H from 0.5 foot to 2.0 feet, but no serious ›r will result from using the table for any of the heads given. **For weirs other lengths** than 1 foot, multiply the tabular discharge by the actual ;th in feet. .01 foot = 0.12 inch = scant ⅛ inch.

l, H, ft.	Cub. ft. per sec.	Head, H, in ft.	Cub ft. per sec.	Head, H, in ft.	Cub. ft. per sec.	Head, H, in ft.	Cub. ft. per sec.	Head, H. in ft.	Cub. ft. per sec.
01	0.003	.51	1.213	1.01	3.380	1.51	6.179	2.01	9.489
02	0.009	.52	1.249	1.02	3.430	1.52	6.240	2.02	9.560
03	0.017	.53	1.285	1.03	3.481	1.53	6.302	2.03	9.631
04	0.027	.54	1.321	1.04	3.532	1.54	6.364	2.04	9.703
05	0.037	.55	1.358	1.05	3.583	1.55	6.426	2.05	9.774
06	0.049	.56	1.395	1.06	3.634	1.56	6.488	2.06	9.846
07	0.062	.57	1.433	1.07	3.686	1.57	6.551	2.07	9.917
08	0.075	.58	1.471	1.08	3.737	1.58	6.613	2.08	9.989
09	0.090	.59	1.509	1.09	3.790	1.59	6.676	2.09	10.062
.0	0.105	.60	1.548	1.10	3.842	1.60	6.739	2.10	10.134
.1	0.121	.61	1.586	1.11	3.894	1.61	6.803	2.11	10.206
.2	0.138	.62	1.626	1.12	3.947	1.62	6.866	2.12	10.279
.3	0.156	.63	1.665	1.13	4.000	1.63	6.930	2.13	10.352
.4	0.174	.64	1.705	1.14	4.053	1.64	6.994	2.14	10.425
.5	0.193	.65	1.745	1.15	4.107	1.65	7.058	2.15	10.498
.6	0.213	.66	1.786	1.16	4.160	1.66	7.122	2.16	10.571
.7	0.233	.67	1.826	1.17	4.214	1.67	7.187	2.17	10.645
.8	0.254	.68	1.867	1.18	4.268	1.68	7.251	2.18	10.718
.9	0.276	.69	1.909	1.19	4.323	1.69	7.316	2.19	10.792
20	0.298	.70	1.950	1.20	4.377	1.70	7.381	2.20	10.866
21	0.320	.71	1.992	1.21	4.432	1.71	7.446	2.21	10.940
22	0.344	.72	2.034	1.22	4.487	1.72	7.512	2.22	11.015
23	0.367	.73	2.077	1.23	4.543	1.73	7.577	2.23	11.089
24	0.392	.74	2.120	1.24	4.598	1.74	7.643	2.24	11.164
25	0.416	.75	2.163	1.25	4.654	1.75	7.709	2.25	11.239
26	0.441	.76	2.206	1.26	4.710	1.76	7.775	2.26	11.314
27	0.467	.77	2.250	1.27	4.766	1.77	7.842	2.27	11.389
28	0.493	.78	2.294	1.28	4.822	1.78	7.908	2.28	11.464
29	0.520	.79	2.338	1.29	4.879	1.79	7.975	2.29	11.540
30	0.547	.80	2.383	1.30	4.936	1.80	8.042	2.30	11.615
31	0.575	.81	2.428	1.31	4.993	1.81	8.109	2.31	11.691
32	0.603	.82	2.473	1.32	5.050	1.82	8.176	2.32	11.767
33	0.631	.83	2.518	1.33	5.108	1.83	8.244	2.33	11.843
34	0.660	.84	2.564	1.34	5.165	1.84	8.311	2.34	11.920
35	0.690	.85	2.610	1.35	5.223	1.85	8.379	2.35	11.996
36	0.719	.86	2.656	1.36	5.281	1.86	8.447	2.36	12.073
37	0.749	.87	2.702	1.37	5.340	1.87	8.515	2.37	12.150
38	0.780	.88	2.749	1.38	5.398	1.88	8.584	2.38	12.227
39	0.811	.89	2.796	1.39	5.457	1.89	8.652	2.39	12.304
40	0.842	.90	2.843	1.40	5.516	1.90	8.721	2.40	12.381
41	0.874	.91	2.891	1.41	5.575	1.91	8.790	2.41	12.459
42	0.906	.92	2.939	1.42	5.635	1.92	8.859	2.42	12.536
43	0.939	.93	2.987	1.43	5.694	1.93	8.929	2.43	12.614
44	0.972	.94	3.035	1.44	5.754	1.94	8.998	2.44	12.692
45	1.005	.95	3.083	1.45	5.814	1.95	9.068	2.45	12.770
46	1.039	.96	3.132	1.46	5.875	1.96	9.138	2.46	12.848
47	1.073	.97	3.181	1.47	5.935	1.97	9.208	2.47	12.927
48	1.107	.98	3.231	1.48	5.996	1.98	9.278	2.48	13.005
49	1.142	.99	3.280	1.49	6.057	1.99	9.348	2.49	13.084
50	1.177	1.00	3.330	1.50	6.118	2.00	9.419	2.50	13.163

Table 13 is an extension of the "original" table published in our first edition, 2. Most of the values now given are taken, by permission, from a table published Messrs. A. W. Hunking and Frank S Hart, of Lowell, Mass., in May, 1884.

(*n*) **Messrs. A. Fteley and F. P. Stearns** * experimented at Boston, Mass., in 1877–79, upon weirs 5 feet and 19 feet long, 3 feet 2 inches and 6 feet 6½ inches high, and under heads from 0.8 inch to 19 inches. For weirs in thin partition and without end contraction, with a rectangular and uniform channel of approach and under heads greater than 0.07 foot or 0.84 inch (other conditions as specified in (*b*) and (*d*)), their formula is:

$$\textbf{Discharge, } Q = 3.31\, L\, H^{\frac{3}{2}} + 0.007\, L \quad = 0.4125\, L\, H \sqrt{2\,g\,H} + 0.007\, L \;\Big\}\dagger \quad \ldots \; (6).$$

In their experiments, the heads were measured six feet back from the weir. The total variation in the values of the coefficients obtained was about 2½ per cent. Compare foot-note § below.

(*o*) **M. Bazin** ‡ experimented at Dijon, France, in 1886–88, with weirs from about 1½ to 6½ feet long, from about 9 inches to 3 feet 9 inches high, and under heads from 2½ to 21 inches. The top of the weir is shown in Fig. 23 *a*. The weirs were placed at different points in a rectangular and regular canal 700 feet long, smoothly lined with cement. Compare foot-note § below.

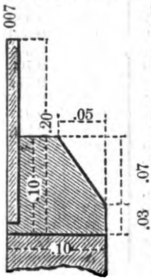


Fig. 23 *a*.
Measurements in meters.

While Mr. Francis and Messrs. Fteley and Stearns provide for the effect of velocity of approach (see Art. 14 *w* and *v*) by modifying the measured *head* H, M. Bazin includes it in the *coefficient* *m* in the formula $Q = m\, L\, H \sqrt{2\,g\,H}$. After comparing his experiments with those of Messrs. Fteley and Stearns, (Art. 14 *n* and *v*), he gives, for *m*, in cases where there is no velocity of approach, the values M in the last column of Table 14, or, very approximately,

$$\left(\begin{matrix}\text{No velocity of approach,}\\ \text{H and H}'\text{ identical}\end{matrix}\right)\; M = 0.405 + \frac{0.003}{H}.$$

When velocity of approach is to be taken into account:

$$\mathbf{m} = M\left[1 + 0.55\left(\frac{H}{H+p}\right)^2\right] \ldots\ldots \; (7);$$

where H is the head actually measured to running water, and *p* is the height *a b* of the weir, Fig. 20. H and *p* must of course both be measured in the same unit, as both in meters, or both in feet, etc.

M. Bazin believes that except in the case of very low weirs (which should be avoided) the values of *m* given by formula (7) and in Table 14 calculated from it, will be found within 1 per cent. of the truth for weirs in thin partition and without end contraction, if the conditions of his experiments are exactly reproduced, and provided especially that the sheet of water is not allowed to expand laterally after passing the crest (Art. 14 (*d*)) and that the air has free access to the space *w*, Fig. 20, behind the falling sheet of water.

For heads between 4 inches and 1 foot, M. Bazin gives, as **sufficiently approximate,**

when there is no velocity of approach, $M = 0.425$,§

and, to allow for velocity of approach, $\mathbf{m} = 0.425 + 0.21\left(\frac{H}{H+p}\right)^2$.

---

* Transactions, American Society of Civil Engineers, Jan., Feb. and March, 1883.

† See correction for Velocity of Approach, Art. 14 (*u*).

‡ Expériences nouvelles sur l'écoulement en déversoir. Extrait des Annales des Ponts et Chaussées, Oct., 1888. Paris, Vve Ch. Dunod, 1888. Translation by A. Marichal and John C. Trautwine, Jr., presented to Engineers' Club of Philadelphia, in 1889, for publication in its Proceedings.

§ This would make $x = 3.41$ (since $x = m\sqrt{2\,g} = 8.025\, m$); whereas Mr. Francis gives $x = 3.33$, which agrees very well with Messrs. Fteley and Stearns, within the limits of Art. 14 (*m*). Yet M. Bazin measured the head 16 feet back from the weir, while the other experimenters measured it only 6 feet back, and the slight increase of head thus obtained by M. Bazin would of itself have made his coefficient *lower* than theirs. Its excess may be largely due to the character of the channel of approach, which in his case was from 50 to 700 feet long, rectangular and regular in cross-section, and smoothly coated with cement. In the other experiments it was much less regular.

Table 14. Values of m, in the formula

$$Q = m\, L\, H \sqrt{2 g H} \; . \; . \; . \text{ or}$$

$$\underset{\text{in cub. ft. per sec.}}{\textbf{Discharge,}} = m \times \text{length, in ft.} \times \text{meas'd head H, ft.}^{*} \times \sqrt{2\, g\, \text{H, ft}};$$

by M. Bazin's formula (7): $m = \text{M}\dagger \left[1 + 0.55 \left(\frac{\text{H}}{\text{H} + p}\right)^2\right]$. For M, see last column of table. Or, very approximately, $\text{M} = 0.405 + \frac{0.003}{\text{H}}$.

NOTE. The coefficient *m*, for any given case, remains the same for English, metric or other measure; provided the head, the length and *g* are all measured in the same unit, and the discharge in the cube of that unit; for *m* is simply $\frac{2}{3}\, c = \frac{2}{3} \times \frac{\text{actual discharge}}{\text{theoretical discharge}}$.

It will be noticed that **below the heavy lines** the head H is greater than ½ height *p*, and thus exceeds the limit laid down in (f) and (m).

Head H, Fig. 24, p. 556.			Height, *p*, Fig. 20, of crest of weir above bed of up-stream channel.										
	approximate		meters	0.20	0.30	0.40	0.50	0.60	0.80	1.00	1.50	2.00	
Meters.	feet.	inches.	approx. feet	0.656	0.984	1.312	1.640	1.969	2.624	3.280	4.920	6.560	In units.
			approx. inches	7.87	11.81	15.75	19.69	23.62	31.50	39.38	59.07	78.76	
				m	m	m	m	m	m	m	m	m	M†
.05	.164	1.97		.458	.453	.451	.450	.449	.449	.449	.448	.448	.4481
.06	.197	2.36		.456	.450	.447	.445	.445	.444	.443	.443	.443	.4427
.07	.230	2.76		.455	.448	.445	.443	.442	.441	.440	.440	.439	.4391
.08	.262	3.15		.456	.447	.443	.441	.440	.438	.438	.437	.437	.4363
.09	.295	3.54		.457	.447	.442	.440	.438	.436	.436	.435	.434	.4340
.10	.328	3.94		.459	.447	.442	.439	.437	.435	.434	.433	.433	.4322
.12	.394	4.72		.462	.448	.442	.438	.436	.433	.432	.430	.430	.4291
.14	.459	5.51		.466	.450	.443	.438	.435	.432	.430	.428	.428	.4267
.16	.525	6.30		.471	.453	.444	.438	.435	.431	.429	.427	.426	.4246
.18	.591	7.09		.475	.456	.445	.439	.435	.431	.428	.426	.425	.4229
.20	.656	7.87		.480	.459	.447	.440	.436	.431	.428	.425	.423	.4215
.22	.722	8.66		.484	.462	.449	.442	.437	.431	.428	.424	.423	.4208
.24	.787	9.45		.488	.465	.452	.444	.438	.432	.428	.424	.422	.4194
.26	.853	10.24		.492	.468	.455	.446	.440	.432	.429	.424	.422	.4187
.28	.919	11.02		.496	.472	.457	.448	.441	.433	.429	.424	.422	.4181
.30	.984	11.81		.500	.475	.460	.450	.443	.434	.420	.424	.421	.4174
.32	1.050	12.60		—	.478	.462	.452	.444	.436	.430	.424	.421	.4168
.34	1.116	13.39		—	.481	.464	.454	.446	.437	.431	.424	.421	.4162
.36	1.181	14.17		—	.483	.467	.456	.448	.438	.432	.424	.421	.4156
.38	1.247	14.96		—	.486	.469	.458	.449	.439	.432	.424	.421	.4150
.40	1.312	15.75		—	.489	.472	.459	.451	.440	.433	.424	.421	.4144
.42	1.378	16.54		—	.491	.474	.461	.453	.441	.434	.425	.421	.4139
44	1.444	17.32		—	.494	.476	.463	.454	.442	.435	.425	.421	.4134
.46	1.509	18.21		—	.496	.478	.465	.456	.443	.435	.425	.421	.4128
.48	1.575	18.90		—	—	.480	.467	.457	.444	.436	425	.421	.4122
.50	1.640	19.69		—	—	.482	.468	.459	.445	.437	.426	.421	.4118
.52	1.706	20.47		—	—	.483	.470	.460	.446	.438	.426	.421	.4112
.54	1.772	21.26		—	—	.485	.472	.461	.447	.438	.426	.421	.4107
.56	1.837	22.05		—	—	.487	.473	.463	.448	.439	.427	.421	.4101
.58	1.903	22.83		—	—	.489	.475	.464	.449	.440	.427	.421	.4096
.60	1.969	22.62		—	—	.490	.476	.466	.451	.441	.427	.421	.4092

Owing to the wide range of the head H and of the height *p* in these experiments, we find in them **a wider divergence** in the values of the coefficient than resulted from the earlier investigations. Thus, the smallest value of *m* above the heavy lines is 0.4092, or about one nineteenth less than the mean, 0.4325; and the greatest is 0.459, or about one sixteenth more than 0.4325.

---

* In these experiments, the head H was measured at a point 5 meters (16.4 ft) back from the weir. The correction for velocity of approach is contained in the coefficient *m*.

† M is the value of *m* when there is no velocity of approach; *i. e.*, where the cross-section of the channel of approach is indefinitely great compared with that of the stream of water passing over the weir.

(*p*) From a comparison of a number of experimental data, **the author** deduced the following

Table 15. Approximate values of the coefficient m in the formula:

$$Q = m\,L\,H\sqrt{2gH},$$

for weirs of several different shapes and thicknesses. (Original.)

Head, H. Feet.	Head, H. Inches.	Sharp Edge.*	2 Inches thick.	3 ft. thick; smooth; sloping outward and downward, from 1 in 12 to 1 in 18.	3 ft. thick; smooth; and level.
		m	m	m	m
.0833	1	.41	.37	.32	.27
.1666	2	.40	.38	.34	.30
.25	3	.40	.39	.34	.31
.3333	4	.40	.41	.35	.31
.4166	5	.40	.41	.35	.32
.5	6	.39	.41	.35	.33
.5833	7	.39	.41	.35	.32
.6666	8	.39	.41	.34	.31
.8333	10	.38	.40	.34	.31
1.	12	.38	.40	.33	.31
2.	24	.37	.39	.32	.30
3.	36	.37	.39	.32	.30

(*q*) **To find the head H, approximately;** having the discharge Q. According to formulæ (3) and (4), Art. 14 (*l*).

$$Q = m\,L\,H\sqrt{2gH} = x\,L\,H\sqrt{H} = m\,L\sqrt{2g}\sqrt{H^3}, = x\,L\sqrt{H^3},$$

Hence

$$H = \sqrt[3]{\frac{Q^2}{m^2\,L^2\,2\,g}} = \sqrt[3]{\frac{Q^2}{x^2\,L^2}} \quad \ldots\ldots \quad (8)$$

or

$$\textbf{Head, } H, \text{ approximately} = \sqrt[3]{\frac{\text{square of discharge of stream, in cub. ft. per sec.}}{m^2 \times \text{length}^2 \times 64.4}}$$

$$= \sqrt[3]{\frac{\text{sq. of discharge}}{x^2 \times \text{length}^2}}.$$

The coefficient m or x itself varies somewhat with the head; but the formula may be usefully employed as an approximation by taking, for sharp-crested weirs, $m = 0.415$ ($m^2 = 0.172$) or $x = 3.33$ ($x^2 = 11$). For other shapes, see Table 15, above.

(*r*) **Submerged weirs,** Fig. 23 *b*, are those in which the surface of the *down*-stream water at *h*, after the construction of the weir, is *higher* than the crest *a*.

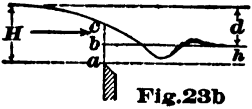


Fig.23b

In a weir discharging freely into the air, as in Fig. 20, Mr. Francis found that with a head of 1 foot the discharge was diminished only about one thousandth part by placing a solid horizontal floor about 6 inches below and in front of the crest of the weir for the water to fall upon. Also, when the head was 10 inches, and the water fell freely through the air into water of considerable depth (as in Fig. 20), the quantity discharged was the same whether the surface of the down-stream water was about 3 inches or about 13 inches below the crest *a*.

In experiments by Mr. Francis and by Messrs. Fteley and Stearns, with air freely admitted underneath the falling sheet of water just below the crest *a*, the discharge was not appreciably affected by a submergence of h = from 0.017 H to 0.023 H. When air was only partially admitted, the discharge was affected (*increased*) by less than one per cent. while h remained less than 0.15 H.

---

* These values are lower than those given in Art. 14 (*m*) and (*n*), and much lower than those in (*o*).

ubuat's formula for submerged weirs. Let

and h = the heads measured vertically from the crest a of the weir to the surface of still water * up-stream and down-stream from the weir, respectively.

: H − h = their difference = the difference in level between the up-stream and down-stream surfaces of still-water;*

$$= \text{coefficient of discharge} = \frac{\text{actual discharge}}{\text{theoretical discharge}}.$$

hen

$$Q = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d};\dagger \quad \ldots\ldots\ldots \quad (9) \text{ or:}$$

$$\textbf{ual discharge} \atop \text{cub. ft. per sec.} = c \times \frac{\text{length of}}{\text{weir in ft.}} \times 8.025\,\sqrt{d \text{ in ft.}} \times \left(h \text{ in ft.} + \frac{2}{3}\,d \text{ in ft.}\right).$$

) **Messrs. Fteley and Stearns** ‡ experimented at Boston in 1877 with **bmerged weirs** under up-stream heads H from about 4 to 10 inches; and . **Francis** § at Lowell in 1883 under heads from about 1 foot to 2 feet 4 hes.

rom these experiments we deduce the following

able 16, of approximate values of the **coefficient c in** the formula for charge over **submerged weirs.**

$$Q = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d}.$$

educed from experiments by Fteley and Stearns and by J. B. Francis. In . Francis' experiments, the value of c for a given value of h + H generally reased as H increased.

	Fteley and Stearns. (H = 0.325 to 0.815 feet.)	J. B. Francis. (H = 1 to 2.32 feet.)
h + H	c	c
.05	. . . . . .	.623 to .632
.10	.625 to .635	.620 to .630
.20	.618 to .628	.610 to .625
.30	.600 to .610	.598 to .615
.40	.590 to .600	.586 to .610
.50	.585 to .595	.585 to .607
.60	.583 to .593	.585 to .607
.70	.580 to .590	.585 to .607
.80	.581 to .591	.585 to .607
.90	.590 to .600	. . . . . .
.95	.610 to .615	. . . . . .

For velocity of approach, see Art. 14 (*u*) etc.

In deducing this formula, the water that passes over the weir between c and b is ımed to flow as over a weir with its crest at b, and with free discharge into the air, ver the crest a in Fig. 20; and for this portion, by formula (2) in Art. 14 (*l*), the harge would be:

$$Q_b = c\,L\,\frac{2}{3}\,d\,\sqrt{2\,g\,d};$$

le the water that passes through the lower portion between b and a is regarded as ing through a submerged vertical *orifice* whose height is $b\,a = h$, under a head . For this lower portion, therefore, the discharge would be:

$$Q_a = c\,L\,h\,\sqrt{2\,g\,d}.$$

is assumed that the coefficient of discharge c is the same for the upper section s for the lower one $a\,b$. Hence, adding these two discharges together, we obtain, he entire disharge:

$$Q = Q_b + Q_a = c\,L\left(h + \frac{2}{3}\,d\right)\sqrt{2\,g\,d}.$$

Transactions, American Society of Civil Engineers, March, 1883, p. 101, etc.

Transactions, American Society of Civil Engineers, Sept., 1884, p. 295, etc.

(*t*) **Mr. Clemens Herschel,**[*] **comparing these experiments with some** earlier ones by Mr. Francis, gives the following:

Having ascertained the depths H and h of the crest below the still-water levels up-stream and down-stream respectively, divide h by H. Find the quotient, as nearly as may be, in the column headed $h \div H$ in Table 17. Take out the corresponding coefficient a, and multiply it by the up-stream head H.[†]

The product a H is the head which would cause the given weir to discharge the same quantity freely *into the air*, as in Fig. 20. Find the discharge into air over the given weir with the head a H; and this discharge will be approximately the same as that of the actual *submerged* weir under the up-stream head H and against the down-stream head h; or (H being the actual up-stream head on the *submerged* weir) the discharge is

$$Q = m\,L\,a\,H\,\sqrt{2\,g\,a\,H} = x\,L\,a\,H\,\sqrt{a\,H} \; . \; . \; . \; . \; . \; . \quad (10).$$

TABLE 17.

$h \div H$	a	$h \div H$	a	$h \div H$	a
.10	1.000 to 1.010	.45	0.894 to 0.930	.72	0.762 to 0.784
.20	0.975 to 0.995	.50	0.874 to 0.910	.74	0.747 to 0.769
.25	0.960 to 0.984	.55	0.853 to 0.889	.76	0.732 to 0.752
.30	0.945 to 0.973	.60	0.829 to 0.863	.78	0.713 to 0.733
.35	0.928 to 0.960	.65	0.803 to 0.833	.80	0.693 to 0.713
.40	0.912 to 0.946	.70	0.775 to 0.799		

(*u*) **Velocity of approach.** See Fig. 24. It is generally impracticable to measure the head H′ to perfectly still water o' up-stream. The head is usually measured at a point o, from 2 or 3 to 6 or 8 feet or more up-stream from the weir, according to the size of the latter. At such points the velocity is generally appreciable, and the surface therefore a little lower than at o'. Hence a formula using the smaller head H so measured, instead of H′, and coefficients based upon H′, will give too small a discharge. Mr. Francis found that a current of 1 foot per second, or nearly 0.7 mile per hour, at the point o to which the head was measured, increased the discharge but about 2 per cent. when the head was 13 inches; and a current of 6 inches per second increased the discharge about 1 per cent. when the head was 8 inches.

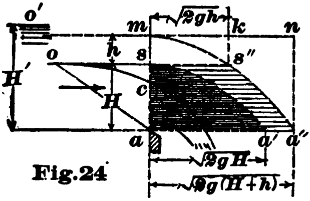


Fig. 24

If, however, the velocity of approach is such as to require consideration, proceed as follows: For the approximate mean velocity of approach, we have:

$$v = \frac{\text{approximate discharge}}{\text{area of entire cross section of stream at } o} = \frac{3.33\ L\ H^{\frac{3}{2}}}{\text{area at } o};$$

and, for the head due to this velocity, $h = \dfrac{v^2}{2g}$

Then, for all practical purposes, we may say: $H' = H + h$; or

$$Q = m\,L\,(H + h)\,\sqrt{2\,g\,(H + h)} = x\,L\,(H + h)^{\frac{3}{2}} \; . \; . \; . \; . \; . \; . \quad (11)$$

although, strictly speaking, the difference of level between o' and o is really (as shown in Fig. 24) somewhat *greater* than h, or than $v^2 \div 2\,g$, because some head is lost in friction between o' and o.

---

[*] Transactions, American Society of Civil Engineers, May, 1885, pp. 189, etc.

[†] Mr. Herschel's table, from which ours is condensed, gives a for every 0.01 foot of $h \div H$; but the values of a intermediate of those we have selected may be taken from our table almost exactly by simple proportion. The range in the coefficient a in the table for each value of $h \div H$ is that indicated by the experiments, which varied similarly. We are not instructed how to select between these extremes; but in most cases their *mean* value is probably nearest right.

) **Messrs. Fteley and Stearns** make $H' = H + 1.5\,h$ for suppressed rs, and $H' = H + 2.05\,h$ for weirs with complete end contractions, as averages; . Mr. Hamilton Smith, Jr.,* after comparing their experiments with others Lesbros, Castel and Mr. Francis, gives $H' = H + 1\frac{1}{3}\,h$, and $H' = H + 1.4\,h$, for two cases respectively.

) On the other hand, Mr. Francis' formula, as modified for velocity of roach,

$$= x\,L\dagger\,\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right) = m\,L\dagger\sqrt{2g}\,\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right)\ddagger \quad . \; . \; . \quad (12),$$

kes the effect of H' *less* than that of $H + h$.

) **Messrs. A. W. Hunking and Frank S. Hart,** Civil and Hy-ulic Engineers, have substituted for the expression $(\sqrt{(H+h)^3} - \sqrt{h^3})$ in nula (12), the equivalent one $K\,\sqrt{H^3}$, in which K is a coefficient deduced from former expression, and therefore depending upon the relation between H h, or, ultimately, upon that between the cross-section $a\,s$ Fig. 24 at the weir . the entire cross-section of the stream at o.

laving found the area of cross-section at o, divide it by $\left(L - n\frac{H}{10}\right)$, which he length of the weir corrected for contraction. See Art. 14 (m). Call the tient D.§ Divide the measured head H by D. Find this last quotient in column $\frac{H}{D}$ of the table. Multiply the approximate discharge, $Q = 3.33$ $- n\frac{H}{10}\Big)\,H^{\frac{3}{2}}$,by the corresponding coefficient K; or

$$\textbf{Actual Discharge}\ Q = 3.33\,K\left(L - n\frac{H}{10}\right)H^{\frac{3}{2}}. \quad . \; . \; . \; . \; . \quad (13)$$

able 18. Coefficient K in formula (13).

	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K
1	1.0000	.09	1.0020	.17	1.0072	.24	1.0143	.31	1.0239
2	1.0001	.10	1.0025	.18	1.0081	.25	1.0155	.32	1.0254
3	1.0002	.11	1.0030	.19	1.0090	.26	1.0168	.33	1.0271
4	1.0004	.12	1.0036	.20	1.0100	.27	1.0181	.34	1.0287
5	1.0006	.13	1.0042	.21	1.0110	.28	1.0195	.35	1.0305
6	1.0009	.14	1.0049	.22	1.0121	.29	1.0209	.36	1.0322
7	1.0012	.15	1.0056	.23	1.0132	.30	1.0224	.37	1.0341
8	1.0016	.16	1.0064						

"Hydraulics," John Wiley & Sons, New York, 1886.

If there are end contractions, L here becomes $\left(L - n\frac{H}{10}\right)$. See Art. 14 ($m$).

This formula is deduced as follows: Let the area of the parabolic segment $a\,s\,a'$, . 24, represent the theoretical discharge over a weir one foot long (as explained in t-note § p. 549) under the measured head $H = a\,s$, as though there were no current . Let $m\,s = h = v^2 \div 2\,g$. The theoretical velocities of the particles passing the ique plane $o\,a$ under their actual heads, will now be represented by horizontal lines ', $a\,a''$, etc., etc., drawn from every point in $s\,a$ to the *outer* curve $s''\,a''$; the line $s\,s''$ resenting v = velocity of approach $= \sqrt{2\,g\,h}$, and $a\,a''$ representing $\sqrt{2\,g\,(H+h)}$. en, area $s\,s''\,a''\,a$

$$\text{area } m\,a''\,a - \text{area } m\,s''\,s = \frac{2}{3}\text{ area of rectangle } a\,n - \frac{2}{3}\text{ area of rectangle } s\,k$$

$$\frac{2}{3}\left(H + h\right)\sqrt{2\,g\,(H+h)} - \frac{2}{3}\,h\,\sqrt{2\,g\,h}, = \frac{2}{3}\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right);$$

d the actual discharge is

$$= c \times \text{length of weir} \times \text{area } s\,s''\,a''\,a = c\,L\,\frac{2}{3}\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right)$$

$$m\,L\,\sqrt{2\,g}\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right) = x\,L\left(\sqrt{(H+h)^3} - \sqrt{h^3}\right).$$

§ In a weir without end contraction, $D = H + p$.

K is very approximately $= 1 + \frac{1}{4}\left(\frac{H}{D}\right)^2$. Hence

$$Q = 3.83\left[1 + \frac{1}{4}\left(\frac{H}{D}\right)^2\right]\left(L - n\frac{H}{10}\right)H^{\frac{3}{2}}$$

$$= \left[3.33 + 0.83\left(\frac{H}{D}\right)^2\right]\left(L - n\frac{H}{10}\right)H^{\frac{3}{2}}. \quad . \quad . \quad . \quad (14)$$

See Journal of the Franklin Institute, Philadelphia, August, 1884, from which we condense the above table.

(y) **M. Bazin,** see Art. 14 (o), provides for the velocity of approach by modifying the coefficient m instead of the head H, making $m = 0.425 + 0.21\left(\frac{H}{D}\right)^2$; while by Messrs. Hunking and Hart's method (based upon Mr. Francis' experiments) m becomes $= 0.415 + 0.10\left(\frac{H}{D}\right)^2$.

Art. 15. Inclined weirs. If the up-stream face of the weir, instead of being vertical, as in Fig. 25, is inclined up-stream, as in Fig. 25 *a*, or down-stream, as in Fig. 25 *b*, the character and amount of the discharge are modified. With an up-stream inclination (Fig. 25 *a*) the lower side of the sheet of water passing over the weir leaps higher, and tends more and more up-stream as the inclination is increased. With a down-stream inclination (Fig. 25 *b*), on the contrary, as the inclination increases the upward leap of the sheet decreases, its profile becomes more and more flattened, and the curve of the upper surface, due to the fall, extends farther up-stream from the crest of the weir.

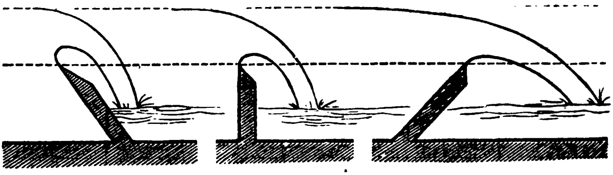


FIG. 25 *a*. **Inclined up-stream.**

FIG. 25. **Vertical.**

FIG. 25 *b*. **Inclined down-stream.**

An up-stream inclination (Fig. 25 *a*) decreases, and a down-stream one (Fig. 25 *b*) increases, the discharge, as is indicated by the following coefficients obtained by M. H. Bazin:*

For the discharge over an inclined weir, having ascertained the discharge over a vertical weir of the same height and head and under similar conditions in other respects, multiply the discharge over the vertical weir by the following approximate coefficients:

	Inclination.		Angle		Coefficient.
	Horizontal.	Vertical.	with hor.	with vert.	
Weirs inclined up-stream, Fig. 25 *a*....	1	1	45°	45°	0.93
	$\frac{2}{3}$	1	56° 19′	33° 41′	0.94
	$\frac{1}{3}$	1	71° 34′	18° 26′	0.96
Vertical weirs, Fig 25...	0	1	90°	0°	1.00
Weirs inclined down-stream, Fig. 25 *b*.....	$\frac{1}{3}$	1	71° 34′	18° 26′	1.04
	$\frac{2}{3}$	1	56° 19′	33° 41′	1.07
	1	1	45°	45°	1.10
	2	1	26° 34′	63° 26′	1.12

---

* "Expériences Nouvelles sur l'Écoulement en Déversoir," 2e Article; "Annales des Ponts et Chaussées," January, 1890, translated in *Proceedings, Engineers' Club of Philadelphia*, vol. ix., 1892.

The discharge will be increased also if the inner corner or edge of the crest rounded off, instead of being left sharp; or if the sides of the reservoir con- rge more or less as they approach the weir, so as to form wings for guiding e water more directly to it: or if $a\,b$, Fig 20, be less than twice $a\,m$. Indeed, many modifying circumstances exist to embarrass experiments on this and nilar subjects that some of those which have been made with great care are ndered inapplicable as other than tolerable approximations, in consequence the neglect to take into consideration some local peculiarity which was not the time regarded as exerting an appreciable effect. Unless, therefore, cir- mstances admit of our combining all the conditions mentioned in Art. 14 (*d*),) and (*m*), pp. 547, 548 and 550, thereby securing *very* approximate results, we ist either resort to an actual measurement of the discharge in a vessel of own capacity; or else be content with rules which may lead to errors of 5, or more per cent. in proportion as we deviate from these conditions. Fre- ently even 10 per cent. of error may be of little real importance.

Remark 1. When the water, after passing over a weir, Fig. 26, instead of fall- g freely into the air, is carried away by a slightly inclined apron or trough, T, e floor of which coincides with the crest *a*, of the weir, then the discharge is t appreciably diminished thereby when the head $a\,m$, is 15 inches or more. t if the head $a\,m$ is but 1 foot, then the calculated discharge must be reduced out one-tenth; if 6 inches, two-tenths; if 2½ inches, three-tenths; and if 1 h, five-tenths, or one-half, as approximations.

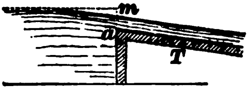


Fig. 26.

Remark 2. Professor Thomson, of Dublin, proposed the use of triangular tches, or weirs, for measuring the discharge; inasmuch as then the periphery vays bears the same ratio to the area of the stream flowing over it; which is t the case with any other form. Experimenting with a right-angled triangular tch in thin sheet-iron, Fig. 26 A, with heads of from 2 to 7 inches, measured rtically from the bottom of the notch to *the level surface of the quiet water*, he ind *discharge* in cubic feet per second = $.0051 \times \sqrt{\text{fifth power of head in inches}}$. 2.54 × $\sqrt{\text{fifth power of head in feet}}$.* Or, in general, if m = coefficient of itraction (Art. 9, p. 541) T = tangent of half the angle of the notch = width water-surface ÷ the depth in the notch. g = the acceleration of gravity = say feet per second, and h = the head, measured as above; then

$$\text{Discharge} = \frac{8m}{15}\,T\,\sqrt{2gh^5}^{\,*} = 4.28\,m\,T\,\sqrt{h^5}.^{*}$$

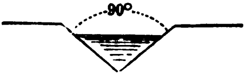


Fig. 26 A.

emark 3. In constructing the irrigating canal, Canale Villoresi, near Milan, 1881–4, the Italian engineer, Cesare Cippoletti,† adopted **a trapezoidal tch**, with its bottom edge horizontal and its ends sloping at $\frac{\text{4 vertical}}{\text{1 horizontal}}$, in er to avoid the necessity of either suppressing or allowing for end contrac- ns. (See Art. 14 *c*, p. 547, and *m*, p. 550.) The contraction was found to affect y the triangular spaces over the sloping ends of the weir, and the effective gth of the weir thus remained constant (and equal to the length of the bot- edge) for all heads. In using these weirs the contraction is complete along bottom as well as at the ends

---

For such roots see p. 68.

See his work, *Canale Villoresi; Modulo per la Dispensa della Acqua, etc.*, Milan, 6; published by Societa Italiana per Condotte d'Acqua. Results summarized by . Carpenter, in Bulletin No. 13, Agricultural Experiment Station, Fort Collins, orado, October, 1890.

ON THE FLOW OF WATER IN OPEN CHANNELS.

Art. 16. The mean velocity of flow is an imaginary uniform one, which, if given to the water at every point in the cross section, would give *the same discharge* that the actual ununiform one does. Or

$$\textbf{mean velocity} = \frac{\text{volume of discharge}}{\text{area of cross section}}$$

In channels of uniform cross section, **the maximum velocity** is found about midway between the two banks, and generally at some dist below the surface. This dist varies in diff streams; but, as an average, it seems to be about one third of the total depth. Where the total depth is great in proportion to the width, (say ½ the width or more), the max vel has been found as deep as midway between surf and bottom; while in small shallow streams it appears to approach the surf to within from .1 to .2 of the total depth. Many experiments upon shallow streams have indeed indicated that the max vel was *at* the surf.

The ratio between the velocities in different parts of the cross section varies greatly in diff streams; so that but little dependence can be placed upon rules for obtaining one from the other. With the same surf vel, wide and deep streams have greater mean and bottom vels than small shallow ones. In order **to approximate roughly to the mean vel** when the greatest surf vel is given, it is frequently assumed that the former is = ⅘ (or .8) of the latter. But Mr. Francis found, in his experiments at Lowell, that *surface* floats of wax, 2 ins diam, floating down the center of a rectangular flume 10 ft wide, and 8 ft deep, actually moved about 6 per cent *slower* than a tin tube 2 ins diam, reaching from a few ins above the surf, down to within 1½ ins of the bottom of the flume; and loaded at bottom with lead, to insure its maintaining a nearly vert position. While the wax *surf* float moved at the rate of 3.73 ft per sec, the rate of the tube (which was evidently very nearly the same as that of the center *vert* thread of water) was 3.98 ft per sec. Also, that in the same flume, with vels of the center tube varying from 1.55 to 4 ft per sec, the vel of the tube was *less* than that of the mean vel of the entire cross section of water in the flume, about as .96 to 1, for the lesser vel; and .98 to 1 for the greater vel. While, in another rectangular flume 20 ft wide and 8 ft deep, with vels varying from 1.16 to 1.84 ft per sec, that of the tubes was *greater* than that of the entire mass of water, about as 1.04 to 1. In a flume 29 ft wide, by 8.1 ft deep, with vels of about 3 ft per sec, it was as 1 to .9; and in a flume 36½ ft wide, by 8.4 ft deep, with vels of about 3½ ft per sec, as 1 to .97.

Charles Ellet, Jr, C E, found in the Mississippi "at diff points on the river, in depths varying from 54 to 100 ft; and in currents varying from 3 to 7 miles an hour that the speed of a float supporting a line 50 ft long, is almost always greater than that of the surf float alone." The same results were obtained with lines 25 and 75 ft long; the excess of the speed of the line floats being about 2 per cent over that of the simple floats: and Mr. Ellet concludes, therefore, that the mean vel of the entire cross section of the Mississippi, instead of being less, is absolutely greater by about 2 per cent, than the MEAN *surf* vel. He, however, employed .8 of the *greatest* surf vel as representing approximately, in his opinion, the mean vel of the *entire* cross section of water. In *shallow* streams, he always found the surf float to travel more rapidly than a line float.

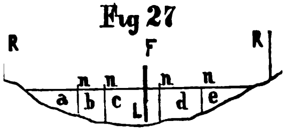
European trials of the *mean* vel of *separate single verticals*, in tolerably deep rivers, have resulted in from .85 to .96 of the surf vel at each vertical. The mean of all may be taken at .9.

Bottom velocity. In streams of nearly uniform slope and cross section, there is a great reduction of vel near the bottom. As a very rough approximation, the deepest measurable vel, in streams of uniform slope etc, appears to be from ½ to ⅔ of the mean vel.

Art. 17. To measure the surface velocity, select a place where the stream is for some dist (the longer the better) of tolerably uniform cross section; and free from counter-currents, slackwater, eddies, rapids, etc. Observe, by a seconds-watch, or pendulum, how long a time a float (such as a small block of wood) placed in the *swiftest* part of the current, occupies in passing through some previously measured dist. From 50 feet for slow streams, to 150 ft for rapid ones, will answer very well. This dist in ft, or ins, div by the entire number of seconds reqd by the float to traverse it, will give the greatest surf vel in ft or ins per sec.

The surf vel should be meas'd in perfectly calm weather, so that the float may not be disturbed by wind; and, for the same reason, the float should not project much above the water. The measurement should be

eated several times to insure accuracy. In very small streams, the banks
d bed may be trimmed for a short dist, so as to present a uniform channel·
y. The float should be placed in the water a little dist above the point for
umencing the observation; so that it may acquire the full vel of the water,
ore reaching that point.



Art. 18. To gauge a stream by means of its velocity. Select a place where the cross-section remains, for a short distance, tolerably uniform, and free from counter-currents, eddies, still water, or other irregularities. Prepare a careful cross-section, as Fig. 27. By means
poles, or buoys, n, n, divide the stream into sections, *a*, *b*, *c*, &c. Plant two range-
es, R. R, at the upper end, and two others at the lower end, of the distance
ough which the floats are to pass; for observing by a seconds watch, or a pendu-
ı, the time which they occupy in the passage. Then measure the *mean* velocity of
h section *a*. *b*. *c*, &c., separately, and directly, by means of long floats, as F L,
ching to near the bottom: and projecting a little above the surface. The floats may
in tubes, or wooden rods; weighted in either case, at the lower end, until they
l float nearly vertical. They must be of different lengths, to suit the depths of
different sections. For this purpose the float may be made in pieces, with screw-
its. The area of each separate section of the stream in square feet, being multi-
d by the observed mean velocity of its water in feet per second, will give the
harge of that section in cubic feet per second. And the discharges of all the
rate sections thus obtained, when added together, will give the total discharge
he stream. And this total discharge, divided by the entire area of cross-section
he stream in square feet, gives the mean velocity of *all* the water of the stream,
eet per second.

em. If the channel is in common earth, especially if sandy,
loss by soakage into the soil, and by evaporation, will frequently abstract so
h water that the disch will gradually become less and less, the farther down
am it is measured. Long canal feeders thus generally deliver into the canal
a small proportion of the water that enters their upper ends.

he double float is used for ascertaining vels at diff depths. It consists
float resting upon the surface of the water, and of a heavier body, or "lower
", which is suspended from the upper float by means of a cord. The depth
he lower float of course depends upon the length of the suspending cord
ich may be increased or diminished at pleasure until the lower float is be-
d to be at that depth for which the vel is wanted), and upon its straight-
, which is more or less affected by the current. Owing to this latter circum-
ce, it is difficult to know whether the lower float is really at the proper
h. Moreover it is uncertain to what extent the two floats and the string
rfere with one another's motions. In deep water the string may oppose a
ter area to the current than the lower float itself does. It thus becomes
tful to what extent the vel of the upper float can be relied upon as indicat-
hat of the water at the depth of the lower one.

rt. 19. Castelli's quadrant, or hydrometric pendulum,
isted of a metallic ball suspended by a thread from the center of a graduated
The instrument was placed in the current, with the arc parallel to the
tion of flow; and the vel was then calculated from the angle formed be-
n the thread and a vert line.

uthey's pressure plate was a sheet of metal suspended by one of its
, about which it was left free to swing. The plate was immersed in the
m, with its face at right angles to the current. The vel was estimated by
s of the weight required to make the plate hang vert in opposition to the
of the current.

tot's tube was originally a simple glass tube, Fig. 27 A, open
th ends and bent in the shape of the letter L. One leg of the
s held horizontal under water, with its open end facing the
nt; and the velocity v at the point o where it was placed was
ured by the vertical height h $\left(\text{theoretically} = \frac{v^2}{2g}\right)$ to which
ater rose in the other leg above the surface of the stream.

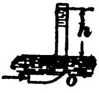


Fig. 27A

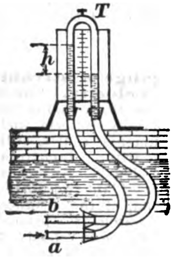


Fig. 27B

As developed by M. Darcy and by Prof. S. W. Robinson,[*] **and** rudely indicated in Fig. 27 B, Pitot's tube consists essentially of *two* horizontal glass or metal tubes *a* and *b*, of very small bore, placed side by side in the current and pointed up-stream. Tube *a* receives the current in its open up-stream end, while *b* is closed at its up-stream end, and has small *lateral* openings only. The other end of each tube communicates, by means of small metal or rubber piping, with one leg of an inverted U-shaped glass gauge fixed in a boat or on shore. For convenience, the two flexible pipes may be joined together into one double pipe. By sucking through a stop-cock T at the top, water is drawn up to any convenient height in the two legs of the gauge. When there is no current, the two columns of course stand at the same height; but in a current, the difference h in their heights is such that $v = \sqrt{2\,g\,h}$, no corrective coefficient being required. The instrument is remarkably simple and accurate, and can be used in very narrow and shallow streams of water or of gas. It measures velocities as low as 4 inches per second.

In practice, *a* and *b* are fixed together in one piece, and placed, when in use, in a metal frame which slides vertically, either upon a wire passing through it and provided with a plummet which rests upon the bottom and keeps the wire stretched, or (in streams shallower than about 20 feet) upon a vertical wooden rod, the lower end of which holds in the bed of the stream. In the former case, the frame is provided with a long vane for keeping the instrument headed up-stream. In either case, means are provided for showing the depth to which the instrument is submerged.

By making the gauge scale adjustable vertically, and placing it (at each change of depth of instrument) with its zero opposite the top of the lower column, we obviate the necessity of observing the height of both columns at each reading; for the reading of the upper column alone then gives the head h at once.

Art. 20. The wheel meter consists of a wheel which is turned by the current, and which communicates its motion, by means of its axle and gearing, to indices which record the number of revolutions. The instrument may be clamped to any part of a long pole reaching to the bottom of a stream, and thus may be used at any depth. The observer, by means of a wire, rod or string reaching down to the instrument, throws the registering apparatus first into, and then out of, gear with the wheel (applying a brake to the former at the instant it is thrown out of gear), and carefully noting the times when he does so. The instrument is then raised, the number of revolutions in the measured time is read off from the indices, and from it the velocity is calculated. But the meter is often made **self-registering**; the wheel, at each revolution, automatically breaking and re-establishing a galvanic current generated by a battery. The wire carrying this current is thus made to operate Morse telegraphic registering apparatus placed in a boat or on shore.

A number of meters, so arranged, can be attached at different points on the same pole at the same time, and thus **simultaneous observations of velocities at different depths** may be made and registered.

Meters are usually so arranged as to swing freely about the long vertical pole to which they are clamped, and are provided each with a vane or tail similar to that of a windmill, for keeping the wheel in the proper position as regards the current. The wheels are generally made like those of a windmill; *i. e.*, with blades set at such an angle as to present a sloping surface to the current; and with the axis of the wheel parallel to the direction of flow. The axis runs in agate bearings. When desired, the rim of the wheel is furnished with an air-chamber, which just counterbalances the weight of the wheel, and thus removes journal friction due to it. Meters provided with electrical registering apparatus sometimes have the gearing and indices, etc., enclosed in a glass case, to prevent them from becoming clogged by weeds, sediment, etc.

A wheel meter is rated by moving it at a known velocity through still water, and noting the effect produced. In this way a coefficient is obtained for each meter, which, when multiplied by the number of revolutions recorded in any given case, gives the velocity for that case.

---

[*] See Van Nostrand's Magazine, March, 1878, and August, 1886.

Art. 21. Kutter's formula for the mean vel of water flowing in open annels of uniform cross section and slope throughout.

Caution. The use of all such formulæ is liable to error arising from the fficulty of ascertaining the exact condition of the stream as regards roughness bed, surface slope,* etc.

Rem. 1. Care must be taken that the bottom vel is not so reat as to wear away the soil. If there is any such danger artificial means must be applied to protect the channel-way; or it may be advisable to reduce the rate of fall, and increase the cross section of the channel; so as to secure the same disch, but with less vel. A liberal increase should also be made in the dimensions of such channels, to compensate for obstructions to the flow, arising from the growth of **aquatic plants,** or deposits of mud from rain-washes, etc; or even from very strong winds blowing against the current.

Rem. 2. Water running in a channel with a horizontal bed, or bottom, cannot have a uniform vel, or depth, throughout its course; because the action of gravity due to the inclined plane of a sloping bottom, is wanting in this case; and the water can flow only by forming its *surface* into an inclined plane; which evidently involves a diminution of depth at every successive dist from the reservoir.

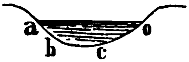


Fig. 28.

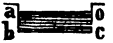


Fig. 29.

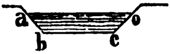


Fig. 30.

Theory of flow. It is generally held that the resistances to the flow of water in a pipe or channel are directly proportional to the area of the bed surface with which the water comes in contact (*i e*, to the product of the "wetted perimeter" as *a b c o* Figs 28, 29, 30 mult by the length of the channel, or of the portion of it under consideration); and to the square of the vel of the flowing water; and, inasmuch as the resistance at any given point in the cross section appears to be inversely as the dist of that point from the bottom or sides, we conclude that the total resistances are inversely as the area of the cross section; because the greater that area, the greater would be the mean dist of all the particles from the bottom and sides. The resistance is *independent of the pressure.*

In short, the resistances are assumed to be in proportion to

$$\frac{\text{vel}^2 \times \text{wet perimeter} \times \text{length}}{\text{area of cross section}} \quad \text{or} \quad \frac{v^2 p l}{a}$$

and the head h'' in feet or in metres etc, required to overcome those resistances, is

$$\begin{matrix}\text{resistance}\\ \text{head}\end{matrix} = \begin{matrix}\text{a coefficient}\\ C\end{matrix} \times \frac{\text{vel}^2 \times \text{wet perimeter} \times \text{length}}{\text{area of wet cross section}} \quad \text{or} \quad h'' = C\frac{v^2 p l}{a}$$

from which we have

$$v^2 = \frac{a h''}{C p l} \quad \text{and} \quad v = \sqrt{\frac{1}{C}\frac{a}{p}\frac{h''}{l}} \quad \text{or} \quad \text{vel} = \sqrt{\frac{1}{C}} \times \sqrt{\frac{\text{area of wet cross section}}{\text{wet perimeter}} \times \frac{\text{resistce head}}{\text{length}}}$$

---

[*] "In measuring the slope of a large river, the ordinary errors of the most careful leveling are a large proportion of the whole fall; the variation of level in the cross section of the surface is often as great as the slope for ten miles or more; the exact point where the level should be taken is often uncertain: the rise and fall of the water makes it extremely difficult to decide when the levels should be taken at the upper and lower points; waves of translation may affect the inclination to a great and uncertain degree, and may even make the surface slope the reverse way." Genl T. G. Ellis. Trans Am Soc Civ Engrs. Aug 1877.

But $\dfrac{\text{area of wet cross section}}{\text{wet perimeter}}$ or $\dfrac{a}{p}$ is the "hydraulic radius" or "mean depth" or "**mean radius,**" R, of the cross section;

and $\dfrac{\text{resistance head}}{\text{length}}$ or $\dfrac{h''}{l}$ is the inclination or slope, S, (frequently denoted by "I") of the hydraulic grade line, or the sine of the angle *w s o* Fig 1½. In open channels, it is = the fall of the surface per unit of length.

We therefore have
$$\text{velocity} = \sqrt{\frac{1}{C}} \times \sqrt{\text{mean radius} \times \text{slope}}$$

or, by using a coeff (c) $= \sqrt{\dfrac{1}{C}}$,
$$\text{velocity} = \text{coefficient } c \times \sqrt{\text{mean radius} \times \text{slope}}$$
$$\text{or} \qquad v = c\sqrt{RS}$$

The earlier hydraulicians gave (each according to the results of his investigations) *fixed* **values for the coeff c,** (generally about 95 to 100 for channels in earth or gravel, as in our early editions), making it, in other words, a *constant*, and independent of the shape, size, slope and roughness of the channel. But more recent investigators have shown that the coefficient c is affected by differences in any of these particulars.

According to the formula of Ganguillet and Kutter (generally called, for convenience, "**Kutter's formula**"*) the value of c is:

For English measure. **For metric measure.**

$$c = \frac{41.6 + \dfrac{.00281}{\text{slope}} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{.00281}{\text{slope}}\right) \times n}{\sqrt{\text{mean rad in } feet}}} = \frac{23 + \dfrac{.00155}{\text{slope}} + \dfrac{1}{n}}{1 + \dfrac{\left(23 + \dfrac{.00155}{\text{slope}}\right) \times n}{\sqrt{\text{mean rad in } metres}}}$$

Tables giving values of c for diff grades, mean radii and degrees of roughness, and for English and metric measures, are given on pp 566, etc.

Here **n** is a "**coefficient of roughness**" of sides of channel as given below. These values of n were obtained from experience, by averaging a large number of experiments made under very different circumstances. They therefore embrace all the disturbing effects arising from obstructions existing upon the bottom and sides of the channel in the cases experimented upon. In small artificial channels of uniform cross section and slope, these obstructions may be said to consist entirely of the comparatively minute roughnesses of the *material* of which the bed of the channel consists. But in rivers and earth canals, even where the *general* direction, slope and cross section are tolerably uniform, (as they were in the cases upon which our list is based), there are still many considerable irregularities in the sides and bottom; and these exert a much greater retarding effect upon the mean vel than the mere *roughness* of the material of the banks. We therefore find larger values given for n in such cases than for small regular artificial channels, although the material of the sides etc was in many cases smooth mud; and we must not apply to such comparatively irregular channels the small values of n obtained by experiments with small and carefully made straight flumes of uniform section and slope, even if we suppose the bottom and sides of the former to be made as smooth as those of the latter.

No general formula is applicable to cases of **decided bends in the course** of a natural stream, or of **marked irregularities in the cross section.** Such cases would require still higher coefficients n than those here given for rivers and canals; but they would have to be ascertained by experiment for each case, and would be useless for other cases. For such streams we must therefore depend upon actual measurements of the velocity, either direct or by means of the disch.

---

* See "Flow of Water," translated from Ganguillet and Kutter, by Rudolph Hering and John C. Trautwine, Jr., New York, John Wiley & Sons, 1889. $4.00.

There is much room for the exercise of judgment in the **selection of the roper coefficient n** for any given case, even where the condition of the annel is well known. It may frequently be necessary to use values of *n* inter-diate between those given; for careless brickwork may be rougher than well ished rubble; side slopes in "very firm gravel" may have very diff degrees roughness; etc etc. The engineer should make lists of values of *n* from his n experience, fully noting the peculiarities of each case, and calculating *n* m the tables.

A given diff in the deg *n* of roughness exerts a much greater effect upon the efficient *c*, and thus upon the velocity, in small channels than in larger ones. is therefore especially necessary in small channels that care be exercised in ding (by experiment if necessary) the proper value of *n*; and, where a large ch is desired, the sides of small channels should be made particularly smooth.

Table of n, or coefficient of roughness.

n any given case the value of **n is the same whether the mean dius is given in English, metric or any other measure.**

Artificial channels of uniform cross section.

s and bottom of channel lined with	n =
well planed timber	.009
neat cement (applies also to glazed pipes and very smooth iron pipes).	.010
plaster of 1 measure of sand to 3 of cement; (or smooth iron pipes).	.011
unplaned timber (applies also to ordinary iron pipes)	.012
ashlar or brickwork	.013
rubble	.017

Channels subject to irregularity of cross section.

als in very firm gravel	.020
als and rivers of tolerably uniform cross section, slope and direction, in moderately good order and regimen, and free from stones and weeds	.025
having stones and weeds occasionally	.030
in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	.035

rt. 22. The following tables give values of the coefficient obtained by Kutter's formula for diff slopes (S) mean radii (R) and degrees ughness (*n*).*

ution. Different values of *c* must be used with English and with metric ures. We give tables for both measures.

t. Having the slope S, the mean rad R and the deg *n* of roughness; **to l the coeff c.** Turn to the division of the table corresponding to the slope S. In the first column find the given mean rad, R. In the same with this R, and under the given *n*, is the proper value of *c*.*

. Having the slope S, the mean rad R and either *c* or the actual or reqd ; **to find the actual, or the greatest permissible, deg n of ghness** of channel. If the vel is given, and not *c*, first find

$$\frac{\text{velocity}}{\sqrt{\text{slope} \times \text{mean radius}}}.$$

Turn to the division of the table corresponding to iven S, and in the first col find the given R. In the same line find the given, or just obtained, for *c*; over which will be found the reqd *n*.*

. Having the slope S, the deg *n* of roughness, and the actual or required **to find the actual or necessary mean rad, R.** Assume a rad; and from the division of the table corresponding to the given S take e value of *c* corresponding to the given *n* and the assumed R. Then say

$$v' = c \text{ so found} \times \sqrt{\text{assumed mean radius} \times \text{slope}}$$

---

is often necessary to interpolate values of S, R, *n* and *c* intermediate of in the tables; this may be done mentally by simple proportion.

If this v' is the same as the given vel, or near enough to it, take the assumed R as the proper one. Otherwise, repeat the whole process, assuming a new R, *greater* than the former one if v' is *less* than the given vel, and *vice versa*.*

4th. Having the dimensions of the wetted portion (*abco* Figs 28, 29, 30,) of the channel, the deg n of roughness, and the actual or reqd vel; **to find the actual or necessary slope, S:**

$$\text{Find the mean rad, R} = \frac{\text{area of wet cross section}}{\text{length, } abco\text{, of wet perimeter}}$$

Assume one of the four slopes of the tables to be the proper one. From the corresponding division of the table take out the value of c corresponding to the given R and n.

If R is 3.28 feet, or 1 metre, the value of c thus found is the proper one (because then c, for any given n, remains the same for all slopes); and the slope, S, may be found at once, thus:

$$\text{Slope, S} = \left(\frac{\text{given velocity}}{c \times \sqrt{\text{mean radius}}}\right)^2$$

But if R is greater or less than 3.28 feet, or 1 metre, say

$$v' = c \text{ thus found} \times \sqrt{\text{mean radius} \times \text{assumed slope}}$$

If this v' is near enough to the given vel, take the assumed S as the proper one. Otherwise, assume a new S, *greater* than the former one if v' is *less* than the given vel, and *vice versa*; and repeat the whole process.*

---

* It is often necessary to interpolate values of S, R, n and c intermediate of those in the tables. This may be done mentally by simple proportion.

Table of coefficient c, for mean radii in *feet*.

Slope S = .000025 per unit of length, = 1 in 40000, = .132 foot per mile.

Mean rad R feet	.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R feet
	Coefficients n of roughness												
	c	c	c	c	c	c	c	c	c	c	c	c	
.1	65	57	50	44	40	33	28	23	17	14	12	10	.1
.2	87	75	67	59	58	45	38	31	24	19	16	14	.2
.4	111	97	87	78	70	59	51	42	32	26	22	19	.4
.6	127	112	100	90	81	69	60	49	38	31	26	22	.6
.8	138	122	109	99	90	77	66	55	43	35	30	25	.8
1	148	131	118	106	97	83	72	60	47	38	32	28	1
1.5	166	148	133	121	111	95	83	69	55	45	38	33	1.5
2	179	160	144	131	121	104	91	77	61	50	43	37	2
3	197	177	160	147	135	117	103	88	70	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	209	188	172	158	146	127	113	96	78	65	56	49	4
6	226	206	188	174	161	142	126	108	88	74	64	57	6
8	238	216	199	184	171	151	135	117	96	82	71	63	8
10	246	225	207	192	179	159	142	124	102	87	76	68	10
12	253	231	214	198	186	165	149	129	107	92	81	72	12
16	263	242	223	208	195	174	157	138	115	100	88	79	16
20	271	249	231	215	202	181	164	144	121	106	94	84	20
30	283	261	243	228	215	193	176	157	133	117	104	95	30
50	297	274	257	241	228	207	190	170	147	130	117	107	50
75	306	284	267	251	238	217	200	180	157	140	127	117	75
100	312	290	273	257	244	223	207	187	163	147	134	124	100

Table of coefficient c, for mean radii in *feet*.—Continued.

Mean rad R feet	Coefficients n of roughness .009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R feet
	c	c	c	c	c	c	c	c	c	c	c	c	
.1	78	67	59	52	47	39	33	26	20	16	13	11	.1
.15	91	79	69	62	56	46	39	31	23	19	16	13	.15
.2	100	87	77	68	62	51	44	35	26	21	18	15	.2
.3	114	99	88	79	71	59	50	41	31	25	21	18	.3
.4	124	109	97	88	79	66	57	46	35	28	24	20	.4
.6	139	122	109	98	90	76	65	53	41	33	28	24	.6
.8	150	133	119	107	98	83	71	59	46	37	31	27	.8
1	158	140	126	114	104	89	77	64	49	40	34	29	1
1.5	173	154	139	126	116	99	87	72	57	47	40	34	1.5
2	184	164	148	135	124	107	94	79	62	51	44	38	2
3	198	178	161	148	136	118	104	88	71	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	207	187	170	156	145	126	111	95	77	64	56	49	4
6	220	199	182	168	156	137	122	105	85	72	63	56	6
8	228	206	189	175	163	144	129	111	91	78	68	61	8
10	234	212	195	181	169	149	134	116	96	82	72	64	10
12	238	217	200	185	173	153	138	120	99	86	75	68	12
16	245	223	206	191	180	160	144	126	106	91	81	73	16
20	250	228	211	196	184	165	149	131	110	96	85	77	20
30	257	236	219	204	192	172	157	139	118	103	92	84	30
50	266	245	228	213	201	181	165	148	127	112	101	93	50
75	272	250	233	218	207	187	171	153	133	119	108	99	75
100	275	254	237	222	210	190	175	158	137	123	112	104	100
.1	90	78	68	60	54	44	37	30	22	17	14	12	.1
.2	112	98	86	76	69	57	48	39	29	23	19	16	.2
.3	125	109	97	87	78	65	56	45	34	27	22	19	.3
.4	136	119	106	95	86	72	62	50	38	31	25	22	.4
.6	149	131	118	105	96	81	70	57	44	35	30	25	.6
.8	158	140	126	114	103	88	76	63	48	39	33	28	.8
1	166	147	132	120	109	93	81	67	52	42	35	31	1
1.5	178	159	144	130	120	103	89	75	59	48	41	35	1.5
2	187	168	151	138	127	109	96	81	64	53	45	39	2
3	198	178	162	149	137	119	104	89	71	59	51	45	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	206	186	169	155	143	125	111	94	76	64	55	49	4
6	215	195	178	164	152	134	119	102	84	71	61	54	6
8	221	201	184	170	158	139	124	107	88	75	66	59	8
10	226	205	188	174	162	143	128	111	92	78	69	62	10
15	233	212	195	181	169	150	135	118	98	85	75	68	15
20	237	216	200	185	173	154	139	122	102	89	79	71	20
30	243	222	206	191	179	160	145	128	108	95	84	77	30
50	249	227	211	197	185	166	151	134	114	100	91	83	50
100	255	234	218	204	191	172	158	140	121	108	98	91	100
.1	99	85	74	65	59	48	41	32	24	18	15	12	.1
.2	121	105	93	83	74	61	52	42	31	25	21	17	.2
.3	133	116	103	92	83	69	59	48	36	29	24	20	.3
.4	143	125	112	100	91	76	65	53	40	32	27	23	.4
.6	155	138	122	111	100	85	73	60	46	37	31	26	.6
.8	164	145	131	118	107	91	79	65	50	41	34	29	.8
1	170	151	136	123	113	96	83	69	54	44	37	32	1
1.5	181	162	146	133	122	105	91	77	60	49	42	36	1.5
2	188	170	154	140	129	111	97	82	64	54	45	40	2
3	200	179	163	149	137	119	105	89	72	59	51	45	3
4	205	185	168	155	143	125	111	94	76	63	55	48	4
6	213	193	176	162	150	132	117	100	82	69	60	53	6
8	218	198	181	167	155	137	122	105	87	73	64	57	8
10	222	201	185	170	158	140	125	108	89	76	67	60	10
15	228	207	190	176	164	145	131	113	95	82	72	65	15
20	231	210	194	180	168	149	134	117	98	85	76	68	20
30	235	215	198	184	172	154	139	122	103	89	80	73	30
50	240	220	203	189	177	158	143	126	108	94	85	78	50
100	245	224	208	194	182	163	148	131	113	99	90	83	100

Table of coefficient c, for mean radii in *feet*.—CONTINUED.

Slope	Mean rad R feet	Coefficients n of roughness.												Mean rad R feet
		.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope S = .0004 per unit of length, = 1 in 2500, = 2.112 feet per mile.	.1	104	89	78	69	62	50	43	34	25	19	16	13	.1
	.15	116	101	90	80	71	59	50	40	29	23	19	16	.15
	.2	126	110	97	87	78	65	54	44	32	25	21	18	.2
	.3	138	120	107	96	87	73	62	50	37	30	24	21	.3
	.4	148	129	115	104	94	79	68	55	42	33	27	23	.4
	.6	157	140	126	113	103	87	75	62	47	38	31	27	.6
	.8	166	148	133	121	110	93	81	67	51	42	35	30	.8
	1	172	154	138	125	115	98	85	70	55	45	37	32	1
	1.5	183	164	148	135	124	106	93	78	61	50	42	37	1.5
	2	190	170	154	141	130	112	98	83	65	54	45	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	94	76	63	55	48	4
	6	211	191	175	161	149	130	116	99	81	69	60	53	6
	10	219	199	183	168	157	138	123	107	88	75	66	59	10
	20	227	207	190	176	164	146	131	115	96	83	73	66	20
	50	235	215	198	184	173	154	139	123	104	91	82	75	50
	100	239	219	203	189	177	158	143	127	108	96	87	80	100
Slope S = .0010 per unit of length, = 1 in 1000, = 5.28 ft. per m.	.1	110	94	83	73	65	54	45	36	27	21	17	14	.1
	.2	129	113	99	89	81	66	57	45	34	27	22	18	.2
	.3	141	124	109	98	89	74	63	51	39	30	25	21	.3
	.4	150	131	117	105	96	80	69	56	43	34	28	24	.4
	.6	161	142	127	115	104	88	76	63	48	39	32	27	.6
	.8	169	150	134	122	111	94	82	68	52	42	35	30	.8
	1	175	155	139	127	116	99	86	71	56	45	38	33	1
	1.5	184	165	149	136	124	108	93	78	62	50	43	37	1.5
	2	191	171	155	142	130	112	98	83	66	54	46	40	2
	3	199	179	163	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	93	75	63	54	48	4
	6	211	190	174	160	149	130	116	99	81	68	59	52	6
	10	218	197	181	167	155	136	122	105	87	74	65	58	10
	20	225	205	188	175	163	144	129	113	94	81	72	65	20
	50	232	212	196	182	170	151	137	120	101	89	79	72	50
	100	236	216	200	186	174	155	141	124	105	94	85	77	100
Slope S = .01 per unit of length, = 1 in 100, = 52.8 feet per mile.	.1	110	95	83	74	66	54	46	36	27	21	17	14	.1
	.15	122	105	93	83	75	62	52	42	31	24	20	17	.15
	.2	130	114	100	90	81	67	57	46	34	27	22	19	.2
	.3	143	125	111	100	90	76	64	52	39	31	25	22	.3
	.4	151	133	119	107	98	82	70	57	44	35	29	24	.4
	.6	162	143	129	116	106	90	77	64	49	39	33	28	.6
	.8	170	151	135	123	112	95	82	68	53	43	35	31	.8
	1	175	156	141	128	117	99	87	72	56	45	38	33	1
	1.5	185	165	149	136	125	107	94	79	62	51	43	37	1.5
	2	191	171	155	142	130	112	99	83	66	55	46	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
	4	204	184	167	154	142	123	109	93	76	63	55	48	4
	6	210	190	173	160	148	129	115	99	81	68	59	52	6
	10	217	196	180	166	154	136	121	105	86	74	65	58	10
	20	225	204	187	173	161	143	128	112	93	80	71	64	20
	50	231	210	194	181	168	150	135	119	100	87	78	71	50
	100	235	214	197	184	172	153	139	122	104	91	82	75	100

For slopes steeper than .01 per unit of length, = 1 in 100 = 52.8 feet per mile, *c* remains practically the same as at that slope. But the *velocity* (being $= c \times \sqrt{\text{mean radius} \times \textit{slope}}$) of course continues to increase as the slope becomes steeper.

Table of coefficient c, for mean radii in ***metres.***

	Mean rad R metres	Coefficients m of roughness.												Mean rad R metres
		.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope = .000025 per unit of length, = 1 in 40000.	.025	34	29	25	22	20	17	14	11	9	7	6	5	.025
	.05	44	38	83	30	27	22	19	16	12	9	8	7	.05
	.1	58	50	44	40	36	30	26	21	16	13	11	9	.1
	.2	72	63	56	51	46	39	34	28	21	18	15	13	.2
	.3	82	72	64	58	53	45	39	33	25	21	17	15	.3
	.4	89	79	71	64	59	50	44	37	29	23	20	17	.4
	.6	99	88	80	72	67	57	50	42	33	28	23	20	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.50	121	109	100	92	85	74	66	57	46	38	33	29	1.50
	2	127	115	106	98	91	80	71	61	50	42	37	32	2
	8	136	124	114	106	99	87	78	68	56	48	42	37	3
	4	142	130	120	111	104	93	83	73	61	52	46	41	4
	6	149	137	127	119	111	100	90	80	67	58	51	46	6
	10	158	145	135	127	120	108	98	88	75	66	59	53	10
	15	164	151	141	133	126	114	104	94	81	72	64	59	15
	20	167	155	145	137	130	118	108	98	85	75	68	62	20
	80	172	160	150	142	135	123	113	103	90	81	74	68	30
Slope = .00005 per unit of length, = 1 in 20000.	.025	40	35	80	26	24	20	17	18	10	8	7	5	.025
	.05	52	44	39	84	31	26	22	18	18	11	9	7	.05
	.1	65	57	50	44	40	84	29	24	18	14	12	10	.1
	.2	79	69	62	55	51	43	87	30	28	19	16	13	.2
	.3	87	77	69	62	57	48	42	35	27	22	18	16	.3
	.4	93	83	74	67	62	53	46	38	30	25	21	18	.4
	.6	102	90	82	74	69	59	52	43	34	28	24	21	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	118	107	97	90	83	73	65	55	45	38	38	28	1.5
	2	123	111	102	94	87	77	68	59	48	41	35	31	2
	8	129	117	108	100	93	83	74	64	53	45	40	35	8
	4	133	121	112	104	97	86	77	68	56	49	48	38	4
	6	138	126	117	109	102	91	82	72	61	58	47	42	6
	10	143	131	122	114	107	96	87	78	66	58	52	47	10
	15	147	135	126	118	111	100	91	82	70	62	56	51	15
	20	150	137	128	120	113	103	94	84	72	64	58	53	20
	30	152	140	131	123	116	105	97	87	76	68	62	57	30
length, = 1 in 10000.	.025	47	40	35	31	28	22	19	15	11	9	7	6	.025
	.05	59	50	44	40	35	29	25	20	15	12	10	8	.05
	.1	72	62	55	50	45	37	32	26	19	16	13	11	.1
	.2	84	74	66	60	54	46	39	32	25	20	17	14	.2
	.3	91	81	73	66	60	51	44	37	28	23	19	17	.3
	.4	97	86	77	70	64	55	48	40	31	25	21	18	.4
	.6	104	92	83	76	70	60	58	45	35	29	25	21	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	117	105	96	88	82	72	64	54	44	37	32	28	1.5
	2	120	109	100	92	85	75	67	57	47	40	34	30	2
	4	128	116	107	99	92	82	73	64	53	46	40	36	4
	6	131	119	110	102	96	85	77	67	56	49	43	39	6
	10	135	123	114	106	100	89	81	71	60	58	47	48	10
	15	137	126	116	109	102	92	83	74	63	55	50	46	15
	30	141	129	120	112	106	95	87	78	67	59	54	50	30
	.025	52	45	40	35	31	25	21	17	12	9	8	6	.025
	.050	63	55	48	43	39	32	27	21	16	12	10	8	.050
	.1	75	66	59	53	48	40	34	27	21	16	13	11	.1
	.2	87	77	69	62	57	48	41	34	26	21	17	15	.2
	.4	99	88	80	72	66	57	49	41	32	26	22	19	.4
	.6	104	93	84	77	71	61	53	45	36	29	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	118	107	98	90	84	74	65	56	46	39	34	30	2
	4	124	113	104	97	90	79	71	62	51	44	39	35	4
	10	130	119	110	102	96	85	77	67	57	50	45	40	10
	30	135	124	114	107	100	90	82	78	62	55	50	46	30

Table of coefficient c, for mean radii in *metres*.—CONTINUED.

	Mean rad R meters	Coefficients **n** of roughness.												Mean rad R metres
		.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope = .0004 per unit of length, = 1 in 2500.	.025	55	47	41	37	33	27	22	17	13	10	8	7	.025
	.050	66	58	51	45	40	33	28	23	17	13	11	9	.050
	.1	78	68	61	55	50	42	35	28	21	17	14	12	.1
	.2	90	80	70	64	59	49	42	35	27	22	18	15	.2
	.3	95	85	76	70	63	54	47	39	30	24	21	17	.3
	.4	99	89	80	73	67	57	50	42	32	27	22	20	.4
	.6	105	94	85	78	72	62	54	45	36	30	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	117	106	97	89	83	73	65	56	45	38	34	30	2
	4	123	111	102	95	88	78	70	61	50	43	38	34	4
	6	125	114	105	97	91	81	72	63	53	46	40	36	6
	10	128	117	108	100	93	83	75	66	55	48	43	39	10
	30	132	121	112	104	98	87	79	70	60	52	48	43	30
Slope = .0010 per unit of length, = 1 in 1000.	.025	57	50	43	38	34	28	23	18	13	11	9	7	.025
	.050	69	59	52	47	42	34	29	23	17	13	11	9	.050
	.1	80	70	63	56	50	42	36	30	22	17	14	12	.1
	.2	90	80	72	65	60	50	43	35	27	22	18	16	.2
	.3	96	86	77	70	64	54	47	39	30	25	21	18	.3
	.4	100	89	81	74	67	58	50	42	33	27	23	19	.4
	.6	104	94	85	78	72	62	54	46	36	30	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	116	106	97	90	83	72	64	55	45	38	33	29	2
	4	121	111	102	94	87	77	69	60	50	42	37	33	4
	6	124	113	104	97	90	80	71	62	52	45	40	36	6
	10	127	115	106	99	92	82	73	64	54	47	42	38	10
	30	130	119	110	102	96	86	77	68	58	51	46	42	30
Slope = .01 per unit of length, = 1 in 100.	.025	59	50	44	39	35	28	24	19	14	10	9	7	.025
	.05	69	60	53	48	43	35	29	24	18	14	11	9	.05
	.1	81	71	63	57	51	43	36	30	22	18	15	12	.1
	.2	91	81	72	65	60	50	44	36	27	22	18	16	.2
	.3	97	86	77	71	65	55	48	40	31	25	21	18	.3
	.4	101	90	81	74	68	58	50	42	33	27	23	20	.4
	.6	106	95	86	78	72	62	54	46	36	30	25	22	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	115	104	94	87	80	70	62	53	43	36	31	27	1.5
	2	117	105	96	89	83	72	64	55	45	38	33	29	2
	4	121	110	101	93	87	76	68	59	49	42	37	33	4
	10	126	114	105	98	91	81	73	64	53	46	41	37	10
	30	129	118	108	101	95	84	77	67	57	50	45	41	30

For slopes steeper than .01 per unit of length, = 1 in 100, the *coefficient* c remains practically the same as at that slope. The *velocity*, however, being $= c \times \sqrt{\text{mean radius} \times \textit{slope}}$, continues to increase as the slope becomes steeper.

To construct a diagram, fig 30 A, from which the **values given by Kutter's formula** may be taken by inspection.

Draw xz hor, and say from 2 to 4 ft long; and oy vert at any point o within say the middle third of xz. On oy lay off, as shown on the left, the values of c for which the diagram will probably be used. If a scale of .05 inch, or .002 metre, per unit of c be used, and be made to include $c = 250$ for English measure, or 150 for metric measure, oy will be **about** 1 ft long. For the sake of clearness we show only the larger divisions in this and in what follows.

On oz lay off, as shown on its upper side, the *square roots* of all the values of the mean rad R for which the diagram is to be used. One inch per ft, or .06 metre per metre, of sq rt, is a convenient scale. *Mark* the dividing points with the respective values of the *mean radii themselves.*

Having decided upon the *flattest* slope to be embraced in the diagram, say

$$w = 41.6 + \frac{.0028}{\text{flattest slope per unit of length}} \quad \text{for English measure.}$$

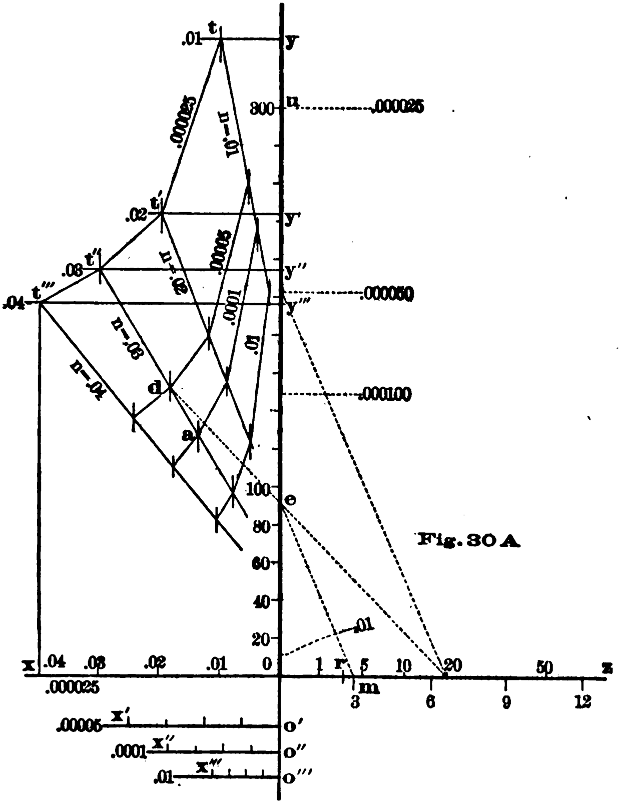


Fig. 30 A

$$w = 23 + \frac{.00155}{\text{flattest slope per unit of length}}$$ for metric measure.

For each value of n to be embraced in the diagram, say

$y - w = \frac{1.811}{n}$ for English measure; or $y - w = \frac{1}{n}$ for metric measure.

To each value of $y - w$, add w, thus obtaining values of y. We take .000025 per unit of length as the flattest slope,* and .01, .02, .03 and .04 for n.† Hence (using English measure) $w = 41.6 + \frac{.0028}{.000025} = 41.6 + 112 = 153.6$.

$y - w = \frac{1.811}{.01}, \frac{1.811}{.02}, \frac{1.811}{.03}, \frac{1.811}{.04};$ = 181.1, 90.5, 60.4, 45.3 respectively, and $y = 181.1 + 153.6$, $90.5 + 153.6$, $60.4 + 153.6$ and $45.3 + 153.6$; or 334.7, 244.1, 214.0 and 198.9 respectively. Lay off these values of y on oy in pencil, as at y, y', y'', and y''', using the scale already laid off for c on oy.

---

* This is about as flat as is likely to occur in practice.

† In most cases many intermediate ones would be used.

From each point, y, y' etc, draw a hor pencil line yt, $y't'$ etc, and mark on it, in pencil, the value of n used in determining its height oy etc.

Next say $x = w \times$ *greatest* value of n. Make $ox = x$ by the scale of *sq rts* of R on oz. In our case $ox = 153.6 \times .04 = 6.144$ by the scale of *sq rts* of R, or $= 6.144^2 = 37.75$ by the scale of R.

Divide ox into as many equal spaces (4 in our case) as .01 is contained in greatest n. Mark the dividing points with the values of n, as in our Fig.

From each dividing mark on ox erect a perpendicular, (xt''' etc) in pencil, to cut that hor line ($y'''t'''$ etc) which corresponds to the same value of n. The intersections are points in a hyperbola. Join them by straight lines $t'''t''$, $t''t'$, $t't$ etc.

From r in oz (corresponding to a mean rad of 3.28 ft, or 1 metre) draw radial lines, rt, rt', rt'' etc. Mark them "$n = .01$", "$n = .02$" etc, the same as their corresponding lines yt, $y't'$ etc.

For each slope (S) to be used in the diagram (except the flattest, for which this has already been done) say

$$x', x'' \text{ etc} = \left(41.6 + \frac{.0028}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for English measure.}$$

$$x', x'' \text{ etc} = \left(23 + \frac{.00155}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for metric measure.}$$

Thus, our slopes are = .000025, .00005, .0001 and .01 per unit of length. Hence,

$$x' = \left(41.6 + \frac{.0028}{.00005}\right) \times .04 = 3.904; \quad x'' = \left(41.6 + \frac{.0028}{.0001}\right) \times .04 = 2.784;$$

$$x''' = \left(41.6 + \frac{.0028}{.01}\right) \times .04 = 1.675.$$

Lay off each value of x', x'' etc from oy on a separate hor pencil line $o'x'$ etc, using the scale of sq rts of R as on oz.

Mark each line $o'x'$ etc in pencil with the slope used in fixing its length.

Divide each dist $o'x'$ etc into the same number of equal parts as ox. From the dividing points (which, like those of ox, represent the values of n) erect perps to cut the radial lines rt''', rt'' etc, each perp cutting that radial line which corresponds to the value of n represented by the point at the foot of the perp. The intersections corresponding to each line $o'x'$ etc form a hyperbolic curve. Mark each curve with the slope of its corresponding line, ox, $o'x'$ etc.

The drawing is now in the shape proposed by Mess Ganguillet and Kutter, and is ready for use in finding either c, n, R or S when the other three are given. Thus:

1st. Having R, S and n, **to find c.** For example let R = 20 ft, S = .00005, $n = .03$. From the intersection d of slope curve .00005 and radial line $n = .03$, draw* d–20 to the point (20) in oz corresponding to the given R. At e, where d–20 cuts oy, is the reqd c, = 96 in this case.

2d. Having R, S and c, **to find n.** For example let R = 20 ft, S = .00005, $c = 96$. Through the points R = 20 in oz, and $c = 96$ in oy, draw* d–20 to cut curve .00005. n (= .03) is found by means of the radial lines nearest to the intersection, d.

3d. Having S, n and c, **to find R.** For example let S = .00005, $n = .03$, $c = 96$. Find curve .00005 and radial line $n = .03$. From their intersection d draw d–20 through the point e showing $c = 96$. Its intersection with oz shows the reqd R, 20 in this case.

---

* Instead of *drawing* these lines, we may use a fine black thread with a loop at one end. Drive a needle either into one of the points R or into one of the intersections, d etc. Slip the loop over the needle. The other end of the thread is held between the fingers, and the thread is made to cut the other points as reqd. The diagram should lie perfectly flat, and the string be drawn tight at each observation, in order that friction between string and paper may not prevent the string from forming a straight line. Or the free end of the string may rest on a pamphlet or other object about ¼ inch thick, to keep the string clear of the diagram. Special care must then be taken to have the eye perp over the point observed.

4th. Having R, c and n, **to find S.** For example let R = 20 ft, c = 96, n = .03. Through R = 20 and c = 96 draw d–20. S (.00005) is found by means of the curves nearest to the point d of intersection of d–20 with radial line n = .03.

The following addition to Kutter's diagram, proposed by Mr Rudolph Hering, Civil and Sanitary Engineer, Philadelphia,* enables us **to read the velocity from the diagram.**

Find the sq rt of the reciprocal of each slope to be embraced in the diagram $= \sqrt{\frac{1}{\text{slope per unit of length}}}$. Lay off these sq rts on the right of oy, using the scale of c already laid off on its left. In our fig we have so proportioned the two scales that $\frac{c}{\sqrt{\text{recip of S}}} = \frac{1.5}{1}$. *Mark* the dividing points with the *slopes per unit of length*.

On oz lay off the vels to be embraced in the diagram, using the scale of sq rts of R already laid off on oz, and making $\frac{\text{vel}}{\sqrt{\text{R}}} = \frac{c}{\sqrt{\text{recip of S}}}$

1st. Having R, S and n; **to find v.** For example let R = 20 ft, S = .00005, n = .03. From R = 20 draw d–20 to the intersection d of curve .00005 with radial line n = .03. d–20 cuts oy at e, where c = 96. With a parallel ruler join R = 20 with S = .00005 on oy. Draw a parallel line through c = 96. It cuts oz at m, giving the reqd vel, 3.03 ft per sec.

2d. Having R, S and v; **to find n.** For example let R = 20 ft, S = .00005, v = 3.03 ft per sec. With a parallel ruler join R = 20 and slope .00005 on oy. Draw a parallel line through v = 3.03. It cuts oy at e, where c = 96. Through R = 20 and c = 96, draw d–20 to cut curve .00005. The point d of intersection, being on radial line n = .03, shows .03 to be the proper value of n.

Any line drawn to the curves from R = 3.28 ft or 1 metre, is one of the radial lines used in making the diagram. It therefore necessarily cuts all the slope curves at points showing the same value of n.

3d. Having S, n and v; **to find R.** For example, let S = .00005, n = .03, v = 3.03 ft per sec. *Assume* a value of R, say 10 ft. Find curve .00005 and radial line n = .03. Join their intersection d with R = 10 ft. The connecting line cuts oy at c = 82. With a parallel ruler join c = 82 with v = 3.03. Draw a parallel line through slope = .00005 on oy. It cuts oz at R = 27.3, showing that a new trial is necessary, and with an assumed R *greater* than 10 ft.

If R thus found is the same as the assumed one, the latter is correct. If they are nearly equal, their mean may be taken.

4th. Having R, n and v; **to find S.** For example, let R = 20 ft, n = .03, v = 3.03 ft per sec. *Assume* a slope (say .0001). Find its curve, and radial line n = .03. Join their intersection with R = 20, and note the value (89) of c where the connecting line cuts oy. With a parallel ruler join c = 89 with v = 3.03. Draw a parallel line through R = 20. It cuts oy at slope .000058, showing that a new trial is necessary, and with an assumed S *flatter* than .0001. If R is 3.28 ft, or 1 metre, the diagram gives the correct S at the first trial, no matter what S was assumed at starting. With any other R, if the diagram gives the same S as that assumed, the latter is correct. If the two differ but slightly, we may take their mean.

---

* Transactions of the American Society of Civil Engineers, January 1879.

Table of vels in Circular Brick Sewers when running full, by Kutter's formula, but taking **n** at .015 instead of his .013, in consideration of the rough character of sewer brickwork generally.

When running only half full the vel will be the same as when full, but this is not the case at any other depth whether greater or less. At greater ones it increases until the depth equals very nearly .9 of the diam, when it is about 10 per cent greater than when either full or half full. From depth of .9 of the diam the vel decreases whether the depth becomes greater or less. At depth of .25 diam the vel is about .78 of that when full; and then diminishes much more rapidly for less depths. All this applies also to pipes.

The vel for any fall or diam intermediate of those in the table can be found by simple proportion. Original.

Fall in ft per mile.	Diameters in feet.								Fall in ft per 100 ft.
	2	3	4	6	8	12	16	20	
	Velocities in feet per second.								
.1	.19	.27	.35	.50	.64	.89	1.10	1.34	.0019
.2	.30	.42	.53	.74	.93	1.26	1.56	1.84	.0038
.4	.46	.65	.80	1.08	1.39	1.81	2.20	2.60	.0076
.6	.59	.81	1.00	1.35	1.70	2.22	2.70	3.18	.0114
.8	.69	.95	1.17	1.57	1.94	2.56	3.08	3.60	.0151
1.0	.79	1.07	1.32	1.77	2.16	2.84	3.43	3.96	.0189
1.25	.89	1.21	1.49	1.98	2.42	3.17	3.8	4.5	.0237
1.50	.98	1.33	1.64	2.18	2.64	3.5	4.2	4.9	.0284
1.75	1.06	1.44	1.78	2.34	2.85	3.8	4.5	5.3	.0331
2.0	1.15	1.55	1.91	2.53	3.1	4.0	4.8	5.6	.0379
2.5	1.32	1.78	2.18	2.85	3.5	4.5	5.4	6.3	.0473
3.0	1.44	1.94	2.38	3.2	3.8	5.0	6.0	6.9	.0568
3.5	1.58	2.10	2.58	3.4	4.1	5.3	6.5	7.4	.0662
4.	1.68	2.2	2.7	3.6	4.4	5.7	6.9	7.9	.0758
5.	1.90	2.5	3.1	4.1	4.9	6.3	7.6	8.7	.0947
6.	2.06	2.7	3.3	4.4	5.4	6.9	8.3	9.6	.1136
7.	2.2	3.0	3.6	4.8	5.8	7.5	9.0	10.4	.1325
8.	2.4	3.2	3.8	5.1	6.2	8.0	9.7	11.1	.1514
9.	2.5	3.4	4.1	5.4	6.6	8.5	10.3	11.8	.1703
10.	2.7	3.5	4.3	5.7	6.9	9.0	10.8	12.5	.1894
12.	2.9	3.9	4.8	6.3	7.6	9.9	11.9	13.6	.2273
15.	3.3	4.4	5.4	7.1	8.5	11.0	13.3	15.3	.2841
18.	3.6	4.8	5.9	7.7	9.3	12.1	14.5	16.7	.3409
21.	3.9	5.1	6.3	8.4	10.0	13.0	15.7	17.9	.3975
24.	4.2	5.5	6.8	8.9	10.8	13.9	16.8	19.2	.4546
27.	4.5	5.9	7.2	9.5	11.4	14.8	17.9	20.4	.5109
30.	4.7	6.2	7.5	9.9	12.0	15.6	18.8	21.5	.5682
35.	5.0	6.7	8.2	10.8	13.0	16.8	20.4	23.2	.6629
40.	5.4	7.1	8.7	11.5	13.9	18.0	21.7	24.8	.7576
45.	5.6	7.5	9.2	12.2	14.8	19.1	23.0	26.3	.8523
50.	5.9	8.0	9.7	12.8	15.5	20.1	24.2	27.7	.9470
60.	6.5	8.7	10.7	14.1	17.0	22.1	26.5	30.3	1.136
70.	7.0	9.4	11.5	15.2	18.4	23.9	28.5	32.8	1.326
80.	7.4	10.1	12.3	16.2	19.7	25.5	31.0	35.0	1.515
90.	7.9	10.7	13.1	17.2	20.9	27.0	32.3	37.1	1.705
100.	8.4	11.3	13.8	18.2	22.0	28.5	34.1	39.1	1.894

A vel of 10 ft per sec = 600 ft per minute = 36000 ft, or 6.818 miles per hour. About 5 ft per sec is as great as can be adopted in practice to prevent the lower parts of the sewers from wearing away too rapidly by the debris carried along by the water.

t. 23. The rate at which rain water reaches a sewer or t, etc. **Burkli-Ziegler Formula.** See "European Sewerage ns," by Rudolph Hering, C. E., in Trans. Am. Soc. C. E., Nov. 1881.

$$\begin{array}{l}\text{ft. per}\\ \text{d per}\\ \text{reach-}\\ \text{ewer}\end{array} = \begin{array}{c}\text{A coef}\\ \text{according}\\ \text{to judgment}\end{array} \times \begin{array}{c}\text{Av. cub. ft. of rainfall}\\ \text{per }\textbf{second}\text{ per }\textbf{acre,}\\ \text{during heaviest fall.}\end{array} \times \sqrt[4]{\frac{\text{Av. slope of ground in feet per 1000 ft.}}{\text{No. of acres drained}}}$$

efficient, for paved streets, 0.75; for ordinary cases, 0.625; for suburbs gardens, lawns, and macadamized streets, 0.31.

e that 1 inch of rainfall per *hour* may be taken as equivalent to 1 cubic foot cond per acre. See Conversion Tables, pp. 235, etc.

ample. If an area of 3100 acres (nearly 5 square miles), with an average of 5 feet per 1000 feet, receives a maximum rainfall of 3 inches per hour, assuming a coefficient of 0.5, the rate at which the water would reach the of a sewer at the lower end of the 3100 acres would be

$$3 \times \sqrt[4]{\frac{5}{3100}} = 0.5 \times 3 \times 0.203 = 0.305 \text{ cubic feet per second per acre;}$$

5 × 3100 = 945.5 cubic feet per second, total.

the grade of the intended sewer be say 4 feet per mile; and, to avoid ive wear of its brickwork by debris swept along by the water, let its y be limited to 6.3 feet per second, which may be permitted on occasions as rains of 3 inches per hour, although, for tolerably constant flow, where to debris, it should not exceed about 5 feet per second.

l, in table opposite, the diameter, 14 ft., corresponding, as nearly as may a velocity of 6.3, and to a grade of 4 feet per mile. The area is 154 feet. Hence, 154 × 6.3 = 970 cubic feet per second = capacity of sewer. w for deposits in the sewer, make the diameter say 14.5 or 15 feet.

le of least velocities and grades for drain-pipes and rs in cities, in order that they may under ordinary circumstances keep lves clean, or free from deposits. (Wicksteed.)

Diam. Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.	Diam. in Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.
4	240	36	146.7	18	180	294	18.0
6	220	65	81.2	21	180	343	15.4
7	220	76	69.5	24	180	392	13.5
8	220	87	60.7	30	180	490	10.8
9	220	98	53.9	36	180	588	9.0
10	210	119	44.4	42	180	686	7.7
11	200	145	36.7	48	180	784	6.8
12	190	175	30.2	54	180	882	6.0
15	180	244	21.6	60	180	980	5.4

ight per foot run **of glazed terra cotta pipes** for drains, etc.; s per foot run adopted by the United Sewer Pipe Makers of the United March, 1887. For discounts, see Price List.

rain pipe, with socket joint					Sewer pipe, with sleeve joint					
Wt	Price	Bore	Wt	Price	Bore	Wt	Price	Bore	Wt	Price
lbs	$	ins	lbs	$	ins	lbs	$	ins	lbs	$
4	0.14	6	18	0.30	15	45	1.25	30	150	5.50
7	0.16	8	22	0.45	18	65	1.70	36	195	7.00
10	0.20	10	30	0.65	21	89	2.50	42	203	8.50
12	0.25	12	33	0.85	24	100	3.25	48	230	10.50

oints are filled with cement mortar; or, when used for drainage only, y. Drain pipes (3 to 12 ins bore) are about ⅝ inch thick. A bend or costs about as much as from 3 to 5 feet of pipe. The 48-inch pipes are ins thick.

24. When the area of cross section of channel is re- l at any point, as by a dam (Fig 33, p. 576) or by narrowing it, either des (Fig 32) or by placing in it a pier etc, Fig 34; a portion at least of e of grav (which would otherwise be giving *vel* to the water up-stream ne point where the obstruction takes place), causes *pressure* against the c. This pres maintains the up-stream water at a higher level than it would otherwise have. Said water is then practically in a reservoir; *i e*, it has less vel and greater pres than before. If the reservoir has no outlet, there is no vel; and *all* of the head, or force of grav, acting on the water is expended in *pres*.

But if there is an outlet, as over the dam, or between the piers etc, a portion *c o*, Figs 31, 33, 34, of this pres or head, is expended in giving *vel* (or an acceleration of vel) to the water escaping by that outlet; after which only so much head (in the shape of surface slope) is needed as will overcome the resistances of the channel down-stream from the obstruction, and so *maintain uniform* the vel given to the water by the head *c o*.

Where a large canal, such as those intended for navigation, is fed from a reservoir, the fall *c o* in feet is approximately

$$= \text{mean velocity}^2 \text{ in canal, in feet per second, } \times .017;$$

and in smaller canals, such as mill courses,

$$= \text{mean velocity}^2 \text{ in canal, in feet per second, } \times .02.$$

The abruptness of the fall may be diminished by rounding off or sloping the edges of the piers, or the corners at the sides of the channel (Fig 32) or the approach to the dam

Fig 33 is a cross section of **Clegg's dam,** across Cape Fear River, N. C. It is from measurements made by Ellwood Morris, C E; by whom they were communicated to the writer. The dam is of wooden cribwork; and its level crest, 8 ft 5 ins wide, is covered with plank; along which the water glides in a smooth sheet, 6 ins deep, (at the time of measurement). At the upper end of this sheet, and in a dist of about 2 ft, a head *c o* of 9 ins forms itself, as in the fig.

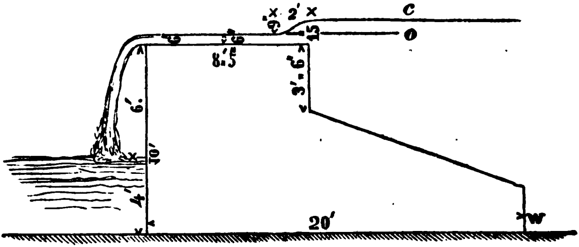


Fig 33

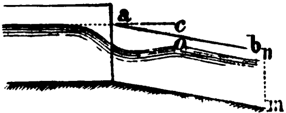


Fig 31

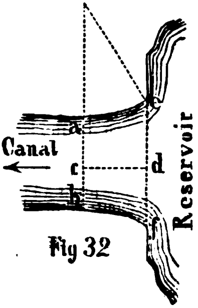


Fig 32

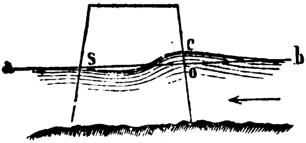


Fig 34

. 25. **Scour.** In a channel of uniform and constant slope and cross 1, the vel of the particles of water immediately adjoining the bottom and s very slight; and but little scouring takes place. But when irregular- 1 the slope or cross section occur, as in the last article, the scour is greatly sed in their immediate neighborhood.

erection of one or two piers in a quite large stream, will frequently pro- n almost incredible amount of scour, if the bottom is at all of a yielding . The greatest scour of course takes place during freshets; and near the ction.

uring action is supposed to be as square of vel.

ding to Smeaton, a vel of 8 miles an hour will not derange quarry rubble stones, not exceed- a cub ft, deposited around piers, &c; except by washing the soil from under them.

1 inch per sec. = 5 ft per min. = .056818 of a mile, or 300 ft per hour.
1 foot per sec, = 60 ft per min. = .681816 of a mile, or 3600 ft per hour.

reduce inches per sec, to feet per minute, multiply by 5.
" " " " " " " hour, " " 300.
" " " " " to miles per hour, divide by 17.6.
nile per hour = 88 ft per min = 1.4667 ft, or 17.6 ins per sec.

two following tables are (with many corrections) from Nicholson's ture; and must be looked upon merely as probable approximations. They suppose the piers, e properly rounded or pointed at their upstream ends, so as to give as free a passage as pos the water. He says that if they are square-ended, the head will be increased about 50 per ct. ject is an extremely intricate one, and admits of no precise solution. If the increased vel way the bottom until the area of water-way becomes as great as it originally was, the head rs; and the vel also becomes reduced to its original rate. This is common in soft bottoms.

LE Of heads produced by obstructions to streams.

l Vel. eam.* Per Hour. Miles.	Kind of Bottom which begins to wear away under Bottom Vel. equal to those in the first three cols.	Proportion of Area of original Water-way, occupied by the Obstructions. 1/12	1/10	1/8	1/6	1/4	1/3	1/2	5/8	3/4
		Head of Water produced at the Obstructions; in Feet.								
.170	Oose, and Mud...	.0003	.0004	.0004	.0006	.001	.0014	.0033	.0067	.0167
.341	Clay............	.0011	.0014	.0017	.0023	.004	.0058	.0133	.0267	.0646
.681	Sand.....	.0045	.0056	.0069	.0091	.015	.0231	.0532	.1069	.2584
1.36	Gravel..........	.0182	.0225	.0276	.0364	.060	.0924	.2128	.4276	1.096
2.04	Small Shingle....	.0409	.0507	.0621	.0819	.135	.2079	.4788	.9621	2.326
2.72	Large "	.0728	.0902	.1104	.1456	.240	.3696	.8412	1.710	4.144
3.41	Soft Shistus......	.1137	.1410	.1725	.2275	.375	.5775	1.320	2.672	6.475
4.09	Stratified Rocks..	.1638	.2030	.2484	.3276	.540	.8316	1.915	3.848	9.304
6.81	Hard Rocks.......	.4550	.5640	.6901	.9100	1.50	2.310	5.280	10.69	25.9

BLE Increased velocities produced at and by ded, or pointed obstructions. If square, these vels must, accord- Nicholson, be increased 1/8 part.

al Vel. eam.* Per Hour. Miles.	Proportion of Area of Water-way, occupied by the Obstructions. 1/12	1/10	1/8	1/6	1/4	1/3	1/2	5/8	3/4
	Velocity produced at the Obstruction in Feet per Second.								
.170	.28	.29	.30	.32	.35	.394	.52	.7	1.05
.341	.56	.58	.60	.64	.70	.788	1.05	1.4	2.1
.681	1.13	1.16	1.20	1.26	1.40	1.58	2.1	2.8	4.2
1.36	2.27	2.33	2.40	2.52	2.80	3.16	4.2	5.6	8.4
2.04	3.39	3.48	3.60	3.78	4.20	4.74	6.3	8.4	12.6
2.72	4.54	4.66	4.80	5.04	5.60	6.32	8.4	11.2	16.8
3.41	5.60	5.80	6.00	6.40	7.00	7.88	10.5	14.0	21.0
4.09	6.78	6.96	7.20	7.56	8.40	9.48	12.6	16.8	25.2
6.81	11.3	11.6	12.0	12.6	14.0	15.8	21.0	28.0	42.0

ry vague expression. Does it refer to the greatest surface vel at mid-channel; or to the mean e entire cross-section?

Art. 26. The resistance of water against a flat surface moving through it at right angles, is nearly as the squares of the vel; and, according to Hutton, its amount in lbs per sq ft approx = Square of vel in ft per sec. **Or like the pres of a running stream** against a perp fixed flat surface, it is = wt of a col of water whose base = pressed surf, and whose ht = head due to the vel

The resist of a sphere is to that of its great circle about as 1 to 2.9.

When the moving surf, instead of being at right angles to the direction in which it moves, forms another angle with it, the resistance becomes less in about the following proportions. Therefore, when the surf is inclined, first calculate the resistance as if at right angles; and then mult by the following decimals opposite the angle of inclination:

90°	1.00	60°	.88	40°	.58	20°	.16
80	.98	55	.83	35	.46	15	.10
70	.95	50	.76	30	.34	10	.06
65	.92	45	.68	25	.24	5	.02

The scour, or abrading power of moving water is considered to be as the square of its vel.

Art. 27. To calculate the horse-power of falling water, on the ordinary assumption that a horse-power is equal to 33000 lbs lifted 1 foot vert per min. That of average horses is really but about ⅔ as much, or 22000 lbs, 1 foot high per min. Mult together the number of cub ft of water which fall per min; the vert height or head in feet, through which it falls; and the number 62.3, (the wt of a cub ft of water in lbs:) and div the prod by 33000. Or, by formula,

$$\text{\textit{The number of horse-powers}} = \frac{\overset{\textit{cub ft}}{\textit{per min}} \times \overset{\textit{vert}}{\textit{height in ft}} \times \overset{\textit{lbs}}{62.3}}{33000.}$$

Ex. Over a fall 16 ft in vert height, 800 cub ft of water are dischd per min. How many horse powers does the fall afford?

$$\text{Here, } \frac{\overset{\text{cub ft}}{800} \times \overset{\text{ft}}{16} \times \overset{\text{lbs}}{62.3}}{33000} = \frac{797440}{33000} = 24.17 \text{ h-pow.}$$

Water-wheels do not realize all the power inherent in the water, as found by our rule. Thus, undershots realize but from ¼ to ⅓; breast-wheels, ½; overshots, from ⅔ to ¾; turbines, ⅔ to .85 of it; according to the skill of design, and the perfection of workmanship. Even when the wheel revolves in a close-fitting casing, or breast, elbow buckets give considerably more power than plain radial or center-buckets. Of the power actually received by a wheel, part is expended in friction, &c; while the remainder does the *useful* or *paying* net work of raising water, grinding grain, sawing, &c.

Observations by Genl Haupt, in 1866, gave the following results for a small **hydraulic ram.** Head of water to ram = 8.812 ft; diam of drive-pipe = 1½ ins; length 15 ft. Diam of delivery-pipe = ⅜ inch; length 200 ft. Vert height to which the water was raised by the delivery-pipe, 63.4 feet. Strokes of ram per min, 170. Quantity of water which worked the ram = 768 cub ins, = 3.31 galls, = 27.73 lbs per min. Quantity raised 63.4 ft high per min, = 48 cub ins, = 1.736 lbs. Hence the power expended per min, was $\overset{\text{lbs water}}{27.73} \times \overset{\text{ft}}{8.812} = \overset{\text{ft-lbs}}{244.35}$. And the useful effect, was $\overset{\text{lbs water}}{1.736} \times \overset{\text{ft}}{63.4} = \overset{\text{ft-lbs}}{110.06}$. Hence the ratio which the *useful effect* bears to the power in this instance, is $\frac{110.06}{244.35}$, or .45. The *actual power* of the ram is, however, greater than this, inasmuch as it has to overcome the friction of the water along the delivery-pipe.*

To find the horse-power of a running stream. Water-wheels with simple float-boards,† instead of buckets, are sometimes driven by the mere force of the ordinary natural current of a stream, without any appreciable fall like that in the foregoing case. In such cases, we must substitute the *virtual* or *theoretic* head; which is that which would impart to it the same vel which it actually has. This virtual head may be taken at once from Table p. 589. Thus, a stream has a vel of 2.386 miles per hour; or 210 ft per min; or 3½ ft per sec; and in the column of heads in Table 10, opposite to 3.5 vel per sec, we find the reqd head .190 of a ft. Having thus found the head, we must now find the quantity of water which passes any given area of the stream in a min. Thus, suppose that the *immersed* part of a float when vert is 5 ft long, and 1 ft wide or deep; then the area of this part which receives the force of the current, is 5 × 1 = 5 square feet. Hence, $\overset{\text{area}}{5 \text{ sq ft}} \times \overset{\text{vel}}{210} = 1050$ cub ft per min. Having now the cub ft per min, and the vert height or head, the number of horse-powers *of the stream* of the given area, is found by the foregoing rule, or formula.

---

* **A committee of the Franklin Institute,** in 1850, gave .71 as the coefficient for a ram at the Girard College, in which the diam of drive-pipe was 2½ ins; its length, 160 ft; fall, 14 ft. Delivery-pipe, 1 inch diam; 2260 ft long; vert rise, or height to which the water was raised, 93 ft. No details of the experiment are given. Some large rams in France give a *useful effect* of from .6 to .65 of the whole power expended. It is an excellent machine for many purposes; and is sometimes used for filling railway tanks at water stations.

† **Such wheels, for floating mills, in Europe,** rarely exceed 15 ft diam. Whatever the diam, they may have about 18 to 20 floats. The floats are from 8 to 16 ft long; and about $\frac{1}{8}$ to $\frac{1}{10}$ as deep as the diam of the wheel. They should not dip their entire depth into the water, but nearly so. They should not be in the same straight line with the radii; but should incline from them 30° up stream, to produce their full effect. All these remarks apply to wheels moving freely in a wide or indefinite channel; as in the case of a floating mill, built on a scow, and anchored out in a stream: but not to wheels for which the water is dammed up, and acts with a practical fall. No great exactness is to be expected in rules on this subject. The best vel for the wheel is about .4 that of the stream.

$$\text{Thus, } \overset{\textit{No of}}{\textit{H. Pow.}} = \frac{\overset{\textit{cub ft per min}}{1050} \times \overset{\textit{vert ht in ft}}{.190} \times \overset{\textit{lbs}}{62.3}}{33000} = \frac{12429}{33000} = .377 \textit{ of a H. Pow.}$$

But in practice the *wheels* actually realize but about 0.4 of this power *of the stream.* Therefore, the actual power of our wheel will be but .377 × .4 = 0.1508 of a horse power; or 33000 × .1508 = 4976 ft-lbs per minute. Making a rough allowance for the friction of the machine at its journals, &c, we should have about 4400 ft-lbs of *useful* power per minute; that is, the wheel would actually raise about 440 lbs 10 ft high; or 44 lbs 100 ft high, &c, per min. The vel of the stream must not be measured at the surface; but at about ½ of the depth to which the floats are to dip, or be immersed. This, however, is necessary chiefly in shallow streams, in which the depth of the float bears a considerable ratio to that of the water.

This power of a running stream, (for any given area of transverse section,) increases as the cubes of the vels; for, as we have seen, the power in ft-lbs per min is found by mult together the weight of water which passes through the section in a min, and the virtual head in ft; and since this weight increases as the vel, and this head as the square of the vel, the prod of the two (or the power) must be as the cube of the vel. Therefore, if the vel in the foregoing case had been 10.5 ft per sec, or 3 times 3.5 ft, the power of the wheel would have been 27 times as great, or .1508 × 27 = 4.07 horse powers.

DREDGING.

DREDGING is generally done by skilled contractors, who own the requisite machines, scows or lighters, &c; and who make it a specialty. It is necessary to specify whether the dredged material is to be measured in place before it is loosened: or after being deposited in the scow: because it occupies more bulk after being dredged. It was found, in the extensive dredgings for deepening the River St Lawrence through the Lake of St Peter, that on an average a cub yd of tolerably stiff mud in place, makes 1.4 yds in the scow; or 1 in the scow, makes .715 in place. Also stipulate whether the removal of bowlders, sunken trees, &c, is to constitute an extra. These often require sawing and blasting under water. The cost per cub yd for dredging varies much with the depth of water: the quantity and character of the material: the dist to which it has to be removed; whether it can be at once discharged from the machine by means of projecting side-shoots or slides; or must be discharged into scows, to be removed to a short dist by poling, or to a greater dist by steam tugs; whether it can be *dropped* or dumped into deep water by means of flap or trap doors in the bottom of the hoppers of the scows; or must be *shovelled* from the scows *into shallow water*. (at say 4 to 8 cts per yd;) or *upon land*, (at say from 6 to 10 or 20 cts for the shovelling alone, or shovelling and wheeling, as the case may be;) whether much time must be consumed in moving the machine forward frequently, as when the excavation is narrow, and of but little depth; as in deepening a canal, &c; whether many bowlders and sunken trees are to be lifted; whether interruptions may occur from waves in storms; whether fuel can be readily obtained, &c, &c. These considerations may make the cost per cub yd in one case from 2 to 4 times as great as in another. The *actual cost* of deepening a ship-channel through Lake St Peter, to 18 ft, from its original depth of 11 ft, for several miles through moderately stiff mud, was 14 cts per cub yd in place, or 10 cts in the scows; including removing the material by steam tugs to a dist of about ½ a mile, and dropping it into deep water. This includes repairs of plant of all kinds, but no profit. It was a favorable case. When the buckets work in deep water they do not become so well filled as when the water is shallower, because they have a more vertical movement, and, therefore, do not scrape along as great a distance of the bottom. Hence one reason why deep dredging costs more per yard; in addition to having to be lifted through a greater height. Perhaps the following table is tolerably approximate for large works in ordinary mud, sand, or gravel; assuming the plant to have been paid for by the company; and that common labor costs $1 per day.

Table of actual cost of dredging on a large scale; including dropping the material into scows, alongside; or into side-shoots, on board. Common labor $1 per day. Repairs of plant are included; but no profit to contractor. (Original.)

Depth in Ft.	Cts per Yard, in place.	Cts per Yard, in scow.	Depth in Ft.	Cts per Yard, in place.	Cts per Yard, in scow.
Less than 10	8.4	6	25 to 30	18.2	13
10 to 15	9.8	7	30 to 35	25.2	18
15 to 20	11.2	8	35 to 40	35.0	25
20 to 25	14.0	10			

For towing of the scows by steam tugs to a distance of ¼ mile, and dropping the mud into deep water, add 4 cts. per yard in the scow; for ½ mile, 6 cts.; for ¾ mile, 8 cts.; for 1 mile, 10 cts. Add profit to contractor. On a small scale work is done to a less advantage; and a corresponding increase must be made in these prices. Also, if the contractor himself furnishes the dredgers and plant, a still further addition must be made. It is evident that the subject admits of no great precision. Small jobs, even in favorable material, but in inconvenient positions, may readily cost two or three times as much per yard as the above; and in very hard material, as in cemented gravel and clay, four or five times as much for the *dredging*. The cost of *towing*, however, will remain as before, if wages are the same.

The cost of dredgers, tugs, &c., will vary of course with their capabilities, strength of construction, style of finish, whether having accommodations for the men to live on board or not, &c. When for use in salt water, the bottoms of both dredgers and scows should be coppered, to protect them from sea-worms; and if occasionally exposed to high waves, both should be extra strong. The most powerful machines on the St. Lawrence cost about

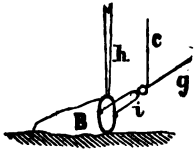
$45,000 each; and removed in 10 working hours on an average about 1800 cubic yards in place, or 2520 in the scows. Good machines, capable, under similar circumstances, of doing as much, may, however, be built for about $25,000 to $30,000. To remove this quantity to a distance of ½ to 1 mile, would require two steam tugs, costing about $6000 to $10,000 each; and 4 to 6 scows (some to be loading while others are away), holding from 30 to 60 cubic yards each, and costing from $800 to $1500 each at the shop. Scows with two hoppers are best. Such a dredger would require at least 8 or 10 men, including captain, engineer, fireman, and cook; each tug 4 or 5 men; and each scow 2 men. The engineer should be a blacksmith; or a blacksmith should be added. In certain cases a physician, clerk, assistant engineer, &c., may be needed.

Dredgers are often built on the principle of the Yankee Excavator, with but a single bucket or dipper, of from 1 to 2 cubic yards capacity. Hull about 25 by 60 feet. Draft 3 feet. Cylinder about 7 or 8 inches diameter; 15- to 18-inch stroke; ordinary working pressure 50 to 80 ℔s. per square inch, according to hardness of material. Cost $8000 to $12,000. Will raise as an average day's work (10 hours) from 200 to 500 yards in place, or 280 to 700 in the scow, according to the depth, nature of the material, &c. Require 5 or 7 men in all aboard, including cook. Burn ½ to 1 ton of coal daily. Tolerably large bowlders and sunken logs can be raised by the dipper.*

When the material is hard and compacted, the buckets of dredgers should be armed with strong steel teeth projecting from their cutting edges. On arriving at such material every alternate bucket is sometimes unshipped. By arranging the buckets so as to dredge a few feet in advance of the hull, low tongues of dry land may be cut away; the machine thus digging its own channel. The daily work in such cases will not average half as much as in wet soil.

On small operations, **dredgers worked by two or more horses,** instead of by steam, will answer very well in soft material; or even in moderately hard, by reducing the size and number of the buckets. A two-horse machine will raise from 50 to 100 yards of ordinary mud in place, or 70 to 140 in the scow, per day, at from 12 to 15 feet depth.

Soft material in small quantity, and at moderate depth, may be removed by the slow and expensive mode of the **bag-scoop,** or **bag-spoon.**



This is simply a bag B, made of canvas or leather, and having its mouth surrounded by an oval iron ring, the lower part of which is sharpened to form a cutting edge. It has a fixed handle *h*, and a swivel handle *i*. One man pushes the bag down into the mud by *h*, while another pulls it along by the rope *g*; and when filled, another raises it by the rope *c*, and empties it. If the bag is large, a windlass may be used for raising it. The men may work from a scow or raft properly anchored. Or a long-handled metal spoon, shaped like a deeply-dished hoe, may be used by only one man; or a larger spoon may be guided by a man, and dragged forward and backward by a horse walking in a circle on the scow, &c., &c.

The weight of a cubic yard of wet dredged mud, pure sand, or gravel, averages about 1⅓ tons; say 111 ℔s. per cubic foot; muddy gravel, full 1½ tons; say 125 ℔s. per cubic foot. Pure sand or gravel dredges easily; also beds of shells. Wet dredged clay will slide down a shoot inclined at from 5 to 1, to 3 to 1, according to its freedom from sand, &c.; but wet sand or gravel will not slide down even 3 to 1, without a free flow of water to aid it; otherwise it requires much pushing.

---

* The writer has seen cases in which a **circular saw** for logs in deep water would have been a very useful addition to a dredger. It should be worked by steam; and be adjustable to different depths. It would cost but about $500.

FOUNDATIONS.

A VOLUME might be occupied by this important subject alone. We have space for only a few general hints; leaving it to the student to determine how far they may be applicable in any given case. In ordinary cases, as in culverts, retaining walls, &c, if excavations, or wells, &c, in the vicinity, have not already proved that the soil is reliable to a considerable depth, it will usually be a sufficient precaution, after having dug and levelled off the foundation pits or trenches to a depth of 3 to 5 ft, to test it by an iron rod, or a pump-auger; or to sink holes, in a few spots, to the depth of 4 to 8 ft farther; (depending upon the weight of the intended structure;) to ascertain if the soil continues firm to that distance. If it does, there will rarely be any risk in proceeding at once with the masonry; because a stratum of firm soil, from 4 to 8 ft thick, will be safe for almost any ordinary structure; even though it should be underlaid by a much softer stratum. If, however, the firm upper stratum is exposed to running water, as in the case of a bridge-pier in a river, care must be taken to preserve it from gradually washing away; or from becoming loosened and broken up by violent freshets; especially if they bring down heavy masses of ice, trees, and other floating matter. These are sometimes arrested by piers, and accumulate so as to form dams extending to the bottom of the stream; thus creating an increase of velocity, and of scouring action, that is very dangerous to the stability both of the bottom and of the structure. When the testing has to be made to a considerable depth, it may be necessary to drive down a tube of either wrought or cast iron, to prevent the soil from falling into the unfinished hole. If necessary, this tube may be in short lengths, connected by screw joints, for convenience of driving; and the earth inside of it may be removed by a small scoop with a long handle.*

Borings in common soils or clay may be made 100 feet deep in a day or two by a common wood auger 1½ inches diameter, turned by two to four men with 3 feet levers. This will bring up samples.

In starting the masonry, the largest stones should of course be placed at the bottom of the pit, so as to equalize the pressure as much as possible; and care should be taken to bed them solidly in the soil, so as to have no rocking tendency. The next few courses at least should be of large stones, so laid as to break joint thoroughly with those below. The trenches should be refilled with earth as soon as the masonry will permit; so as to exclude rain, which would injure the mortar, and soften the foundation. It is well to ram or tread the earth to some extent as it is being deposited.

If the tests show that the soil (not exposed to running water) is too soft to support the masonry, then the pits should be made considerably wider and deeper; and afterward be filled to their entire width, and to a depth of from 3 to 6 or more ft, (depending on the weight to be sustained,) with rammed or rolled layers of sand, gravel, or stone broken to turnpike size; or with concrete in which there is a good proportion of cement. On this deposit the masonry may be started. The common practice in such cases, of laying planks or wooden platforms in the foundations, for building upon, is a very bad one. For if the planks are not constantly kept thoroughly wet, they will decay in a few years; causing cracks and settlements in the masonry.

Some portions of the brick aqueduct† for supplying Boston with water gave a great deal of trouble where its trenches passed through running quicksands and other treacherous soils. Concrete was tried, but the wet quicksand mixed itself with it, and *killed* it. Wooden cradles, &c, also failed; and the difficulty was finally overcome by simply depositing in the trenches about two feet in depth of strong gravel.‡ Sand or gravel, *when prevented from spreading sideways*, forms one of the best of foundations. To prevent this spreading, the area to be built on may be surrounded by a wall; or by squared piles driven so close as to touch each other; or in less important cases, by short sheet piles only. But generally it is sufficient simply ve the trenches a good width; and to ram the sand or gravel (which are all etter if wet) in layers; taking care to compact it well against the *sides* of rench also. Under heavy loads, some settlement will, of course, take place, the case in all foundations except rock. If very heavy, adopt piling, &c. RILLAGE.

---

* Subterranean caverns in limestone regions are a frequent source of trouble, against which it is difficult to adopt precautions.

† The Cochituate aqueduct, built 1846–48; egg-shape, 6 feet 4 inches × 5 feet, with semicircular invert.

‡ Smeaton mentions a stone bridge built upon a natural bed of gravel only about two feet thick, overlying deep mud so soft that an iron bar 40 feet long sank to the head by its own weight. One of the piers, however, sank while the arches were being turned, and was restored by Smeaton. Although a wretched precedent for bridge-building, this example illustrates the bearing power of a thick layer of well-compacted gravel.

hen an unreliable soil overlies a firm one, but at such a h that the excavation of the trenches (which then must evidently be made r, as well as deeper) becomes too troublesome and expensive; especially (as generally happens in that case) water percolates rapidly into the hes from the adjacent strata, we may resort to piles. When ing deep foundation-pits in **damp clay,** we must remember that this rial, being soft, has, to a certain degree, a tendency to press in every direc- like water. This causes it to bulge inward at the sides, and upward at the om. The excavations for tunnels, or for vertical shafts, often close in all nd, and become much contracted thereby before they can be lined; there- they should be dug larger than would otherwise be necessary. The bottoms nal and railroad excavations in moist clay are frequently pressed upward he weight of the sides. **Dry clay** rapidly absorbs moisture from the air, swells, producing effects similar to the foregoing. Its expansion is attended reat pressure; so that retaining-walls backed with dry rammed clay will n danger of bulging if the clay should become wet. It is a treacherous rial to work in. **For concrete foundations,** see pp. 946, &c.

to the greatest load that may safely be trusted on an earth founda- Rankine advises not to exceed 1 to 1.5 tons per square foot. But experi- proves that on good compact gravel, sand, or loam, at a depth beyond spheric influences, 2 to 3 tons are safe, or even 4 to 6 tons if a few inches of ement may be allowed, as is often the case in isolated structures without ors. Years may elapse before this settlement ceases entirely. Pure clay, ially if damp, is more compressible, and should not be trusted with more 1 to 2.5 tons, according to the case. All earth foundations must yield *some-* . **Equality of pressure** is a main point to aim at. **Tremor** in- es settlements, and causes them to continue for a longer period, especially eak soils. Great care must be taken not to overload in such cases, even if l. **Foundations in silty soils** will probably settle, in years, at the of from 3 to 12 inches per ton (up to 2 tons) per square foot of *quiet* load, t on piles.

ure 2 shows an easy mode of obtaining a foundation in certain cases. It e "**pierre perdue**" (lost stone) of the French; in English, "**ran- n stone,**" or **rip-rap.**

is merely a deposit of rough angular quarry stone thrown into the water; largest ones being at the outside, to resist disturbance from freshets, ice, ing trees, &c. A part of the interior may be of small quarry chips, with gravel, sand, clay, &c. When the bottom is irregular rock, this process s the expense of levelling it off to receive the masonry. For 2 or 3 feet w the surface of the water, the stones may generally be disposed by hand, so lie close and firmly. Small spawls packed between the larger ones will e the work smoother, and less liable to be displaced by violence. Cramps hains may at times be useful for connecting several of the large stones her for greater stability. **Rip-rap, however, is apt to settle.**

the bottom is so yielding as to be liable to wash away in ets, it may, in addition, be protected, as in Fig 2, by a covering of the same kind of stones, as at *c*: extending all around the structure. Or the main pile of stones may be extended as per dotted line at *d*; so that if the bottom should wash away, as per dotted line at *o*, the stones *d* will fall into the cavity, and thus prevent further damage. Sheet-piles, *s s*, may be as an additional precaution. For greater security, the bed of the river may dged or scooped under the entire space to be covered by the main deposit, as tted lines in Fig 3, to as great a depth as any scouring would be apt to reach;

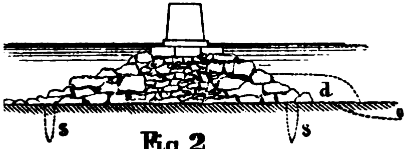


Fig 2

this excavation also to be filled with stone. Such foundations are evidently best adapted to quiet water. The masonry should rest on a strong platform.

Large deposits of stone, as in these two figs, greatly increase the velocity, and the scouring action of the stream around them, especially in freshets; unless the bottom on each side from the deposit be dredged out to such an extent that the original area of water shall not be reduced. If the bottom is treacherous, this should be done before depositing the covering stones c, Fig 2. Judgment and experience are necessary in such matters, as in all others connected with engineering. Mere study will not guard against constant failures. Theory and practice must guide each other.

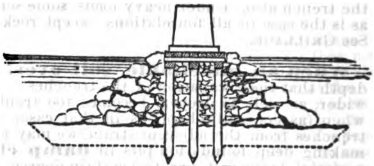
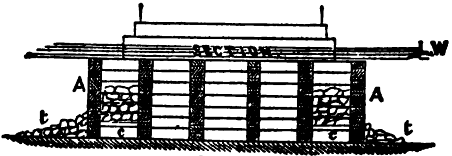


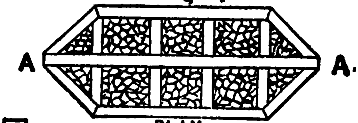
Fig 3

Fig 3 is another simple method; and when it does not create too great an obstruction to the navigation of the stream, or to the escape of its waters in time of high freshets, is a very effective one. Here the piles are first driven into the river bottom, for the support of the pier; then the deposit of stone is thrown in, for the support and protection of the piles; preventing them from bending under their loads; and shielding them from blows from floating bodies. The tops of the piles being cut off to a level, a strong platform of timber is laid on top of them, as a base for the masonry. The top of the platform should not be less than about 12 or 18 ins below ordinary low water, to prevent decay. Mitchell's iron screw pile; or hollow piles of cast iron, may be used instead of wooden ones.

Figs 4 represent a convenient method of establishing a foundation in water, by means of a **timber crib, A A, without a bottom.** It should be built of squared timbers, notched together at their crossings, as shown at Fig 5; each notch being ¼ of the depth of the stick. By this means each timber is supported throughout its entire length by the one below it; and resists pulling in both directions. Bolts also are driven at the intersections; at least in the sides of the crib, to prevent one portion from being floated off from the other. The crib is thus divided into square or rectangular cells, from 2 to 4 or 5 ft on a side, according to the requirements of the case. The partitions between the cells are put together in the same manner as those at the sides of the cribs; and consequently, like the latter, form solid wooden walls.



Figs 4



PLAN.

Fig. 5½.

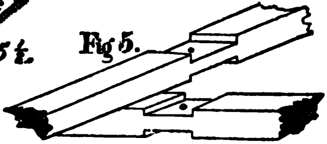


Fig 5.

The crib may be framed afloat, at any convenient spot; and when finished, may be towed to its final place, where it is carefully moored in position, and then sunk by throwing stone into a few cells provided with platforms, as at c c, for that purpose. These platforms should be placed a little above the lower edge of the cells, so as not to prevent the crib from settling slightly into the soil, and thus coming to a full bearing upon the bottom. After it has been sunk, all the cells are filled with rough stone. A stout top platform may be added or not, as the case may be; also, a protection, t t, of random stone, to prevent undermining by the current. If the sides are exposed to abrasion from ice, &c, they may be covered in whole or in part with plank, or plate iron; and the angles strengthened by iron straps, &c. In deep water, a foundation may be made partly of random stone, as in Figs 2 and 3; and on top of this may be sunk a crib, with its top about 2 ft under low water, as a base for the masonry. This is much safer than random stone alone.

On uneven rock bottom it may be necessary to scribe the bottom of the crib to fit the rock; or the crib may first be sunk by means of a loaded platform on its top, or by filling some of its cells, until its lowest timbers are within a short distance above the bottom. Being there kept in a horizontal position, small stones may be thrown into the cells, and allowed to find their way under the timbers of the crib, thus forming a level support for it. The cells may then be filled; and rip-rap deposited outside around the crib to prevent the small stones from being displaced.

crib with only an outside row of cells for sinking it may be t; and the interior chamber may be filled with concrete under water. The masonry may then on the concrete alone. If the crib rests upon a foundation of broken stone, the upper interstices his stone should first be levelled off by small stone or coarse gravel to receive the concrete of the r chamber.

r a crib like Fig. 4 may be sunk, and piles be driven in the cells, which afterward be filled with broken stone or concrete. The masonry may then rest on the piles only. h in turn will be defended by the crib. If the bottom is liable to scour, place sheet-piles or ap around the base of the crib.

y all means avoid a crib like *e*, Fig 5½, much higher at one part n at another, *if the superstructure s is to rest on the timber of the crib* instead of on , or on concrete independent of the timber; for the high part of the crib will compress more under oad than the low part, and will thus cause the superstructure to lean or to crack.

crib either straight sided or circular, with only an outer row of cells for **puding may be used as a cofferdam** (see cofferdams, p. 586). The joints een the outer timbers should be well caulked; and care be taken, by means of outside pile-planks, el, &c, to prevent water from entering beneath it.

he cast-iron Bridge across the Schuylkill at Chestnut St, ila, Mr. Strickland Kneass, Engineer, affords a striking example of crib dation. The center pier stands on a crib, an oblong octagon in plan; 31 by 87 feet at base; 24 0 ft at top; and (with its platform) 29 ft high. Its timbers are of yellow pine, hewn 12 ins are; and framed as at Fig 5. The lower timbers were carefully cut or scribed to conform to the gularities of the tolerably level rock upon which it rests. These were ascertained (after the 8 ft h of gravel had been dredged off) in the usual manner of mooring above the site a large floating den platform, composed of timbers corresponding in position with all those of the lower course he intended crib, both longitudinal and transverse. Soundings were then taken close together g all these lines of timber. Most of the cells are about 3 by 4 ft on a side, in the clear. A few hem had platforms at the level of the second course from the bottom, for receiving stone for sink. the crib; the others are open to the bottom.

he crib was built in the water; and was kept floating, during its construction, with its unfinished continually just above water, by gradually loading it with more stone as new timbers were added. stone required for this purpose alone was 800 tons. When the crib was towed into position, and red, 150 tons more were added for sinking it. All the cells were afterward filled with rough dry e, and coarse gravel screenings; making a total of 1666 tons. A platform of 12 by 12 inch squared ber covered the whole; its top being 2½ ft below low water. The pier alone, which stands on this , weighs 3255 tons; and during its construction it compressed the crib 6½ ins. The weight of erstructure resting on the pier, may be roughly taken at 1000 tons more.

n ordinary caisson is merely a strong scow, or a box with- a lid; and with sides which may at pleasure be readily detached from its bottom. It is built on , and then launched. The masonry may first be built in it, either in whole or in part, while at; and the whole being then towed into place, and moored, may be sunk to the bottom of the r, to rest upon a foundation previously prepared for it, either by piling, if necessary; or by ely levelling off the natural surface, &c. The bottom of the caisson constitutes a strong timber platform, upon which the masonry rests; and is so arranged, that after it is sunk, the sides may be detached from it, and removed to be rebottomed for use at another pier, if needed. This detaching may be effected by some such contrivance as that shown in Fig 6, where P P *w* is the bottom of the caisson, to which are firmly attached at intervals strong iron eyes *t*; which are taken hold of by hooks *d*, at the lower end of long bolts E *n*, reaching to the top timbers S of the crib, where they are confined by screw nuts *n*. By loosening the nuts *n*, the hooks *d* can be detached from the eyes *t*; and the sides can then be removed from the bottom; there being no other connection between the two. These hooks and eyes are usually placed outside of the caisson; the screw nuts *n* being sustained by the projecting ends of cross pieces, as *t t*, Fig 9. The improper position given them in our Fig was merely for convenience of illustrating the principle. It will sometimes be necessary to have side detachable from the others, in order to float the caisson away clear from the finished pier; ess it be floated away before the masonry has been built so high as to render the precaution use- . Fig 6 shows one of many ways of constructing a caisson; with sides consisting of upright ner-posts, I; cap pieces S, on top; and sills *g* at bottom, resting on the bottom platform P P *w*; rmediate uprights T, framed into the caps and sills; the whole being covered outside by one or thicknesses of planking B, which, as well as the platform, should be well calked, to prevent king. Tarpaulin also may be nailed outside to assist in this. The greatest trouble from leaking here the sides join the platform. On top of the platform is firmly spiked a timber *o o*, extending around it just inside of the inner lower edge of the sides of the caisson. Its use is to prevent sides from being forced inward by the pressure of the water outside. The details of construction of course vary with the requirements of the case. In deep caissons, inside cross-braces or struts n side to side, as at *c c*, Fig 7, will be required to prevent the sides from being forced inward by pressure of the water, as the vessel gradually sinks while the masonry is being built within it. the masonry is carried up, the struts are removed; and short ones, extending from the sides of caisson to the masonry, are inserted in their place. When the caisson is shallow, only the upper rse of braces will be required; they also support a platform for the workmen and their materials. leep caissons, in order not to be in the way of the masons, the outer planking of the sides may, part, be gradually built up as the masonry progresses It may sometimes be expedient to build masonry hollow at first, with thin transverse walls inside to stiffen it if necessary; and to com-

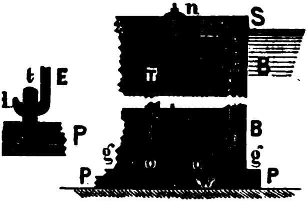


Fig 6.

plete the interior after sinking the caisson. Indeed, masonry or brickwork, in cement, may thus be built hollow at first, resting on the platform; the masonry itself forming the sides of the caisson. Or the sides may consist of a water-tight casing of iron, or wood, of the shape of the intended pier, &c. This casing being confined to the platform, becomes, in fact, a mould, in which the pier may be formed, and sunk at the same time by filling it with hydraulic concrete. **For concrete foundations,** see pp 946 &c.

On rock bottom the under timbers of the platform may be cut to suit the irregularities as already stated under "Cribs." Or the bottom may be levelled up by first depositing large stones around the area upon which the caisson is to rest; and then filling between these with smaller stones and gravel; testing the depth by sounding. Or a level bed of cement concrete may, with care, be deposited in the water. If there are deep narrow crevices in the rock, through which the concrete may escape, they may be first covered with tarpaulin. Diving bells may often be used to advantage, in all such operations. But in the case of very irregular rock, it will often be better to resort to coffer-dams.

Valves for the admission of water for sinking the caisson are usually introduced. If, after sinking, it should be necessary to again raise the whole, it is only necessary to close the valves, and pump out the water. Guide piles may be driven and braced alongside of the caisson, to insure its sinking vertically, and at the proper spot. Or it may be lowered by screws supported by strong temporary framework.

Assuming the uprights I, T, &c. Fig 6, to be sufficiently braced, as at *c c*, Fig 7, the following table will show the thickness of planking necessary for different distances apart of the uprights, (in the clear,) to insure a safety of six against the pressure of the water at different depths; and at the same time not to bend inward under said pressure, more than $\frac{1}{480}$ part of the distance to which they stretch from upright to upright; or at the rate of ¼ inch in 10 ft stretch; ⅛ inch in 5 ft, &c. Such a table may be of use in other matters.

Table of thickness of white pine plank required not to bend more than $\frac{1}{480}$ part of its clear horizontal stretch, under different heads of water. (Original.)

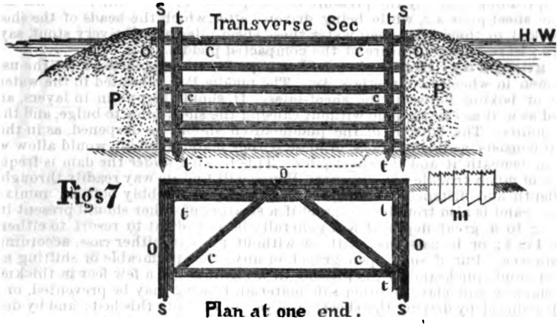
Stretch in Ft.	HEADS IN FEET.				
	40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22¼	20	17½	14	11

Coffer-dams are enclosures from which the water may be pumped out, so as to allow the work to be done in the open air. Their construction of course varies greatly. In still shallow water, a mere well-built bank of clay and gravel; or of bags partly filled with those materials when there is much current, will answer every purpose; or (depending on the depth) a single or double row of sheet-piles; or of squared piles of larger dimensions, driven touching each other; their lower ends a few feet in the soil; and their upper ones a little above high water, and protected outside by heaps of gravelly soil or puddle, (as at P in Fig 7,) to prevent leaking. The sheet-piles may be of wood; or of cast iron, of a strong form.

The sufficiency of a mere bank of well-packed earth in still water, is shown by the embankments or levees, thrown up in all countries, to prevent rivers from overflowing adjacent low lands. The general average of the levees along 700 miles of the Mississippi, is about 6 ft high; only 3 ft wide on top; side-slopes 1½ to 1. In floods the river rises to within a foot or less of their tops; and frequently bursts through them, doing immense damage. They are entirely too slight.

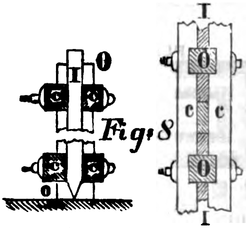
The method of a single row of 12 by 12 inch squared piles, driven in contact with each other, (*close piles,*) and simply backed by an outer deposit of impervious soil, is very effective; and with the addition of interior cross-braces or struts, like *c c*, Fig 7, to prevent crushing inward by the outside pressure of the water and puddle when pumped out, has been successfully employed in from 20 to 25 ft depth of water, in which there was not sufficient current to wash away the puddle. The cross-braces are inserted successively, as the water is being pumped out; beginning, of course, with the upper ones. The ends of these braces may abut on longitudinal timbers, bolted to the piles for the purpose. Another method is **a strong crib,** composed of uprights framed into caps and sills; and covered outside with squared timbers or plank, laid touching each other, and well calked; as in the caisson, Fig 6; but without a bottom. Between the opposite pairs of uprights are strong interior struts, as *c c*, Fig 7, reaching from side to side, to prevent crushing inward. The upper series of these usually supports a platform for the workmen, windlasses, &c. The crib having been built on land, is launched, taken to its final place, and sunk by piling stones on a temporary platform resting on the cross-struts; the bottom of the stream having been previously levelled off, if necessary, for its reception.

To prevent leaking under the bottom of the crib, sheet-piles may be driven around it, their heads extending a few feet above its bottom; or a small deposit of outside puddle may be placed around it, as shown at the stone deposits *t t*, Fig 4. Or a broad flap of tarpaulin may be closely nailed around and a little above the lower edge of the crib; so arranged that it may be spread out loosely on the river bottom, to a width of a few feet all around the outside of the crib; and the puddle may be placed upon it. Such a tarpaulin is also very useful in case the river bottom is somewhat irregular, and cannot be levelled off without too great expense; in which case the crib cannot come to a full bearing upon it; and consequently the water would leak or flow beneath freely. It is especially adapted to uneven rock; where sheet-piles cannot be driven. An artificial stratum of impervious soil may, however, be deposited on bare rock: in which case the sinking of the crib, and the subsequent operations will be the same as on a natural stratum. These expedients are evidently more or less applicable in other cases, where, to avoid repetition, they are not specially mentioned.



Figs 7

Fig 7 is another crib coffer-dam; in which the sides, instead of being planked longitudinally, as in the last instance, are sheathed with vertical sheet-piles *s*, driven after the crib is sunk. It is much inferior to the last, owing to its greater liability to leak. In one of this description, Fig 7, successfully used in 16 ft water, he dimensions of the crib were 34 ft by 80 ft. Along each long side were 7 uprights *t, t*, 19 ft long, 12 ins square, 12¾ ft apart. Into each opposite pair of these were notched, nd held by dog-irons, 6 cross-braces *c c*, of 12 ins square. The distance between the wo upper ones was 3 ft in the clear; gradually diminishing to 18 ins between the wo lower ones, on account of the increased pressure of the water in descending On he outside of the uprights, and opposite the ends of the braces, were bolted longi-dinal timbers to support the outside pressure against the 3-inch sheet-piling *s s*. her longitudinal pieces *o o*, confine the heads of the sheet-piles to the top of the ib after they are driven. The feet of the sheet-piles were cut to an angle, as at *m*; make them draw close to each other at bottom in driving.



Figs 8

SECTION. **PLAN.**

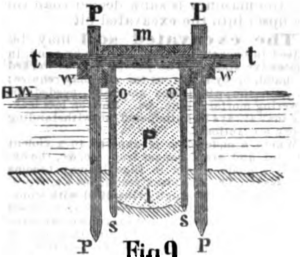
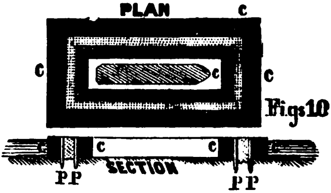


Fig 9

The sheet-piles will drive in a far more regular and satisfactory manner, with the rrangement shown in Figs 8. Here *o o* are the uprights; *c c* are pairs of longitudinal pieces, notched and bolted to the uprights, near both their tops and their feet; and at as many intermediate points as may be desired. The sheet-piles I, are inserted between these; and of course are guided during their descent much more perfectly than in Fig 7.

When the current is too strong to permit the use of outside puddle, P, Fig 7, the principle of coffer-dam shown in Fig 9, is generally used; in which both sides of the puddle are protected from washing away. The space to be enclosed by the dam is surrounded by two rows of firmly-driven main piles *p p*, on which the strength chiefly depends. They may be round. In deciding upon their number, it must be remembered that they may have to resist floating ice, or accidental blows from vessels, &c. With reference to this, extra *fender*-piles may be driven. A little below the tops of the main piles are bolted two outside longitudinal pieces *w w*, called *wales;* and opposite to them two inner ones, as in the fig. The outer ones serve to support cross-timbers *t t*, which unite each pair of opposite piles, and steady them; and prevent their spreading apart by the pressure of the puddle P. The inner ones act as guides for the sheet-piles *s s*, while being driven; after which the heads of the sheet-piles are spiked to them. In deep water these sheet-piles must be very stout, say 12 ins square; to resist the pressure of the compacted puddle.

A gangway m, is often laid on top of the cross-pieces *t t*, for the use of the workmen in wheeling materials, &c. The puddle P is deposited in the water in the space, or boxing, between the sheet-piles. It should be put in in layers, and compacted as well as can be done without causing the sheet-piles to bulge, and thus open their joints. The bottom of the puddle-ditch should be deepened, as in the fig, in case it consists, as it often does, of loose porous material which would allow water to leak in beneath it and the sheet-piles. This leaking *under* the dam is frequently a source of much trouble and expense. Water will find its way readily through almost any depth and distance of clean coarse gravelly and pebbly bottom, unmixed with earth. Sand is also troublesome; and if a stratum of either should present itself extending to a great depth, it will generally be expedient to resort to either simple cribs, Fig 4; or to caissons; with or without piles in either case, according to circumstances. But if such open gravel, or any other permeable or shifting material, as soft mud, quicksand, &c, is present in a stratum but a few feet in thickness, and underlaid by stiff clay, or other safe material, leaking may be prevented, or at least much reduced, by driving the sheeting-piles 2 or 3 ft into this last; and by deepening the puddle-trench to the same extent. It may sometimes be better, and more convenient, to dredge away the bad material entirely from all the space to be enclosed by the dam, and for a short distance beyond, before commencing the construction of the latter. If the dam, Fig 9, is (as it should be) well provided with cross-braces, like *c c*, Fig 7, extending across the enclosed area, the thickness or width *o o* of the puddle, need not be more than 4 or 5 feet for shallow depths; or than 5 to 10 ft for great ones; because its use is then merely to prevent *leaking*. But if there are no braces, it must be made wider, so as to resist *upsetting* bodily; and then, with good puddle, *o o* may, as a rule of thumb, be ¾ of the vertical depth *o l* below high water; except when this gives less than 4 ft; in which case make it 4 ft; unless more should be required for the use of the workmen, for depositing materials, &c. Or if the excavation for the masonry is sunk deeper than the puddle, the dam must be wider; else it may be upset into the excavated pit.



Figs 10

The excavated soil may be raised in buckets by windlasses, or by hand, in successive stages. The pumps may be worked by hand, or by steam, as the case may require; as also the windlasses generally needed for lowering mortar, stone, &c. More or less leaking may always be anticipated, notwithstanding every precaution.

Where a coffer-dam is exposed to a violent current, and great danger from ice, &c, the expensive mode shown in Figs 10 may become necessary. The two black rectangles *c c*, represent two lines of rough cribs filled with stone, and sunk in position; one row being enclosed by the other; with a space several feet wide between them. Sheet-piles *p p* are then driven around the opposite faces of the two rows of cribs; and the puddle is deposited within the boxing thus provided for it, as shown in the fig.

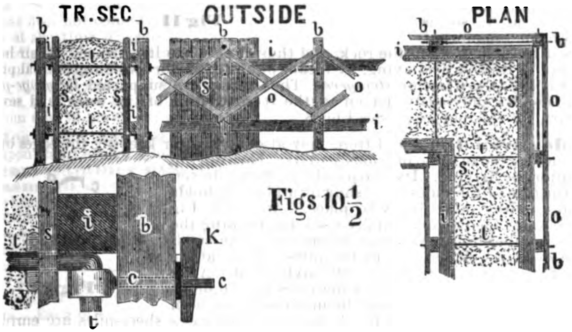
Where the current is not strong enough to wash away gravel backing, we may, on rock especially, enclose the space to be built on, by a single quadrangle of cribs sunk by stone; and after adopting precautions to prevent the gravel from being pressed in beneath the cribs, apply the backing.*

Figs 10½ show the plan, outside view, and transverse section, to a scale of 20 ft to an inch, of a coffer-dam on rock, in 8 to 9 ft water, used successfully on the Schuylkill Navigation.

---

* A pure clean coarse gravel is entirely unfit for such purposes. A considerable proportion of h is essential for preventing leaks.

Coffer-dam on rock. Uprights *b*, about 1 ft square, and 10 ft apart from center to center along the sides of the dam; and 10 ft in the clear, transversely of the dam, support two lines of horizontal stringers, *i i*; inside of which are the two lines of sheeting-piles, *s s*, enclosing between them a width of 7 ft of gravel puddle. Two flat iron bars (*t t*, of the transverse section) tie together each pair of uprights *b b*. These bars are ½ inch thick, by 2½ ins deep, and 9 ft long. Their hooked ends fit into eye-bolts *c*, which pass through the uprights *b*; outside of which they are fastened by keys, *k*, (see detail sketch.) Between the keys and *b*, were washers. At the corners of the dam (see plan) were additional tie-bars, as shown. A small band of straw, as seen at *y*, wrapped around the tie-bars just inside of the sheet-piles; and kept in place by the puddle; effectually prevented the leaking which generally proves so troublesome in such cases. The stout oblique braces, *o o*, were merely spiked to the outside faces of the uprights *b*. They are not shown in the transverse section. This dam was built on shore; in sections 30 to 40 ft long. These were floated into place, and weighted down, sheet-piled, and puddled with gravel. The dam had sluices by which water was admitted when necessary for preventing the outside head from exceeding 9 ft. The lengths of the uprights *b b* were first found by careful soundings.



Figs 10½

The mooring of large caissons or cribs, preparatory to sinking them, is sometimes troublesome, especially in strong currents. It may be necessary to drive clumps of piles; or to temporarily sink rough cribs filled with stone, to which to attach the long guide-ropes by which the manœuvring into position, &c, s done. Frequently dams are left standing after the work is done; if not in the way of navigation, or otherwise objectionable; inasmuch as the materials are rarely worth the expense of removal. But if removed, the piles should not be *drawn* out of the ground; but be *cut off* close to river bottom; for if drawn, the water entering their holes may soften the soil under the masonry. It is often expedient to drive two rows of piles from the dam to the shore, for supporting a gangway for the workmen; or even for horses and carts; or for a railway for the easy delivery of large stones, &c.

Coffer-dams may be sunk through a soft to a firm soil, in shape of a box of cribwork, either rectangular or circular, and without a bottom. This being strongly put together, and provided with proper temporary internal bracing, (to be gradually removed as the masonry is built up,) is floated into place; and after being loaded so as to rest on the soft bottom, is sunk by dredging out the soft material from inside. Additional loading will sometimes be required for overcoming the friction of the soil against the outside; or it may even become necessary to dredge away some of the outer material also. **On rock** it may at times be expedient to drill holes in deep water, for receiving the ends of piles, or of iron rods, &c. This may be done by means of long drill-rods, working in an iron tube or pipe unk as a guide to the rod; with its lower end over the spot to be bored. Or a diving-ell may be used. Or a cylinder of staves 4 to 12 inches thick, long enough to each above the surface, and having a broad tarpaulin flap or apron around its lower dge, to be covered with gravel to prevent leaking; may be sunk, and the water mped out, to allow a workman to descend, and work in the open air.

Piles. When driven in close contact, as in Fig 11, for preventing leakage; for onfining puddle in a coffer-dam: or for enclosing a piece of soft or sandy ground, to revent its spreading when loaded; or if the outside soil should wash away from around them, &c, they are called **sheet-piles.** Generally these are thinner than they are wide; but frequently they are square; and as large as bearing piles; and are then called **close piles.** To make them drive tight together at foot, they are cut obliquely as at *f*. Occasionally, when driven down to rock through soft soil, their feet are in addition cut to an edge, as at *i*, so as to become somewhat bruised when they reach the rock, and thus fit closer to its surface. Their heads are kept in line while driving, by means of either one or two longitudinal pieces *a* and *o*, called **wales** or *stringers*. These wales are supported by *gauge-piles*, or *guide-piles*, previously driven in the required line of the work, and several ft apart, for this purpose. See Figs 8.

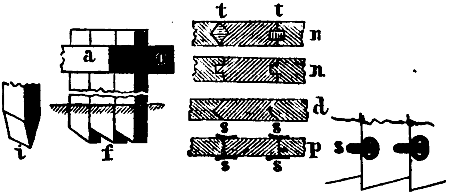


Fig 11

A dog-iron d, of round iron, may also be used for keeping the edges of the piles close at top to those previously driven, both during and after the driving. Its sharp ends, *c c*, being driven into the tops of the wales *w w*, (shown in plan,) it holds the descending pile *o* firmly in place. At *n*, *d*, *p*, Fig 11, are other modes occasionally used for keeping the piles in proper line. At *p*, the letters *s s* denote small pieces of iron well screwed to the piles, a little above their feet, to act as guides; very rarely used. At *m* are shown wooden tongues *t t*, sometimes driven down between the piles after they themselves have been driven; to assist in preventing leaks. In some cases sheet-piles are employed without being driven. A trench is first dug to their full depth for receiving them; and the piles are simply placed in these, which are then refilled. Closer joints can be secured in this manner than by driving.

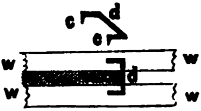
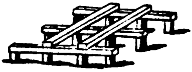


Fig 12

When piles are intended to sustain loads on their tops, whether driven all their length into the ground, or only partly so, as in Fig 8, they are called **bearing piles.** They are generally round; from 9 to 18 ins diam at top; and should be straight, but the bark need not be removed. White pine, spruce, or even hemlock, answer very well in soft soils; good yellow pine for firmer ones; and hard oaks, elm, beech, &c, for the more compact ones. They are usually driven from about 2½ to 4 ft apart each way, from center to center, depending on the character of the soil, and the weight to be sustained. **A tread-wheel** is more economical than the winch for raising the hammer, when this is done by men. Morin found that the work performed by men working 8 hours per day, was 3900 foot-pounds per man, per minute by the tread-wheel; and only 2600 by a winch.

After piles have been driven, and their heads carefully sawed off to a level, if not under water, the spaces between them are in important cases filled up level with their tops with well rammed gravel, stone spawls, or concrete, in order to impart some sustaining power to the soil between the piles. Two courses of stout timbers (from 8 to 12 ins square, according to the weight to be carried) are then bolted or treenailed to the tops of the piles and to each other, as shown in the Fig, forming what is called a **grillage.** On top of these is bolted a floor or platform of thick plank for the support of the masonry; or the timbers of the upper course of the grillage may be laid close together to form the floor. The space below the floor should also, in important cases, be well packed with gravel, spawls, or concrete. **If under water,** the piles are sawed off by a diver, or by a circular saw driven by the engine of the pile-driver, and the grillage is omitted. Instead of it the masonry or concrete may be built in the open air in a caisson, which gradually sinks as it becomes filled; or on a strong platform which is lowered upon the piles by screws as the work progresses. Or a strong caisson may first be sunk entirely under water, and then be filled with concrete, up to near low water; the caisson being allowed to remain. Or the caisson may form a cofferdam, to be first sunk, and then pumped out. If the ground is liable



› wash away from around the piles, as in the case of bridge piers, &c, defend it y sheet-piles, or rip-rap, or both.

The cost of a floating steam pile driver, scow 24 ft by 50 ft, draft ³ ins, with one engine for driving, and one (to save time) for getting another ile ready; with one ton hammer, is about $6000; and $500 more will add a cir- ılar saw, &c, for sawing off piles at any reqd depth. Requires engineman, cook, ıd 4 or 5 others. Will burn about half a ton of coal per day. Driving 20 feet ıto gravel, and sawing off, will average from 15 to 20 piles per day of 10 hours. ı mud about twice as many. On land about half as many as in water.

In the gunpowder pile driver invented by the late Mr. Thomas Shaw, ' Philadelphia, the hammer is worked by small cartridges of powder, placed one r one in a receptacle on top of the pile; and exploded by the hammer itself. can readily make 30 to 40 blows of 5 to 10 ft per minute; and, since the ımmer does not come into actual contact with the piles, it does not injure their ›ads at all; thus dispensing with iron hoops, &c, for preserving them. When ıly a slight blow is required, a smaller cartridge is used. To drive a pile 20 ft ıto mud averages about one-third of a pound of powder; into gravel, 4 times as ıch. This machine does not assist in raising the pile, and placing it in ›sition, as is done by ordinary steam pile drivers; the latter, however, average ıt from 6 to 14 blows per minute.

Piles have been driven by exploding small charges of **dynamite** id upon their heads, which are protected by iron plates.

Steam-hammer pile drivers, operating on the principle of that devised ' **Nasmyth** about 1850, are economical in driving to great depths in difficult ils where there are say 200 or more piles in clusters or rows, so that the machine n readily be moved from pile to pile.

The steam **cylinder** is upright, and is confined between the upper ends of two rtical and parallel I or channel **beams** about 6 to 12 ft long and 18 ins apart, e lower ends of which confine between them a hollow conical "**bonnet cast- ıg,**" which fits over the head of the pile. This casting is open at top, and through the hammer, which is fastened to the foot of the piston-rod, strikes the head of e pile. Each of the vertical beams encloses one of the two upright guide-timbers, "leaders," of the pile driver, between which the driving apparatus, above de- ribed, is free to slide up or down as a whole.

When a pile has been placed in position, ready for driving, the bonnet casting is ıced upon its head, thus bringing the weight of the beams, cylinder, hammer, and sting upon the pile. This weight rests upon the pile throughout the driving, the paratus sliding down between the leaders as the pile descends.

The steam is conveyed from the boiler to the cyl by a flexible pipe. When it is mitted to the cyl, the hammer is lifted about 30 or 40 ins, and upon its escape the mmer falls, striking the head of the pile. About 60 blows are delivered per min- ɘ. The hammer is provided with a trip-piece which automatically admits steam the cylinder after each blow, and opens a valve for its escape at the end of the -stroke. By altering the adjustment of this trip-piece, the length of stroke (and us the force of the blows) can be increased or diminished. The admission and ;ape of steam, to and from the cyl, can also be controlled directly by the attendant. e number of blows per minute is increased or diminished by regulating the sup- r of steam.

[n making the up-stroke, the steam, pressing against the lower cyl head, of course ›sses downward on the pile and aids its descent.

The chief advantage of these machines lies in the great rapidity th which the blows follow one another, allowing no time for the disturbed earth, ıd, &c, to recompact itself around the sides, and under the foot, of the pile. This ables the machines to do work which cannot be done with ordinary pile drivers. ey have driven Norway pine piles 42 ft into sand. They are less liable than ıers to split and broom the pile, so that these may be of softer and cheaper wood. e bonnet casting keeps the head of the pile constantly in place, so that the piles not "dodge" or get out of line. Their heads have, in some cases, been set on fire the rapidly succeeding blows.

These machines consume from 1 to 2 tons of **coal** in 10 hours, and **quire a crew** of 5 men. They work with a **boiler pressure** of from to 75 lbs per sq inch.

Rules for the Sustaining Power of Piles.

They differ very much. No rule can apply correctly to all conditions. The ground itself between the piles, in most cases, supports a part of the load; although the whole of it is usually assigned to the piles. Again, in very clayey soils, there is greater liability to sink somewhat with the lapse of time, in consequence of the admission of water between the pile and the clay; thus diminishing the friction between them. The less firm the soil, the more will the piles be affected by tremors; which also tend in time to cause sinking. In some cases this sinking will not be that of the piles settling deeper into the earth *around* them; but that of the entire compacted mass of piles and earth into which they were driven, settling down into the less dense mass *below* them. Piles are sometimes blamed for settlements which are really due to the crushing (flatways) of the timbers which rest immediately upon their heads.

In the fine **London bridge** across the Thames, each pile under some of the piers sustains the very heavy load of 80 tons. They are driven but 20 feet into the stiff, blue London clay; and are placed nearly 4 ft apart from center to center; which is too much for such piers and arches. At 3 ft apart scant, they would have had but 45 tons to sustain. They are 1 ft in diam at the middle of their length. Ugly settlements, some of them to the extent of about a ft, have occurred under these piers. **Blackfriars bridge,** in the same vicinity, exhibits the same defect. By some this is ascribed in both cases to the gradual admission of water between the clay and the piles, perhaps by capillary action of the piles themselves; or perhaps by direct leaking. It may, however, be owing in part to the crushing of the platforms on top of the piles; or to a bodily settlement of the entire mass of piled clay, into the unpiled clay beneath, under the immense load that rests upon it. This here amounts to 5½ tons per sq foot of area covered by a pier; and is probably too much to trust upon damp clay, when even the slightest sinking is prejudicial.

Maj J. Sanders, U. S. Engs, experimented largely at Fort Delaware in river mud; and gave the following in the Jour. Franklin Inst, Nov 1851. For the **safe** load for a common wooden pile, driven until it sinks through only small and nearly equal distances, under successive blows, divide the height of the fall in ins, by the small sinking at each blow in ins. Mult the quot by the weight of the hammer, ram, or monkey, in tons or pounds, as the case may be. Divide the prod by 8. He does not state any specific coefficient of safety.

Example. At the Chestnut St Bridge, Philada, the greatest weight on any pile is 18 tons. Mr Kneass had the piles driven until they sank ¾, or .75 of an inch under each blow from a 1200 lb hammer, falling 20 ft. Was he safe in doing so? Here we have the fall in ins = 20 × 12 = 240. And $\frac{240}{.75} = 320$; and 320 × 1200 = 384000 lbs; and $\frac{384000}{8} = 48000$ lbs, = 21.4 tons safe load by Maj Sanders' rule. The soil was river mud.

Our own rule is as follows. Mult together the cube rt of the fall in ft; the wt of hammer in lbs; and the decimal .023. Divide the prod by the last sinking in ins. + 1. The quotient will be the extreme load that will be just at the point of causing more sinking. For the safe load take from one twelfth to one half of this, according to circumstances. Or, as a formula,

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{\begin{matrix}\text{Cube rt of}\\ \text{fall in feet}\end{matrix} \times \begin{matrix}\text{Wt of hammer}\\ \text{in pounds}\end{matrix} \times .023.}{\text{Last sinking in inches} + 1}$$

Example. The same as the foregoing at Chestnut St Bridge. Here the cube rt of 20 ft fall is 2.714 ft. Hence we have

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{2.714 \times 1200 \times .023}{.75 + 1} = \frac{74.9}{1.75} = 42.8 \text{ tons.}$$

Or say half of this, or 21.4 tons, the load for a safety of 2. Major Sanders' rule makes the **safe** load 21.4. The actual one is 18 tons.

A safety of 2 is not enough for river mud.

But although Major Sanders' rule and our own agree very well in this instance *if a safety of 2 be taken for each*, they differ widely in some others. Thus at **Neuilly Bridge,** France, Perronet's heaviest hammer weighed 2000 lbs, fall 5 ft, sinkage .25 of an inch in the last 16 blows; or say .016 inch per blow. The piles sustain 47 tons each. Our rule gives 38.8 tons for a safety of 2; while Sanders' rule gives 515 tons safe load! If, as we think probable, there was no actual sinking at the last blow, then our rule gives 39.3 tons for a safety of 2; while Sanders' gives infinity.

At the **Hull Docks,** England, piles 10 ins square, driven 16 ft into alluvial mud, by a 1500 lb hammer, falling 24 ft, sank 2 ins per blow at the end of the driving. They sustain at least 20 tons each, or according to some statements 25 tons. Our rule gives 33.2 tons for the extreme load; or 16.6 for a safety of only 2. Sanders gives for *safety* 12.06 tons. As before remarked, 2 is not safety enough for mud. In mud, it is not primarily the piles, but the piled **soil** that settles, bodily, for years.

At the **Royal Border Bridge,** England, piles were very firmly driven from 30 to 40 ft in sand and gravel, in some cases wet. Pine was first tried, but it split and broomed so badly under the hard driving, that American elm was substituted, with success. They were driven until they sank but .05 inch per blow, under a 1700 lb monkey, falling 16 ft. They support 70 tons each. Our rule gives 47 tons for a safety of 2; while Sanders gives 364 tons safe load!

It is the writer's opinion, however, that the piles did not actually sink, as was (and always is, in such cases) taken for granted by the observers; but that they were merely compressed or partially crushed by overdriving. Most of the piles were driven until they sank (?) only an inch under 150 blows; but we doubt whether they were any safer, or farther in the ground, than when they had received only one of them; and consider such extreme precaution worse than useless.

In some experiments (1873) at Philada, a trial pile was driven 15 ft into soft river mud, by a 600 lb hammer; its last sinking being 18 ins under a fall of 36 ft. Only 5 hours after it was driven was loaded with 6.5 tons; which caused a sinking of but a very small fraction of an inch. Our rule e 6.4 tons as the extreme load. Under 9 tons it sank .75 of an inch; and under 15 tons, 5 ft. By Sanders' rule its safe load would be 2.14 tons.

U. S. Govt trial pile, about 12 ins sq, driven 29 ft through layers of silt, sand, and clay, ham- 910 lbs, fall 5 ft, last sinking .375 of an inch, bore 26.6 tons; but sank slowly under 27.9 tons. rule gives 26 tons extreme load.

'rench engineers consider a pile safe for a load of 25 tons, when it is driven to the refusal of lbs, falling 4 ft; our rule gives 24.2 tons for safety 2. They estimate the refusal by its not sink- more than .4 of an inch under 30 blows. In many important bridges &c they drive until there is sinking under an 800 lb hammer, falling 5 ft. Our rule here gives 31.5 tons extreme load; or 15.7 safety 2.

s to the proper load for safety, we think that not *more* than one-half the extreme load given our rule should be taken for piles *thoroughly* driven in *firm* soils: nor more than one-sixth when river mud or marsh; assuming, as we have hitherto done, that their feet do not rest upon rock.

f liable to tremors, take only **half** these loads.

'iles may be made of any required size as regards either length or cross section, by bolt- and fishing together sidewise and lengthwise, a number of squared timbers.

'iles with blunt ends. At South Street Bridge, Phila, 1200 stout piles of Nova Scotia spruce h blunt ends were driven 15 to 35 ft, partly in strong gravel, by a common steam pile driver, at a l cost (piles and driving) of $7 to $8 each. At **Wilmington Harbor,** Cal, Mr. C. B. Sears, S. Army, (Jour. Am. Soc. C. E., Dec 1876) found that in firm compact wet sand, after the first few ws the piles would not penetrate more than .5 to 1.5 ins at a blow, no matter how far the 2400 lb nmer fell. The unpointed ones of which there were many thousands, drove quite as readily to aver- depths of 15 ft in this sand as the pointed ones, and with much less tendency to cant. As a high had no farther effect than to batter the heads he reduced it to 10 ft, which drove an average of ut .72 inch to a blow. To insure straight driving, the ends must be at right angles to the length.

stead of driving piles to moderate depths it may at times be better to merely **plant** them butt n in holes bored by an auger like **Pierce's Well Borer.**

'he ultimate friction of piles even with the bark on, and driven about 3 ft apart from cen en probably never much exceeds about 1 ton per sq ft even when well driven into dense moist d or loamy gravel; nor more than .5 to .75 of a ton in common soils and clays; or than .1 to .2 ton in silt or wet river mud depending on the depth and density.

'he friction of cast iron cylinders seems to be about .3 that of piles.

There is a great difference in the penetrability of different ls. Thus, in the Lary bridge, no special difficulty was found in driving piles 35 ft into deep wet l; while, in other wet localities, piles of very tough wood, well shod with iron, cannot be driven into sand, without being battered to pieces. The same difference has been found in the case of w-piles. At the Brandywine light-house these could not be forced more than 10 ft into the clean sand. Stiff wet clay (and clean gravels) also differ very much in this respect. Generally they penetrable to any required depth with comparative ease; but we have seen stout hemlock piles ered to pieces in driving 6 ft through wet gravel; and Mr. Rendel found that at Plymouth he uld not by any force drive screw-piles more than about 5 ft into the clay, which is not as stiff as London clay," on which the forementioned new London and Blackfriars bridges were founded; into which even ordinary wooden piles were driven 20 ft without special difficulty.

mixture of mud with the sand or gravel facilitates driving very much; but before beginning an nsive system of piling, a few experimental ones should be driven, to remove doubt as to the ble and expense that may be anticipated. Mere boring will often be but a poor substitute for this. a general rule, a heavy hammer with a low fall, drives more pleasantly than a light one with a fall. Where a hammer of ⅔ ton (1500 lbs) falling 25 ft, in a very strong ground, shattered the ; one of 2 tons, (4500 lbs,) with 7 ft fall, drove them satisfactorily. More blows can be made in same time with a low fall; and this gives less time for the soil to compact itself around the piles een the blows. At times a pile may resist the hammer after sinking some distance; but start n after a short rest; or it may refuse a heavy hammer, and start under a lighter one. It may e slowly at first, and more rapidly afterward, from causes that may be difficult to discover. The ing of one sometimes causes adjacent ones previously driven, to spring upward several feet. A is in the most favorable position when its foot rests upon rock, after its entire length has been en through a firm soil, which affords perfect protection against its bending like an overloaded mn; and at the same time creates great friction against its sides; thus assisting much in sus- ing the load, and thereby relieving the pressure upon the foot. A pile may rest upon rock, and be very weak; for if driven through very soft soil, all the pressure is borne by the sharp point; the pile becomes merely a column in a worse condition than a pillar with one rounded end. See l. Strength of Iron Pillars. In such soils the piles need very little sharpening; indeed, better be driven without any; or even butt end down.

ıe driving of a pile in soft ground or mud will generally cause an adjacent one previously driven, an outwards unless means be taken to prevent it.

piling an area of firm soil it is best to begin at its center and work outwards; otherwise the soil become so consolidated that the central ones can scarcely be driven at all.

Elastic reaction of the soil has been known to cause entire piled areas se, together with the piles, before they were built upon.

n very firm soil, especially if stony; or n in soft soil, if the piles are pointed, and to be driven to rock; their feet should protected by **shoes** of either wrought , as at *a*, *s*, and *b*, Figs 13; spiked to the by means of the iron straps *n*, forged hem; or of cast iron, as at *c*, where the e is a solid inverted cone, the wide flat e of which affords a good bearing for the bottom of the pile-point. The dotted is a stout wrought-iron spike, well se- ed in the cone, which is cast around it; holds the shoe to the pile. Regular

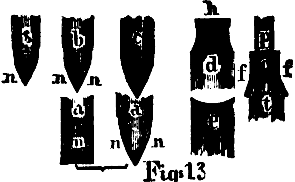


Fig 13

wrought-iron shoes will generally weigh 18 to 30 lbs; but sheet iron may be used when the soil is but moderately compact; plate iron when more so; and solid iron or steel points, from 2 to 4 ins square at the butt, and 4 to 8 ins long, when very compact and stony. **Holes may be drilled in rock** for receiving the points of piles, and thus preventing them from slipping: by first driving down a tube, as a guide to the drill, after the earth is cleaned out of the tube. **To preserve the heads** to some extent from splitting under the blows of the hammer, they are usually surrounded by a hoop *h*, Fig *d*; from ½ to 1 inch thick; and 1½ to 3 ins wide. These are, however, sometimes but imperfect aids; for in hard driving the head will crush, split, and bulge out on all sides, frequently for many feet below the hoop: moreover, the hoops often split open. The heads, therefore, often have to be sawed, or pared off several times before the pile is completely driven; and allowance must be made for this loss in ordering piles for any given work; especially in hard soil. Capt Turnbull, U S Top Eng, states that at the Potomac aqueduct, his pileheads were preserved from injury by the simple expedient of dishing them out to a depth of about an inch, and covering them by a loose plate of sheet iron; as shown in section at *s*, Figs 13. A very slight degree of brooming or crushing of the head, materially diminishes the force of the ram. Piles may be driven through small loose rubble without much labor. Shaw's driver does not injure the heads. Piles which foot on sloping rock may slide when loaded.

To drive a pile head below water a wooden punch, or follower, as at *p*, Figs 13, may be used. The foot of this punch fits into the upper part of a casting *f f*, round or square, according to the shape of the pile; and having a transverse partition *o o*. The lower part of the casting is fitted to the head of the pile *t*; and the hammer falls on top of the punch. When driving piles vertically in very soft soil, to support retaining-walls, or other structures exposed to horizontal or inclined forces, care must be taken that these forces do not push over the piles themselves; for in such soils piles are adapted to resist vertical forces only, unless they be driven at an inclination corresponding to the oblique force.

A broken pile may be drawn out, or at least be started, if not very firmly driven, by attaching scows to it at low water, depending on the rising tide to loosen it. Or a long timber may be used as a lever, with the head of an adjacent pile for its fulcrum. Or a crab worked by the engine of the pile driver. In very difficult cases the method devised by Mr J. Monroe, C E, may be used. A 4 inch gas pipe 15 ft long, shod with a solid steel point, and having an outer shoulder for sustaining a circular punch, was thereby driven close to and 2 or 3 ft deeper than two piles driven 12 ft, in 37 ft water, and broken off by ice. Four pounds of powder were then deposited in the lower end of the pipe, and exploded, lifting the piles completely out of place. It will often be best to let a broken pile remain, and to drive another close to it. May be drawn by hydraulic press.

Ice adheres to piles with a force of about 30 to 40 lbs per sq inch, and in rising water may lift them out of place if not sufficiently driven.

Iron piles and cylinders. Cast iron in various shapes has been much used in Europe for sheet piles; especially when intended to remain as a facing for the protection of concrete work, filled in behind and against them.* Cast-iron cylinders, open at both ends, may be used as bearing piles; and may be cleaned out, and filled with concrete, if required. The friction in driving is greater than in solid piles, inasmuch as it takes place along both the inner and the outer surfaces. This may be diminished by gradually extracting the inside soil as they go down. They require much care, and a lighter hammer, or less fall than wooden ones, to prevent breaking: to which end a piece of wood should be interposed between the hammer and the pile; or the ram may be of wood. But it is better to use them in the shape of **screw cylinders,** which, moreover, gives them the advantage of a broad base, as in the following.

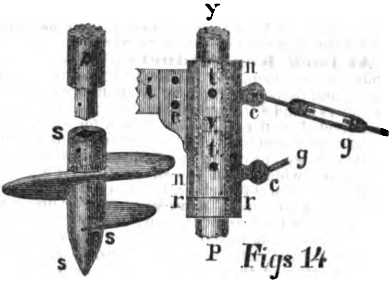
Brunel's process. He experimented with an open cast-iron cylinder, 3 ft outer diam; 1½ ins thick; in lengths of 10 ft, connected together by internal socket and joggle joints, secured by pins, and run with lead. It had a sharp-edged hoop or cutter at bottom: and a little above this, one turn of a screw, with a pitch of 7 ins, and projecting one foot all around the outside of the cylinder. By means of capstan bars and winches, he screwed this down through stiff clay and sand, 58 feet to rock, on the bank of a river. In descending this distance the cylinder made 142 revolutions; sinking on an average about 5 ins at each. The time occupied in actually screwing was 48½ hours; or about $1\frac{2}{10}$ ft per hour. There were, however, many long intervals of rest for cleaning away the soil in the inside. After resting, there was no great difficulty in restarting. The next fig will give an idea of the arrangement of the screw.

The screw-pile of Alex. Mitchell, Belfast, consists usually of a rolled iron shaft A, Figs 14, from 3 to 8 ins diam; and having at its foot a cast-iron screw S S S, with a blade of from 18 ins to 5 ft diam. The screws used for light-houses, exposed to moderate seas, or heavy ice-fields, are ordinarily about 3 ft diam, have 1½ turns or threads, and weigh about 600 lbs. The round rolled shafts are from 5 to 8 ins diam. They are screwed down from 10 to 20 ft into clay, sand, or coral, by about 30 to 40 men, pushing with 6 to 8 capstan bars, the ends of which describe a circle of about 30 to 40 ft diam. For this purpose a platform on piles has frequently to be prepared. In quiet water, this may be supported on scows; or a raft well moored may be used when the driving is easy; or the deck of a large scow with a well-hole in the center for the pile to pass through. Roughly made temporary cribs, filled with stone and sunk, might support a platform in some positions. The platform must evidently be able to resist revolving horizontally under the great pushing force of the men at the capstan bars; and on this account it is difficult to drive screws to a sufficient depth, in clean compact sand, by means of a floating platform. The feet of the piles must be firmly secured to the screws, to prevent

---

* **Cast iron, intended to resist sea-water,** should be close-grained, hard, white metal. In such, the small quantity of contained carbon is chemically combined with the metal; but in the darker or mottled irons it is mechanically combined, and such iron soon becomes soft, (somewhat like plumbago,) when exposed to sea-water. Hard white iron has been proved to resist for at least 40 years without any deterioration: whether constantly under water, or alternately wet and dry. Copper and bronze are but slightly and superficially affected by sea-water; but destructive galvanic action takes place if diff metals are in contact.

ıeir being lifted out of them by the upward force of waves against the super-
ructure. At *y p*, Figs 14, is shown a mode of splicing or uniting the different
ngths or sections of a pile. The point of junction is at *v*; *r r* is a stout iron ring
rged on to the lower pile *p*,
bout a foot or 18 ins below its
p *v*. A strong cylindrical cast-
g *n n*, enclosing the ends of
e sections, rests on this ring,
d is pinned through the piles,
at *t t*. On this casting are
so cast projections *c c c*, for at-
ching rods *g g*, and beams *i*, &c,
ecessary for bracing the struc-
re from pile to pile. The time
tually required for driving a
rew is from 2 to 10 hours, in
vorable circumstances.



Figs 14

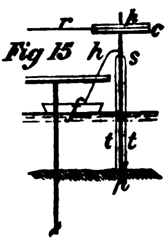
At the Brandywine lighthouse, on
sand-bank of very pure sand, cov-
ed 6 or 8 ft at low water, and from
to 13 ft at high, they could not be
rced down, from a fixed platform,
r more than 10 ft. At other places 20 ft in sand is reached without much trouble, where the sand
ntains a good deal of mud, but its **bearing power** is then less. This (ultimate) ranges between
out 1 and 6 tons per sq ft according to purity, depth, compactness, &c, of the sand. In important
ses the bearing power should be tested.

Mitchell's piles have been screwed about 40 feet into a mixture of clay and sand, with screws
ft diam. They pass through small broken stone and coral rock without much difficulty; and will
ısh aside bowlders of moderate size. Ordinarily, clay or sand will present no great obstruction;
ıt occasionally either of them will do so. Perfectly pure clean sand, as a general rule, gives most
fficulty. At the Brandywine shoal the driving was aided by a spur and pinion placed as low as the
ater permitted; and the levers were worked by 30 men. The danger of twisting off the shaft is
e limit for screwing them. They are much used for the anchoring of chains for mooring buoys, &c.
n land, small screws, with short hollow shafts, make good durable supports for depot pillars, cranes,
ooden telegraph poles, station signals in marine surveying, &c, &c. They can readily be unscrewed
r removal. Horses or oxen may be used in driving large screws. The Brandywine light-house
ands on 9 screw-piles, which are surrounded by 30 others of 5 ins diam, as fenders. They have to
sist not only moderate seas, but immense fields of floating ice, miles in extent. An unfinished
ructure was destroyed by ice, which at times injures the bracing of the standing one.

Test borings should be made to ensure that the screws do not stop just
ove a very weak stratum which may endanger their bearing power. So with *any* piles.

By means of a jet of water forcibly impelled through a tube by a force
ımp, the most obstinate sands (but not stiff clay or cemented gravel) will be loosened, and the sink-
g of screw-piles, or wooden ones, or even the largest cylinders, be greatly facilitated. In a govern-
ent **pier at Cape Henlopen** in very compact sand, in which 6 out of 7
rews previously broke before reaching 10 ft, the use of the jet was found to remove more than
ree-fourths of the resistance.* The pile *p* to be sunk having first been placed in position as in Fig
, the lower open ends *t t* of a bent iron tube *t s t* of one and a
arter ins bore were stood upon the upper face of the screw disk, and
ere held firmly by 3 or 4 men while the pile was being screwed down
the capstan *c*, which was worked by a leading rope *r*. From the
nd *s* of the pipe, a hose *h*, 2 ins diam. led to the force pump, the
linder of which was 5 ins bore, and 9 ins stroke, and worked about
full strokes per minute, by a mule walking on a tread wheel on a
ating platform *f*. There was now no trouble in screwing the piles to
y required depth. Previous trials by playing the jet *beneath* the disk
ve unsatisfactory results.

Fig 15



In Mobile Bay several thousands of wooden piles,
om 18 to 48 ins diam, were sunk from 10 to 20 ft into obstinate sand,
the average sinking rate of about 1 ft per second, entirely by means
jets. The jet was propelled by a city steam fire engine, on a steam-
at, through its own hose, with a one and a quarter inch nozzle.
uring the descent the nozzle *n n* was held loosely in its place near
e foot of the pile, by two staples *s s* and by a string *t* reaching to the
rface. The piles were suspended by their heads from shears, by the
ckle of which their descent was regulated. The sand settled firmly
ound the piles in a few minutes after they were sunk.†

At Tensas River, Alabama, for iron cylinders 6 ft diam
closing piles, in deep light shifting sand, the jet was forced by a small
ary pump of 200 to 300 revolutions per minute, through a canvas hose 3 ins diam,
o a central conical cast iron vessel 10 ins diam, from which radiated 12 gas pipes 1
h diam, and about 30 ins long. At the outer end of each of these radii was an
ow to which was attached a long vertical pipe reaching down into the cylinder,
d made in 10 ft lengths with screw ends for prolonging them as the cylinder went
wn. This apparatus was raised and lowered by a light block and line; and by it
ne each cylinder was sunk about 16 ft into the light sand in a few hours.‡

---

* Report Sec of War 1872. † John W. Glenn, C E, Van Nostrand, June 1874.
‡ Gabriel Jordan, C E; Trans Am Soc C E, Feb 1874.

At the Levan Viaduct, Mr James Brunlee, England, in a light sandy marl of great depth, sunk hollow cast iron cylinders of 10 ins outer diam, to a depth of 20 ft, by means of a jet pipe 2 ins diam passing down inside of the cylinder, and through a hole in its base, which was a cast iron disk 30 ins diam, and 1 inch thick, strengthened by outside flanges. The connecting flanges of the cylinder sections are *outside*, thus impeding the descent, as did also the broad bottom disk; still 3 or 4 hours usually sufficed for the sinking of each, to 20 ft depth. Actual trial showed that their safe sustaining power was about 5 tons per sq ft of bottom disk.

At Lock Ken viaduct each pier consists of two cylinders, open at both ends; of cast iron, 8 ft in diam; 1⅛ ins thick; in lengths of 6 ft, weighing 4 tons each; and bolted together by inside flanges, with iron cement between them. The cylinders stand 8 ft apart in the clear; and are in 36 ft water. "A strong staging was erected; and 4 guide-piles driven for each cylinder. The several lengths being previously bolted together, these were lowered into their places. Each cylinder sank by its own weight one or two ft through the top mud, and then settled upon the sand and gravel which form the substratum for a great depth. Into this last they were sunk about 8 or 9 ft farther, by excavating the inside earth under water, by means of an inverted conical **screw-pan,** or dredger, of ¼ inch plate iron. This was 2 ft greatest diam, and 1 ft deep; and to its bottom was attached a screw about 1 ft long, for assisting in screwing it down into the soil. Its sides had openings for the entrance of the soil; and leather flaps, opening inward, to prevent its escape. From opposite sides of the pan, 3 rods of ¾ inch diam projected upward 4 feet, and were there forged together, and connected by an eye-and-bolt joint to a long rod or shaft, at the upper end of which was a four-armed cross-handle, by which the pan was screwed down by 4 men on the staging."

"When a pan was full, a slide which passed over the joint at the bottom was lifted; and the pan was raised by a tackle. This pan raised about 1 cub ft at a time. A smaller one of only 1 ft diam, and 1 ft deep, raising about ¼ cub ft, was used when the material was very hard. By this means the cylinders were sunk at the rate of from 2 to 18 ins per day. The slow rate of 2 ins was caused by stones. some of them of 50 lbs. These were first loosened by a screw-pick, which was a bar of iron 3 ft long, with circular arms 12 ins long projecting from the sides. After being loosened by this, the stones were raised by the pan. The expense of all this apparatus was very trifling; and the excavation was done easily and cheaply. After the excavation was finished, and the cylinder sunk, before pumping out the water, concrete (gravel 2, hydraulic cement 1 measure) was filled in to the depth of 12 feet, by means of a large pan with a movable bottom; and about 12 days were left it to harden. The water was then pumped out, and the masonry built in open air. In some of the cylinders, however, the water rose so fast, notwithstanding the 12 ft of concrete, that the pumps could not keep them clear; and 6 ft more of concrete had to be added in those. Finally random-stone, or rough dry rubble, was thrown in around the outsides of the cylinders, to preserve them from blows and undermining."* The masonry extends 20 ft above the cylinders, and above water.

The vacuum and plenum processes. We can barely allude to the general principles of these two modes of sinking large hollow iron cylinders. In the **vacuum process** of Dr. Lawrence Holker Potts, of London, the cylinder *c*, Fig 16, while being sunk, is closed air-tight at top, by a trap-door, opening upward. A flexible pipe *p*, of India-rubber, long enough to adapt itself to the sinking of the cylinder, and provided with a stopcock *s*, leads from the cylinder to a vessel *v*; which may be placed on a raft, or a scow, or on land, as may suit circumstances. The cylinder being first stood up in position, as in the fig, the water is pumped out, and the interior soil removed if the cylinder has sunk some distance by its own weight. The cock *s* is then closed, and the air is drawn out from the vessel *v* by an air-pump. The cock is then opened, and most of the air in the cylinder rushes into the void vessel *v*; thus leaving the cylinder comparatively empty, and therefore less capable of resisting the downward pressure of the external air upon its top This pressure, as is well known, amounts to nearly 15 lbs on every sq inch; or nearly 1 ton per sq ft of area of the top. Consequently the cylinder is forced downward in the bed of the river, by this amount of pressure, in addition to its own weight. At the same time, the pressure of the air upon the surface of the water is transmitted through the water to the soil around the open foot of the cylinder; so that if this soil be soft or semi-fluid, it will be pressed up into the nearly void cylinder, in which is no downward pressure to resist it. The descent varies from a few inches, to 4 or 5 ft each time. The process is then repeated, by admitting air again into the cylinder, opening the trap-door, removing the water and soil, as before, &c. Additional lengths of cylinder may be bolted on, by means of interior flanges.

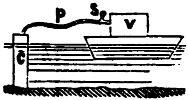


Fig 16

It is **adapted only to soft soils,** and to wet sandy ones; but is not sufficiently powerful in very compact ones; nor does it answer where obstructions from bowlders, logs, &c. occur;

***Hollow Iron Piles** either cast or wrought with solid pointed feet, to be driven by the hammer falling *inside* of them and striking against the top of the solid foot, are a recent device of great use in many cases. They are made in sections of which enough can be gradually united to reach any required depth. They avoid the danger of bending which attends striking the top. The iron feet are swelled outwardly a little to diminish earth-friction against the pile above them.

removal of which requires men to enter the cylinder to its foot; which they cannot do in the rarefied . The pipe *p* should be of sufficient diam to allow the air to leave the cylinder rapidly, so that the er pressure may act upon the top as suddenly as possible.

.t the Goodwin Sands light-house, England, hollow cylinders 2½ ft in diam, were sunk 34 ft into d by this process, in about 6 hours; where a steel bar could be driven only 8 ft by a sledge-ham- :. Others, 12 ins in diam, have been sunk 16 ft into sand within less than an hour. In this last tance the air-pump had two barrels, 4½ ins diam, 16 inch stroke, worked by 4 men. The pipe *p* of lead, and only ½ inch diam.

'he plenum process, invented by Mr Triger, France, consists in forcing air into the cylinder :, Fig 17, to such an extent as to force out the ter, compelling it to escape beneath the open foot, o the surrounding water. The interior of the cylin- being thus left dry to the bottom, men pass down it loosen and remove the soil at and below its base. When s is done, they leave; the compressed air is allowed to ape; and the cylinder, being no longer sustained by upward pressure of the compressed air beneath its , sinks into the cavity, or the loosened material at its t. Fig 17 shows the simple arrangement by which rkmen are enabled to enter or leave the cylinder, :hout allowing the compressed air to escape; as well the general principle of the entire process.

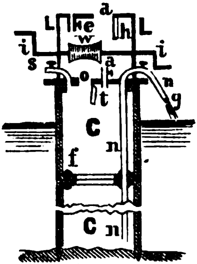


Fig 17

L is a separate small chamber, **the air-lock,** which is ioved when a new length of pipe is to be added; and afterward laced and firmly bolted on. This chamber has a small air-tight r *d*, by which it can be entered from without; and another, *o*, ning into the cylinder. The flaps, *t*, *h*, of both doors, open in- 'd, or toward the cylinder. This chamber also has two stopcocks; one, *a*, in its floor, communi- ng with the cylinder; and one *e*, above, communicating with the open air. At *s* is a bent tube, with a cock, which passes air-tight through the side and the bottom of the air-lock. Through ie compressed air is forced into the cylinder, by an air force pump or condenser; and through it same air is allowed to escape at a later period. A siphon is shown at *n n n*. A drum *w* is used ioisting the excavated material from the bottom, to the air-lock; its axle *i i* passes air-tight through ling-boxes in the sides of the lock; the hoisting being done by men outside. This is the general ingement employed by Mr W. J. McAlpine, C E, of New York, at Harlem bridge; and from his :ription of it, ours has been condensed. The cylinders were there 6 ft diam, 1½ ins thick, and in ;ths of 9 ft, bolted together through inside flanges *f*, as the sinking went on. The air-lock is 6 ft n, by nearly 6 ft high; with sides of boiler iron; and top and bottom of cast iron.

ow suppose the cylinder C C to be let down, and steadied in position, as in the fig; and the air- : L L to be adjusted on top of it. The next process is to force in air through the curved tube *s*; flap *t* of the lower door *o*, and the cock *a*, being previously closed. As the compressed air accu- ates in the cylinder, it forces out the water; which escapes partly beneath the bottom of the cyl- er, and partly by rising through the siphon *n n*, and flowing out at *g*. The door *o* being already ed, and that at *d* open, the air in the air-lock is in the same condition as that outside; so that kmen can enter it readily. Having done so, they close the door *d*, and the cock *e*; and open the : *a*, through which condensed air from the cylinder rushes upward, soon filling the air-lock. en this is done, the flap *t* is opened, and the men descend through the door *o* by a ladder, or by a ket lowered by the drum *w*, to the bottom. Here they loosen and excavate the material as deep hey can; and, filling it into a bucket or bag, they signal to those outside, who raise it to the air- . When done, they ascend to the air-lock, close the door *o*, and the cock *a*; and open the cock *e*, ough which the condensed air in the lock soon escapes, leaving the internal air the same as that ide. The door *d* is then opened, the buckets of earth are removed, and the men go out. Finally cock at *s* is opened, the condensed air in the cylinder escapes through it to the outside air, and cylinder sinks by its own weight into the cavity and loosened soil prepared for it at its base, and ch is new forced up into the cylinder by the rush of the returning water. The process is then eated. The sinking will often vary from 0 to 10 or more feet at one operation. Until depths of r 50 ft, most men can endure the pressure of the condensed air; but as the depth increases this omes more difficult, and positively dangerous to life. Cast-iron cylinders 15 ft diam; and great sons, Fig 18, have been thus suuk; but at times at great expense and trouble.

'he cylinder should be guided in its descent by a strong frame, which be supported by piles. Otherwise it will be apt to tilt, and thus give great trouble to settle it n its exact place. Have been sunk in deep water by divers undermining inside.

The plenum process as applied at the South St bridge, Philada, Ir. John W. Murphy, contracting engineer, differs materially from that described above; and eover deserves notice on account of the great simplicity and efficacy of his plant. This consisted :ly of two canal boats, decked, each 100 ft long, by 17½ ft wide, and 8 ft depth of hold. They e anchored parallel to each other, 15 ft apart. Supported by the boats, and over the space between n, was a strong four-legged shears about 50 ft high; at the top of which was attached tackle for idling the cast iron cylinders. In the hold of one of the boats was a **Burleigh mpressor** having two pistons of 10 ins diam, and 9 ins stroke; together with boiler. On the deck of the same boat stood a vertical **air-tank or regulator,** : long, by 2 ft diam, made of quarter inch boiler iron. This served to maintain a supply of com ;sed air in the submerged cylinder in case of an accidental stopping of the compressor; which rwise would probably be fatal to the laborers in the cylinder. The condensed air flowed from air-tank to the air-lock of the cylinder through a hose 4 ins diam, made of gum elastic and can- and so long, and so placed, as to extend itself as the cylinder went down, thus maintaining the munication at all times. Entirely across both boats, and across the interval between them, ex- ded two heavy wooden **clamps,** each 3 ft wide by 18 ins high; each composed of three pieces of 12 × 18 inch timber strongly bolted together. At the centers of these clamps the two inner vertical sides which faced each other were hollowed out to the depth of a foot by concavities corresponding to the curve of the cylinders. The distance apart of the clamps was regulated by two strong iron rods, having screws and nuts at their ends for that purpose. Thus when a section of a cylinder was hoisted by means of the shears into its position over the space between the two boats, the two concavities of the clamps were brought into contact with it, and the nuts being then screwed up, the cylinder was firmly held in place by the clamps. The shears could then be used to raise another section of the cylinder to its place upon the first one, that the two might be bolted together. By repeating this process the height of the cylinder would soon become too great to allow the shears to place another section upon it; in which case the nuts of the screws were slightly loosened, and the cylinder was allowed to slip down slowly into the water until its top was but a little above the surface. The screws were then again tightened, and the cylinder again held fast until other sections were added and bolted to it. When there was danger that the upward pressure of the condensed air might lift a cylinder, the clamps were raised by the shears clear of the boats; then tightened to the cylinder, and a platform of planks laid upon them, and loaded with stone.

The **air-lock** was so arranged as not to require to be removed when a new section was to be bolted on. This was effected as follows. Sections of the cylinder were bolted together in the manner just described, until its foot rested on the bottom, with its top a few feet above high water. A heavy cast iron **diaphragm** 1¼ inches thick, to form the floor of the air-lock, was then placed on top. Then was added another 10 ft high section of the cylinder, to form the chamber of the air-lock. These were bolted together; and then another diaphragm was added at top to form the roof of the air-lock. These diaphragms were furnished with openings, and with doors and valves corresponding with those shown in Fig 17. and remained permanently in the cylinders when the work was finished. If the depth of soil to be passed through before reaching rock is so great as to require other sections of cylinder to be bolted on above the top of the air-lock, this may be done to any extent, inasmuch as it is immaterial whether the air-lock is under water or not. **To keep the cylinder both air- and water-tight** the faces of the flanges before being bolted together were smeared with a mixture of red and white lead and cotton fiber.

At the South St bridge the cylinders were 4, 6, and 8 ft diam; in lengths or sections 10 ft long. They were all 1¼ inch thick. Inside flanges 2¾ ins wide, 1¼ thick, with bolt-holes 1¼ inch diam, by 5 ins apart from center to center. The bottom edge has no flange. A 10 ft section of an 8 ft cylinder weighs 14600 lbs; of a 6 ft one, 10800; of a 4 ft one, 6800. An 8 ft diaphragm, 2800 lbs; 6 ft, 1600; 4 ft, 783. The rock under the soil was quite uneven in places; but was levelled off as the cylinders went down. These were then bolted to it by cast iron brackets. **The work went on, day and night,** summer and winter: with no interruption from the tides, floods, or floating ice; and the thirteen columns were sunk, filled with concrete, and completed in 11 months; much of which was consumed in levelling off the rock, and bolting the brackets. The want of guides caused much tilting, trouble, and delay.

Rise and fall of tide about 7 ft. Greatest depth of soil, gravel, &c, passed through, 30 ft: least, 6 ft. Depth of water about 25 ft. The work was under charge of John Anderson, a very skillful and energetic superintendent of such matters. **The entire neat cost** of the cylinders in place, and filled with hydraulic concrete, was approximately $92 per foot of total length for the 8 ft ones; $64 for the 6 ft; and $40 for the 4 ft diams. There were three gangs of workmen; and each gang worked 4 hours at a time. See a full and very instructive description with engravings, by D. M. Stauffer, Superintending Engineer for the city, in the Journal of the Franklin Inst, Nov., 1872. From it the above few items are taken. Mr. Anderson's firm (Anderson & Barr, Tribune Building, N. Y.) have since successfully sunk a number of such cylinders, including (1884-5) four of wrought-iron, 8 ft diam, 66 ft long, at an angle of 45° with the hor, intended as struts to prevent the movement of one of the abut piers of Chestnut St bridge, Phila.

Cast iron cylinders **have cracked through,** around their entire circumference, in many parts of the U. S. in very cold weather; owing to the diff of contraction of the iron, and of the concrete filling. Ignorant use of them may be attended by great danger.

The shaded part of Fig 18 shows a transverse section of the **caisson of yellow-pine** timber and cement, for the **Brooklyn tower of East River** (N Y) suspension bridge, of 1600 ft clear span. It is 168 ft long at bottom, and 102 ft wide. A longitudinal section resembles the transverse one, except in being longer, and in showing more shafts J. Of these there are 6, arranged in pairs, for expedition and as a precaution against accident. Namely, two water-shafts J, each 7 ft by 6½ ft across, for removing by buckets and hoisting apparatus, the material excavated beneath the caisson; together with such water as may accumulate at *o o*; two air-shafts of 21 ins diam, through which air is forced from above, to expel the water from the chamber C S S D below the caisson, so as to allow the laborers to work there at undermining; the expelled water escaping under the foot C D of the caisson, into the river; and two supply shafts of 42 ins diam, for admitting laborers, tools, c. The several shafts of course have air-chambers on top, on the same principle as 17, to prevent the escape of the compressed air in *s s*.

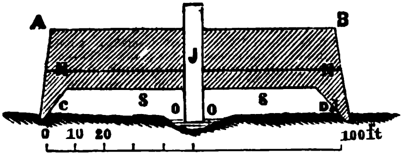


Fig 18

The shafts are of ½ inch boiler iron. The foot C D, nine timbers high, is continuous, extending entirely around the caisson; its bottom is shod with cast iron; its four corners are strengthened by wooden knees 20 ft long.

From the bottom, up to the line N, N, 14 ft, the caisson is built of horizontal layers of timbers one foot square; the layers crossing each other at right angles; and the timbers of each layer touching each other well forced and bolted together; and all the joints filled with pitch. To aid in preventing leakage, the nuts and heads of the screws have India-rubber washers; also all outside seams, as well as all the seams of the layer of timbers N, N, are thoroughly calked; and a layer of tin, enclosed between two layers of felt, is placed outside of each outer joint; and over the entire top of the layer next below N, N.

When the caisson was built up to N, N, on land, it was launched, floated into position, and anchored; after which were added for sinking it, fifteen courses of timbers one ft square; and laid one ft apart in the clear; with the intervals filled with concrete. The top course A B is of solid timber, to serve as a floor for supporting machinery, &c. It was sunk some feet below the very bottom of the river, in order to avoid the teredo.

Cribs are sunk outside of the caisson, to form temporary wharves for boats carrying away excavated material; and for vessels bringing stone, &c.

When the caisson was sunk, and the water forced out from the chamber or space C S S D, workmen began to excavate uniformly the enclosed area of river bottom, so as to allow the caisson to descend slowly until it reached a firm substratum. The space C S S D, as well as the shafts, was then filled up solid with concrete masonry. A coffer-dam was built on top of the caisson; and in it the regular masonry of the tower was started. The total height of this tower including the caisson, is about 300 ft. For full details see report, 1873, of W. A. Roebling the chief engineer.

Hollow cylinders, or other forms of brickwork or masonry, with a strong curb or open ring of timber or iron beneath them, may be gradually sunk by undermining and excavating from the inside; and form very stable foundations. Under water this may be done by properly shaped scoops, with or without the aid of the diving-bell, according to the depth, &c. On land it will often be the most economical and satisfactory mode, especially in firm soils. The descent may be assisted by loading them. If, as sometimes happens, the friction of their sides against the earth outside prevents their sinking by their own weight. A brick cylinder, 46 ft outer diam, walls 3 ft thick, has been sunk 40 ft in dry sand and gravel, without any difficulty. It was built 18 ft high, (on a wooden curb 21 ins thick,) and weighed 300 tons before the sinking was begun. The interior earth was excavated slowly, so that the sinking was about 1 ft per day; the walls being built up as it sank. Tunnel shafts are at times so sunk.

On the Rhine for a coal shaft, a brick cylinder 25½ feet diam was first thus sunk by its own weight 76 ft through sand and gravel; then an interior one, 15 ft diam, was sunk in the same way to the depth of 256 ft below the surface; of which depth all the 180 ft below the first cylinder was a running quicksand. At 256 ft friction rendered the cylinder immovable. The quicksand was removed by boring; no pumping was done; but the water was permitted to keep the cyl full.

The entire foundation for a large pier of masonry has been sunk in this manner, in a single mass; a sufficient number of vertical openings being left in it for the workmen to descend, or for tools to be inserted for undermining. This is generally a very slow and tedious operation, especially under water. It may often be expedited by diving-bells or by diving-dresses. It will generally be better to make the mass wider at bottom than above it, so as to diminish friction against the outside earth. On land, water may at times be used for softening the bottom earth. By keeping the interior of such hollow masonry dry, it may even be *built downward* from the surface; by undermining only a portion of its circumference at a time, filling said portion with masonry, and then removing and filling the other portion; and so on in successive stages of 2 or 3 ft downward at a time. This mode may be adopted also when friction has stopped the sinking of a mass by its own weight when undermined.

The sand pump as used at the St Louis bridge will often be of service in raising sand from cylinders while being sunk in water. With a pump pipe of 3.5 ins bore, and a water jet under a pressure of 150 lbs per sq inch, 20 cub yds of sand per hour were raised 125 feet. A jet of air has also been successfully used in the same way, as at the East River, N Y, suspension bridge, &c.

Fascines. On marshy or wet quicksand bottoms, foundations may be laid by first depositing large areas of layers of fascines, or stout twigs and small branches, strongly tied together in bundles from 6 to 12 ft long, and from 6 ins to 2 ft in diam.

The layers or strata of bundles should cross each other. A kind of floating raft or large mattress is first made of these, and then sunk to the bottom by being loaded with earth, gravel, stones, &c. In this manner the abutments and piers of the great suspension bridge at Kieff, in Russia, with spans of 440 ft, were founded in 1852, on a shifting quicksand. There the fascine mattresses extend 100 ft beyond the bases of the masonry which rests upon them.

Fascines may be used in the same way for sustaining railway embankments, &c, over marshy ground, but they will settle considerably.

Sand-piles. We have already alluded to the use of sand well rammed in layers into trenches or foundation pits; but it may also be used in soft soils, in the shape of piles. A short stout wooden pile is first driven 5 to 10 feet or more, according to the case. It is then drawn out, and the hole is filled with wet sand well rammed. The pile is then again driven in another place, and the process repeated. The intervals may be from 1 to 3 ft in the clear. Platforms may be used on these piles as on wooden ones. If the sand is not put in wet, it will be in danger of afterward sinking from rain or spring water. In this case, as with fascines, it is well to test the foundation by means of trial loads. Some settlement must inevitably take place until all the parts come to a full bearing: but it will be *comparatively* trifling. The same occurs in every large work to some extent; as in a roof or arch of great span, whether of wood, iron, or masonry; so also with all tall piers, walls, &c, &c. Sandy foundations under water should be surrounded by stout well-driven sheet-piling, to prevent the enclosed sand from running out in case the outer sand is washed away; and should also be defended by a deposit of random-stone.

On bad bottoms under water, **small artificial islands** of good soil have been deposited; and the masonry founded upon them. Canal locks and other structures may at times be advantageously founded in this way in marshy soils. If necessary, a depth of several feet of the bad soil may be dredged out before the firmer soil is deposited; and the latter may be weighted by a trial load to test its stability.

The mode of laying a foundation under water, by building the masonry upon a timber platform above water, upheld by **strong screws,** and lowered into the water as the work is finished in the open air, a course or two at a time, has of late been much employed with entire success, in large bridge-piers in deep water. It however is not new. It was suggested more than 100 years ago by Belidor.

Piles are driven 6 to 10 ft apart around the space to be occupied by the pier; having their tops connected by heavy timber cap-pieces. These last uphold the screws, which work through them. The whole is braced against lateral motion.

A CLUMP OF PILES WELL DRIVEN; and then enclosed by an iron cylinder sunk to a firm bearing, and filled with concrete, is an excellent foundation. The piles may extend to the top of the cylinder, and thus be enclosed in the concrete. Such an arrangement has been patented by S. B. Cushing, C. E., Providence, R. I. The cylinder and concrete serve to protect the piles from sea-worms, and from decay above low water; and are not intended to support the load above them.

STONEWORK.

Where work is done on a large scale, blasting can sometimes be done at from 10 to 20 per cent less cost per cubic yard by means of **machine drills and dynamite,** than by **hand drills and gunpowder.** Ordinarily, however, **the cost is about the same,** and the advantage of the newer methods consists rather in economy of time, convenience, and having the work more entirely under control. In ordinary railroad work in average hard rock, and when common labor costs $1 per day of ten hours, the cost per cubic yard, for loosening, will ordinarily range between 30 and 60 cts, including tools, drilling, powder, &c.

Holes for blasting, drilled by hand, are generally from 2½ to 4 ft deep; and from 1½ to 2 ins diam. **Churn-drilling** is much more expeditious and economical than that by *jumping*, mentioned below. The churn-drill is merely a round iron bar, usually about 1¼ ins diam, and 6 to 8 ft long; with a steel cutting edge, or bit, (weighing about a ℔, and a little wider than the diam of the bar,) welded to its lower end. A man lifts it a few inches; or rather catches it as it rebounds, turns it partially around; and lets it fall again. By this means he drills from 5 to 15 feet of hole, nearly 2 ins diam, in a day of 10 working hours, depending on the character of the rock. From 7 to 8 ft of holes 1¾ ins diam, is about a fair day's work in hard gneiss, granite, or compact siliceous limestone; 5 to 7 ft in tough compact hornblende; 3 to 5 in solid quartz; 8 to 9 in ordinary marble or limestone; 9 to 10 in sandstone; which, however, may vary within all these limits. When the hole is more than about 4 ft deep, two men are put to the drill. Artesian, and oil wells, in rock, are bored on the principle of the churn-drill.

The **jumper,** as now used, is much shorter than the churn-drill. One man (the *holder*) sitting down, lifts it slightly, and turns it partly around, during the intervals between the blows from about 8 to 12 ℔ hammers, wielded by two other laborers, the *strikers.* It can be used for holes of smaller diameters than can be made by the churn-drill; because the holder can more readily keep the cutting end at the exact spot required to be drilled. It is also better in conglomerate rock; the hard siliceous pebbles of which deflect the churn-drill from its vertical direction, so that the hole becomes crooked, and the tool becomes bound in it. The coal conglomerates are by no means hard to drill with a jumper. The jumper was formerly used for large deep holes also, before the churn-drill became established.

Either tool requires resharpening at about each 6 to 18 inches depth of hole; and the wear of the steel edge requires a new one to be put on every 2 to 4 days. With iron jumpers, the top also becomes battered away rapidly. As the hole becomes deeper, longer drills are frequently used than at the beginning. The smaller the diameter of the hole, the greater depth can be drilled in a given time; and the depth will be greater in proportion than the decrease of diam. Under similar circumstances three laborers with a jumper will about average as much depth as one with a churn-drill.

The **hand-drill,** in which the same man uses both the hammer and the short drill, is chiefly used for shallow holes of small diam. With it a fair workman will drill about as many feet of hole from 6 to 12 ins deep, and about ¾ inch diam, as one with a churn-drill can do in holes about 3 ft deep and 2 ins diam, in the same time. Only the jumper or the hand-drill can be used for boring holes ch are horizontal, or much inclined.

Cost of quarrying stone. After the preliminary expenses of purchasing the site of a good quarry; cleaning off the surface earth and disintegrated top rock; and providing the necessary tools, trucks, cranes, &c; the total neat expenses for *getting out* the rough stone for masonry, per cub yard, ready for delivery, may be roughly approximated thus: Stones of such sizes as two men can readily lift, measured *in piles*, will cost about as much as from ¼ to ½ the daily wages of a quarry laborer. Large stones, ranging from ½ to 1 cub yd each, got out by blasting, from 1 to 2 daily wages per cub yd. Large stones, ranging from 1 to 1½ cub yds each, in which most of the work must be done by wedges, in order that the individual stones shall come out in tolerably regular shape, and conform to stipulated dimensions; from 2 to 4 daily wages per cub yard. The smaller prices are low for sandstone, while the higher ones are high for granite. Under ordinary circumstances, about 1⅓ cub yds of good sandstone can be quarried at the same cost as 1 of granite; or, in other words, calling the cost of granite 1, that of sandstone will be ¾; so that the means of the foregoing limits may be regarded as rather full prices for sandstone; rather scant ones for granite; and about fair for limestone or marble.

Cost of dressing stone. In the first place, a liberal allowance should be made for **waste.** Even when the stone wedges out handsomely on all sides from the quarry, in large blocks of nearly the required shape and size, from ⅙ to ¼ of the rough block will generally not more than cover waste when well dressed. In moderate-sized blocks, (say averaging about ½ a cub yard each,) and got out by blasting, from ¼ to ⅓ will not be too much for stone of medium character as to straight splitting. About the last allowance should also be made for well-scabbled rubble. The smaller the stones, the greater must be the allowance for waste in dressing. In large operations, it becomes expedient to have the stones dressed, as far as possible, at the quarry; in order to diminish the cost of transportation, which, when the distance is great, constitutes an important item—especially when by land, and on common roads.

A stonecutter will first take out of wind; and then fairly patent-hammer dress, about 8 to 10 sq ft of plain face in hard granite, in a day of 8 working hours; or twice as much of such inferior dressing as is usually bestowed on the beds and joints; and generally on the faces also of bridge masonry, &c, when a very fine finish is not required. In good sandstone, or marble, he can do about ⅓ more than in granite. Of *finest* hammer finish, *granite*, 4 to 5 sq ft.

Cost of masonry. Every item composing the total cost is liable to much variation; therefore, we can merely give an example to show *the general principle* upon which an approximate estimate may be made; assuming the **wages of a laborer** to be $2.00 per day of 8 working hours; and $3.50 for a mason. **The monopoly of quarries** affects prices very much.*

Cost of ashlar facing masonry. Average size of the stones, say 5 ft long, 2 ft wide, and 1.4 thick; or two such stones to a cub yd. Then, supposing the stone to be granite or gneiss, the cost per cub yd of masonry **at such wages** will be,

Getting out the stone from the quarry by blasting, allowing ⅓ for waste in dressing; 1⅓ cub yds, at $3.00 per yard	$4.00
Dressing 14 sq ft of face at 35 cts	4.90
" 52 " beds and joints, at 18 cts	9.36
Neat cost of the dressed stone at the quarry	18.26
Hauling, say 1 mile; loading and unloading	1.20
Mortar, say	.40
Laying, including scaffold, hoisting machinery, superintendence, &c	2.00
Neat cost	21.86
Profit to contractor, say 15 per ct	3.28
Total cost	25.14

Dressing will cost more if the faces are to be rounded, or moulded. If the stones are smaller than we have assumed, there will be more sq ft per cub yd to be dressed, &c.

If in the foregoing case, the stones be *perfectly* well dressed on all sides, including the back, the cost per cub yd would be increased about $10; and if some of the sides be curved, as in arch stones, say $12 or $14; and if the blocks be carefully wedged out to given dimensions, $16 or $18; thus making the neat cost of the dressed stone *at the quarry* say $28, $31, or $35 per cub yd.

---

* The blocks of granite for **Bunker Hill monument** averaging 2 cub yds each, were quarried by wedging, and delivered at the site of the monument, at a neat actual cost of $5.40 per cub yd; by the Monument Association; from a quarry opened by themselves for the purpose. The Association received no profit; their services being voluntary. The average contract offers for the same, were $24.30! The actual cost of getting out the rough blocks at the quarry was $2.70. Loading upon trucks at quarry, about 15 cts. Transportation 8 miles by railway and common road, $2.55. Total, $5.40. In 1825 to 1845; common unskilled labor averaging $1 per day.

The item of laying will be much increased if the stone has to be raised to great heights; or if it has to be much handled; as when carried in scows, to be deposited in water-piers, &c. Almost every large work presents certain modifying peculiarities, which must be left to the judgment of the engineer and contractor. The percentage of contractors' profit will usually be less on large works than on small ones.

Cost of ashlar facing masonry. If the stone be **sandstone** with good natural beds, the getting out may be put at $3.00 per cubic yard. Face dressing at 26 cts per sq ft: say $3.64 per cubic yd. Beds and joints 13 cts per sq ft; say $6.76 per cub yd. The neat cost, laid, $17.00.

And **the total cost of large well scabbled ranged sandstone masonry in mortar,** may be taken at about $10 per cub yd.

Cost of large scabbled granite rubble, such as is generally used as backing for the foregoing ashlar; stones averaging about ½ cub yd each:

Labor at $1 per day.	Cost per cub yd of masonry.
Getting out the stone from the quarry by blasting, allowing ⅙ for waste in scabbling; $1\frac{1}{7}$ cub yds at $3.00	$3.43
Hauling 1 mile, loading and unloading	1.20
Mortar; (2 cub ft, or 1.6 struck bushels quicklime, either in lump or ground; and 10 cub ft, or 8 struck bushels of sand, or gravel; and mixing)	1.50
Scabbling; laying, including scaffold, hoisting machinery, &c	2.50
Neat cost	8.63
Profit to contractor, say 15 per ct	1.30
Total cost	9.93

Common rubble of small stones, the average size being such as two men can handle, costs, to get it out of the quarry, about 80 cts per yard of pile; or to allow for waste, say $1.00. Hauling 1 mile, $1.00. It can be roughly scabbled, and laid, for $1.20 more; mortar as foregoing, $1.50. Total neat cost, $4.70; or, with 15 per ct profit, $5.40, *at the above wages for labor.*

With smaller stones, such as one man can handle, we may say, stone 70 cts; hauling $1; laying and scaffold, tools &c, $1; mortar $1.50. Making the neat cost $4.20; or with 15 per ct profit, $4.83. Neat scabbled irregular range-work costs from $2 to $3 more per yd than rubble; according to the character of the stone &c. The laying of thin walls costs more than that of thick ones, such as abutments &c.

The cost of plain 8 inch thick ashlar facings for dwellings &c in Philada, in 1888, is about as follows per square foot showing, put up, including everything. Sandstone, $1.50 to $2.25. Pennsylvania marble, $2.50. New England marble, $2.75 to $3.25. Granite, $2.25 to $2.75. If 6 ins thick, deduct one-eighth part. **First class artificial stone** could be made and put up at one-third the price. **North River blue stone flags,** 3 ins thick, for footwalks, put down, including gravel &c. 70 cts per sq foot. **Belgian street pavement,** with gravel, complete, $3.50 per sq yard in Eastern cities.

When dressed ashlar facing is backed by rubble, the expense per cub yard of the entire mass will of course vary according to the proportions of the two. Thus, if ashlar at $12 per yd, is backed by an equal thickness of rubble at $5, the mean cost will be ($12 + $5) ÷ 2 = $8.50; or if the rubble is twice as thick as the ashlar then ($12 + $5 + $5) ÷ 3 = $7.33, &c. **Such compound walls are weak** and apt to separate in time, as also walls of cut stone backed by concrete, or by brick; from unequal settlement of the two parts.

At times the contractor must be **allowed extra** in opening new quarries; in forming short roads to his work; in digging foundations; or for pumping or otherwise draining them, when springs are unexpectedly met with; for the centers for arches, &c; unless these items are expressly included in the contract per cub yd.

RETAINING-WALLS.

Art. 1. A retaining-wall is one for sustaining the pressure of earth, sand, or other *filling* or *backing*, deposited behind it after it is built; in distinction to a **face-wall,** which is a similar structure for preventing the fall of earth which is in its undisturbed natural position, but in which a vert or inclined face has been excavated. The earth is then in so consolidated a condition as to exert little or no lateral pres, and therefore the wall may generally be thinner than a retaining one.

This, however, will depend upon the nature and position of the strata in which the face is cut. If the strata are of rock, with interposed beds of clay, earth, or sand; and if they dip or incline toward the wall, it may require to be of far greater thickness than any ordinary retaining-wall; because when the thin seams of earth become softened by infiltrating rain, they act as lubrics, like soap, or tallow, to facilitate the sliding of the rock strata; and thus bring an enormous pres against the wall. Or the rock may be set in motion by the action of frost upon the clay seams; or, as sometimes occurs, by the tremor produced by passing trains. Even if there be no rock, still if the strata of *soil* dip toward the wall, there will always be danger of a similar result; and additional precautions must be adopted, especially when the strata reach to a much greater height than the wall. **A vertical wall** has both *c o* and *d s* vert.

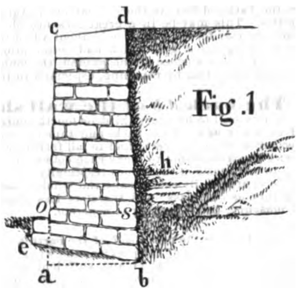


Fig. 1

Experience, rather than theory, must be our guide in the building of both kinds of wall. We recommend that the hor thickness *a b*, Fig 1, at the base of a vert or nearly vert retaining-wall *c d b a*, which sustains a backing of either sand, gravel, or earth, level with its top *c d*, as in the fig, should not be less than the following, in railroad practice, when the foundations are not more than about three feet deep.

When the backing is deposited loosely, as usual, *as when dumped from carts, cars, &c.*

Wall of cut-stone, or of first-class large ranged rubble, in mortar.....a b...	.35	*of its entire vert height* d b.
" *good common scabbled mortar-rubble, or brick.*	.4	" " " "
" *well-scabbled dry rubble*..............................	.5	" " " "

With good masonry, however, we may take the height *d s* instead of *d b*, and then the above proportions of *d s* will give a sufficient thickness at the ground-line *o s*.

When the backing is somewhat consolidated in hor layers, each of these thicknesses *may* be reduced, but no rule can be given for this.

The offset *o e*, in front of the wall, is not included in these thicknesses.

When, however, the backing is a pure clean sand, or gravel, we should use only the full dimensions; inasmuch as the tremor, caused by passing trains, would neutralize any supposed advantage from ramming materials so devoid of cohesion. Such sand may be rammed with much advantage for the purpose of compacting it in foundations; but a diff principle is involved in that case. When it is done even with cohesive earths, with a view of saving masonry in retaining-walls, it is probable that the expense will generally be found quite equal to that of the masonry saved.

The base *a b* in Fig 1, is $\frac{4}{10}$ of the height *b d*. In the foregoing thicknesses at base, the back *d b* of the wall is supposed to be vert; and the face *c a* either vert, or battered (sloped or inclined backward) to an extent not exceeding about 1½ inches to a foot; which limit it is rarely advisable to exceed in practice, owing to the bad effect of rain, &c, upon the mortar when the batter is great. The base of a vert wall need not in fact be as thick as one with a battered face; but when the batter does not exceed 1.5 inches to a foot, the diff is very small. See Table, Art 7.

Rem. 1. **A mixture of sand, or earth, with a large proportion of round bowlders** paving pebbles, &c, will weigh considerably more than the materials ordinarily used for backing; and will exert a greater pres against the wall; the thickness of which should be increased, say about one-eighth to one-sixth part, when such backing has to be used.

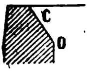
Rem. 2. The wall will be stronger if all the **courses of masonry be laid with an inclination inward,** as at *o e b*; especially if of dry masonry, or if time cannot be allowed (as it always should be, when practicable) for the mortar to set properly, before the backing is deposited behind it. The object of inclining the courses, is to place the joints more nearly at right angles to the direction *f* P, Figs 6, 7, and 8, of the pres against the back of the wall; and thus diminish the tendency of the stones to slide on one another, and cause the wall to bulge.

When the courses are hor, there is nothing to prevent this sliding, except the friction of the stones, one upon the other, when of dry masonry; or friction and the mortar, when the last is used. But if, as is frequently the case, (especially in thick and hastily built walls,) this has not had time to harden properly, it will oppose but little resistance to sliding. But when the courses are inclined, they cannot *slide*, without at the same time being *lifted up* the inclined planes formed by themselves. In retaining-walls, as in the abuts of important arches, the engineer should place as little dependence as possible upon mortar; but should rely more upon the position of the joints, for stability.

An objection to this inclining of the joints in *dry* (without mortar) walls, is that rain-water, falling on the battered face, is thereby carried inward to the earth backing: which thus becomes soft, and settles. This may be in a great measure obviated by laying the outer or face-courses hor; or by using mortar for a depth of only about a foot from the face. The top of the wall should be protected by a coping *c d*, Fig 1, which had better project a few ins in front. After the masonry has been built up to the surface of the ground, the foundation pit should be filled up; and it is well to consolidate the filling by ramming, especially in front of the wall.

The back *d b* of the wall should be left rough. In brickwork it would be well to let every third or fourth course project an inch or two. This increases the friction of the earth against the back, and thus causes the resultant of the forces acting behind the wall to become more nearly vert; and to fall farther within the base, giving increased stability. It also conduces to strength not to make each course of uniform height throughout the thickness of the wall; but to have some of the stones (especially near the back) sufficiently high to reach up through two or three courses. By this means the whole masonry becomes more effectually interlocked or bonded together as one mass; and therefore less liable to bulge. Very thick walls may consist of a facing of masonry, and a backing of concrete.

REM. 3. It is the pres itself of the earth against the back, that creates the friction, which in turn modifies the action of the pres; as the wt or pres of a body upon an inclined plane produces friction between the body and the plane, sufficient, perhaps, to prevent the body from sliding down it. A retaining-wall is *overthrown* by being made to revolve around its outer toe or edge *e*, Fig 1, as a fulcrum, or turning-point; but in order thus to revolve, its back must first plainly rise; and in doing so must rub against the backing, and thus encounter and overcome this friction. The friction exists the same, whether the wall stands firm or not; as in the case of the body on an inclined plane; the only diff is that in one case it *prevents* motion; and in the other only *retards* it.



Where deep freezing occurs the back of the wall should be sloped forwards for 3 or 4 ft below its top as at *c o*, which should be quite smooth so as to lessen the hold of the frost and prevent displacement.

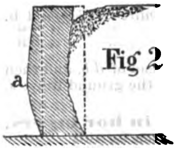


Fig 2

REM. 4. When the wall is too thin, it will generally fail by **bulging** outward, at about ⅓ of its height above the ground, as at *a*, in Fig 2. A slight bulging in a new wall does not *necessarily* prove it to be actually unsafe. It is generally due to the newness of the mortar, and to the greater pres exerted by the fresh backing; and will often cease to increase after a few months. It need not excite apprehension if it does not exceed ¼ inch for each foot in thickness at *a*. See Remark 3, Art 7.

Art. 2. The young engineer need not in practise concern himself particularly about the PRECISE SP GRAV OF HIS BACKING, or about the ANGLE OF SLOPE at which it will stand; for the material which he deposits behind his wall one day, may be dry and incoherent, so as to slope at 1½ to 1; the next day rain may convert it into liquid mud, seeking its own level, like water; the next it may be ice, capable of sustaining a considerable load, as a vert pillar.

Moreover, he cannot foretell what may be the nature of his backing; for, as a general rule, this must consist of whatever the adjacent excavation may produce from time to time; sand to-day, rock to-morrow, &c. Retaining-walls are therefore usually built before the engineer knows the character of their backing; so that in practice, these theoretical considerations have comparatively but little weight. Theory, uncontrolled by observation and common sense, will lead to great errors in every department of engineering; but, on the other hand, no amount of experience alone will compensate for an ignorance of theory. The two must go hand-in-hand.

Again, **the settlement of the backing under its own wt,** aided by the tremors produced by heavy trains at high speed; its expansion by frost, or by the infiltration of rain; the hydrostatic pressure arising from the admission of the latter through cracks produced in the backing during long droughts; as well as its lubricating action upon it, (diminishing its friction, and giving it a tendency to slide,) &c, exert at times quite as powerful an overturning tendency as the legitimate theoretical pres does. The action of these agencies is gradual. Careful observation of retaining-walls year after year, will often show that their battered faces are becoming vertical. Then they will begin to incline outward; and eventually the wall 'l fail. Theory omits loads that may come on backing increasing its pres.

Assuming the theoretical views advanced by Professor Moseley to be correct as theories, the thicknesses which we have recommended in Art 1, for mortar walls, correspond to from 7 to 14 times; and for dry walls about 10 to 20 times, the pres assigned by him; and we do not consider ours greater than experience has shown to be necessary. See Table 3. Retaining-walls designed by good engineers, but in too close accordance with theory, (which assumes that a resistance equal to twice the theoretical pres is sufficient,) have failed; and the inference is fair that many of those which stand have too small a coefficient of safety.

The fact is, (or at least so it appears to us,) there must be defects in the theoretical assumptions of some of the most prominent writers who give practical rules on this subject. Thus Poncelet, who certainly is at their head, states that his tables, for practical use, give thicknesses of base for sustaining $1\frac{8}{10}$ times the theoretical pres; and this he considers amply safe. Yet, for a vert wall of cut granite, his base for sustaining dry sand level with the top, as in Fig 1, is .35 of the vert height; and for brick, .45. But the writer found that *when not subject to tremor*, a wooden model of a vert wall, weighing but 28 lbs per cub ft, and with a base of .35 of its height, balanced perfectly dry sand sloping at 1½ to 1, and weighing 89 lbs per cub ft.

Now, THE RESISTANCE OF SIMILAR WALLS, OF THE SAME DIMENSIONS, VARIES AS THEIR SPECIFIC GRAVITIES; and, since granite weighs about 165 lbs per cub foot, or 6 times as much as our model, it follows, we conceive, that a wall of that material, with a base of .35 of its height, must have a resistance of 6 times any *true theoretical* pres, instead of only 1.8 times; and that his brick wall must have about 5 times the mere balancing resistance. Our experiments were made in an upper room of a strongly built dwelling; and we found that the tremor produced by passing vehicles in the street, by the shutting of doors, and walking about the room, sufficed to gradually produce leaning in walls of considerably more than twice the mere balancing stability while quiet: and it appears to us that the injurious effects of a heavy train would be comparatively quite as great upon an actual retaining-wall, supporting so incohesive a material as dry sand.

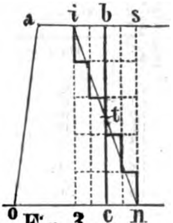


Fig 3

Since, therefore, Poncelet's wall is in this instance sufficiently stable or *practice*, it seems to us that his *theory*, which neglects the effect of remors, &c, must be defective. He also gives $\frac{1}{5}$ of the height as a sufciently safe thickness for a vert granite wall supporting *stiff earth*; but re suspect that very few engineers would be willing to trust to that proortion, when, as usual, the earth is dumped in from carts, or cars; espe lly during a rainy period. If deposited, and consolidated in layers, ieory could scarcely assign any thickness for the wall; for the backing thus becomes, as it were, a ass of unburnt brick, exerting no hor thrust; and requiring nothing but protection from atmospheric ifluence, to insure its stability without any *retaining*-wall. It is with great diffidence, and distrust our opinions, that we venture to express doubts respecting the assumptions of so profound an instigator and writer as Poncelet; and we do so only with the hope that the views of more compent persons than ourselves, may be thereby elicited. Our own have no better foundation than exriments with wooden and brick models, by ourselves; combined with observation of actual walls.

Art. 3. After a wall *a b c o*, Fig 3, with a vert back, has been proportioned by r rule in Art 1, it may be **converted into one with an offsetted ack, as** *a i n o*. This will present greater resistance to overturning; and yet con in no more material. Thus, through the center *t* of the back, draw any line *i n*; m *n* draw *n s*, vert; divide *i s* into any *even* number of equal parts; (in the fig re are 4;) and divide *s n*, into *one more* equal parts; (in the fig there are 5.) From points of division draw hor, and vert lines, for forming the offsets, as in the fig. n the offsetted wall, the cen of grav is thrown farther back from the toe *o*, than the other, thus giving it increased leverage and resistance: but within ordinary ctical limits, the diff is very small; and since the triangle of supported earth is ater than when the back is vert, its pres is also greater; so that probably no apciable advantage attends *that* consideration. **The increase of thickness ar the base, diminishes, however, the erage** ***v a*, Fig 3, of the pres** *f* P, **of the rth against** the back. The *center of pressure* of pres is in both cases at ⅓ the vert height, meas l from the bottom; and it is therefore plain that farther back from the front it is applied, the shorter t *v a* become. Moreover, in the offsetted back, the *tion* of the pres becomes more nearly vert than the back is upright. It is to these causes, rather to the throwing back of the cent of grav, that offsetted wall owes its increase of stability over vith a vert back.

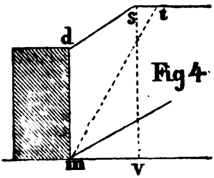


Fig 4

t. 4. When, as in Fig 4, the backing is higher than the l, and slopes away from its *inner* edge *d*, at the natural slope *d s*, of 1½ to 1, we onfident that the following thicknesses at base will at least be found *sufficient* for vert walls with sand. They are deduced from the experiments just alluded to, and are but rude approximations, with no scientific basis. We should not have inserted them, but for the fact that we know of no others for this case.

The first column contains the vert height *s v*, of the earth, as compared with the vert height of the wall; which latter is assumed to be 1; so that the table begins with backing of the same height as the wall, as in Fig 1. These vert walls may be changed to others, with battered faces, by Art 8; or without any such proceeding, their faces may be battered to any extent not exceeding 1½ inches to a foot, or 1 in 8, without sensibly affecting their stability, without increasing the base.

TABLE 1. (Original.)

Total height of the earth, compared with the height of the wall above ground.	Wall of Cut Stone, in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.	Total height of the earth, compared with the height of the wall above ground.	Wall of Cut Stone in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.
	Thickness at Base, in parts of the height.				Thickness at Base, in parts of the height.		
1.	.35	.40	.50	2.	.58	.63	.73
1.1	.42	.47	.57	2.5	.60	.65	.75
1.2	.46	.51	.61	3.	.62	.67	.77
1.3	.49	.54	.64	4.	.63	.68	.78
1.4	.51	.56	.66	6.	.64	.69	.79
1.5	.52	.57	.67	9.	.65	.70	.80
1.6	.54	.59	.69	14.	.66	.71	.81
1.7	.55	.60	.70	25. or more	.68	.73	.83
1.8	.56	.61	.71				

Art. 5. But when the slope *n r*, Fig 5, of 1½ to 1, starts from the *outer* edge *n* of the wall, greater thickness is required. Poncelet gives the following for this case, for *dry sand.*

TABLE 2.

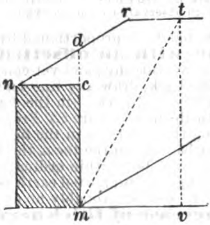


Fig. 5.

Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.	Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.
1	.35	.452	2.4	.762	1.02
1.1	.393	.498	3.0	.811	1.11
1.2	.439	.548	4.0	.852	1.18
1.3	.485	.604	6.0	.883	1.25
1.4	.532	.665	11.0	.909	1.28
1.5	.579	.726	21.0	.922	1.31
1.6	.617	.778	31.0	.926	1.32
1.7	.645	.824	Infinite.	.934	1.34
1.8	.668	.847			
1.9	.690	.903			
2.0	.707	.930			

When the earth reaches above the top of the wall, as in Figs 4 and 5. the wall is **surcharged;** and the earth that is *above* the top, is called the SURCHARGE. When the surcharge is carefully deposited above the wall, so as to slope back at a steeper angle than 1½ to 1, as say at 1 to 1, theory does not require the wall to be as thick. Notwithstanding Poncelet's high position, the writer cannot imagine that the base of a brick wall need be so great as 1⅓ times its height for any height of sand whatever.

Art. 6. On the theory of retaining-walls. Let *b c a m*, Fig 6, be such a wall, upholding backing or filling *c s m g;* the upper surf *c s* of which is hor, and level with the top *b c* of the wall; and let *m s* represent the nat slope of the earth which composes the backing; *m g* being hor.

Abundant experience on public works shows that this slope, whether for sand, gravel, or earth, when dry, may be practically taken at 1½ to 1; that is, 1½ hor, to 1 of vert measurement; which corresponds to an angle *s m g* of 33° 41′ with the hor; which is also about the angle at which bricks and roughly dressed masonry begin to slide on each other. This angle, however, varies considerably; being greatly influenced by the degree of dryness, or dampness, of the material; so that moderately damp sand or earth will stand at a slope of 1 to 1, or at an angle of 45°. Whatever it may be, it is called THE ANGLE OF NAT SLOPE of the material under consideration. In theoretical calculations for walls, it is safest to assume (as we have done throughout) that the backing is perfectly dry, since

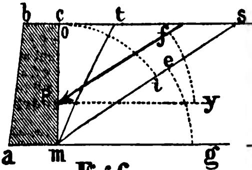


Fig. 6

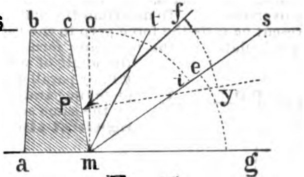


Fig. 7

its pres is then greatest; unless it be supposed to be so wet as to possess some degree of fluidity. The triangle *c m s* of earth above the nat slope *m s*, tends to slide down said slope, but is prevented from so doing by the wall.

It is assumed in all cases, that the wall is secured from *sliding* along its base, Art 9, that it is thick enough to prevent failure by *bulging;* and that it will fail only by *overturning*, by rotating around its toe, *a*, as a fulcrum. The thickness necessary to insure safety against the last will also be sufficient to prevent bulging. Now referring only to Fig 6 with a vert back, if the angle *o m s*, contained between the natural slope *m s*, and a vert line *m o*, drawn from the inner bottom edge *m* of the wall, be divided by a line *m t*, into two equal angles, *o m t*, *t m s*, then the angle *o m t* is called THE ANGLE, and *m t* THE SLOPE, OF MAXIMUM PRESSURE. The triangular prism of earth, of which *o m t* is a section, or an end view, is called THE PRISM OF MAX PRES; because, if considered as a wedge acting against the back of the wall, it would produce a greater pres upon it than would the entire triangle *c m s* of earth, considered as a single wedge. For although the last is the heaviest, yet it is more supported by the earth below it. Calculation shows that if we consider the earth *o m s* to be thus div into wedges by *any* line *m t*, the wedge that will press most against the wall is that formed when *m t* divides the angle *o m s*, or the arc *o t*, into two equal parts. But see Art 11.

Since *m g* is hor, and *m o* vert, the two form an angle of 90°; consequently the angle of max pres is plainly found by taking the angle *s m g* of nat slope from 90°, and div the rem by 2. Thus a nat slope of 1½ to 1, or 33° 41′, taken from 90°, leaves 56° 18′; and $\frac{56°\ 18'}{2}$ = **28° 9′, the corresponding angle o m t of max pres.**

For ease of calculation, only one foot of the *length* of the wall, and of its backing, is usually considered. The number of *cub* ft of wall, or of backing, is then equal to that of the *square* feet in heir respective profiles, or cross-sections.

Now, according to Moseley, if we assume the particles of earth composing the acking to be perfectly dry, and devoid of cohesion, (or tendency to stick to each ther,) which is very nearly the case in pure sand; and if we suppose the wall to be uddenly removed, then the triangle of earth *c m t*, comprised between the slope *m t* f max pres, and the vert back *c m* of the wall, Fig 6, would slide down, under the inuence of a force which may be represented by *y* P, acting in a *direction* *y* P, at right ngles to the face *c m* of the triangle of earth; (or in other words, at right angles the back of the vert wall,) its *center of force* being at P, distant ⅓ way between and *c*, measured from the bottom; and its *amount* equal to either of the following:

$$\left.\begin{array}{ll}\textbf{No 1.} & \begin{array}{c}\textit{Perp pres}\\ y\ P\end{array} = \dfrac{\textit{Wt of the triangle of earth}\ c\ m\ t \times o\ t}{\textit{vert depth}\ o\ m};\ \text{or}\\[2ex] \textbf{No 2.} & \begin{array}{c}\textit{Perp pres}\\ y\ P\end{array} = \dfrac{\begin{array}{c}\textit{Wt of a single cub}\\ \textit{ft of the backing}\end{array} \times \textit{sq of}\ o\ t}{2.}\end{array}\right\}\ \text{See Art. 11.}$$

n view of the great uncertainty involved in the matter of the actual pressure o th against retaining-walls in practice (see Art 2.), and in order to furnish mple rule which, although entirely unsupported by theory, is still (in the writer's nion) sufficiently approximate for ordinary practical purposes, we shall assume t No 1 of the two foregoing formulas applies near enough to walls **with inned backs** *c m*, also, as Figs 7 and 8, (precisely as they are lettered,) at least **til the back of the wall inclines forward as much as 6 ins r, to 1 foot vert**, or at an angle *c m o* of 26° 34′. **What follows on aining-walls will involve this incorrect assumption, and st be regarded merely as giving safe approximation.**

me appear to assume this perp pres to be the only one acting against the back e wall; and hence arrive at erroneous practical conclusions. For when, in to prevent this force from causing the triangle of earth to slide, we place a ning-wall in front of it, then, instead of *motion*, the force will produce *pres* of earth against the wall, causing *friction* between the pressed surfaces of the earth and wall. That is, if a wall were to begin to overturn around its toe *a* as a fulcrum, its back *c m* must of course rise, and in so doing must rub against the earth filling in contact with it; and this rubbing would evidently act to *impede* the overturning. So long as the wall does not move, the same friction assists in *preventing* overturning. To ascertain the amount and effect of this friction, let *y* P, Fig 8, represent *by scale* the force perp to the back *c m;* and supposed to have been previously calculated by the foregoing formula No 1. Make the angle *y* P *f* equal to the angle of wall friction,* draw *y f* at right angles to *y* P, or parallel to *m c;* make P *x* equal to *y f*, and complete the parallelogram P *y f x*. Then will *x* P represent by the same scale, **the amount of the friction against the back of the wall.**

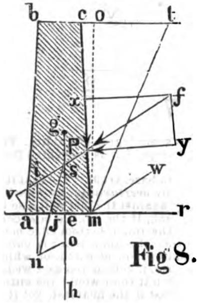


Fig 8.

Hence we have acting at P, two forces; namely, the perp force *y* P, and the friction *x* P; consequently, by comp and res of force, the diag *f* P of the parallelogram P *y f x*, if measured by the same scale, will give us the amount of their resultant; **which is the approx single theoretical force, both in amount and in direction, which the wall has to resist,** including the wall friction.

But this force, *f* P, is also always equal to the perp force *y* P, mult by the nat sec of the angle *y* P *f* of the wall friction; (or divided by its nat cosine) and of course may be ascertained thus:

$$\begin{array}{c}\textit{Approx theoreti-}\\ \textit{cal pres f P}\end{array} = \frac{\begin{array}{c}\textit{wt of triangle}\\ \text{c m t}\end{array} \times \text{o t} \times \begin{array}{c}\textit{nat sec of angle y P f}\\ \textit{of wall friction}\end{array}}{\textit{vert depth } \text{o m}} = \frac{\begin{array}{c}\textit{wt of}\\ \text{c m t}\end{array} \times \text{o t}}{\textit{cos } \text{y P f} \times \text{o m}}$$

Or finally, if it is assumed, as we do throughout, that the earth is perfectly dry (inasmuch as its pressure is then the greatest) and that the angles of nat slope, and of wall friction are then each 33° 41′ or 1.5 to 1, then in Figs 6, 7 and 8, if the angle *c m o* between the back *c m* and the vert *o m* does not exceed about 26° 34′ we may assume

$$\begin{array}{c}\textbf{Approx theoretical}\\ \textbf{pres f P}\end{array} = \textbf{wt of triangle c m t} \times .643$$

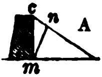
which includes the action of the friction of the earth against the back of the wall.

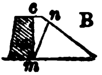
Rem. 1. **When the back of the wall is offsetted or stepped,** as in Fig 3, instead of being simply battered, as in Figs 7 and 8, the direction of the pres of the earth will be the same as if the back had the batter *i n*.

Rem. 2. **Now to find both the overturning tendency of the earth,** and the resistance of the wall against being overturned around its toe *a* as a fulcrum, first find the cen of grav *g* of the wall and through it draw a vert line *g h*. Prolong *f* P towards *v* and draw *a v* perp to it. By any scale make *s o* = wt of wall, and *s i* = calculated pres *f* P. Complete the parallelogram *s i n o*, and draw its diagonal *s n*, which will be the resultant of the pres *f* P and of the wt of the wall; and should for safety be such that *a j* be not *less* than about one-fifth of *a m*, *even with best masonry and unyielding soil.* Otherwise the great pressure so near the toe *a* may either fracture the wall or compress the soil near that point so that the wall will lean forward. In walls built by our rule, Art 1, or by table, p 610, *a j* will be more than one-fifth of *a m*. The pres *f* P if mult by its leverage *a v* will give the moment of the pres about *a*; and the wt of the wall mult by its leverage *e a* will give that of the wall. The wall is safe from overturning in proportion as its moment exceeds that of the pres. It is assumed to be safe against *sliding*, *breaking*, or *settling* into the soil.

---

* **This angle of wall friction** is that at which a plane of masonry must be inclined to the horizontal so that dry sand or earth would slide down it. It is about the same as the nat slope, or 33° 41′, or 1.5 to 1; and its nat secant is 1.202, and its nat cos .832.





lem. 4. If the earth slopes downward from C, as A or B, instead of being hor as in Figs 6, 7, 8, use the wt of the th *c m n* instead of *c m t*, *m n* being the slope of max pressure. A the point of application will still be at P (at one-third of ;) as in 6, 7, 8; but in B it will be a little higher as explained ow for Fig 9.

iurcharged walls are those in which the earth backing ends above the tops of the walls.

ιccording to theory, when as in Fig 9, there is a surcharge *k* of backing, sloping away from *c* at its natural slope *c v*, › max pres against the wall is ained when the earth reaches to ‹ level of *d*, where the slope *m t d* max pres intersects the face of the ; slope *c v*; so that if afterward the th is raised to *v*, or to any greater ght, no additional pres is thereby own against the back of the wall. also if the earth slopes from *b*, or m between *c* and *b*, except that n the slope *m d* of max pres must end up to meet this other slope.

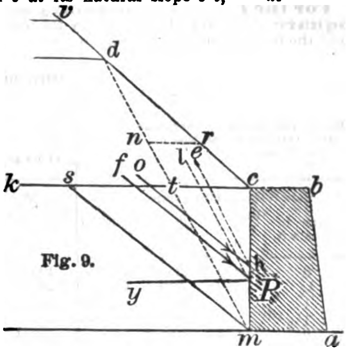


Fig. 9.

ſhe approximate amount the oblique pres, when the wall is charged, (as in any of the Figs 4, ,) may be found on the same prinle as when the earth is level with top; namely, instead of the *trian-c m t* of earth, Figs 6, 7, 8, 9, find : wt of *all* the earth *d s m t*, Fig 4, *ι t r*, Fig 5, or *c d m*, Fig 9 (if the charge reaches to *d* or *v*, or higher), ween the slope *m d*, Fig 9, *m t*, Figs 4 and 5, of max pres, the back of the wall, and front slope; omitting any which, like *d c n*, Fig 5, rests on the top of the wall d thus adds to its stability) when the slope starts in front of *c*. Having found ; weight, then for dry backing the

$$\left.\begin{array}{r}\textbf{Pressure}\\ \textbf{approximately}\end{array}\right\} = \textbf{Wt of the earth} \times \textbf{.648},$$

luding the action of the friction of the earth against the back of the wall; near ugh (in the writer's opinion) for practical purposes in so uncertain a matter; ; essentially empirical.

ſhe direction of the pressure thus found will be the same as when the th is level with the top *b c*; namely, as in Figs 6 and 7, first draw a line, as P *y*, p to the back *c m*, whether vert or inclined. Then draw another line, as P *f*, king the angle *y* P *f* = the angle of wall friction, which we all along assume to 33° 41′, or 1.5 to 1. Then P *f* will give the direction of the pressure. But its nt of application will not always be at P (one-third of the height of the wall ve *m*) as heretofore; for in all cases it will be at that point P, or at some **gher** one as *h*, where the back is cut by a line *l* P or *e h*, Fig 9, drawn from the of grav of the sustained earth (omitting any that rests immediately on the top , and parallel to the slope *m d* of max pres; and such a line will strike at one-d the height of the wall only when the sustained earth *t c m* or *d c m* **forms a mplete triangle,** one of whose angles is at the inner top edge *c* of the wall. **all other cases** said line for a surcharge will strike above P.

Art. 7. On page 603, Fig 1, we recommend that the base *o s* at the ground-line of well built **vertical** walls should not be less than .35, or .4, or .5 of the height *d s* above said line, depending on the kind of masonry. But a wall with a **battered** (inclined) front or face as found by Art 8, (by which the following table was prepared), will be as strong, and at the same time contain less masonry than a vert wall, although the battered one will have the thickest base *o s*.

Table 3, of thicknesses at base *o s*, Fig 1, and at top *c d*, of walls with battered faces, so as to be as strong as vertical ones which contain more masonry.

For the cub yds of masonry above *o s* per foot run of wall, mult the **square** of the vert height *d s* by the number in the column of cub yds. Then add the foundation masonry below *o s*.

(Original.)

	All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .35 of its ht *d s*.			All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .4 of its ht *d s*.			All the walls below have the same strength as a vert one whose base *o s*, fig 1 = .5 of its ht *d s*.		
	Cut stone.			Mortar rubble.			Dry rubble.		
Batter, in ins to a ft.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run.
0	.350	.350	.01296	.400	.400	.01482	.500	.500	.01852
½	.352	.310	.01226	.401	.359	.01407	.501	.459	.01778
1	.355	.270	.01158	.403	.320	.01339	.503	.420	.01709
1½	.359	.234	.01098	.408	.283	.01280	.506	.381	.01643
2	.364	.197	.01039	.413	.246	.01220	.510	.343	.01580
2½	.371	.163	.00989	.419	.210	.01165	.516	.308	.01526
3	.379	.129	.00941	.425	.175	.01111	.522	.272	.01470
3½	.389	.096	.00898	.435	.143	.01070	.528	.236	.01415
4	.400	.066	.00863	.445	.110	.01028	.537	.204	.01372
5	.425	.007	.00800	.468	.051	.00961	.555	.138	.01283
Triangle	.429	.000	.00794	.490	.000	.00907	.612	.000	.01133

Moseley and others quote Gadroy, for a DRY SAND SLOPING AT 21°. It would be better to cease from circulating such evident mistakes. Dry sand will stand at no less angle for a savant than for anybody else. For practical purposes, we may say that dry sand, gravel and earths, slope at 33° 41′ or 1½ to 1; as abundant experience on railroad embkts proves. Poncelet gives tables for walls to support dry earth sloping at 1 to 1, or 45°; but as we do not believe in the existence of such earth, we omit such tables. Sand, gravel, and earths may be moistened to diff degrees, so as to stand at any angle between hor and vert; and by moistening and ramming, the earths may be converted into compact masses, exerting little or no pres; and may even so continue after they become dry; being then, in fact, a kind of air-dried brick. It is sometimes difficult to know whether earth or sand is perfectly dry or not; and an *exceedingly* small degree of moisture will cause them to stand at 1 to 1, in *small heaps*, such as have probably been observed by the authorities on the subject. The writer found that fine sand from the sea-shore, and under cover, would stand at 1½ to 1 during warm dry weather, and at 1 to 1 when the air was damp. Yet no diff whatever in its degree of moisture was perceptible to the feeling. Its susceptibility to dampness was of course owing to salt. A few handfuls of dry earth may perhaps be coquetted into standing at 1 to 1 on a table; but so far as our observation extends, when it is dumped in large quantities from carts and wheelbarrows, its slope is about 1½ to 1; and this we consider the proper one to be used in practical calculations, where safety is the consideration of paramount importance.

The less the nat slope, the greater is the pres; and since the slope is least when the backing is perfectly dry, (omitting of course its condition when so absolutely *wet* as to become partially fluid,) we have, on the score of safety, confined our tables to dry backing. As stated in Art 1, we cannot recommend dimensions less than those there given, when we consider the rough treatment to which masonry is exposed on public works.

In carrying a road along dangerous precipices, we should rather be tempted at times to make thicker walls. We imagine, for instance, that the centrifugal force of a heavy train, whirling around a sharp curve, convex on the dangerous side, should not be overlooked in designing walls for such localities. This force is hor; and is applied near the top of the wall; and, consequently, its leverage may be considered as equal to the height; whereas the theoretical pres of the earth is oblique; and is applied at ⅓ of the height from the bottom; so that its leverage about the toe of the wall is very short. Moreover, the simple *weight* of the train, produces pres against the wall; as well as that of the backing. All such considerations are omitted by theorists. The dangerous pres caused by tremors, &c, cannot be assumed to be applied at $\frac{1}{3}$ of the height from the bottom; nor indeed, can it be calculated at all.

REM. 2. **Wharf walls** are an instance where the thickness should be increased, notwithstanding that the pres of the water in front helps to sustain them. The earth behind such walls, is not only liable to be very heavily loaded when vessels are discharging; but is apt to become saturated with water, especially below low-water level; and thus to exert a very great pres against the walls. Moreover, the water gets under the wall; and by its upward pressure virtually reduces its weight, and consequently its stability. The same cause of course diminishes the friction of the wall upon its base. Such walls are, therefore, very liable to slide, if the foundation is smooth, and horizontal; and have done so even when the foundation had a considerable inclination backward, as in Fig 1. See Art 9.

REM. 3. A retaining-wall is usually in greater danger for a few months after its completion, than after time has been allowed for the mortar to harden perfectly; and for the backing to settle. When there are suspicions of the safety of a new wall, it would be well to place strong temporary shores against it, at about $\frac{1}{3}$ to $\frac{1}{2}$ of its height above ground. In some cases, permanent buttresses of masonry may be built for the purpose. They should be well bonded into the wall.

REM. 4. The pres of the earth backing will be much reduced, if the first few feet of its height be made up in thin hor layers, to be consolidated by being used by the masons instead of scaffolding; as shown at *h*, Fig 1. Frequently this can be done without inconvenience; and at very trifling cost.

Art. 8. To change a vert retaining-wall, into one with a battered face, which shall present an equal resistance against overturning; although requiring less masonry.

This is sometimes termed a **transformation of profile.** (Original.)

Let *a b o i*, Fig 10, be the vert wall. Mult its base *o i*, by 1.225; (1.22475 is nearer;) the prod will be the base *o e*, of a triangular wall *b o e*, possessing the same stability; and yet not requiring much more than half the masonry of the vert one. See Rem 1. This being done, suppose a wall to be desired with a face batter, of say 3 ins to a ft; or 1 in 4. From the point *n*, where the face of the triangular wall intersects that of the vert one, step off vert any 4 short equal spaces; and from the upper one *m*, step off one space hor, to *v*. Through *v* and *n* draw the dotted line *s t*, which evidently will batter 1 in 4. Then is *b s t o* approximately the reqd wall; but a little thicker than necessary. To reduce it, from *t* draw the dotted line *t b*. Mark the point *c*, where this line intersects the face *a i*, of the vert wall; and through *c* draw *d l*, parallel to *s t*. Then is *b d l o* the reqd wall. Our fig is drawn in an exaggerated manner, so as to avoid confusion in the lines. The base *o e* of the triangular wall, would not in reality be near so great as it is represented.

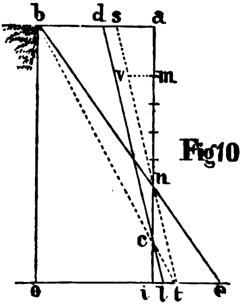


Fig 10

It will be observed that as the base increases, the quantity of masonry diminishes.

REM. 1. **The battered wall will in fact be safer than the vert one.** The battered wall has the same moment of stability as the vert one; and the pres of the earth against it also remains unchanged, but the *moment* or *tendency* of the pres to upset the wall has become less. For let *a b m n*, Fig 11, represent a vertical wall; and *f o* the *amount* and direction of pres behind it. (For ease of illustration, we have placed *o above* the true cen of pres of the earth filling, which would be at one-third of *a n* above *n*.) Now, the leverage with which this pres tends to overturn the wall around its toe *m*, is the dist *m s*, measured from the toe or fulcrum *m*, and at right angles to the direction *f o s c* of the pres; and this leverage mult by the force *f o*, gives the overturning tendency or moment of said force. See "Moments and leverage." Again, let *a n y*, represent a triangular wall of the same stability as the other, as found by our rule. Here we still have the same amount *f o*, and direction *f o s c*, of pres force against the wall; but it now acts to overturn the wall *a n y* around the toe *y*; and therefore, with the reduced leverage *y c*. Consequently, its overturning tendency is less than before. Therefore, in ordinary language, we may say that the wall is stronger than before, although its moment of stability, or standing tendency, has in itself undergone no change. If the pres *f o* against the vert back were hor, as in the case of water, then its leverage would evidently be the same in both walls; and the proportion between the overturning moment of the pres, and the moments of stability of the two walls, would be constant.

REM. 2. In attempting to reduce the masonry by adopting a wall, *o b e*, Fig 10, of a triangular section; or of one nearly approaching a triangle, special attention should be given to the quality of the masonry near the thin toe *e*; which will otherwise be apt to crack, or fail under the pres.

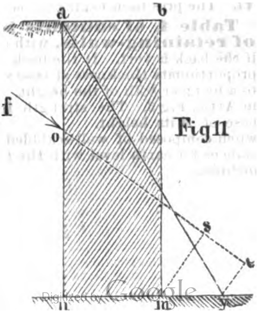


Fig 11

REM 3. MOREOVER, WHEN COMMON MORTAR IS USED WITHOUT AN ADMIXTURE OF CEMENT, which it never should be, in retaining-walls, where durability is an object, a great batter is objectionable; inasmuch as the rain, combined with frost, &c, soon destroys the mortar. In such cases, therefore, the batter should not exceed 1 or 1½ ins to a ft; and even then, at least the pointing of the joints, and a few feet in height of both the upper and the lower courses of masonry, should be done with cement, or cement-mortar. We have observed a most marked diff in the corrosion of the mortar, where, in the same walls, with the same exposure, one portion has been built with a vert face; and another with a batter of but 1½ inch to a foot. Common mortar will never set properly, and continue firm, when it is exposed to moisture from the earth. This is very observable near the tops and bottoms of abuts, retaining-walls, &c; the lime-mortar at those parts will generally be found to be rendered entirely worthless. A profile somewhat like Fig 12, may at times prove serviceable, instead of the triangular. This is the form of the Gothic buttress; which probably had its origin in the cause just spoken of.

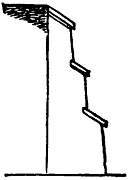


Fig 12

Art. 9. A retaining-wall may slide, without losing its verticality; and, indeed, without any danger of being overturned. This is very apt to occur if it is built upon a hor wooden platform; or upon a level surf of rock, or clay, without other means than mere friction to prevent sliding. This may be obviated by inclining the base, as in Fig 1; by founding the wall at such a depth as to provide a proper resistance from the soil in front; or in case of a platform, by securing one or more lines of strong beams to its upper surf, across the direction in which sliding would take place. On **wet clay,** friction may be as low as from .2 to ⅓ the weight of the wall; on dry earth, it is about ½ to ⅔; and on sand or gravel, about ⅔ to ¾. The friction of masonry on a wooden platform, is about $\frac{6}{10}$ of the wt, if dry; and ¾ if wet.

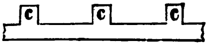


Fig 13

Counterforts, shown in plan at *c c c*, Fig 13, consist in an increase of the thickness of the wall, *at its back*, at regular intervals of its length. We conceive them to be but little better than a waste of masonry. When a wall of this kind fails, it almost invariably separates from its counterforts; to which it is connected merely by the adhesion of the mortar; and to a slight extent, by the bonding of the masonry. The table in Art 7 shows that a very small addition to the base of a wall, is attended by a great increase of its strength; we therefore think that the masonry of counterforts would be much better, and more cheaply employed in giving the wall an additional thickness, along its entire length; and for the lower third of its height. Counterforts are very generally used in retaining-walls by European engineers; but rarely, if ever, by Americans.

Buttresses are like counterforts, except that they are placed *in front* of a wall instead of behind it; and that their profile is generally triangular, or nearly so. They greatly increase its strength; but, being unsightly, are seldom used, except as a remedy when a wall is seen to be failing.

Land-ties, or long rods of iron, have been employed as a makeshift for upholding weak retaining-walls. Extending through the wall from its face, the land ends are connected with anchors of masonry, cast-iron or wooden posts; the whole being at some dist below the surface.

Retaining walls with curved profiles are mentioned here merely to caution the young engineer against building them. Although sanctioned by the practice of some high authorities, they really possess no merit sufficient to compensate for the additional expense and trouble of their construction.

Art. 10. Among military men, a retaining-wall is called a **revetment.** When the earth is level with the top, a **scarp revetment;** when above it, a **counterscarp** revetment, or a *demi-revetment*. When the face of the wall is battered, a *sloping;* and when the back is battered, a *countersloping* revetment. The batter is called the **talus.**

Art. 11. The pres against a wall Fig 6, from sand etc level with its top, is not diminished by reducing the quantity of sand, until its top width *c s* becomes less than that (*c t*) pertaining to the angle *c m t* of maximum pres. The pres then begins to diminish, but in practice *the diminution is not appreciable until the width is reduced to* about one sixth of that (*c s*) pertaining to the angle *c m s* of natural slope, or about half of *c t*. The pres then begins to decrease rapidly as the width is further reduced.

Table 4, of contents in cub yards for each foot in length of retaining-walls, with a thickness at base equal to .4 of the vert height, if the back is vert. If the back is stepped according to the rule in Art 3, the proportionate thickness at base will of course be increased. Face batter, 1½ inches to a foot; or ⅛th of the height. Back either vert, or stepped according to the rule in Art 3, Fig 3. The strength is very nearly equal to that of a vert wall with a base of .4 its height. Experience has proved that such walls, when composed of well-scabbled mortar rubble, are safe under all ordinary circumstances for earth level with the top. Steps or offsets, *o e*, at foot, Fig 1, are not here included.

TABLE 4. (Original.)

Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.
1	.018	10½	1.38	20	5.00	29½	10.9	48	28.8	74	68.5
½	.028	11	1.51	½	5.25	30	11.3	49	30.0	76	72.2
2	.050	½	1.65	21	5.51	31	12.0	50	31.3	78	76.1
½	.078	12	1.80	½	5.78	32	12.8	51	32.5	80	80.0
3	.113	½	1.95	22	6.05	33	13.6	52	33.8	82	84.1
½	.158	13	2.11	½	6.33	34	14.5	53	35.1	84	88.4
4	.200	½	2.28	23	6.61	35	15.3	54	36.5	86	92.5
½	.253	14	2.45	½	6.90	36	16.2	55	37.8	88	96.8
5	.313	½	2.63	24	7.20	37	17.1	56	39.2	90	101.8
½	.378	15	2.81	½	7.50	38	18.1	57	40.6	92	105.8
6	.450	½	3.00	25	7.81	39	19 0	58	42.1	94	110.5
½	.528	16	3.20	½	8.13	40	20.0	59	43.5	96	115.2
7	.613	½	3.40	26	8.45	41	21.0	60	45.0	98	120.1
½	.703	17	3.61	½	8.78	42	22.1	62	48.1	100	125.0
8	.800	½	3.83	27	9.12	43	23.1	64	51.2	102	130.1
½	.903	18	4.05	½	9 45	44	24.2	66	54.5	104	135.2
9	1.01	½	4.28	28	9.80	45	25.3	68	57.8	106	140.5
½	1.13	19	4.51	½	10.2	46	26.5	70	61.3		
10	1.26	½	4.73	29	10.5	47	27.6	72	64 8		

STONE BRIDGES.

Art. 1. In an arch *s t s*, Fig 1, the dist *e o* is called its **span**; *t a* its **rise**; *t* its **rown**; its lower boundary line, *e a o*, its **soffit**, or **intrados**; the upper one, *t r*, its **back**, or **extrados.** The terms soffit and back are also applied to the ntire lower and upper curved *surfaces* of the whole arch. The ends of an arch, or he showing areas comprised between its intrados and extrados, are its **faces**; thus he area *s t s a* is a face. The inclined surfaces or joints, *r e*, *r o*, upon which the *feet* f the arch rest, or from which the arch *springs*, are the **skewbacks.** Lines evel with *e* and *o*, at right angles to the faces of the arch, and forming the lower dges of its feet, (see *n n*, Fig 2½,) are the **springing lines**, or **springs.** The locks of which the arch itself is composed, are the **arch-stones**, or **voussoirs.** he center one, *t a*, is the **keystone**; and the lowest ones, *s s*, the **springers.** he term *archblock* might be substituted for voussoir, and like it would apply to rick or other material, as well as to stone. The parts *t r*, *t r*, are the **haunches**; nd the spaces *t r l*, *t r b*, above these, are the **spandrels.** The material deposited n these spaces is the **spandrel filling**; it is sometimes earth, sometimes masonry; or partly of each, as in Fig 1.

In large arches, it often consists of several parallel SPANDREL-WALLS, *l l*, Fig 2½, running lengthwise the roadway, or astraddle of the arch. They are covered at top either by small arches from wall to all, or by flat stones, for supporting the material of the roadway. They are also at times connected gether by vert cross-walls at intervals, for steadying them laterally, as at *t t*, Fig 2½. The parts *p e n*, *g p o n*, Fig 1, are the ABUTMENTS of the arch; *e n*, *o n*, the *faces*; *g p*, *g p*, the backs; and *n*, *p n*, the *bases* of the abuts. The bases are usually widened by *feet*, *steps*, or *offsets*, *d d*, for distibuting the wt of the bridge over a greater area of foundation; thus diminishing the danger of setement. The distance *t a* in any arch-stone, is called its *depth*.

The only arches in common se for bridges, are the circular, often called segmental); and he elliptic.

Art. 2. To find the epth of keystone for irst-class cut-stone rches, whether circular or elliptic.*

Find the rad *c o*, Fig 1, which ill touch the arch at *o*, *a*, and Add together this rad, and *alf* the span *o e*. Take the sq of the sum. Div this sq rt y 4. To the quot add $\frac{2}{10}$ of a . Or by formula,

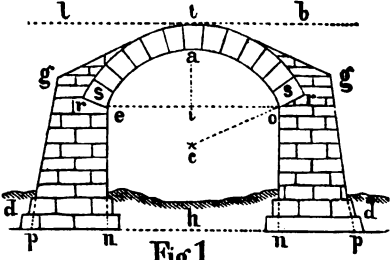


Fig 1

---

* Inasmuch as the rules which we give for arches and abuts are entirely original and novel, it may t be amiss to state that they are not altogether empirical; but are based upon accurate drawings

$$\text{Depth of key in feet} = \frac{\sqrt{\text{Rad} + \text{half span}}}{4} + 0.2\ \text{foot.}$$

For second-class work, this depth may be increased about ⅛th part; or **for brick or fair rubble,** about ⅓rd. See table of Keystones.

In large arches it is advisable to increase the depth of the archstones toward the springs; but when the span is as small as about 60 to 80 or 100 feet, this is not at all *necessary* if the stone is good; although the arch will be stronger if it is done. In practice this increase, even in the largest spans, does not exceed from ¼ to ½ the depth of the key; although theory would require much more in arches of great rise.

REM. **To find the rad** *c o*, whether the arch be circular or elliptic. Square *half* the span *s o*. Square the whole rise *i a*. Add these squares together; div the sum by *twice* the rise *i a*. Or it may be found near enough for this purpose by the dividers, from a small arch drawn to a scale.

Amount of pressure sustained by archstones. In bridges of the same width of roadway; if all the other parts bore to each other the same proportion as the spans, the *total* pres would increase as the squares of the spans, while the pressure *per square foot* would increase as the spans. But in practice the depth of the archstones increases much less rapidly than the span; while the thickness of the roadway material, and the extraneous load per sq ft, remain the same for all spans. Hence the *total* pressures, at key and at spring, increase less rapidly than the *squares* of the spans; but *more* rapidly than the simple *spans;* as do also the pressures *per square foot.* Thus in two bridges of the same width, but with spans of 100 and 200 ft, with depths of archstones taken from our table and uniform from key to spring; supposed to be filled up solid with masonry of 160 lbs per cub ft, to a level of about 15 inches above the crown, (including the stone paving of the roadway); with an extraneous load of 100 lbs per sq ft; the pressures will be approximately as follows:

	Span 100 ft.				**Span 200 ft.**			
	AT KEY.		AT SPRING.		AT KEY.		AT SPRING.	
	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.
Rise.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
⅙	42¾	13¾	56	18¾	126	29¼	179	42
⅕	36¾	12¼	57	19	112	27¼	181	44
¼	31¼	11	57½	20	97	24½	188	47½
⅓	25	9	61½	22½	80¼	21	207	54¼
½	18	6¾	67½	25	57½	15¾	230	61½

It will be seen that with the same span, the pres at the key becomes less, while that at the spring becomes greater, as the rise increases. Also that when the archstones are of uniform depth, the pres at either spring of a semicircular arch is about 4 times as great as at the key; whereas when the rise is but one-sixth of the span, the pres at spring averages but about one-third greater than at the key. These proportions vary somewhat in different spans.

The greater pres per sq ft at the springs may be reduced by increasing the depth of the archstones towards the springs. This however is not *necessary* in moderate spans, inasmuch as good stone will be safe even under this greater pres.

By using parallel spandrel walls, see Fig 2½, or by partly filling with earth instead of masonry, the pres on the archstones may be diminished, say, as a rough average, about ⅙ part.

---

and calculations made by the writer, of lines of pres, &c, of arches from 1 to 300 ft span, and of every rise, from a semicircle to $\frac{1}{12}$ of the span. From these drawings he endeavored to find proportions which, although they might not endure the test of strict criticism, would still apply to all the cases with an accuracy sufficient for ordinary practical purposes.

Table 1. Of some existing arches, with both their actual and their calculated depths (by our rule) of keystone. Where two depths are given in the column of keys the smallest is for first-class cut-stone, and the largest for good rubble, or brick. Those also which are not specified are of first-class cut-stone. C stands for circular, E for elliptic. For 2d class work, add about ⅙th part; and for brick, or fair rubble, about ⅛th.

		Span.	Rise.	Radius.	Actual Key.	Key by our Rule.	Engineer.
		Ft.	Ft.	Ft.	Ft.	Ft.	
Cabin John. Washington Aqueduct	C	220	57.25	134.25	4 16	4.11	Meigs.
Grosvenor Bridge, Chester, England	C	200	42	140	4	4.07	Hartley.
London Bridge, England	E	152	29.5	112.6	4.75	3.62	Rennie.
Gloucester, across the Severn, England	E	150	35	98	4.50	3.49	Telford.
Dora Riparia, Turin, Italy	C	148	18	160.1	4.92	4.03	Mosca.
Pont-y-Prydd, Wales. Rough rubble	C	140	35	87.5	1.5	3.3 4.2	Edwards.
Pont de l'Alma, Paris, France. Small rough rubble in cement	E	141.4	28.2	102.8	4.92	3.5 4.44	Darcel.
Souppes, France. An experimental arch of cut-stone in Portland cement mortar *	C	124.3	6.97	280.5	2.62 !	4.82	Vaudrey.
Waterloo. Across the Thames at London	E	120	32	72.2	4.5	3.07	Rennie.
Tongueland, England. Turnpike	C	118	38	64.8	3.5	3.0	Telford.
Pont National, formerly Pont Napoleon. R. R. viaduct, Paris. Small rough rubble in cement	C	115.5	14.75	120	4	3.54 4.48	Couche.
Holy Trinity, Florence. Very slightly pointed		96.5	15.1	83.9	2.75	3.07	Ammanati.
Dean Bridge, Scotland. Turnpike	C	90	30	48.9	3	2.62	Telford.
Dunkeld " "	C	90	30	48.9	3.17	2.62	Telford.
Licking Aqueduct. Chesapeake & Ohio Canal	C	90	15	75	2.83	2.94	Fisk.
Posen Viaduct, Germany. Brick in cement	C	80	16	58	4.66	2.68 3.40	
Allanton, Scotland. Turnpike	C	75	11.5	67	2.5	2.75	Stevenson.
Falls Bridge, Philadelphia & Reading R R	C	78	25	43	3	2.46	
Staines, England. Turnpike	C	74	9.3	78	3†	2.98	Rennie.
Brent R R Viaduct, England. Brick in cement	E	70	17.6	43.6	3	2.42 3.02	Brunel.
Bow Bridge. Turnpike. England	E	66	13.75	46.5	2.50	2.42	Walker.
Royal Border Viaduct, England. Brick in cement	C	61.5	30.75	30.75	2.66	2.16 2.88	
Bewdly. Turnpike. England	C	60	20	38	2.2	2.18	Telford.
Chestnut Street Bridge, Philadelphia. Brick in cement	C	60	18	34	2.5	2.2 2.8	Kneass.
Monocacy Aqueduct, Chesapeake & Ohio Canal	E	54	9	45	2.5	2.32	Fisk.
Llanrwast, Wales. Turnpike	C	58	17	33	1.5	2.17	Inigo Jones.
Avon Viaduct, England. Brick in cement	E	50	15	28.3	2	2.03 2.67	Vignoles.
James River Aqueduct, Virginia	C	50	7	47	2.66	2.32	Ellet.
Tonoloway Culvert, under Ches & Ohio Canal. Rubble in cement	C	45	15	21	2	1.8 2.3	Fisk.
Philadelphia & Reading R R	C	44	8	34.3	2.5	2.08	Steele.
" " " "	C	31.9	5	26.8	1.66	1.83	Steele.
Edinburg & Dalkeith R R. Scotland	E	24	4	20	.75	1.62	Jardine.

* See Experimental Arch at Souppes, France.

† Staines bridge. Some authorities give 2 ft 4 ins as the depth at key.

Experimental Arch at Souppes, France. See Table.
Span = about 18 × rise.

	Span.	Rise.	Radius of intrados.	Depth of arch-stones			Width.
				at spring.	at key		
					on faces.	betw faces.	
Meters.....	37.886	2.125	85.5	1.10	1.10	0.80	3.5
Feet........	124.30	6.97	280.52	3.61	3.61	2.624	11.5

Arch of granite. The centers rested (for four months) on sand in 16 cylinders, 1 ft diameter, 1 ft high, of $\frac{1}{32}$-inch sheet iron. The unloaded arch settled 15 millimeters (0.59 inch) on striking the centers. The additional settlements under extraneous loads were as follows:

Extraneous load.			Increase of settlement.	
	Kilograms.	Pounds.	Millimeters.	Inches.
Distributed.................	367000	809000	21	0.8
Center..............	4975	11000	0.3	0.012
Distributed................	132600	292000	1.2	0.047

With the distributed load of 367000 kilog, a load of 4975 kilog, falling 0.3 m (11.8 ins) on key, caused vibrations of 2.8 mm (0.11 inch). *Annales des Ponts et Chaussées*, 1866 *Part* 2, 1868 *Part* 2.

The arch on the BOURBONNAIS RAILWAY, is probably the boldest;[*] and THE CABIN JOHN ARCH, by Capt, now Gen'l M. C. Meigs, U S Army, the grandest stone one in existence. PONT-Y-PRYDD, in Wales, is a common road bridge, of very rude construction; with a dangerously steep roadway. It was built entirely of rubble, in mortar, by a common country mason, in 1750; and is still in perfect condition. Only the outer, or *showing* arch-stones, are 2.5 ft deep; and that depth is made up of two stones. The inner arch-stones are but 1.5 ft deep; and but from 6 to 9 inches thick. The stone quarried with tolerably fair natural beds; and received little or no dressing in addition. The bridge is a fine example of that ignorance which often passes for boldness. PONT NAPOLEON carries a railroad across the Seine at Paris. The arches are of the uniform depth of 4 ft, from crown to spring. They are composed chiefly of *small rough quarry chips*, or *spawls;* well washed, to free them from dirt and dust; and then thoroughly bedded in good cement; and grouted with the same. It is in fact an arch of cement-concrete. The PONT DE ALMA, near it, *and built in the same way*, has elliptic arches of from 126 to 141 ft span; with rises of $\frac{1}{5}$ the span. Key 4.9 ft. These two bridges, considering the want of precedent in this kind of construction, on so large a scale, must be regarded as very bold; and as reflecting the highest credit for practical science, upon their engineers, Darcel and Couche. Some trouble arose from the unequal contraction of the different thicknesses of cement. They show what may be readily accomplished in arches of moderate spans, by means of small stone, and *good hydraulic cement* when large stone fit for arches is not procurable. In Pont Napoleon the depth of arch is less than our rule gives for second class cut-stone.

Art. 3. The keystones for large elliptic arches by the best engineers, are generally made about ⅓ part deeper than our rule requires; or than is considered necessary for circular ones of the same span and rise; in order to keep the line of pres well within the joints; although the elliptic arch, with its spandrel filling, has slightly less wt; and that wt has a trifle less leverage than in a circular one; and consequently it exerts less pres both at the key, and at the skewback. See London, Gloucester, and Waterloo bridges, in the preceding table.

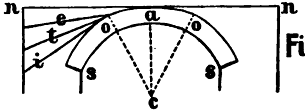


Fig. 1½

Rem. Young engineers are apt to affect shallow arch-stones; but it would be far better to adopt the opposite course; for not only do deep ones make a more stable structure, but a thin arch is as unsightly an object as too slender a column. According to our own taste, arch-stones fully ⅓ deeper than our rule gives for first-class cut stone, are greatly to be preferred when appearance is consulted. Especially when an arch is of rough rubble, which costs about the same whether it is built up as arch, or as spandrel filling, it is mere folly to make the arches shallow. Stability and durability should be the objects aimed at; and when they can be attained even to excess, without increased cost, it is best to do so.

---

[*] Built like that at Souppes.

Table 2. Depths of keystones for arches of first-class cut stone, by Art 2. For second class add full one-eighth part; and for superior brick one-fourth to one-third part, if the span exceeds about 15 or 20 ft. Original.

Span. Feet.	Rise, in parts of the span. $\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.
2	.55	.56	.58	.60	.61	.64	.68
4	.70	.72	.74	.76	.79	.83	.88
6	.81	.83	.86	.89	.92	.97	1.03
8	.91	.93	.96	1.00	1.03	1.09	1.16
10	.99	1.01	1.04	1.07	1.11	1.18	1.28
15	1.17	1.19	1.22	1.26	1.30	1.40	1.50
20	1.32	1.35	1.38	1.43	1.48	1.59	1.70
25	1.45	1.48	1.53	1.58	1.64	1.76	1.88
30	1.57	1.60	1.65	1.71	1.78	1.91	2.04
35	1.68	1.70	1.76	1.83	1.90	2.04	2.19
40	1.78	1.81	1.88	1.95	2.08	2.18	2.33
50	1.97	2.00	2.08	2.16	2.25	2.41	2.58
60	2.14	2.18	2.26	2.35	2.44	2.62	2.80
80	2.44	2.49	2.58	2.68	2.78	2.98	3.18
100	2.70	2.75	2.86	2.97	3.09	3.32	3.55
120	2.94	2.99	3.10	3.22	3.35	3.61	3.88
140	3.16	3.21	3.33	3.46	3.60	3.87	4.15
160	3.36	3.44	3.58	3.72	3.87	4.17	
180	3.56	3.63	3.75	3.90	4.06	4.38	
200	3.74	3.81	3.95	4.12	4.29		
220	3.91	4.00	4.13	4.30	4.48		
240	4.07	4.15	4.30	4.48			
260	4.23	4.31	4.47	4.66			
280	4.38	4.46	4.63				
300	4.53	4.62	4.80				

Art. 4. To proportion the abuts for an arch of stone or brick, whether circular or elliptic. (Original.)

The writer ventures to offer the following rule, in the belief that it will be found to combine the requirements of theory with those of economy and ease of application, to perhaps as great an extent as is attainable in an endeavor to reduce so complicated a subject, to **a simple and reliable working rule for practical bridge-builders.** This is all that he claims for it. Notwithstanding its simplicity, it is the result of much labor on his part. It applies equally to the smallest culvert, and to the largest bridge; whatever may be the proportions of span and rise; and to any height of abut whatever. It applies also to all the usual methods of filling above the arch; whether with solid masonry to the level *v f*, Fig 2, of the top of the arch; or entirely with earth; or partly with each, as represented in the fig: or with parallel spandrel-walls extending to the back of the abut, as in Fig 2½. Although the stability of an abut cannot remain precisely the same under all these conditions, yet the diff of thickness which would follow from a strict investigation of each particular case, is not sufficient to warrant us in embarrassing a rule intended for popular use, by a multitude of exceptions and modifications which would defeat the very object for which it was designed. We shall not touch upon the theory of arches, except in the way of incidental allusion to it. Theories for arches, and their abuts, omit all consideration of passing loads; and consequently are entirely inapplicable in practice when, as is frequently the case, (especially in railroad bridges of moderate spans,) the load bears a large ratio to the wt of the arch itself. Hence the theoretical line of thrust has no place in such cases. Our rule is intended for common practice: and we conceive that no error of practical importance will attend its application to any case whatever; whether the arch be circular or elliptic.

It gives a thickness of abut, which, without any backing of earth behind it, is safe in itself, and in all cases, against the pres. when the bridge is unloaded. Moreover, in very large arches, in which the greatest load likely to come upon them in practice is small in comparison with the wt of the arch itself, and the filling above it, our abuts would also be safe from the *loaded* bridge, without any dependence upon the earth behind them; but as the arches become less, and consequently the wt of the load becomes greater in proportion to that of the arch, and of the filling above it, we must depend more and more upon the resistance of the earth behind the abuts, in order to avoid the necessity of giving the latter an extravagant thickness. *It will therefore be understood throughout that, except when parallel spandrel walls are used, our rules suppose that after the bridge is finished, earth will be deposited behind the abuts, and to the height of the roadway, as usual.*

In small bridges and large culverts of first class railroads, subject to the jarring of heavy trains at high speeds, the comparative cheapness with which an excess of strength can thus be given to important structures, has led, in many cases, to the use of abutments from one-fourth to one-half thicker than by the following rule.
If of rough rubble add 6 ins to insure full thickness in every part.

$$\left.\begin{array}{c}\textit{Thicks } \mathrm{o\,n} \textit{ of abut at spring}\\ \textit{in ft, when the height } \mathrm{o\,s}\\ \textit{does not exceed } 1\tfrac{1}{2} \textit{ times the}\\ \textit{base } \mathrm{s\,p}\end{array}\right\} = \frac{\textit{Rad in ft}}{5} + \frac{\textit{rise in ft}}{10} + 2\,\textit{ft.}$$

Mark the points *n* and *y* thus ascertained. Next, from the center *i*, of the span or chord *e o*, lay off *i h*, equal to $\frac{1}{24}$ part of the span. Join *a h*; and through *n*, and parallel to *a h*, draw the indefinite line *g n p* of the abut. Do the same with the other abut. Make *y m* and *n g* each equal to half the entire height *i t* of the arch; and from *g* draw a straight line *g x*, touching the back of the arch as high up as possible; or still better, as shown at *t m*, with a rad *d t* or *d m*, (to be found by trial,) describe an arc *t m*. Then *g x* or *t m* will be the top of the masonry filling above the arch;* and this should be completed before striking the centers; before which, also, the embkt should be finished, at least up to *y n*.

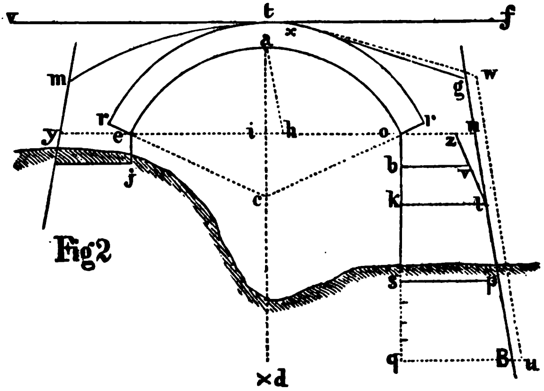


Fig 2

Now find by trial the point *s*, Fig 2, at which the thickness *s p* is equal to two-

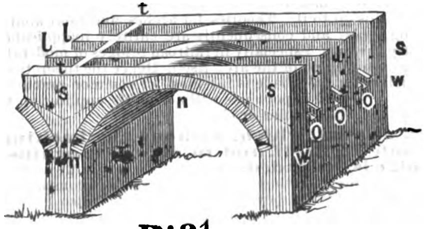


Fig 2½

---

* EXCEPT WHEN THE RISE IS BUT ABOUT $\frac{1}{5}$ OF THE SPAN, OR LESS; in which case carry the masonry up solid to the level *v t f*, of the top of the arch. Or if the arch is a large one, exceeding say about 60 ft span; and especially if its rise is greater than about $\frac{1}{5}$ of its span, it is better to economise masonry by the use of parallel interior spandrel-walls, *l l*, Fig 2½, carried up to *v t f*, Fig 2. Indeed, such interior walls may often be advantageously introduced in much smaller arches. When high, they are steadied by occasional cross-walls, as *t t*, Fig 2½. Their feet should be spread by offsets, as shown at *o o o*, so as to bear upon the whole surf of the back of the arch: thus equalizing the pres upon it. On top of the walls flagstones may be laid, or small arches may be turned from wall to wall, for supporting the ballast, &c, of the roadway. The spaces below are left hollow. In Fig 2½ the dark part *w w* is supposed to be a section across an abutment; but omitting the second cross-wall, similar to *t t*. In R R bridges, put a spandrel-wall, *l l*, under each rail.

thirds of the corresponding vert height os, and draw sp. Then will the *thickness* on or ey be that at the springing line of the given circular or elliptic arch of any rise and span; and the line gp will be the *back* of the abut; provided its height os does not exceed $1\frac{1}{2}$ times sp; or in other words, provided sp is not less than $\frac{2}{3}$ of os. In practice, os will rarely exceed this limit; and only in arches of considerable rise. But if it should, as for instance at oq, then make the base qu equal to sp, added to one-fourth of the *additional* height sq; and draw the back uw, parallel to gp; and extending to the same height, &c, as in Fig 2. If, however, this addition of $\frac{1}{4}$ of sq should in any case give a base qu, less than *one-half* the total height oq, (which will very rarely happen in practice,) then make qu equal to half said total height; drawing the back parallel to gp, and extending it to the same height as before. The additional thicknesses thus found below sp, have reference rather to the pres of the earth behind the abut, than to the thrust of the arch. In a very high abut, the inner line gp would give a thickness too slight to sustain this earth safely.

When the height ob, Fig 2, of the abut is less than the thickness on at spring, a small saving of masonry (not worth attending to, except in large flat arches) may be effected by reducing the thickness of the abut throughout, thus: Make ok equal to on, and draw kl. Make oz equal to $\frac{3}{4}$ of on, and draw lz. Then, for any height ob of abut less than on, draw bv, terminating in lz. This bv will be sufficient base, if the foundations are firm. The back of the abut will be drawn upward from v, parallel to gp, and terminating at the same height as g or w.

Rem. 1. All the abuts thus found will (with the provisions in Art 6) be safe, without any dependence upon the wing-walls; no matter how high the embkt may extend above the top of the arch. If the bridge is narrow, and the inner faces of the wing-walls are consequently brought so near together as to afford material assistance to the abuts, the latter may be made thinner; but to what extent, must depend upon the judgment of the engineer.

We, however, caution the young practitioner to be careful how he adopts dimensions less than these given by our rule. There are certain practical considerations, such as carelessness of workmanship; newness of the mortar; danger of undue strains when removing the centers; liability of derangement during the process of depositing the earth behind the abuts, and over the arch; &c, which must not be overlooked; although it is impossible to reduce them to calculation.

Whenever it can be done, the centers should remain in place until the embkt is finished; and for some time afterward, to allow the mortar to set well. But for more on this see Rem 4. p 633

Rem. 2. A good deal of liberty is sometimes taken in reducing the quantity of masonry above the springing line of arches of considerable rise, and of moderate spans. When care is taken to leave the centers standing until the earth filling is completed above the arch, and behind its abuts, so that it may not be deranged by accident during that operation; and when good cement is used instead of common mortar, such experiments may be tried with comparative safety; especially with culvert arches, in which the depth of arch-stones is great in proportion to the span. They must, however, be left to the judgment of the engineer in charge; as no specific rules can be laid down for them. They can hardly be regarded as legitimate practice, and we cannot recommend them. We have known nearly semicircular arches, of 30 to 40 ft span, to be thus built successfully, with scarcely a particle of masonry above the springs to back them. Such arches, however, are apt to fall, if at any future period the earth filling is removed, without taking the precaution to first build a center or some other support for them. Even when the embkt can be finished before the centers are removed, we cannot recommend (and that only in small spans) to do *less* than to make ng, Fig 2, equal to $\frac{1}{6}$ of the total height tt of the arch; and from g so found, to draw a straight line touching the back of the arch as high up as possible.

Rem. 3. We have said nothing about **battering the faces of the abuts,** because in the crossing of streams, the batter either diminishes the water-way; or requires a greater span of arch. Such a batter, however, to the extent of from $\frac{1}{2}$ to $1\frac{1}{2}$ ins to a ft, is useful, like the offsets, for distributing the wt of the structure, and its embkt, over a greater area of foundation; especially when the last is not naturally very firm; or when the embkt extends to a considerable height above the arch. In our tables, Nos 3 and 5, of approximate quantities of masonry in semicircular bridges of from 2 to 50 ft span, the faces are supposed to be vert.

Art. 5. Abutment-piers. When a bridge consists of several arches, sustained by piers of only the usual thickness, if one arch should by accident of flood, or otherwise, be destroyed, the adjacent ones would overturn the piers; and arch after arch would then fall. To prevent this, it is usual in important bridges to make some of the piers sufficiently thick to resist the pres of the adjacent arches, in case of such an accident; and thus preserve at least a portion of the bridge from ruin. Such are called abutment-piers.

Our formula of $\frac{\text{Rad}}{5} + \frac{\text{Rise}}{10} + 2$ ft, for the thickness at spring; with the back battering as before, at the rate of $\frac{1}{24}$ of the span to the rise; face vert; will of itself (*without any modification for great heights*) give a perfectly safe abut-pier, for any unloaded bridge; and to any height whatever; due regard being had, however, to the consideration alluded to in the next Art. Thus, for an abut-pier as high as oq, Fig 2; or of any greater height; it is only necessary first to find the thickness on at spring as before; and then draw the battered back gnp: extending it down to the base at B; without adding $\frac{1}{4}$ of the additional height sq. This addition is made in the case of abuts, that they may be secure from the pres of the *earth* behind them; as well as from the pres of the arch; a consideration which does not apply to abut-piers; in which only the pres of the arch is to be resisted.

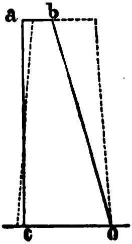


Fig 3

But although the abut-pier thus found by our formula, would be abundantly safe, yet its shape *a b c o*, Fig 3, is inadmissible. In practice it would be changed to one somewhat like that shown by the dotted lines; having an equal degree of batter on both faces. This of course requires more masonry, with but little increase of stability; but that cannot be avoided.

WHEN AN ABUT-PIER IS BUILT IN DEEP WATER, or in a shallow stream subject to high freshets, care must be taken that water cannot find its way *under* the pier, and thus produce an upward pres, which will either diminish, or entirely counteract its efficiency as an abut. See Remark 2, Art 4, of Hydrostatics.

Art. 6. Inclination of the courses of masonry below the springs of an arch. Although our foregoing rule gives a thickness of abut which cannot be *overturned*, or *upset*, by the pres of the arch, yet if the arch be of large span, and small rise, its great hor thrust may produce a *sliding* outward of the masonry near the level of the springs, if the stones are laid in *hor* courses; especially if the mortar has not set well.

This danger, it is true, could be avoided by confining the courses together by iron bolts and cramps; or by increasing considerably the thickness of the abuts; but the expense of doing either of these, leads to the cheaper expedient of inclining the masonry, as shown between *o* and *n*, Fig 4; the courses near *o* being steeper; and gradually becoming less steep near *n*. By this process the arch is virtually prolonged into the body of the abut, so far that when the inclination of the lower masonry ceases, as at *n*, the direction of the theoretical *line of thrust*, or *of pres* of the arch, (rudely represented by the dotted curved line *o n*) is nearly at right angles to the joints of the hor masonry below *n*; and consequently, said thrust is unable to produce sliding at that point. Between *o* and *n*, the line of pres is everywhere so nearly at right angles to the variously inclined joints, as to preclude the possibility of sliding in that interval also.

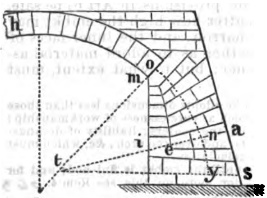


Fig. 4

The abut being thus safe throughout from both overturning and sliding, can fail only from defective foundations; or from the inferiority of the stone of which it is built; and which, if soft, may be crushed.

This inclination of the masonry is as necessary in an elliptic arch, Fig 4½, as in a circular one.

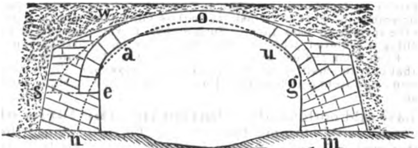


Fig. 4½

The elliptic form is plainly unfavorable for uniting the arch-stones with the inclined masonry near the springs, so as to receive the thrust properly; or about at right angles to its resultant. In ordinary cases this difficulty may be overcome by making the joints of only the outside or showing arch-stones to conform to the elliptic curve; as between *e* and *s*; while the joints of the inner or hidden ones, may have the directions shewn between *g* and *u*, nearly at right angles to the line of thrust. It will rarely happen, however, that the young engineer will have to construct elliptic arches of sufficient magnitude to require either this, or any equivalent expedient. For spans less than 50 ft, with rises not less than about $\frac{1}{6}$ of the span, nothing of the kind is actually necessary, if the mortar is good, and has time to harden.[†]

In order to incline the masonry of any abut with sufficient accuracy, it would be necessary first to trace the curved line of pres of the given arch, so as to arrange the bed joints about at right angles to it at every point of its course; but we offer the following process as sufficing for all ordinary practical purposes; while its simplicity places it within the reach of the common mason. In actual bridges the direction of the actual thrust changes as the load is passing; therefore, in practice no given degree of inclination of the abut masonry can conform to it precisely during the entire passage. Consequently, any excess of refinement in this particular, becomes simply ridiculous; especially in small spans.

Rule for inclining the beds of the masonry in the abuts. Add together the rad *c m*, Fig 4; and the span of the arch. Div the sum by 5. To the quot add 3 ft. Make *o t*, on the rad, equal to the last sum. Then is *t* a central point, toward which to draw the directions of the beds, as in the fig. Draw *t s* hor, and from *t* as a center, describe the arc *o y*; *o* being the center of the depth of the springers. From *y* lay off on the arc the dist *y n*, equal to one-sixth part of *t y*; draw *t n a*. It will never be *necessary* to incline the masonry below this *t n a*. Neither *need* the inclination extend entirely to the face *m i* of the abut; but may stop at *e*, about half-way between *i* and *n*. From *e* upward, the inclination may extend forward to the line *e m*.

Cement should be freely used, not only in the arches themselves, and in the masonry above them, as a protection from rain-soakage; but in abuts, wing-walls, retaining-walls, and all other important masonry exposed to dampness. The entire backs of important brick arches should be covered with a layer of good cement, about an inch thick. The want of it can be seen throughout most of our public works. The common mortar will be found to be decayed, and falling down from the soffits of arches; and from the joints of masonry generally, within from 3 to 6 ft of the surface of the ground. The moisture rises by capillary attraction, to that dist above the surf of the nat soil; or descends to it from the artificial surf of embankments, &c; therefore, cement-mortar should be employed in those portions at least. The mortar in the faces of battered walls, even when the batter is but 1 to 1½ inches per foot, is far more injured by rain and exposure, than in vert ones; and should therefore be of the best quality. See MORTAR, &c.

We have, however, seen a quite free percolation of surface water through brick arches of nearly 3 ft in depth, even when cement was freely used. In aqueduct bridges, we believe that cement has not been found to prevent leaks, whether the arches were of brick, or even of cut-stone. May not this be the effect of cracks produced by settlement of the arch; or by contraction and expansion under atmospheric influence? Cement at any rate prevents the joints from crumbling.

[†] The feet of both elliptic and semicircular arches are always made hor; but it is plain from Fig 4½, that this practice is at variance with correct principles of stability in the case of the ellipse. It is the same in the semicircle. In ordinary bridges of the latter form, the vert pres, or weight resting on each skewback, is (roughly speaking) usually about from 3½ to 4 times the hor pres on the same: and the total pres is about 4 times as great as the pres on the keystone. Therefore, theoretically, the skewback should usually be about 4 times as deep as the keystone; and its bed, instead of being hor, should be inclined at the rate of about 1 vert to 4 hor.

When the arch is flat, this inclination may become so steep, especially in the upper parts, that struts, or shores of some kind, must be used for preventing the masonry from sliding down, until the completion of the arch secures it from doing so. The hor courses between the face $m t$, and the line $o a$, will aid somewhat in this respect.

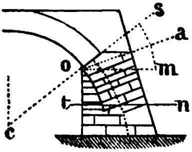


Fig 5

This method should be applied to *all very large arches* whose rise is one-third, or less, of the span. As before remarked, it is not actually necessary in arches not exceeding about 50 ft span, and not flatter than $\frac{1}{5}$ of the span. Indeed, if the earth filling can be deposited before the centers are removed, these limits may be considerably extended without danger. Still, since a certain degree of inclination is attended with very little trouble or expense, we would recommend for even such arches, a process somewhat like the following: From *half* the span take the rise. Div the rem by 3. Make $o t$, Fig 5, equal to the quot. Draw $t n$, and $o m$, hor. Div the angle $s o m$ into two equal parts, by the line $o a$. Incline the masonry so as to be parallel to $o a$, as far down as $t n$. The inclined courses may extend out to the face $o t$, or not, at pleasure.

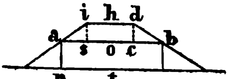


Fig 7

Rem. 1. To find the length (ab, Fig 7) **from face to face of a culvert.** From the height $h t$ of the embkt, take the *above-ground* height $n a$ of the culvert; the rem will be the height $h o$ of the embkt *above* the culvert. Then the reqd length $a b$ is plainly equal to the top width $i d$ of the embkt, added to the two dists $a s$, $c b$, which correspond to its steepness of side-slopes. Thus, if the side-slope is, as usual, 1½ to 1, then $a s$ and $c b$ will each be equal to 1½ times $o h$; or the two together will be 3 times $o h$. So that if the width $i d$ is 14 ft, and $h o$ 5 ft, the length $a b$ will be

$$14 + (5 \times 3) = 14 + 15 = 29 \text{ ft.}$$

Art. 7. The following tables, 3, 4, and 5, of quantities, will be found useful for expediting preliminary estimates; for which purpose chiefly they are intended; hence no pains have been taken to make them scrupulously correct, but rather a little in excess of the truth. The first column of Table 3 contains the total vert height $o c$, Fig 6, from the crown o of a semicircular arch, to the foundation or base $g m$ of its abut. The other columns give approximately the number of cub yds contained in each running foot, or foot in length of the culvert or bridge, measured from end to end (face to face) of the arch proper; and including only the arch and its abuts, as shown in Fig 1; or in the half section $o p m g y$ in Fig 6; including footings to the abuts, but omitting the wing-walls ($w n$), and the spandrel-walls (s), Figs 6 and 2½. At the foot of each column is the approximate content in cub yds of the two spandrel-walls by themselves; one over each face of the arch.

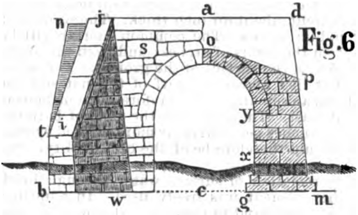


Fig.6

These spandrel-walls are calculated on the supposition that their thickness at base, at their junction with the wing-walls, where their height is greatest, is equal to $\frac{4}{10}$ of their height at that point: except where that proportion gives a less thickness at top than 2½ ft; and that they extend 2 ft ($o e$) above the top o of the arch. At the top of the arch, they are all supposed to be 2½ ft thick at top; that being assumed to be about the least thickness admissible in a rubble wall in such a position. Both the back and the face are supposed to be vert. The contents of these spandrel-walls will vary somewhat, however, even in the same span, with the height of the abut and the arrangement of the wings. They, however, constitute so small a proportion of the entire contents given in Table 5, that this consideration may be neglected in preliminary estimates. They are so firmly bonded into the masonry of the wings at their highest points, and so strongly connected by mortar with the backing of the arch at their bases, that they require no greater thickness however high the emb may be.

The contents of the four wing-walls, of which $n j w b$, Fig 6, is one, will be found in a table (No. 4) immediately following that for the body of the culvert. We have also added a table (No. 5) for complete semicircular culverts of various lengths, including their spandrel and wing walls.

Rem. 1. Although the thickness of wing-walls increases in all parts with their height, they are not made to *show* thicker at *n j* than at *t i*, Fig 6; but (as seen in the fig) are offsetted at their back *t n*, a little below their slanting upper surf *i j*, so as to give a uniform width for the steps or flagstones, as the case may be, with which they are covered. In the fig the covering is supposed to be of flagstones; but steps are preferable, being less liable to derangement. To prevent the flagstones from sliding down the inclined plane *j i*, the lower stone *i* should be deep and large, and laid with a hor bed. The flags are sometimes cramped together with iron, and bolted down to the wall. Steps require nothing of that kind, as seen at *s*, Fig 11.

Rem. 2. **The tables show the inexpediency of too much contracting the width of water-way,** with a view to economy, by adopting a small span of arch, when a culvert of greater span can be made, *of the same total height.*

For the wings must be the same, whether the span be great or small, provided the total height is the same in both cases; and since the wings constitute a large proportion of the entire quantity of masonry, in culverts of ordinary length, the span itself, within moderate limits, has comparatively little effect upon it. Thus, the total masonry in a semicircular culvert of 3 ft span, 8 ft total height, and 60 ft long between the faces of the arch, is, by Table 5, 151¼ cub yds; while that of a 5 ft span, of the same height and length, is 152.4. A semicircular bridge of 25 ft span, 24 ft total height, and 40 ft between the faces of the arch, contains 1031 cub yds; while one of 35 ft span, of the same height and length, contains 1134 yds; so that in this case we may add nearly 50 per cent to the water-way, by increasing the masonry of the bridge but $\frac{1}{10}$th part.

Rem. 3. Partly for the same reason, and partly because **the culverts for a double-track road are not twice as long as those for a single-track one,** the quantity of culvert masonry for the former will not average more than about from ¼ to ⅙ part more than that for the latter; so that it frequently becomes expedient to *finish* the culverts at once to the full length required for a double track, although the embkts may at first be made wide enough for only a single one, with the intention of increasing them at a future time for a double one.

Thus, the average size of culverts for a single track may be roughly taken at 6 ft span, 30 ft long from face to face, and 10 ft total height; and such a one contains, by Table 5, 140 cub yds. For a double track, it would require to be about 12 feet longer; and we see by Table 3 that this will add 2.67 × 12 = 32 cub yds; making a total of 172 yds instead of 140; thus adding rather less than ¼ part. When the culverts are under very high embkts, and consequently much longer, the addition for a double track becomes comparatively quite trifling.

Table 3, of approximate numbers of cub yds of masonry per foot run, contained in the arches and abutments only. as shown in Fig 1 (omitting wings, and the spandrel-walls over the faces of the arches) of semicircular culverts and bridges, of from 2 to 50 ft span, and of different total heights, *h t*, Fig 1, or *o c*, Fig 6. It will be seen that in many cases, a bridge of larger span contains less masonry than one of smaller span, *when their total heights are the same.* There is a liberal allowance for footings or offsets at the bases of the abuts.

TABLE 3. (Original.)

Total Height.	Span 2 ft.	Span 3 ft.	Span 4 ft.	Span 5 ft.	Span 6 ft.	Span 8 ft.	Span 10 ft.	Span 12 ft.	Span 15 ft.
Feet.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.
2	.42	....	....	....	....	....	....	....	....
3	.60	.63	.67	....	....	....	....	....	....
4	.79	.83	.87	.92	.97	....	....	....	....
5	.99	1.04	1.08	1.15	1.21	....	....	....	....
6	1.28	1.28	1.28	1.37	1.46	1.58	1.69	....	....
7	1.62	1.59	1.55	1.64	1.72	1.85	1.97	2.12	....
8	2.01	1.96	1.91	1.95	1.99	2.13	2.26	2.38	....
9	2.45	2.38	2.31	2.29	2.27	2.42	2.56	2.65	3.02
10	2.94	2.85	2.76	2.72	2.67	2.77	2.87	2.98	3.34
11	....	3.38	3.26	3.19	3.12	3.16	3.19	3.25	3.67
12	....	3.96	3.82	3.72	3.62	3.57	3.52	3.55	4.01
13	....	....	4.42	4.29	4.17	4.10	4.02	3.86	4.36
14	....	....	5.08	4.90	4.77	4.67	4.57	4.41	4.72
15	....	....	....	5.57	5.42	5.30	5.17	5.01	5.09
16	....	....	....	6.30	6.12	5.97	5.82	5.56	5.69
17	....	....	....	....	6.87	6.70	6.52	6.26	6.34
18	....	....	....	....	7.69	7.48	7.27	7.01	7.04
19	....	....	....	....	....	8.32	8.07	7.71	7.69
20	....	....	....	....	....	9.20	8.92	8.54	8.49
21	....	....	....	....	....	....	9.82	9.46	9.34
22	....	....	....	....	....	....	10.8	10.3	10.2
23	....	....	....	....	....	....	....	11.3	11.1
24	....	....	....	....	....	....	....	12.3	12.1
25	....	....	....	....	....	....	....	......	13.2
26	....	....	....	....	....	....	....	......	14.2

Contents of the two spandrel-walls, over the two ends of the arch, in cu. yds.

2.9	3.7	4.4	5.2	5.8	7.9	9.8	12.	16.

TABLE 3. (Continued.)

Total Height.	Span 20 ft.	Span 25 ft.	Total Height.	Span 35 ft.	Total Height.	Span 50 ft.
Feet.	Cub. y.	Cub. y.	Feet.	Cub. y.	Feet.	Cub. y.
12	4.60	....	20	10.5	27	18.0
13	4.98	....	21	11.0	28	18.7
14	5.37	6.10	22	11.6	29	19.4
15	5.77	6.41	23	12.2	30	20.1
16	6.18	6.76	24	12.7	31	20.9
17	6.60	7.16	25	13.3	32	21.6
18	7.03	7.61	26	13.8	33	22.4
19	7.47	8.10	27	14.5	34	23.1
20	8 12	8.60	28	15.1	35	23.9
21	8.82	9.02	29	15.7	36	24.7
22	9.57	9.72	30	16.3	37	25.5
23	10.4	10.4	31	17.0	38	26.3
24	11.3	11.2	32	18.1	39	27.1
25	12.2	12.1	33	19.2	40	28.0
26	13.1	13.0	34	20.4	41	28.8
27	14.1	14.0	35	21.7	42	30.0
28	15.2	15.0	36	23.0	43	31.5
29	16.3	16.1	37	24.3	44	33.0
30	17.4	17.2	38	25.7	45	34.6
31	18.6	18.4	39	27.2	46	36.3
32	19.9	19.6	40	28.7	47	38.1
33	21.2	20.9	41	30.2	48	39.8
34	22.6	22.2	42	31.8	49	41.6
35	24.0	23.6	43	33.5	50	43.6
36	25.4	25.0	44	35.2	51	45.5
37	26.9	26.5	45	36.9	52	47.4
38	28 5	28.0	46	38.7	53	49.4
39	30.1	29.5	47	40.6	54	51.6
40	31.7	31.2	48	42.5	55	53.7
41	....	32.8	49	44.4	56	55.9
42	....	34.5	50	46.4	57	58.1
43	....	36.3			58	60.4
44	....	38.1			59	62.7
45	....	40.0			60	65.1
Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.	28.	42.		85.		195.

Art. 8. The **following table** of contents of wing-walls, or wings, will, like the preceding one, be useful in making preliminary estimates. The wings *wo*, *no*, shown in plan at Fig 8, are supposed to form an angle *aoc*, of 120°, with the face, or end *oo* of the culvert. Their outer or small ends *nn*, are all assumed to be of the dimensions shown on a larger scale at E. Thickness at base at every part equal to $\frac{4}{10}$ of the height of the wall at said part; except when that proportion becomes too small to allow the width or thickness at top to be 2.5 ft; in which case it is enlarged at such parts sufficiently for that purpose. See Remark 2. This happens only when the height *m m*, Fig. E, of the wing, becomes less than 9 ft. Batter of face, 1½ ins. to a ft.; or 1 in 8. Back vert.; but offsetted, if necessary, for a short dist. below the top, so as to give a uniform *showing* top thickness of 2½ ft. The masonry is supposed to be good well-scabbled mortar rubble. The height given in the first column is the greatest one; or that at *o o* (or *w j*, Fig. 6), where the wing joins the face of the culvert. In the table no allowance is made for footings (offsets or steps) at the base of the wings; as these are frequently omitted in wings on good founda-

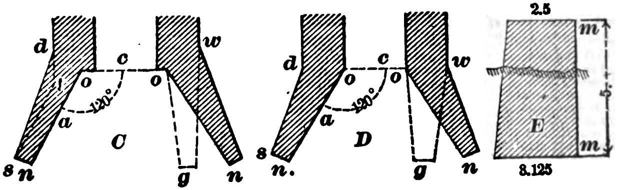


Fig. 8

tions. In taking out quantities from the table, bear in mind that the height of the wings is usually a little greater than that of the culvert itself.

The plan shown at C is the common one, but that at D is greatly preferable for culverts; for the shoulders at *o o* in Fig. C, apart from their greater liability to catch branches of trees, etc., floating down stream, offer of themselves a much greater resistance to the flow of the water into the culvert than do the mere corners at *o o*, Fig. D.

Table 4, of approximate contents, in cub yds, of the four wing-walls of a culvert, or bridge. (Original.)

The heights are taken where greatest; as at *j w*, Fig 6

Height of wing.	Length of one wing.	Cub. yds. in 4 wings.	Height of wing.	Length of one wing.	Cub. yds. in 4 wings.
Feet.	Feet.		Feet.	Feet.	
6	1.73	4.04	30	43.3	818
7	3.46	8.85	32	46.8	997
8	5.20	14.6	34	50.3	1192
9	6.93	21.5	36	53.7	1414
10	8.66	30.2	38	57.2	1661
11	10.4	40.9	40	60.7	1928
12	12.1	53.7	42	64.2	2220
14	15.6	85.2	44	67.6	2552
16	19.1	128	46	71.1	2912
18	22.5	183	48	74.6	3306
20	26.0	247	50	78.0	3741
22	29.5	329	55	86.7	4942
24	32.9	426	60	95.3	6404
26	36.4	541	65	104	8131
28	39.8	672	70	113	10155

To reduce cub yds to perches of 25 cub ft, mult by 1.080.
To reduce perches to cub yds, mult by .926, or div by 1.08.

The contents for heights intermediate of those in the table may be found approximately by simple proportion.

Rem. 1. It is not recommended to actually prolong all wings until their dimensions become as small as shown at E, in Fig 8. In large ones it will generally be more economical to increase their end height *m m*, a few feet. The contents, however, may be readily found by the table in that case also. Thus suppose the height of the wings at one end to be 30 ft, and at the other end 8 ft; we have only to subtract the tabular content for 8 ft high, from that for 30 ft high. Thus, 818 — 14.6 = 803.4 cub yds required content.

Rem. 2. It might be supposed that inasmuch as the wings of arches often have to sustain the pressure from embankments reaching far above their tops, they should, like ordinary retaining-walls, be made much thicker in that case. But the fact that they derive great additional stability from being united at their high ends to the body of the bridge or culvert, renders such increase unnecessary when proportioned by our rule; no matter how far the earth may extend above them; as shown by abundant experience.

Relying upon this aid, we may indeed, when the earth does not extend above the top, reduce the base at *o* to one-third of the ht, as shown at *o t*; and by dotted line *t s*. Experience shows that we may also do the same even when the earth reaches to a great height above the top; provided that the wings, instead of being splayed or flared out, as at *o n*, *o n*, merely form straight prolongations of the abutments of the arch, as shown by the dotted lines at *o g w*. In this case the pressure of the earth against the wings is less than when they are splayed. We have known the thickness at *o* to be reduced in such cases to less than one-third the height, when the wings were 15 ft high, and the height of the embankment *above their tops* 16 feet in one case, and 36 ft in another. In another instance, similar wings 25½ ft high, and with 29 ft of embankment above their top, had their bases at *o* rather less than $\frac{1}{5}$ of the height. In all these cases, the uniform thickness at top was 2.5 feet; backs vertical. We mention them because this particular subject does not seem to be reducible to any practical rule. The last wall appears to us to be too thin; especially if the earth is not deposited in layers; and after allowing the mortar full time to set. The labor, however, required in compacting the earth carefully in layers, may cost more than is thereby saved in the masonry. The young practitioner must bear this in mind when he wishes to economise masonry by such means: and also that the thin wall may bulge, or fail entirely, if the earth backing is deposited while the mortar is imperfectly set.

Table 5. Approximate contents in cubic yards, of complete semicircular culverts and bridges of from 2 to 50 feet span; including the 2 spandrel walls; and the 4 wings; all proportioned by the foregoing directions; and taken from the two preceding tables. The height in the second column, is from the top of the *keystone* to the bottom of the foundation. The wings are calculated as being 2 ft higher than this, including the thickness of the coping. The wings are frequently carried only to the height of the top of the arch; thus saving a good deal of masonry. Table 4, of wings alone, will serve to make the proper deduction in this case.

The several lengths are from end to end, or from face to face, of the *arch proper.* The contents for intermediate lengths may be found exactly; and those for intermediate heights, quite approximately, by simple proportion. In this table, as in No. 3, it will be observed that *when the heights are the same in both cases,* a larger span frequently contains less masonry than a smaller one. A semicircular culvert or bridge contains less masonry than a flatter one, when the total height is the same in both cases; therefore, the first is the most economical as regards cost; but it does not afford as much area of water-way; or width of headway.

(Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.	Cub.Y.
2	5	27	32	42	52	72	92	112	132	152	172	192	212
	6	37	43	56	69	94	120	146	171	197	222	248	274
	7	49	57	73	89	122	154	187	219	251	284	316	349
	8	63	73	93	113	153	193	233	273	313	353	393	433
	10	101	116	145	175	234	291	351	410	469	527	586	645
3	5	28	34	44	54	75	96	117	138	158	179	200	221
	6	38	44	57	70	95	121	146	172	198	223	249	275
	7	49	57	73	89	121	153	184	216	247	280	312	343
	8	63	73	93	112	152	191	230	269	308	348	387	426
	10	101	115	143	172	229	286	343	400	457	514	571	628
	12	149	169	208	248	328	407	487	567	646	726	806	885
4	5	30	35	46	57	78	100	122	143	165	186	208	229
	6	38	45	58	70	96	122	147	173	198	224	250	275
	7	49	57	73	88	119	150	181	212	243	274	305	336
	8	63	73	92	111	149	188	226	264	302	340	379	417
	10	100	114	141	169	224	279	335	390	445	500	555	611
	12	147	166	204	243	319	395	472	548	625	701	777	854
	14	209	234	285	336	437	539	641	742	844	945	1047	1149
5	6	41	47	61	75	102	130	157	184	212	239	267	294
	7	52	60	76	93	125	158	191	224	257	289	322	355
	8	65	75	94	114	153	192	231	270	309	348	387	426
	10	100	114	141	168	223	277	331	386	440	495	549	603
	12	146	165	202	239	314	388	463	537	611	686	760	835
	14	207	231	280	329	427	525	623	721	819	917	1015	1113
6	7	53	62	79	96	131	165	200	234	268	303	337	372
	8	66	76	96	116	156	196	236	276	316	356	396	436
	10	100	113	140	167	220	274	327	380	434	487	541	594
	12	146	164	200	236	308	381	453	526	598	670	743	815
	14	206	219	277	325	420	516	611	706	802	897	993	1088
	16	281	311	373	434	556	679	801	923	1046	1168	1291	1413
8	7	57	67	85	104	141	178	215	252	289	326	363	400
	8	70	81	102	124	166	209	251	294	337	379	422	464
	10	104	118	145	173	228	284	339	395	450	505	561	616
	12	147	135	200	236	308	379	450	522	593	664	736	807
	14	206	230	276	323	416	510	603	696	790	883	977	1070
	16	281	310	370	430	549	669	788	908	1027	1146	1266	1385
	18	367	405	480	554	704	854	1003	1153	1302	1452	1602	1751
10	8	74	85	108	131	176	221	266	311	357	402	447	492
	10	107	121	150	179	236	294	351	408	466	523	581	[illegible]
	12	148	166	201	236	306	377	447	518	588	658	729	799
	14	207	229	275	321	412	504	595	686	778	869	961	1052
	16	280	309	368	426	542	659	775	891	1008	1124	1241	1357
	18	366	402	475	548	693	839	984	1129	1275	1420	1565	1711
12	10	110	125	154	183	242	301	359	418	476	535	594	652
	12	151	168	204	239	310	381	452	523	594	665	736	807
	14	206	228	272	317	405	493	581	669	758	846	934	1022
	16	279	306	362	418	529	640	751	862	974	1085	1196	1307
	18	364	399	469	540	680	820	960	1100	1241	1381	1521	1661
	20	470	512	598	684	855	1026	1197	1368	1540	1711	1882	2053

Table 5—(Continued.) (Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
15	12	162	182	222	262	342	422	502	583	663	743	823	903
	14	215	239	286	336	427	522	616	711	805	899	994	1088
	16	285	318	370	427	541	654	768	882	996	1110	1223	1337
	18	369	404	474	545	686	826	967	1108	1249	1390	1530	1671
	20	473	515	600	685	855	1024	1194	1364	1534	1704	1873	2043
	22	596	646	748	850	1054	1258	1462	1666	1870	2074	2278	2482
20	14	237	264	317	371	478	586	693	801	908	1015	1123	1230
	16	304	335	397	458	582	706	829	953	1076	1200	1324	1447
	18	381	416	486	556	697	838	978	1119	1259	1400	1541	1681
	20	479	520	601	682	844	1007	1169	1332	1494	1656	1819	1981
	22	598	646	741	837	1028	1220	1411	1603	1794	1985	2177	2368
	24	739	795	908	1021	1247	1473	1699	1925	2151	2377	2603	2829
25	16	327	360	428	496	631	766	901	1036	1172	1307	1442	1577
	18	403	441	517	594	746	898	1050	1202	1355	1507	1659	1811
	20	500	543	629	715	887	1059	1231	1403	1575	1747	1919	2091
	22	614	663	760	857	1051	1246	1440	1635	1829	2023	2218	2412
	24	751	807	919	1031	1255	1479	1703	1927	2151	2375	2599	2823
	26	909	974	1104	1234	1494	1754	2014	2274	2534	2794	3054	3314
	28	1085	1160	1310	1460	1760	2060	2360	2660	2960	3260	3560	3860
35	22	685	748	859	975	1207	1439	1671	1903	2135	2367	2599	2831
	24	817	880	1007	1134	1388	1642	1896	2150	2404	2658	2912	3166
	26	969	1033	1181	1309	1585	1861	2137	2413	2689	2965	3241	3517
	28	1130	1205	1356	1507	1809	2111	2413	2715	3017	3319	3621	3923
	30	1327	1408	1571	1734	2060	2386	2712	3038	3364	3690	4016	4342
	32	1549	1639	1820	2001	2363	2725	3087	3449	3811	4173	4535	4897
	35	1946	2054	2271	2488	2922	3356	3790	4224	4658	5092	5526	5960
50	30	1494	1594	1795	1996	2398	2800	3202	3604	4006	4408	4810	5212
	32	1711	1819	2035	2251	2683	3115	3547	3979	4411	4843	5275	5707
	34	1956	2071	2302	2533	2995	3457	3919	4381	4843	5305	5767	6229
	36	2228	2350	2597	2844	3338	3832	4326	4820	5314	5808	6302	6796
	38	2519	2650	2913	3176	3702	4228	4754	5280	5806	6332	6858	7384
	40	2835	2975	3255	3535	4095	4655	5215	5775	6335	6895	7455	8015
	42	3197	3347	3647	3947	4547	5147	5747	6347	6947	7547	8147	8747
	45	3818	3991	4337	4683	5375	6067	6759	7451	8143	8835	9527	10219
	50	5063	5281	5717	6153	7025	7897	8769	9641	10513	11385	12257	13129

Art. 9. Especial pains should be taken to **secure an unyielding foundation for culverts and drains under high embkts;** otherwise the superincumbent weight, especially under the middle of the embkt, may squeeze them into the soil below, if soft or marshy; and thus diminish the area of waterway, or at least cause an ugly settlement at the midlength of the culvert. Also, in soft ground, the embkt may press the side walls closer together, narrowing the channel. This may be prevented by an inverted arch, or a bed of masonry, between the walls. A stratum from 3 to 6 ft thick, of gravel, sand, or stone broken to turnpike size, will generally give a sufficient foundation for culverts in treacherous marshy ground; or quicksand, with but a moderate height of embkt. It should extend a few feet beyond the masonry in every direction, and should be rammed; the sand or gravel being thoroughly wet, if possible, to assist the consolidation. Piling will sometimes be necessary. If the masonry is built upon timber platforms, or a smooth surface of rock, care must be taken to prevent it from sliding, from the pres of the earth behind it. This same pres may even overthrow the piles, if they are not properly secured against it.

Art 10. Drains. Drains of the dimensions in Fig 11, contain 1 perch, of 25 cub ft; or .926 of a cub yd, per ft run.

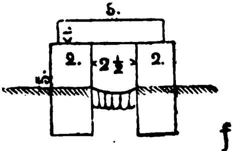
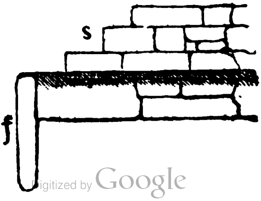


Fig 11



They are frequently built of dry scabbled rubble, and paved with spawls. When there is much wash through them, with a considerable slope, it is better to continue the foundation solid clear across. This is often done without those causes, inasmuch as the additional masonry is a mere trifle; and the excavation of a single broad foundation-pit is less troublesome than that of two narrow ones. A deep flag-stone *f* at the entrance, and others at short dists of the length, may be introduced in both drains and culverts, to protect from undermining.

These drains extend under the entire width of the embkt, from toe to toe; and may terminate in steps, as in the side view at S. They are of course better when built with mortar, with an admixture of cement to prevent the water when full from leaking into and softening the embankment.

Sometimes two or three such drains may be placed parallel to each other, instead of a culvert. When two are so placed, they contain only 1½ times the masonry of one; still their use will generally involve no saving of masonry over a culvert. A man can crawl through Fig 11 to clean it.

Art. 11. The drainage of the roadways of stone bridges of several arches, is generally effected by means of open gutters, which descend slightly from the crowns of the arches, each way, until they reach to near the ends of the respective spans.

There they discharge into vertical iron pipes built into the masonry. The upper ends of the pipes should be covered by gratings. When inconvenience would result from the water falling upon persons passing under the arches, these pipes may be carried down the entire height of the piers; but when such is not the case, they may extend only to the soffit, or under face of the arch; allowing the water to fall freely through the air from that height.

Table 6, of approximate contents, in cub yds, of a solid pier of masonry, 6 ft by 22 ft on top; and battering 1 inch to a ft on each of its 4 faces. The contents of masonry of such forms must be calculated by the prismoidal formula; and not by taking the length and breadth of the pier at half its height as an average length and breadth, as is sometimes done. This incorrect method would give only 6492 cub yds as the content of the pier 200 ft high; instead 7178 yds, its true content. High piers may for economy be built hollow, with or without interior cross-walls for strengthening them, as the case may require; and the batter is generally reduced to ½ inch or less to a foot. Hollow piers require good well-bedded masonry.

(Original.)

Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards	Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards	Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards.
6	23.	7.	32.5	52	30.67	14.67	537	128	43.33	27.33	2759
7	.17	.17	38.6	54	31.	15.	570	130	.67	.67	2848
8	.33	.33	44.9	56	.33	.33	605	132	44.	28.	2940
9	23.5	7.5	51.3	58	.67	.67	641	134	.33	.33	3032
10	.67	.67	58.	60	32.	16.	679	136	.67	.67	3126
11	.83	.83	64.8	62	.33	.33	717	138	45.	29.	3222
12	24.	8.	71.7	64	.67	.67	757	140	.33	.33	3320
13	.17	.17	79.	66	33.	17.	798	142	.67	.67	3420
14	.33	.33	86.4	68	.33	.33	840	144	46.	30.	3521
15	24.5	8.5	94.	70	.67	.67	884	146	.33	.33	3623
16	.67	.67	102	72	34.	18.	928	148	.67	.67	3728
17	.83	.83	110	74	.33	.33	978	150	47.	31.	3836
18	25.	9.	118	76	.67	.67	1021	152	.33	.33	3944
19	.17	.17	127	78	35.	19.	1070	154	.67	.67	4056
20	.33	.33	135	80	.33	.33	1120	156	48.	32.	4168
21	25.5	9.5	144	82	.67	.67	1171	158	.33	.33	4284
22	.67	.67	153	84	36.	20.	1224	160	.67	.67	4402
23	.83	.83	163	86	.33	.33	1278	162	49.	33.	4520
24	26.	10.	172	88	.67	.67	1334	164	.33	.33	4640
25	.17	.17	182	90	37.	21.	1392	166	.67	.67	4763
26	.33	.33	192	92	.33	.33	1451	168	50.	34.	4887
27	26.5	10.5	202	94	.67	.67	1510	170	.33	.33	5014
28	.67	.67	212	96	38.	22.	1569	172	.67	.67	5143
29	.83	.83	223	98	.33	.33	1631	174	51.	35.	5275
30	27.	11.	234	100	.67	.67	1695	176	.33	.33	5409
31	.17	.17	245	102	39.	23.	1761	178	.67	.67	5545
32	.33	.33	256	104	.33	.33	1829	180	52.	36.	5680
33	27.5	11.5	268	106	.67	.67	1899	182	.33	.33	5820
34	.67	.67	280	108	40.	24.	1968	184	.67	.67	5962
35	.83	.83	292	110	.33	.33	2041	186	53.	37.	6106
36	28.	12.	304	112	.67	.67	2115	188	.33	.33	6252
38	.33	.33	329	114	41.	25.	2191	190	.67	.67	6400
40	.67	.67	356	116	.33	.33	2269	192	54.	38.	6552
42	29.	13.	383	118	.67	.67	2346	194	.33	.33	6704
44	.33	.33	411	120	42.	26.	2424	196	.67	.67	6859
46	.67	.67	441	122	.33	.33	2504	198	55.	39.	7016
48	30.	14.	472	124	.67	.67	2587	200	.33	.33	7178
50	.33	.33	504	126	43.	27.	2672	202	.67	.67	7339

Art. 12. Brick Arches. Since even good brick fit for large arches has far less crushing strength than good granite or limestone, and is inferior even to good sandstone, while its weight does not differ very materially from stone, it is plain that it cannot be used in arches of as great span as stone can. Some of those already built, and which have stood for many years, have a theoretical co-efficient of safety of but about 3; whereas the authorities direct us not to trust even stone with more than one-twentieth of its crushing load. This last, however, appears to the writer to be one of those hasty assumptions which, when once admitted into professional books, are difficult to be got rid of. It is his opinion that with good cement, and proper care in striking the centers, one-tenth of the ultimate strength is sufficiently secure against even the abnormal strains caused by the settling at crown, and rising at the haunches when the centers are struck. It is useless to attempt to fix limits of safety for bad materials poorly put together.

Rem. 1. The common practice of building brick arches in a series of **concentric rings,** as at *a c e e*, Fig 12, with no other bond between them than that afforded by the mortar, is censured by authorities, on the ground that the line of pressure in passing from the extrados to the intrados tends to separate the rings, and thus weaken the arch by, as it were, splitting it longitudinally. The reason for using these rings, instead of making the radial joints continuous throughout the depth *m n* of the arch, as at *b*, is to avoid the thick mortar-joints at the back of the arch, and shown in the Fig. If the center of an arch built as at *b* be struck too soon, the soft mortar in these thick joints will be so much compressed as to cause great settlement at the crown, throwing the arch out of shape, and creating such inequality of pressure as might even lead to its fall, especially if flat. As a compromise between rings and continuous joints, they are sometimes employed together, so as to get rid of **some** of the long radial joints; and at the same time to break **at intervals** the continuity of the rings. Thus in Fig 12, which is supposed to be brick-and-a-half deep, beginning at the abutment *a*, we may lay half-brick rings as far as say to *e o e*; then **cutting away the brick** *o* to the line *e e*, we may lay from *e e* to *m n* a block of bricks with continuous radial joints, the same as at *b*; and then start again with three rings; and so on alternately. A still better, but more expensive, mode would be to fill *e e*, *m n* with a regular cut-stone voussoir.

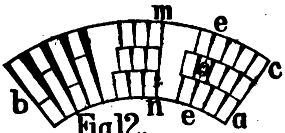


Fig.12.

The proper intervals for changing from rings to blocks will depend upon the number of the rings and the depth *c a* of the arch; reference being also had to reducing the amount of brick cutting as much as possible.

These points can be best decided on from a drawing of a portion of the arch on a scale of 3 or 4 ins to a foot. Generally the rings are made only half-brick, or about 4 to 4.5 ins thick, as at *a c*; and in Brunel's Maidenhead viaduct of two elliptic brick arches of 128 ft span, and 24.25 ft rise; the boldest brick arches yet attempted; but which have been estimated to have a co-efficient of safety of but three against crushing at the crown.

So many others of from 70 to 100 ft span have been successfully built entirely in rings of either half or whole brick thick, as to justify us in attaching but little weight to the above theoretical objection, provided first class cement be used, and time allowed it to become nearly or quite as hard as the bricks themselves, before striking the centers. Under such circumstances we should not object to a series of rings even 1.5 bricks thick, laid alternately header and stretcher, as at *b*.

If the bricks were voussoir-shaped, that is, a little thicker at one end than the other, then rings a whole-brick thick could be used without any increase in thickness of mortar-joint at the back of each ring. Still with more than one ring, the radial joints would not be continuous, as at *b*, but broken as at *ac*. Such bricks however would be more expensive to make; and moreover, in order fully to answer the intended purpose, they would have to be made of many patterns, so as to conform to the many radii used in arches; and even to the radii of the different rings, when the depth of the arch required several of them.

Rem. 2. Wet the bricks before laying.

Rem. 3. When the ends or faces of a brick arch are to be finished with **cut stone voussoirs,** these had better not be inserted until some time after the completion of the brickwork, the hardening of the mortar, and a partial easing of the centers; lest they be cracked or spawled by the unequal settlements of themselves and the bricks.

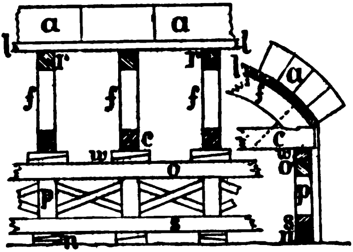
Rem. Brick arches, from their great number of joints are apt to settle much more than cut stone ones when the centers are removed, and thereby to derange the shape of the arch, and at times, without due care, even to endanger its safety, especially if it be large and flat. When the span exceeds about 30 to 35 ft, and particularly if flat, use only brick of superior quality in good **cement** mortar. With even best materials and work we advise the young engineer not to attempt brick arches for railroad bridges of greater spans than about the following. Considerably larger ones than some of them have been built, and have stood; but their coefs of safety are not in all cases satisfactory. In this table the rise is in parts of the span.

R.	S.	R.	S.	R.	S.	R.	S.	R.	S.
.5	100	⅓	88	.225	68	⅙	50	.134	35
.4	97	.29	82	⅕	60	.155	45	⅛	30
.36	93	¼	75	.183	55	⅐	40		

On the **Filbert Street Extension of the Penna R R, in Phila,** are four brick arches of 50 ft 1 inch span, and with the very low rise of 7 ft. They are 2 ft 6 ins thick, except on their showing faces, where they are but 2 ft. The joints are in common mortar, and about ¼ inch thick. These four arches, about 200 yards apart, with a large number of others of 26 ft span, form a viaduct. The piers between the short spans are 4 ft 3 ins thick. Those at the ends of the 50-ft spans, 18 ft 6 ins. The springing lines of all the arches are about 6 to 8 ft above the ground. One of the 50-ft arches settled 3 ins upon prematurely striking the centers; but no further settlement has been observed, although the viaduct has, since built (1880) had a very heavy freight and passenger traffic, at from 10 to 20 miles per hour. Roadbed, about 100 ft wide, giving room for 9 or 10 tracks.

CENTERS FOR ARCHES.

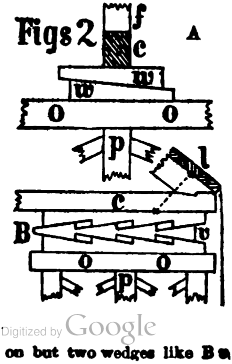
Art. 1. A center is a temporary wooden structure (built lying flat, on a full size drawing, on a fixed platform, under cover or not) for supporting an arch while it is being built. It consists of a number of trusses or **frames,** *f*, *f*, Fig. 1, placed from 1 to 6 ft apart from cen to cen, and covered with a flooring *l*, *l*, of rough boards or planks, usually laid close, and called the **sheeting** or **lagging,** immediately upon which the archstones are laid. In Fig 3, the lagging is not laid close. There is no great economy in placing the frames very far apart, on account of the greater required amount of lagging, the thickness of which increases rapidly. For the thickness of lagging see Rem 9.



Figs 1.

The centers rest by the ends of their chords, *c*, upon wooden **striking wedges** *w*, Fig 1, supported by **standards** composed of posts *p*, whose tops are connected by **cap-pieces** *o;* and whose feet rest on **stringers** *s;* the whole being braced diagonally as shown.

If the ground is very firm, and the arch light, the standards may rest on it, with the interposition of **adjusting-blocks,** *n*, below the stringer, to accommodate irregularities of the surface of the ground, as in the Fig. These blocks should be somewhat double-wedge-shaped, so that by driving them the standard may be raised at any point in case it should settle a little into the ground. But for heavy arches the standards must rest on a much firmer foundation, such as short blocks of brickwork sunk a few feet into the ground, or some other device adapted to the case. Frequently projecting offsets or footings, or at times recesses, are provided in the masonry of the abutments and piers for this express purpose; and with a view to this it is well to design the center at the same time as the arch.



Figs 2

Up to spans of 50 or 60 ft a single row of posts (one under each end of each frame) will suffice; but for much larger ones two or three rows, 2 or more feet apart may become expedient, as in the lower Fig 2.

The striking or lowering-wedges before alluded to are for striking or lowering the center after the completion of the arch. They consist of pairs of wedge-shaped blocks, *w w*, at A, Figs 2, of hard wood, from 1 to 2 ft long, about half as wide, and a quarter or more as thick, (sufficient to lower the center from say 2 to 6 or more inches, according to span and other circumstances,) resting on the cap *o*, of the standard, while the chord *c* of the frame rests on *them*. When the end of a frame is supported by two or more posts *p*, as at B, Fig 2, instead of upon one, the striking-wedges are sometimes made as there shown; and where B *v* is one long wedge at right angles to the abutment, and acting as four wedges which may all be lowered together by blows against the end B.

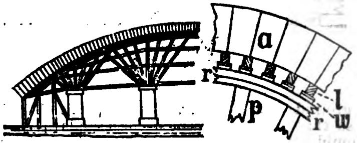
Up to spans of 60 or 80 ft, all the frames may rest on but two wedges like B v

each so long as to reach **transversely** across the entire arch. Then all the frames can be lowered at one operation, as described near end of Art 9.

If we had to consider only the friction of dry wood against dry wood, the taper of these wedges might be as steep as 1 vert to 3 hor, without any danger of their sliding upon each other of their own accord; and they would then require very moderate blows to start them, or even to entirely separate them, when the center had finally to be lowered. But it is of the utmost importance, especially in large arches, that the **centers should be lowered very slowly,** otherwise the momentum acquired by so heavy a body as the arch in descending suddenly even but 2 or 3 ins, might possibly affect its shape, or even its safety.

Therefore the wedges should not have a taper steeper than about 1 in 6 or 8 for arches of less than about 50 ft span; or than 1 in 8 or 10 for larger spans. Vertical lines at equal dists apart should be drawn on the long sides of the wedges as a guide for lowering them all to the same extent at a time; and this should not exceed in all about half an inch a day in intervals of about an eighth of an inch, for 50 ft spans; or about .1 to .25 of an inch per day in all, for spans over 100 ft. Slowness is especially to be recommended in **brick arches,** not only because their greater number of joints exposes them to greater derangement of shape, but because even good brick has much less than the average crushing strength of good granite, limestone, or sandstone, and therefore is far more liable than they to crack, or even to crush (as the writer has seen) when the strains are thrown almost entirely upon their edges, as described in Art 3.

At Gloucester Bridge, England, of first class cut stone, span 150 ft, rise 35 ft, the centers were entirely struck within the very short space of 3 hours; and the crown of the arch descended 10 ins! **At Grosvenor Bridge,** England, of first class cut stone, span 200 ft, rise 42 ft, such care was taken in easing the centers that the crown of the arch settled but 2.5 ins. This case however was marked by two or three peculiarities, all of which contributed to this favorable result. Namely, the center instead of being a series of frames supported as usual by their ends, and of course involving an appreciable, although small, degree of sagging or settlement, consisted essentially of vertical and inclined posts or struts, see Fig 3, footing on four temporary piers of masonry, 7 or 8 feet thick, built in the river, parallel to the abutments, and as long as they. These piers supported six frames (or rather six series) about 7 ft apart cen to cen, of such struts, footing on cast iron shoes. Fig 3 shows half of one series. Each frame or series consisted of four fan-like sets of posts, all in the same vertical plane. The long horizontal pieces seen extending from side to side of the arch were bolted to the struts to increase their stiffness; and other pieces for the same purpose united the six series transversely. Here each strut sustains its own share of the weight of the archstones, and transfers it directly to the unyielding foundation of the pier; whereas in the usual trussed centers, the entire load rests upon the frames, and is finally transferred to the comparatively unstable support of the posts at their ends.



Figs 3.

The tops *p* of the posts of a series varied about from 5 to 8 ft apart cen to cen; and were connected by a continuous curved rib, *r r*, of two thicknesses of 4 inch plank, bent to conform approximately to the curve of the arch. On this rib were placed pairs of striking-wedges *w* like Fig 2, about 16 ins long, 10 to 12 ins wide, and tapering 1.5 ins, so near together (varying about from 2.5 to 3.5 ft cen to cen) that there was a pair under each joint of the archstones, *a a*. On these wedges, and extending over all six of the frames, were the lagging pieces *l*, 4.5 ins thick.

This peculiar arrangement of the striking-wedges and lagging has, in large spans, great advantages over the usual one of placing them only at the ends of the frames. In the last the entire center and the entire arch are lowered together, without giving an opportunity to rectify any slight derangements of shape or inequality of bearing that may have occurred in the arch during its construction, This center, designed by Mr. Trubshaw, admits of lowering either the whole equally, or any one part a little more or less than the others. He had much experience in large arches, and stated that during the striking he found that he had an arch under better control, or could humor it better, by keeping the haunches a little down and the crown a little up, until near the end of the operation.

Rem. 1. Instead of piers of masonry for supporting the feet of the posts, wooden cribs or piles may often be used if the arch is over water.

The principle of supporting even trussed frames by struts at points of the chord as far from the abutments as circumstances will admit of (in addition to those at the very ends) should always be applied when possible, in order to reduce their sagging to a minimum. **Steps or offsets in the masonry** of the abutments and piers may be provided for receiving the feet of such struts, when they are inclined.

Rem. 2. Screws may be used instead of wedges for lowering centers. At the Pont d'Alma, Paris, ellipse of 141.4 ft span, and 28.2 ft rise, the frames were supported by wooden pistons or plungers, the feet of which rested on **sand confined in plate-iron cylinders** 1 ft in diam and height, and having near the bottom of each a plug which could be withdrawn and replaced at pleasure, thus regulating the outflow of the sand and the descent of the center. This device succeeded perfectly, and is well worthy of adoption under arches exceeding about 60 ft span. When much larger than this the driving of the wedges on striking requires heavy blows, and becomes a somewhat awkward operation, requiring at times a battering-ram, even when the wedges are lubricated. In railroad cuttings crossed by bridges, **the earth under the arch** has been made to serve as a center, by dressing its surface to the proper curve, and then embedding in it curved timbers a few feet apart, and extending from abut to abut, for supporting the close plank lagging.

Rem. 3. All centers must yield or settle more or less under the wt of the arch, especially when supported only near their ends; and since the arch itself also settles somewhat not only when the centers are struck, but for some time after, it is advisable to make them at first a little higher than the finished arch is intended to be. This extra height, when the supports are at the ends, may be from 2 to 4 ins per 100 ft of span for cut stone arches (according to time of striking, character of masonry, workmanship, etc.), and about twice as much in brick ones.

Rem. 4. The proper time for striking centers is a disputed point among engineers, some contending that it should be done as soon as the arch is finished and sufficiently backed up; and others that the mortar should first be given time to harden. It is the writer's opinion that inasmuch as in cut-stone arches the mortar joints should be very thin; and since, in such, the mortar is at best of very little service, it is of no importance when they are struck; provided the masonry backing, and the embkt up to *y n* Fig 2, p 618, have been completed; but that in brick or rubble, the numerous joints of both of which require much mortar, (which for hardness should consist largely of cement,) 3 or 4 months, or longer, if possible, should be allowed it to harden sufficiently to prevent undue compression and consequent settlement when the centers are struck. The continuance of the centers need not interfere with traffic over the bridge.

Art. 2. The pressure of archstones against a center is very trifling until after the arch is built up so far on each side that the joints form angles of 25° or 30° with the horizontal. Theoretical discussions on this pressure make no allowance for accidental jarrings in laying the archstones, or by the accumulation of material ready for use, laborers working on it, &c. Without going into any detail, we merely advise on the score of safety not to assume it at less than about the following proportions or ratios to the weight of the entire arch, namely, in a semicircular arch .47; rise .35 span, .61; rise .25 span, .79; rise .2 span, .86; rise .167 span, or less, 1, or equal to the wt of the arch. This gives the pressure of a semicircular arch upon its centers rather less than half its wt. **The wt of the centers themselves** when supported only near the ends must be considered as part of the load borne by them.

Art. 3. We have seen that as an arch *a a a* is being gradually built upward on both sides, after passing the points *e, e*, Fig 4, where its joints form angles *a s e*, of about 30° with the horizontal *a s*, the arch begins to press more and more upon the centers; thereby tending to flatten them at the haunches, as shown at *h* in the dotted line; and consequently to raise them at the crown, as shown at *c*. But as the building goes on still higher, the added stones press much more heavily upon the centers than those below had done, and thereby tend to a final derangement of the centers just the reverse of that caused by the lower ones; namely to depress them at the crown *a*, as at *o*; and consequently to raise the haunches as at *n*; and this the more because the upper stones actually tend to lift or ease the lower ones from the lagging. In some cases where this tendency has been increased by **forcing** the keystones into place by too hard driving, the lagging under the haunches could be drawn out without any trouble before the centers were eased at all. On striking the centers this tendency to sink at crown and rise at haunches is very apt to exhibit itself more or less dangerously in the arch-stones themselves, as in Fig 5, causing those near the crown to press very hard together at the extrados, and to separate from each other at the intrados; while near the haunches the reverse takes place. Hence the angles of the stones are frequently split and spawled off near *c* and *h* by this unequal pressure. These

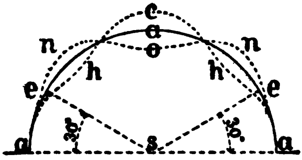


Fig. 4.

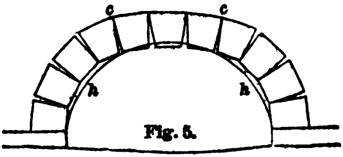


Fig. 5.

derangements are of course much more likely to be serious in high arches than in flat ones, especially if their spandrels are not sufficiently built up before lowering the centers.

In the Grosvenor bridge, before alluded to, of 200 ft span, this dangerous excess of pressure near *c* and *h* was prevented by covering the skewback joint of the springing course at each abutment with a wedge of lead 1.5 ins thick at the intrados of the arch, and running out to nothing at the extrados. Beside this a strip 9 ins wide of sheet lead was laid along the intrados edge of every joint until reaching that point at which it was judged that the line of pressure would pass from the intrados to the extrados; after which similar strips were laid along the extrados edges of the joints, up to the crown. Hence when the centers were struck, this excess of pressure merely compressed the lead, and was thus enabled to distribute itself more evenly over the entire depth of the joints. See Trans Ins Civ Eng London, vol i. See also top of p 921.

At the bridge at Neuilly, France (of 5 elliptic arches of 120 ft span, and 30 ft rise), the centers were so radically defective in design that the arches sank 13.25 ins at crown during the time of building; and 10.5 ins more during and immediately after the striking; or say 2 ft in all. Their construction made the striking very tedious and hazardous; greatly endangering the lives of the workmen and the existence of the arches. Some of the joints at the extrados at the haunches opened an inch each; and those at the intrados of the crown .25 of an inch. By the exercise of great care and humoring in lowering the centers, these openings were much reduced.

Rem. 1. Chamfering the edges of the archstones diminishes the danger of their spawling off from unequal pressure; as does also the **scraping out of the mortar of the joints** for an inch or two in depth before striking the centers.

Rem. 2. It is evident that in order to prevent, or at least to diminish the alternate derangements of the center, those of its web members which at first acted as **struts** near the haunches, Fig. 4, to prevent them from sinking as at *h*, must afterwards act as **ties** to prevent them from rising as at *n*; while those which at first acted as ties near the crown *a*, to prevent it from rising as at *c*, must afterwards act as struts to prevent it from sinking as at *o*. In other words, the principle of **counterbracing** must be attended to as well in a frame or truss for a center, as in one for a bridge.

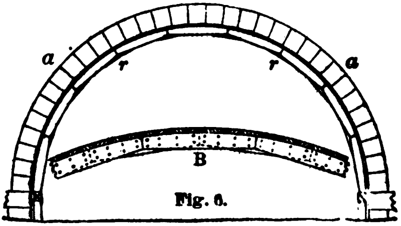


Fig. 6.

Art. 4. From the foregoing it is plain that **a simple unbraced wooden**

arch, or curved rib is, on account of its great flexibility, about as unfit a form as could be chosen for a center, except for very small spans, where a great proportional depth of rib can be readily secured. Still the writer has seen it used for a cut-stone semicircular arch of 25 ft span, with archstones 2 ft deep. Fig 6 shows one rib *r r*, and the arch, *a a*, drawn to a scale. Each rib consisted of two thicknesses of 2 inch plank in lengths of about 6.5 ft, treenailed together so as to break joint, as at B. Each piece of plank was 12 ins deep at middle, and 8 ins at each end; the top edge being cut to suit the curve of the arch. The treenails were 1.25 ins in diam; and 12 of them showed to each length. These ribs were placed 17 ins apart from cen to cen, and steadied together by a bridging piece of inch board, 13 ins long, at each joint of the planks, or about 3.25 ft apart. Headway for traffic being necessary under the arch, there were no chords to unite the opposite feet of the ribs. The ribs were covered with close board lagging, which also assisted in steadying them together transversely. As the arch approached about two-thirds of its height on each side, the ribs began to sink at the haunches, as at *h*, Fig 4; and to rise at the crown, as at *c*. This was rectified by loading the crown with stone to be used in completing the arch; which was then finished without further trouble.

A still more striking example of the use of a simple unbraced wooden rib, was in the old National Turnpike bridge over Wills Creek, at Cumberland, Md. This bridge, of which one arch with its center is shown in Fig 7 drawn to a scale, consisted of two elliptic cut stone arches 26.5 ft wide across roadway, and of 60 ft span, and 15 ft rise. The archstones were 3 ft deep at crown, and 4 ft deep at skewbacks. **Each frame of the center** was a simple rib 6 ins thick, composed of three thicknesses of 2 inch oak plank in different lengths (about 7 to 15 ft) to suit the curve, and at the same time to preserve a width of about 16 ins at the middle of each length, and 12 ins at each of its ends. The thicknesses were well treenailed together, breaking joint and showing from 10 to 16 treenails to a length.

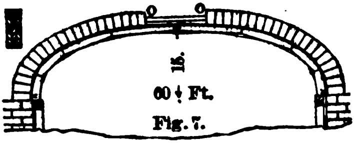


Fig. 7.

Here, as in Fig 6, there were no chords, owing to the violence of the floods in the creek. These ribs were placed 18 ins from cen to cen, and steadied against one another by a board bridging-piece 1 ft long, at every 5 ft. These were of course assisted by the lagging.

When the archstones had approached to within about 12 ft of each other near the middle of the span, the sinking at the crown, and the rising at the haunches had become so alarming that pieces of 12 × 12 oak, 00, were hastily inserted at intervals, and well wedged against the archstones at their ends. The arch was then finished in sections between these timbers, which were removed one by one as this was done.

Rem. 1. Such instances of partial failure are very instructive. It is indeed by such, rather than by theoretical deductions, that the proper dimensions are arrived at in a vast number of cases pertaining to engineering, machinery, &c.* Thus we might with entire confidence of no serious mishap, apply ribs of the foregoing dimensions to spans only half as great.

Rem. 2. Assuming the rib-planks to be 12 ins wide, it would, as a matter of detail, be better to make them about 10 ins wide at the ends instead of the 8 ins in Fig 6 making top curve 2 ins. To secure this, their lengths, depending on the **radius** of the rib, must not exceed those in **the following table:**

Rad of Arch.	Greatest Length.	Rad of Arch.	Greatest Length.
Feet.	Feet and Ins.	Feet.	Feet and Ins.
5	2 " 5	30	6 " 4
10	3 " 4	35	7 " 0
15	4 " 2	40	7 " 6
20	5 " 0	45	7 " 10
25	5 " 9	50	8 " 2

---

* The young engineer should make and preserve full notes in detail of all such as may fall within his notice.

If cut 1½ times as long as this table, they will be very approximately 8 ins wide at ends; or each will on top curve 4 ins.

Art. 5. In cases where all possible headway is essential during the building of the arch, as in the two foregoing ones, the writer would suggest the expedient rudely illustrated by Fig 8; namely to **place the centers above the arch**, instead of **below** it; and after the arch is completed in sections, *a a*, instead of **lowering** the centers, **to take them apart.**

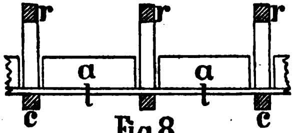


Fig 8.

Fig 8 is a transverse section through part of the center, and of the arch *a a*. Here *r c*, *r c*, *r c*, are frames of the center say 5 or 6 ft apart; and of any depth and construction whatever that may be necessary to insure absolute safety; and *l l* is the lagging. Having built the arch from abutment to abutment in a series of sections *a*, *a*, *a*, necessarily separated say a foot or more by the deep frames, we may take the centers apart, and then fill in the narrow intermediate sections upon a lagging suspended by iron rods from the already completed sections. Good concrete might be used for these narrow sections. In some cases it might be well to use **deep plate-iron ribs** of I section, resting the lagging on the lower flange. Part of the web might be left remaining embedded in the masonry; and the upper part and both flanges removed after the arch is finished.

Art. 6. Centers with hor chords *c c* Fig 9 are objectionable (notwithstanding their strength) in large spans of great rise, as on right side of the Fig, on account of the excessive length required for the web members; and hence it will in such cases usually be found expedient to adopt something analogous to what is shown on the left hand of the Fig. Here a truss *f*, shorter and shallower than that on the right hand, is substituted for the latter. At its ends provision must be made for supporting not only itself, but the archstones below it. As the pressure of these lower archstones is comparatively small, this may usually be effected by resting the end of the frame *f* upon another and shallower frame *o a*. This may in large spans be aided by either inclined or vertical struts, either single or braced together; or as the trestles on p 755. Sometimes one shallow truss like *f* is sustained upon another truss throughout its entire length. The striking-wedges for these various supports may be placed at either their tops or their feet, as may be most convenient.

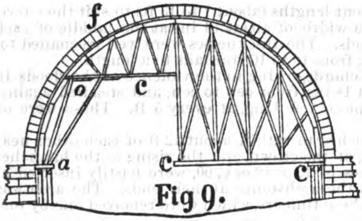


Fig 9.

Art. 7. For flat arches of 10 feet clear span, a mere board *o o* Fig 10, 12 ins deep, by 1.5 ins thick, with another piece *c* of the same thickness on top of it, trimmed to the curve, and confined to *o o* by nailing on two cleats of narrow board, will answer every purpose, with intervals of 18 ins from cen to cen. If the upper piece also is as much as 12 ins deep at its center, the clear span may be extended to 15 ft.

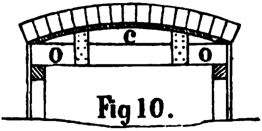


Fig 10.

For spans of 10 to 15 ft, and of any rise, two thicknesses of plank from 1 to 2 ins thick according to span; 8 to 12 ins wide at middle of each piece, in lengths as per table, Rem 2, Art 4, well **nailed** or **spiked** together, according to span, breaking joint as in Fig 6, will answer for distances of 2 to 3 ft apart cen to cen. For greater dists apart increase the thickness of the planks proportionally.

If the centers have to be moved from place to place, to serve for other arches, then, to preserve them from injury in handling, their feet should be united by nailing on one or both sides of each frame a chord piece of about 1 inch board; and also a vertical piece or pieces of the same size from the center of the chord to the top of the frame.

Even when they are not to be moved, the chord pieces are useful even in so small spans, inasmuch as they render the striking easier, by not allowing the feet of the ribs to give trouble by spreading outward and pressing against the abutments.

For spans of 15 to 30 ft, and for any rise not less than one sixth of the span, the following dimensions, varying with the span, may be used for distances apart of 3 ft from cen to cen. See Fig 11. **For the bow** *b*, two thicknesses of 1 to 2 inch plank from 9 to 12 ins wide at the middle; and from 7 to 10 ins at each end, well spiked together breaking joint as at B, Fig 6. **For the chord** *c*, two thicknesses of plank of same size as the bow at its middle; placed on outsides of bow, and well spiked to its ends. **A vertical** *v*, in one piece as wide as a bow plank, and twice as thick. Its top is placed under the bow, and is confined to it by two pieces, *o, o*, of bow plank twice as long as the bow plank is deep, and spiked to both *v* and the bow. The foot of *v* passes between the two thicknesses of the chord *c*, and is spiked to them. **Two oblique tie-struts,** *s*, each of two pieces of bow plank, outside of the bow and vertical *v*; footing against each other; and spiked to bow and *v*. These with *v* divide the bow into 4 parts.

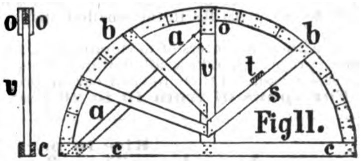


Fig11.

Rem. 1. The above dimensions are suitable to a rise of one sixth. If the rise is one fourth, the **thickness only** of the planks may be reduced one third part; and for a rise of one third or more, we may reduce to one half.

Rem. 2. If in the larger of these spans the struts *s* should show any inclination to bend sideways, nail on some pieces *t* from frame to frame. Also in the larger ones with rises exceeding one third, insert four double struts *s*, instead of two; thus dividing the bow into 6 parts, as at left side of Fig. 11. For spans of 25 to 35 ft, add also two struts like *a a*, of same size as *v*.

Art. 8. For spans greater than about 30 ft, the writer believes that as a general rule (liable to modifications according to the judgment of the engineer in charge) the following ideas will lead to safe practice. Namely, to adopt a bowstring truss with a simple Warren or triangular web, as at *f* on the left side of Fig 9. The bow to rest on the chord, and each to be of a single thickness. The web members (especially in large spans) to be also of single thickness, and placed below the bow, resting on the chords, and well strapped to both, so as to act as either ties or struts. In smaller spans the web members may each be in two thicknesses, one bolted or treenailed to each side of the bow and chord. Other modes will suggest themselves; but we have not space for such details.

Or a web of the Howe, or of the Pratt system, as on the right side of Fig 9 may be used. But in reference to both of these it may be remarked that **the use of long iron rods in centers** of large spans is highly objectionable, owing to the different rates of expansion between iron and wood. Therefore if these systems are used, all the members should be of wood. **The lattice** may be used.

Even when the rise of the **arch** exceeds .25 of the span, it is better not to let that of the **centers exceed** that limit; but adopt the expedient shown at the left side of Fig 9, with a rise of about one sixth of the span.

Rem. 1. To fix on the number of web triangles in a Warren truss or frame for a center, find the square root of the span, and to it add one tenth of the span. Divide their sum by 2, and call the quotient *n*. Divide the span by *n*. If this quotient is a whole number use it; or if the quotient is partly decimal, use the whole number nearest to it, as a distance in feet to be stepped off along the chord; thus dividing the chord into a number of equal parts. All the points thus found on the chord, are the places for the **feet** of the triangles. Next, from half way between each two of these points, draw vertical lines to the bow. The points thus found along the bow, are the places of the **tops** of the triangles. This rule will be used in connection with the following Table of Areas of Bows, as the two are dependent on each other.

In large arches **the timber of the bow should not be wasted** by trimming its upper edges to the curve of the arch, but should be left straight; and separate pieces so trimmed, like *c* in Fig. 10, should be spiked on top of them.

The transverse area of the bow, in square inches, may be taken from the following table; and may in practice be assumed to be uniform throughout its entire length; which in fact it is quite approximately. See Rem 2.

TABLE FOR BOWSTRING CENTERS.

Table of areas in square inches at the crown of each Bow, of **properly trussed** Bowstring frames for centers of stone or brick arches. The frames to be placed 5 feet apart from cen to cen. With these areas, the combined weights of arch, center (of oak), and lagging, will in no case in the table strain the Bow at crown of the greatest spans quite 1000 lbs per square inch; diminishing gradually to 600 or 700 lbs in the smallest spans, which are more liable to casualties.

Although centers of moderate span are usually made of white or yellow pine, spruce, or hemlock, all of which are considerably lighter than oak, we have for safety assumed them to be of oak, in preparing our table.

For spans of from 10 to 20 feet use the same sizes as for 20 feet.

Span in feet.	Rise in parts of the Span.							Original.
	.5	.4	.35	.3	.25	.2	.15	.1
	Areas of transverse section of Bow, in square inches.							
20	14	17	19	21	24	29	38	59
25	18	22	25	28	33	40	53	80
30	23	28	32	37	43	51	71	103
35	28	34	40	45	54	64	87	125
40	34	41	48	55	65	77	106	150
45	40	49	57	65	76	92	126	175
50	47	57	66	76	89	107	146	203
55	53	64	75	87	102	121	166	233
60	60	73	85	99	115	135	187	263
65	68	81	95	110	129	151	209	294
70	75	90	105	122	143	168	233	325
75	83	99	115	133	157	184	256	357
80	91	108	125	145	171	201	279	390
85	99	117	136	157	185	218	302	423
90	108	127	147	169	199	235	325	457
95	115	136	158	181	214	252	348	490
100	123	146	169	194	229	270	372	524
110	133	166	191	219	260	307	420	592
120	155	187	213	246	291	345	470	660
130	172	208	237	274	323	384	520	
140	190	230	263	303	357	424	572	
150	209	252	289	333	393	466		
160	229	276	315	365	430	509		
170	250	299	343	399	469			
180	272	323	373	435	511			
190	294	347	403	472				
200	318	372	435	509				

Rem. 2. The square root of any of these areas gives in inches the **side of a square bow** of that area. The distances apart of the triangles which form the web of the frame, having first been found by Rem 1 (for said Rem and this table are dependent on each other), the above areas for bows 5 ft apart from cen to cen, suffice not only to resist the pressure along the bow, but also, as **square beams,** to sustain with a safety in no case less than about 5, the load of arch-stones resting upon them between the adjacent tops of two triangles; and with very trifling deflections. It is therefore unnecessary to deepen the ribs for that purpose; although it may be done (preserving the same area) in case considerations of detail should render it desirable.

As before suggested, it will generally be best, in spans exceeding 30 or 40 ft, to give the bow a rise not exceeding about one fifth or one sixth of the span; and to support the frames as at *f*, Fig 9.

The size of the chord may be the same as that of the bow; and like it uniform from end to end; care however being taken that it be not materially weakened by footing the bow upon its ends; or (when too long for single timbers) by the splicing necessary to prevent its being stretched or pulled apart by the thrust of the bow. When, however, the chord can be placed at, or a little below the springs of the arch, all danger of this kind may be avoided by simply wedging its ends well against the faces of the abutments.

As to the size of the web members, when a bowstring truss is **fully loaded** on top of the bow, (as is approximately the case with a center and its archstones,) the strains on the web members are quite insignificant, and arise chiefly from the weight of the center itself; **but while it is being so loaded,** they are not only greater, but are constantly changing, not only in amount, but also in character—being at one period compressive, and at another tensile.

Hence it would be very tedious to calculate the dimensions of the web members. Fortunately the necessity for doing so is in a great measure obviated by the fact that a center being but a temporary structure, the timber composing it is not ultimately wasted if a greater quantity of it is used than is absolutely required. Moreover facility of workmanship is secured by not having to employ timbers of many different sizes.

Hence the writer will venture to suggest, entirely as a rule of thumb, to **give each web member half the transverse area of the bow,** taking care to make each of them a **tie-strut.**

Rem. 3. As to details of joints, we refer to the Figs on pages 735, 736; merely suggesting here the use of long and wide iron shoes where timbers are subjected to great pressure sideways.

Rem. 4. To prevent the thrust of the bow when its rise is small, from splitting off the ends of the chords, the two may be united by many more bolts than are employed in roof trusses, &c, where only one is generally placed near each end of the chord. But they may when required be inserted at intervals extending to many feet from the ends. They should have strong large washers; and may have about the same inclination as the shortest web member.

Another way of securing the same end in smaller spans, is by completely encasing the two sides of the bow and chord, to a distance of a few feet from their ends, in short pieces of board or plank spiked to both of them, and having about the same inclination as just suggested for bolts.

Rem. 5. Build up both sides of the arch at once, in order to strain the centers as little as possible.

Rem. 6. When a bridge consists of more than one arch, and they are to be built one at a time, there must be at least two centers; for a center must not be struck until the contiguous arches on both sides are finished, for fear of overturning the outer unsupported pier. Therefore if there are but two arches, they must be built at once, requiring two centers.

Rem. 7. Always use supports either vertical or inclined (and provided with striking-wedges) under the frames, and intermediate of the end supports, when possible; even if they can extend out but a few feet from the abutments, as at the left side of Fig 9.

Rem. 8. The weight of large centers and their lagging is greater for flat arches than for high ones of the same span; and also approaches nearer to that of the supported arch.

Rem. 9. Thickness of lagging. The following table gives thicknesses which will not bend more than an eighth of an inch under the weight of any probable archstones **adapted to the respective spans;** and generally not so much.

TABLE OF LAGGING.—Original.

Distance apart of frames, in the clear.	Span of center in feet.					
	10.	20.	50.	100.	150.	200.
	Thickness of close lagging not to bend more than ⅛ inch.					
Feet.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
6	3¼	3⅝	4¼	4¾	5	5¼
5	2½	2⅞	3⅝	3⅝	3⅝	4
4	1⅞	2⅛	2½	2¾	2⅞	3
3	1¼	1⅜	1¾	1⅞	2	2
2	¾	⅞	1	1⅛	1⅛	1¼

With thicknesses three quarters as great as these, the bending may reach a full quarter inch; which may be allowed in dists apart of 3 or more ft.

Rem. 10. Centers are framed, or put together, (like iron bridges) on a firm, level temporary floor or platform, on which a full-size drawing of a frame is

first made. As each frame is finished, it is removed to its place on the piers or abuts.

Art. 9. The Wissahickon Bridge of the Reading R R, at Philadelphia, has five arches of 65 ft span, 23 ft rise, 28 ft wide (archstones 3 ft deep, with dressed beds and joints, in cement mortar); with four cutstone piers 9.5 ft thick at top, and from 35 to 50 ft high. It contains about 15400 cub yds of masonry.* **Each center** consisted of 7 frames or trusses of hemlock timber, of the Bowstring pattern, with lattice web-members; and as nearly as may be, of the same span and rise as the arches. They were placed 4.5 ft apart from center to center; and were supported near each end *f*, Fig 13 (a transverse section to scale) by a hemlock post *p*, 12 ins square. **The bow** was of two thicknesses *b b* of hemlock plank, 6 ins apart clear, in lengths of 6 ft, with their upper edges cut to suit the curve of the arch. Each piece was 4 ins thick, by 13.5 ins deep at its middle, and 12 ins at its ends. These pieces did not break joint; but at each joint were four ¾ inch bolts, with nuts and washers, uniting them with chocks or filling-in pieces. The bow, *b b*, footed on top of the ends of the chords *f;* and the angle formed by their meeting (seen only in a side view) was (for about 2.5 ft horizontal and 5.5 ft vertical) filled up solid with vertical pieces, to afford a firmer base for resting the frame on *n;* beyond which it extends (in a side view) about 18 ins.

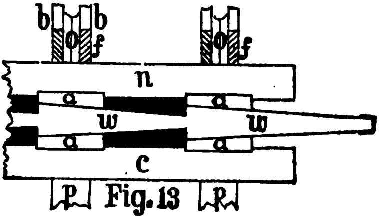


Fig. 13

The chords *f* were of two thicknesses of 4 × 12 hemlock plank, 6 ins apart clear, and most of them in two or three lengths; breaking joint, and with two ¾ inch bolts, with nuts and washers, at each joint, for bolting them together, and to filling-in pieces. **The web members** of each frame were 26 lattices, *o*, of 3 × 12 inch hemlock, crossing each other about at right angles, at intervals of about 3.5 ft from center to center, and passing between the two thicknesses *b b* of the bow, and *f f* of the chords. A few of the lattices were in two lengths, and the joints were not at the crossings. The lattices were connected at each crossing by two hard wood treenails 9 ins long, and 2 ins diam; and one such, 18 ins long, passed through the intersection of each end of a lattice with a bow or chord. The first lattice foots about 4 ft from the end of a chord. They do not extend above the top of the bow. All the spaces between the two thicknesses of bow or chord, where not occupied by the ends of lattices, were completely filled by chocks, well spiked.

Each frame contained about 360 cub ft of timber; and weighed about 5 tons. They were very flexible laterally until in place, and braced together by 4 transverse horizontal planks spiked to their chords; and by 5 others above them, spiked to the lattices.

Until the keystones were placed, all the joints of the frames continued tight, under the pressure from the arch, and from the unfinished backing to the height of about 14 ft above the springing line; but after the keystones were set, all the joints of the chords alone opened from .25 to .75 of an inch; and at the same time the lagging under the *haunches* of the arches became slightly separated from the soffit of the masonry.

Each center sank but a full inch at the middle, under the pressure from the arch and 14 ft of backing.

The portion of the bridge above the piers was about two thirds completed before the centers were struck.

There was one wedge *w*, *w*, (32.5 ft long, of 12 × 12 inch *oak*) under each end of a center. It was trimmed to form 7 smaller ones *w*, *w*, each 4.5 ft long, and tapering 7 ins; one under each end of each frame *f*. They played between tapered blocks *a*, *a*, of *oak*, 2 ft long, 1 ft wide, let 1 inch into the cap *c*, or into the piece *n*, on which last the frames *f*, *f*, rested. The sliding surfaces were well lubricated with tallow when put in place.

The wedges were struck with ease, at one end of a center at a time, by an oak log battering-ram 18 ft long, and nearly a ft in diam, suspended by ropes, and swung and guided by 4 men. They generally yielded and moved several inches at the second blow with a 3 or 4 ft swing. Although each wedge was loosened *entirely* within 2 or 3 minutes, thus lowering the centers *very suddenly*, yet on account of the good character of the masonry, not the slightest crack of a mortar joint could afterwards be detected in any part of the work. After three days the average sinking of the keystones was only .35 of an inch; the least was $\frac{1}{4}$; and the greatest $\frac{5}{8}$ of an inch. The heads and feet of the posts *p* compressed the hemlock caps *c*, and the sills, about $\frac{3}{8}$ of an inch each, showing that for arches of this size the caps and sills had better be of some harder wood, as yellow pine or oak; although probably the compression was facilitated by the large mortices, 3 by 12 ins, and 6 ins deep.

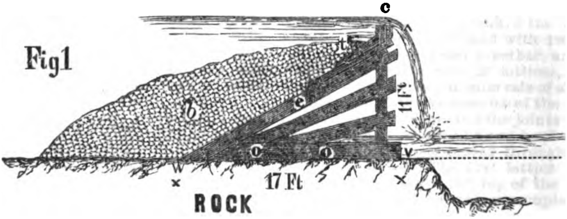
---

* This bridge, finished without accident, in 1882, reflects much credit on the late William Lorenz, Esq, Ch. Eng; on Mr. Charles W. Buchholz, Assistant in Charge: and on the skilful and energetic contractors, William & James Nolan, of Reading, Penna. These last most cordially assisted the writer in making observations during the entire progress of the work.

TIMBER DAMS.

Primary requisites, in the erection of dams, are, a foundation sufficiently firm to prevent them from settling, and thus leaking; the prevention of leaks through their backs, or under their bases; and the prevention of wear of the bottom of the stream in front of the dam, by the action of the falling water. For the first purpose, hard level rock bottom is of course the best; and should be chosen, if possible. In that case, thick planks, *tt*, Fig 6, (single or double, as the case may be,) closely jointed, and reaching from the crest, *c*, to the back lower edge *w*, (where they should be scribed down to the rock;) with a good backing, *b*, of gravel, will suffice to prevent leaks. Gravel, or rather very gravelly soil, is far better than earth for this purpose; for if the water should chance to form a void in it, the gravel falls and stops it. To prevent this backing from being disturbed near the crest of the dam, by floating bodies swept along by freshets, a rough pavement of stones, about 15 to 18 inches deep, as shown in Fig 7, should be added for a width of about 10 to 20 feet; or until its top becomes 3 to 5 feet below the crest *c* of the dam, according to circumstances.

Fig1

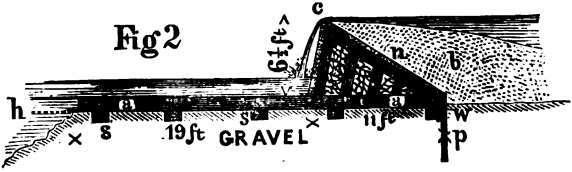


In Fig 1, (a dam on the Schuylkill navigation,) the upper timbers, *e*, are all close jointed, and laid touching, so as not to require planking in addition.

But if the bottom of the stream is gravel or earth, there must in addition to these be used two thicknesses of sheet piles, *p*, Fig 2, &c, close driven, breaking joint, to a depth of several feet, to prevent leaking through the soil beneath the base of the dam. Frequently but one thickness is used. If the bottom is soft or open for a depth of only a few feet, it is at times better to remove it, and base the dam on the firmer stratum below; still, however, using the sheet piles. Old decayed timber and other rubbish should be removed from the base. In very bad soils of greater depth, it may be necessary to support the dam entirely upon a platform resting on bearing piles. Here great precautions are necessary against leaks; but the case occurs so rarely, that we shall not stop to consider it.

As to the wearing away of the bottom of the stream by the water falling over the front of the dam, precautions should be used in all cases except that of very hard rock, or of medium rock protected by a considerable depth of water. The dam, Fig 1, was built upon a tolerably firm micaceous gneiss in nearly vertical strata, covered by about 2 feet of water in ordinary stages. In 89 years the rock was

Fig2



worn away in front of the dam, as shown in the fig, to the average depth of 3 feet; or very nearly 1 inch per year. The depth of water on the crest *c*, was usually from 6 to 18 ins; rarely 5 or 6 ft during freshets; and but a few times during the whole period, 8 or 9 ft.

At Jones's dam, on Cape Fear River; height of dam, 16 ft; front vert; fall, usually 10 ft, into 6 ft depth of water; the soft shale rock, in vert strata, was, in the course of a few years, worn away 16 ft; and the dam was undermined to such an extent as to fall into the cavity.

In another case, dam 36 ft high; front vert; the water falling upon nearly vert strata of hard shale rock, usually covered by but about 2 ft of water; in about 20 years wore it to an irregular depth of from 10 to 20 ft; and extending from the very face of the dam, to 70 or 80 ft in front of it.

In Fig 2, upon a stream subject to very violent freshets, the gravel was washed away for a considerable width and depth beyond the apron, as at A. To prevent a repetition, the cavity was filled with cribwork full of stone, clear across the river.

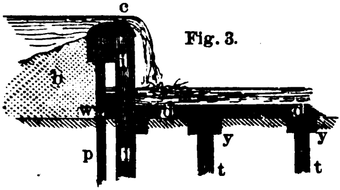


Fig. 3.

A deposit of blocks of loose stone, of even a ton weight or more, will not serve as a protection in front of a dam exposed to high freshets; but will soon be swept away. A common precaution against this wear, in low dams, is an *apron*, *a a*, Fig 2; or *d d*, Fig 3; of either rough round tree trunks, or of hewn timber, laid close together; extending under the entire base of the dam, and from 15 to 30 ft in front of its face. These are sometimes bolted to pieces, *s s*, Fig 2; or *y y*, Fig 3; laid under them across the stream. In Fig 3, with very soft bottom, these pieces *y y* are supposed to be bolted to short piles *t t*, driven for that purpose.

At times a distinct wide low timber crib, filled with stone, and covered on top with stout plank, has been placed in front of the dam, to receive the fall of the water; and is effective in protecting the bottom. Also, in some cases, a dam of less height, and of cheap character, has been built at a short distance down stream from the main one, in order to secure at all times a deep pool in front of the latter for breaking the force.

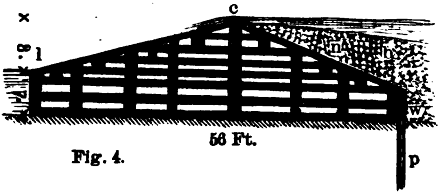


Fig. 4.

Another precaution is to substitute a sloping front like *c l*, Fig 4, or such as Figs 1 and 2 would form if reversed, for the nearly vert one of the other figs; thus to some extent reducing the force of the water. This, however, is but a partial remedy, especially for soft bottoms in shallow water; for the sliding sheet still descends with great force. The best form of dam, perhaps, in such cases, is that shown in Fig 5, in which the front consists of a series of steps of about 1 vert, to 3 or 4 hor. These effectually break the force of the water; and, with the addition of an apron *o o*, secure a satisfactory result. It is objected against this form, as also against Figs 4 and 6, that their fronts are liable to be torn by descending trees, ice, and other bodies swept along during freshets; but experience shows that this objection has but little weight; for when such bodies pass, the sheet of water is thicker than usual; and protects the front timbers. On the Sch Nav, the timbers *c l*, Fig 6, scarcely wear thin at the rate of an inch in 10 to 15 years.

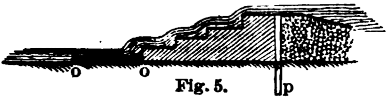


Fig. 5.

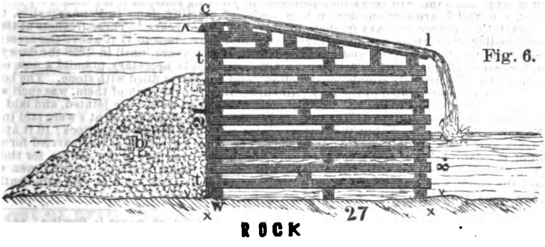


Fig. 6.

The forms of wooden dams are many; (see the figs, which show those most used;) varying with the circumstances of the case, and with the fancy of the designer. In the United States they are usually of cribwork, of either rough round logs with the bark on, or of hewn timber; in either case about a foot through. These timbers are merely laid on top of each other, forming in plan a series of rectangles with sides of about 7 to 12 ft. They are not notched together, but simply bolted by 1 inch square bolts (often ragged or jagged) about 2 to 2½ feet long, through two timbers at every intersection. These are not found to rust or wear seriously, even when exposed to a current. Square bolts hold best. Round logs are flattened where they lie upon each other. Experience shows that firmer but more expensive connections are entirely unnecessary. The cribs are usually, but not always, filled with rough stone. In triangular dams, disposed as in Figs 1, 2, and 7, this stone filling is not so essential as in other forms; because the weight of the water, and of the gravel backing, tends to hold the dam down on its base. Still, even in these, when the lower timbers are not bolted to a rock bottom, or otherwise secured in place, some stone may be necessary to prevent the timbers from floating away while the work is unfinished, and the gravel not yet deposited behind it. On rock, the lowest timbers are often bolted to it, to prevent them from floating away *during construction;* and when the water is some feet deep, this requires coffer-dams. Or, the cribs may be built at first only a few feet high; then floated into place, and sunk by loading them with stone; for the reception of which a rough platform or flooring will be reqd in the cribs, a little above their lowest timbers. The bolting to the rock may then be dispensed with. The water may flow through the open cribwork as the building higher goes on; attention being paid to adding stone enough to prevent it floating away if a freshet should happen. Or, cribs shown in plan at *c c*, Fig 8, loaded with stone, may be sunk, leaving one or more intervals, like that at *o o o o*, between them, for the free escape of the water. These openings to be finally closed by floating into them closing-cribs shaped like *n*.

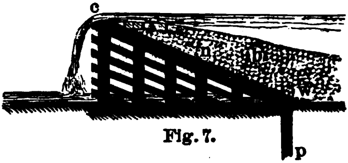


Fig. 7.

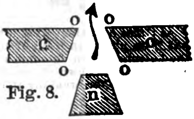


Fig. 8.

The workmanship of a dam in deep water can of course be much better executed in coffer-dams, than by merely sinking cribs. The joints can be made tighter: the stone filling better packed; the sheet piling more closely fitted, &c.

When a very uneven rock bottom in deep water, or the introduction of sluices in the dam, or any other considerations, make it expedient to build dams within coffer-dams, both should be carried on *in sections;* so as to leave part of the channel-way open for the escape of the water. Commencing at one or both shores, the first section of the coffer-dam may reach say quarter way or more across the stream. In the section of the dam itself built within this enclosing coffer-dam, ample sluices should be left for the water to flow through when we come to build the *closing* section of the coffer-dam. When the dam has been finished, these sluices may be closed by **drop-timbers***. Before removing one section of coffer-dam, the outer end of the enclosed section of dam itself must be firmly finished in such a manner as to constitute a part of the inner end of the next section of coffer-dam. It is impossible to give details for every contingency; the engineer must rely upon his own ingenuity to meet the peculiarities of the case before him. In some cases of shallow water, mere mounds of earth may answer for coffer-dams; or rough stone mounds, backed with earth or gravel.

After the water has passed beyond the crest, *c* in the figs, there is no necessity for preventing its leaking down among the crib timbers: on the contrary, the thick sheeting planks, (or squared timbers, as occasion may require.) *c l*, Figs 4 and 6, which form the slopes along which the water then flows in some dams, are usually not laid close together, but with open joints of about ½ inch wide between them, for the express purpose of allowing part of the water to fall through them, so as to keep the timbers beneath them partially wet; which, to some extent, renders them more durable. In Figs 1, 4, 6, and 7, the water of the lower pool flows freely back among the crib timbers, and rough quarry stones with which the cribs are filled either partly or entirely. In Figs 4 and 6, these stones are not shown. In the dam, Fig 1, none were used. In Fig 2, they were as shown.

A substantial, and not very expensive dam of the form of Fig 7, may be built of rough stone in cement. Some hewn timbers should be firmly built horizontally into the masonry of the sloping back *c n w*, at a few feet apart, with their tops level with the surf of the masonry. To these must be well spiked close-jointed sheeting-plank *c n w*, for protecting the masonry from the action of the water, and of floating bodies. The gravel backing *b*, may be omitted; but the sheet piles *p*, and an apron in front of the dam, will be as indispensable in yielding soils, as if the dam were of timber.

Figs 1, 2, 4, 6, and 7, are sections drawn to a scale, of existing dams in Pennsylvania, that have stood successfully the force of heavy freshets for a long series of years.† These freshets at times carry along large bodies of ice, trees, houses, bridges, &c.; and have risen to 11 ft above the crests. Fig 1, on the Sch Nav, was built in 1819, and served perfectly for 39 years, until in 1858 the decay of much of its timber, especially of the close-laid top ones, *e*, rendered it necessary to build a new one just in front of it. It was of extremely simple construction; and was never filled with stone. The bottom timbers, *o o*, 10 ft apart, were bolted to the rock; and immediately over each of them, was such a series of inclined timbers as is shown in the fig. The top ones, *e*, however, were close-jointed, and laid touching, so as to form the top sheeting, instead of thinner planks. The short pieces at *t* were laid in the same way. No coffer-dam was used; but the bottom pieces were first bolted to the rock; 10 ft apart; then the stringers and the sloping pieces were added. The close covering (*e*) was carried forward from each end of the dam, until at last a space of only about 60 ft was left in the center, for the water to pass. The close covering for this space being then all got ready, a strong force of men was set to work, and the space was covered so rapidly that the river had not time to rise sufficiently high to impede the operation.

---

* Timbers ready prepared for closing an opening through which water is flowing; and suddenly *dropped* into place by means of grooves or guides of some kind for retaining them in position. Several such timbers may at times be firmly framed together, and then be all dropped at once; closing the opening or sluice at one operation; especially when it is of small size. In some cases, a crib may be sunk on the up-stream side of such an opening, for closing it.

† Those on the Schuylkill Navigation were obligingly furnished by James F. Smith, Esq, chief engineer and superintendent of that work. Other valuable information from the same source will be found in different parts of this volume.

Fig 2 is a canal feeder dam on the Juniata. Here *s s* are timbers stretching clear across the stream, (about 300 ft,) and sustaining the apron *a a*, of stout hewn timbers laid touching. This dam was filled with stone, for the retention of which the front sheeting planks were added.

Fig 6 is on the Sch Nav; was built in 1855. It is a form much approved of on that work, for such situations; namely, firm rock foundation, with a considerable depth of water in front. The highest dam (32 ft) on the Sch Nav, is very similar to it; built in 1851. All the dams on this work are of hewn timber, chiefly white and yellow pine. The water occasionally runs from 8 to 12 feet deep over their crests; and then overflows and surrounds many of the abuts. The vertical back allows the overflowing water to leak down among *all* the lower timbers of the dam, and thus tend to their preservation.

Fig 4 shows the dams on the Monongahela slackwater navigation; W. Milnor Roberts, eng. They are of round logs, with the bark on; flattened at crossings. The longest ones in the fig are 10 feet apart along the length of the dam. Experience shows that such dams possess all the strength necessary for violent streams. On rock, the lowest timbers are bolted to it.

Fig 7 has been successfully used to heights of 40 ft.*

Fig 8 is intended merely as a hint for a very low dam on yielding bottom. Its main supports are piles *i i*, from 4 to 8 ft apart, according to the height of the dam; and other circumstances; and *t t* are short piles for sustaining the apron *d d*. It may be extended to greater heights by adding braces in front; which may be covered by stout planks, to form an inclined slide for the overfalling water. Many effective arrangements of piles, and sloping timbers for dams on soft ground, will suggest themselves to the engineer. Thus, at intervals of several feet, rows of 3 or more piles may be driven transversely of the dam; the top of the outer pile of each row being left at the intended height of the crest, while those behind are successively driven lower and lower; so that when all are afterward connected by transverse and longitudinal timbers, and covered by stout planking, and gravel, they will form a dam somewhat of the triangular form of Fig 7. It would be well to drive the piles with an inclination of their tops up stream.

There is much scope for ingenuity both in designing, and in constructing dams under various circumstances; and in turning the course of the water from one channel to another, by means of ditches, pipes, or troughs, &c., at diff heights; aided at times by low temporary dams or mounds of earth; or of sheet piles, &c; or by coffer-dams; so as to keep it away from the part being built. Each locality will have its peculiar features; and the engineer must depend on his judgment to make the most of them.

Abutments of dams as a general rule should not contract the natural width of the stream; or, if they must do so, as little as possible; for contractions increase the height, and violence of the overflowing water in time of freshets; during which a great length of overfall is especially desirable. They should be very firmly connected with the ends of the dams; and should, if the section of the valley admits of it, be so high, and carried so far inland, that the high water of freshets will not sweep either over them, or around their extremities; and thus endanger undermining, and destruction. In wide, flat valleys they cannot be so extended without too much expense; and the only alternative is to found them so deeply and securely as to withstand such action; making their height such that they will, at least, be overflowed but seldom. Their ends adjacent to the dam, should be rounded off, so as to facilitate the flow of the water over the crest.

They are best built of large stone in cement; for although sufficient strength may be secured by timber, that material decays rapidly in such exposures. If of earth only, they are very apt to be carried away if a freshet should overtop them.

Sluices should be placed in every important dam, in order that all the water may be drawn off, if necessary, for the purpose of repairs; or of removing mud deposits; or finding lost articles of importance, &c. They may be merely strong boxings, with floor, sides, and top of squared timbers; and passing through the breadth of the dam, just above the bottom. To prevent trees, &c, from entering and sticking fast in them, some kind of strong screen is expedient. In common cases a sluice should not *exceed* about 3½ ft by 5 ft in cross-section; otherwise it becomes hard to work. Two or more such openings may be used when much water is to be voided. They should be near the abutments. The gates or valves for opening and shutting them, should be at the up-stream end; for if at the lower one, accumulations of mud, &c, will fill the sluices, and prevent them from working. They are usually of timber; and slide vertically in rebates; being raised and lowered by rack and pinion; but in very important dams they may be of cast iron. Two sets of sluices are desirable; that one may be always ready for use if the other is stopped for repairs.

The part of the apron in front of the sluice should be particularly firm, so as not to be deranged by the water rushing out under a high head.

Dams are sometimes, but rarely, built in the form of an arch; convex up stream. This form is strong; and when the shores are of rock it may be expedient to use it; but if the banks are soft, they will be exposed to wear by the current thrown against them at the abuts of the arch.

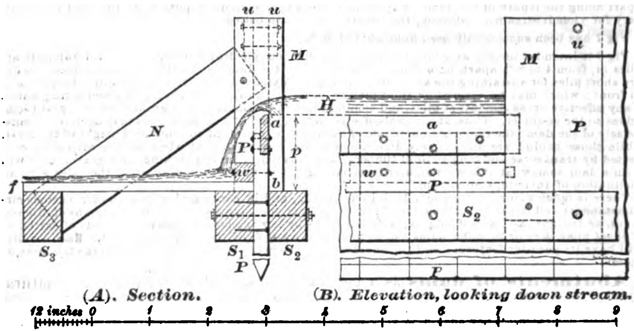
At times dams are built obliquely across the stream, with the object of increasing the length, and consequently reducing the depth of water over the crest in times of freshets. The argument, however, appears to the writer to be of but little weight, inasmuch as the reduction of depth would extend but a trifling distance up stream from the dam; and would therefore scarcely have an appreciable effect in diminishing the injury to the overflowed district above Moreover, the increased expense is probably always more than commensurate with any advantage gained.

---

* **Cost of crib dams.** With common labor at $1.50 per day; lumber, $20 per 1000 ft. board measure, delivered; stone for filling, $1 per cub. yard; gravel 50 cents per cub. yd.; iron for bolts, etc., 4 cts. per lb.,—such dams in shallow water usually cost, complete from 9 to 12 cents per cubic foot, or $2.43 to $3.24 per cubic yard of crib.

Figs. 9 and 10 are **designs for small measuring weirs,** suitable for shallow streams up to say 100 feet wide; Figs. 9 for earth or gravel bottom, and Figs. 10 for rock.

In the former, the 8 × 10 inch hemlock sills S_1 and S_3 are first laid across the bottom of the stream, which is trenched where necessary; care being taken to lay S_1 in a true line. The sills should extend say from 5 to 10 feet into each bank of the stream. Tongued and grooved sheet piling P, of 3 × 10 inch hemlock, is then driven close behind the upper sill S_1 to a depth of from two to four feet, and spiked to S_1. A third sill, S_2, of the same length as S_1 and S_3, is then laid behind the sheet piling; and the two sills S_1 and S_2 and the sheet piling P are then secured together, as shown, by 1 inch bolts, spaced about 2 feet apart. The tops of the sheet piling project about a foot above the sills, and are stiffened by 4 × 4 inch timbers *w*, bolted in front of them and resting upon the flooring *f* of 2 × 10 inch spruce. This flooring, like the sills, extends several feet beyond each end of the weir into the bank, and is there loaded to its full capacity with heavy stones. Any spaces left underneath it by unevenness of the bottom should also be leveled up with stones or gravel.



(*A*). *Section.* (*B*). *Elevation, looking down stream.*

Figs. 9.—Measuring Weir on Earth or Gravel Bottom.

A 10 × 10 inch yellow pine post M, 3 feet high, is tenoned between sills S_2 and S_1 at each end of the overflow, and braced by an 8 × 10 inch yellow pine strut N, tenoned to it and to the sill S_3. Beyond these posts the sheet piling P extends as high as the top of the posts, and is carried, at that height into the bank; the tops of the piles being held in line by two 2 × 8 inch waling pieces *u u* bolted to them, one on each side.

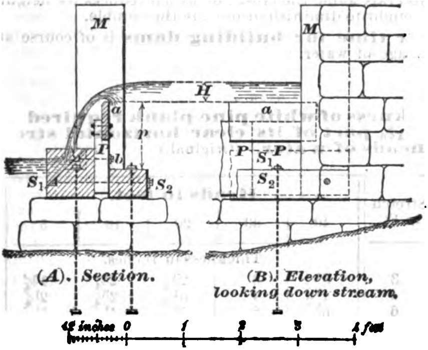
In Figs. 10, the hemlock sills, S_1 of 10 × 10 inch, and S_2 of 6 × 8 inch, rest upon a Portland cement masonry wall, of varying height to accommodate the inequalities of the rock bottom; and are secured to it by 1 inch bolts spaced about 4 feet apart. These bolts pass down through the masonry, as shown, and a foot or more into the rock below.

Between the two sills are bolted upright 3 × 10 inch tongued and grooved hemlock planks P, 15 inches long. At each end of the weir, a 10 × 10 inch yellow pine post M is tenoned between the sills, as in Figs. 9, and built into the masonry ends of the dam, which last extend well into the banks of the stream.

In both Figs. the crest-piece *a*, is of 2 × 8 inch oak, beveled so as to leave a horizontal top face ½ inch wide. The crest-piece is let in flush with the back of the piles or boards P, to which it is bolted, and is let into the end posts M about 2 or 3 inches. At low stages of water, the flow may be confined to a *portion* of the length of the overfall by flash-boards placed along the rest of the dam.

A crest-piece made of 8 × ¼ inch bar iron is preferable to one of wood. It requires of course much less cutting away of the sheet-piling, and its upper edge is less subject to abrasion by drift passing over the weir. The top edge, and the abutting ends of the several lengths, should be planed smooth and square; the former to insure a sharp inner corner at *a* for the water to pass over, and the latter in order to avoid leakage. As a further precaution against leakage, a strip or butt-strap of $8 \times \frac{1}{8}$ inch iron, about a foot long, may be let in, *between the crest-piece and the sheet piling*, opposite each joint of the former, and overlapping both the adjoining ends, the piling being cut away $\frac{1}{8}$ inch deeper at those points, in order to accommodate them. Such butt-straps, if placed on the *up*-stream side of the crest-piece, would break the continuity of the sheet of water passing over the weir, and thus interfere somewhat with the correctness of the gauging. Such iron is obtainable in any commercial center, in lengths of about 16 feet. $8 \times \frac{1}{4}$ weighs $6\frac{2}{3}$ pounds per running foot; $8 \times \frac{1}{8}$, $3\frac{1}{3}$ pounds.

All the joints should be caulked with oakum. To apply the usual weir formulæ (see Art. 14 *f*, p. 548) the back of the weir should be vertical for a depth *p* below



Figs. 10.—Measuring Weir on Rock Bottom.

the crest *a* equal at least to twice the head H on the weir. It is therefore better to protect the back of the weir by tarpaulin rather than resort to puddling, except close to the bottom.

In a long weir with a low fall, it is difficult to secure a sufficiently free access of air to the space behind the falling sheet of water, especially when the stream is low and the sheet tends to hug the face of the dam. In such cases a partial vacuum * forms between the falling sheet and the face of the dam, and increases the discharge, thus vitiating the results. It is therefore important, in designing measuring weirs, to arrange (as far as possible) so that the sheet of water may fall clear through the entire distance between the up-stream and down-stream levels without striking any portion of the weir itself, for such striking would diminish the clear space behind the sheet and increase the difficulty of preventing a vacuum there.

---

* Such a vacuum causes the down-stream water near *w*, Figs. 9, to rise behind the sheet. When the rarefaction of the air behind the sheet has proceeded to a certain extent, the external air breaks in and relieves the vacuum. Then another vacuum forms, and is in turn relieved, and so on, alternately. At such times it has been noticed that light bodies, such as chips, etc., floating in the down-stream water near the ends of the weir, are drawn into the space behind the sheet and carried toward the middle of its length, and then in turn ejected at the point where they entered, thus traveling back and forth along the space behind the sheet.

Tremblings in Dams. Dams over which the water falls in a long, smooth, unbroken sheet of considerable height, are more or less subject to tremblings, caused apparently by alternate compression and rarefaction of the air by the falling sheet, especially in the space (W, Fig. 20, p. 547) behind the sheet, where a partial vacuum is often formed, because the air there is entangled in the falling water and given off again by it down stream in the shape of foam.

Such tremblings sometimes cause a rattling of windows half a mile or more away. We have known this to be stopped (in one case unintentionally) by building a well-covered wide crib apron, a few feet high, against the front of the dam, for preventing the abrasion of the bottom. In other cases a series of oblique timbers placed against the front of the dam, and part way up it, at a slope of about 1½ to 1, and covered with plank, has been perfectly effective in stopping it. In short, any device which admits air more freely behind the falling sheet, or destroys the continuity of the latter (such as flash boards of different heights or placed at intervals along the crest), or which reduces its height and its continuous length, ought to diminish or obviate the trouble.

The proper time for building dams is of course at the longest period of low stage of water.

Table of thickness of white pine plank required not to bend more than $\frac{1}{150}$ part of its clear horizontal stretch, under different heads of water. (Original.)

Stretch in Ft.	Heads in feet.				
	40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22½	20	17½	14	11

WATER SUPPLY.

Consumption of water. Owing largely to the proper extension of the use of water in dwellings, the quantity required in cities increases faster than the population. In other words, the per capita consumption increases.

Use. Abundant experience shows that a supply of 50 gallons (or say 7 cubic feet) per capita per day is abundant for all the needs and luxuries of well-to-do families in American cities. The manufacturing consumption, of course, bears no fixed relation to the population. In cities it is generally much less than the domestic consumption.

Waste. In American cities, the waste often amounts to two or three times the quantity really used. Of the 116 gallons per capita per day, delivered in New York in 1899, Mr. Freeman* estimates that from 31 to 56 gallons were used, 10 unavoidably wasted, and from 50 to 75 avoidably wasted.

In Philadelphia, investigations by means of the Deacon waste-water detector, on 142 modern seven-room, two-story dwellings, with bath, etc., on two intermediate streets, showed that, of 222 gallons per capita per day, furnished through 782 fixtures, 192 gallons, or 86.5 per cent. were wasted, and only 30 gallons, or 13.5 per cent., were used. The City is now building enormous works for the purpose of pumping, filtering, conveying, repumping, storing, and distributing the water wasted, as well as the smaller quantity used. Of the total cost,† less than half would have sufficed for the water used and unavoidably wasted.

Sources of waste. The waste is caused by heedlessness; by allowing water to run to waste in order to prevent it from freezing in winter and in order to get cooler water in summer; by leaky and otherwise defective fixtures; by unsuspected leaks in mains and service pipes, etc.

As a "guess, tempered by judgment," Mr. Freeman* classifies the 50 to 75 gallons per capita per day, wasted in New York, as follows:

Leaks in mains	10 to 15	gals per capita per day.
" service pipes	10 to 15	" " "
" defective plumbing	15 to 25	" " "
Careless and wilful waste	14 to 17	" " "

The avoidable waste is usually perpetrated by a small fraction (say from one-fifth to one-third) of the population, the remainder using water reasonably. In the Philadelphia case, above cited, of the 782 fixtures, 22 were found to be "leaking slightly," and 32 "turned on continually."

Waste restriction. Water meters. Waste is best restricted by making its avoidance a pecuniary object to the consumer; and this is best accomplished by the use of the water meter, at least on all services (domestic, industrial, and public) where waste is found to be going on. The meters should be owned and maintained by the corporation supplying the water.

Minimum charge. In order to encourage the liberal *use* of water, while discouraging its *waste*, and thus avoid undue economy (tending to uncleanliness) each consumer should be charged a minimum periodical rate, sufficient to cover amply all the water he can possibly use and enjoy.

Mr. Freeman* estimates the average cost of domestic meters, for New York and Brooklyn, mostly 5-8 inch and 3-4 inch, with a few of larger sizes, at $12.50 each, and the cost of installation by the city, working systematically and on a large scale, at $2.50 each, or a total of $15.00 each. He assumes "the average life of the ordinary domestic meter, of a good type, well cared for, and with occasional repairs and renewal of worn parts," at "not far from 20 years"; and annual expenses as follows:

	Providence, R. I. Actual, approx.	New York. Assumed.
Interest on cost of meter and setting	$0.50	$0.45
Depreciation and renewal of meter (life assumed 20 years)	0.75	0.75
Maintenance and repairs, testing and resetting	0.46	0.70
Reading meters and computing bills	0.42	0.60
Total annual cost, per meter	$2.13	$2.50

---

* Report upon New York's Water Supply, made to Bird S. Coler, Comptroller, by John R. Freeman, Civil Engineer, 1900.

† The total cost may reach $20,000,000.

Free water for fire protection. Cities sometimes give to manufacturers a free supply of water through special connections, to be used for fire protection only; the manufacturer giving bond not to use such connection for any other purpose, and the city placing a meter on the connection for the detection of any illicit use of the water for other purposes.

Water for city use should not be drawn from the very bottom of the reservoir, because it will then be apt to carry along the sediment; which not only injures the water, but creates deposits within the pipes; thus obstructing the flow. In fixing upon the necessary capacity of a reservoir, this must be taken into consideration; inasmuch as all the water below the level for drawing off, must be regarded as lost. When circumstances justify the expense, it is well to curve up the reservoir end of the service main, so as to provide it with valves at different heights; for drawing off only the purest stratum that may be in the reservoir. With this view, the valve-tower generally has such valves communicating with the water in the reservoir; and by this means only the purest is admitted into the tower: and from it, into the city pipes. This refinement, however, is rarely practicable. Such valves must of course be worked by watchmen.

Art. 1. Reservoirs. In important reservoirs of earth, for storing water to moderate depths for cities, experience appears not to sanction **dimensions** *bolder* than 10 feet thick at top; inner slope 2 to 1; outer slope 1½ to 1.[*] A top width of 15 feet to 20 feet, and inside slopes of 3 to 1, are adopted in some important cases; with outer slopes of 2 to 1. Both slopes, however, are at times made only 1½ to 1. The level water surface should be kept at least 3 or 4 feet below the top of the embankment; or more, if liable to **waves.** In a large reservoir, a quite moderate breeze will raise waves that will run 3 feet (measured vertically) up the inner slope. A low wall, or close fence, *w*, Fig. 37, is sometimes used as a defence against them. The top and the outer slopes should be protected at least by sod or by grass. To assist in keeping the top dry, it should be either a little rounding, or else sloped toward the outside.[†] The soft soil and vegetable matter should be carefully removed from under the entire base of the embankments; which should be carried down to soil itself impervious to water, in order that leakage may not take place *under* them. To aid in this, a double row of sheet piles, or a sunk wall of cement masonry, carried to a suitable depth below the bottom, may be placed along the inner toe in bad cases. If there are springs beneath the base, they must either be stopped, or led away by pipes. The embankment should be carried up in layers, slightly hollowing toward the center, and not exceeding a foot in thickness; and all stones, stumps, and other foreign material, such as *clean* gravel, sand, and decomposed mica schists, &c, that may produce leakage, carefully excluded. These layers should be well consolidated by the carts; and the easier the slopes are, the more effectively can this be done. The layers, however, should not be distinct, and separated by actual plane surfaces; but each succeeding one should be well incorporated with the one below. This has sometimes been done by driving a drove of oxen, or even sheep, repeatedly over each layer; in addition to the carting. Rollers are not to be recommended, as they tend to produce seams between the layers. This might possibly be obviated by projections on the circumference of the roller.

Gravelly earth is an excellent **material,** perhaps the best. The choicest material should be placed in the slope next to the water; and should be deposited and compacted with special care in that portion, so as to prevent the water from leaking into the main body of the dam, and thus weakening it. It is not amiss to introduce a bench, *b*, Fig 37, in the outer slope, to diminish danger from rainwash by breaking the rapidity of its descent.

If the bottom of the reservoir itself is on a leaky soil, or on fissured rock, through the seams of which water may escape, it must be carefully covered with from 1½ to 3 feet of good puddle; which, in turn, should be protected from abrasion and disturbance, by a layer of gravel; or of concrete, either paved or not, according to circumstances.

---

[*] The writer suggests that a top width equal to 2 feet + twice the square root of the height in feet, will be safe for any height whatever of reservoir properly constructed in other respects.

[†] Some engineers slope the top toward the *inside.*

Reservoirs constructed with the foregoing dimensions, and with care, may remain safe for an indefinite period; but where serious damage would result from failure, the following **additional precautions** should be taken. The inner slopes should be carefully faced up to the very top, with at least a close dry rubble-stone pitching, not less than 15 to 18 inches thick; as a protection against wash, and against muskrats. These animals, we believe, always commence to burrow under water. If the slopes are much steeper than 2 to 1, this dry pitching will be apt to be overthrown by the sliding down of the softened earth behind it, if the water in the reservoir should for any cause be drawn down rather suddenly. It will be much more effective, but of course more costly, if laid in hydraulic cement; and still more so if laid upon a layer a few inches thick of cement-and-gravel concrete; especially if this last be underlaid by a layer about 1½ to 3 feet thick of good puddle, spread over the face of the slope; the great object being to protect the inner slope from actual contact with the water. If this can be effectually accomplished, slopes as steep as 1½ to 1 will be perfectly secure; for the danger does not arise from any want of weight of the earth for resisting overthrow. **Special care should be bestowed upon the inner toe of the slope,** to prevent water from finding its way beneath it, and softening the earth so as to undermine the stone pitching. Near the top, reference should be had to danger of derangement by ice, frost, rain, and waves. Flat inner slopes tend not only to prevent the displacement of the pitching; but increase the stability of the embankment, by causing the pressure of the water (which is always at right angles to the slope) to become more nearly vertical; and thus to hold the embankment more firmly to its base than if there were no water behind it. Sometimes the toes of both the inner and outer slopes abut against low retaining-walls in cement. This gives a neat finish, and tends to preservation from injury.

Many engineers, in order to prevent leaking, either through or beneath the embankment, construct a **puddle-wall,** *p*, Fig. 37, of well-rammed impervious soil, (gravelly clay is the best,) reaching from the top to several feet below the base. This wall should not be less than 6 or 8 feet thick on top, for a deep reservoir; and should increase downward by *offsets* (and not by slopes. or batters) at the rate of about 1 in total thickness, to 3 or 4 in depth. Other engineers object to these puddle-walls; and contend that leakage should be prevented by making both the inner slopes and the bottom of the reservoir, water-tight, by means of puddle, concrete, and stone facing in cement, as just alluded to. They argue that if the embankment is well constructed, it is itself a puddle-wall throughout.

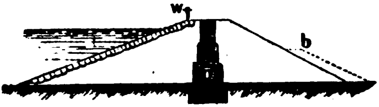


Fig. 37.

Near San Francisco, Cal, are two earthen reservoir dams built about 1864. one 95 feet high, 26 on top, inner slope 2.75 to 1, outer 2.5 to 1. The other 93 high, 25 on top, inner slope 3.5 to 1, outer 3 to 1. In each the puddle-wall is carried 47 feet deeper than the base. No stone facing.

It is difficult to prevent water under high pressure from finding its way through considerable distances along seams where earth is in contact with smooth rock, wood, or metal; as, for instance, along the surfaces of iron pipes laid under reservoir embankments; or along the tie-rods sometimes used through the puddle of coffer-dams; and the same is apt to occur under the bases of embankments which rest on smooth rock. Special care should be taken that the earth used in such positions is not of a porous nature; and that it is thoroughly compacted all along the seam; and the straight continuity of the seam should be interrupted or broken as frequently as possible by projections. Faucets or flanges do this to a limited extent in the case of iron pipes; and something similar, but on a larger scale, should at short intervals be constructed in the shape of collars or yokes of cement stonework, in the case of rock or masonry.

It is usually advisable to **divide reservoirs into two parts.** so that while the water in one part is being drawn off for use, that in the other may purify itself by settling its sediment. Also, one part may remain in use. while the other is being cleaned or repaired. Many days, or even two or three weeks, sometimes, are required for the complete settlement of the very fine clayey particles in muddy water; depending on the depth of the reservoir. One or more flights of steps to the bottom of the reservoir should be provided.

Mud in Reservoirs. The reservoirs of the New River Water Co, London, England, were uncleaned for 100 years. during which mud 8 feet deep was deposited, or about an inch annually. At Philadelphia it is about .25 inch per annum from the Schuylkill, and 1 inch from the Delaware River. At St. Louis, Missouri, about 3 to 4 feet per year! Vegetation is apt to take place in shallow reservoirs and near the edges of deep ones, especially in very warm weather; and the plants, on decaying, injure the water.

Water flowing through marsh lands is sometimes unfit for drinking purposes. That, for instance, in some sections of the Concord River, Massachusetts, was reported by the eminent hydraulic engineer, Loammi Baldwin, of Boston, to be absolutely *poisonous* from this cause.

The construction of a large deep reservoir is not only a very costly, but a very hazardous undertaking. With every watchfulness and care, it is almost impossible entirely to prevent leaking; although this may not manifest itself for months, or even years. Should a break occur, especially near a city, it would probably be attended by great loss of life and property. If the water once finds its way in a stream, either across the unpaved top, or through the body of the embankment, the rapid destruction of the whole becomes almost certain.

Art. 1a. Storing Reservoirs. The entire annual yield of a stream may be much more than sufficient for supplying a certain population with water; and yet in its natural condition the stream may not be available for this purpose, because it becomes nearly dry in summer, when water is most needed; while, at other seasons, the rains and melted snows produce floods which supply vastly more than is required; and which must be allowed to run to waste. A storing reservoir is intended to collect and store up this excess of water, so that it may be drawn off as required during the droughts of summer, and thus equalize the supply throughout the entire year. This, when the locality permits, is effected by building a dam across the stream, to form one side of the reservoir; while the hill-slopes of the valley of the stream form the other sides. The stream itself flows into this reservoir at its up-stream end. When the stream is liable to become nearly dry during long summer droughts experience shows that the **capacity** of the reservoir should be equal to from 4 to 6 months' supply, according to circumstances. During the construction of the dam, a free channel must be provided, to pass the stream without allowing it to do injury to the work. If the dam were built precisely like Fig 37, entirely of earth, it would plainly be liable to destruction by being washed away in case the reservoir should become so full that the water would begin to flow over its top. To provide against this we may, by means of masonry, or of cribs filled with broken stone, or otherwise, construct either the whole, or part of the dam, to serve as an **overfall**, or a **waste-weir.** Or a side channel (an open cut, pipes, or a culvert, &c) may be provided at one or both ends of the dam, and in the natural soil, at such a level as to carry away the surplus flood water before it can rise high enough to overtop the earthen dam. Besides these, and the pipes for carrying the water to the town, there should be an outlet, with a valve or gate, at the level of the bottom of the reservoir; in order that, if necessary for repairs, or for cleaning by scouring, all the water may be drawn off. The entrances to the city pipes should be protected by gratings, to exclude fish, &c.

To facilitate repairs or renewals of all valves, &c, which are under water, the reservoir ends of the pipes or culverts to which they are attached, may be surrounded by a water-tight box or chamber, which will usually be left open to the reservoir; but may be closed when repairs are required. Access may then be had to them by entering at the outer end, after the water has flowed away from inside. In case the outlet is through a long line of pipes which cannot thus be entered, a special entry for this purpose may be cast in the pipe itself, near the outer toe of the embankment; to be kept closed except in case of repairs. Sometimes a better, but more expensive means of access to such valves, is secured by enclosing them in a **valve-tower** of masonry. This is a hollow vertical water-tight chamber, like a well; but near the toe of the inner slope; having its foundation at the bottom of the reservoir; whence the tower rises through the water to above its surface. This chamber is provided with valves or gates usually left open to the reservoir; but which may be closed when repairs are needed; and the water in the tower allowed to escape from it through the open valves of the outlets. This done, workmen can descend through the tower by ladders from the aperture at its top.

At times the outlets for the discharge of surplus flood water are, like those for scouring, placed at, or just above, the level of the bottom of the reservoir. In order that these may work in case of a sudden flood at night, &c, they must be furnished with self-acting valves, which will open of their own accord when the flood is about to rise too high. This may be effected by attaching them to floats, the rising of which, when the water is high, will pull them open. All such outlets should be large enough to let men enter them for repairs. They should by no means be laid through the artificial earthen body of the dam itself, without being supported upon masonry reaching down to a firm natural foundation; otherwise they are very apt to be broken by the subsidence of the embankment. It is usually safer to carry them through the firm natural soil near one end of the dam. Their valves, if only single, should be at their inner or reservoir end, so as to leave the outlets themselves usually empty, for inspection; but it is better to have two valves, so that one may be used when the other needs repair; and in this case one may be placed at each end. Reservoirs which are supplied by pumps, need no precautions against overflow; because the pumping is stopped when they are filled to the proper height. Large storing reservoirs necessarily submerge more or less land, which has therefore to be purchased. By intercepting the descending water, they frequently prevent spring floods from injuring low lands farther down stream. If there are mills down stream from the reservoir, they would evidently be deprived of water for driving them, unless a portion of that stored in the reservoir be devoted to that purpose. Water thus applied to *compensate* for the loss of the natural stream, is called **compensation water**; and the reservoir, a *compensating* one.

Art. 1b. Distributing reservoirs. Frequently a valley fit for a storing reservoir can be found only at a long dist (sometimes many miles) from the town; and it then becomes expedient to construct also an additional one of smaller size than the storing one, near the town; and at as great an elevation above it as circumstances will permit; but lower than the storing one. This is called, by way of distinction, a *distributing* reservoir, because from it the water, after having flowed into it from the storing reservoir, through the long *supply pipe* which connects them, is distributed in various directions through the town, by means of the street mains, or pipes. This small reservoir should hold a supply sufficient at least for a few days; a few weeks would be better; and the end of the supply pipe which terminates in it, should be provided with a valve for shutting off the supply from the storing reservoir. These precautions permit repairs to be made along the line of supply pipe without depriving the town of water in the mean time. With a view to such repairs; as well as to scouring out sediment from the supply pipe, this last should be provided with **outlet valves** at various low points along the entire interval between the two reservoirs; especially at those at which the valves may disch into natural watercourses. On opening these valves, the outrush of the water carries away sediment; and leaves the pipe empty for inspection.

In fixing upon the diams of pipes for supplying cities, it is necessary to bear in mind, that by far the greater portion of the 24 hours' yield is actually drawn from them during only 8 to 12 hours of daylight; and therefore the capacity of the pipes must be sufficient to furnish the daily supply in much less than 24 hours. Again, during the hot summer months, much more water is used than during the winter ones; and this consideration necessitates a still larger diam.

Art. 2. Systems of street pipes for supplying cities. The writer knows of no practical rules for proportioning the diams for such systems. The various complications involved, render a purely scientific investigation of little or no service. With much hesitation, he ventures the following purely empirical rules of his own; based on such limited observations as have casually fallen under his notice.

Rule 1. *When, at no point in a system of city pipes, is the head, or vert dist below the surface of the reservoir, compared with the hor dist from the reservoir, less than at the rate of 50 ft per mile, then the population in the last column of the following Table A, may be abundantly supplied, for all city purposes, by either one pipe of the inner diam or bore in the 1st col; or by 2, 3, &c, pipes of the diams in the other cols.* These diams are given to the nearest safe ⅛ inch. The supply is assumed to be about 60 gallons per day to each inhabitant.

TABLE A. (Original.)

NUMBER OF PIPES.								
1	2	3	4	6	8	12	24	Population
Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	
6	4⅝	3⅞	3½	3	2⅝	2⅜	1¾	1647
8	6⅛	5¼	4⅝	4	3½	3⅛	2⅜	3465
10	7⅝	6½	5⅞	5	4⅜	3⅞	3	5908
12	9⅛	7¾	7	5⅞	5¼	4½	3½	9324
14	10⅝	9⅛	8⅛	7	6⅛	5⅜	4⅛	13706
16	12⅛	10⅜	9¼	7⅞	6⅞	6	4⅝	19141
18	13⅝	11⅝	10½	8⅞	7⅞	6¾	5¼	25677
20	15¼	13	11⅝	9⅞	8¾	7½	5¾	33426
22	16¾	14¼	12¾	10⅞	9⅝	8¼	6¼	42433
24	18¼	15½	13⅞	11¾	10½	9	6⅞	52671
26	19¾	16¾	15	12¾	11⅜	9¾	7½	64447
28	21¼	18⅛	16¼	13¾	12¼	10½	8	77565
30	22¾	19⅜	17⅜	14¾	13⅛	11¼	8⅝	91680
32	24⅜	20⅝	18½	15⅝	14	11⅞	9	108160
34	25⅞	22	19⅝	16¾	15	12¾	9¾	125840
36	27⅜	23¼	20⅞	17¾	15¾	13½	10¼	144480
40	30½	25⅞	23⅛	19⅝	17½	15	11½	188320
44	33½	28⅜	25⅜	21⅝	19¼	16½	12⅝	239600
48	36½	31	27¾	23⅝	21⅛	18	13¾	297600
54	41	34⅞	31¼	26½	23⅝	20⅛	15⅜	391200
60	45⅝	38¾	34⅝	29⅜	26⅜	22⅜	17⅛	511200
66	50⅛	42⅝	38⅛	32⅜	29⅛	24⅝	18¾	650400
72	54¾	46½	41⅝	35⅜	31⅝	26⅞	20½	800000
80	60⅝	51⅝	46⅛	39¼	35¼	29¾	22¾	1064000

It is well to allow in addition from ½ inch to 1 inch, or more, (depending on the character of the water,) to each diameter; for deposits and concretions.

The water, after reaching the city through one or more large main pipes from the reservoir, must be distributed through the streets by means of smaller mains branching from the larger ones. The diameters of these smaller ones also may be found by Table A. Thus, if a street, with its alleys, &c, contains about 6000 persons, (the rate of head being, as before, not less than 50 feet to a mile at any point of the system,) then we see by the table that a 10-inch pipe will answer. It would be well to lay no city street pipes of less than 6 inches diameter.

Mains which cross each other should be connected at some of their intersections, to allow the water a more free circulation throughout the entire system; so that if the supply at any point is temporarily cut off from one direction by closing the valves for repairs, or is diminished by excessive demand, it may be maintained by the flow from other directions.

Avoid dead ends when possible, as the water in them becomes foul and unwholesome.

RULE 2. *With the same diameters, different rates of head will supply the proportionate populations in column* 3 *of Table B. Or, to find the diameters which at different rates of head will supply the same populations given in the last column of Table A,* multiply the diameter given in Table A, by the corresponding number in column 4 of Table B; or (approximately) do as directed in column 5.

TABLE B. (Original.)

COL. 1.	COL. 2.	COL. 3.	COL. 4.	COL. 5.
Rate of Head, in Feet per Mile.	Rate of Head, compared with that in Table A.	Proportionate Populations.	Proportionate Diam. to supply the Populations in Table A.	Remarks.
5	.1	.32	1.58	
10	.2	.45	1.37	
12½	.25	.50	1.32	Add one-third.
15	.3	.55	1.27	Add full one-fourth.
20	.4	.64	1.20	Add one-fifth.
25	.5	.71	1.14	Add one-seventh.
30	.6	.78	1.11	Add one-ninth.
35	.7	.84	1.07	Add one-fourteenth.
37½	.75	.87	1.06	Add one-sixteenth.
40	.8	.90	1.05	Add one-twentieth.
45	.9	.95	1.02	Add one-fiftieth.
50	1.0	1.00	1.00	
75	1.5	1.23	.92	Deduct one-thirteenth.
100	2.0	1.41	.88	Deduct one-eighth.
125	2.5	1.59	.83	Deduct full one-sixth.
150	3.0	1.73	.80	Deduct one-fifth.
200	4.0	2.00	.76	Deduct nearly one-fourth.
250	5.0	2.25	.73	Deduct nearly two-sevenths.
300	6.0	2.46	.69	Deduct three-tenths.
400	8.0	2.83	.66	Deduct full one-third.
500	10.0	3.18	.63	

Example. By Table A we see that with the rate of head of 50 feet per mile, a 30-inch pipe will supply a population of 91580; but with three times that rate of head, or 150 feet per mile, we see by column 3, Table B, that the same pipe will supply 1.73 times as many persons, or 91580 × 1.73 = 158433 persons. But if, at this greater rate of head, we still wish to supply only 91580 persons, then we find in column 4, Table B, that we may diminish the diameter of the pipe from 30, down to 30 × .80 = 24 inches; or, by column 5, we have 30 — 6 = 24 inches.

Again, after the water has reached the city by the 30-inch pipe of Table A, if we wish to distribute it through the city by say eight branches or smaller mains, we see by column 6, Table A, that each of them must have at least 13⅛ inches diameter. From these eight, other smaller ones may branch off into the cross streets, alleys, &c; and in estimating the supply required for any particular street main, we must evidently add what is required also for such cross streets, &c, &c, as are to be fed from said main.

If certain limited parts of a city pipe system have considerably less rates of head than most of the remainder, it may become expedient to supply the former by a special separate main of larger diameter; which may start either directly from the reservoir; or as a branch from the grand leading main which feeds the lower parts, according to circumstances.

It must be remembered, that although by increasing the diameters, an abundant supply may be obtained under a small rate of head, as well as under a great one, yet the water will not rise to as great a height in the service pipes for supplying the different stories of dwellings, &c. Even with the diameters in Table A, the water, under ordinary use, will not rise in these pipes to the full height of the surface of the reservoir; and if an unusual drawing-off is going on at the same time at many parts of the system, as in case of an extensive fire, or frequently during the hot summer months, it may not rise to even one-half of that height.

Art. 3. The following has been found very effective for preventing concretions in water pipes. Formerly in Boston, cast-iron city pipes, 4 inches diameter, became closed up in 7 years; and those of larger diameter became seriously reduced in the same time. But later, during 8 years, in which this varnish was used, no concretions formed.*

Coal-pitch varnish to be applied to pipes and castings, made for the Water Department of Philadelphia, under the following conditions:

First. Every pipe must be thoroughly dressed and made clean, free from the earth or sand which clings to the iron in the moulds: hard brushes to be used in finishing the process to remove the loose dust.

Second. Every pipe must be entirely free from rust when the varnish is applied. If the pipe cannot be dipped immediately after being cleansed, the surface must be oiled with linseed oil to preserve it until it is ready to be dipped: no pipe to be dipped after rust has set in.

Third. The coal-tar pitch is made from coal tar, distilled until the naphtha is entirely removed, and the material deodorised. It should be distilled until it has about the consistency of wax. The mixture of five or six per cent of linseed oil is recommended. Pitch which becomes hard and brittle when cold, will not answer for this use.

Fourth. Pitch of the proper quality having been obtained, it must be carefully heated in a suitable vessel to a temperature of 300 degrees Fahrenheit, and must be maintained at not less than this temperature during the time of dipping. The material will thicken and deteriorate after a number of pipes have been dipped; fresh pitch must therefore be frequently added; and occasionally the vessel must be entirely emptied of its old contents, and refilled with fresh pitch; the refuse will be hard and brittle like common pitch.

Fifth. Every pipe must attain a temperature of 300 degrees Fahrenheit, before it is removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip.

All pipes of 20 inches diameter and upward, will require to remain at least thirty minutes in the hot fluid, to attain this temperature; probably more in cold weather.

Sixth. The application must be made to the satisfaction of the Chief Engineer of the Water Department; and the material be subject at all times to his examination, inspection, and rejection.

Seventh. Payment for coating the pipes will only be made on such pipes as are sound and sufficient according to the specifications, and are acceptable independent of the coating.

Eighth. No pipe to be dipped until the authorized inspector has examined it as to cleaning and rust; and subjected it thoroughly to the hammer proof. It may then be dipped, after which, it will be passed to the hydraulic press to meet the required water proof.

Ninth. The proper coating will be tough and tenacious when cold on the pipes, and not brittle or with any tendency to scale off. When the coating of any pipe has not been properly applied, and does not give satisfaction, whether from defect in material, tools, or manipulations, it shall not be paid for; if it scales off or shows a tendency that way, the pipe shall be cleansed inside before it can be recoated or be receivable as an ordinary pipe.

---

* Mr. Dexter Brackett, of Boston, informs us, 1892, that while tubercles form there in uncoated pipes to a thickness of about three-quarters of an inch, rendering 4-inch pipes of little or no value for fire supply, yet no actual stoppage has been known to occur from this cause during the twenty-three years of his connection with the City Engineering Department. He states also that even their coated pipes, taken up after being in the ground for ten or fifteen years, are generally found to be pitted on their inner surfaces.

Art. 4. The pipes are laid to conform to the vertical undulations of the street surfaces. The tops of the pipes are laid not less than 3½ feet below the surface of the street; but in 3-inch pipes the water has at times been frozen at that depth.

In Philada., in 1885, there were about 784 miles of street pipes; or about 1 mile to every 1100 inhabitants. The population was about 860,000; residing in about 150,000 dwellings. **Berlin,** 1887–8; 1,400,000 inhabitants, in 20,000 houses (average 70 persons per house). Mean consumption per head, 17 U. S. gallons per day; maximum, 24; minimum, 12½; all approximate. 25,000 wheel meters in use.

No galvanic action has been observed where lead pipes or brass unite with cast-iron ones. **No pipe** less than 6 inches diam should be laid in cities; and even they only for lengths of a few hundred feet. Their insufficiency is chiefly felt in case of fire. 8 ins would be a better minimum. No more leakage occurs in winter than in summer; except from the bursting of *private service-pipes* by freezing.

To compact the earth thoroughly against the pipes excludes air, and greatly impedes rust. Pipes may be corroded by the leakage of gas through the body as well as through the joints of **adjacent gas-pipes.**

WEIGHT OF CAST-IRON WATER-PIPES,

As used in Phila., and tested by hydraulic press before delivery to an internal pres of 300 lbs per sq inch. This table includes spigots, and faucets or bells. The pipes are required to be made of remelted strong tough gray pig iron. such as may be readily drilled and chipped; and all of more than 8 ins diam to be cast vertically, with the bell end down. Deviations of 5 per cent above or below the theoretical weights, are allowed for irregularities in casting, which it seems impossible to avoid.

The pipes are in lengths from 3 to 3½ ins longer than 12 ft; so that when laid they measure 12 ft from the mouth, *f*, Fig 38, of one bell to that of the next.

Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
3	5/16	158	16	5/8	1322	36	15/16	4334
4	3/8	211	20	"	1654	36	1 1/16	4862
6	7/16	385	20	11/16	1798	36	1 3/16	5366
8	"	460	30	[illegible]	3313	48	1 1/8	7282
10	1/2	667	30	.9	3610	48	1 3/8	8667
12	9/16	899	30	1	3964	48	1 1/2	9878

The following sizes **of lap-welded wrought-iron water-pipe** are made by the National Tube Works Co., McKeesport, Pa., and fitted with their "**Converse patent lock-joint.**" One end of each length of pipe has the lock-joint permanently attached (leaded) to it at the works before shipping. The "weights per foot" include these joints. The weight of "lead per joint" given is that required to be poured in *laying* the pipe, or that for one side only of the joint.

Outer **diam**, ins	2	3	4	5	6	8	10	12	16
Weight per ft, lbs	1.86	3.48	5.26	7.33	8.76	13.20	17.08	25.12	47.70
Lead per joint, lbs	5/8	1 9/16	2 1/8	3 1/4	3 9/16	5 3/4	6	8 3/4	16
Average **car load:**									
Number of lengths	800	380	275	145	126	128	80	56	40
" " feet	11500	5600	4500	2600	2000	2000	1200	800	630

The pipes are tested for a bursting pressure of 500 lbs per square inch, or higher if desired. They are furnished either coated with asphaltum, or "**kalameined;**" or, if desired, first kalameined and then coated with asphaltum. Kalameining consists in "incorporating upon and into the body of the iron a non-corrosive metal alloy, largely composed of tin." The surface thus formed is not cracked by blows, or by bending the pipe, either hot or cold.

The joint, or coupling, is of cast-iron, and has internal recesses which receive and hold lugs on the outside of each length of pipe, near each of its ends. The joint is then poured with lead in the usual way (see next page), either with clay collars, or with a special pouring clamp furnished by the Co. This clamp resembles the "jointer," Figs 39 &c, except that it is in two rigid semi-circular pieces, connected together by a hinge-joint, and furnished with handles like those of a lemon-squeezer, and has a hole in one side for pouring. The coupling forms a flush inner surface with the pipe at the joint, thus avoiding much of the resistance of cast-iron pipes to flow. For cases where it may be necessary to make frequent changes, the couplings are made in two pieces, which are bolted together by flanges.

Wrought-iron, for pipes, has the great **advantages** over cast-iron of lightness, toughness, and pliability. The lightness of wrought-iron pipes renders them easier to handle, and cheaper *per foot* notwithstanding that their cost *per ton* is about 25 per cent greater. They are not liable to breakage in transportation or from rough handling, and they may be bent through angles up to about 25°. They therefore require no special bend castings for such angles. The National Co supply bending machines, to be worked by two men. One machine can, by changing the dies, be used in bending all sizes of pipe. The pipes are in lengths of from 15 to 18 feet, instead of 12 feet, as in the case of cast-iron, so that **fewer joints are required per mile.**

The Co furnish special "service clamps" and tapping machines for **attaching service pipes to mains.** This may be done (as in the case of the Payne machine, while the main is under pressure. The service clamp is a cast-iron saddle, which, before the main is tapped, is attached to it by means of a U bolt, and which remains permanently so attached after the tapping. A sheet-lead gasket is placed between clamp and main. The clamp has a tapped cylindrical opening through it, into which the corporation stop is screwed before the pipe is tapped. The drill of the tapping machine passes through the stop, and through the cylindrical opening in the clamp, and drills through the lead gasket and through the side of the main.

The Co furnish also pipe-cutting machines, and special castings (reducers, crosses, &c, &c) fitted with the Converse joint.

Art. 5. Wrought-iron pipes corrode much more rapidly than cast.

A gutta-percha pipe, ⅛ inch thick, and ¾ inch bore, has sustained safely an internal pres of more than 250 lbs per sq inch; equal to nearly 600 feet head. It merely swelled slightly at 337 lbs. In 1851 a tube of that material, 2½ ins bore, about ½ inch thick, and 1350 ft long, was sunk in the East River, New York, to carry the Croton water to Blackwell's Island. It was held down by weights. It proved unsatisfactory owing to abrasion caused by tidal currents, and injury from the anchors of dragging vessels. A wrapping of canvas, confined by spun yarn, was useful in preventing the former, but not the latter. This pipe was replaced in 1870 by wrought-iron pipes.

Ball's patent iron and cement pipe, is made by The Patent Water and Gas Pipe Co, of Jersey City, N. J. It is formed of riveted sheet-iron, and each length is dipped into, and coated with, a hot mixture of coal tar and asphalt. The lining of hydraulic cement is then applied. This ranges, in thickness, from ⅜ inch for 12-inch pipes to 1 inch for 20-inch pipe. This pipe is made up to diams of 36 ins. It is laid in a bed of cement mortar, and completely covered with the same. Suitable means are provided for making all the attachments, &c, required in city pipes for water and gas. More than 1300 miles of it are in use in various towns, some of it for 35 years; and it appears to give general satisfaction. Tubercles do not form in these pipes, as they are apt to do in cast-iron ones. There is every reason to suppose that they are durable. The trenches being dug, the Jersey City Co furnish pipes and lay them (including the cement).

A. Wyckoff & Son, Elmira, N. Y., make **wooden water pipes.** For pressures of 15 to 20 lbs per sq inch, they furnish either plain pipes, 3½ to 7 ins square externally, and from 1½ to 4 ins internal diam; or round pipes, 1 inch to 16 ins bore, coated externally with asphaltum cement. At their ends, both the square and the round pipes are banded with iron. For pressures from 40 to 160 lbs per sq inch, the round wooden pipes, before being coated with cement, are spirally wrapped, by steam power, with hoop iron, which is first passed through a preparation of coal-tar. The iron is wound so tightly as to be imbedded in the pipe, leaving its outer surface flush with that of the wood. The ends of each length of pipe receive extra banding. The asphaltum cement coating is then applied. These pipes have been extensively and successfully used for both water and gas. Suitable arrangements are provided for joints and connections.

Water pipes of bored oak and pine logs, laid in Philada 50 to 60 years ago, are frequently quite sound, and still fit for use, except where outer sap wood is decayed. When this is removed, many of these old pipes have been relaid in factories, &c. Clay *well packed* around wooden pipes, excludes the contact of air, and thus contributes greatly to their durability. Loose porous soils, such as gravel, &c, on the contrary, are unfavorable.

Pipes made of bituminized paper, prepared under great pressure, have been used for both water and gas. They are much less liable to break than cast-iron, and do not weigh or cost more than about half as much. Pipes of 5 ins bore and ½ inch thick, have resisted test strains of 220 lbs per sq inch; equal to a water head of 507 ft.

Costs of water pipe and laying. The following figures are deduced from a table kindly furnished by Mr. Allen J. Fuller, General Superintendent, Bureau of Water, Philadelphia. They represent average conditions for straight pipe, laid in earth, in that city. The cost, in any given case, may differ materially from these figures, according to circumstances.

"Laying" includes all handling of materials, after their delivery on the ground, for laying them in the trench, making joints, calking, etc. Calkers receive $2.50, lead men $2.00, and laborers $1.75 per day of 8 hours.

The costs of materials are taken as follows: Pipe castings, 1.2 cts.; lead, 5 cts.; gasket, 3½ cts.; coke, 0.27 cts., per pound; blocking, 1.7 cts. per ft. B. M.

Add for stops, branches, fire hydrants, special castings, repaving, damages, foremen's wages, cost of tools, etc. For rough estimates, to cover fixtures, rock excavation, additional depth required for trench, wear and tear of tools, and ordinary repaving, but not including damages, asphalt repaving, or trestling, the costs in the table may be increased as follows:

Diameter of pipe	4	6	8	10	12	16 to 48 inches.
Add	80	70	65	60	50	40 per cent.

MATERIALS.

Pipe.		Per length of 12 feet.											Per lineal foot.
Diam.	Thickness.	Iron.		Lead.		Gasket.		Coke.		Block'g.		Total.	
Ins.	Ins.	lbs.	$	lbs.	$	lbs.	$	lbs.	$	Ft. B M	$	$	$
4	3/8	214	2.57	6	0.30	0.2	0.01	4	0.01	2	0.03	2.92	0.24
6	7/16	367	4.40	10	0.50	0.2	0.01	4	0.01	2	0.03	4.96	0.41
8	7/16	490	5.88	12	0.62	0.3	0.01	4	0.01	2	0.03	6.56	0.55
12	9/16	918	11.02	18	0.90	0.6	0.02	5	0.01	2	0.03	11.98	1.00
18	5/8	1510	18.12	30	1.50	0.8	0.03	7	0.02	2	0.03	19.70	1.64
24	3/4	2458	29.50	40	2.00	1.0	0.03	8	0.02	3	0.05	31.60	2.63
30	13/16	3325	39.90	75	3.75	1.3	0.05	9	0.02	4	0.07	43.79	3.65
"	1	4009	48.11	75	3.75	1.3	0.05	9	0.02	4	0.07	52.00	4.33
36	15/16	4610	55.32	115	5.75	2.0	0.07	10	0.03	5	0.09	61.25	5.10
"	1 1/16	5680	68.16	115	5.75	2.0	0.07	10	0.03	5	0.09	74.09	6.17
48	1 1/4	8038	96.46	150	7.50	2.4	0.08	12	0.03	8	0.14	104.21	8.68
"	1 1/2	9455	113.46	150	7.50	2.4	0.08	12	0.03	8	0.14	121.21	10.10

EARTHWORK.

Pipe.		Trench.			Earthwork, per lineal foot.			
Diam.	Thickness.	Width, feet.		Depth, feet.	Excavation.		Back fill and removing surplus.	Total.
Ins.	Ins.	Top.	Bottom.		Cubic yards.	$	$	$
4	3/8	2.50	2.25	4.50	0.40	0.09	0.02	0.11
6	7/16	2.50	2.25	4.50	0.40	0.09	0.02	0.11
8	7/16	2.50	2.25	4.50	0.40	0.09	0.02	0.11
12	9/16	2.75	2.50	4.50	0.47	0.11	0.06	0.16
18	5/8	2.75	2.50	4.50	0.48	0.11	0.09	0.20
24	3/4	3.70	3.00	4.75	0.64	0.14	0.15	0.30
30	13/16	4.25	3.25	5.00	0.76	0.17	0.23	0.40
"	1	4.25	3.25	5.00	0.76	0.17	0.23	0.40
36	15/16	5.00	3.75	5.50	0.97	0.22	0.30	0.52
"	1 1/16	5.00	3.75	5.50	0.96	0.22	0.30	0.52
48	1 1/4	7.00	4.50	6.50	1.47	0.44	0.56	0.99
"	1 1/2	7.00	4.50	6.50	1.47	0.44	0.56	0.99

HAULING, LAYING, RECAPITULATION.

Pipe.		Hauling, per lineal foot, at 75 cents per ton.			Laying, per lineal foot.	Recapitulation of Cost, per lineal foot.				
Diam.	Thickness.	Pipe.	Miscellaneous.	Total.		Materials.	Earthwork.	Hauling.	Laying.	Total.
Ins.	Ins.	$	$	$	$	$	$	$	$	$
4	3/8	0.01	0.01	0.02	0.03	0.24	0.11	0.02	0.03	0.40
6	7/16	0.01	0.01	0.02	0.04	0.41	0.11	0.02	0.04	0.58
8	7/16	0.01	0.01	0.02	0.04	0.55	0.11	0.02	0.04	0.72
12	9/16	0.03	0.01	0.04	0.05	1.00	0.16	0.04	0.05	1.25
18	5/8	0.04	0.01	0.05	0.06	1.64	0.20	0.05	0.06	1.95
24	3/4	0.07	0.01	0.08	0.08	2.63	0.30	0.08	0.08	3.09
30	13/16	0.09	0.02	0.11	0.08	3.65	0.40	0.11	0.08	4.24
"	1	0.11	0.02	0.13	0.08	4.33	0.40	0.13	0.08	4.94
36	15/16	0.13	0.02	0.15	0.08	5.10	0.52	0.15	0.08	5.85
"	1 3/16	0.16	0.02	0.18	0.09	6.17	0.52	0.18	0.09	6.96
48	1 1/4	0.22	0.03	0.25	0.12	8.68	0.99	0.25	0.12	10.04
"	1 1/2	0.26	0.03	0.29	0.12	10.10	0.99	0.29	0.12	11.50

Art. 6. Cast-Iron Pipe Joints. Philadelphia standard. The clear distance, *d*, between the spigot and the faucet, is nearly uniform for all sizes of pipe, varying only from $\frac{5}{16}$ inch for 4-inch pipe, to $\frac{7}{16}$ inch for 30-inch pipe. The depth, *m n*, of the faucet varies from 3 ins in 4-inch pipe, to 4 ins in 30-inch pipe.

The small beads at *s* and *m*, *s'* and *m'* on the spigot end of the pipe, project about ¼ inch; and prevent the calking material from entering the pipe. The calking consists of about 1 to 2 ins in depth of well-rammed, untarred gasket, or rope yarn; above which is poured melted lead, confined from spreading by means of clay plastered around the joint. The lead is afterwards compacted by a calking hammer.

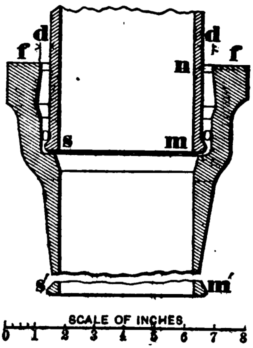
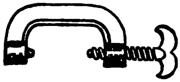


Fig 38

The lead is poured through a hole left in the clay on the upper side of the pipe. In large pipes, two additional holes are left in the clay, one at each side of the pipe, and lead is first poured into the side holes by two men at once, one man pouring into each side hole until the joint is half full. The side holes are then stopped, and, after the lead already poured has hardened, the two men finish the pouring by means of the top hole. This course is necessary, because the great weight of melted lead in the entire large joint would press away the clay at the lower side of the joint, and thus escape.

The moisture in the clay is liable to freeze in cold weather, and to render it too hard to be used. It is also liable, at all times, as is also any dampness in the pipe, to be converted into steam by the heat of the melted lead. The steam sometimes breaks out, or "blows" through the clay, allowing the lead to escape.

Art. 7. The Watkins patent "**Pipe Jointer**" avoids these difficulties by dispensing with the ring of clay. It consists of a ring R, Figs 39 and 40, of square cross-section, and made of packing composed of alternate layers of hemp cloth and India rubber. This ring is encircled by one or more thin strips of spring steel, which are riveted to it at intervals, as shown. E E are iron-elbows riveted outside of the steel bands. After the gasket has been rammed into its place, the ring is placed around the spigot near the faucet, in the position shown in Fig 40, and is held loosely by the clamp, Fig 41, one point of which enters a small pit in each of the elbows, E E. The ring is then, by means of a hammer, driven close up against the end, *f*, of the faucet, Fig 38; the screw of the clamp is tightened somewhat, so as to bring the ring close to the spigot; a small dam of clay is placed in front of the aperture between the two elbows, E E; and the joint is ready for pouring. After the lead has hardened, the "jointer" is removed, and is ready for use at another joint. Upon its removal the lead is found smooth, requiring no chipping. One can be used for several hundred joints. They dispense with the services of the men who prepare the clay collars, and supply them to the pourers. Thos. Watkins, Johnstown, Pa.

Fig. 41.


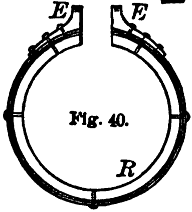


Fig. 40.

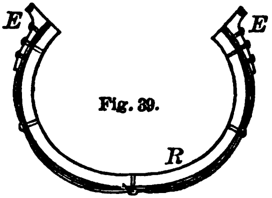


Fig. 39.

Art. 8. As a further preventive against the **escape of any of the gasket into the pipe,** a ring of lead pipe is sometimes placed in the joint before the gasket is inserted. This lead pipe is of such diameter that it can just be pushed through the space, *d*, Fig 38, between the spigot and the faucet; and of such length as just to encircle the water-pipe. It is driven as closely as possible into the narrow annular space at *o o*, Fig 38. The gasket is then rammed in, and the lead poured, as usual.

Art. 9. In John F. Ward's flexible joint, Fig. 42, for cast-iron pipes laid across the irregular beds of streams, a portion, *a o*, of the inside of the bell B is accurately turned to form the middle zone of a sphere, with center at C, and the narrow surface *m m*, on the outside of the spigot S, is turned to form a segment of a sphere accurately fitting the former. The lead is poured while the two adjacent lengths of pipe, resting on suitable vessels or floats, are in a straight line, or nearly so. The lead occupies the space *m n*, shown black, and is held in place on the spigot by the depression *d d*. As fast as the joints are thus filled, the floats are moved forward, and the pipes, if small, are passed into shallow water without further care. Suitable apparatus is used for lowering large pipes into deep water without undue strain on the joints. The joint permits a deflection of 15°, as shown; but further deflection, which would be liable to split the bell, is prevented by the stops at *o o* on the bell and *v v* on the spigot. In some cases preliminary dredging may be expedient, to diminish abrupt irregularities of the bottom. Over thirty lines of pipe furnished with this joint have been successfully laid, of diameters up to 3 feet.

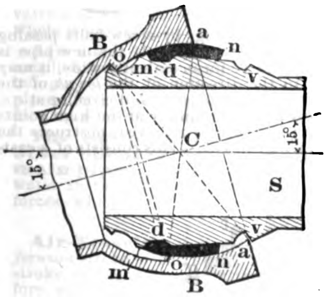
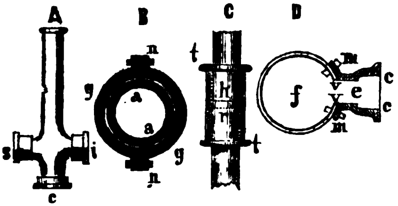


Fig. 42.

Art. 10. In Figs 43, A is a **double branch;** which is a pipe having, in addition to the faucet, *c*, at one end, two others, *s* and *i*, to which pipes leading in opposite directions (as at cross-streets) may be attached. If either *s* or *i* be omitted, the pipe is a **single branch.** The pipe is stronger when these extra faucets are near its end, than if they were at its middle. In a long line of pipes, for the sake of expedition, different gangs of men are frequently laying detached portions some distance apart; and when two ends of different portions are brought near enough together to be united, as *h* and *r*, Fig *C*, their junction cannot be effected by the usual spigot-and-faucet joint. In this case a cast-iron **sleeve,** *t t*, is used, which is first slid upon one of the pieces of pipe; and (after the other piece also is laid) is slid back into the position in the fig, so as to cover the joint. Sleeves are usually about a foot long; as thick as the pipe; and their diam is sufficient to allow the usual joint of gasket and lead. There is of course such a joint at each end of the sleeve.

Art. 11. When a crack occurs in a pipe, *a a*, Fig B, already in use, it is repaired by means of a cast-iron sleeve, *g g*, made in two parts, bolted together by means of flanges as at *n n*. In other respects it is like the preceding sleeve. The intermediate white ring is the lead joint. If the crack is too long, or otherwise too bad to be remedied by a sleeve, the pipe is broken to pieces; and the lead joints at its ends melted out, so as to allow of its removal. Then, since an entire new pipe cannot now be inserted, owing to the overlapping of the spigot-and-faucet ends, two short pieces must be substituted for it. One end of each of these is lead-jointed to the pipes already laid; while the other two ends, which will probably be a few inches apart, are covered by a sleeve, *t t*, Fig. C.



Figs. 43

Cracks may at times be temporarily repaired in an emergency, by a wrapping of folds of canvas thoroughly saturated with white-lead paint; and tightly confined to the pipe by a spiral banding of thin hoop-iron or wire. Or, by an iron band, made in two parts, B B, Fig 44, and clamped together by screw-bolts, S S. Such bands are useful, also, for strengthening pipes that are considered to be in danger of bursting.

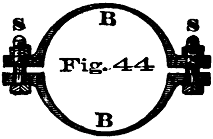


Fig. 44

Art. 12. To attach a pipe, *e*, Fig 43, to one, *f*, already in use, but in which no provision has been made for such attachment, a piece may be cut out of *f*, as at *v v*, and a casting, *e*, furnished with flanges, *m m*, bolted over the opening, by screw-bolts passing through female screws tapped in the thickness of the pipe. If the new pipe is so large that the opening, *v v*, if circular, would be inconveniently wide, it may be made *oval*, with the longest diameter in the direction of the *length* of the pipe, *f*. In that case the casting *e* will be oval at its flanges; and circular at *c c*.

Art. 13. Air valves. Air is apt to collect gradually at the high points of vertical curves along the supply pipes; and, unless removed, obstructs the flow. This may be prevented by an air valve, Fig 44A. This consists of a cast-iron box, *c c d d*, confined to the main pipe *m m*, by screw-bolts passing through its flange *d d*. It has a cover *g n g*, confined to it by screws *t t;* and at the top of which is an opening *n*, for the escape of air from within. In this box is a float *f*, which may be a close tin or copper vessel, or of layers of cork, as supposed in the fig; or &c. This float has a spindle or stem *s s*, fast to it; which passes through openings in the bridge-bars *a a*, and *o;* thereby allowing the float to rise and fall freely, but preventing it from moving sideways. When the pipe *m m* is empty, the float is down; its base *y* resting on the cross-bar *a a*. The stem *s s* has fixed to it a valve *v*, which rises and falls with it and the float. Suppose the pipe *m m* to be empty, and consequently the float and the valve *v* down. Then, if water be admitted into the pipe, it will rise and fill also the box as far up as *e;* and in doing so will lift the float *f*, and the valve *v*, to the position in the fig; thus preventing egress to the outer air by closing the opening at *v*. Now, air carried along by the water, will, on account of its lightness, ascend to the highest points it meets with.

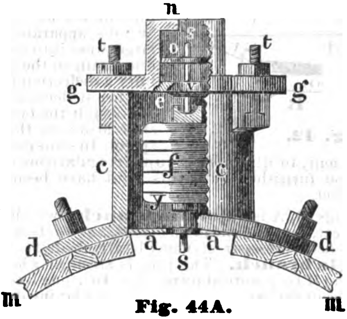


Fig. 44A.

Hence, when such air arrives under the opening *a a*, it will rise through it, and ascend to *e*; the closed valve preventing it from going farther. Thus successive portions of air ascend, and in time accumulate to such an extent as gradually to force much of the water downward out of the box. When this takes place, the float, which is held up only by the water, of course descends also; and in doing so, pulls down with it the valve *v*. The accumulated air then instantly escapes through the openings at *v* and *n*, into the atmosphere; and the water in the pipe *m m*, immediately ascends again into the box, carrying with it the float; and thus again closing the valve *v*. The valve, and the valve-seat *e*, are faced with brass, to avoid rust, and consequent bad fit. The whole is protected by an iron or wooden cover, reaching to the level of the street.

Air valves are no longer used in city pipes; their place being supplied by the fireplugs at average distances of about 150 yards apart. These, being placed as much as possible at the summits of undulations in the lines of pipes, for convenience of washing the streets, and being frequently opened for that purpose, permit also the escape of accumulated air.

The escape of compressed air through an air valve, or other opening, has been known to produce bursting of the main pipes; for the escape is instantaneous, and permits the columns of water in the pipes on both sides of the valve, to rush together with great forces, which arrest each other, and react against the pipes.

Air-Vessels. Motion is imparted to the water in a line of pipes, by the *forward* stroke of the piston of a single-acting pump; but during the *backward* stroke, this motion is stopped; and the water in the pipes comes to rest. Therefore, at the next forward stroke, all the water has to be again set in motion; and the force that must be exerted by the pump to do this is much greater than would be required if the motion previously imparted had been maintained during the time of the backstroke. The addition of an air-vessel secures this maintenance of motion, and thus effects a great saving of power; besides diminishing the danger of bursting the pipes at each forward stroke. It is merely a tall and strong air-tight iron box, usually cylindrical, strongly bolted on top of the pipes just beyond the pump, and communicating freely with them through an opening in its base. It is full of air. The forward stroke of the piston then forces water not only along the pipes, but also into the lower part of the air-vessel, through the opening in its base; thus compressing its contained air. But during the backstroke, this compressed air, being relieved from the pressure of the pump, expands; and in so doing presses upon the water in the pipes, and thus keeps it in motion until the next forward stroke; and so on. An air-vessel also acts as an *air-cushion;* permitting the piston to apply its force to the water in the pipes gradually: thus preserving both the pipes and the pump from violent shocks. The air in the vessel, however, becomes by degrees absorbed and taken away by the water; and its action as a regulator then ceases. To prevent this, fresh air must be forced into the vessel from time to time by a condenser, or forcing air-pump. A *double-acting* pump does not *so much* need an air-vessel. There is no particular rule for the size or capacity of air-vessels. In practice it appears to vary from about 5 to 50 times that of the pump; with a height equal to two or more times the diameter. A stand-pipe (see below) is sometimes used instead of an air-vessel.

A stand-pipe is sometimes used for the same purpose as an air-vessel (see above). It is a tall pipe, open to the air at top; and communicating freely at its foot with the water-pipe, in the same manner as in an air-vessel. Its top must be somewhat higher than that to which the pump has to force the water through the system of pipes; otherwise the water would be wasted by flowing over its top. The area of its transverse section should be *at least* equal to that of the pipe or pipes which conduct the water *from* it; but it is at times better to have it much larger, as a stand-pipe may then answer, especially in a small town, *as a reservoir,* if the pumping should cease for a few hours. A stand-pipe should be cylindrical, not conical; for if thick ice should form on top of the water in a conical one, a sudden forcing of it upward by the pump might strain the stand-pipe seriously. The stand-pipes connected with the Philadelphia Water-Works are from 125 to 170 feet high; 5 feet diameter; and made of riveted boiler-iron about $\frac{5}{8}$ inch thick near the base, and about $\frac{1}{4}$ inch near the top. They have no protection from the weather; nor are they braced in any manner; but retain their positions by their own inherent strength, although exposed at times to violent winds.

Art. 14. The service-pipes for supplying single dwellings, are of lead; and of ½ to ⅝ inch bore. They are connected with the street mains, *n n*, Fig 45, by a **brass ferrule,** *f*, here shown at ⅓ real size. The dotted lines show its ½ inch bore. The tapering ferrule is merely hard driven into a cylindrical hole reamed out of the main, as at *s*. The lead pipe, *o*, is attached to the other end of the ferrule; overlapping it about 1½ ins; and the joint soldered, *t*. The extra thickness near *f*, is for giving proper shape and strength for hammering the ferrule into the main. The pipe and solder are shown in section. Besides the stopcocks attached to each service-pipe, and to its branches through the house, there is an underground one by which the city authorities can stop off the water in case of delinquency in payment of dues; and another by which the plumber can stop it off when so required during indoor repairs. **Galvanized iron tubes** are being much used for service-pipes, especially for hot water; being less subject to contraction and expansion, which produce leaks.

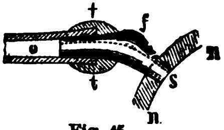


Fig. 45.

Art. 15. The so-called "**corporation stops**" or "corporation cocks" are inserted into the pipe by a special machine, Fig 46. Their great advantage over the ferrule, Fig 45, is that **they can be inserted into a pipe when the latter is full of water under pressure.** Besides, inasmuch as they are *screwed* into the pipe, they are in no danger of being forced out of it by any pressure within it. As their name implies, they are furnished with a stop-valve, which is kept closed while the valve is being inserted into the pipe, and is then opened, and generally remains open permanently.

Pipe-tapping machines, for drilling and tapping the pipe, and for attaching these stops, are made in a variety of forms. Fig 46 shows one made by **Walter S. Payne & Co.,** Fostoria, Ohio. Each of these machines is furnished with a number of malleable cast-iron saddles, which fit the various diams of pipe with which it is to be used. The saddle is not shown in the fig. It is fastened to the pipe by a chain slung around the latter. The chain is tightened by a bolt and nut at each end.

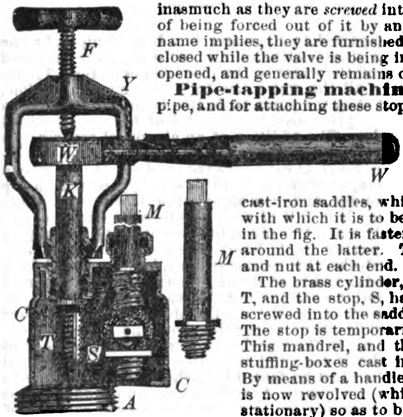


Fig. 46.

The brass cylinder, C C (into which a tap-and-drill, T, and the stop, S, have first been inserted), is then screwed into the saddle by means of the thread at A. The stop is temporarily screwed on to a mandrel, M. This mandrel, and the drill-shank, K, pass through stuffing-boxes cast in one with the head of the cyl. By means of a handle, not shown in the fig, this head is now revolved (while the body of the cyl remains stationary) so as to bring the drill, T, and stop, S, into the respective positions shown in the fig. When the cyl head comes to the proper position, it is stopped by a lug inside of the cyl. The drill is then immediately over the center of a large circular opening in the base of the cyl, C C, and over a similar opening, through the saddle, to the surface of the pipe to be tapped. It is then pushed down until it touches said pipe. The ratchet-wrench, W W, is then set on the square head of the drill-shank, K; the feeder-yoke, Y, with feed screw, F, is put in position as shown; and the pipe is drilled and tapped by working the wrench; whereupon the water in the pipe, if under pressure, rushes out through the hole thus made, and fills the cylinder.

By reversing the position of the switch on the ratchet in the wrench, W, and by working the latter, the tap is now withdrawn from the hole, but remains in the cyl. The cyl head is now revolved so as to reverse the positions of S and T; the lug inside of the cyl stopping the head when the *stop* is immediately over the hole. By means of the ratchet-wrench, applied to the square head of the mandrel, M, the stop, S (*the valve of which must be closed*), is now screwed into the hole, but only far enough to hold securely, and thus prevent the further escape of the water from the pipe when the machine is now removed. The stop is now screwed firmly into place by means of a wrench applied to a square on the stop itself. When the pres in the pipe exceeds about 200 lbs per sq inch, the feeding apparatus, as used with the *drill* (see [illegible]), may be also used in aiding the insertion of the stop.

The mandrel, M, is made in two lengths (one of which screws into the other) in order that the upper part may be out of the way of the wrench-handle while drilling. It has three or more diff threads at its foot, to suit diff sizes of stop. Stops, made to suit the machine, are furnished as wanted.

The machine can work in any direction radial to the pipe, and can therefore be used for tapping a pipe in any part of its circumference.

After the stop is inserted, the service-pipe is attached to its outer end by a coupling nut passing over the thread there shown.

The machines are guaranteed to tap under a pressure of 600 ℔s per square inch.

Art. 15 a. The pneumatic dome Figs. 46 *a*, and 46 *b*, invented by Mr. N. Monroe Hopkins, of Washington, D. C., is designed to prevent the bursting of water-pipes in freezing weather.

In unprotected pipes, the water, in freezing, is unable to expand longitudinally, and therefore frequently bursts the pipe in expanding laterally. The domes, being placed in the pipe, as shown, at intervals of about 12 feet, where freezing is to be apprehended, permit the longitudinal expansion, which pushes the ice in both directions toward each dome, where it compresses the air-cushion there provided. In the horizontal dome, Fig. 46 *a*, the double inclined planes, cast in its lower side at *c*, compel the two horizontal columns of ice to rise into the dome, instead of merely abutting endwise against each other.

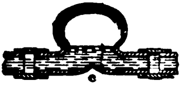


FIG. 46 *a*.

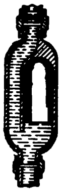


FIG. 46 *b*.

In order to insure that the domes in a system (such as those for a house or mill or on a bridge) shall not be deprived of their air by the flow of the water in the pipes below them, an inspirator is placed in the pipe at the entrance to the system. The inspirator consists essentially of a constriction in the pipe, which increases the velocity of flow at that point, and thus causes an indraft of air through a valve provided for the purpose (see Venturi meter, page 532). The air, thus introduced, is carried along the pipe in bubbles between the surface of the water and the top of the pipe, and is entrapped by the domes. When, by closing faucets, etc., the flow in the pipe is checked, the increase of pressure closes the valve.

Severe tests of both large and small pipes (4-inch and ¾-inch), protected by these domes, have shown them to be always effective in preventing rupture.

Art. 16. Stop-valves, or **gates,** opening vertically in grooves, are placed across the street pipes at intervals of from 100 to 300 yds. Their use is to shut off the water from any section during repairs; the water of such sections being allowed to run to waste, and to soak into the ground.

The details are much varied by diff makers.

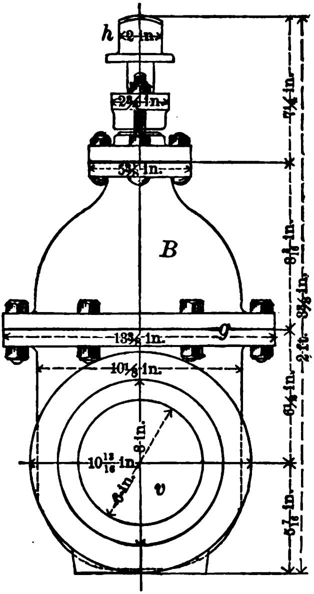


Fig. 47.

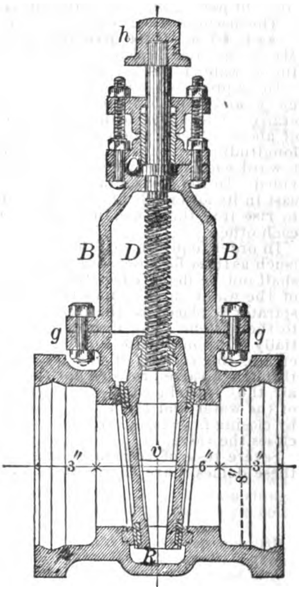


Fig. 48.

Figs 47 and 48 show such a gate made by Chapman Valve Mfg Co, Indian Orchard, Mass. The valve, *v*, is cast in one piece. When down, as in the figs, it closes the pipe. As in other styles, it opens vert by means of a screw, D, the valve rising into the cast-iron case or box, B B, and leaving, when all the way up, an opening of the full diam of the pipe. The screw is turned by a wrench fitting on its square head, *h*. The screw, D, itself, is prevented from moving vert by the collar, C.

The two principal castings which compose the box or cover are bolted together by means of flanges, *g*. The joint faces of the castings are carefully smoothed; and a thin strip of lead is inserted between them, as a precaution against leaks. The recess, R, admits small particles of foreign matter which might otherwise prevent the gate from closing perfectly. The valve seats are faced with Babbitt metal. At the top of the cover, the screw stem passes through a stuffing-box, which prevents leaking at that point. Very careful workmanship is required throughout.

Chapman Bell-end Water-gates.

Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.	Bore. ins.	Wt. lbs.
2	32	6	195	12	600	20	1700
3	55	7	245	14	843	24	2750
4	116	8	290	16	1080	30	6400
5	135	10	439	18	1475	36	8300

Art. 17. Fig 49 shows an arrangement **with outside screw** for raising and lowering the valve. Here the screw, D, does not revolve, but is attached to the valve, and rises and falls with it, being raised or lowered by turning the wheel, W, at the center of which is a nut through which the screw passes. The nut is fixed in the wheel, and is so confined that it, and the wheel, cannot move vertically.

Art. 18. A four-way stop, or **four-way valve,** Figs 50 and 51, is placed at the intersection of two mains; the four ends of which are attached, respectively, to the four openings, M M M M. At the bottom is an additional opening, connecting, by means of an elbow, H, with pipes running to a fire-hydrant at the street curb. See Arts 20 and 21. Two or more of such bottom openings may be made, if desired, for the supply of as many fire-hydrants. All of the openings are opened or closed at one time by raising or lowering the valve or plug, P, by means of a wrench or key applied to the square head, S, of the screw stem. As in Figs 47 and 48, the screw turns, but is prevented from rising and falling, and the plug moves up and down on the screw.

Inasmuch as all sediment escapes into the bottom opening which leads to the fire-hydrant, the valve is not liable to clogging through this cause. The fire-hydrant, being fed from both of the mains, obtains a fuller supply than would be possible if it were fed, as usual, through only one main.

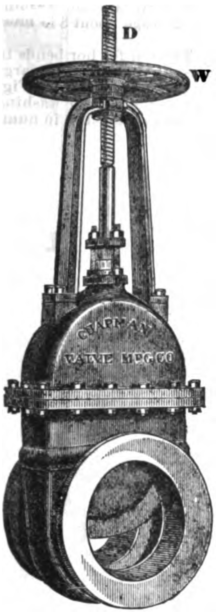


Fig. 49

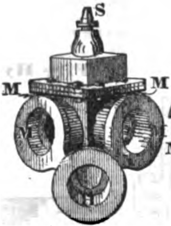


Fig. 50

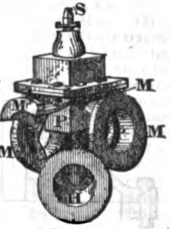


Fig. 51

Art. 19. Whatever the style of the gate may be, it is, when attached to the pipe, **protected by a surrounding box,** generally of plank or cast-iron, with four sides, which taper so that the box is of smaller hor section at top than at bottom. It is open at bottom, but has a movable iron top, level with the street. This top is taken off when the valve is to be opened or closed, or inspected. Two of the opposite sides of the box of course have openings for the passage of the pipes to or from the valve.

The gates, especially of large mains, must be closed very slowly. Otherwise, the too sudden arresting of the momentum of the flowing water would be apt to break either them or the covers; or burst the pipes. As a precaution against this, the covers for very large valves are cast with outside strengthening ribs.

No self-acting air-valves (Fig 44 A) are now placed at street summits, to allow confined air to escape. The fire-plugs answer instead. The rad for hor bends in mains is if possible not less than about 12 times their diams; they are made as large as the widths of the streets will admit; usually about 50 ft. **Fire-plugs,** Figs 52, &c, are placed as much as possible at summits, so as to serve also for washing the streets; and for the escape of accumulated air. They average about 8 in number to each mile of pipe; or 1 to each block of buildings.

confined air to escape. The fire-plugs answer instead. The rad for hor bends in mains is if possible not less than about 12 times their diams; they are made as large as the widths of the streets will admit; usually about 50 ft. **Fire-plugs,** Figs 52, &c, are placed as much as possible at summits, so as to serve also for washing the streets; and for the escape of accumulated air. They average about 8 in number to each mile of pipe; or 1 to each block of buildings.

Fire Hydrants.

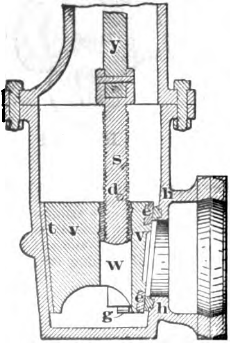


Fig.53

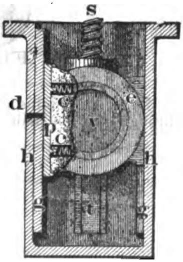


Fig. 54

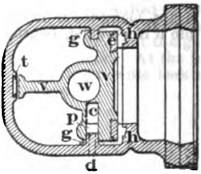


Fig.55

Art. 20. Fig 52 represents a common street **fire-plug, or fire-hydrant.**

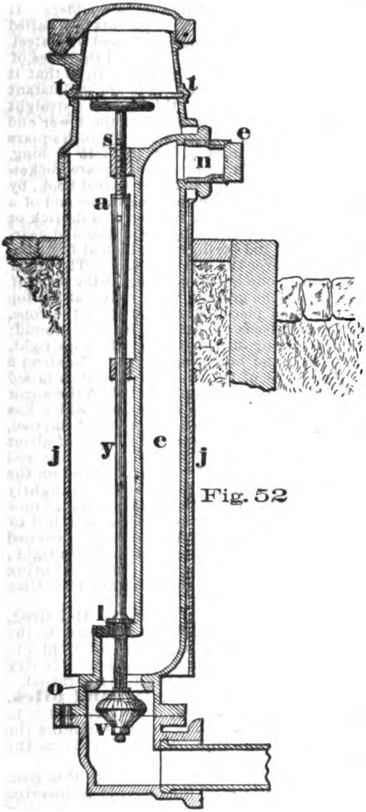


Fig. 52

The valve, *v*, is of layers of well-hammered sole-leather; and, when closed, shuts against a brass ring seat, *o*, which is confined to its place by a lead joint. The valve is opened by working down the screw, *s*, which, by means of a swivel joint at *a*, can revolve without turning the valve-rod, *y*. When the valve, *v*, is closed, after the plug has been in use, the chamber, *c*, is full of water, which, if allowed to remain, would be in danger of freezing, and of bursting the plug. But, in closing the valve, we raise the flange, *l*, on the rod, and thus allow the water to escape through the opening at *l*, whence it runs to waste into the ground, through the open lower end of the **frost-jacket,** *j j*, which is a hollow cast-iron cylinder surrounding the working parts of the hydrant. Being free to slide vert it rises and falls when the level of the ground is disturbed by frost, and the hydrant is thus protected against injury from this cause.

The top, *t*, of the hydrant case, is cast in one piece with the chamber, *c*.

The stopper, *e*, screws on over the nozzle, *n*.

Art. 21. In the **Chapman fire-hydrant,** Figs 53, 54, and 55, made by the Chapman Valve Mfg Co, Indian Orchard, Mass, **the valve, v v, is a *sliding* one.** The stem, *y*, Fig 53, to which the screw, *s*, is attached, is, like that in Figs 47 and 48, prevented from moving *vert* by a collar, fast to it near its top, and confined in a circular groove. When the rod and screw are made to revolve, by means of a wrench applied to the square head of the former, the valve slides up or down on the screw, admitting the water to, or shutting it off from, the hydrant. The valve slides on the two guides, *g g*, which are cast in one piece, respectively, with the two *vert* sides of the lower part of the hydrant. Its circular face, where it comes into contact with the hydrant case, *h*, is faced with a gun-metal ring, *e e*, which bears against a similar ring, made of Babbitt metal, let into the hydrant case.

The water, left in the hydrant case after closing the valve, **escapes** through a cylindrical hole, *d*, about ⅜ inch diam, bored through the guide, *g*, and case, *h*. This hole is at such a ht as to be just above the top of the loose plate, *p*, when the valve is closed, as in Fig 53. This plate lies in a vert groove in the side of the valve-casting, and is pressed against the side of the case, *h*, by two spiral springs confined in cavities, *c c*, in the valve casting. When the valve begins to rise, the hole, *d*, is closed by the plate, *p*, and remains so until the valve is again entirely closed.

Test Borings. Figs 1 and 2 show a tool **for boring into soils, clay, sand,** or **gravel,** even when quite indurated, or when frozen. It will not bore through hard rock, or through large boulders. It consists of two sheet-iron cylindrical segments S S, called "pods," having their lower or cutting edges shod with steel. These edges project (as shown in Fig 1) beyond the sides of the auger, and thus make the hole larger than it, so that it cannot bind or stick. The two cutting edges are equidistant from the vert cen line of the tool, and this insures a straight and vert hole. At *a* the auger is attached to the lower end of a vert boring rod composed of a number of 1½-inch square iron bars, or 2½-inch iron tubes, about 10 to 15 ft long, jointed together at their ends by means of square socket-joints. At the top of this boring-rod is a swivel-hook, by means of which the entire apparatus is hung to the end of a rope, which passes over a pulley at the top of a derrick or tripod, and down to a drum worked by a windlass and gearing. By means of this drum and rope, the auger and boring-rod (which at first consists of only one bar) are lifted, and suspended over the intended hole. The auger is then lowered, and rotated hor by two men or one horse, working at the ends of levers which grip the boring-rod a few ft above the ground. The swivel at the top of the boring-rod permits this rotation to take place without twisting the rope. The shape of the auger is such that its rotation feeds or screws it into the ground; and the man at the windlass has, during the boring, merely to keep the rope tight, so as to prevent the auger from boring too fast, and becoming clogged. In about 8 revolutions the auger fills with earth. By means of the windlass it is then raised to about 2 ft above the ground; and by unkeying and removing the band *b* the auger is opened like a pair of tongs, and the earth emptied into a wooden box which has in the meantime been placed over the hole. The box is then removed and emptied, and the boring proceeds as before. When the boring has reached a depth of about 10 ft, a second bar must be added to the top of the rod. For this purpose the rod and auger are raised a few inches; a slight frame-work of boards is placed on the ground, close to the boring-rod and surrounding it; and a flange is clasped tightly to the rod just above, and close to, the framework. The framework and flange now support the rod and auger; the swivel-hook and rope are removed, and attached to the upper end of the second bar, which is then raised, and its lower end is fastened into the socket-joint upon the top of the first one. The rope is then drawn tight; the flange removed; the auger lowered to the bottom of the hole; and the boring resumed. Additional lengths of boring-rod are attached in the same way from time to time, as required by the descent of the auger.

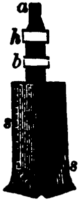


Fig 1.

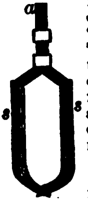


Fig 2.

The borers may be made from 6 to 18 inches in diameter, or larger. If desired, the boring may be made from 24 to 36 ins diam by attaching a reamer to the auger. This auger will bore to a depth of 100 ft or more at the rate of from 5 to 20 ft per hour. It removes stones as large as half the diam of the hole. In dry soils a bucketful of water is poured into the hole each time the auger is raised.

This borer may be advantageously used in boring the holes for **sand piles,** and at times, **instead of driving wooden piles,** it may be better to *plant* them (butt down if preferred) in holes bored by this auger; ramming the earth well around them afterwards. This will save adjacent buildings from the jarring and injury done by a pile driver.

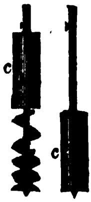
If sand, mud, or loose gravel is reached in boring with this tool, the hole is reamed out 4 ins larger, and a **tubing** of inch boards is inserted into the hole, and driven into and through the sand or gravel, which is then removed from within the tubing by means of a **sand-pump,** consisting of a hollow iron cylinder, about 5 ins diam × 30 ins long, with a valve at its foot, opening upward. The sand-pump is lowered to the bottom of the hole; covered with water to a depth of 2 to 4 ft, and churned quickly up and down 4 to 6 ins, by hand, 20 or 30 times, during which the sand fills the pump, which is then drawn up and emptied. From 10 to 20 ft in depth of sand, mud, &c, per hour can thus be taken from a 6 to 18-inch hole. This pump is also used for removing broken earth, &c, from a hole bored in compact earth by the borer first described.

The cost, with derrick, boring-rods, rope, sand-pump, &c, &c, complete, is about $175. **The auger weighs** from 150 to 200 lbs, according to size. Boring-rod 1½ ins sq, 3½ lbs per ft. Derrick, 150 lbs.

The sand-borer, Figs 3 and 4, like the sand-pump just described, is used inside of tubing, and for the same purpose. The hollow iron cylinder C, 10 ins diam × 30 ins long, slides vertically on the rod, but the screw is fast to the rod. While boring, the sand below and around the cyl keeps it in the position shown in

Fig 3. Six revolutions of the rod and screw fill the cyl with sand. The rod is then lifted. This first draws the screw up into the cyl, as in Fig 4; and a valve at the foot of the screw closes the bottom of the cyl, and prevents the sand from falling out when the borer is lifted from the hole. The rod is hollow, and open at top and bottom. This allows passage of the air, and thus prevents resistance from suction in withdrawing the borer. This tool is rotated and withdrawn in the same way as the earth borer first described. Cost about $30.

Steel prospecting augers, from 2 to 4 ins diam, and 2 ft long, are used for boring holes from 2½ to 6 ins diam, and to depths of 10 to 50 ft, into **clay, sand,** or **fine gravel,** of all of which they bring up samples. They are turned by wrenches, and by man or horse power. See also p 674.



FIGS 3, 4

The boring tool shown in vert section by Fig 6, and in hor cross section by Fig 5, is very useful **for boring shallow holes by hand through surface soils,** clay, and gravel, and bringing up samples. The borer proper consists of a cylinder of spring steel, 3 or 4 ins diam, and 4 or 5 ins high, with sides ¼ inch thick, having a vert slit (see cross section) throughout its height, and beveled to a cutting edge all around its foot, as shown in the vert section. At its top it is riveted, as shown, or welded, to the inverted-v-shaped forging, which, by means of the socket at its top, is screwed to a length of gas-pipe which serves as a handle, and to which other pieces are joined by sockets as boring proceeds.

The boring is done by two men, who grasp the handle, and, holding the tool vert, drive it into the ground by repeatedly lifting it and forcibly bringing it down upon the same spot. As the tool strikes the ground, the beveled shape of its cutting edge causes it to open slightly, and when the downward pres is relieved in lifting it, it springs back and grasps the earth which has entered it. It soon fills; and the men, finding that it ceases to penetrate readily, lift it to the surface and empty it. The character of its contents from different depths, measured along the handle, is noted from time to time.

In six days of 8 hours each, three men (one resting at intervals) using one such auger between them, bored 20 holes, averaging 9½ ft each, in loam, gravel, clay, and decomposed mica schist, at a cost of 22 cts per foot. Wages of each man, $2 per day.

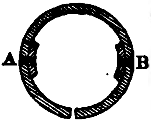


FIG 5.

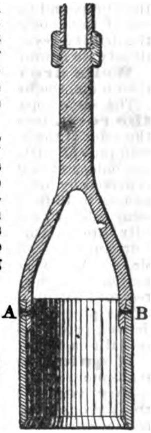


FIG 6.

For work in loam, clay, or non-running sand, **an effective screw-auger** can be made by any good blacksmith, by merely forming a one-inch sq bar of iron or steel into corkscrew shape about 2 ft long, with 6 complete turns 6 ins in diam; its lower end sharpened to form a vertical cutting edge, which should project say .5 of an inch beyond the spiral of the screw, in order to diminish friction. It will bring up full samples. Requires a derrick, or some other simple mode of lifting, when the screw is full.

Artesian Well Drilling. Deep vert holes in earth and rock, 6 and 8 ins in diam, such as are reqd for artesian wells for water and oil, and for mining explorations, are drilled by repeatedly lifting and dropping, in the same vert line, a heavy iron bit, Fig 1, p 673, with a steel cutting-edge. The bit is partly revolved horizontally after each blow, to insure roundness of hole. The length of the cutting-edge of the bit is a little greater than the diam of the bit, and the hole is thus made sufficiently large to prevent the bit from binding in it.

The bit is the lowest one of a series of iron and steel bars, &c, screwed together at their ends, and called a "**string of tools.**" The string of tools varies in length from 25 to 60 ft, according to the size and depth of the hole, and the hardness of the rock; and its diam throughout (above the cutting-edge) is an inch or two less than that of the hole. Its weight is from 800 to 4000 lbs. Its upper member is always a "rope-socket," Fig 4 (*without* a swivel), to which the lower end of the supporting rope **cable** is attached. This cable passes up out of the hole to a **hor lever,** which, by means of a horse-power or steam-engine, is kept constantly moving up and down with a see-saw motion. The string of tools, with the cutting-edge of the bit at its lower end, is thus alternately lifted from 2 to 4 ft, and let fall, from 30 to 50 times per minute, and so drills its way into the rock or earth. From 4 to 10 ft in depth of water are kept in the hole, to facilitate the drilling and the removal of debris. After water is reached, the drilling may be continued, even if the hole is full of water; but a great depth of water of course diminishes the force of the blows of the bit. A suitable arrangement must be provided for **paying out the rope** as the boring tool descends. A clamp is attached to the cable; and the man in charge, by turning the clamp, twists the rope, and thus **turns the bit horizontally** about one-fifth of a revolution after each stroke, until six or eight complete revolutions have been made in one direction. He then reverses the motion, and makes an equal number of turns, at the same rate, in the opposite direction.

After drilling a few feet, the string of tools is lifted out of the hole by means of the cable, to allow the **removal of the debris** which has accumulated in the hole. This is done by means of a **sand-pump,** which is a sheet-iron cylinder, say 4 ins diam, and 4 to 6 ft long, provided, at its foot, with a valve opening upward. The pump is lowered to the bottom of the hole, and filled with the mixed water and debris by churning it up and down a number of times. Sometimes, in addition to the valve, the pump is fitted with a plunger, which is at the foot of the pump when the latter is let down to the bottom of the hole. The plunger is then drawn up into the pump, and the debris follows it. In either case, the pump, when filled, is lifted out of the hole and emptied; the string of tools is again lowered into the hole, and the drilling resumed. The debris must be removed after every 3 to 5 ft of drilling. Otherwise it would interfere too greatly with the action of the bit.

Wells are usually drilled from 6 to 8 ins diam. For diams less than 6 ins, the tools are so slender that they are liable to be broken in a deep hole.

The same apparatus is used for **drilling through the earth above the rock,** before the latter is reached. This is called "spudding." In this case the sides of the hole must be prevented from caving in. For this purpose a wrought-iron pipe of such diam as to fit the hole closely, and ¼ inch thick, is inserted into the hole, and is driven down from time to time as the drilling proceeds. The pipe is driven by means of a heavy maul of oak, or other hard wood, 14 to 18 ins square, and 10 to 16 ft long. This maul is attached, by one end, to the lower end of the same cable which, during drilling, supports the string of tools. It is thus repeatedly lifted, and dropped upon the head of the tube, which is protected by a cast-iron "driving-cap." The foot of the tube is shod with a steel cutting-edge ring, or "steel shoe." When the tube has been driven as far as it will readily go, the maul is removed from the end of the rope; the string of tools substituted; and the drilling resumed within the pipe.

The pipe is put together in lengths of from 8 to 18 ft, and the drilling and pipe-driving proceed alternately until the rock is reached, and the foot of the pipe forced into it to a depth of a few ins, or far enough to shut off quicksand or surface water.

If quicksand is encountered, the string of tools is removed, and the sand-pump is used inside of the pipe.

For reaming out, or **enlarging, holes,** or for **straightening** crooked ones, &c, special tools, such as reamers, &c, are substituted in place of the boring bit.

Special care must be taken to have **all the rubbing surfaces thoroughly lubricated.** The pulley in the mast-head, and the pinion-wheels of the horse power (if such be used) should be well oiled every two or three hours.

In very **cold or wet weather, a shed of rough boards,** or a covering of canvas, about 8 ft high, should be erected, to protect the men; and, if steam is used, 2 or 3 boards should be used as a covering for the belt, which will slip if wet.

The following description is based upon the machines made by the Pierce Well Engineering and Supply Co.*

For holes from 200 to 1000 ft deep, **portable drilling machines,**† worked by horse or steam power, are used. In these machines, the drill-rope, extending from the string of tools up out of the hole, passes over a sheave at the top of a wooden mast; down to, and around, a pulley fast to the working lever; and thence, by way of a pulley fixed at the foot of the mast, to a drum upon which it is wound. To this drum a friction and ratchet wheel is attached, for paying out the cable as the tools descend.

The mast is hinged six feet above its foot, so that its upper part may be laid hor when the machine is to be moved. When at work, it is held in position by two timber struts or braces, bolted to it near its top, and having their lower ends fastened to the "**drill-jack,**" which is a light and strong framework, 9 ft long, 8 ft wide, and 4 ft high, at the foot of the mast, containing the working lever which raises the rope and lets it fall, the drum on which the rope is wound, the shaft and cam which work the lever, &c. The operator stands at the foot of the mast, and by means of foot- and hand-levers within his reach, regulates all the movements of the machine. One of these governs the pawl and ratchet wheel regulating the paying out of the cable. By letting the ratchet-wheel of the drum move one notch, the bit is fed down quarter of an inch.

---

* See Business Directory, No. 484.

† See Price-list, 3.67.

The operator, by moving a slide with his foot, holds the working lever down, out of reach of the cam, thus stopping the up-and-down motion of the rope and tools. By means of another lever he can now put the rope-drum in gear with the main driving-shaft, so that the rope is wound up on the drum, and the tools drawn up out of the hole. Another lever controls the separate reel on which the light rope, carrying the sand-pump, is wound. All these operations are performed by the same power (horse or steam, as the case may be), which works on without stopping; the various changes being made by merely throwing the different parts into, or out of, gear with the main driving-shaft.

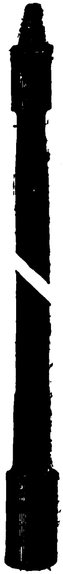


FIG. 2.

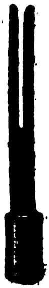


FIG. 4.

One of these portable machines requires two horses or a small steam-engine, a man to attend the same, and another man to operate the machine, empty the sand-pump, change the tools, &c. It can be transported on a farm wagon over any common road. Two men can unload it, set it up, and commence drilling, in two hours; and, unless steam is preferred, the two horses used for its transportation furnish the motive power. The machine can be taken down and reloaded in the wagon in two hours.

Figs 1 to 4 show the **tools used with these machines.** For the different sizes of machine they differ chiefly in their dimensions and weights.

Fig 1 shows the **drilling bit,** which is 30 to 36 ins long, and weighs about 100 lbs. Its lower or cutting edge is 6 ins long. Its top is screwed into the foot of the "**auger-stem,**" Fig 2, which is of 3-inch round iron, 12 ft long, and weighs 350 lbs. Its use is that of a weight, giving additional force to the blows of the bit. Its top is screwed into the foot of the "**drill-jars,**" Fig 3; and to the top of these is screwed the "**rope-socket,**" Fig 4, to which the drilling cable is attached. **If the bit, or auger-stem, becomes wedged in the hole by any means,** the operator stops the churning motion of the tools, and the rope is let out about 12 ins. This permits the upper link U of the drill-jars, Fig 3, to slide down about 12 ins in the slot S in their lower link. The churning motion is then started again, and the upward jerk of the link U against the upper end of the slot loosens the tools.

These machines are made in a number of sizes, to drill holes from 200 to 1000 feet deep. The string of tools weighs from 800 to 1800 lbs.: and the machine complete including tools, rope, mast, etc., but exclusive of power. from 1800 to 4500 lbs. They cost from $700 to $1500 exclusive of power. The smaller sizes may be worked by horse power. A horse power weighs about 800 lbs., and costs about $75. Steam engine, 1600 to 3600 lbs., $150 to $300.

For wells from 1000 to 3000 feet deep, a stationary machine, with a walking-beam, is used, similar to those employed in the oil regions of Pennsylvania. A square pyramidal derrick is erected, 74 feet high, 20 feet square at base, 4 feet square at top. Each of its 4 corner legs is of 2 inch × 8 inch and 2 inch × 10 inch planks, spiked together so as to form a 10 inch × 10 inch angle-piece, 2 inches thick. The legs are braced together by horizontal and diagonal timbers. The walking-beam is of timber, 26 feet long, 12 inches wide, and 26 inches deep at the middle of its length, where it is pivoted to the top of a wooden post 18 inches square and 12 feet high, called a "Samson post." This post, at its foot, is dovetailed into the main sill of the machine, which is 18 inches wide × 24 inches deep.

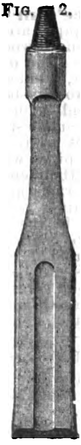


FIG. 1.

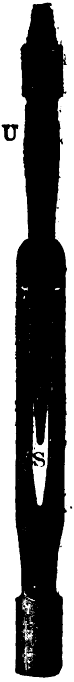


FIG. 3.

The motive power is a 15-hp steam-engine, which, by means of a belt and pulley, crank and pitman, working at one end of the walking-beam, gives to the latter its see-saw motion. To the other end of the beam, and immediately over the well, is suspended, by means of a hook, a "temper-screw." This last is composed of two bars of iron, about ⅝ × 2 ins, 5 ft long, hung 2 ins apart, fastened together at their top ends, at which point there is an eye, which is suspended on the walking-beam hook. At the bottom of the two bars there is a sleeve-nut, and between the two bars and passing through the nut, is a screw 5 ft long, at the bottom of which there is a head, which carries a swivel, set-screw, and a pair of clamps. These grasp the cable, 2 or 2¼ ins diam, which carries at its lower end the **string of tools.** This, for a 2000-ft hole, consists of a steel bit, 3 or 4 ft long, weighing 200 to 400 lbs; an auger-stem of 4 or 5-inch round iron, from 24 to 30 ft long, and weighing from 1200 to 2100 lbs; steel-lined drill-jars 8 ft long, weighing 600 to 700 lbs; a sinker-bar of round iron of same diam as the auger-stem, 12 to 15 ft long, and weighing from 600 to 1100 lbs; and a rope-socket, 2½ ft long, weighing 200 lbs. Total length of string of tools, 50 to 60 ft, total weight, 3000 lbs; or, for an 8-inch hole in the hardest rock, 4000 lbs. **The sinker-bar** is added to give additional wt, and thus to assist in pulling the cable down through the water, either in lowering the string of tools or in working the drill-jars. The shapes of the other tools are given by Figs 1 to 4. **Special tools** are used for recovering articles that may be accidentally dropped into the hole.

The drilling cable is wound on a drum, called a bull-wheel shaft, at the foot of, and inside of, the derrick. While drilling is going on, it passes from the bull-wheel shaft loosely over the sheave at the top of the derrick, and down to the clamps at the lower end of the temper-screw on the end of the walking-beam. As the drilling progresses, the temper-screw is turned or fed out by the man in charge, who also, by means of a clamp, twists the rope, so as to change the position of the bit after each stroke.

When the tools are to be lifted out of the hole, the cable is disengaged from the clamps on the temper-screw, and is wound upon the bull-wheel shaft, which, for this purpose, is thrown into gear with the steam-engine; the pitman being at the same time removed from the crank-pin, so that the walking-beam is at rest. As in the portable machines, the sand-pump is also raised by the same power which does the drilling.

About 10000 ft b m of rough lumber are reqd for the derrick, walking-beam, sills, &c, and about 3000 ft more for sheds over the boiler, engine, and belt.

In ordinary hard limestone rock, such a **machine will drill** about 1½ ft per hour under the most favorable circumstances. **Two men are required;** one to attend to the boiler, sharpen the bits, &c, and one to operate the machine.

In Pierce's machine* for test-boring, mineral prospecting and well boring, the pipes are driven by an iron ram, like that of a pile driver, but bushed with hard wood on its lower or striking end. The ram is worked by a hand winch. The pipes are in lengths of 5 to 10 feet. After each length is driven, water,† under pressure, is forced, by a hand pump, through a hollow drill rod, into the bottom of the hole, while the drill rod is churned up and down by hand. The water forces the drillings (mud, sand, gravel, etc.) to the surface. The smallest machine drives 2 to 3 inch pipes; the largest, 2 to 8 inch. The machines are in detachable parts, weighing from 10 to 65 lbs each. Four upright iron pipes, which carry the head casting and crown pulley, act as guides for the ram, their ends fitting into sockets in castings at their heads and feet. The driving rams are made in sections which are bolted together. In the smaller machines the weight of the ram may thus be made from 100 to 200 pounds, and, in the larger machines, from 100 to 2,000 pounds, as required. Borings can be made to depths of 100 to 400 feet. These machines have been extensively used in Nicaragua by the Isthmian Canal Commission. If desired, the machines can be furnished with special tools for boring in rock and for taking out solid cores (as with the diamond drill), with others for taking out dry cores in earth, and with sand-pumps and mud sockets for bringing up mud, fine sand, gravel, and detached pieces of rock and minerals.

---

* Business Directory, No. 484.

† In Alaska, where the frost extends to great depths, boiling or hot water is used. This is obtained by melting ice or snow in iron tanks about 4 ft square and 2 ft deep.

MACHINE ROCK-DRILLS.

Art. 1. Machine Rock-drills bore much more rapidly than hand drills, and more economically, provided the work is so great as to justify the preliminary outfit. They drill in any direction, and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam directly; or by air, compressed by steam or water power into a tank called a "receiver," and thence led to the drills through iron pipes. The air is best for tunnels and shafts, because, after leaving the drills, it aids ventilation.

Art. 2. Such drills are of two kinds; rotating drills and **percussion drills.** In the former, the drill-rod is a long tube, revolving about its axis. The end of this tube, hardened so as to form an annular cutting-edge, is kept in contact with the rock, and, by its rotation, cuts in it a cylindrical hole, generally with a solid core in the center. The core occupies the core-barrel, Art 8. The drill-rod is fed forward, or into the hole, as the drilling proceeds. The debris is removed from the hole by a constant stream of water, which is led to the bottom of the hole through the hollow drill-rod, and which carries the debris up through the narrow space between the outside of the drill-rod and the sides of the hole.

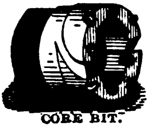
In **percussion drills,** the drill-rod is solid, and its action is that of the churn drill.

Art. 3. In the **Brandt** (European) **rotary drill,** the cutting-edge at the end of the tubular drill-rod is armed with hardened steel teeth. It is pressed against the rock under enormous hydraulic pressure, and makes but from 5 to 8 revolutions per minute.

Art. 4. The Diamond drill is the only form of rotary rock-drill extensively used in America. In it, the boring-rod consists of a number of tubes jointed rigidly together at their ends by hollow interior sleeves.

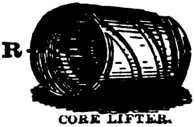
Art. 5. The boring-bit, Fig 1, is called a "core-bit." Its cutting-edge has imbedded in it a number of **diamonds** as shown. These are so arranged as to project slightly from both its inner and outer edges. Annular spaces are thus left between core and core-barrel, and between the latter and the walls of the hole. These spaces permit the ingress and egress of the water used in removing the debris from the hole, and, at the same time, prevent the core from binding in the barrel, or the latter in the hole.

Fig. 1



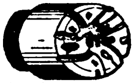
CORE BIT.

Fig. 2



CORE LIFTER.

Fig. 3



BORING HEAD.

Art. 6. Just above the "core-bit," the **"core-lifter,"** Fig 2, is screwed to the barrel. This is a tube about 8 ins long and of the same outer diam as the barrel. Inside it is slightly coned, with the base of the cone upward, and furnished with a loose split-ring, R, toothed inside, and similarly coned. While the drilling is going on, this ring encircles the core closely, and remains loose from the outer cylinder; but when the drilling is stopped, and the drill-rod begins to be raised, the ring is caught and raised by the outer cylinder; and, by reason of its beveled shape, is pressed hard against the core of rock, which is pulled apart close to its foot by the power which lifts the drill-rod.

Art. 7. This power is supplied by a **rope-drum.** fastened to the top of the frame which supports the drill and worked by the same engine which rotates the drill-rod. The rope from the drum passes up to a pulley at the top of a **derrick,** and thence down to the upper end of the drill-rod. The considerable height of the derrick enables from 40 to 50 feet of the drill-rod to be removed in one piece.

Art. 8. Above the "core-lifter" is the **"core-barrel."** This is a wrought-iron tube from 8 to 16 ft long. It is spirally grooved outside, to permit the ascent of the water and debris from the hole; and is sometimes set with diamonds on its outer surface, to prevent wear. The bit, lifter, and barrel are of uniform outer diam, a little less than the diam of the hole. The outer diam of the drill-rod varies from about 1⅞ ins for 2-inch barrel to 5⅛ ins for 12 inch barrel.

Art. 9. Where it is not desired to preserve the core intact, a "**boring-head,**" Fig 3, may be used instead of the "core-bit," Fig 1. This is a solid bit (except that it is perforated with holes which allow the water to pass out from the drill-rod), and is armed with diamonds, some of which project beyond its circumference.

Art. 10. The drill-rod **revolves at a speed** of from 200 to 400 revolutions per minute. **The engine,** by which it is rotated, consists usually of two cylinders, either fixed or oscillating, operated by steam or compressed air, and working at right angles to each other. By means of cranks they turn a shaft, which communicates its motion, through bevel gearing, to the drill-rod. The latter is **fed down,** as the hole progresses, either by other **bevel gearing** driven by the same engine; or by being attached to a cross-head which connects the piston rods of 2 **hydraulic cylinders,** the piston rods being parallel with the drill rod.

Art. 11. The diamond drill bores perfectly **circular holes, in straight lines and in any direction, to great depths;** from 300 to 1500 feet being not uncommon. This, with the fact that **it brings up unbroken cores,** from 8 to 16 ft long, which show the precise nature and stratification of the rock penetrated, renders it very valuable in test-boring, prospecting of mines, &c. They are also furnished of sufficient size to bore holes from 6 to 15 ins diam, for **artesian wells.** The roundness of the holes bored enables the use of casing of nearly as great diam as that of the hole; and their straightness is advantageous in case a pump has to be used.

Art. 12. In soft rock a bit may drill through 200 ft or more without resetting. On the other hand, in very hard rocks, similar drills will wear out in 10 ft or less. In 1883-4, a diamond drill by the Am'n Diamond Rock Boring Co, weighing complete about 1400 lbs, and costing about $2800, bored, in 1428 hours of actual boring, 58 holes of 2 ins diam, and aggregating 9141 lineal ft. Average length of hole 172.5 ft. **Average rate,** 6.4 lin ft per hour; greatest, 12.8. **Average total cost,** about 96 cts per lin ft. The rock was principally limestone, with some quartz and sandstone. The holes were bored at angles varying from 0° to 45° with the vertical.

As a rough average we may say that **in ordinary rocks,** as granite, limestone, and hard sandstone, these drills will bore deep holes, 2 to 3 ins diam, at **from 1 to 2 ft per hour,** and at a cost of from **$1 to $2 per ft.**

Art. 13. These drills are made of many widely **different sizes,** and with **different mountings,** depending upon the nature of the work to be done.

They are sold under restrictions as to the location and extent of the territory in which they are to be used. **The prices** depend, to a great extent, upon the nature of these restrictions. The card prices for some of the leading sizes, are as follows: Discount, see price list.

Diam of hole.	Diam of core.	Depth of hole.	Boiler H. P. required.	Card price.	
				Drill.	Pump
Ins.	Ins.	Feet.	H. P.	$	$
2$\frac{9}{16}$	2	4000	25	4000	....
2$\frac{1}{16}$	1$\frac{1}{2}$	1500	15	2500	3400
"	"	1000	12 to 15	1900	2800
1$\frac{9}{16}$	1	600	10	1400	1900
"	"	400	hand	425	....

Art. 14. In **percussion drilling machines,** the drill-bar is driven forcibly against the rock by the **pressure of steam or of compressed air,** acting upon a piston, P, Fig 4, moving in a cylinder, C C, Figs 4 and 5; and makes about 300 strokes per minute. The rotation of the drill-bar is accomplished automatically, as explained in Art 27.

Art. 15. The cylinder, C C, is free to slide longitudinally in the fixed frame or shell, S S, Fig 5, to which it is attached, and which, in turn, is fixed to the tripod or other stand (see Arts 18 and 19) upon which the machine is supported.

Art. 16. The drill-rod, R, corresponding to the churn drill, is fastened, by an appropriate chuck, K, to the end of the piston-rod, O. The drilling is begun with a short drill-rod, and with the cylinder as far from the hole as the length of the shell, S, will permit. As the bit penetrates the rock, the cylinder is fed forward,* either automatically or by hand (see Art 28), as far as the length of the shell permits. The drilling is then stopped, by shutting off the steam,[*] and the cylinder is run back, by reversing the motion of the feeding apparatus. The short drill-bar is then removed, and, if the drilling is to be continued, a longer one is substituted in its place, and the process repeated.

---

* By **forward,** or **downward,** we mean *toward* the hole which is being drilled. By **backward,** or **upward,** *from* the hole.

Art. 17. Inasmuch as the act of drilling wears the edges of the bit, thus reducing its diam somewhat, **the hole will of course be tapering,** or of slightly less diam at bottom than at top. The second bit must therefore be of slightly less diam than the first; say from $\frac{1}{16}$ to $\frac{1}{8}$ inch less; the third must be less than the second, and so on. On the other hand, in long holes, the drill-bar will seldom be in a perfectly straight line, so that the bit, instead of striking always in the same spot, will describe a circle, and thus enlarge the hole.

Art. 18. The shell, S, in which the cylinder slides, is provided with an arrangement by which it may be clamped, either to a **tripod,** as in Fig 5, or to a long **bar or column,** along which it may slide. The column, if hor, may rest upon two pairs of legs; or it may be braced, in any position, against the opposite sides of a narrow cut, or against the floor and ceiling of a tunnel-heading, &c, in which case one of its ends is provided with a screw which is run out so as to cause the two ends of the col to press firmly against the opposite rock walls; or rather against wooden blocks which are always placed between each end of the col and the rock. In any case, the supports of the drill are so jointed that it can bore in any direction.

Art. 19. Frequently the drill is **clamped to a short arm,** which, in turn, is clamped to the column, and projects at right angles from it. The arm may be slid lengthwise of the column, and may be revolved around it, and thus may be placed in any desired position, and there clamped. This gives the drill a greater range of motion, and enables it to bore holes over a greater space than would otherwise be possible without moving the column.

Art. 20. In tunnels, one or more drills may be mounted upon a **drill-carriage,** travelling upon a railroad track running longitudinally of the tunnel. Upon this track the carriage is moved up to the work, or run back from it when a blast is to be fired. The gauge of the track may be made wide enough to admit of a second track, of narrower gauge, running underneath the drill-carriage. Upon said narrower track the cars are run which carry away the debris. Drill-carriages are less commonly used in this country than in Europe.

Art. 21. The pressure used in the cylinders of percussion drills is usually from about 60 to 70 lbs per sq inch. **In an hour, one will drill** a hole from 1 to 2 ins diam, and from 3 to 10 ft deep, depending on the character of the rock and the size of the machine at from 10 to 25 **cts per lin ft** with labor at $1 per day. **A bit requires sharpening** at about every 2 to 4 ft depth of hole. One blacksmith and helper can sharpen drills for 5 or 6 machines.

Art. 22. The bits are of many different shapes, varying with the nature of the work to be done. For uniform hard rock, the bit has two cutting-edges, forming a cross with equal arms at right angles to each other. For seamy rock, the arms of the cross are equal, but form two acute and two obtuse angles with each other, as in the letter X. For soft rock, the cutting-edge sometimes has the shape of the letter Z.

Art. 23. Each drill requires one man to operate it. Two or three men are required for moving the heavier sizes from place to place. One man can attend to a small air-compressor and its boiler.

Art. 24. Figs 4 and 5 represent the "**Eclipse**" percussion drill of the Ingersoll-Sergeant Drill Co. Havemeyer Building, New York. Fig 5 shows the drill, mounted (as is most frequently the case) upon a tripod. Fig 4 is a longitudinal section through the cylinder, valve-chest, and piston.

Art. 25. The cylinder, C, is provided at each end with a **rubber cushion,** N, for deadening the blows of the piston, which, in all percussion drills, is liable, at times, to strike either cylinder-head. The side of each cushion nearest the piston is protected by a thin iron plate. The cushions have to be renewed from time to time.

Art. 26. The valve, V, is shaped somewhat like a spool. The bolt, B, passes loosely through its center and guides it. Steam is admitted from the boiler to the steam-chest, and occupies all of the space between the two end flanges of the valve, except s. It drives the valve alternately from one end of the valve-chest to the other, and back, according as one end or the other is relieved from opposing pressure by being put into communication with the exhaust, E, by way of the passages, D D′ and F F′. D and D′ communicate with the ends of the steam-chest through passages not shown; while F and F′ communicate, through similar passages, with the exhaust, E. The piston has an annular channel, L L′, encircling it. Whatever the position of the piston, one of the passages, D or D′, is always, by means of this channel, in communication with its corresponding passage, F or F′, leading

---

[*] To avoid repetitions, we will use the word *steam* to signify either steam or compressed air whichever happens to be used.

to the exhaust. Thus, one or the other end of the valve-chest is always in communication with the open air; and to that end the valve is driven by the pres of the steam surrounding it, admitting steam to the cyl, C, from the other end.

Art. 27. The rotation of the piston, and, with it, that of the drill-bar, is effected thus: The spirally-grooved, cylindrical steel bar, A, called a **rifle-bar,** passes through and works in, the **rifle-nut,** H, which is firmly fixed in the end of the piston. and has spiral grooves corresponding with those on the rifle-bar. Said bar is fixed, at its upper end, to the ratchet-wheel, J, the pawls of which are so arranged that, on the *down* stroke of the piston, the rifle-nut, H, acting upon the grooves on the rifle-bar, causes it, and, with it, the ratchet-wheel, to revolve about their common axis. The weight and momentum of the piston, &c, are such that it thus readily turns the ratchet-wheel without itself turning. Thus the bit is prevented from rotating while delivering its blow. But, on the *up* stroke, the tendency of the rifle-nut is to turn the rifle-bar and ratchet-wheel in the *opposite* direction; and as this is prevented by the pawls, the *rifle-bar* remains *stationary*, while the *piston*, *piston-rod*, and *drill* are *made to revolve* about their common axis.

Art. 28. The feed-screw, M, is collared, at its upper end, to the fixed frame, Q. It is thus prevented from moving longitudinally when revolved by means of the crank fixed to its top. Its lower end works in a nut, T, fixed to the cylinder, which last is thus moved longitudinally backward or forward as the crank is turned.

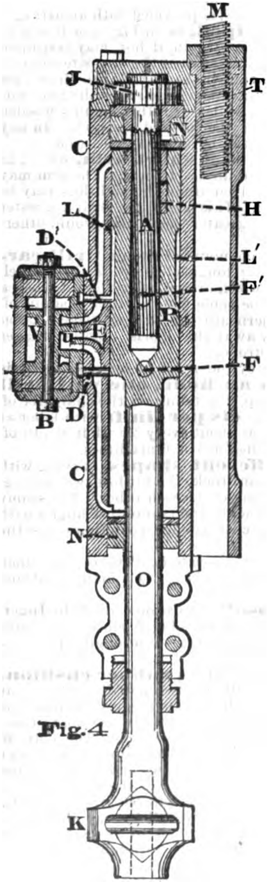


Fig. 4

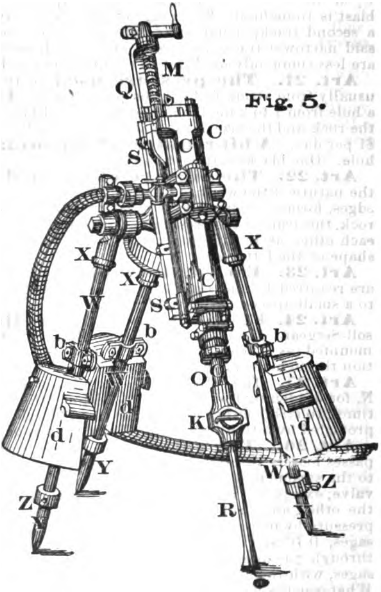


Fig. 5.

Large drills are frequently furnished with an ***automatic* feeding arrangement** in addition to the hand-crank. In this arrangement, when the cylinder requires feeding forward, and when, consequently, the piston is running nearly to the forward limit of its stroke, the piston presses against a cam projecting into the cyl near the forward end, and presenting an inclined plane to it. The motion of this cam, by means of an exterior axle, running alongside of the cyl and furnished at its top with a dog, turns a ratchet-wheel fixed to the feed-screw. When desired, the automatic feed may be thrown out of gear, and the feed moved by hand.

Art. 29. **The tripod legs** consist of wrought-iron tubes, W W. These are screwed at their upper ends into sockets, X X. At their lower ends, they receive the pointed and tapering steel bars, Y Y, about 2 or 3 ft long. The legs may be lengthened or shortened by turning the set-screws, Z Z, thus regulating the distance to which the bars, Y Y, can enter the legs. The clamps, *b b*, have L-shaped hooks of ½ inch to 1 inch round iron forged to them. On these hooks **the weights,** *d d*, are hung, which hold the machine down against the upward reaction of its blows.

Art. 30. The following **table** gives the principal **dimensions** of these drills, with the **diams and lengths of holes** to which each is adapted. Size H is used for submarine work, heavy tunneling, and deep rock cutting. G and F for tunneling, street grading, quarrying, and sewer work. E, D, and C for general mining purposes. B is adapted only for very light work. In asking for estimates on drills and compressors, give the fullest possible description (accompanied by a sketch) of the work to be done, stating its present and proposed extent. State whether the work is on the surface or underground. State how far the steam or compressed air will be carried. Give depth of holes to be drilled, nature of rock, &c. Percussion drills are sold without restriction as to the purpose or extent to which they are to be used.

	Letter designating the size of the machine.							
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Inner diameter of cylinder........ins.	1¾	2½	2¾	3	3⅛	3½	4¼	5
Length of full stroke .... "	3	4	5	6	6	6½	7	7
Length of feed "	12	20	24	24	24	26	34	34
Length of machine*..... "	36	34	36	40	42	53	60	60
Wt of machine, unmounted, lbs.	80	155	195	230	250	345	605	670
Wt of tripod, without wts. "	‡	125	125	125	125	150	275	275
Wt of 3 wts for tripod legs, "		250	250	250	250	350	400	400
Wt of column, arm & clamp "		200	280	280	280	420	420	420
Diam of hole drilled...........ins.	½ to ⅞	¾ to 1¼	1 to 2	1 to 2	1 to 2	1½ to 2½	2 to 4	3 to 6
Max depth of vert † hole.......ft.	‡	4	8	10	12	16	30	40

* From top of handle of feed-crank to lower end of piston at the end of the down stroke.

† For greatest advisable lgth of *hor* holes, deduct one-fourth from these depths.

‡ Machine **A** is mounted on a small frame.

Art. 31. **The drills of different makers differ chiefly** in the methods by which the valve is operated. In some this is done, as in the Ingersoll "Eclipse" drill, Art 26, by the **pres of steam.** In others, the valve is moved by a **lever or tappet,** which projects into the cylinder so as to come into contact with, and be moved by, the piston at each stroke. As these strokes are made with great force some 300 or more times per minute, such valve-gear is necessarily subject to great wear.

Art. 32. In the "**Little Giant Drill,**" made by the **Rand Drill Co.,** the valve, V, Fig 6, is slid backward and forward, in the same direction in which the piston is moving, by the tappet, T, which is pivoted at *p*. The inclined lower corners of this tappet ride up as they come, alternately, in contact with the shoulders, *s s*, of the piston.

Art. 33. In the "**Economizer**" and the "**Slugger**" (Rand Drill Co.) the valve, as in the Ingersoll "Eclipse" drill, is moved by steam, but upon a quite different principle. In these two drills, there is no steam cushion for the piston to strike against on the down stroke, the force of which is thus more completely expended upon the rock. The cushion behind or above the piston, on the return stroke, is formed by exhaust steam. Both of these drills cut off steam before the completion of either stroke, thus using the steam expansively. On the down stroke, the "Economizer" cuts off earlier than the "Slugger." Hence its name. In both machines the point of cut-off is fixed when the machine is made.

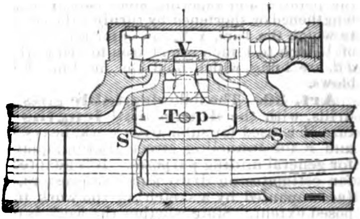


Fig. 6.

Art. 34. In the improved **Burleigh** drill, the valve, V, Fig 7, is moved by **two** tappets, T T′, which are alternately struck by the ends of the piston, P.

Art. 35. In the "Dynamic" rock-drill, invented by **Prof De Volson Wood,** the valve is attached to a valve-piston, V, Fig 8, which is moved backward and forward by steam, which is admitted so as to act alternately upon its two ends. The admission of this steam is controlled by a small auxiliary valve, *a*. A hub on the back of the auxiliary valve fits in the spiral groove shown on the plug, *n*. This plug is constantly pressed downward (as the Fig stands) by steam pressing upon its upper shoulder, but it is lifted at each forward stroke by the conical surface of the piston, P, pressing against its foot. It thus moves constantly up and down, carrying the valve, *a*, with it. By turning the plug, *n*, by means of the adjusting-stem, *s*, the hub of the valve is made to occupy a higher or lower point in the spiral groove, and thus the stroke of the piston may be varied, or may be confined to any part of the cylinder.

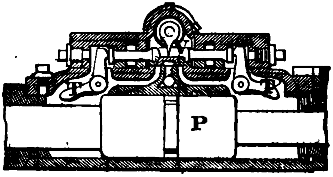


Fig. 7.

In this drill, unlike the Ingersoll, Art. 27, the **piston rotates** while making the *downward* stroke. **The piston-rod, *o*, is made lighter than in other drills.** This gives a greater surface under the piston for the pressure of the steam on the up stroke, and, consequently, greater lifting power. This is useful when the drill sticks in the hole.

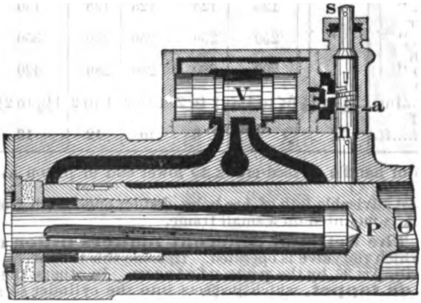


Fig. 8.

The **tripod legs** are of bar iron. Their length is adjustable.

Art. 36. The **Pierce hand rock-drill** is a percussion drill worked by a crank which turns a disc about 2 ft in diam. The disc has a semi-circular slot, in which works the arm which raises the drill-rod. This arm, in rising, compresses a coil-spring, which, on the down stroke, drives the drill against the rock. An iron ball, weighing 30 lbs or more, is furnished with each machine. This ball may be screwed to the top of the drill-rod, for giving greater force to the blows of the drill. The ball may be used without the spring, by disengaging the latter.

The drill makes about 40 strokes of 10 or 12 ins per minute; and bores holes from ¾ to 2½ ins diam. It can be arranged to drill to depths of 30 ft and over. For sharpening the bits, it has an emery wheel attached, which is turned by the crank. The latter, at such times, is thrown out of gear with the disc.

The drill is mounted on a rectangular two-legged frame, about 5 ft high by 2 ft wide, made of iron tubes. To the top of this frame a third leg is attached, by adjusting which the angle of the drill-rod with the vert may be changed. Like other percussion drills worked by hand-power, this one ceases to work to advantage when said angle exceeds about 45°.

Art. 37. **Channeling** consists in making long, deep, and narrow cuts in the rock. In this way large blocks can be gotten out without blasting and the consequent danger of fracture. This is ordinarily done by boring a row of holes about an inch apart in the clear, and then breaking down the intermediate spaces by means of a blunt tool, called a **broach.** This is called **broach channeling.** For this purpose a steam drilling machine is mounted upon a hor bar resting upon two pairs of legs. The hor bar is placed over the intended row of holes, and the drill is slid along upon it from one hole to the next. In using the *broach*, the rotating apparatus is thrown out of gear, so that the edge of the broach maintains its position in line with the row of holes.

Art. 38. **The Saunders patent channeling machine**, of the Ingersoll Co, consists of a rock-drilling machine, having, in place of the usual drilling-bit, a gang of tools consisting of a number of chisels, clamped together side by side, and thus forming a cutting tool about 7 ins long by ¾ inch wide. This tool has as many cutting-edges (each as long as the tool is wide) as there are chisels. The machine is supported upon a carriage, moving on a track parallel with the channel to be cut. The tool is of course not rotated; but the rifle-bar, A, Fig 4, is employed to move the carriage along the track about an inch after each blow. The carriage remains stationary while a blow is being struck. Under favorable circumstances **this machine has cut** from 80 to 100 sq ft of channel per day of ten hours. Its **weight,** including carriage, is about 5000 lbs.

A valve is provided, by which, if desired, the **steam may be shut off** from the piston on the down stroke, so that said stroke may be made with only the *weight* of the piston, rod, and drill.

Art. 39. The Ingersoll Co have a special appliance, designed by Mr. W. L. Saunders, C E, for **drilling and blasting rocks under water,** even when they are covered by a considerable depth of mud.

Art. 40. **Air compressors** for rock-drills, as made and used in this country, are mostly hor, direct-acting engines. That is, the axes of the steam- and air-cylinders are hor; and the piston-rod passes directly from the steam-cylinder into the air-cylinder. A fly-wheel is attached, by a crank and connecting-rod, to the piston-rod. Sometimes the steam-engine is separate from the compressor, and the power is conveyed to the latter by belts or gearing; or water-power may be used in the same way. The air is forced into a receiver, which is generally a plate-iron cylinder, 3 or 4 ft in diam, and 5 to 12 ft long.

If the air- or pumping-cylinder of the compressor is so arranged as to take in air on one stroke only, and force it out into the receiver upon the return stroke, it is "**single-acting.**" If, at each stroke, it both takes in and forces out air, it is "**double-acting.**" If the compressor has only one air-cylinder, it is "**single.**" If it has two, and thus practically consists of two single compressors, it is "**duplex.**"

The **valves** may be either "**poppet**" valves, held in place by springs, and operated by the pressure of the air itself; or **slide** valves, operated by eccentrics and rods, as in steam-engines.

The compression of the air develops **heat.** This is removed either by causing cold water to circulate through the air-piston, and through jackets surrounding the air-cylinder; or by injecting it into the air-cylinder in the form of spray. Or both methods may be used together.

Art. 41. The following partial list of Clayton compressors, compiled from data given by the makers, shows the **dimensions** and **performance** of each. We give also a list of their **receivers.**

CLAYTON DOUBLE-ACTING AIR-COMPRESSORS. Partial List.

	Number, designating the size of the machine.			
	1	2½	4	7
Duplex Direct-acting* Compressors.				
Diam of steam-cylinders ins.	8	10	14	18
" air " ins.	8	10	14	18
Length of stroke ins.	12	13	15	24
Number of revolutions per minute	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute Actual.	136	210	438	900
Approximate wt of compressor lbs.	3000	7000	15000	25000
Approx number of rock-drills with 3-inch cyls supplied with air at 60 to 80 lbs per sq inch	2	4	8	18
Single Direct-acting* Compressors.				
Diam of steam-cylinder ins.	8	10	14	18
" air " ins.	8	10	14	18
Length of stroke ins.	12	13	15	24
Number of revolutions per minute	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute Actual.	68	105	219	450
Approx wt of compressor	1650	3850	8250	13750
Approx number of rock-drills with 3-inch cyls supplied with air at 60 to 80 lbs per sq inch	1	2	4	9

* The price of a compressor alone, to be worked by a separate steam-engine or water-power, is of course less than that of the above compressor and engine combined.

Air-Receivers; vertical and horizontal.

Diameter inches.	Length, Feet.	Approximate weight, lbs.	Diameter, Inches.	Length, Feet.	Approximate weight, lbs.
33	5	700	40	8	1675
30	7	890	40	10	1900
36	8	1560	40	11	2000
40	6	1600	40	12	2100

The Air-Receivers have brass-face pressure-gauge, glass water-gauge, safety-valve, blow-off valve, try-cocks, flanges and connections to automatic feed on compressor.

TRACTION.

Traction on common roads, and canals; or the power reqd to draw vehicles and boats along them. In connection with this subject read the preceding and the following one.

The following table shows tolerable approximations to the force in lbs per ton, reqd to draw a stage coach and passengers, up ascents on the Holyhead turnpike road in England, (a fine road,) by horses; as ascertained by means of a dynamometer. The entire weight was 1¼ tons; but in the table, the results are given per single ton. From the nature of such cases, no great accuracy is attainable.

Proportional Ascent.	Ascent in Ft. per Mile.	At 4 Miles per Hour.	At 6 Miles per Hour.	At 8 Miles per Hour.	At 10 Miles per Hour.
		Lbs.	Lbs.	Lbs.	Lbs.
1 in 15½	340.7	210	216	225	240
1 " 20	264.	196	202	212	229
1 " 26	206.1	155	160	166	175
1 " 30	176.	137	142	147	154
1 " 40	132.	114	120	124	130
1 " 64	82.5	109	115	120	126
1 " 118	44.7	102	107	113	120
1 " 138	38.3	99	103	109	117
1 " 156	33.9	98	101	106	112
1 " 245	21.6	93	96	101	107
1 " 600	8.8	81	86	91	96
Level.	0.	76	80	86	91

The following results, most of them with the same instrument, are also in lbs per ton; with a four-wheeled wagon, at a slow pace, on a level; and the roads in fair condition.

On a cubical block pavement	32 lbs per ton	to 50.
" McAdam road, of small broken stone	62 " " "	probably to 75.
" gravel road	140 " " "	
" Telford road, of small stone on a paving of spawls	46 " " "	" " 75.
" broken stone, on a bed of cement concrete	46 " " "	" " 75.
" common earth roads	200 to 300.	On a plank road 30, to 50 lbs.

The tractive power of a horse diminishes as his speed increases; and perhaps, within certain limits, say from ¾ to four miles per hour, nearly in inverse proportion to it. Thus, the average traction of a horse, *on a level*, and actually pulling for 10 hours in the day, may be assumed approximately as follows:

Miles per hour.	Lbs. Traction.	Miles per hour.	Lbs. Traction.
¾	333.33	2¼	111.11
1	250.	2½	100.
1¼	200.	2¾	90.91
1½	166.66	3	83.33
1¾	142.86	3½	71.43
2	125.	4	62.50

If he works for a smaller number of hours, his traction may increase as the hours diminish; down to about 5 hours per day and for speeds of about from 1¼ to 3 miles per hour. Thus, for 5 hours per day his traction at 2½ miles per hour will be 200 lbs, &c. When ascending a hill, his power diminishes so rapidly, from having partially to raise his own weight, (which averages about 1000 to 1100 lbs,) that up a slope of 5 to 1, he can barely struggle along without any load. On such an ascent. he must exert a force equal to 439 lbs per ton, or of 196 lbs for the 1000 lbs of his own weight. Assuming that on a level piece of good turnpike, he would when hauling a cart and load, together weighing 1 ton, have to exert a traction of 60 lbs; then on ascending a hill of 4° inclination, (or 1 in 14.3; or 369¼ ft per mile,) he would have to exert 156 lbs, against the gravity of the 1 ton: and 67 lbs, against that of his own weight; or 223 lbs in all. He may, for a few mins, exert without injury, about twice his regular traction. This calculation shows that up a hill of 4°, an average horse is fully tasked in drawing a total load of one ton; and should, therefore, be allowed, in such a case, to choose his own gait; and to rest at short intervals. A fair load for a single horse with a cart, at a variable walking pace, working 10 hours per day, on a common undulating road in good order, is about half a ton, in addition to the cart, which will be about half a ton more. With two horses to this same cart, the *load* alone may be about 1¼ tons.

Rem. Since the action of gravity is the same on good roads and bad ones, it follows that **ascents become more objectionable the better the road is.** Thus, on an ascent of 2°, or 184.4 ft per mile, gravity alone requires a traction of 78 lbs per ton;

which is about 10 times that on a level railroad at 6 miles an hour; but only about equal to that on a level common turnpike road, at the same speed. Therefore, (to speak somewhat at random,) it would require 10 locomotives instead of 1; but only 2 horses instead of 1. A grade of 1 in 35; or 150 ft to a mile; or 1° 38′, is about the steepest that permits horses to be driven down a hard smooth road, in a fast trot, without danger. It should, therefore, not be exceeded except when absolutely necessary, especially on turnpikes.

On canals and other waters, the liquid is the resisting medium that takes the place of friction on level roads. But unlike friction, its resistance varies as the squares of the vels; at least from the vel of 2 ft per sec, or 1.364 miles per hour; to that of 11½ ft per sec, or 7.84 m per h. As the speed falls below 1⅛ m per h, the resistance varies less and less rapidly; and this is the case whether the moved body floats partly above the surface; or is entirely immersed. In towing along stagnant canals, &c. the vel is usually from 1 to 2½ m per h; for freight most frequently from 1½ to 2. Less force is required to tow a boat at say 2 m per h, where there is no current, than at say 1¼ m per h, against a current of ¾ m per h, because in the last case the boat has to be *lifted up* the very gradual inclined plane or slope which produces the current.

The force required to tow a boat along a canal depends *greatly* upon the comparative transverse sectional areas of the channel, and of the immersed portion of the boat. When the width of a canal at water-line is at least 4 times that of the boat; and the area of its transverse section as great as at least 6½ times that of the *immersed* transverse section of the boat, the towing at usual canal vels will be about as easy as in wider and deeper water. With less dimensions, it becomes more difficult. (D'Aubuisson.) Much also depends on the shape of the bow and other parts of the boat; and on the proportion of its length to its breadth and depth. Hence it is seen that the mere weight of the load is by no means so controlling an element as it is on land. The whole subject, however, is too intricate to be treated of here. Morin states that naval constructors estimate the resistance to sailing and steam vessels at sea, at but from about .5 to .7 of a lb for every sq ft of immersed transverse section, when the vel is 3 ft per sec, or 2.046 miles per hour. It is far greater on canals.

On the Schuylkill Navigation of Pennsylvania, of mixed canal and slackwater, for 108 miles, the regular load for 3 horses or mules, is a boat of very full build; and no keel: 100 ft long, 17½ ft beam; and 8 ft depth of hold; drawing 5½ ft when loaded.* Weight of boat about 65 tons; load 175 tons of coal, (2240 lbs;) total weight 240 tons, or 80 tons per horse or mule. On the down trip with the loaded boats, for 4 days, the animals are at work, *actually towing*, (except at the locks,) for 18 hours out of the 24; thus exceeding by far the limits of time usually allowed for continuous effort.

On the canal sections, (which have 60 ft water-line; and 6 ft depth,) the speed is 1¾ miles per hour; and on the deep wide pools, 2 miles.

On the up trip with the empty 65-ton boats, the average speed is about 2½ miles per hour. The empty boats draw 16 to 18 ins water; and frequently keep on without stopping to rest day or night through the entire distance of 108 miles. The animals generally have 2 or 3 days' rest at each end of the trip; but are materially deteriorated at the end of the boating season.

If our preceding assumption of 143 lbs traction of a horse at 1¾ miles per hour, is correct, the traction of the loaded boats on the canal sections is $\frac{143 \text{ lbs}}{80 \text{ tons}} = 1.83$ lbs per ton.

The intelligent engineer and superintendent of the Sch Nav, James F Smith, gives as the results of his own extensive observation, that one of these large boats loaded (240 tons in all) may, without distressing the animals, be drawn along the canal sections, for 10 hours per day, as follows: By one average horse or mule, at the rate of 1 mile; by two animals, at 1½ miles; and by three, at 1¾ miles per hour. When four animals are used the gain of time is very trifling. At a time of rivalry among the boatmen, one of them used 8 horses; but with these could not exceed 2½ miles per hour in the canal portions. Two or more horses together cannot for hours pull as much as when working separately.

If our preceding short table of the traction of a horse at diff vels for 10 hours is correct, then the traction of the above loaded coal boats (240 tons) on the canal sections of the navigation, is as follows: The last column shows the traction in lbs per sq ft of area of immersed transverse section where largest; viz, about 95 sq ft.

Horses.	Miles per Hour.		Lbs. per Ton.	Lbs. per Sq Ft.
1	1	$\frac{250}{240}$	1.04	2.63
2	1½	$\frac{333}{240}$	1.39	3.50
3	1¾	$\frac{428}{240}$	1.78	4.50
3 on pools	2	$\frac{375}{240}$	1.56	3.95
3	2½	$\frac{800}{240}$	3.33	8.42
3 up-trip	2½	$\frac{300}{65}$	4.61	12.60

Lachine Canal, Canada, 120 ft wide at water-line; 80 ft at bottom; depth on mitre sills 9 ft; 6 horses tow loaded schooners with ease.

Before the enlargement of the **Erie canal,**† its dimensions were 40 ft water-line; 28 ft bottom; 4 ft depth of water. The average weight of the boats was about 30 tons. With 75 tons of load, or 105 tons total, they were towed by 2 horses, at the rate of about 2 miles per hour; which by our table gives a traction of nearly 2.4 lbs per ton. The boats were about 80 ft long; 14 ft beam; full 3½ ft draught loaded; hence the traction by our table would be about 5.7 lbs per sq ft of immersed transverse section.

---

* **Cost of boats,** 1884, (Schuylkill Canal) about $1800. Annual repairs about $85. Boats last 16 to 20 years. Length, 102 ft; beam, 17½ ft; draft, 1½ to 5½ ft; capacity, 180 tons; weight, about 58 tons; speed, with 3 mules, 1¾ miles per hour.

† Length 363 miles; cost $19680 per mile. The enlarged canal has 70 ft; 42 ft; and 7 ft of water; and cost $90800 per mile for the enlargement only. The cost of the several canals in Pennsylvania has ranged between $23000 and $50000 per mile.

While, for 80-ton loaded boats on a smaller canal, (the boats nearly touching bottom,) the traction at 1½ miles, would be 3½ lbs per ton; or about twice as great as the above 1.78 lbs. It also would be 5.7 lbs per sq ft of immersed section.

ANIMAL POWER.

Art. 1. So far as regards horses, this subject has been partially considered under the preceding head, Traction. All estimates on this subject must to a certain extent be vague, owing to the diff strengths and speeds of animals of the same kind; as well as to the extent of their training to any particular kind of work. Authorities on the subject differ widely; and sometimes express themselves in a loose manner that throws doubt on their meaning. We believe, however, that the following will be found to be as close approximation to practical averages as the nature of the case admits of with our present imperfect knowledge. We suppose a good average trained horse, weighing not less than about ½ a ton, well fed and treated. Such a one, when actually walking for 10 hours a day, at the rate of 2½ miles per hour, on a good *level* road, such as the tow-path of a canal, or a circular horse-path,* **can exert a continuous pull, draught, power, or traction, of 100 lbs.**

Now, 2½ miles per hour, is 220 ft per min, or 3⅔ ft per sec; and since 10 hours contain 600 min, his day's work of actual hauling on a level, at that speed, amounts to

$$\overset{\text{min}}{600} \times \overset{\text{ft}}{220} \times \overset{\text{lbs}}{100} = 13\,200\,000 \text{ ft-lbs per day.}$$

Or, 22000 ft-lbs per min, or 366⅔ ft-lbs per sec.† Which means that he exerts force enough during the day to *lift* 13 200 000 lbs 1 foot high; or 1 320 000 lbs 10 feet high; or 132 000 lbs 100 ft high, &c. He may exert this force either in *traction* (hauling) or in *lifting* loads. If he has to raise a small load to a great height, the machinery through which he does it must be so geared as to gain speed, at the loss (commonly but improperly so expressed) of power. Whether he lifts the great weight through a small height, or the small weight through a great height, he exerts precisely the same amount of force or power.

Experience shows that within the limits of 5 and 10 hours per day, (the speed remaining the same,) **the draft of a horse may be increased in about the same proportion as the time is diminished;** so that when working from 5 to 10 hours per day, it will be about as shown in the following table. Hence, the total amount of 13 200 000 ft-lbs per day may be accomplished, whether the horse is at work 5, 6, or 8, &c, hours per day.‡ This, of course, supposes him to be actually lifting or hauling *all the time;* and makes no allowance for stoppages for any purpose.

Table of draft of a horse, at 2½ miles per hour, on a level.

Hours per day.	Lbs.	Hours per day.	Lbs.
10	100	7	142⁶⁄₇
9	111⅑	6	166⅔
8	125	5	200

Experience also shows that at speeds between ¾ and 4 miles an hour, his force or draught will be inversely in proportion to his speed. Thus, at 2 miles an hour, for 10 hours of the day, his draught will be

$$\overset{\text{miles}}{2} : \overset{\text{miles}}{2\tfrac{1}{2}} :: \overset{\text{lbs}}{100} : \overset{\text{lbs}}{125} \text{ draught.}$$

At 1½ miles, it would be 166⅔ lbs; at 3 miles, 83⅓ lbs; and at 4 miles, 62½ lbs; as per table in Traction.

Therefore, in this case also, the entire amount of his day's work remains the same; § and within all the foregoing limits of hours and speed, may be practically taken to be about 13 200 000 ft-lbs per day; or 22000 ft-lbs per min of a day of 10 hours. But it does not follow that the horse can always in practice actually *lift loads* at that rate; because generally a part of his power is expended in overcoming the *friction* of the machinery which he puts in motion; and moreover, the nature of the work may require him to stop frequently; so that in a *working day* of 8 or 10 hours, the horse may not actually be *at work* more than 5, 6, or 7 hours.

---

* To enable a horse to work with ease **in a circular horse-walk,** its diam should not be less than 25 ft; 30 or 35 would be still better.

† **A nominal horse-power** is 33000 ft-lbs per minute; this being the rate assumed by Boulton and Watt in selling their engines; so that purchasers wishing to substitute steam for horses, should not be disappointed. Their assumption can be carried out by a very strong horse day after day for 8 or 10 hours; but as the engine can work day and night for months without stopping, which a horse cannot, it is plain that a one-horse engine can do much more work than any one such horse. Hence many object to the term horse-power as applied to engines; but since everybody understands its plain meaning, and such a term is convenient, it is not in fact objectionable. Boulton and Watt meant that a one-horse engine would at any moment perform the work of a very strong horse. An *average* horse will do but 22000 ft-lbs per min.

‡ It is plain that although the *day's* labor will be the same, that of an hour, or of a min, will vary with the number of hours taken as a day's work. It must be remembered that a *working day* of a given number of hours, by no means implies, in every case, that number of hours of *actual work;* but includes intermissions and rests.

§ **This remark about speed will not apply to loads towed through the water.** Thus, if his draught at 2 miles an hour be 125 lbs; and at 4 miles, 62½ lbs; he will *on land* draw loads in these proportions; but in hauling a boat *through the water* at the greater speed, he has to encounter the increased resistance of the *water* itself; which resistance at 4 miles is much more than twice as great as at 2 miles; probably 4 times as great. Therefore, at 4 miles on a canal, his draught of 62½ lbs would not suffice for *a load* half as great as he could tow with his draft of 125 lbs at 2 miles.

As a rough approximation, to allow for the waste of force in overcoming the friction of hoisting machinery, and the weight of the hoisting chains, buckets, &c, we may say that **the useful or paying daily net work of a horse, in hoisting by a common gin,** is about 10 000 000 ft-lbs. That is, he will raise equivalent to 10 000 000 lbs net of water, or ore, &c, 1 foot. The load which he can raise at once, including chains, bucket, and an allowance for friction, will be as much greater than his own direct force, as the diam of the horse-walk is greater than that of the winding drum; and it will move that much slower than he does. His own direct force will vary according to the number of hours per day that he may be required to work, as in the foregoing table. With these data, the size of the buckets can be decided on; and of these there should be at least two, so that the empty one at the bottom may be filled while the full one at top is being emptied; so as to save time. The same when the work is done by men.

Art. 2. A practised laborer hauling along a level road, by a rope over his shoulders; or in a circular path, pushing before him a hor lever, at a speed of from 1½ to 3 miles per hour, exerts about ⅙ part as much force as a horse; or 2 200 000 ft-lbs per day; or 3666⅔ ft-lbs per min of a day of 10 hours of actual hauling or pushing.

But laborers frequently have to work under circumstances less advantageous for the exertion of their force than when hauling or pushing in the manner just alluded to; and in such cases they cannot do as much per day. Thus in turning a winch or crank like that of a grindstone, or of a crane, the continual bending of the body, and motion of the arms, is more fatiguing. **The size of a winch should not exceed 18 ins,** or the rad of a circle of 3 ft diam; and against it a laborer can exert a force of about 16 lbs, at a vel of 2½ ft per sec, or 150 ft per min, making very nearly 16 turns per min; for 8 hours per day. To these 8 hours an addition must be made of about ¼ part, for short rests. Or if a *working day* is taken at 8, or 10, &c, hours, $\frac{1}{5}$ part must generally be taken from it for such rests. On the foregoing data an hour's *work* of 60 min of *actual hoisting* would be

$$\overset{\text{lbs}}{16} \times \overset{\text{ft}}{150} \times \overset{\text{min}}{60} = 144000 \text{ ft-lbs};$$

or, deducting $\frac{1}{5}$ part for rests, 115200 ft-lbs per hour of *time, including rests*. In practice, however, a further deduction must be made for the fric of the machine, and for the wt of the hoisting chains; and in case of raising water, stone, ore, &c, from pits, for the wt of the buckets also. As a rough average we may assume that these will leave but 100 000 ft-lbs of paying, or useful work per hour; that is, that **a man at a winch will actually lift equivalent to 100000 lbs of water, ore, &c, 1 foot high per hour's time, including rests.** This is equal to 1666⅔ ft-lbs per min of a day of 10 hours, including rests. Therefore, in a day of 10 working hours he would raise 1 000 000 lbs net, 1 foot high; **or just $\frac{1}{10}$ part of what a horse would do with a gin** in the same time. We have before seen that in hauling along a level road, he can at a slow pace perform about ⅙ of the daily duty of a horse. He may also work the winch with greater force, say up to 30 or even 40 lbs; but he will do it at a proportionately slower rate; thus, accomplishing only the same daily duty. **With a gin, like those for horses,** but lighter, with 2 or more buckets, a practised laborer will in a working day of 10 hours, raise from 1 200 000 to 1 400 000 ft-lbs net of water, ore, &c. With a shallow well or pit, more time is lost in emptying buckets than in a deep one; but the deep one will require a greater wt of rope. To save time in all such operations on a large scale, there should be at least two buckets; the empty one to be filled while the full one is being emptied. It is also best to employ 2 or more men to hoist at the same time, by winches, at both ends of the axis; and the men will work with more ease if the winches are at right angles to each other. Each winch handle may be long enough for 2 or 3 men. An extra man should be employed to empty the buckets. He may take turns with the hoisters. The same remarks apply in some of the following cases.

On a treadwheel a practised laborer will do about 40 per cent more daily duty than at a winch; or in a working day * of 10 hours, including rests, he will do about 1 400 000 ft-lbs. And he can do this whether he works at the outer circumf of the wheel, stepping upon foot-boards, or tread-boards, on a level with its axis; or walks inside of it near its bottom. In both cases he acts by his wt, usually about 130 to 140 lbs; and not by the muscular strength of his arms. When at the level of the axis, his wt acts more directly than when he walks on the bottom of the wheel; but in the first case he has to perform a slow and fatiguing duty resembling that of walking up a continuous flight of steps; while in the second he has as it were merely to ascend a very slightly inclined plane; which he can do much more rapidly for hours, with comparatively little fatigue: and this rapidity compensates for the less direct action of his wt. Therefore, in either case, as experience has shown, he accomplishes about the same amount of daily duty. Treadwheels may be from 5 to 20 ft in diam, according to the nature of the work. They are generally worked by several men at once, and may at times be advantageously used in pile-driving, as well as in hoisting water, stone, &c.

By a good common pump, properly proportioned, a practised laborer will in a day of 10 working hours, raise about 1 000 000 ft-lbs of water, net.†

Bailing with a light bucket or scoop, he can accomplish about 200 000 ft-lbs net of water. **By a bucket and swape,** (a long lever rocking vertically, and weighted at one end so as to balance the full bucket hung from the other; often seen at country wells,) 600 000 to 800 000. In the last he has only to pull down the empty bucket, and thereby raise the counterweight. **By 2 buckets at the ends of a rope suspended over a pulley,** 500 000 to 600 000. Here he works the buckets by pulling the rope by hand.

---

* The working day must be understood to include necessary rests, and such intermissions as the nature of the work demands; but does not include time lost at meals. A *working day* of 10 hours may, therefore, have but 8, 7, or 6, &c hours of *actual labor*. This will be understood when we hereafter speak of a working day, or simply a day.

† Desaguliers's estimates of daily work of men and horses exceed the above, but are entirely too great

By a tympan, or tympanum,* worked by a treadwheel, about 1 200 000 to 1 400 000.

By a Persian wheel,† a chain-pump, a chain of buckets,‡ or an Archimedes screw, all worked by a treadwheel, from 800 000 to 1 000 000 ft-lbs. Of these four, the first three lose useful effect by either spilling, leaking, or the necessity for raising the water to a level somewhat higher than that at which it is discharged.

When any of the five foregoing machines are worked by men at winches, the result will be about ⅕ less than by treadwheels. They are all frequently worked also by either steam, water, or horse-power.

By walking backward and forward, on a lever which rocks on its center, a man may, according to Robison's Mech Philosophy, perform a much greater duty than by any of the preceding modes. He states that a young man weighing 135 lbs, and loaded with 30 lbs in addition, worked in this manner for 10 hours a day without fatigue; and raised 9¼ cubic feet of water, 11½ ft high per min. This is equal to 3 984 000 ft-lbs per day of 10 hours; or 6640 ft-lbs per min; or nearly $\frac{4}{10}$ of the net daily work of a horse in a gin.

A laborer standing still, can barely sustain for a few min, a load of 100 lbs, by a rope over his shoulder, and thence passing off hor over a pulley. And scarcely as much, when (facing the load and pulley) he holds the end of a hor rope with his hands before him. He cannot push hor with his hands at the height of his shoulders, with more than about 30 lbs force.

Weisbach states from his own observation, that 4 practised men raised a **dolly** (a wooden beetle or rammer, of wood; with 4 hor projecting round bars for handles) weighing 120 lbs, 4 ft high, at the rate of 34 times per min, for 4½ min; and then rested for 4½ min: and so on alternately through the 10 hours of their working day. Therefore, 5 of these hours were lost in rests; and the duty performed by each man during the other 5 hours, or 300 mins, was

$$\frac{120 \times 4 \times 34 \times 300}{4} = 1\,224\,000 \text{ ft-lbs.}$$

In the old mode of driving piles, where the ram of 400 to 1200 lbs suspended from a pulley, was raised by 10 to 40 men pulling at separate cords, from 35 to 40 lbs of the ram were allotted to each man, to be lifted from 12 to 18 times per min, to a height of 3¾ to 4¾ feet each time, for about 3 min at a spell, and then 3 min rest. It was very laborious; and the gangs had to be changed about hourly, after performing but ½ an hour's actual labor.

Hauling by horses. See Traction. When working all day, say 10 working hours, the average rate at which a horse walks while hauling a full load, and while returning with the empty vehicle, is about 2 to 2¼ miles per hour; but to allow for stoppages to rest, &c, it is safest to take it at but about 1.8 miles per hour, or 160 ft per min. The time lost on each trip, in loading and unloading, may usually be taken at about 15 min. Therefore, to find the number of loads that can be hauled to any given dist in a day, first find the time in min reqd in hauling one load, and returning empty. Thus: div twice the dist in ft to which the load is to be hauled; or in other words, div the length in ft, of the round trip, by 160 ft. The quot is the number of min that the horse is in motion during each round trip. To this quot add 15 min lost each trip while loading and unloading; the sum is the total time in min occupied by each round trip. Div the number of min in a working day (600 min in a day of 10 working hours) by this number of min reqd for each trip; the quot will be the number of trips, or of loads hauled per day.

Ex. How many loads will a horse haul to a dist of 960 ft, in a day of 10 working hours, or 600 min? Here, $960 \times 2 = 1920$ ft of round trip at each load. And $\frac{1920}{160} = 12$ min, occupied in walking. And $12 + 15$ in loading, &c) $= 27$ min reqd for each load. Finally, $\frac{600}{27} = \frac{\text{min in 10 hours}}{\text{min per trip}} = 22.2$, or say 22 trips, or loads hauled per day.

Table of number of loads hauled per day of 10 working hours. The first col is the distance to which the load is actually hauled; or half the length of the round trip. The cost of hauling per load, is supposed to be for one-horse carts; the driver doing the loading and unloading; rating the expense of horse, cart, and driver at $2 per day.

---

[*] The tympan revolves on a hor shaft; and is a kind of large wheel, the spokes, arms, or radii of which are gutters, troughs, or pipes, which at their outer ends terminate in scoops, which dip into the water. As the water is gradually raised, it flows along the arms of the wheel to its axis, where it is dischd. The scoop wheel is a modification of it. It is an admirable machine for raising large quantities of water to moderate heights. We cannot go into any detail respecting this and other hydraulic machines.

[†] A kind of large wheel with buckets or pots at the ends of its radiating arms; revolves on a hor axis; discharges at top. The buckets are attached loosely, so as to hang vert, and thus avoid spilling until they arrive at the proper point, where they come into contact with a contrivance for tilting and emptying them. The noria is similar, except that the buckets are firmly held in place, and thus spill much water. It is therefore inferior to the Persian wheel.

[‡] An endless revolving vert chain of buckets. D'Aubuisson and some others erroneously call this the noria. It is an effective machine.

Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Miles.	No. of Loads.	Cost per Load.
		Cts.			Cts.			Cts.
50	38	5.26	1500	18	11.11	1	7	28.57
100	37	5.41	2000	15	13.33	1¼	6	33.33
200	34	5.88	2500	13	15.39	1½	5	40.00
300	32	6.25	3000	11	18.18	2	4	50.00
400	30	6.67	3500	10	20.00	3	3	66.67
600	27	7.41	4000	9	22.22	4	2	100.00
1000	22	9.09	5000	7	28.57	9	1	200.00

If the loading and unloading is such as cannot be done by the driver alone; but requires the help of cranes, or other machinery, an addition of from 10 to 50 cts per load may become necessary. Hauling can generally be more cheaply done by using 2 or 3 horses, and one driver, to a vehicle. The neat load per horse, in addition to the vehicle, will usually be from ½ to 1 ton, depending on the condition, and grades of the road. From 13 to 15 cub ft of solid stone; or from 23 to 27 cub feet of broken stone, make 1 ton. **In estimating for hauling rough quarry stone for drains, culverts, &c,** bear in mind that each cub yard of common scabbled rubble masonry, requires the hauling of about 1.2 cub yds of the stone as usually piled up for sale in the quarry; or about ¾ of a cub yd of the original rock in place. **A cub yd of solid stone, when broken into pieces, usually occupies about 1.9 cub yds perfectly loose;** or about 1¾ when piled up. A strong cart for stone hauling, will weigh about ¾ ton; or 1500 lbs; and will hold stone enough for a perch of rubble masonry; or say 1.2 pers of the rough stone in piles. The average weight of a good working horse is about ½ a ton.

Morin gives the following results from careful experiments made by him for the French Government. The draft of the same wheeled vehicle on a road, may in practice be considered to be,

1st. On hard turnpikes, and pavements; in proportion to the loads; inversely as the diams of the wheels; and nearly independent of the width of tire. It increases to uncertain extents with the inequalities of the road; the stiffness (want of spring) of the vehicle; and the speed; (considerably less than as the square roots of the last.)

2d. On soft roads, the draft is less with wide tires than with narrower ones; and for farming purposes he recommends a width of 4 ins. With speeds from a walk to a fast trot, the draft does not vary sensibly.

TRUSSES.

INTRODUCTION.

General Principles.

1. Truss Design a Specialty. The design, construction and erection of trusses have become a specialty, to which persons confine themselves more or less exclusively, and thus attain a degree of expertness beyond the reach of the general engineer.* The latter, however, should have a knowledge of the subject, sufficient at least to enable him to form a well-grounded opinion of the general merits of a design and to guard him against the adoption of one involving serious imperfections. In a volume like this we can discuss only general principles.

2. The Truss Principle. Theoretically, a truss consists of a number of straight bars, joined, near their ends, by perfectly flexible joints, loaded only at these joints, and so arranged that all its internal stresses are sustained by its members, and only the vertical † pressures, due to the weights of the truss and its load, are transmitted to the abutments.

3. Distinction between Beams and Trusses. When a solid beam (Fig. 2, ¶ 7, Transverse Strength) bends, under its own weight or under that of its load, all the fibers above the neutral axis are compressed, while all those below are extended; and the resulting change of length, in each fiber, is proportional to the distance of the fiber from the neutral axis; but, in a truss, the loads (including the weight of the truss itself) are theoretically regarded as divided into portions which are concentrated at the joints between the members and which act through the cens of grav of their cross-sections. So placed, the stresses caused by them could not act transversely of the members, as in a beam, causing so-called secondary stresses, but must act longitudinally or axially of the members, and must be uniformly distributed over their entire cross-sectional areas. This is the distinguishing feature of all trusses.

4. In such a truss the material would be used most economically, and the stresses in each piece and in each part of such piece could be readily and accurately determined.

5. In the truss of a well-designed bridge or roof, this ideal condition is approximated by using, for the principal members, straight and rather slender pieces, and by so distributing the extraneous load that it shall be applied only at the joints between the members, thus subjecting them chiefly to forces acting at their ends and in the directions of their lengths. In pin-connected trusses (see ¶ 175) the joints are practically flexible.

Most of the trusses in common use consist of two long members, usually horizontal (but see ¶ 49), called **chords,** extending throughout the span and connected by **web members,** which are sometimes all inclined, and sometimes alternately vertical and inclined. Inclined web members are called **diagonals.**

6. Ties and Struts. A member sustaining tension is called a rod or tie. One sustaining compression is called a strut or post. One capable of sustaining both tension and compression is called a tie-strut or a strut-tie.

7. The dimensions of a truss are usually measured along the center lines of its members; and, in pin-connected trusses, the pins are placed at the intersections of these lines. Hence, the measurements are usually made from "center to center of pins."

8. In a plate girder, the flanges are usually regarded as performing the function of the chords of a truss, and the web as performing that of the web members of a truss.

---

* Railroad companies and municipal corporations frequently prepare their own bridge specifications; but the general proportions, number of panels, etc., are often left to the judgment of the bidders.

† We here suppose the truss to be loaded vertically. If the load is otherwise applied, as in the case of the wind pressure upon a horizontal bracing truss, the pressure on the supports may be horizontal, or otherwise inclined to the vertical, but all the internal stresses are still sustained by the truss members.

Loading.

9. Dead and Live Load. In bridges, we distinguish between the "dead" and the "live" load; the dead load comprising the weight of the permanent structure—*i. e.*, of the bridge itself, with its trusses, bracing and floor system; while the live load comprises any temporary and extraneous loads, such as engines, cars, horses, vehicles, foot passengers, etc., which may come upon the bridge.

10. The dead load is usually distributed uniformly along the span, but the loaded chord (that carrying the roadway) of course usually receives a greater share of it than the unloaded chord. The live load comes only upon the loaded chord. In determining stresses, it is usual to consider the weight of live load and of floor system as being on the loaded chord, and the rest of the dead load as divided equally between the two chords. It sometimes happens, however, that both the upper and the lower chords carry roadways. They must then, of course, both be treated as "loaded," though not necessarily *equally* loaded; for one may carry a railway while the other carries only a highway.

Unsymmetrical Loading. Counterbracing.

11. Unsymmetrical Loading. In Figs. 2 to 10, the loads are supposed to be placed symmetrically.

12. If this could be the case in practice, the compression members would never be called upon to resist tension, or the tension members to resist compression; and the trusses in Figs. 2 to 10 would suffice (supposing each member to have sufficient strength), even though the compression members were incapable of resisting tension and vice versa. Thus, the tension members might be flexible chains, and the compression members might be posts, merely abutting against supports at their ends.

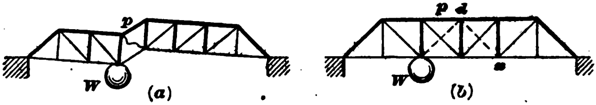


Fig. 1.

13. But in a truss, Fig. 1 (*a*), with a flexible tie in the panel, *p*, as shown, the load W, unsymmetrically placed, would cause failure, as indicated.

14. Counterbracing. To prevent this, those members which, under moving loads, may be subjected alternately to both tension and compression, may be so constructed as to be able to resist both kinds of stress. That is to say, the tension members may be so stiffened as to be capable of acting as posts, and the ends of the compression members so connected to the chords that those members can also act as ties. This is the expedient usually employed in trusses without vertical web members.

15. Counters. In trusses with rectangular panels, the distortion, Fig. 1 (*a*), caused by unsymmetrical loading, is usually prevented by the introduction of additional members called counterbraces, or counters, in distinction from the "main" members, which last are designed to resist the normal stresses due to uniformly or symmetrically distributed loads. Thus, in Fig. 1 (*b*) the unsymmetrical load, W, tends to convert the rectangle, *p*, into a rhomboid, by lengthening its diagonal, W *d*; and this may be prevented by the introduction of an oblique tension member (counter) in the line of that diagonal, as shown by dotted line. For a similar reason, such a counter is inserted also in the corresponding panel, *x d*.

16. Triangles. It will be noticed that the introduction of counters reduces the truss to a framework made up exclusively of *triangles*.

17. It might at first sight appear that the several parts of a bridge truss must be most strained when covered from end to end with its maximum load; but this is true only of the chord and of the *main* diagonals and verticals near the *ends* of the truss. The other web members may be more strained by a *part* of the load, placed unsymmetrically on the truss; so that, although correctly proportioned for a full load, they may be too weak for a partial one. If all be made as strong as the end ones, they will, it is true, be safe for a passing load; but this would require an expense of material that would be justified only in the case of moderate spans, especially of wood, in which the additional trouble and expense of getting out and fitting together pieces of many different sizes may more than counterbalance the saving in material.

18. In large bridges, where the live load is small, relatively to the dead load, but little counterbracing is needed, and that at and near the center only; whereas, in a very light bridge, the counters should extend from the center, where they are most strained, to near the ends, where the strain upon them is least.

Cross-bracing.

19. **Bracing between Trusses.** Advantage is taken of the proximity of the two or more trusses of a bridge, standing side by side, to connect them by cross-bracing, thus giving to the entire structure far greater lateral stability than would be possible in the single trusses.

20. Thus, **lateral bracing,** Fig. 39, consists of horizontal trusses placed between the two upper chords of the main trusses, or between the two lower chords, or both; the chords of the main trusses acting also as the chords of the lateral trusses. The lateral bracing prevents lateral deflection of the chords.

21. **Sway bracing,** Fig. 64 (*c*) (called also diagonal, cross, vibration and wind bracing), consists of short trusses (usually vertical) crossing the bridge transversely and thus connecting the two trusses. The sway bracing has its own chords, but uses parts of the posts of the main truss as its end posts.

22. **Portal bracing,** Fig. 54 (*a*), consists of sway bracing (usually in an inclined plane) joining the tops of the end posts in trusses of sufficient depth to permit its use. The portal bracing, with the end posts, forms a portal through which trains, etc., enter the bridge.

Types of Trusses.

23. **The simplest form of truss** consists of a single triangle, Figs. 2 (*a*) and (*b*). In Fig. (*a*) the load produces compression in the rafters, tension in the chord or tie rod,[*] and compression (= the tension in the chord) between the heads of the rafters; in Fig. (*b*) vice versa.

24. The truss shown in Fig. 2 (*a*) is in common use for roofs of small span, as in dwellings. In practice, it is of course loaded along the rafters, and not only at the apex as in Fig. (*a*); but, in calculating the stresses in truss members, we commonly first assume that the loads are concentrated at the *intersections* of the members. The effect of their actual distribution *along* the members is then determined separately, treating the members as beams.

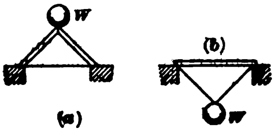


Fig. 2.

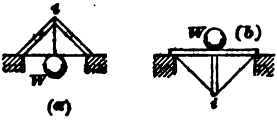


Fig. 3.

25. In Fig. 3 (*a*) (called a King truss), the vertical tie (improperly called a King post), and in Fig. 3 (*b*) the vertical post, simply carries the weight of the load to the apex, *t*, where it produces the same effect as in Figs. 2 (*a*) and (*b*).

26. Hence, neglecting the weights of the vertical tie and other members, the stresses, caused by a given load, W, in the diagonals, and in the horizontal tie, Fig. 3 (*a*), are the same (not only in character, but also in amount) as those produced by an equal load, W, in Fig. 2 (*a*). Similarly, those in Fig. 3 (*b*) correspond with those in Fig. 2 (*b*).

---

[*] In Figs. 2 to 12, and 14 to 17, double or heavy lines indicate posts or struts, and light lines indicate ties.

27. Figs. 4, 5, and 6, giving modifications of the simple forms shown in Figs. 2 and 3, illustrate in principle most of the bridge trusses in common use for spans up to 300, 400 or even 500 feet. See Figs. 7 to 10, ¶¶ 35, etc.

28. In Figs. 4, 5, and 6, there is an upper chord, in compression, and a lower chord, in tension; the shorter chord sustaining the compression between the heads of the rafters, Figs. 2 (*a*) and 3 (*a*), or the horizontal tension between the feet of the diagonals, Figs. 2 (*b*) and 3 (*b*). Figs. 4 (*a*) and 5 (*a*) are modifications of Figs. 2 (*a*) and 3 (*a*); Figs. 4 (*b*) and 5 (*b*) of Figs. 2 (*b*) and 3 (*b*); Fig. 6 (*a*) of Fig. 2 (*a*), and Fig. 6 (*b*) of Fig. 2 (*b*)

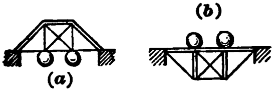
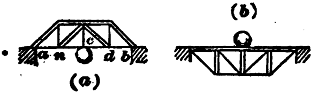

(*a*) (*b*)

Fig. 4.

29. Figs. 4 (*a*) and 4 (*b*) may be regarded as showing Figs. 3 (*a*) and 3 (*b*) respectively, with the vertical member, as well as the load, split in two, and the two parts separated by horizontal straining pieces. If the loads are placed symmetrically, so that the horizontal pressures, Fig. 4 (*a*), or tensions, Fig. 4 (*b*), on the two ends of the shorter chord, are equal, the two diagonal counters in the center are unnecessary.



(*a*) (*b*)

Fig. 5.

30. Howe and Pratt Systems. In Fig. 5 (*a*) the vertical web members are in tension, and the diagonals are in compression, embodying the "Howe" principle, used in bridges with wooden diagonals; while in Fig. 5 (*b*) the verticals are in compression, and the diagonals in tension, embodying the "Pratt" principle, used in bridges with metal diagonals. In such bridges long compression members are objectionable.

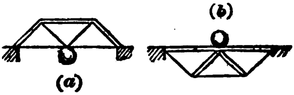

(*a*) (*b*)

Fig. 6.

31. Warren or Triangular Trusses. In Fig. 6, illustrating the "Warren" or "triangular" truss, the web members are all diagonal, and are alternately in tension and in compression. They divide the truss profile into *isosceles* triangles.

32. Through, Deck and Pony Spans. Figs. 4 (*a*), 5 (*a*) and 6 (*a*), with the roadway on the lower chords, are called "through" spans, and Figs. 4 (*b*), 5 (*b*) and 6 (*b*), with the roadway on the upper chord, are called "deck" spans. The deck span permits the use of sway bracing (see ¶ 21) between, and throughout the depth of, the two or more trusses forming the bridge, while the through span of course does not; but the use of the through span is often required, in order to give sufficient head-room for boats, floods, trains on crossing roads, etc., below the bridge. A truss, loaded on the lower chord, but too shallow for lateral bracing (see ¶ 20) between the upper chords, is called a "pony" truss (or "pony through" truss).

33. Panels. The points where the vertical web members meet the chords, in Figs. 4 and 5, are called **panel points**; and the rectangular spaces, *a n*, *n c*, *c d*, etc., Fig. 5 (*a*), between the verticals, are called panels.

34. The Warren truss, Fig. 6, has no verticals, as essential parts of it. See ¶¶ 45 and 46. Its subdivisions are called simply triangles; and a panel is a length of truss equal to the width of a triangle. A panel of either chord, however, is that portion of it between two panel points.

35. Further modifications of these designs, with more numerous panels, are shown in Figs. 7 to 10. Figs. 7 (*a*) and 8 (*a*), with verticals in tension, represent the Howe truss of Figs. 4 (*a*) and 5 (*a*), while Figs. 7 (*b*), 8 (*b*), 9 (*a*) and 9 (*b*), with diagonals in tension, represent the Pratt truss of Figs. 4 (*b*) and 5 (*b*). Figs. 10 represent the Warren truss of Fig. 6.

36. Fig. 8 (*a*) represents simply Fig. 7 (*a*), lowered so as to become a deck, instead of a through bridge; while Fig. 8 (*b*) represents Fig. 7 (*b*) converted from a deck to a through span by being carried on vertical end posts.

37. In Figs. 8, the vertical end posts, and the horizontal piece at each end of the loaded chord, form no part of the truss proper. The latter simply act as beams, supporting the load during its passage from the abutment to the truss and vice versa. The end post in Fig. (*a*) supports only one end of this beam, while that in Fig. (*b*) supports half the *truss*.

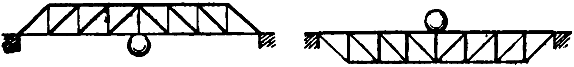

(*a*) **Through Howe.** (*b*) **Deck Pratt.**

Fig. 7.

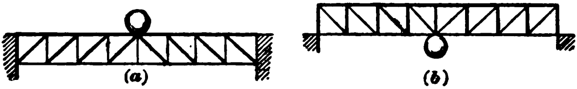

(*a*) **Deck Howe.** (*b*) **Through Pratt.**

Fig. 8

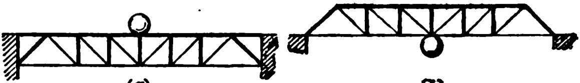

(*a*) **Deck Pratt.** (*b*) **Through Pratt.**

Fig. 9.

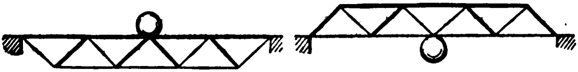

(*a*) **Deck Warren.** (*b*) **Through Warren.**

Fig. 10.

38. In Figs. 8 the middle vertical carries no part of the load. Theoretically it serves merely to prevent deflection of the two unloaded middle chord panels under their own weight; but in practice such members are often inserted for the purpose of obtaining convenient connections for lateral pieces, such as floor beams.

39. In Figs. 9 (modifications of Fig. 7 (*b*)) and in Fig. 10 are shown, in principle, the most common forms of metal bridge truss, used as deck and as through spans respectively.

40. In Fig 9 (*a*), as in Fig. 8, the vertical end posts and the horizontal pieces at the ends of the loaded chord form no part of the truss; and in Fig. 9 (*b*), as in Figs. 8, the middle vertical supports only the unloaded middle chord panels.

41. Intersections. In deep trusses, two or more sets of web members are sometimes combined in one truss, with one pair of chords. Thus the two simple Pratt trusses shown in Figs. 11 (*a*) and (*b*) combine to make the "Whipple" or "double intersection Pratt" truss, Fig. 11 (*c*), recently in general use.

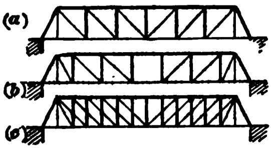


Fig. 11.

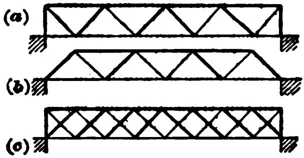


Fig. 12.

42. Similarly the two simple Warren trusses, in Figs. 12 (*a*) and (*b*), combine to form the double intersection Warren of Fig. 12 (*c*).

43. A combination of *four* systems is called a "quadruple intersection" truss. See Fig. 59 (*t*).

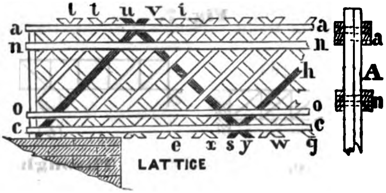
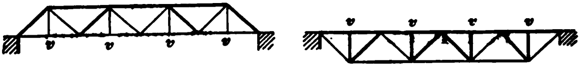


Fig. 13.

44. The old Towne "lattice" truss, Fig. 13, consisting of planks crossing each other (usually at right angles) and bolted or tree-nailed together at their intersections, may be regarded as a combination of several Warren trusses.



(*a*) **Fig. 14.** (*b*)

45. Sub-verticals. In deep trusses, where the horizontal spread of the panels is considerable, sub-verticals, *v*, Figs. 14 and 15, are often used, especially in Warren trusses, to support the segments of the loaded chord. See also Figs. 59 (*i*), (*r*), and (*s*).

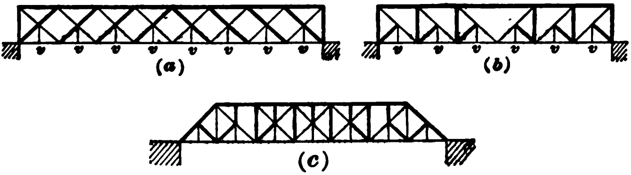


Fig. 15.

In the **"Baltimore" truss,** Fig. 15 (*b*), each diagonal is braced, at its middle point, by a short diagonal strut inclined in the opposite direction, and a sub-vertical is suspended from their junction. With very long panels, sub-verticals are sometimes used for the panels of the unloaded chord also. See Fig. 15 (*c*).

46. Collision struts, or collision posts, S, Figs. 59 (*k*), (*m*), (*o*), and (*t*), and 73 (*a*), are used for bracing long diagonal end posts against a blow from a derailed train.

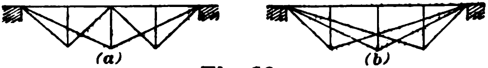

(*a*) (*b*)

Fig. 16.

47. Fink and Bollman Trusses. Figs. 16 show two obsolete modifications of Fig. 3 (*b*), viz.: the Fink, Fig. 16 (*a*), and the Bollman, Fig. 16 (*b*). The large bridge over the Ohio River at Louisville, Ky., completed 1870, is of the Fink type. The Bollman was largely used on the Baltimore and Ohio Railroad years ago.

48. In the Fink and in the Bollman truss there was but one chord, as shown. This chord usually carried the roadway. Where the roadway was placed lower, it gave the truss the appearance of having two chords. Under uniformly distributed loads, in the Fink, and under all circumstances in the Bollman, the stress in this chord was uniform throughout. In the Bollman (see Fig.), the longitudinal stresses in the chord were all applied at its ends. Each type may be regarded as a combination of several suspension trusses like Fig. 3 (*b*). In the Bollman, the simple trusses were all of the same span and depth; and each vertical post, except the central one, divided its simple truss eccentrically. The Fink principle is still largely used in metal roof trusses. See Figs. 26.

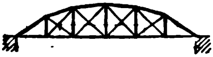


Fig. 17.

49. Curved Chords. Trusses with curved or "broken" chords, Fig. 17, are frequently used for long spans. The members themselves, between panel points, are always straight. In the **bowstring,** Fig. 17, the panel points of the upper chord lie in a curve, convex upward. In the **crescent** truss, the lower chord also is convex upward. The bowstring truss has the advantage, over those with horizontal upper chords, of making all the chord and web stresses more nearly equal, thus simplifying the construction and reducing the weight of the trusses. It has the disadvantage of permitting no overhead bracing near the ends of the span. If the curve of the upper chord is made parabolic, the dead load stress is uniform throughout the lower chord, and in each vertical (now in tension) the stress is equal to the dead load on the lower chord. The diagonals receive no dead load stress, but are called into action only by eccentric loads.

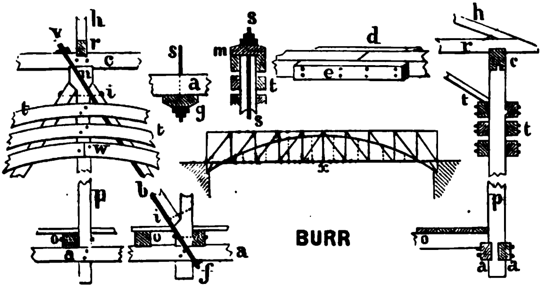


Fig. 18.

50. The Burr truss, Fig. 18, at one time much used for wooden bridges, was a combination of a Howe truss and an arch.

Camber.

51. Camber. In practice, the members of the upper and lower chords of bridges are not placed perfectly in line, but so that the chords curve slightly, with the convex side upward. This curve is called the camber. Its object is to prevent the truss from bending down below a horizontal line when heavily loaded. When the chords are cambered (see $y\,s$ and $c\,d$, Fig. 19), they become approximately concentric arcs of two large circles, of which the center is at t; and the upper one plainly becomes longer than the lower. The verticals, instead of remaining truly vertical, become portions of radii of the arcs mentioned; and, although their lengths remain unchanged, yet their tops are farther apart than their feet; and this renders it necessary to lengthen the diagonals. See ¶¶ 211–214.

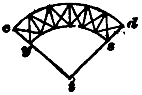


Fig. 19.

Cantilevers.

52. The cantilever principle is shown in Fig. 20, where A and B represent counterweights or anchorages. Fig. 21 shows the Niagara cantilever bridge. It consists of two cantilever trusses, ab, $a'\,b'$, connected by an ordinary truss, ba', which is suspended, by a vertical link at each end, from the ends of the cantilever trusses. The weight of the truss is counterbalanced by anchorages, A and B, or by weights, or both. The principal advantage of the cantilever is that it may be built outward from the piers across the channel. It thus greatly facilitates erection in cases where false-work cannot well be used.

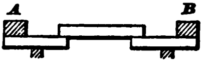


Fig. 20.

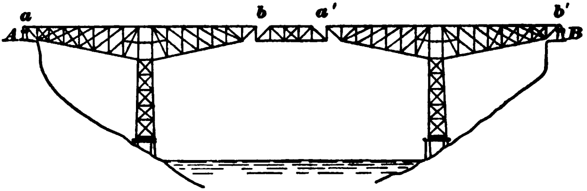


Fig. 21.

Movable Bridges.

53. Movable bridges, including draw, swing and lift bridges, are of three general classes; one in which the movable part slides horizontally, one in which it swings horizontally, and one in which it swings vertically. Ordinarily, the movable span is pivoted near the middle, and swings horizontally on the central "swing" or "pivot" pier, as in Fig. 22. In such cases it is

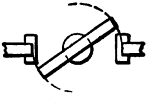


Fig. 22.

usually mounted on a central pivot, or on a nest of rollers or wheels running on a circular track. Such a bridge must be so designed that, when it is swung open, or if it is not brought to a bearing at the ends when closed, it shall sustain not only its own weight, but also any other loads that may come upon it. In addition, each half must be able to act as a bridge supported at both ends, with all possible live loads; for, as an unbalanced live load comes on either end, that end will be brought to a bearing. In elaborate bridges, provision is made for raising the ends of the draw span, when closed, thus bringing both ends to a firm bearing, and the floor flush with that on the adjacent abutment or fixed span. This raising is usually made sufficient to relieve the middle pier of only a portion of the load. The bridge then acts like a "continuous" girder (see Transverse Strength, ¶¶ 78, etc.) supported at three or at four points, depending upon the arrangement of the bearing on the pivot pier.

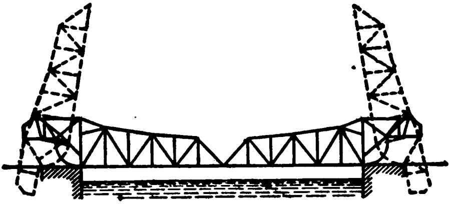


Fig. 23.

54. Drawbridges in which the movable part swings vertically, may either revolve about a pivot, or they may roll, as in the Scherzer rolling lift bridge, Fig. 23.

55. Skew bridges are used where a channel, road, etc., is crossed obliquely, and where it is inconvenient to have the abutments perpendicular to the trusses. For simplicity in making floor connections, etc., the truss is usually so designed as to bring the panel points opposite each other, as in Figs. 24 and 25. Where the skew is but slight, this necessitates a difference

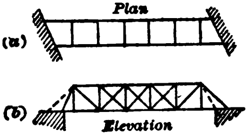


Fig. 24.

in inclination between the two end posts, as in Fig. 24, involving complication in the connections for the portal bracing. But where the skew is greater, it may be possible to make it just equal to one or more even panels, adjusting

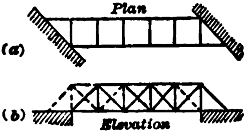


Fig. 25.

the panel length to suit, and thus leaving each truss symmetrical, as in Fig. 25. In each figure, those members which belong only to the *farther* truss (the upper one in the plan) are shown by *dotted* lines.

Roof Trusses.

56. Roof trusses are made in a great variety of forms. Those shown in Figs. 26 are common. In Fig. 26 (*a*), part of load, at *d*, compresses the rafter from *d* to *a*, while the remainder compresses the strut, *d h*, and pulls the rod *h i* and the part-chord *h a*. Similarly, part of *c* passes through *c a* to *a*, and the remainder through *c k d h i* to the apex *i*. Thus each load is eventually carried by the members, part to the apex and part along a rafter to an abutment. It will be seen that the greatest stresses in the rafters and in the chord occur near the ends.* Sometimes the members shown vertical in Fig. (*a*) are inclined, or the lower chord is "broken," being usually convex upward. Roof trusses are often composed, as in Figs. (*b*) and (*c*), of two Fink trusses, inclined, and leaning against each other, their feet being held in position by a *tie*, *m n*, and the rafters forming the upper chords of the Fink trusses.

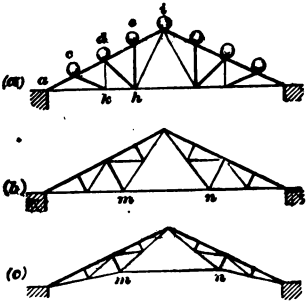


Fig. 26.

STRESSES IN TRUSS MEMBERS.

General Principles.

57. Conditions of Equilibrium. In trusses, as in beams, it is necessary and sufficient, for equilibrium, that the internal stresses, and their moments, shall balance the external forces and their moments. The external forces (viz., the loads and the end reactions) and the resulting moments and shears, are discussed under Statics, ¶¶ 285, etc. We here discuss the determination of the internal stresses. For the fundamental distinction between beams and trusses, see Trusses, ¶ 3.

58. In general, the stresses in the members are found by means of the principles of moments (Statics, ¶¶ 301, etc.), and of Shears (Statics, ¶¶ 325, etc.), making use of the force parallelogram (Statics, ¶¶ 35, etc.) or force triangle (Statics, ¶¶ 46, etc.), the force and cord polygons (Statics, ¶¶ 72, etc., 86, etc.) and the influence diagram (Statics, ¶¶ 339, etc.).

59. A very convenient method, and one in common use, is that described more fully in ¶¶ 67, etc., below, where the truss is considered as being cut through by a section. We then seek to ascertain what stresses, in the members so cut, would be required to preserve equilibrium.

60. Before the stresses can be calculated, and the truss proportioned to those strains, its weight must be known; for this constitutes a load, and therefore affects the stresses. But, on the other hand, we cannot learn its weight until we know the sizes of its different members. In this dilemma we must assume for it an approximate weight, based upon our knowledge of somewhat similar trusses already built. This becomes the more necessary as the truss increases in size, so that its own weight becomes greater in proportion to that of the load.

---

* If the diagonals were parallel, their stresses, and those in the verticals, would be greatest at the center of the span, and least at the abutments.

61. To distinguish between ties and struts; from the point, *o*, Figs. 27, where the force is applied, draw *o c* to represent the applied force, in the direction in which that force tends to move the point, *o*; and upon *o c* as a diagonal construct the force parallelogram, *a b*. Through *o* draw *i i* parallel to the other diagonal *a b*. Then, if a piece be on the same side of *i i* with *o c*, it is a strut; while, if it be on the opposite side, it is a tie.

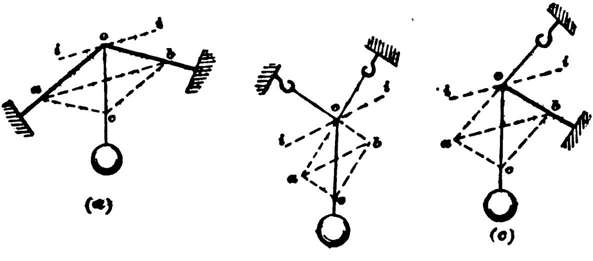


Fig. 27.

62. Ties and struts may often (as in Fig. 27) be readily distinguished by inspection, by imagining the piece to be flexible, like a rope or chain. If it is seen that it would then resist the force acting upon it, the member is a tie; if not, it is a strut. Or, suppose that the piece is not secured at its ends. If, then, it is seen that it would resist the force acting upon it, the member is a strut; if not, it is a tie.

63. Or we may proceed as follows: In Fig. 28 (*a*), representing joint *a*, we begin with the known net vertical reaction, R[*]; and find the unknown stresses in the chord and in the end post by means of the force triangle, making their arrows follow the known direction of R. Transferring these arrows to the respective truss members, Fig. (*d*), we find that the chord pulls away from *a*, and is therefore a tie; while the end post pushes toward *a*, and is therefore a strut.

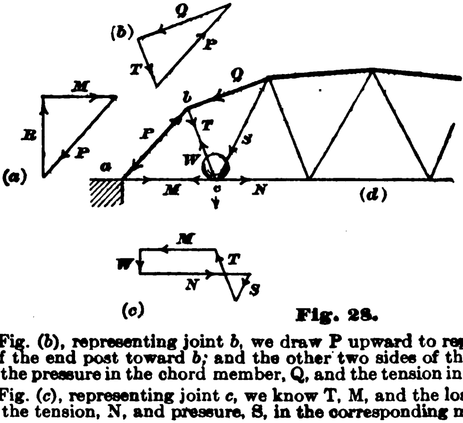


Fig. 28.

64. In Fig. (*b*), representing joint *b*, we draw P upward to represent the pressure of the end post toward *b*; and the other two sides of the force triangle give the pressure in the chord member, Q, and the tension in the tie, T.

65. In Fig. (*c*), representing joint *c*, we know T, M, and the load, W, and we obtain the tension, N, and pressure, S, in the corresponding members.

---

[*] Inasmuch as half of each end panel rests directly upon a support, and thus adds nothing to the stresses in the members, we must, in determining those stresses, use only the

net reaction = reaction — half panel load.

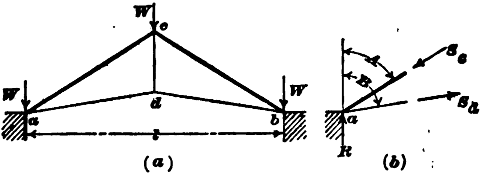
66. *Tensile* stresses, because they tend to *elongate* a member, are conventionally regarded as *positive*, and designated by +, while *compressive* stresses are regarded as *negative*, and designated by —.

Method by Sections.

67. Let Fig. 29 (*a*) represent a roof truss, with three equal loads, W, of 2 tons each, applied at *a*, *c* and *b*, respectively, and let it be required to find the stresses produced, by those loads alone, in the members *a c* and *a d*. Suppose the portion shown in Fig. 29 (*b*) to be separated from the rest of the truss, as shown, by cutting through the members *a c* and *a d*. The lower portions of those members, shown in (*b*), are, however, supposed to be held in their original positions by the stresses S_c and S_d, exerted in these members themselves. Taking moments about the right support, *b*, Fig. 29 (*a*), we have, for the upward reaction of the left abutment, *a*,

$$R = \frac{3\,W}{2}.$$

68. We have, then, at *a*, Fig. 29 (*b*), four forces, as follows: two known forces, viz.: W, vertically downward, = 2 tons, and R, vertically upward, $= \frac{3\,W}{2}$; and two unknown forces, S_c and S_d. Now S_c makes a known angle, A, and S_d a known angle, B, with the vertical. The vertical forces, W and R, have, of course, no horizontal resolutes (see Statics, ¶¶ 54, etc.); and their vertical resolutes are the forces themselves.



(*a*) (*b*)

Fig. 29.

69. The horizontal resolutes of the inclined forces, S_c and S_d, are, respectively: S_c.sinA, and S_d.sinB; and their vertical resolutes are: S_c.cosin A, and S_d.cosin B.

70. We see, by inspection, that the stress, S_c, in the rafter, *a c*, is compression, and that the stress, S_d, in the lower member, is tension; but, for convenience, we may at first assume, in advance, that all of the unknown stresses are tensions or +. Then those which finally appear as + are known to be tensions, and vice versa. Their horizontal resolutes, in this case, are therefore both taken, for the present, as being right-handed, or positive; and their vertical components upward or positive also. It will be remembered (see ¶ 66) that we regard tensions as positive, and compressions as negative.

71. Now, in order that the four forces at *a*, viz.: W = 2 tons, downward, $R = \frac{3\,W}{2}$, upward, S_c and S_d, may be in equilibrium, it is necessary:

(1) that the sum of their horizontal resolutes be zero, or
$S_c.\sin A + S_d.\sin B = 0$;
(2) that the sum of the vertical resolutes be zero, or
$R - W + S_c.\text{cosin } A + S_d.\text{cosin } B = 0.$

Thus, let $A = 45°$, $\sin A = 0.707$; $\text{cosin } A = 0.707$.
$B = 75°$, $\sin B = 0.966$; $\text{cosin } B = 0.259$.

Then $0.707\,S_c + 0.966\,S_d = 0$;

$$R - W + 0.707\,S_c + 0.259\,S_d = 0;$$

$$S_c = \frac{-\,0.966\,S_d}{0.707} = \frac{-\,0.259\,S_d - R + W}{0.707}$$

$$0.966\,S_d - 0.259\,S_d = 0.707\,S_d = R - W.$$

72. Again, in Fig. 30, with section *uv*, stress in *ed* $= W_1 - R = 6 - 15 = -9$. With section *vy*, stress in *ef* $= \dfrac{W_1 - R}{\cos\theta} = \dfrac{-9}{\cos\theta}$. With section *ux*, stress in *gd* $= \dfrac{W_1 + W_2 - R}{\cos\theta} = \dfrac{-3}{\cos\theta}$. It will be seen that these forces, all acting *downward* on the part truss to the *left* of the section, give *tension* in *ed*, and *compression* in *ef* and *gd*.

With section *ux* we cut *two* web members, *gd*, and *gc;* but the stress in *gd* has already been found $= \dfrac{3}{\cos\theta}$, the vertical component of which is $= 3$. Hence, stress in *gc* $= W_1 + W_2 + W_3 + 3 - R = 6$.

73. It is, however, evident from inspection that the middle vertical bears simply the middle load, $W_3 = 6$; for, cutting the truss by a curved section, as at *c*, and examining the small portion thus cut out, we see that we have but two vertical forces—viz., the central load, W_3, and the stress in the vertical member; and, for equilibrium, these two must be equal.

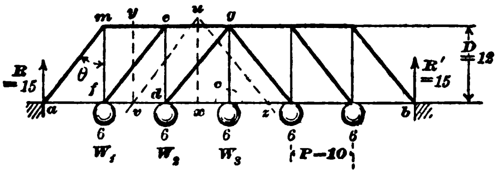


Fig. 30.

Chord Stresses, Moments.

74. For the chord stresses, Fig. 30, let P = panel length = 10 ft. Then the bending moment at the panel point, *d*, is

$$\begin{aligned} M &= 2\,R\,P - W_1 P \\ &= 15 \times 20 - 6 \times 10 \\ &= 300 - 60 = 240. \end{aligned}$$

Cutting the truss by section *uv*, we find that, of the three members cut, only the upper chord member, *eg*, has a moment about *d*. Call its stress S. Its leverage is the depth, D, of the truss, $= 12$; and, for equilibrium, $S\,D = M$. Hence, $S = \dfrac{M}{D} = \dfrac{240}{12} = 20$.

75. Similarly, taking moments about *e*, we find the stress, in the lower chord member, *f d*, cut by the section, *u v*, to be $\dfrac{240}{12} = 20$, or the same as the stress in the upper chord panel cut by the same section. Inspection shows the correctness of this result; for the diagonal strut, *e f*, evidently delivers to the upper chord panel, *e g*, a compressive stress or "chord increment" (see ¶ 77) = the tensile stress which it delivers to the lower chord panel, *f d*.

76. If the chord members are inclined, their lever arms must of course be measured *perpendicularly to them;* and we can no longer use the vertical depth of the truss as the lever arm.

77. Chord Increments. Fig. 30. Each diag delivers a comp stress to the upper chord, and, in trusses with parallel chords, an equal tensile stress to the lower chord. Find the shear, or vertical component, V_1, V_2, etc., of the stress in each diagonal, beginning with the end post. Then the "chord increments," h_1, h_2, etc., or the stresses in the chord members, *af* and *me*, *fd* and *eg*, etc., due to the several diagonals separately, are

$$\begin{aligned} h_1 &= V_1 \tan\theta \\ h_2 &= V_2 \tan\theta \\ h_3 &= V_3 \tan\theta \end{aligned}$$

and, for the *total* stress in each chord member, we have, $H_1 = h_1$; $H_2 = h_1 + h_2$; $H_3 = h_1 + h_2 + h_3$; and so on.

Shear.

78. Any portion of a shear diagram applies to all those members through which its shear travels, up to the *panel point* where the shear undergoes a change. Thus, in Fig. 31 the shear diagram on the right of the Fig. includes the vertical, *tu;* that on the left includes the diagonal, *mo;* and that between the loads includes the diagonal, *t n*, and the vertical, *m n*.

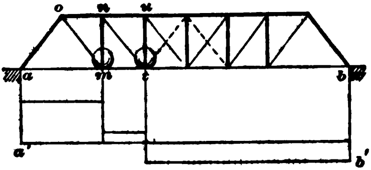


Fig. 31.

79. **Shear Influence Diagram.** See Statics, ¶¶ 325, etc. In a *truss*, Fig. 32, the ordinates, *c' g*, etc., to the line *a'' b'* (constructed as in Fig. 156, Statics, ¶ 349), give the left end *reactions;* and those, *c' h*, etc., to the line *a'b''*, give the right end *reactions*, for *any position* of the load; and the resulting *shears* for a load, W (not shown), *at any panel point;* but the *shears in a panel*, *cd*, for a load, W, *between the panel points*, are modified by the action of the stringers in distributing the load between said adjacent panel points, as indicated by the influence lines, *qh*, etc., for the several *panels*. Thus, with W at *c* and at *d*, respectively, the shear, in the panel *cd*, is represented respectively by *c'h* (negative) and by *d'q* (positive); and, as the load passes from *c* to *d*, the shear in the panel changes from *c'h* to *d'q*.

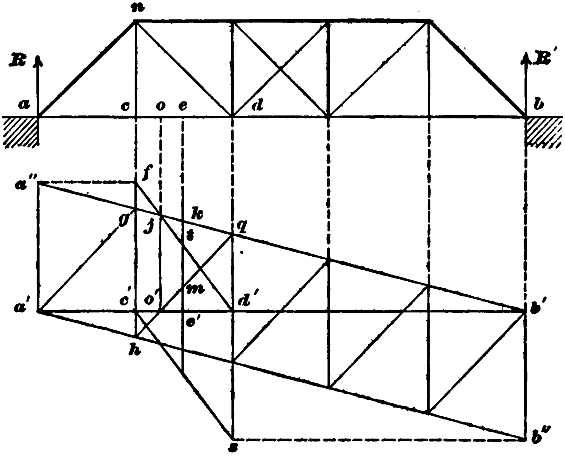


Fig. 32.

But when W is placed at any point, *e*, between *c* and *d*, its load is distributed between the panel points, *c* and *d*, by the stringer, acting as a beam.

80. Thus, drawing, for this beam, *c d* (as for the whole beam in Statics, Fig. 156), the *panel* influence lines *fd'* and *c's*, we see that, as W moves from *d* into the panel *cd*, as to *e*, the truss reaction at *a* is thereby slightly increased from *d'q* to *e'k;* but at the same time a portion of W, represented by *e't*, is

carried by the stringer to *c*, where it diminishes the shear *c'k* (due to the truss reaction at *a*), leaving *tk* as the value of the shear in the panel. As we place W successively at other points, farther from *d*, and approaching *o*, the load carried by the stringer to *c*, and represented by the ordinates from *c'd'* to *fd'*, continue to increase faster than does the left end truss reaction, R, represented by the ordinates from *a'b'* to *a''b'*; and the resulting shears in the panel are represented by the ordinates from *jd'* to *jq*. At *o*, the part load, *o'j*, carried to *c*, is = the left truss reaction, and the shear in the panel is zero. With the load between *o* and *c*, the part loads carried to *c*, and represented by the ordinates from *o' s'*, to *ff*, are *greater* than the corresponding left end truss reactions; and the result is a *negative* shear in the panel, indicated by the ordinates from *j g* to *jf*. It will be noticed that the resulting shears throughout, both positive and negative, are indicated by the ordinates from *c'd'* to *hq*.*

Reversing the process, a similar argument may be applied to the panel influence line *c's*, beginning with the load at *c*, with negative shear in panel = *c'h*, and supposing it moved across the panel to *d*, where positive shear in the panel becomes = *d'q*.

81. In the case of a **uniform load,** extending on to the span from the right support, *b*, the point *o* is the position of head of load for maximum positive shear in the panel, *cd;* for, in the case of a uniform load, the shear, with head of load at *e*, is represented by the *area* (sum of all the ordinates) *e'mqb'e'*; and manifestly this area increases as the head of the load approaches *o;* but when it reaches *o*, the area above *a'b'* can increase no further, and when it *passes* *o*, the *negative* shears, represented by the ordinates from *o'c'* to *o'h*, begin to *reduce* the resultant positive shear.

82. Having found, by any method, the maximum shear, *d'q*, due to a concentrated load at *d*, for the diagonal, *d n*, Fig. 32, and the reverse maximum shear, *c' h*, due to the same load at *c*, we may draw an influence line, *h q*, which gives, as before, the point, *o*, of position of head of uniform load for maximum stress in the diagonal, *d n*, from which (as above) we find the corresponding position of the head of a series of concentrated loads.

In practice, the influence line for shear is of value chiefly in thus finding the position of load producing maximum stress, and the resulting stresses, in trusses with *curved* chords, such as Fig. 17. In such a truss, owing to the inclination of the members of the upper chord, those members take some of the shears in their respective panels, and the stress in the diagonal is therefore less than the shear in the panel.

Graphic Determination of Dead Load Stresses.

83. Construct first a diagram of the truss, as in Fig. 33 (*a*), lettering the spaces between the members, and those between the arrows representing the dead loads. Call the end post, 1–3, between A and B, "AB," the stress in it "*ab*," the load at 2, "*cd*," etc., using capital letters for panels and truss members, and small letters for loads and stresses. Adopt a suitable scale of forces, and construct the diagram, Fig. 33 (*b*), as follows:

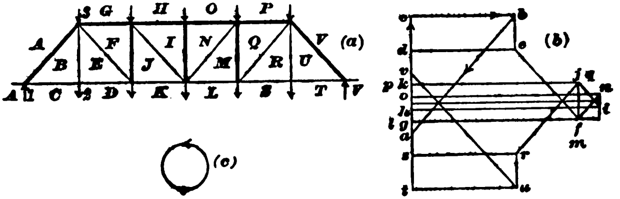


Fig. 33.

84. Consider first the point 1, Fig. 33 (*a*). There are here three forces in equilibrium, viz., *ac*, *ab* and *bc*. Find the net end reaction, R = *ac*, and lay it off upward (since it acts upward on 1) from any convenient point, *a*, to *c*, Fig. 33 (*b*). From *a* draw an indefinite line *ab* parallel to AB and from *c*

---

* Since *fh*, *jo'*, *qd'* and *ke'* are parallel, *me'* = *kt*, and *c'h* = *gf*.

draw *cb* parallel to BC, obtaining the force triangle *acb* of the point 1. The lengths of *cb* and *ba* then give the stresses by scale.

85. In Fig. 33 (*b*), the arrow on *ac* indicates the upward direction of that force. Following around the triangle, we affix arrows (in the same direction) to *cb* and *ba*. Supposing these arrows now to be transferred to the corresponding members in Fig. 33 (*a*) we see that *b c* pulls from the point 1, showing that *b c* is tensile, or +, while *b a* pushes toward 1, showing that *b a* is compressive, or —.

86. The characters of the stresses may be found more quickly as follows: Draw a circle, Fig. 33 (*c*), and place on it arrows pointing around in the direction (counter-clockwise in this case) followed around the truss in constructing the load line. See ¶ 92, below. Then consider any panel point, Fig. 33 (*a*), and follow the letters in the spaces around that point in the direction of the arrows on the circle. Note the order of the letters, and follow the corresponding equilibrium polygon, Fig. 33 (*b*), around in the same direction. This will give the directions in which the forces respectively act on that point.

87. Thus, consider the panel point 2. Following around 2 in the direction of the circle, we read B, C, D, E. Turning now to Fig. 33 (*b*), and reading *b, c, d, e,* we find that on *bc* we go from right to left (or opposite to the direction indicated by the arrow drawn for point 1); hence *bc* acts to the left on 2, and BC is therefore in tension, and its stress *bc* is +.

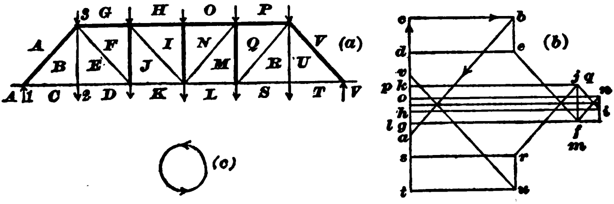


Fig. 33 (repeated).

88. Given now the stress, *bc*, in BC, construct, on *bc*, the force polygon *bcde* for the four forces acting on the point 2. Thus, from *c* lay off *cd* downward, to represent the dead load on the lower chord at 2. Since *bc* acts as a pull from the left on 2, and since the forces must follow each other around the polygon, *cd* must evidently be drawn downward from *c* and not from *b*. From *d* draw an indefinite line parallel to DE, and from *b* another, parallel to BE. They will intersect at some point, as *e*, and *eb* and *de* will then represent the stresses in BE and DE.

89. Inspection would show that *be* = *cd*, since *cd* is the only force acting on 2 with a vertical component, and that *bc* = *de*; but the construction of the force polygon *bcde* is necessary for the completion of the diagram.

90. Having now found the stresses in DE, BE, and AB, and knowing the panel load (= *g a*) at the point 3, construct the polygon *g a b e f g*. This gives *e f* and *f g*, and from these the process may be continued and the diagram completed.

91. It will be noticed that, in some cases, a point on the diagram, Fig. 33 (*b*), is given more than one letter. Ordinarily this is simply a coincidence, arising from overlapping of the force polygons. In some cases, however, the coincidence of the letters shows that the stress in the member is zero.

92. In practice it is usual to construct first the entire load line *ct*, thus: draw first the net reaction, *ac*, upward; then, following around the truss counter-clockwise, draw all the other exterior (dead load) forces in their proper order, thus *cd, dk, kl, ls, st, tv, vp, po, oh, hg, ga*. The stress diagram may then be constructed, as before.

Live Loads.

93. It might at first be supposed that each member of the truss would receive its maximum stress when the train completely covered the bridge; but this is true only of the chord members. In the truss shown in Fig. 33 (*a*) each web member receives its maximum stress when the greatest possible shear occurs in a section cutting that member.

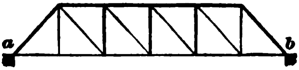


Fig. 34.

94. In Fig. 34, the *main* diagonals to the *left*, and the *counters* to the *right* of the center, C, are shown. Any one of these members receives its maximum stress from a uniformly distributed load when the load extends from it to the *right* support *b*, with head of load at a point, *o*, Fig. 32 (*a*), found as in ¶ 81; and *vice versa* for the diagonals inclined in the *opposite* direction. Each *vertical* receives its maximum stress when the load extends from the *farther* support to a point, *o* (see ¶ 81), in the panel beyond the vertical. This statement must be slightly modified when the concentrated wheel loads are considered. See ¶¶ 97, etc.

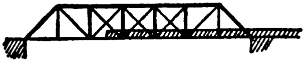


Fig. 35.

95. Assumed Uniform Live Load. As a crude approximation, the engine and train are sometimes considered as a uniform load crossing the bridge, Fig. 35; but this method, ignoring, as it does, the great concentration of weight in modern locomotives, is apt to be either unsafe or wasteful of material. This assumption is proper in connection with wind pressure on train. See ¶ 121.

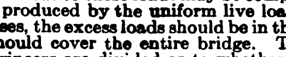


Fig. 36.

96. Concentrated Excess Loads. Again, to provide for the locomotive loads, one or more concentrated excess loads, Fig. 36, are sometimes employed. The stresses due to these loads may be computed separately, and added to the stresses produced by the uniform live loads. To produce the maximum chord stresses, the excess loads should be in the middle of the span, and the train load should cover the entire bridge. This method is fairly approximate, and engineers are divided as to whether this method of concentrated excess loads should be used, or that of the actual or "typical" locomotive wheel loads as explained below.

97. Standard or Typical "Wheel" Loadings. In the method of wheel loads, the actual stresses, produced by the heaviest engines likely to cross the bridge, are considered. Even in engines of nearly the same weight, the loads may be differently spaced, and spaced at intervals of odd fractions of an inch, rendering computation very laborious. For this reason, and in order to provide for the use of heavier engines in the future, it is customary to consider an imaginary or "typical" engine, with loads and spacing given in round numbers, the stresses from which shall at least be equal to those produced by the heaviest engines likely to be used during the life of the bridge.

The live loads are ordinarily taken as consisting of two typical locomotives with their tenders, followed by a uniform train load. See Digests of Specifications.

98. The following is an example of the computation of live load stresses by the method of locomotive wheel loads:

Fig. 37 (*b*) represents the loads on one rail, corresponding to Cooper's Standard,* Class E 40, which consists of two coupled consolidation locomotives, followed by a train considered as equivalent to a uniform load of 4000 lbs. per linear foot. In the diagram, Fig. 37 (*a*), all loads are figured in thousands of pounds, moments in millions of foot-pounds, and distances in feet.

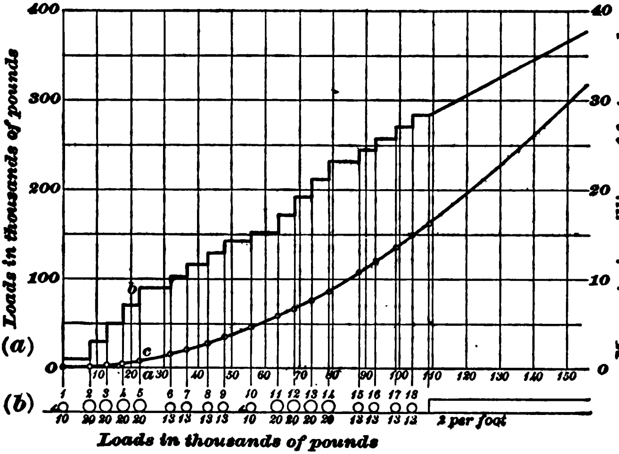


Fig. 37.

99. Live Load Web Stresses. The maximum live load stresses will occur in the web members of any panel of the truss in Fig. 34 or 38, when the live load produces the maximum shear in that panel. It can be shown that this will occur when $P = \frac{W}{n}$, where P = the live load on the panel cut by the section; W = the total live load on the truss, and n = the number of panels in the truss. This equation is called the criterion for maximum shear.

100. The following table is based upon this relation. The second column is obtained by adding successive wheel loads to P. In this case, W = 6 P, since our truss has 6 panels. Let any wheel be at a panel point. Then, by moving the wheel a little to the left or right, it will be included in or excluded from P. Hence P and W have each a minimum and a maximum value for each wheel at the panel point.

No. of wheel at any given panel point.	Value, P, of load on panel to left of given point.	Corresponding value of W for maximum shear in panel.
1	0 to 10,000	0 to 60,000
2	10,000 to 30,000	60,000 to 180,000
3	30,000 to 50,000	180,000 to 300,000
4	50,000 to 70,000	300,000 to 420,000
5	70,000 to 90,000	420,000 to 540,000

101. The correct position of live load, for maximum shear in any panel, is found by successive trials. When the correct position is found, the moment about the right support is computed, and from this the shear is obtained. For example, see below.

---

* "Transactions Am. Soc. Civ. Engrs.," vol. XLII, No. 858, Dec., 1899, p. 227. See Digests of Specifications.

102. These operations may be performed by computation, with or without the aid of graphic methods. As the method of computation alone is rather tedious, particularly when the form of the truss is complicated by curved chords or sub-panels, and as the graphic method is abundantly accurate for all practical purposes, and has the advantage of direct appeal to the eye, only the latter is given herewith.

103. The "wheel diagram," Fig. 37 (*a*),* gives (1) a stepped "load line" or "shear diagram," and (2) a curved "moment diagram" or "equilibrium polygon." See Statics, ¶¶ 359, etc.

104. The load line gives the total live load to the left of, and including, any point.

105. The moment line gives, at any point, the (left-handed) live load moment, about that point, of all loads to the left of and including that point. Thus, to the left of and including wheel No. 5 we have

Wheel.	Load.	Distance from wheel 5.	Moment about 5 in ft.-lbs.
1	10,000	23	230,000
2	20,000	15	300,000
3	20,000	10	200,000
4	20,000	5	100,000
5	20,000	0	0
Total,	90,000		830,000

and the ordinates, *ab* to the load curve, and *ac* to the moment curve, under wheel 5, measure 90.0 and 0.830 respectively.

106. Fig. 38 represents the truss, to the same scale as Fig. 37. We may call this a "truss diagram." †

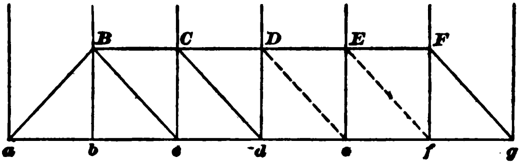


Fig. 38.

107. Example. To compute the maximum shear in the panel *bc*, Fig. 38, first find that position of the load which will produce that maximum shear. As a guess, place the truss diagram, Fig. 38,† with its point *c* under wheel 2, Fig. 37. Examining the load diagram, over the right end, *g*, of the span, we see that we now have a total load, W, of 284,000 lbs. on the span; and the load diagram, over wheel 2 (placed at point *c*) shows (see also table, ¶ 100) that the load, P, on the panel, *b c*, is now somewhere between 10,000 and 30,000 lbs.; but, for maximum shear in the panel, *b c*, the load, P, on that panel must be (see ¶¶ 99 and 100) $= W \div n = 284{,}000 \div 6 > 30{,}000$ lbs. Hence, P must be increased by moving the train diagram, Fig. 37, to the left (or, which is the same thing, by moving the truss diagram, Fig. 38, to the right) until wheel 3 is over *c*. We now have $W = 292{,}000$ lbs.; P = anywhere between 30,000 and 50,000; and required value of P, for maximum shear, $= W \div n = 292{,}000 \div 6 = 48{,}667$ lbs. Hence, the conditions are satisfied, and panel *b c* receives its

---

[*] Method published by Ward Baldwin, "Engineering News," vol. XXII, Sept. 28, 1889, p. 295. See also letter, "Eng. News," Dec. 28, 1889, p. 615.

[†] For the following discussion it will be found convenient to make a copy of Fig. 38, or simply of the lower chord, on a separate piece of paper which may be applied, in different positions, to Fig. 37.

maximum shear, when wheel 3 is at c. The moment diagram shows, vertically over g, the live load moment about the right support = 17,516,000 ft.-lbs.; and the moment at c = 233,000 ft.-lbs.

108. Let M = the (left-handed) live load moment at the right abutment, due to all the loads on the span.
L = the span.
m = the (left-handed) live load moment at the panel point on the right of the panel in question.
l = the length of the panel.
V = the shear in the panel.

$$\text{Then } V = \frac{M}{L} - \frac{m}{l} = \frac{17{,}516{,}000}{150} - \frac{230{,}000}{25} = 107{,}600 \text{ lbs.*}$$

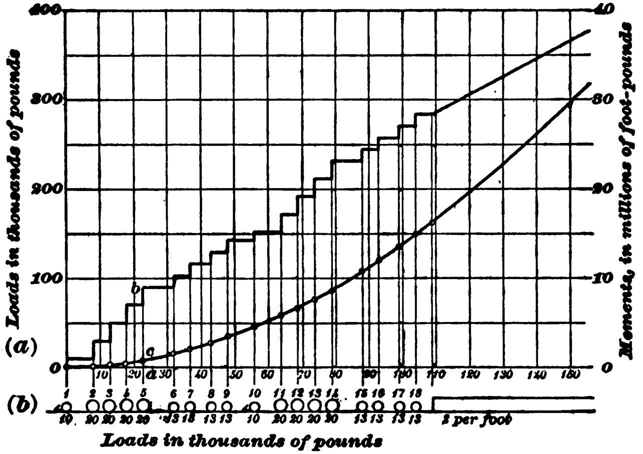


Fig. 37 (repeated).

109. The maximum live load shears in the other panels, similarly computed, are as follows, the load being, in each case, so placed as to give said maximum shear:

Panel, No. and Position of Wheel.	Mom. M at Rt. End of Truss. Ft.-lbs.	Mom. m at Rt. End of Panel. Ft.-lbs.	Shear, Pounds. $V = \frac{M}{L} - \frac{m}{l}$.	Member.	Stress, Pounds. $S = \frac{V}{\cos\theta}$.†
ab 4 at *b*	27,176,000	480,000	162,000	*a*B	—217,200
bc 3 at *c*	17,516,000	230,000	107,600	B*c*	+144,200
cd 3 at *d*	10,816,000	230,000	63,000	C*c*	—63,000
				C*d*	+84,500
de 2 at *e*	4,936,000	80,000	29,700	D*d*	—29,700
				D*e*	+39,800
ef 2 at *f*	1,743,000	80,000	8,400	E*f*	+11,250

---

* Because $\frac{M}{L}$ = the reaction of the left support, a, and $\frac{m}{l}$ = so much of the panel load as goes to the left end of the panel.

† θ = angle between diagonal and vertical.

110. The live load stress in the hip suspender, B*b*, is due entirely to loads upon the two lower-chord panels, *a b* and *b c*. Thus, with wheel 4 at *b*, panel length = *a b* = *b c* = 25 ft., we have:

On *a b*

Wheel.	Load, W.	Dist, *d*, from *a*.	Stress on B*b* = *wd* ÷ 25.
1	10,000	7	2,800
2	20,000	15	12,000
3	20,000	20	16,000
Total,	50,000		30,800

On *b c*

Wheel.	Load, W.	Dist, *d*, from *c*.	Stress on B*b* = *wd* ÷ 25.
5	20,000	20	16,000
6	13,000	11	5,720
7	13,000	6	3,120
Total;	46,000		24,840

Total load on *a b* = 50,000
" " " *b c* = 46,000
Wheel 4 = 20,000
Total load on *ac* = 116,000

Stress in B*b* from *ab* = 30,800
" " " " *b c* = 24,840
" " " " wheel 4 = 20,000
" " " Total, 75,640

111. For any given set of loads on *a c*, the *maximum* stress in B*b* occurs when the load on *a c* is *equally* divided between *a b* and *b c*; and this ordinarily occurs while some wheel (to be found by trial) is passing *b*. Thus, with wheel 4 just to the *right* of *b*, we have, on *a b*, wheels 1, 2 and 3, = 50,000; and, on *b c*, wheels 4, 5, 6 and 7, = 66,000 lbs.; but, with wheel 4 just to the *left* of *b*, we have, on *a b*, wheels 1, 2, 3 and 4, = 70,000; and, on *b c* (neglecting wheel 8, which now enters *b c*), wheels 5, 6 and 7, = 46,000 lbs. Hence, while wheel 4 is passing *b*, there is an instant when the loads on *a b* and on *b c* are equal, and at that instant the stress in B*b* reaches its maximum (75,640 lbs., see ¶ 110) for the given set of loads.

112. Live Load Chord Stresses. The criterion; for position of load for maximum bending moment in any section, and hence for maximum stress in the chord members at that section, is $\frac{W}{w} = \frac{L}{l}$, or $l = L.\frac{w}{W}$; where W = total load on the truss, w = load to the left of the section, L = span of bridge, and l = length of segment to the left of the section.

113. To find the position of load for maximum moment in any panel, by means of the moment diagram; Fig. 37; place a wheel, say wheel 2, at the panel point at the right of the given panel. From the intersection on the load line (usually coinciding with the *x*-axis) vertically over the left support, lay a ruler or stretch a thread to the intersection of the load line with a vertical from the right support. If the line so constructed recrosses the load line at a point vertically over the section in question, the position is a correct one; if not, it is incorrect. To facilitate this work, it is well to use a truss diagram, Fig. 38, drawn on a sheet of tracing paper, with the verticals carefully extended from the panel points as far up as the load or moment lines are likely to extend.

114. It will often be found that more than one position satisfies the criterion, and that some one of these may give greater moments than the others. Hence it is well to look for all possible positions. When these are found, determine the moments, thus: On the moment curve find the two points corresponding (vertically) with the left and the right support respectively, and join these points by a straight line. When the head of train has not reached the left support, the point corresponding with the left support is in the *x*-axis, produced.

115. The required moment is measured by the vertical ordinate distance along the section, between the moment curve and the straight line just constructed. The stress in the chord members affected is equal to the moment divided by the depth or the truss. Using these methods, the following results are obtained:

Section.	Wheel.	Moment, ft.-lbs.	Stress, lbs.	Members.
B*b*	4	4,049,333	144,600	*ab* = *bc*
C*c*	7	6,211,667	221,800	*cd* = —BO
C*c*	8	6,207,667	Not max.	
D*d*	11	7,044,000	Not max.	
D*d*	12	7,056,500	252,000.	—CD = —DE

Wind Loads.

116. A complete bridge is subjected not only to *vertical* loads, due to dead load, to live load, and to impact caused by inequalities in track and in rolling stock, but also to *horizontal* loads. These horizontal loads are due to the transverse action of wind, or of centrifugal forces produced by the train in passing around a curve on the bridge, and to the longitudinal traction or "drag" caused by stopping or starting a train on the bridge. Hence it is necessary to supply horizontal bracing, which, with the two upper chords or the two lower chords of the two vertical trusses, form horizontal trusses, known as the upper and lower lateral systems, Figs. 39 (*a*) and 39 (*b*), and sway and portal bracing, ¶¶ 21 and 22.

117. The wind is considered as blowing at right angles to the bridge.

118. The wind produces several effects, and these must be ascertained separately, and their joint effect then determined. Among these effects are:

(1) Direct stresses in both the upper and lower lateral systems, by pressing directly upon the chords; acting horizontally as a uniformly distributed load.

(2) Additional direct stresses on the lateral system of the loaded chord when a train is on the bridge, owing to pressure of wind against the train.

(3) An overturning moment upon the bridge as a whole, thus increasing the dead and live load stresses in the leeward and diminishing those in the windward vertical truss.

(4) A similar overturning effect upon the train and its wheels, which similarly modifies their pressures upon the floor beams and thus the stresses in the main trusses.

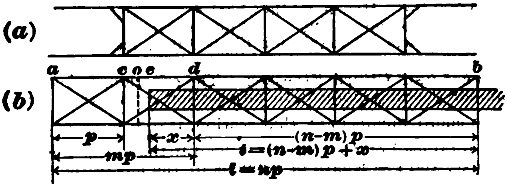


Fig. 39.

119. The wind load, acting directly upon the bridge, is assumed to be equally divided between the upper and lower chords, and between the windward and leeward trusses.

120. (1) The direct wind stresses in the lateral bracing, due to the pressure of the wind on the truss, are found as are the stresses in the main trusses, due to dead load; the horizontal transverse struts of the lateral bracing corresponding to the verticals of the main trusses.

121. (2) Direct stresses in lateral system of loaded chords, Fig. 39 (*b*), due to wind on train.

Examining any panel, as $c\,d$, let

w = wind pressure, in lbs. per lineal foot of train;
p = panel length, in feet;
$w\,p$ = wind pressure, in lbs., per panel fully occupied by train;
n = number of panels in span (= 6 in this case);
$l = n\,p$ = span, in feet;
m = number of panels from left support, a, to and including the panel, $c\,d$, under consideration;
$m\,p$ = distance, $a\,d$, in feet;
x = length, in feet, of that portion of the panel, $c\,d$, which is occupied by train;
t = $(n - m)\,p + x$ = that portion of the span which is occupied by train, in feet;
= wind pressure on train for a pressure of 1 lb. per lineal foot;
R = truss wind reaction,* at a, = $w\,t^2 + 2\,l$;
r = panel wind reaction,* at c, = $w\,x^2 + 2\,p$;
S = wind shear * in panel, $c\,d$, = $R - r = w\,t^2 + 2\,l - w\,x^2 + 2\,p$.

---

* See foot-note (†), ¶ 2.

The horizontal *truss* reaction,* at a, due to a concentrated horizontal pressure, $= 1$, acting at any distance, y (not shown), from d, is $= \frac{(n-m)\,p + y}{n\,p}$; and the horizontal *panel* reaction, at c, due to the same pressure, is $= \frac{y}{p}$. The maximum wind shear,* in the panel, $c\,d$, due to wind on train, occurs (see ¶¶ 79 to 81) when the head of the train reaches that point, o, at which, if the concentrated load be placed, these two reactions will be equal, or $\frac{y}{p} = \frac{(n-m)\,p + y}{n\,p}$. With head of train at o, we have $x = y = \frac{(n-m)\,p}{n-1}$.

Under any conditions, the wind shear,* S, in the panel, is $= \mathrm{R} - r$, where $\mathrm{R} = \frac{w[(n-m)\,p + x]^2}{2\,n\,p}$; and $r = \frac{x^2}{2\,p}$.

Substituting here the value of x, just found, for maximum shear, we obtain, as the maximum value of the wind shear * in the panel,

$$\mathrm{S}_{max} = \frac{w\,p}{2\,(n-1)}\,(n-m)^2.$$

122. (3) **Stresses in main truss members, due to overturning moment of wind on truss.**

Overturning moment = (wind panel load at top chord) × (number of panel points in span) × height of truss.

$$\text{Vertical reaction at one support} = \frac{1}{2}\,\frac{\text{overturning moment}}{\text{width between trusses}}.$$

Since the upper lateral system carries all wind loads to the ends of the bridge, the end posts and the chords (which take the horizontal components of the end post thrusts) are the only main truss members affected.

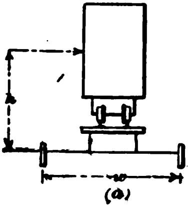


Fig. 40.

123. (4) **Stress in main truss members, due to overturning moment of wind on train, Fig. 40.** Let h = height from center of gravity of lateral system of loaded chord to center of pressure of wind on train, p = wind pressure per lineal foot of train, w = width between centers of gravity of trusses, m = overturning moment, per lineal foot of train, v = added vertical load on leeward truss, per lineal foot of truss. Then $m = h\,p$, and $v = \frac{m}{w}$.

Impact, Etc.

124. The effects of impact, due to inequalities of the track; those of "drag," due to the starting and stopping of trains; and those of centrifugal force of the train on curves, are not susceptible of rigorous calculation, and engineers differ in their requirements respecting provision for them. See Digests of Specifications.

---

* See foot-note (†), ¶ 2.

Determination of Maximum and Minimum Stresses.

125. Where specifications make allowable unit stresses depend upon the relation between the maximum and minimum stresses in any given member, both must be computed.

In computing the maximum and the minimum stress in any member, bear in mind that a condition which, of itself, would have a certain effect upon the stress, may bring with it other conditions which produce a greater effect of the opposite kind. Thus, although the action of wind on train would, of itself, reduce the stresses in certain members, this action can take place only with train on bridge, and the vertical action of the train load would ordinarily increase those stresses more than the wind action would diminish them.

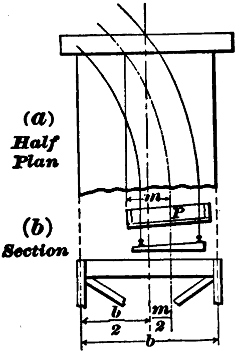
In computing minimum stresses, although the live load is usually to be neglected, we must of course not neglect the dead load, which is always present.

Curves on Bridges.

126. When the track on a bridge is curved, it is usually so laid that the center line of the bridge bisects the middle ordinate, *m*, of the curve. See Fig. 40 (*a*). The center of gravity of a panel load, P, at the center of the span (supposing it to stand over the center of the track) is thus thrown out a distance $= \frac{1}{2} m$ from the center line of the bridge, or a distance $= \frac{1}{2} b + \frac{1}{2} m$ from the inner truss, where b = width of bridge between centers of trusses. Taking moments about the center of the inner truss, we have, therefore, for the load, W, on the outer truss, due to P,

$$W = P \frac{\frac{1}{2} b + \frac{1}{2} m}{b} = P \frac{b + m}{2 b}.$$

It is customary (see Digests of Specifications) to proportion the outer truss on the safe assumption that its share of the live load, at each panel point, is determined by the formula just given, and to design the inner truss like the outer one.



Figs. 40 (a) and (b).

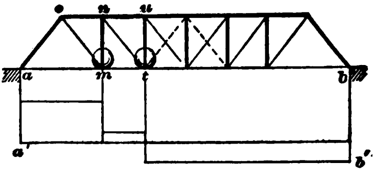


Fig. 31 (repeated).

Counterbracing.

127. In a truss of any ordinary form (like that in Fig. 31), under the action of a uniformly distributed dead or live load, or of a live load distributed symmetrically as regards the center of the span, the shears in each panel on the *left* of the center of the span are *positive*, while those in the panels on the right are *negative;* and the stresses throughout the truss are such that the ties sustain tension, and the struts, compression; the tendency, in each panel, being to *elongate* that diagonal occupied by a *tie*, and to *shorten* that diagonal occupied by a *strut*.

But the tendency of an *eccentric* load, such as those shown in Fig. 31, is to *reverse* the shears in the panels between it and the center; and, if this ffort, relatively to the other forces, is of sufficient magnitude to reverse the final shear in any panel, the tendency will be to *shorten* the diagonal occupied by a *tie*, see Fig. 1 (*a*), and to *lengthen* any diagonal occupied by a *strut*.

As explained in ¶¶ 14 and 15, this condition is met, in the Warren or triangular truss, by making each web member capable of resisting both tension and compression; and, in trusses with both vertical and diagonal web members, by inserting counters.

In a drawbridge or swing bridge, not only the web stresses, but also the chord stresses, are reversed when the draw is opened or closed.

To provide against possible further increase in live loads, over those now in use, specifications sometimes require that, wherever the live and dead load stresses are of opposite character, only 70 per cent. of the dead load stress shall be considered as effective in counteracting the live load stress. For other methods of making similar provision, see Digest of Specifications for Steel Railroad Bridges.

Roof Trusses.

128. In roof trusses, **the dead load,** *i. e.*, the weight of the truss itself and that of the purlins, roof covering, etc., and the snow load, are usually taken as uniformly distributed. In many cases the sum of the dead and snow loads is divided equally between the two supports. In other words, the end reactions are equal.

129. **The weights of steel trusses,** in pounds per square foot of building space covered, may be taken, for preliminary estimate, at (0.05 to 0.08) × span in feet, according to design and loading. Those of **wooden trusses,** with wooden, iron or steel tension members, may be taken at from one-tenth to one-fifth less.

If it is found that the weight of a truss, as designed, considerably exceeds the weight assumed for it in advance, it should be redesigned, assuming a new weight slightly greater than that obtained from the design.

130. **The weights of purlins,** of steel or wood, may be taken at from 2 to 3 lbs. per square foot of building space covered.

131. **The weight of roof covering** may be taken approximately as follows:

Corrugated iron, ...	2 to 3	lbs. per sq. ft. of roof surface.
Slate, ...	7 to 9	“ “ “ “
Shingles, on laths, ...	2 to 3	“ “ “ “
If on boards, add, ...	3	“ “ “ “
If plastered below the rafters, add,	6	“ “ “ “

132. **The snow load,** in States north of lat. 35°, may be taken as varying (chiefly with latitude) from 10 to 30 lbs. per sq. ft. of horizontal projection of roof surface.

133. **The purlins, stringers, etc.,** should be so arranged as to carry the weight of roof covering and of snow directly to the panel points, and thus avoid transverse stresses in the rafters.

134. Each truss bears, besides its own weight, half the weight of roof and snow between the two trusses (or truss and wall), adjacent to it, and each panel point bears half the load between two panel points (or panel point and end support) adjacent to it.

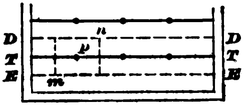


Fig. 41.

Thus, Fig. 41, truss TT carries a weight = that on the surface between the two dotted lines, DD and EE; and panel point *p* carries a weight = that on the rectangle *mn*.

135. The wind is regarded as blowing horizontally upon one side of the roof and as exerting a uniformly distributed normal pressure upon that side. In the following table of assumed normal pressures against sloping surfaces, under horizontal wind pressures of 40 lbs. per sq. ft., the values in the last column are based upon Hutton's experiments. Here *a* is the angle between the sloping roof surface and a horizontal plane.

Assumed normal wind pressure, P, in lbs. per sq. ft. Horizontal wind pressure = 40 lbs. per sq. ft. *a* = angle between roof surface and horizontal plane.

		P				P	
a.	sin *s*.	40.sin *a*.	Hutton.	*a*.	sin *a*.	40.sin *a*.	Hutton.
5°	0.087	3.5	5.1	35°	0.574	22.9	30.1
10°	0.174	7.0	9.6	40°	0.643	25.7	33.3
15°	0.259	10.4	14.2	45°	0.707	28.3	36.0
20°	0.342	13.7	18.4	50°	0.766	30.6	38.1
25°	0.423	16.9	22.6	55°	0.819	32.8	39.4
30°	0.500	20.0	26.5	60°	0.866	34.6	40.0

136. The directions and amounts of the end reactions and of the stresses in the members, due to wind, depend upon whether one or both supports are fixed. If both ends are fixed, their reactions are parallel to the normal wind pressure—*i. e.*, they are at right angles to that side of the roof upon which the wind is blowing; but, if one end is free to slide longitudinally of the truss, its reaction is taken as vertical and that of the other is more nearly horizontal than the normal wind pressure. When one end is free, the stresses must be determined for wind blowing on the fixed side (in which case it tends to flatten the roof) and also for wind blowing on the free side, in which case its horizontal component tends to shorten the tie-rod and to raise the apex. The stresses in the members of roof trusses are conveniently found by means of the method by sections, ¶¶ 67, etc., or graphically, as below.

137. Fig. 42 (*a*) illustrates the graphic treatment of wind stresses for Fig. 42 (*b*), under the three conditions named, viz.:—case 1, with both ends fixed; case 2, wind blowing against the fixed side; and case 3, wind blowing against the free side.*

In Fig. 42 (*a*), the segments *ab*, *bc*, *cd* and *de* represent the normal wind pressures at the panel points AB, BC, CD and DE respectively, and *ae* therefore represents the total normal wind pressure on the roof, all being exerted against the left side.*

138. In case 1 (both ends fixed) the segments *fa* and *ef* of the solid line *ea* represent the left and right reactions respectively.

139. In case 2 (wind blowing against fixed side) the reactions are represented by the dash line *ef'a;* and in case 3 (wind blowing against free side) by the dash-and-dot line *ef''a*.

140. The segments *f'f* and *ff''* represent the horizontal components of the right and left wind reactions respectively in case 1; and *f'f''* that of the wind reaction of the fixed end in case 2 or case 3, or that of the total wind reaction.

141. Having found, by moments, the end reactions *ef* and *fa* for case 1; where, Fig. 42 (*b*),

$$ef = ae\,\frac{A\,s}{\text{Span}};$$

the vertical reactions, *ef'* and *af''*, for cases 2 and 3 respectively, are found by dropping perpendiculars from *e* and from *a*, Fig. 42 (*a*), upon *gf* produced. The reactions of the fixed ends are then given by the closing line, *f'a*, in case 2, and by *ef''* in case 3.

---

* To avoid the necessity of showing two skeleton figures and two diagrams, we have supposed the wind to blow always in one direction (viz., against the left side) and first one end of the truss and then the other end to be fixed. In practice, of course, the reverse of this is the case; *i. e.*, one end or the other of the truss (if not both) is fixed, and remains so; and the wind may blow against either side. The figure and diagram will, however, answer for this latter condition also. Thus, if the wind blow against the left side, as shown, and if, as in case 2, that side is fixed, then the diagram, using the broken lines, *e f' a*, gives the stresses in the members, as they are lettered. But now (the left end remaining fixed) suppose the wind to blow against the free side; *i. e.*, from the right. We may nevertheless suppose the right end fixed, and the wind blowing against the left side, as in Fig. (*b*), and find the stresses in the members from Fig. (*a*) as it stands, using the dash-and-dot diagram, *e f'' a;* but we must then remember that the stresses thus found for BG, GF, etc., on the left of the truss, Fig. (*b*), really apply to the corresponding members, QE, QF, etc., on the right, and vice versa

142. The stresses in the web members, GH, MN, etc., Fig. (*b*), and those in the several members, BG, EM, etc., of the rafters, are given by the corresponding lines, gh, mn, bg, em, etc., in the diagram, Fig. (*a*).

143. In the leeward rafter, in this case, the stress in the three segments, ME, PE, QE, is uniform throughout, and moreover it is the same in each of the three cases, being $= me = pe = qe$.

In the four web members, LM, MN, NP, PQ, to the leeward of the center, the stress, in this case, is zero, being represented by the point, $lmnpq$, Fig. (*a*).

144. The stresses in the several segments, GF, JF, LF, NF, and QF, of the horizontal tie rod, Fig. (*b*), are represented, in Fig. (*a*),

in case 1 (both ends fixed) by gf, jf, lf, nf and qf;
in case 2 (wind against fixed side) by gf', jf', etc.;
in case 3 (wind against free side) by gf'', jf'', etc.

In each of the three cases there is uniform tension in the three leeward segments, LF, NF and QF, of the horizontal tie rod. This uniform tension is

in case 1 (both ends fixed) $= lf = mf = nf$;
in case 2 (wind against fixed side) $= lf' = mf' = nf'$;
in case 3 (wind against free side) $= lf'' = mf'' = nf''$.

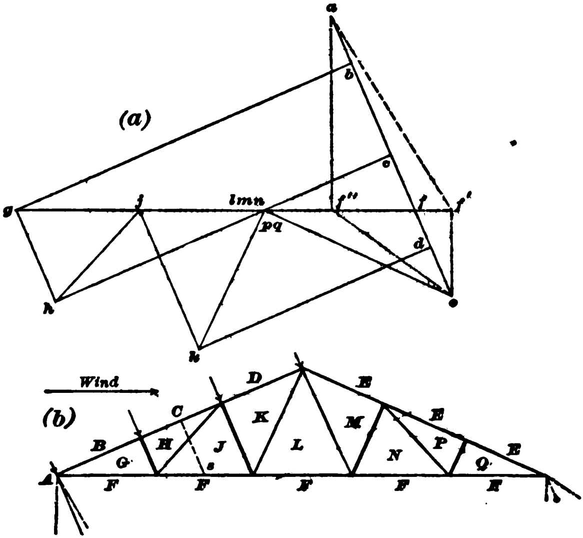


Fig. 42.

145. It is thus seen that, in our Fig., with horizontal tie rod, the difference in the manner of supporting the ends affects only the horizontal stresses in the members of that rod, and, through them, the manner in which the horizontal component $f'f''$ of the wind stress is distributed between the two supports.

146. If the lower chord were not straight, however, the stresses in the rafter and web members would be affected by the difference between the three cases.

147. The final or resultant stress, in any member, is the algebraic sum of the dead, snow and wind loads for that member. In some cases, the wind load may diminish or even reverse the stresses due to dead and snow loads.

148. In timber roof trusses of short span, Figs. 43 to 47, for roofs of dwellings and other small buildings, we may, with sufficient accuracy, make a liberal assumption for load, to include wind pressure. In discussing these figures, we investigate the stresses by means of the force parallelogram. For dimensions of such trusses, see ¶ 266.

149. In the wooden roof truss, Fig. 43, uniformly loaded along each rafter, let H I = the weight of one rafter and its load. Then E I = the horizontal pressure of the head and of the foot of that rafter (the latter being the tension in the chord), and H E = the inclined pressure at its foot.

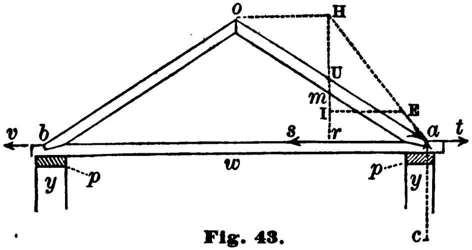


Fig. 43.

150. In Fig. 44, make G R = H I. Then G L is the transverse pressure of the load against the rafter as a beam, and L R is a longitudinal pressure along the rafter, forming a *part* of the *total* longitudinal pressure.

151. If G R were concentrated at G, L R would be uniform from G to *a*, and would not be exerted above G; but, as G R represents a load uniformly distributed along the rafter, from top to foot, the pressure represented by L R increases uniformly, from nothing, at the top, *o*, to L R, at the foot, *a*.

152. Of the transverse pressure, G L, one half, = *o p*, is sustained at the top, *o*, of the rafter, and the other half, = *a q*, at the foot, *a*. At the top, *o p* is resolved into the horizontal pressure, *o b* = E I, against the head of the other rafter, and a uniform thrust, *o z*, along *o a*.

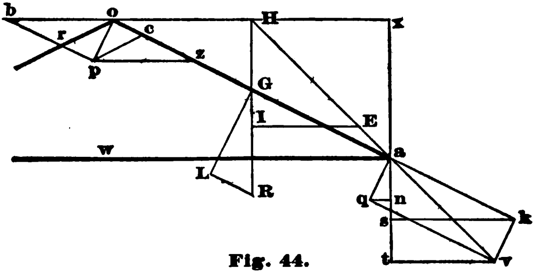


Fig. 44.

153. It is immaterial whether we thus resolve *o p directly* into *o b* and *o z* (as though the head of the rafter rested against a *vertical wall* at *o*), or whether we first resolve it between the two rafters, into *o c* and *o r*. For in the latter case we must add to *o c* a thrust (= *o r* = *c z*) produced in *o a* by the transverse pressure (similar to *o p*) of the head of the *other* rafter; and the sum of these two (*o c* and *o r*) is = *o z*.

154. The total longitudinal thrust in the rafter increases uniformly from *o z*, at the top, to $o\,z + \mathrm{L\,R} = a\,k$, at the foot, where it combines with *a q* (= half the transverse load) to form $a\,v = \mathrm{H\,E}$.

155. Tension in chord $= \mathrm{I\,E} = t\,v = s\,k - n\,q$. Vertical pressure on support $= \mathrm{H\,I} = a\,t = a\,s + a\,n = a\,s + s\,t$.

156. In Fig. 45, having found, as for Fig. 43, the stresses, etc., due to the rafters and their loads, remember that the king rod, $o\,n$, supports its own weight plus the portion $y\,y$ of the chord $= \frac{1}{2}$ the chord. Making $o\,t =$ this combined weight, we have $o\,m = o\,d =$ an additional pressure, uniform throughout each rafter, and $c\,m = c\,d =$ an additional tension on the chord.

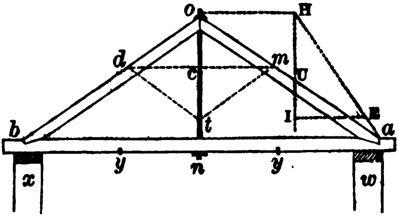


Fig. 45.

157. In Fig. 46, assuming, for safety, that the rafter, $j\,b$, is divided at its center, U, make $e\,o =$ the weight of $z\,r$ and its load ($z\,r =$ half the rafter).

158. Then $e\,i =$ an additional pressure on U b, $e\,k =$ pressure on U c; $s\,i = s\,k =$ additional tension on half chord, $c\,b$, and $e\,o = 2\,e\,s =$ load of and on $z\,r$, $=$ additional tension on king post due to both struts. Then make $a\,g = e\,o +$ weight of king rod $+$ weight of two struts $+$ weight of and on $y\,y$, and proceed as in Fig. 45.

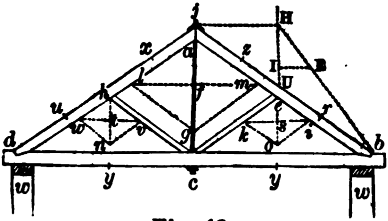


Fig. 46.

159. Each strut will thus bear *half* of the weight of and on $z\,r$, or $x\,u$, only when, as in Fig. 46, the inclination of the strut is the same as that of the rafter. If the strut is steeper than the rafter, it will bear more than half; but if it is less steep than the rafter, it will bear less than half; the remainder being in every case borne by the rafter.

160. The introduction of the struts converts each rafter, considered as a beam, into two beams of shorter span and bearing less loads.

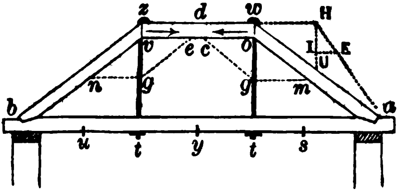


Fig. 47.

161. In the queen truss, Fig. 47, make $o\,g =$ total tension in queen rod $+$ half weight of and on the "straining beam," $z\,w$. Longitudinal pressure on $z\,w =$ tension in chord, $b\,a$, $=$ I E $+ o\,c$.

Deflections.

162. The total deflection of a truss * comprises (1) the elastic or temporary deflection, due to the stretch† of its several members under the loading applied to the truss, and (2) the non-elastic or permanent deflection, due to looseness of its joints. In good construction the latter is relatively negligible in moderate spans.

The total elastic deflection, D, of a truss, at any point, *c*, is made up of partial elastic deflections, *d*, *d*, etc., at *c*, each due to the stretch, *k*,† in some member.

Let it be required to find the deflection at a panel point, *c* (usually the center of a span or the end of the arm of a swing bridge or other cantilever); and, for any load or system of loads, let

D = the total elastic deflection at *c*;
d = the partial elastic deflection, at *c*, due to the stretch, *k*,† in any member;
p = the unit stress in that member;
P = the total stress in that member;
l = the length of that member;
k = the stretch † in that member $= \frac{p\,l}{E}$;
W = that load which, applied at *c*, would produce the stress, P, in that member;
u = $\frac{P}{W}$;
E = the modulus of elasticity of the material, $= p \div \frac{k}{l} = \frac{p\,l}{k}$.

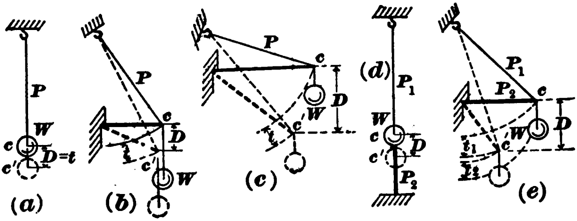


Fig. 48.

163. Equivalence of Work. In Fig. 48, let any load, W, be applied at any point, *c*, of a truss or bar. Then, for a small deflection,‡ such as may be permitted in trusses, the external work, W d,* of a partial deflection, d, due to the stretch, k, in any member, is practically = the internal work, P k, of overcoming the resisting stress, P, in that member, through the distance, k; or

---

* See "The Application of the Principle of Virtual Velocities to the Determination of the Deflection and Stresses of Frames," by Geo. F. Swain, Jour. of the Franklin Institute, vol. LXXXV, 1883; "Trusses with Superfluous Members," by Wm. Cain, Van Nostrand's Magazine, vol. XXVII, No. 4, October, 1882; "The Graphical Solution of the Distortion of a Framed Structure," by David Molitor, Jour. Ass'n Eng'ng Societies, vol. XIII, No. 6, June, 1894; and "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan and F. E. Turneaure, New York, John Wiley & Sons.

† For brevity we here use the word "stretch" to signify any change of length, including the shortening due to compression, as well as the elongation due to tension.

‡ For the sake of clearness, the stretches and deflections, in our Figs., are exaggerated beyond the limit within which the ratio, $\frac{P}{W}$, would remain even approximately constant.

$$W\,d = P\,k.$$

Hence,

$$d = \frac{P}{W}\,k = u\,k; \quad \text{or} \quad \frac{k}{d} = \frac{W}{P}.$$

In words, the stretch, k, in any one member, is, to the resulting partial deflection, d, at c, inversely as is any stress, P, in that member, to a load, W, which, if applied at c, would cause that stress.

Thus, in Fig. 48 (a), where k is in the same direction as D,† P = W, and D = k.

In Figs. (b) and (c),‡ suppose the strut incompressible. Then D is due solely to the elongation of the tie, and $D = \frac{P}{W}\,k = u\,k$.

In Fig. (c), $\frac{P}{W}$ is greater, and (for a given stretch, k, in the tie) D is therefore greater, than in Fig. (b).

164. Deflection Independent of Nature of Cause of Stretch. Now it is evident that the deflection, d, at c, depends solely upon the amount and character of the stretch, k, in the member, and is independent of the nature of the cause of that stretch. That is to say, any change, k (however caused), in the length of the member, necessarily contributes its fixed quota, $d = \frac{P}{W}\,k$, to the total deflection, D, at c. In other words, since d and k are mere distances, and since u is simply a ratio, the relation between d and k is a purely geometrical one, and is therefore not confined to deformations produced by applied loads, but is applicable also to those produced by changes of temperature, to intentional lengthening or shortening of members, or to any other cause.

Hence, if a member be *in any way* lengthened or shortened, by a length, k, a corresponding change, $d, = \frac{P}{W}\,.\,k = u\,k$, takes place in the deflection at c.

For instance, if we place any system of loads upon a truss, and, by the principles of statics, determine the resulting total stress, P, and unit stress p, in any member; we have, for the partial deflection, at c, due to the stretch, k, in that member, under the given system of loads,

$$d = u\,k; \qquad \text{(For } u\text{, see ¶ 165.)}$$

and, since $k = \frac{p\,l}{E}$,

$$d = \frac{p\,u\,l}{E}.$$

165. To obtain the ratio, $u, = \frac{P}{W}$, for each member, we suppose a concentrated load applied at c; and, by the principles of statics, find the resulting total stresses, P, P, etc., in the several members. If the supposed load, at c, be taken = unity, the stresses, P, P, etc., so found, are the desired ratios, u, u, etc.

---

* Strictly speaking, with a load increasing gradually from 0 to W, and with resulting stress increasing gradually from 0 to P, we should deal with the *mean* load, = ½ W, and with the *mean* stress, = ½ P, in each member; but it will be seen that this would not affect the equations derived.

† Where, as in Figs. (a), (b) and (c), only one member is supposed to change its length, D = d.

‡ See note (‡) on preceding page.

166. Summation of Deflections. The total deflection, D, at c, under the given system of loads, being = the sum of the partial deflections, d, d, due (but not necessarily equal) respectively to the stretches, k, k, in the several members, we have

$$D = \Sigma\, d = \Sigma \frac{p\,u\,l}{E}.$$

Thus, in Figs. (d) and (e), we assume the tie extensible and the strut compressible. In Fig. (d), $W = P_1 + P_2$; and $W\,D = P_1\,k + P_2\,k = (P_1 + P_2)\,k$. Hence $D = \Sigma\, u\, k = \frac{P_1}{W}\,.\,k + \frac{P_2}{W}\,.\,k = \frac{P_1 + P_2}{W}\,.\,k = \frac{W}{W}\,.\,k = k$. In Fig. ($e$), $D = \Sigma u\,k = \frac{P_1}{W}\,.\,k_1 + \frac{P_2}{W}\,.\,k_2 = u_1\,k_1 + u_2\,k_2$.

167. Positive and Negative Stretches. In some cases it may happen that the change of length of a member diminishes, instead of increasing, the total deflection at the point, c, in question, and must therefore be taken as negative in summing up the values of $u\,k = \frac{p\,u\,l}{E}$; but when c is the middle point of a span, or the end of a cantilever, all the changes in length of the members ordinarily contribute to the deflection, and must therefore be taken as positive.

Theoretically, the formula, $D = \Sigma \frac{p\,u\,l}{E}$, applies also to the deflections of arches, dams and other structures composed of blocks; but, owing to the uncertainty of the values of E, and to the relative inaccuracy of finish in masonry work, it is of but little practical utility in such cases.

168. Redundant or Statically Indeterminate Members. Trusses frequently contain members whose stresses cannot be found by the principles of statics. Thus, in Fig. 11 (c), the two diagonal tension members meeting at the top of either end post are said to be redundant, or statically indeterminate, because the principles of statics do not enable us to determine what proportion of the total load goes to the supports through each of the two systems, Figs. 11 (a) and (b), composing Fig. 11 (c). But the deflection formula, just given, enables us to determine the stresses in such members; for, by means of it, we may find, separately, the deflection in each of the two systems, Figs. 11 (a) and (b); and the part load, transmitted to the supports through each of these two systems, is inversely proportional to their deflections.

BRIDGE DETAILS AND CONSTRUCTION.

General Principles.

169. In general, a truss **bridge** consists essentially of two or more vertical **trusses**, AB, CD, Fig. 49, placed side by side, and connected by the **floor system**, which, in turn, they support; and bracing (forming a "**lateral system**") is supplied between opposite chords, where practicable, in order to maintain the trusses parallel.

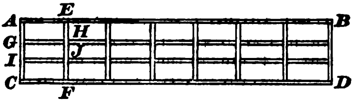


Fig. 49.

170. The **floor system** consists ordinarily of **floor beams** and **stringers**. The floor beams, AC, EF, etc., Fig. 49, are placed transversely to the bridge, and are attached to the trusses at opposite panel points. Connected with these and perpendicular to them or parallel to the trusses, are the stringers, GH, IJ, etc. In railroad bridges, there are usually two or more stringers placed side by side and running the length of the bridge, to support the ties. In city highway bridges, these stringers are usually spaced at smaller intervals, and support buckled plates or other form of flooring, on which the paving is laid. For country highway bridges, the stringers are frequently of wood, placed quite near together, and the planks of the floor are nailed or spiked directly to them.

Solid floors (see Pencoyd floor sections) add to the rigidity and permanence of a bridge, and give increased protection to traffic below, against injury from falling bodies or in case of derailment. Their shallowness is an advantage where head-room is an object.

171. Any load, then, **is carried** first from the ties or floor, etc., to the stringers, then by the stringers to the floor beams, and finally by the floor beams to the panel points of the bridge, where it is carried through the trusses to the supports.

172. Pedestals, shoes or bed plates, Fig. 62, bolted to the piers, support the ends of the trusses. When the bridge is of long span, so that the expansion and contraction due to heat and cold are considerable, **expansion bearings,** Figs. 60, 62, must be provided at one end. See ¶¶ 205, etc. For cross-bracing, see ¶¶ 19, etc.

General Character of Design.

173. Flexible and Rigid Tension Members. Adjustable Counters. Until recently, **eye-bars** have generally been used for the tension members of trusses. These are long flat bars, liable to yield laterally under compression, and furnished, at their ends, with eyes or openings, through which pass pins connecting them with the other members of the bridge; but rigid built members, capable of sustaining some compression, as well as tension, are now much used for tension members. **Counters** were usually made in two lengths, and were **adjustable,** the two lengths being connected by turnbuckles; but these rendered it possible to bring undue and dangerous stresses in the panels, and they are now giving place to counters made each in one length.

174. Compression members are ordinarily **"built up"** of angles and plates, or of channels and plates with latticing, in hollow shapes, bringing most of the material as far as possible from the neutral axes of the cross-section and thus increasing its resisting moment.

175. Pin and Riveted Connections. The web members are connected with the chords either by pins or by rivets. In the former case the truss is said to be pin-connected; in the latter case, riveted.* Until recently, pin-connected trusses have been typical of American practice; but the Americans are now largely using riveted trusses, for spans up to from 150 to 175 ft., while the Europeans are in some cases using pins. The principal advantage claimed for the riveted joint is that it makes a stiffer bridge and one that will not rattle, and that a riveted truss, computed as if pin-connected, will have an additional margin of safety on account of added stiffness. In the pin-connected bridge, on the other hand, the stresses can be much more accurately determined, and deflection may take place without producing twisting or bending stresses in the connections themselves.

176. Tendency to Greater Rigidity. There is a growing tendency to use stiffer bracing, to design at least all short braces for compression, and to make even the longer tension members of channels or angles, forming a rigid member. Unless pin-connected eye-bars are of exactly equal length, some of them will receive more than their share of the total stress.

177. Floor-beam Connections. In the United States, floor-beam connections were formerly made by hanging the floor beams from the pins by means of hangers; but now, where possible, the ends of the floor beams are riveted directly to the inner sides of the posts.

178. In tension members, rivets are so arranged as to reduce the net effective section as little as possible.

179. Compression members are so designed as to place most of the material as far as possible from their neutral axes, and they are sometimes strengthened by auxiliary ties or posts supporting them at their middle points, in cases where the resulting saving in material for the member will be considerably greater than the expenditure of material in the auxiliary member.

---

* Riveted trusses are unfortunately called, also, "lattice girders," "lattice trusses," "riveted lattice girders" and "riveted lattice trusses." The term "lattice" is often applied to shallow trusses with numerous panels.

180. So far as possible, compression members are made equally strong against bending about either of the two principal axes, AB and XY, Fig. 52, of their cross-sections.

181. Where the same member occurs many times in a bridge, and where, therefore, an excess of material in the design of such member would involve a large total waste, the computation of the member is repeated many times until the most economical section is found.

182. In metal trusses the shorter members are usually made to withstand compression, and the longer ones tension, this being more economical of material. Thus, the Pratt truss, with diagonal tension members, is used for steel bridges, while the Howe truss is now built only with wooden diagonals.

Tension Members.

183. In eye-bars, area of cross-section $= \dfrac{\text{maximum tensile stress}}{\text{allowable unit tension}}$.

184. The dimensions of the heads of eye-bars are usually determined by the manufacturers, and are so designed as to give ample excess of strength at the pin-holes; so that, if tested to destruction, fully two-thirds of the number of bars tested shall break in the body of the bar, this being usually required by specifications. It is important that the proportions of eye-bar heads should be such as to ensure thorough working of the metal in the upsetting process.

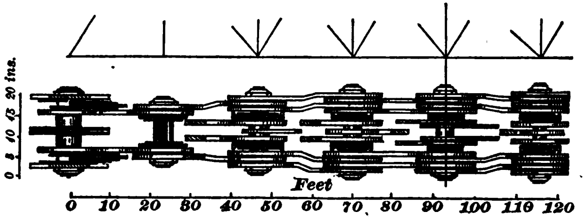


Fig. 50.

185. Fig. 50 shows, to two different scales, the "packing" (arrangement of pins and eye-bars) in the left half of the lower chord of a 150 ft. through (skew) span built by the Phœnix Bridge Co. in 1900 for the Philadelphia and Reading Railway Co. near Reynolds, Pa.

186. Built Sections. Hip vertical hangers, non-adjustable counters and their corresponding mains, are usually built up of rolled steel shapes. A section in common use, shown in Fig. 51, consists of four angles, connected, at intervals, by small narrow flat bars, riveted to the angles and running across zigzag from one to the other. When single, as in Fig. 51, this is called "lacing"; when double, as in Fig. 52 (*b*), "latticing." The shaded area of the angles, Fig. 51, minus that of the rivet holes, is taken as the effective section.

187. Minimum Sections. Specifications (see Digests) usually require the use of some minimum section. Thus, in a counter in which the stress is 58,000 lbs., 3.5 square inches of cross-section would suffice; but specifications frequently forbid the use, in such sections, of any angle smaller than 3½ × 3½ × ⅜, which gives 9.20 square inches gross; or, deducting one rivet hole from each angle, 7.68 square inches net section.

Compression Members.

188. The computation of a compression member consists of a series of approximations; for the unit stress depends upon the radius of gyration, the radius of gyration on the area of section and disposal of material with regard to the axes, and this, in turn, on the unit stress. See Pillars, under Strength of Materials.

189. Fig. 52 (*a*) shows a form often used for posts, and consisting of **two channels,** placed with their backs outward and riveted together by lacing. In Fig. 52 (*b*) the channels are placed with their backs inward. For economy, the channels should be so spaced as to make the radius of gyration the same about either axis, A–B or X–Y. The radius of gyration is given in the hand-books of mfrs of structural shapes. See pp. 892, etc.

190. The upper chord section is frequently **built up** of two channels and a plate, or in some such form as shown in Fig. 53, consisting of two vertical plates or "webs," a horizontal top plate or "cover," four "angles," and flat pieces or bars on each side of the bottom. Lattice bracing, or lacing, is provided along the bottom, except at panel points, where it is omitted in order that the post and the ties may enter the chord from below. In pin-connected trusses the axis of the pin lies in the line AB.

191. The interior width, *w*, depends chiefly on the space required by the post and ties which meet at the panel point, and also upon the height of the inside rivet heads. Usually, for convenience of construction, the greatest width, *w*, required is kept constant throughout the upper chord. The height, H, depends chiefly on the size of eye-bar head, and is kept constant. The thicknesses of the web plates, and sometimes also those of the bars and angles, are varied, along the chord, in order to provide, at each point, sufficient area to withstand the stress.

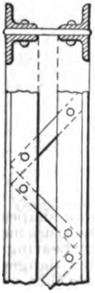
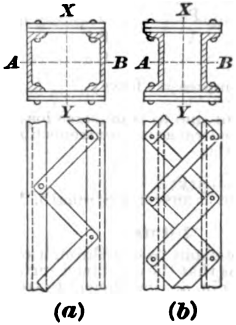


Fig. 51.



(a) (b)

Fig. 52.

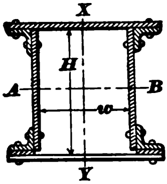
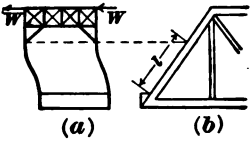


Fig. 53.

192. The **end post** is to be considered not only as a column, but also as a beam subject to shear, on account of the wind blowing against the top of the side of the truss. The design of this built-up form is much the same in principle as that given above for a post. Certain sections are tried, and then changed if necessary.

193. The end post must be safe, not only against bending about the axis, A–B, Fig. 53, under compression, but also against bending about the other axis, X–Y, under the combined effect of compression and the bending moment, due to the wind blowing against the top of the truss. See Fig. 54 (*a*).



(a) (b)

Fig. 54.

194. The portal bracing, above, and the floor beam, below, are assumed to prevent bending of the parts of the posts adjacent to them; and we may consider either half of either post ($= \frac{1}{2} l$) as a vertical cantilever, fixed at one end, and loaded, at the other end (which is at the middle of the post) with a load = wind pressure on half the upper portion of one truss.

195. The maximum stresses, due to compression, of course occur about the middle of the post, while those due to the wind occur near the ends. Hence it would be unreasonable to require the post to resist all of both effects simultaneously throughout its length; and specifications therefore usually allow the unit stress, due to dead, live, impact and wind loading combined, to be increased to 21,000 lbs. per sq. in., properly reduced by formula for compression.

196. The formula for the strength of long compression members is as follows, where

P = average permissible unit load on column, in lbs. per sq. inch;
l = length of column;
r = radius of gyration of its cross-section:

$$P = \frac{21{,}000}{1 + \frac{l^2}{9000\, r^2}}.$$

197. The formula for extreme fiber stress due to combined compression and bending, is

$$S = \frac{P}{A} + \frac{M_b T}{I - \frac{P l^2}{E c}}.$$

Where P = longitudinal compressive force;
A = area of section;
M_b = bending moment due to transverse load;
T = distance from neutral axis to extreme fibers;
I = moment of inertia;
l = length of beam;
E = modulus of elasticity;
c = coefficient. See Transverse Strength, ¶ 103.

Joints.

198. Pin Plates. Where a pin passes through one or more shapes of some member, it often happens that the combined surfaces of the truss members alone, in contact with the pin, are insufficient to transmit, by bearing, all of the stresses to be delivered to the member. There is then danger of crushing the material which presses against the pin. To obviate this, other shapes, usually flats and called pin plates or reinforcing plates, are riveted to the member; giving, in all, sufficient bearing surface for the pin. See Fig. 55; where the letters denote:

AA, angles,
C, cover,
B, bar,
W, web,
P, pin,
J, jaw,
F, filler,
O, outside pin plate,
T, batten plates.

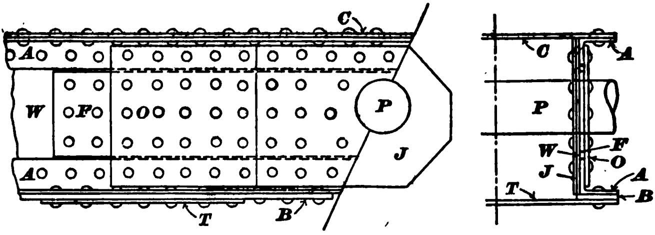


Fig. 55.

199. In Fig. 56, the two channels form the *whole* member (except the latticing, which cannot be included to resist compression) and the pin passes through both channels. In the case of a **built-up chord section,** or of an **end post,** Fig. 53, however, the webs form only a part of the section;

while the cover, the angles and the flats can receive no stress directly from the pin, but must receive it indirectly from the web and from the pin plates attached to it.

200. Where a pin plate is placed on each side of the web, the outside one must, according to most specifications, cover the angles; and there must, in addition, be a "filler" between it and the web.

201. Engineers differ as to the manner in which the stresses are actually transferred through the several parts of a pin connection. We may assume that the stresses in the pin plates are delivered almost directly to the shapes of the member. Thus, the outer reinforcing plate probably delivers most or all of its stress to the angles, and little or none to the web.

202. In each angle, those rivets which pass through the inner pin plate must transmit, by means of their bearing against the angle, the sum of the stresses which they take by shear from inside and from outside. In other words, these rivets are in double shear.

Pins.

203. The pin must be designed to resist bending stresses from the members through which it passes. It is also subject to shear, but this is seldom a critical point.

204. The pin requiring the greatest cross-section is usually either the one at the middle of the span and in the lower chord, where the chord stresses are greatest, or the one at the joint between the end post and the top chord; but, as the pins are relatively small members, all the other pins are, for the sake of uniformity, usually made of the same size with it.

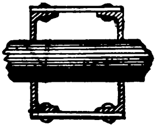


Fig. 56.

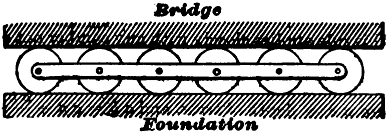


Fig. 57.

Expansion Bearings.

205. Expansion bearings usually consist of a nest of carefully turned rollers placed between two planed surfaces, shown in principle in Fig. 57.

206. The rollers are steel cylinders, from 3 to 6 ins. diam.; and 1 to 4 ft. long; planed smooth. From 4 to 8 or more of these are connected together by a frame, and one such frame is placed under at least *one end* of the truss. The rollers rest upon a strong planed cast bed-plate; bolted to the masonry below. Under the end of the truss is a similar plate by which it rests on the rollers. Since a truss of even 200 ft. span will scarcely change its length as much as 3 ins. by extremes of temperature, the play of the rollers is but small. They are kept in line by flanges cast along the side of the bed-plate. Flanges should also project downward from the upper bed-plate, so as completely to protect the rollers from dust, rain, etc.

207. The total displacement, allowed for the free end of the truss, is usually specified (see Digests); otherwise it may be taken as

$$D = \frac{(T - t) \text{ span}}{145{,}000},$$

where T and t = the max. and min. temps. respectively, in degrees F. The min. temp. to be expected may be obtained from Weather Bureau records of temps. in the shade, but the max. should be taken 20° or 30° higher than that of the Weather Bureau; because, in bright sunshine, the bridge will become much hotter than the air.

208. Rockers. In order to restrict the length of the bearing, where the displacement is moderate, rockers, Fig. 62, are often used instead of rollers.

209. For other regulations and suggestions regarding design of roller bearings, see Digests of Specifications, and Figs. 60 and 61.

Loads, Etc.

210. Loads, Clearance, etc., for Highway Bridges. See also Digests of Specifications for Bridges.

Weights of crowds. At the Chelsea bridge, London, picked men, packed upon the platform of a weigh-bridge, gave a load of only 84 lbs. per sq. ft. At Buckingham Palace, men, wedged as closely as possible upon a space 20 ft. in diameter, the last man lowered from above, among the others, gave 120 lbs. per sq. ft. But modern experimenters have easily obtained loads of from 140 to 150 lbs. per sq. ft. With picked men, averaging 163.2 lbs. each, all facing one way, carefully packed, and confined within an enclosure 6 ft. square (0.9 sq. ft. per man), Prof. L. J. Johnson, at Harvard University, obtained a maximum of 181.3 lbs. per sq. ft.* See also pages 755, etc.

Where the enclosure of the space is such that portions of the persons, standing against the enclosure, may project beyond it, the load, per unit of space, is of course increased; and, with small areas, this increase may be relatively important.

Camber.

211. Amount of Camber. If we divide the span in feet, by 50, the quot. will ordinarily be a sufficient camber, in inches. This amounts to 1 in 600. The camber to be used is, however, usually stipulated in the specifications. See Digests. A well-built bridge, of good design, should not, under its greatest load, deflect more than about one inch for each 100 feet of its span, or 1 in 1200. Indeed, the deflection is frequently much less than this.

212. The excess of length of the upper chord over that of the lower one, given the span, the depth of truss and the camber, will be =

$$\frac{8 \times \text{depth} \times \text{camber}}{\text{span}}.$$

This rule applies closely with any camber not exceeding 0.02 of the span.

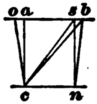


Fig. 58.

213. Length of diagonal $c\,b$, Fig. 58, $c\,b = \sqrt{a\,c^2 + a\,b^2}$; where $a\,c$ = depth of truss, and $a\,b = c\,n + \frac{o\,b - c\,n}{2}$.

214. Sometimes the elongation or shortening, produced by the loading, is computed for each member, and the length of each member affected is correspondingly changed. See Deflections, ¶¶ 162, etc.

Examples.

215. Figs. 59 (*a*) to (*u*), to a uniform scale of 1 inch = 60 feet, serve to indicate current practice respecting the types selected for different spans, the relation between span, panel length and depth, the spacing of stiffeners in plate girders, the arrangement of chord and web members, the use of rigid and flexible members, counters and turnbuckles, in trusses, and, approximately, the dimensions of rigid members and of gusset plates, as seen in elevation.

216. In each case the left half of the span is shown, and the center line of the span is indicated by a dot-and-dash line. Through spans and deck spans are distinguished by the elevation of the roadway, as approximately indicated at the left support.

217. In Figs. *h* to *q*, representing trusses, rigid members are indicated by double lines, flexible members in verticals and main diagonals by single lines, and counters by dotted lines. In pin spans, to avoid confusion, the rigid members are shown cut off near the pins.

218. Figs. (*a*) to (*o*) represent standard designs, from 25 to 200 ft. span, by Mr. Ralph Modjeski, C.E., for the Northern Pacific Railway Co.; Fig. (*p*) a 250 ft. railroad span designed by the Pencoyd Works of the American Bridge Co.; Fig. (*q*) a 368 ft. railroad span by the Phœnix Bridge Co.; Figs. (*r*) to (*t*) designs for riveted trusses by the Elmira Works of the American Bridge Co.; and Fig. (*u*) a riveted railroad bridge, of 102 ft. span, designed by the Pencoyd Works.

---

* Journal, Ass'n Engng Socs, Jan., 1905.

219. Fig. (*a*) represents a beam girder; Figs. (*b*) to (*g*) plate girders; and Figs. (*h*) to (*q*) riveted and pin trusses; Figs. (*h*) and (*i*) being riveted, and Figs. (*j*) to (*q*) pin.

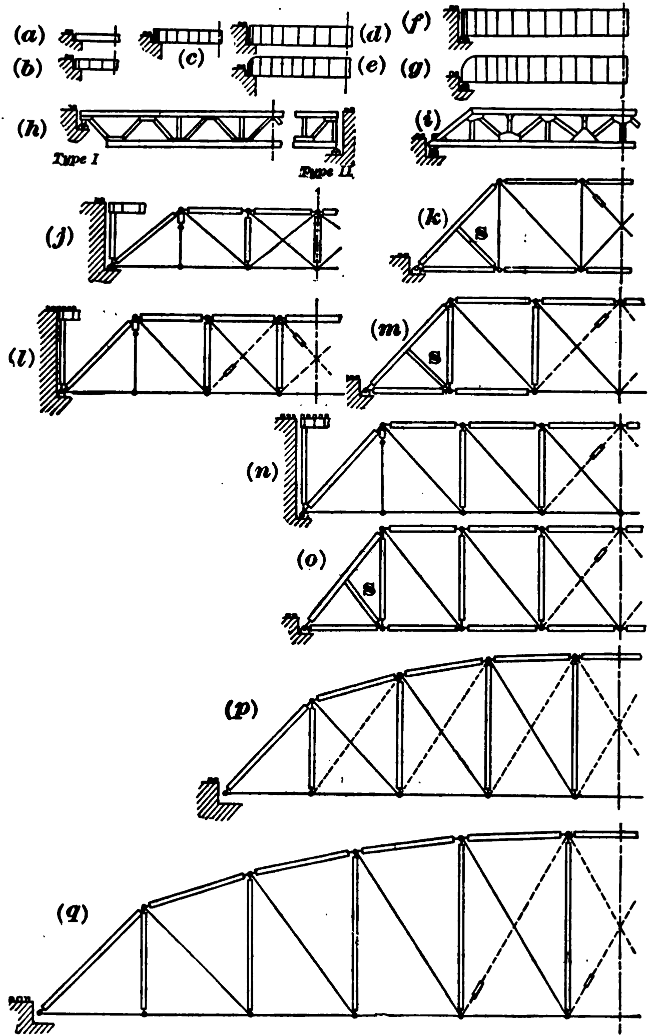


Fig. 59 (*a* to *q*).

220. Fig. (*r*) represents a 128 ft. span for the New York Central and Hudson River R. R., and Figs. (*s*) and (*t*) spans of 143 ft. and 160 ft. respectively for the Delaware, Lackawanna and Western R. R. Figs. (*r*) and (*s*) are modifications of the Baltimore truss, Fig. 15 (*b*), and Fig. (*t*) is a quadruplex Warren truss. Fig. (*s*) is a skew pan. Fig. (*u*) is a "pony" span.

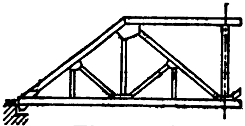


Fig. 59 (*r*).

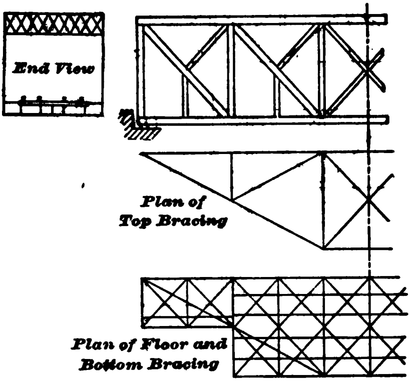


Fig. 59 (*s*).

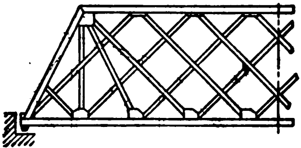


Fig. 59 (*t*).

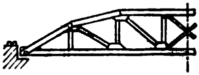


Fig. 59 (*u*).

221. Details. Figs. 60 to 65 show a few details of trusses and of plate girders.

222. Figs. 60 and 61 show left end connections of two through truss bridges (with roller bearings) designed by the Pencoyd Works; Fig. 61 representing the 250 ft. through pin span shown in Fig. 59 (*p*), and Fig. 60 a 124 riveted through span, showing the portal bracing.

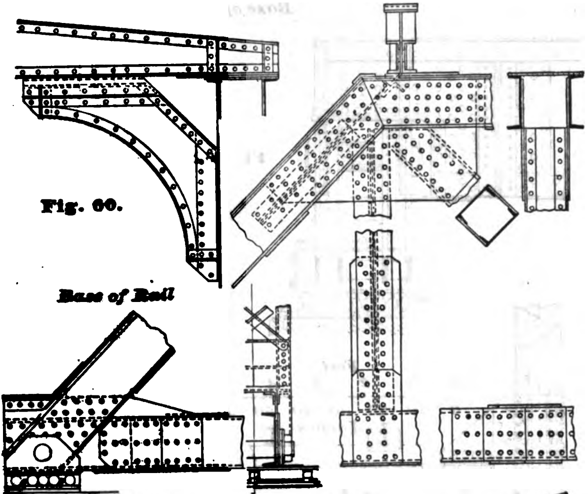


Fig. 60.

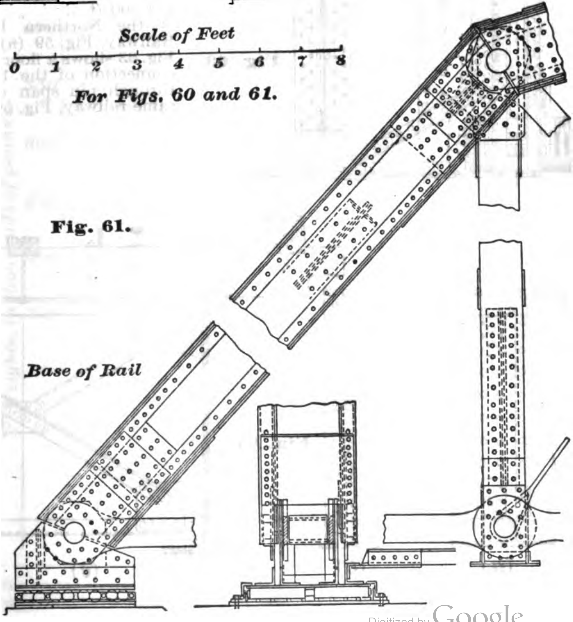


Fig. 61.

For Figs. 60 and 61.

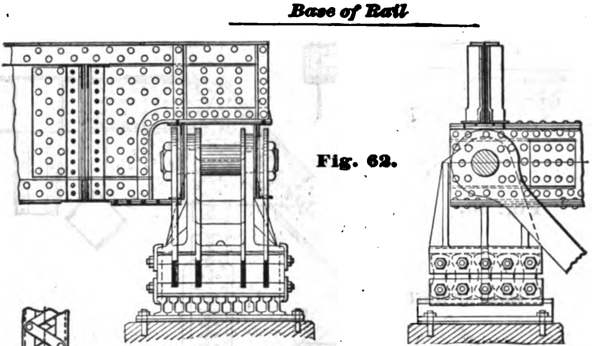


Fig. 62.

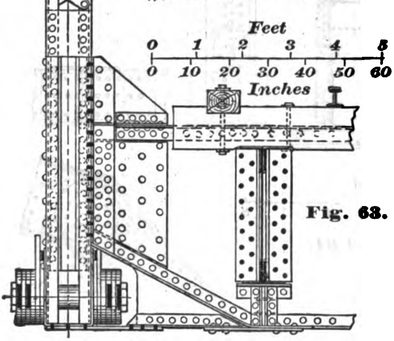


Fig. 63.

Scale for Figs. 62, 63, 64.

223. Fig. 62 shows the left end bearing (with rockers) and a floor beam connection for the standard 200 ft. deck pin span of the Northern Pacific Railway, Fig. 59 (*n*); and Fig. 63 shows a floor beam connection of the 160 ft. through pin span of the same railway, Fig. 59 (*m*).

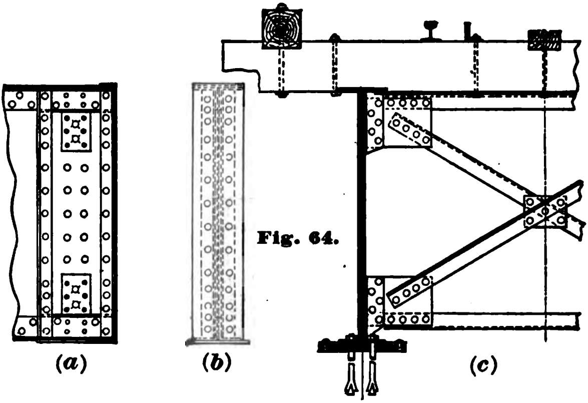


Fig. 64.

224. Figs. 64 and 65 represent respectively a 50 ft. deck plate girder of the N. Pac. Ry. and an 85 ft. through plate girder by the Pencoyd Works.

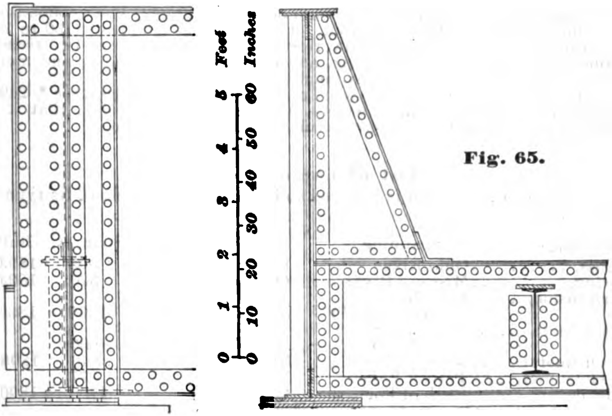


Fig. 65.

225. Weights of Steel Railroad Bridges. Fig. 66, based upon the practice of the Northern Pacific Railway, 1902,* shows the approximate

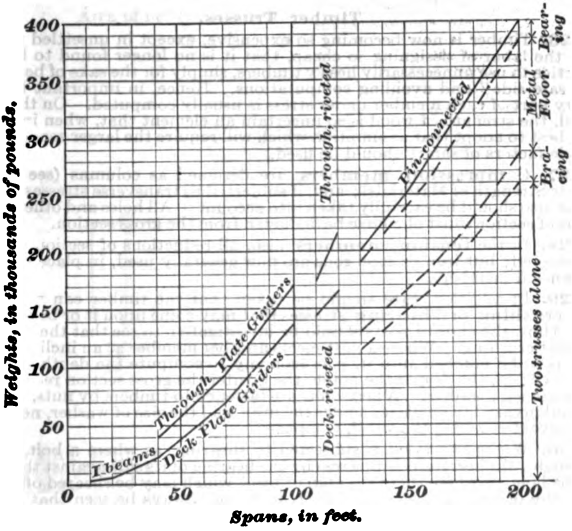


Fig. 66.

---

* See paper by Ralph Modjeski, C.E., in "Journal of the Western Society of Engineers," Chicago, Feb., 1901, vol. VI, No. 1.

weights of steel railroad bridges designed for two locomotives, of 146 tons each, and a uniform train load of 4000 lbs. per foot of track. The weights include the two beams, girders or trusses of one single-track span, with their bracing, metal floor system and end bearings. For wooden floor system, add 400 lbs. per lineal foot. For pin-connected spans (130 to 200 ft.), the three dash diagrams show, respectively, the weights of two trusses alone, of two trusses and bracing, and of two trusses, bracing and metal floor. The solid curve includes weights of end bearings.

For weights of combination (wood and iron) railroad bridges, see ¶ 249.

Highway bridges differ so widely, as to service and design, that it is scarcely practicable to give here useful data as to their weights.

List of Large Bridges.

Each bridge here given is believed to be (1902) the largest of its type in the world.

Type	Spanning	At	Span, ft.	Built
Truss,	Ohio River	Louisville	553	1893
Swing,	Missouri River	Omaha	520	1894
Suspension,	East River ("Brooklyn")	New York	1595	1883
Suspension,	East River ("New")	New York	1600	*
Arch (metal),	Niagara River	Niagara Falls	840	1898
Arch (stone),	Petruff Valley	Luxembourg	277	*
Cantilever,	Firth of Forth	Queensferry	1700	1890

The highest viaduct is the Gokteik Viaduct, in Burmah, with a maximum height of 320 ft., and a total length (composed of short spans) of 2260 ft., built in 1901.

Timber Trusses.

226. Timber is now becoming so expensive, except in unsettled regions, and the labor of designing so cheap, that it is no longer found to be good practice to use unnecessarily heavy timbers, simply for the sake of being "on the safe side" and avoiding computations. Hence, in important bridges, every part of each member under stress is usually computed. On the other hand, the strength of wood is so uncertain an element that, when in doubt, it is best to adopt that assumption which will require the larger section; and ample factors of safety should be used.

227. Compression members are designed as columns (see **Pillars**, under Strength of Materials); and, if subjected to transverse stresses as well, these also should be carefully taken into account. All holes and other reductions of section must of course be deducted from the gross section.

228. In the **tension members** also, all reductions of section must be considered; but iron or steel rods are now generally used, in place of wood, in tension members.

229. In addition, care should be taken that the timber can withstand any **crushing** or **shearing** stresses that may come upon it or be set up in it. Thus, the ends of posts should be investigated, to see that they are safe against crushing. Where a post meets another member at an inclined angle and is to be notched into it, it is economy to compute the depth of notch required; as, the deeper the notch, the greater the gross section required for the notched member. Where bolts are fastened to timbers by nuts, washers should invariably be placed under the nuts, and the size of washer, necessary to prevent crushing the wood, computed.

230. Where the wood is subjected to shearing, as where a bolt, passed through a timber, transmits stress by the bearing of its side against the inside of the hole, or where there is a step or table which may be sheared off by the pressure of another piece against it, it should always be seen that there is sufficient surface along the grain of the wood to take the shear, and some allowance should be made for the possibility that the grain may run out to the surface or to some hole before all the stress can be transferred.

---

* Under construction, 1902.

231. Cross-section of Upper Chord. Since it would be inconvenient, in practical construction, to change the section of a timber upper chord at different points, it is designed throughout to withstand the *maximum* stress occurring between any two panel points. Assume width of chord member. Find r^2 = (least radius of gyration)$^2 = \frac{(\text{width})^2}{12}$. Find allowable unit stress according to column formula given in specifications or adopted, using the given maximum stress. Find area required for this unit stress. Find the resulting depth, which for a horizontal or inclined member is preferably somewhat greater than the width, to allow for bending moment due to its own weight. If this does not give a good commercial size, it may be well to revise, in order to obtain a better section.

232. Struts are preferably made as wide as the upper chord. Each strut must be designed separately. Obtain r^2, allowable unit stress, etc., as for the upper chord. For economy, the struts should average nearly square, even though it should be necessary to alter the section of the upper chord in order to prevent wide deviation from a square section.

233. The **vertical ties** (of iron) may now be designed. Area of cross-section $= \frac{\text{maximum stress}}{\text{allowable unit stress}}$. But see Minimum Section, ¶ 187. The size of a nut is usually fixed by the diameter of the rod, but the washers should be so designed as not to crush the wood.

234. The **bearings** or **indentations**, required in the upper and lower chords to hold the inclined members in place, may now be computed. The component (in the strut) perpendicular to the face or faces against which it presses, is computed, and the necessary depth obtained, assuming the width of the lower chord the same as that of the upper chord and the struts.

235. The section of the **lower chord** may now be decided upon, since the reduction of section, due to indentations, is known.

$$\text{Area of net cross-section} = \frac{\text{maximum stress}}{\text{allowable unit stress}}.$$

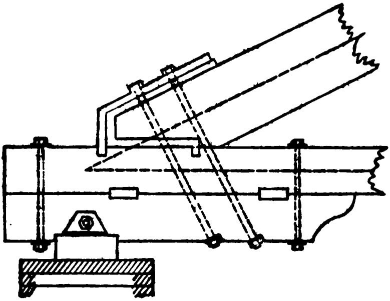


Fig. 67.

236. End Joint. Fig. 67. Many different designs for end joints have been made, proposed and discussed. The ends of the straps should enter notches in the lower chord, to such a depth that the total stress, taken by the end fibers of the sides of the notches, is equal to the stress that the ends of the straps can resist by bending. This depth can be found by successive trials, or by means of two algebraic equations, in which the maximum allowable pressure and the depth of notch are the two unknown quantities.

Determine the shearing surface required to transmit this stress to the body of the lower chord. This will also determine the space between the notches and the end of the lower chord. Compute the stress (if any) that remains to be transferred, and design the long inclined bolts and their washers accordingly. Compute also the compressive area and depth of vertical face of lower end of upper chord, required to transmit the horizontal component of its thrust. See also that the lower bearing presents sufficient surface to resist the vertical component. The keys, between the bolster and the lower chord, must be designed to carry the horizontal component of the wind. For safety, **friction** between the two parts should be neglected.

237. Figs. 68 show **joints** adapted to most of the cases that occur in practice with wooden beams, etc. They need but little explanation. Fig. (*a*) is a good mode of splicing a post; in doing which the line *o o* should never be inclined or sloped, but be made parallel to the axis of the post; otherwise, in case of shrinkage, or of great pressure, the parts on each side of it tend to slide along each other, and thus bring a great strain upon the bolts. When greater strength is required, iron hoops may be used, as at *b*, *h* and *j*, instead of bolts. Fig. (*b*) shows a post spliced by 4 fishing pieces; which may be fastened either by bolts, as in the upper part; or by hoops, as in the lower. The hoops may be tightened by flanges and screws as at *s*; or thin iron wedges may be driven between them and the timbers, if necessary. Fig. C shows a good, strong arrangement for uniting a straining-beam *k*, or rafter *l*, and a queen-post *u;* by letting *k* and *l* abut against each other, and confining them between a double queen-post *t t; n n* are two blocks through which the bolts pass. A similar arrangement is equally good for uniting the tie-beam *w*, with the foot *v*, of the queens; with the addition of a strap, as in the figure. Fig. (*e*) is a method of framing one beam into another, at right angles to it. **An iron stirrup,** as at *f*, may be used for the same purpose; and is stronger. Figs. *g h*, *i j* are **built beams.** When a beam or girder of great depth is required, if we obtain it by merely laying one beam flat upon another, we secure only as much strength as the two beams would have if separate. But if we prevent them from *sliding* on one another, by inserting transverse blocks or keys, as at *g;* or by indenting them into one another, as at *i j;* and then bolt or strap them firmly together to create friction; we obtain nearly the strength of a solid beam of the total depth; which strength is as the *square* of the depth. See Horizontal Shear, ¶ 51, under Transverse Strength.

238. The strength of a built beam is increased by increasing its depth at its center, where it is most strained; as in the upper chords of a bridge. This may be done by adding the triangular strip *y y* between the two beams.

239. A piece of plate-iron may be placed at the joints of timbers when there is a great pressure; which is thus more equally distributed over the entire area of the joint; or cast iron may be used.

240. Frequently a simple strap will not suffice, when it is necessary to draw the two timbers very tightly together. In such cases, one end of each strap may, as at *x*, terminate as a screw; and after passing through a cross-bar Z, all may be tightened up by a nut at *x*. Or the principle of the *double key*, shown at K, may be applied. Sometimes, as at A, the hole for the bolt is first bored; then a hole is cut in one side of the timber, and reaching to the bolt-hole, large enough to allow the screw nut to be inserted. This being done, the hole is refilled by a wooden plug, which holds the nut in place Then the screw-bolt is inserted, passing through the nut. By turning the screw the timbers may then be tightened together.

241. When the ends of beams, joists, etc., are inserted into walls in the usual square manner, there is danger that, in case of being burnt in two, they may, in falling, overturn the wall. This may be avoided by cutting the ends into the shape shown at *m*.

242. When a strap *o*, Fig. R, has to bear a strain so great as to endanger its crushing the timber *p*, on which it rests, a casting like *v* may be used under it. The strap will pass around the back *r* of the casting. The small projections in the bottom, being notched into the timber, will prevent the casting from sliding under the oblique strain of the strap. The same may be used for oblique bolts, and below a timber as well as above it. When below, it may become necessary to bolt or spike the casting to the under side of the timber. When the pull on a strap is at right angles to the timber, if there is much strain, a piece of plate-iron, instead of a casting, may be inserted between the strap and the timber, to prevent the latter from being crushed or crippled; see I and I.

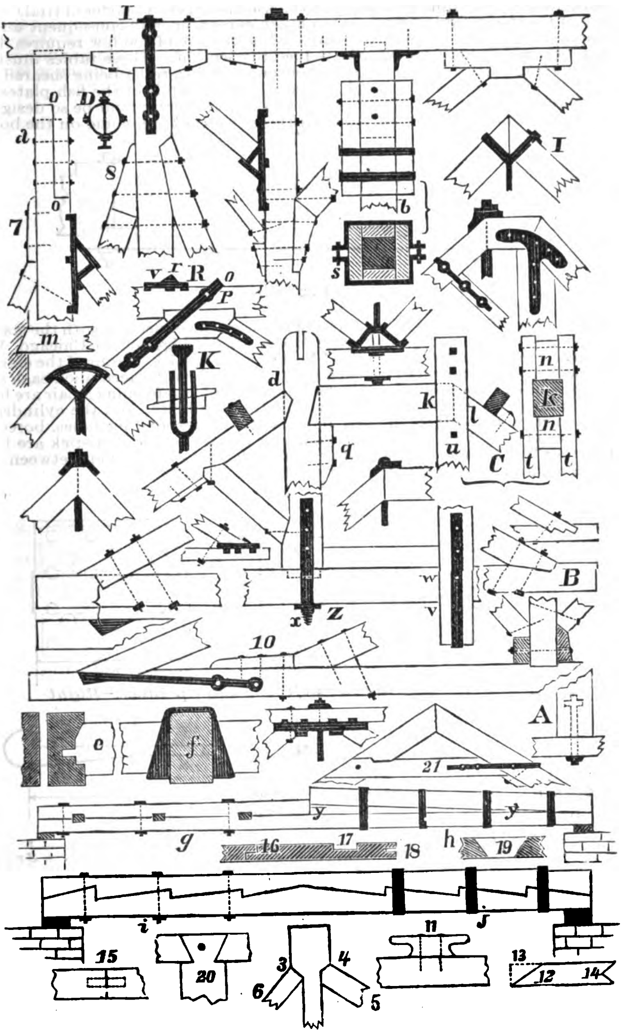

I
o
D
d
8
7
o
b
I
s
v r R
o
P
m
K
d
k
l
q
u
C
n
k
n
t
t
w
v
B
x
z
10
A
c
f
21
y
y'
g
16
17
18
h
19
i
j
11
15
20
3
4
6
5
13
12
14

Fig. 68.

243. Lower Chord Splice. Owing to the length of the lower chord, it may be necessary to splice it, as in Fig. 69, where the splice is a tabled fish-plate joint. The number of tables to be used is largely a matter of trial. The use of too many tables involves too much carpentry, and consequent uncertainty as to distribution of pressures, while the use of too few requires deep notches, which may too greatly reduce the section. These tables must be designed to resist bearing against their ends, and to resist being sheared off. Bolts should be passed through, especially at the ends of the fish-plates, to prevent them from bending outward; and the washers should be so designed as to transmit safely to the wood all the stress that can come on the bolt.

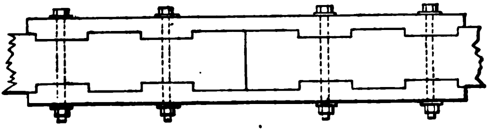


Fig. 69.

244. Fig. 70 shows a lower chord splice used in connection with the standard combination (timber and iron) Howe truss bridges of the Chicago, Milwaukee & St. Paul Railway See ¶¶ 249–251, Figs. 73. Four of the clamp-blocks shown are required for each joint, a block being placed upon each side of each stick to be spliced. The two opposite blocks forming a pair are held together, and against the stick, by four through bolts; and the cylindrical lugs, cast on the surface of each block, enter corresponding holes, bored in the face of the stick. The two blocks on the same side of a stick are held together by the hooked clamp-bar. The clamp-key is driven between the left hook and the beveled key-seat on the left block.

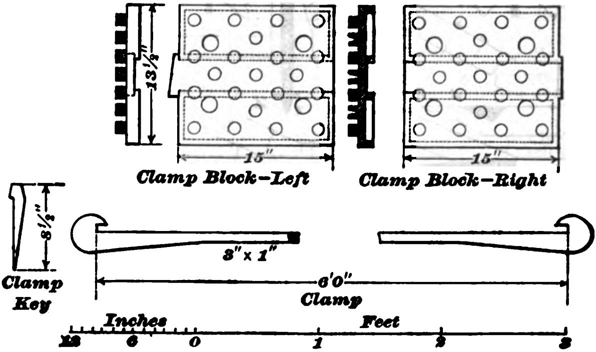


Fig. 70.

245. Figs. 71 illustrate a small wooden **Howe truss bridge.** The top and bottom chords are each made up of three or more parallel timbers *c c c*, placed a small distance apart, to let the vertical tie-rods *r r* pass between them. The main braces, *o o*, are in pairs or in threes. The pieces composing them abut, at top and at bottom, against triangular **angle blocks,** *s*; which, if of hard wood, are solid; and, if of cast iron, hollow, Fig. (*d*), and strengthened by inner ribs. These blocks extend across the three or more chord pieces. Against their centers abut also the counterbraces *e*. These are single pieces in small bridges; or in pairs, in large ones; and pass between the pieces which compose a main brace. Where the wooden braces and counters cross, they are bolted together. In Fig. (*d*), the dotted lines show the strengthening ribs; and the lug, *x*, keeps the block in place. The vertical tie-rods *r r*, of iron, are in pairs, threes, or fours, etc., according to size of bridge; with a screw and nut at each end. The heads and feet of the braces and counters abut square against the angle blocks; and are often kept in place only by tightening the screws of the vertical ties. The end posts, *p* and *d*, the end ties, *i c* and *b y*, and the horizontal extensions, *g i* and *w b*, of the upper chord, form no part of the truss proper.

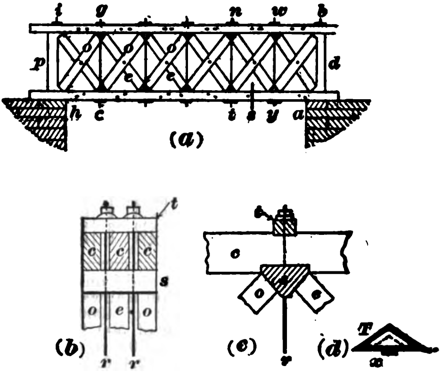


Fig. 71.

246. In **large spans**, to prevent the pressure at the ends of the diagonals from crushing the chords, the angle blocks are sometimes cast with deep projecting flanges under their bases. These flanges, passing between the pieces which compose a chord, extend to the opposite face of the chord. There the flanges bear upon broad washers at the ends of the vertical rods.

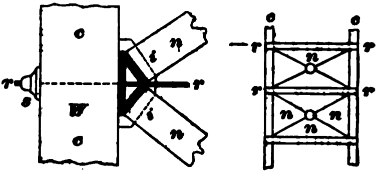


Fig. 72.

247. A common form of **lateral bracing**, Fig. 72, resembles a Howe truss laid flat on its side. In it the diagonals of the cross are struts of timber; and the pieces *r r* are round rods. One of the struts is whole, with the exception of a slight mortise on each vertical side, at its center, for receiving tenons cut on the inner ends of the two pieces which compose the other diagonal. At the sides of the chords, the ends of the diagonals rest upon a ledge (shown by the dotted line *i i*), about 1½ inches wide, cast at the bottom of the cast-iron angle block. The tie-rod *r r*, passing through the chords of both trusses, being tightened by means of the nut *s*, holds the diagonals firmly in place; and, in case of their shrinking a little in time, they can be again tightened up by the same means.

The cast angle block is as deep as a brace; its thickness need not exceed half an inch, in a large bridge. It has holes for the passage of the rod *r r*.

248. Dimensions for each of two trusses of a single track Howe bridge for highway or suburban electric railway, with cars weighing 20 to 25 tons each. Timber not to be strained more than 800 lbs. per sq. inch; nor iron more than 5 tons per sq. inch. Iron supposed to be of rather superior quality, requiring from 25 to 27 tons (60,480 lbs.) per sq. inch to break it. The rods to be *upset* at their screw-ends. To each of the two sides of each lower chord is supposed to be added, and firmly connected, a piece at least half as thick as one of the chord pieces, and as long as three panels, at the center of the span.

Clear Span. Feet.	Rise. Feet.	No. of Panels.	An upper Chord.		A lower Chord.		An End Brace.		A Center Brace.		A Counter.		End Rod.		Center Rod.	
			No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Rods.	Diam.	No. of Rods.	Diam.
				Ins.		Ins.		Ins.		Ins.		Ins.		Ins.		
25	6	8	3	5×6	3	5×12	2	5×8	2	5×6	1	5×6	2	1½	2	1
50	9	9	3	6×9	3	6×14	2	6×9	2	5×8	1	5×8	2	1⅞	2	1⅛
75	12	10	3	6×12	3	6×14	2	6×11	2	6×8	1	6×8	2	2¼	2	1⅜
100	15	11	3	6×14	3	6×16	2	8×12	2	6×10	1	6×10	2	2½	2	1½
125	18	12	4	6×14	4	6×16	2	9×14	2	6×12	1	6×12	2	3	2	1⅜
150	21	13	4	8×14	4	8×18	3	8×14	3	6×10	2	6×10	3	2¾	3	1⅜
175	24	14	4	10×16	4	10×20	3	8×15	3	8×10	2	8×10	3	3	3	1½
200	27	15	4	12×16	4	12×20	3	9×16	3	8×14	2	8×14	3	3½	3	1⅜

249. Standard "combination" (wood and iron) Howe truss railroad bridges, Chicago, Milwaukee & St. Paul Railway, 1891–1902.

The bridges are designed for a train load of 4000 lbs. per lineal foot. Wind stresses are computed for a pressure of 500 lbs. per lineal foot of train. Compressive stresses in braces, max. 505, min. 92 lbs. per sq. inch. Tensile stresses; in main truss rods, max. 12,450, min. 8500 lbs. per sq. inch of net area at root of thread. Lateral rods, max. 15,000 lbs. per sq. inch of net section.

Timber. Cross-ties and guard rails, white oak; top chord packing blocks, white pine; all other timber, white or Norway pine or Douglas fir.

Combination Bridges, Chicago, Milwaukee & St. Paul Railway:

Total length, feet,*	82	93	103	114	125	136	147
Span, feet,*	77	88	99	110	121	132	143
Panels,	7	8	9	10	11	12	13
Upper chord,	4, 7x10	4, 7x10	4, 8x10	4, 8x10	4, 8x10	4, 8x10	4, 8x12
Lower chord,†	4, 7x14	4, 7x14	4, 8x14	4, 8x14	4, 8x15	4, 8x15	4, 8x15
Main braces, M:							
At center,‡	2, 10x10	2, 8x10	2, 10x10	2, 10x10	2, 10x10	2, 10x10	2, 10x10
At ends,	2, 10x12	2, 12x12	2, 12x12	2, 12x14	2, 12x14	2, 14x14	2, 14x14
Counters, C:							
At center,‡	2, 8x10	1, 10x12	1, 8x10	1, 8x10	1, 8x10	1, 10x12	1, 8x10
At ends,	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8	1, 8x8
Vertical rods:							
At center,‡	3, 1½	3, 1½	3, 1⅜	3, 1½	3, 1⅝	3, 1½	3, 1⅝
At ends,	3, 2⅛	3, 2⅜	3, 2½	{3, 2⅛ 2, 2	3, 2¼ 2, 2	3, 2⅜ 2, 2	3, 2½ 2, 2
Weight, lbs.,§	130,300	155,500	182,000	209,400	233,100	259,900	286,500

---

* Lengths and spans are given to the nearest foot. The panel length is 10 ft. 11¼ ins. Each span is longer, by one panel, than the next preceding. Depth, in all cases, 25 feet in clear of chords. Width, 14 ft. 6 ins. in clear between chords.

† For lower chord splice, see Fig. 70, ¶ 244.

‡ Owing to the fact that the spans have alternately odd and even numbers of panels, there is some ambiguity as to the dimensions of web members at center of span.

§ Weight includes the two trusses, bracing and floor system for one single-

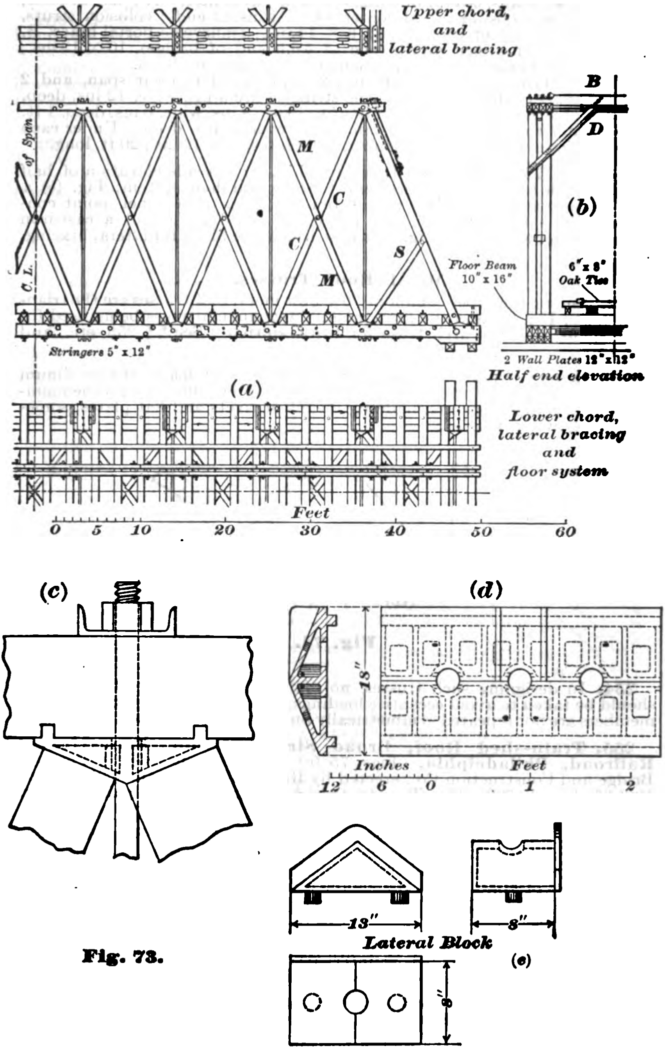


Fig. 73.

---

track span. Of the total weight of the bridge, the trusses have about 53 per cent., lateral systems 26, floor system 18, and wall plates 3 per cent.

250. In all spans, lateral bracing, 6 × 6 ins. at center, 8 × 8 ins. at ends of span; lateral rods, 1⅛ ins. at center, 1½ to 1⅝ ins. at ends; collision struts, S (one at each end of each truss), 6 × 14 ins.; transverse portal brace, B, between ends of upper chords (one at each end of bridge), 10 × 12 ins.; diagonal portal braces, D (two at each end of bridge), 6 × 8 ins.

The floor beams are 10 × 16 ins., 20 ft. long, 14 ft. clear span, and 2 ft. 6 ins. apart center to center. The stringers are 5 ins. wide, 12 ins. deep, placed as shown in the end view, Fig. (*b*). Ties, 8 ins. wide, 6 ins. deep, 1 ft. apart, center to center. Guard rail, 8 ins. wide, 5 ins. deep. Under each end of the lower chord are two timber wall plates, 12 × 12 ins., 20 ft. long.

251. Figs. 73 show the 99 ft. span. Fig. (*a*) is a side elevation of half the span with top and bottom lateral bracing and floor system; Fig. (*b*) a half end elevation, showing portal bracing; Fig. (*c*) a panel point connection (the same for upper and for lower chord); Fig. (*d*) a cast-iron angle block for same; and Fig. (*e*) a cast-iron angle block for lateral bracing.

Metal Roof Trusses.

252. Among the types commonly used for metal roof trusses are the triangular truss, Fig. 26, and the arch truss; the triangular truss being used for short spans, and the arch truss for long spans. See ¶¶ 255, etc., and Fig. 75.

253. In roof trusses of short span, carrying light loads, the minimum sections prescribed in bridge specifications will often suffice for all the members. The connections are made by means of rivets and flat plates, somewhat as shown in Fig. 74.

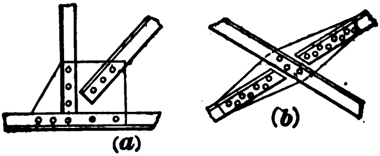


Fig. 74.

254. In designing such trusses, no matter how light the stresses, care should be taken to avoid eccentric loadings, which are apt to occur where the members are not repeated symmetrically on both sides of the flat plates.

255. Train-shed Roof, Broad Street Station, Pennsylvania Railroad, Philadelphia. Figs. 75 (*a*) to (*f*). Built 1893 by Pencoyd Bridge and Construction Co., erected by Railroad Co. Span, 300′ 8″; rise, 108′ 6″; length, 589′ 2½″. Twenty trusses, arranged in 10 pairs, 3 pairs shown in Fig. (*c*).

256. Each truss consists of two arched **rafters**, A B and B C, and a horizontal chord, A C, with three pin joints, A, B and C, Fig. (*a*).

Each rafter is composed of two chords, 14 radial braces and 26 diagonals. For the sake of appearance, the chords are extended across the top panels, occupied by the two triangular apex members, and are there connected by a sliding joint.

257. The horizontal chord AC, Fig. (*a*), lies below the floor of the train-shed, and is suspended, at intervals, from girders which support that floor and which are carried by iron columns in the lower story.

258. End bearings. One foot of each truss rests upon a fixed shoe, bolted down to the pier; the other on a nest of 11 steel rollers.

259. At each end of the roof, a horizontal wind truss, WW, Fig. (*a*), is suspended from the rafters, its ends resting upon brackets riveted to their lower chords.

260. Between these horizontal wind trusses and the rafters, and just behind the glass curtain closing each end of the roof, are placed vertical wind trusses (not shown), with horizontal and diagonal bracing, in planes normal to the main trusses. These vertical trusses are hung from the lower panel points of the rafters, Fig. (*a*). They resist the wind pressure on the curtain, and transmit it, through the horizontal wind trusses, to the lower chords of the rafters.

261. Two rafters, AB and BC, Fig. (*a*), of one truss, together weigh 138,000 lbs.; chord (two lines of eye-bars) 16,000, total 154,000 lbs. One pair of trusses, 308,000. Framework of entire train-shed, including trusses, purlins, rafters and wind bracing, about 7,000,000 lbs.

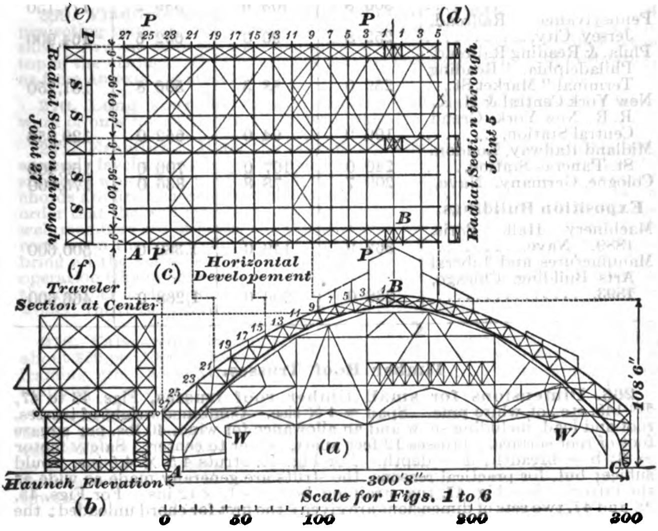


Fig. 75.

262. Assumed extraneous loads, in lbs. per sq. ft. of ground plan: rafters 9.5; laterals and purlins, 7.5; ventilator and skylight framing, 9.5; covering and skylights, 15; snow, 17; wind, 35.

263. The traveler, Figs. (*a*) and (*f*), was of timber, and was so designed as to permit the use of the old train-sheds while the new roof was being erected.

264. The top section, from apex to joint 9, Figs. (*a*) and (*c*), was riveted up at mill and placed in position as a whole, as was also the foot section from joint 27 to the top of the triangular foot. The remainder was riveted on the traveler. To erect one pair of trusses, with the purlins, etc., connecting it with the pair last erected, required about 10 days. Shifting the traveler through a distance about equal to its own length, to receive the next pair of trusses, required about 3 minutes, and was done by means of the engines used for hoisting.

265. Dimensions of Arched Roofs of Large Span.

	Arches.		Roof.	
	Span.	Rise.	Length.	Area Covered.
Train-sheds.	ft. ins.	ft. ins.	ft. ins.	sq. ft.
Pennsylvania Railroad, Philadelphia. Broad Street Station,..........	300 8	108 6	589 2	177,150
Pennsylvania Railroad, Jersey City,............	252 8	90 0	652 6	164,900
Phila. & Reading Railroad, Philadelphia. "Reading Terminal," Market St....	259 0	88 3	506 8	131,250
New York Central & H. R. R. R., New York. Grand Central Station,.........	199 2	94 0	652 0	129,856
Midland Railway, London. St. Pancras Station,....	240 0	107 0	706 0	169,400
Cologne, Germany. Nave,.	209 7	78 8	836 0	175,200
Exposition Buildings.				
Machinery Hall, Paris, 1889. Nave,...........	362 9	149 0	1,380 0	500,600
Manufactures and Liberal Arts Building, Chicago, 1893, ...	368 0	206 0	1,268 0	466,600

Timber Roof Trusses.

266. Dimensions for small timber roof trusses, Figs. 43 to 47, ¶¶ 148, etc., of white pine. Span = 4 × rise. Combined weight of trusses, roof and load, including snow and an allowance for wind, 40 lbs. per square foot of roof surface. Trusses 12 feet apart, center to center. Safety factor = 3; b = breadth; d = depth. For Fig. 46, struts 4.5 × 4.5 ins. would suffice; but, for practical reasons, the struts are generally made as wide as the rafters. For Fig. 47, the straining beam is 12 × 12 ins. For Figs. 45, 46 and 47, two sets of dimensions are given; the first for chord unloaded; the second for chord loaded with 100 lbs. per square foot.

Fig.	Span Ft.	Rise. Ft.	Rafters. Ins.		Chord.			King or Queen Rod. Ins. Diam.	Chord.
					Timber. Ins.		Iron. Ins. Diam.		
			b.	d.	b.	d.			
43	30	7.5	5	10	5	10	1	..	unloaded
45	30	7.5	6.5	9	6.5	9	1	1	unloaded
			8.5	11	8.5	11	..	1¼	loaded
46	40	10	6	8	6	8	1.5	1	unloaded
			8	10	8	15	..	2	loaded
47	60	15	10	12	10	12	1.5	½	unloaded
			12	14	12	12	..	2	loaded

TRANSPORTATION AND ERECTION.

267. **Girders** should be loaded for **transportation** on flat cars, with web vertical, and with bearings at the points distant ⅕ span from the ends. Where too long for two cars, one or more idle spacing cars are used and the points of support are pivoted.

268. **Girders** may be **erected** by means of gin poles, derricks, gallows, etc., or they may be skidded from the cars and lowered to place by jacks and blocking. Gin poles should have at least four guys, with tackles, for easy adjustment. Hoisting may often be done by means of a locomotive running on the tracks of that part of the structure which is already completed. Ropes are used at about ⅕ their ultimate strength.

269. **Viaducts** are usually built from above, by means of an overhead projecting traveler. Sometimes by means of a cableway; but this method is slow. In some cases the traveling tower is on the ground, and reaches to the top of the viaduct. Or, the viaduct may be erected by means of false-work, or from an existing structure.

270. **Long span bridges** are usually built upon a platform of false-work or on a row of trestle bents, well braced.

271. **Erection.** Upon the false-works the lower chords are first laid, as nearly level as may be. The upper chords are then raised upon temporary supports which foot upon the one that carries the lower chord. The upper chords are first placed a few inches higher than their intended positions, in order that the web members may readily be slipped into place. When the web members are in place, the upper chords are gradually lowered until all rests upon the lower chords. The screws are then gradually tightened, to bring all the surfaces of the joints into their proper contact; and by this operation (the upper chord members having the necessary excess of length), the camber is formed, and the lower chords are lifted clear of the false-works; the truss now resting only upon its permanent supports.

272. **False-work** is ordinarily constructed of hemlock or pine, costing about $20 per 1000 ft. board measure. Allow about $15 per 1000 ft. B. M. for framing, etc. $5 to $15 per 1000 ft. B. M. may usually be obtained for old material ("salvage").

The main members are usually 12 × 12 ins., and the diagonals 3 × 12. Bolts, ⅞ inch. Owing to the temporary nature of false-work and to the salvage which may be obtained for it if it is not too badly cut up, it is advisable to use plenty of material of good standard sizes, especially in longitudinal bracing, which may be placed between alternate pairs of bents, forming towers.

273. **In soft bottoms,** the false-work may rest upon piles, to which the uprights of the false-work may be notched and bolted, or banded. Not less than 4 piles per bent should be used. As many as 24 have been used. Piles should be braced below water-mark. Bents should be built in stories of from 12 to 30 feet each. Connections should be made by means of side pieces or fish-plates.

274. **With rock bottom,** in a strong current, it may be expedient to sink cribs filled with stone, as a foundation for the false-work.

275. The erection of **cantilevers and suspension bridges** requires much time; but their use is often necessitated by the impossibility of erecting false-work.

276. **Renewal** of bridges may be accomplished by displacement; either by protrusion, where the new span is skidded longitudinally along the track; by transverse displacement, where both old and new spans are placed on tracks running normally to the bridge; by vertical displacement, either rising or descending; or by pivotal displacement, the old span swinging out about a pivot, and the new span swinging into place about another pivot.

277. **Cautions.** In erection and renewal, consider dead weight of bridge, effect of impact of current of stream, impact of boats, ice, drift, etc., especially when floods are to be apprehended, and strains of hoisting tackle.

For rigidity, a liberal safety factor must be used. Drift may pile up and form a dam. Trestle bents, in the water, increase the velocity and scour of the current, and may thus cause undermining. False-work may be protected against drift by fender piles. Provide against eccentricities of wind stress. Numerous accidents have shown the expediency of guarding the unfinished truss itself against high winds. All lateral and other wind bracing should be in place, and secured, before the false-works are removed and the trusses allowed to rest upon their final bearings.

Avoid dropping tools, etc. Even very small pieces, falling from a great height, are dangerous to life and even to the bridge. Hooks in tackles are liable to break or to pull out. Travelers should be well guyed and clamped, and carefully watched.

DIGESTS OF SPECIFICATIONS FOR BRIDGES AND BUILDINGS.

(1) Steel railroad, highway and electric railroad bridges.
(2) Combination (wood and steel) railroad bridges.
(3) Steel roof trusses, framework and buildings.

The following Digests of Specifications for Bridges and Buildings are intended primarily to give a general view of the essential features of current practice in such matters, and only secondarily to indicate the practice of any particular company.

(1) DIGEST OF SPECIFICATIONS FOR STEEL RAILROAD AND HIGHWAY BRIDGES.

List of Specifications Used.

A, American Bridge Company,
General Specifications for Steel Railroad Bridges, 1900.
Aa, American Bridge Company,
General Specifications for Steel Highway Bridges, 1901.
B, Baltimore & Ohio Railroad Company,
General Specifications for Railroad and Highway Bridges, Roofs, and Steel Buildings, 1901.
C, Cooper, Theodore —,
General Specifications for Steel Railroad Bridges and Viaducts, 1901.
Cc, Cooper, Theodore —,
General Specifications for Steel Highway and Electric Railway Bridges and Viaducts, 1901.
D, Delaware, Lackawanna & Western R. R. Company,
Specifications for Steel Railroad Bridges, October, 1899; revised to July, 1900.
E, Erie Railroad Company,
General Specifications for Bridges, 1900.
G, General practice.
Oo, Osborn Engineering Company,
General Specifications for Highway Bridge Superstructures, 1901.
P, Pennsylvania Railroad Company,
Standard Specifications for Steel Bridges, January 1, 1901.
R, Philadelphia & Reading Railway Company,
Specifications for Steel Bridges, 1898; revised February, 1901.
Y, New York Central & Hudson River R. R. Leased and Operated Lines,
General Specifications for Steel Bridges, 1900.

I. GENERAL DESIGN.

Limiting Spans for Different Types.

BEAMS AND GIRDERS.	A ft	Aa ft	B ft	C ft	Cc ft	D ft	Y ft
Rolled beams, solid floor, etc.,	up to 20	up to 40	up to 20	up to 20	up to 40	up to 20	up to 25
Plate girders,	20 to 100	25 to 80	20 to 100	20 to 120	20 to 80	20 to 100	25 to 100
TRUSSES.							
Riveted trusses,*	100 to 140	40 & over	100 to 120	75 to 150	40 & over	90 to 160	100 to 200
Pin trusses,	over 140	over 140	over 120	over 120	over 120	150 & over	200 & over

Riveted trusses,* under 100 ft; pin trusses, over 100 ft, **Oo.**
Depth of truss, min, = one-eighth of span, **Oo.**

---

* Unfortunately called "lattice girders," **A**; "lattice trusses" and "riveted lattice girders," **C**; "riveted lattice trusses," **D, Y.**

Classification of Highway and Electric Railroad Bridges.

Aa	Cc	
A	A1*	City bridges having buckle-plate floors, and paving on concrete base.
	A2*	City bridges having plank flooring.
B	B*	Suburban or interurban bridges for heavy electric cars.
C	C	Town or country bridges for light electric cars or heavy loads.
D	D	Country bridges for ordinary highway traffic.
E1	E1	Bridges for heavy electric or motor cars only.
E2	E2	" " light " " "

Camber.

Top chord panels longer than lower chord panels by one-eighth of an inch in 10 ft = 1 in 960, **A, B, C, E, R, Y.**

Highway bridges, three-sixteenths of an inch to every 10 ft, **Cc.**

About three-fourths of an inch in 100 ft = 1 in 1600, **D.**

Sufficient to bring joints of compression chord to a square bearing when truss is fully loaded. Each member built longer or shorter in proportion to the stress to which it is subject under a full dead and a full live load, so that under full loading it will have its normal length, **Oo.**

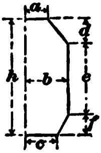
Cross Section of Bridge.

Gage, usually, 4 ft 8½ ins. **Distance, cen to cen of tracks,** 12 to 13 ft.

Width between trusses or girders in deck spans. Pin spans, min, 0.05 span. Riveted trusses (**D**), 10 ft. Plate girders (**G**), 5 to 7 ft. Spans not over 60 ft, 7 ft; 60 to 100 ft, 8 ft; over 100 ft, about one-twelfth of span, **D.** Plate girders (**Y**), over 60 ft, in proportion to height.

Clearance in through spans, on tangents, G.

a	3 to 3½ ft
b	7 ft
c	5 to 5½ ft
d	4 to 6 ft
e	10 to 14 ft
f	1 to 5 ft
h	20 to 22 ft



Minimum Clearance on Curves.

Same min clearance as on tangents, **A;** Ditto for car 74 ft long, 48 ft cen to cen of trucks, 10 ft wide, **B;** Ditto for car 75 ft long, 54 ft cen to cen of trucks. Additional clearance = 0.8 **d** ins on each side; = 1.6 **d** ins between tracks; where **d** = degree of curvature = central angle subtended by a chord of 100 ft, **C;** increase lateral clearance at top of car 2.5 ins for each inch of superelevation of outer rail, **C.** Cen line of bridge bisects middle ordinate and is parallel to chord, **Y.**

Highway Bridges. Headway, 14 ft, **Oo.** For classes A, B, C, and E, min = 15 ft, **Aa, Cc;** for a width of 6 ft over each track, **Aa.** For Class D, 12.5 ft, **Aa, Cc. Horizontal clearance.** Min 14 ins greater than width of roadway between wheel guards, **Aa.** For electric cars, 6.5 ft from cen of track, **Aa;** 7 ft, **Cc.** On curves, provide for a car of 45 ft extreme length, 8 ft wide, 20 ft between truck centers, **Aa.** Width between centers of trusses, min = 0.05 span, **G.**

Tension Members.

In general, hip verticals and one or two panels of lower chord at each end of span are required to be of rigid section, so as to resist both tension and compression.

Angles, used as tension members, must be fastened by both legs, **C, D;** or the section of one leg only will be considered effective, **C.**

Adjustment.
Avoid adjustable members, **A, Aa, D** (**P** except in counters). Avoid

---

* **Cc,** Classes A and B shall be designed to carry, at any future time, a double track electric railway.

adjustable counters, C. Counter rods and ties in pin spans adjustable, Y; screw ends upset, C, E, Y, screw threads U. S. Standard, C, Cc, D, E; diameter, at base of thread greater than in body of bar by one-sixteenth of an inch, D; about 10 per cent, Y; 17 per cent, Oo.

Rods with welded heads must be of wrought iron, Oo. Loops must develop full strength of bar, Oo.

Compression Members.

End posts and upper chords have 2 webs, a cover plate on top flange, batten or tie plate, and lacing on bottom flange, G

Not more than one plate, and that not thicker than one-half of an inch (in highway bridges three-eighths of an inch), shall in general be used as a cover plate, C, Cc. Cover plate must not extend more than 4 ins beyond outer row of rivets, D.

Joints between sections spliced on all sides with at least 2 rows of closely pitched rivets on each side of joint, C. Abutting surfaces faced A (D, P, except in top flanges of girders), E, R, Y. No reliance on abutting surfaces, E.

Lattice Bars.

Width, from 1.5 to 2.5 ins. Thickness, in single lattice, one-fortieth, distance between rivets, in double lattice, one-sixtieth, G. If over seven-sixteenths of an inch, use angles, Oo. Angle with axis of member; in single lattice 60°, double 45°, G.

Pitch, width of channel + 9 ins, C, Cc; 8 × least width of segment, P, R.

Double lattice bars riveted together at their intersections, C, D.

Batten Plates. (Tie Plates, Stay Plates.) Min length generally = from 0.75 to 1.5 × its own width. Min width, 9 ins, or 0.66 × own length, or = least width of member, Oo. Min thickness = three-eighths of an inch or one-fiftieth to one-sixtieth of distance between centers of rivets, G. Rivet spacing (D) max 4 ins cens.

Pin Joints.

Eye Bars. Thickness, min, 0.625 inch, or 0.2 × width of bar, Oo. Heads upset, rolled or forged, A, R. No welds allowed A, D, except (R) to form loops of laterals, counters or sway rods, R. Upset and die-forged, Y. Heads not more than one-sixteenth of an inch thicker than body, D, P. Bars annealed, G; before boring, D. No forge work after boring, R. Bars to be placed side by side must be bored at the same temperature, A, Aa, Y. Pins must pass through without driving. Eye bars working together must be clamped together, and bored at one operation, Oo.

Distance between pin-holes max variation, one sixty-fourth to one thirty-second of an inch; or one sixty-fourth of an inch in from 20 to 25 ft, G.

Built Tension Members. Net section through pin hole = 1.25 to 1.50 × net section through body of member. Net section back of pin hole = 0.75 × net section through pin hole, or = 0.80 to 1.00 × net section through body of member, G; proportion for double shear on section from back of pin to end of plate, Oo; length of plate back of pin, min, 2.5 ins, Oo. Distance, back of eye to back of member, greater than radius of pin, Y.

Pin Holes. Clearance between pin and hole, from one-fiftieth to one thirty-second of an inch, G.

Pin Plates or Reinforcing Plates. At least one plate on each side must extend not less than 6 ins beyond edge of batten plate, G.

Pins. Up to 7 ins diam, rolled, P; over 7 ins, forged, C.

Diameter, min, from 0.66 × to 0.85 × largest dimension of any of its eye bars, G.

Plate Girders.

Min depth; about one-ninth to one-twelfth of span, G.

Proportions of Web and Flange. Bending moments resisted entirely by the flanges, shear resisted entirely by the web plate, C, Cc, R; except when the web is made in one length or is fully spliced to resist the bending stresses, in which case one-sixth of area of cross section of web plate may be considered effective as flange area, Oo, One-eighth of gross area of web included in flange, A, Aa, B; if length = 90 ft or over, E; if length is less than 50 ft, only the cover plate and the horizontal legs of the flange angles are to be included in the flange area, E; no part of web included in flange, C, Cc, D, P, R, Y.

Web. Thickness, min, three-eighths of an inch, G; in highway bridges (Cc, Oo), five-sixteenths of an inch.

Total shear, acting on side next to abutment, to be taken as transferred into flange angles within distance = depth of girder, A, Aa, B, E, Oo, P.

Web Splices. A plate on each side of web, G; at least three-eighths inch thick, A, B; at least five-sixteenths inch, or three-fourths as thick as web, and wide enough for 2 rows of rivets on each side of splice, Oo.

Stiffeners. Generally required at ends and at points of concentrated load. Intermediate stiffeners required usually when unsupported distance between flange angles exceeds 50 to 60 × web thickness; when shear exceeds $10{,}000 - 75 \times \frac{\text{depth}}{\text{thickness}}$, C; in highway bridges when shear exceeds $12{,}500 - 90 \times \frac{\text{depth}}{\text{thickness}}$, Cc; when shear exceeds $\dfrac{20{,}000 \times 12\,t}{1 + \dfrac{d^2}{3{,}000\ t^2}}$, where t = web thickness, d = distance between flanges, Oo.

Spacing usually = depth, or = 5 or 6 ft.

Unit stress, max, $10{,}000 - 45\,\frac{l}{r}$, C; in highway bridges, $12{,}000 - 55\,\frac{l}{r}$, where l = length of stiffener, r = its least radius of gyration, Cc.

Dimensions of angles usually $3\frac{1}{2} \times 3 \times \frac{5}{16}$ to $5 \times 3\frac{1}{2} \times \frac{3}{8}$.

Flanges.

Unbraced length of flange (compression flange, C, P) max = 12 × width, B, P, R, Y; 16 × width, A, C, Cc; 20 × width, D; in highway bridges = 20 × width, Aa, = 25 × width, Oo.

Comp. flange has same gross area as tension flange, A, Aa, B, C, Cc, Oo, P.

Cover plates must not extend more than 5 ins, or 8 × thickness of first plate, beyond outer line of rivets, A, Aa, C, Cc. If of unequal thickness, the heaviest plates are next the angles, and the lightest outside, A, Aa, B, C, Cc, Y. One must extend full length of girder, B, E. Others must be long enough to take 2 extra rows of rivets at each end, C, P.

Bracing, Riveting, Bearings. See Bracing, Riveted Joints, and Bearings, below.

Beam Girders.

Beams in groups of 2, 3 or 4 for each rail, 10-inch channel separators, about 3 ft apart, riveted to webs, B.

Bracing.

Composed of rigid members, riveted, A, Aa, C, Cc, Y; members intersect each other, and other members to which they are connected, on common center lines, passing through all centers of gravity. Attachments riveted symmetrically in all directions, Y.

For Beam Girders. Ten-inch channel strut at each end; with 2 or 3 beams, angle bracing between girders; with 4 beams, angle struts about 6 ft apart. Connections to have at least 3 rivets, B.

Lateral.

B, in through spans. Top bracing; portal struts at ends; intermediate struts as deep as the chords; single angles, $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$, intersecting in each panel, for single track; double angles, latticed, for double track.

B, in through spans, bottom bracing; end bottom strut and intermediate angles, riveted to each other and to stringers at each intersection. Not less than 4 rivets at each intersection and at each end connection.

B, in deck bridges, complete upper and lower systems at each panel.

Oo, bottom end struts in all spans, whether deck or through.

Y, top and bottom lateral bracing in all deck bridges and in all through idges having sufficient head room. Lower lateral bracing in all through bridges. Upper system in all stringers framed between and riveted to floor beams where length of stringers exceeds 15 times width of stringer flange.

Y, in deck bridges without metal floor system, upper bracing of cross-struts at each panel point, composed of 4 angles latticed, with same depth as upper chord; stiff diagonals intersecting in each panel and riveted to each other at each intersection.

For Plate Girders.
Deck. Between upper flanges, angles with at least 4 rivets at connections. Through; lower lateral bracing of angles, intersecting in each panel, riveted to each other and to stringers at each intersection, **B.**
Lateral bracing angles generally of same size as those in stiffeners, **R.**

Cc, in highway bridges, a buckle plate floor may be considered as the required system of lateral bracing at the floor level.

Sway (diagonal, cross, vibration or wind) and **Portal.**

Proportioned to resist unequal loading of trusses in double track spans, **E, R**; end sway bracing to transmit all horizontal forces to abutment, **E**; to carry half the max stress increment due to wind & centrif. force, **A, Aa, B, P.**
In deck spans, at each panel point, **A, Aa, D, E, P, R, Y.**
Overhead bracing in through spans whose depth exceeds 25 ft, **C, D, P, Y**; in highway bridges, 20 ft, **Cc**; 25 ft, **Oo.**
In pony trusses and through plate girders, at ends and at each floor beam or cross strut, **A, Aa, R**; at every panel point, **D.**
In through and half through plate girders, at each floor beam and at each end, or, if there is a solid floor, not over 8 ft apart, **Y.**
In deck plate girders, rigid cross frames at ends and max 20 ft apart, **Y**; sway frames of at least 4 angles at ends and at points 12 to 14 ft apart, **B**; through, not more than 12 × flange width along top flange, **B.**

Riveted Joints.

Rivet Holes. In I beams, must be drilled, **B.**
May be punched; in steel not over ⅝ to ¾ inch thick, **G.**
Sub-punched one-eighth of an inch smaller, and reamed to one-sixteenth of an inch larger, than rivet, in steel over five-eighths to three-fourths of an inch thick; in connections for floor beams and stringers to main trusses or girders, **E.**
No drifting allowed, **A, B, C, D, E, R.**
No interchange of pieces after reaming, **D, P.**
Hole larger than rivet by one-sixteenth of an inch, **G.**
Die larger than punch by max one-sixteenth of an inch, **G.**
Distance from edge of plate to center of rivet. Min, 1.25 to 1.5 ins, or 1.5 to 2 diams of rivet. Max, 4 to 5 ins, or 8 × thickness of plate, **G.**
Pitch Min = 3 × diam of rivet, general; preferably 4 × diam, **Oo.**
Max pitch in line of stress, 5 to 6 ins, or 16 × thickness of thinnest outside plate connected; normal to stress, 30 to 50 × thickness of thinnest outside plate connected. At ends of compression members (or of built members in tension, **B**); for a length of 1.5 to 2 × width or depth of member, 3.5 to 4 × diam of rivet, **G.**
In plate girders, for rivets connecting web to a top flange supporting the track, max = 3 ins, **R.**

Rivets. Diam generally three-fourths or seven-eighths inch. Heads hemispherical, **G.** Height of head, min = 0.6 diam, **R.**

Driving. Avoid hand riveting. Machines, direct-acting, worked by steam, hydraulic pressure or compressed air, capable of maintaining applied pressure after upsetting, **G.**

Floor.

Floor Beams. Depth, min, = ⅙ × length, **Y.** In railroad bridges and important highway bridges, riveted to posts of trusses or to webs of plate girders, **G.** Given also a bearing on lower flange of girder or on a bracket, **G.** In default of such bearing, increase number of rivets by 25 per cent, **R.**
Hangers, when permitted, not adjustable, **C, Cc.** Hangers made of plates or shapes, **Oo.**

Stringers. Depth, min, = ⅛ × length, **Y.** In highway bridges, Classes

A1 and A2, of steel; Classes B, C, and E, track stringers of steel; Class D, of wood or steel, **Cc.** In railroad bridges, and preferably in highway bridges, riveted to webs of floor beams and supported by their flanges or by brackets, **B, R.** Value of this bearing neglected in determining number of rivets required, **R.**

Spacing, cen to cen, 6 ft, 6 ins, **A, B, C, D, Y;** 5 ft, **E;** double track, through, generally 6 ft, **R;** single track, 8 ft, **R.**

Trough Floors. Troughs rectangular, built of plates and angles, and riveted to main girders or trusses by angles, and, when practicable, by bracket angles under the lower horizontal plates. Gusset plates riveted to girders and troughs at distances of not over 8 ft, **Y.**

Bottom filled with a binder composed of 1 cu ft of clean, sharp gravel, screened to one-fourth of an inch, to 1½ gallons of No. 4 asphalt paving composition or enough to fill voids. Gravel first heated to 300° F and the whole mixed at that temperature, **Y.**

Wooden Floor. Continued over abutments, **A, B, C, R.**

Ties or **Floor Beams.** Long leaf yellow pine or white oak, **G.**

Width, 8 ins, **A, B;** 9 ins. **R.**

Depth, 8 ins, for 7 ft span of tie, to 14 ins for 12 ft, **B;** 12 ins, **R;** 10 ins, **Y.**

Notched down one-half of an inch; max 1½ ins, **B.**

Spacing. Usually 6 ins clear; 16 ins cen to cen, **R.** Every 3d, 4th, or 5th tie fastened to stringer by ¾ inch bolt or lag screw, **G.**

Wooden Joists in Highway Bridges. Width, min 3 ins or 0.25 × depth; spacing, max, 2 to 2.5 ft. Ends of joists lap past each other at bearings on floor beams, with 0.5 inch space between them for circulation of air.

Wooden Floor Beams for Electric Railroad Bridges, Classes E1 and E2 Min 6 × 6 ins, spacing max 6 ins, notched down one-half of an inch and secured to girders by three-fourths-inch bolts not more than 6 ft apart. From center of span toward end, so notched (**Cc**) as to reduce camber.

Guard Rails. 6 × 8 ins, yellow or white pine, **G.** Inner face not less than 3 ft 3 ins from center of track, **A;** 3 ft 7½ ins, **B;** 5 ft 4 ins, **Y;** 7 ft 1¼ ins apart, clear, **R.** Notched ½ to 1½ ins over ties, **G.** Fastened to every 3d or 4th tie (to each tie, **R**) and at splices by three-fourth-inch bolt or lag screw, **G.** Splices over floor timbers, with half-and-half joints of 6 ins lap, **G.**

Wheel Guards and Curbs in Highway Bridges. Wheel guards 6 × 4 ins, blocked up from floor plank by blocks 2 × 6 ins, 12 ins long, not more than 5 ft apart, bolted to stringers through blocking pieces, three-fourths-inch bolts, **G.** In electric railroad bridges (**Cc,** Class E) guard timbers min 5 × 7 ins, notched 1 inch over floor timber and secured by three-fourths-inch bolt to every third floor timber and at each splice.

Buckle Plates. Min five-sixteenths of an inch thick for roadway, one-fourth of an inch for footwalk, crown 2 ins, for widths of 4 ft under roadway, 5 ft under footwalks. Preferably in continuous sheets of panel lengths. May be pressed or formed without heating.

Bearings on Abutments and Piers.

Permissible load on masonry foundations, max, pounds per sq inch. 400, **A, Aa, P;** 300, **B;** 250, **C, Cc, D, E, R;** dead load, 500; live load, 250, **Y.**

Bed Plates. Of medium steel, **C, Cc.** Min thickness, three-fourths to 1 inch; in highway bridges, one-half of an inch. Max fiber stress 12,000 lbs per sq inch, **E.**

Where ends of two spans rest on one pier, spans are tied together, or have bed plate, three-eighths to three-fourths of an inch thick, continuous under both, **G.**

Sheet lead, one-eighth to one-fourth of an inch thick, between bed plate and masonry, **G.**

Anchor bolts, 1 to 1.25 ins diam, 9 to 12 ins in masonry, **G;** fastened with sulphur, **R;** with cement, **C, Cc, Y.**

Pedestals. Of riveted plates and angles, **C;** or cast steel, **Y.** Base plate and connecting angles, min three-fourths to seven-eighths of an inch thick, **B, C, Y.** 2 rows of rivets in vertical legs, **C, Y.**

Expansion Bearings. Provide for temperature range of 150° F, **A, C, E, P, R**; for expansion of 1 inch in 100 ft, **D, Y.**

One end sliding, usually in spans less than 60 to 90 ft, **G.**

One end on friction rollers, usually in longer spans, **G**; in all trusses, **Y.**

Hinged bolster at each end, in spans from 80 to 100 ft, **G.**

Rollers rest on bars 3 × 1 inch, spaced 2 ins and riveted to bed plate, **B.** Free ends anchored against lifting and against moving sideways, **C. Y.**

Rollers. Of machinery steel, **C, Cc.** Min diam, 3 to 4 ins, **A, B, D, E, P, R**; 3 ins up to 100 ft span, 1 inch for each additional 100 ft, **C, Y.** Max pressure on rollers, in lbs per linear inch, 700 $\sqrt{d}$, **R**; 1200 $\sqrt{d}$, **A, B, P**; 300 d, **C, D, E**; 600 d, **Oo**; d = roller diam, ins. Length, ins, = 900 $\sqrt{d}$, **Y.**

II. MATERIAL.

Rolled and Cast Steel and Iron.

Rolled steel in superstructures in general.

Cast steel in bed plates in special cases, in machinery of movable bridges.

Rolled iron in loop-welded rods, **P**; in laterals and unimportant members, **R.**

Cast iron in bed plates in special cases and in machinery of movable bridges.

Rolled Steel, Grades.

Soft. In general, in all principal parts.

Medium. In pins, friction rollers, lateral bolts, bearing plates, eye-bars, sliding plates and bed plates; permissibly (**C**) in compression in chords, posts, and pedestals.

Rivet. In rivets.

Machinery. In expansion rollers, **C.**

Rolled Steel. Manufacture.

(**See also** Digest of Specifications of Internat'l Ass'n for Testing Materials.)

All to be made by open-hearth process.

Slabs for rolling plates are hammered or rolled from ingots of at least twice their cross-section, **A, B.**

Plates up to 36 ins wide rolled in universal mill, **D, R**; or have edges planed, **D.**

Rolled Steel. Manipulation.

Annealing. Eye-bars heated to uniform dark red and allowed to cool slowly, **P**; members worked at blue heat are heated to a uniform bright red (not exposed to direct flame) and allowed to cool slowly, **B.**

Steel must not be welded, **B.** No reliance upon welded steel, **C.**

No work put upon steel at or near blue heat, or between boiling-point and point of ignition of hardwood sawdust, **C.**

Rolled Steel. Shop Work.

Sheared edges of steel thicker than five-eighths inch shall be planed, **B.**

All sheared edges (in medium steel, **D**) shall be planed off to a depth of 0.25 inch, **D, Y**; except web plates of girders over 36 ins deep when covered by flange plates, and fillers where sheared edges are not seen, **D.** Grinding not accepted as equivalent of planing, except for lattice bars, **Y.**

No sharp or unfilleted re-entrant corners permitted, **D, Y.** Where a plate, angle or shape has been cut into, the fillet, as well as the cut, must be finished with sharp cutting tool, or with chisel and file, so that no sign of the punched or sheared edge remains, **D.**

Angles or bent plates, used as end connections on girders, floor beams or stringers, must be accurately fitted, so that when the member is milled to length not more than one-sixteenth of an inch will be taken off these connections at their roots, **D.**

Material bent by punching must be straightened before bolting up, **R.**

Web plates, if buckled, must be cold-rolled to remove the buckles, **D.**

Spliced chord sections must be assembled and strung out in shop in lengths of not less than three sections, and, after being drawn up into contact at joints and lined up with splice plates in place, the field rivet holes shall be reamed to a fit before taking apart, and the assembled parts, with their splice plates, match-marked, D.

Riveted members must have all parts pinned up and drawn together before riveting up, D.

In cases of skew work, or of complicated connections, or of a large number of pieces of one and the same kind, the work shall be set up and fitted together in the shop, sufficiently to insure against any misfit, D.

Abutting surfaces at ends of sections of compression members, and ends of members to be framed together, are usually required to be faced.

Rolled Steel. Requirements.

See also Digest of Specifications of International Association for Testing Materials.

TENSILE TESTS.

Specimens of Medium, Soft and Rivet Steel. For tests of full size eye-bars see below.

Ultimate Strength, u, and Elastic Limit, el, in thousands of lbs per sq inch. Elongation, s (stretch), and reduction of area, a, in percentages of original dimensions. Elongation measured in a length of 8 ins.

	Medium or "Pin" Steel.				Soft or "Bridge" Steel.				Rivet Steel.			
	u	el	s	a	u	el	s	a	u	el	s	a
A, Aa ..	60–70	0.5 u	22	..	52–62	0.5 u	25	..	48–58	0.5 u	26	..
B	..	..	..	..	58–63	30	25	..	51–56	27	26	..
C, Cc ...	60–68	0.5 u	22	..	54–62	0.5 u	25	..	50–58	0.5 u	26	..
D	62–70	0.5 u	22	..	54–62	0.5 u	26	..	48–56	0.5 u	28	..
E ...	..	..	..	..	56–64	0.58 u	27	45	..	..	..	..
Oe ..	60–70	35	22	..	52–62	32	25	..	50–60	30	26	..
P	62–70	33	17	40	52–62	28	25	50	48–56	28	28	56
R ...	60–68	0.5 u	20	..	52–60	{0.5 u 28}	25	..	48–56	28	28	..
Y	62–70	0.6 u	25	45	56–64	0.6 u	26	50	48–56	..	28	55

Specimens from metal over five-eighths of an inch thick, el = 0.56 u, Y.
“ “ eye-bars, same requirements as for medium steel, D.
“ “ “ u = 63, B.
“ “ “ over 1.5 ins thick, deduct from el 1 for each one-eighth of an inch; el, min = 20, C.
“ “ “ and pins u = 62–70, l = 0.6 u, s = 25, a = 45, Y.
“ “ pins, s = 15, C, Cc.
“ “ “ (medium, soft or rivet) s, 5 per cent less, A.
“ “ “ (soft) s = 20, B.
“ “ “ and rollers, s = 10, D.
“ “ rollers and bearing plates, u = 70–78, s = 22, Y.

BENDING TESTS.

In medium steel, specimen to bend through an angle of 180° around a bar of diam = 1 to 1½ × thickness of specimen, without showing fracture on outside of bend; in soft and rivet steel, to bend flat upon itself.

NICKING TEST.

When nicked and bent around a bar of diam = thickness of rod, rivet steel shall show a gradual break and a fine, silky, homogeneous fracture, D.

DRIFTING TEST.

Center of hole as in ordinary practice, or 1.5 to 1.87 ins or 2 diams from edge of plate; enlarge to 1.25 to 1.50 diam, G.

ANGLE TEST.

Angles of all thicknesses must open flat. Angles not over one-half of an inch thick must bend shut, cold, under hammer blows without sign of fracture, **B.**

TEST PIECES.

Minimum section, usually one-half sq inch. Length, min, 8 to 12 ins. Tests are usually required for each melt or blow.

TESTS OF FULL SIZE EYE-BARS.

	Ultimate lbs per sq inch min	Elastic limit lbs per sq inch min	Elongation per cent min
A, Aa	5,000 less than small specimen		10 between necks
B	55,000	0.5 ult	12 between necks § 10 between necks ‖
C, Cc	56,000		10 between necks
D	58,000	30,000	12 in 10 ft
E			15
Oo	55,000		12.5 in 15 ft
P	48,000	27,000	14¶ in 10 ft
R	58,000 * 56,000 †	0.5 ult	13 between necks 10 between necks
	48,000 ‡	27,000	15 between necks
Y	58,000	33,000	10 in 20 ft

In general not over 4 per cent of total number of bars in bridge will be tested, **R**; at least 4 per cent, and not less than 3 bars, **B.**

75 per cent of fracture must be silky, the remainder fine granular, **R.**

Break in head shall not be cause for rejection—

(a) if bar develops 10 per cent elongation (12.5 per cent in 15 ft, **Oo**) and the required ultimate strength (ultimate 56,000, **C**, 55,000, **Oo**) and if not more than one-third of all the bars tested break in the head, **A, C, Oo.**

(b) if bar stretches 14 per cent and if a second bar breaks in body and the average stretch of the two bars is not less than 16 per cent, **P.**

Company pays for bars which meet requirements, less scrap value, **G.**

TESTS OF COMPLETED STRUCTURE.

Specified loads, or their equivalent, passed over structure (in railroad bridges at a speed of not over 60 miles per hour, and brought to a stop at any point by means of air or other brakes) or maximum load rested upon structure for 12 hours. After test, structure must return to its original position and must show no permanent change in any part, **C.**

COMPOSITION.

Phosphorus, max percentage.
In acid steel, 0.06 to 0.08; in basic steel, 0.04 to 0.06; in castings 0.08.
Sulphur, max percentage, 0.04 to 0.06.

MAXIMUM PERMISSIBLE VARIATION FROM SPECIFIED CROSS-SECTION OR WEIGHT.

2.5 per cent, **G**, except in extra wide plates, **D, Oo, P.**

In plates over 40 ins wide, in proportion to width, up to 5 per cent in plates 90 ins or wider, **D.**

1.5 per cent; where plates 36 ins and wider form 40 per cent of total, 2 per cent in excess, **Y.**

Long plates, ⅛ inch out of line in 20 ft, ¼ inch in 40 ft, **R.**

Shapes or plates, 3 per cent short in thickness; plates 80 ins wide, 5 per cent, **R.**

---

* † Medium steel. * Bars not over 10 sq ins. † 20 sq ins. Proportional values for intermediate areas. ‡ Soft steel. § In bars not longer than 20 ft between necks. ‖ In bars longer than 20 ft between necks. ¶ Max, 16.

Steel Castings.

Manufacture. Open hearth, **A, Aa, D, P, Y**; acid, **Y**; annealed, **P, R, Y.** Carbon, per cent, 0.25 to 0.40, **G.**
Phosphorus, per cent, max, 0.08, **B, Y.**

TENSILE TESTS.

	Size of test piece, ins.	Ultimate strength, lbs per sq inch, min.	Elastic limit, lbs per sq inch, min.	Elongation per cent, in 2 ins min.	Reduction of area per cent.
C, D, E, P, R ..	½ square or ¾ round	65,000 to 70,000	33,000 or 0.5 ult	10 to 15	20, **P**
Y	¾ round about 6 long	(a) 55,000 to 65,000	...	20	
		(b) 72,000 to 80,000	0.5 ult	15	25

BENDING TEST.

Y (a), for general purposes, bed plates, pedestals, etc., to bend 90°, to a radius = diameter of test piece.
Y (b), for drawbridge rollers, etc.

Rolled Iron.

Requirements in Osborn's specification for highway bridges, **Oo.** Made from puddled iron or rolled from fagots or piles of No. 1 wrought iron scrap, alone or with muck bar. Tensile strength, min, 48,000 lbs per sq inch (50,000, **R**); yield point, 25,000 lbs per sq inch (26,000, **R**); elongation, 20 per cent in 8 ins; in sections weighing less than 0.654 lb per lineal ft, 15 per cent. Specimens cut from bar as rolled must bend through an angle of 180° under a succession of light blows; when nicked and bent, fracture shall be generally fibrous and free from coarse crystalline spots; not over 10 per cent of the fractured surface shall be granular; specimens heated bright red shall bend through an angle of 180° under a succession of light blows not delivered directly on the bend; must not show red-shortness. In flat and square bars, one-thirty-second of an inch, in round iron 0.01 inch, variation either way in size will be allowed, **Oo.**

Cast Iron.

Tough gray iron, **A, D, E, R**; unless otherwise specified, **A, R.**
Transverse strength. Bar 1 inch square, 12 ins span, to bear 2,500 lbs, center load. Must deflect 0.15 inch before rupture, **G.** Bar 1 inch square, 4.5 ft span, to bear 500 lbs center load, **E, R.**

Phosphor Bronze.

1 inch cube, under compression, elastic limit, 20,000 lbs. Under 100,000 lbs, permanent set max one-sixteenth of an inch, **B.**

Timber.

Sap wood not allowed in more than 10 per cent of the pieces of one kind, and no piece will be accepted showing sap covering more than 0.25 × the width of the piece on any face at any point, or more than half the thickness of any plank at its edge, at any point, **Oo.**

III. LOADS.

1. Vertical Loads.

(Dead and Live Loads and Impact.)

Dead Loads in Steam Railroad Bridges.

Dead load = weight of metal + n lbs per lineal ft of track, **C**, **D**, **E**, **R**, **Y**. n = 400, **C**, **D**, **E**; n = 500, **R**; n = 620, **Y**.

Timber taken at 4.5 lbs per ft, B.M., **G**. Ballast, 110 lbs per cu ft, **C**.

Rails, splices, and joints taken at 100 lbs per lineal ft of track, **A**, **B**, **C**.

Rails, splices, guard rails, etc., at 160 lbs per lineal foot of track, **P**.

Two-thirds of dead load assumed to be carried by loaded chord, **Y**; in spans less than 300 ft, **B**; in longer spans calculate distribution, **B**.

Dead Loads in Highway and Electric Railroad Bridges.

Iron, 3.33 lbs per lineal ft of bar 1 sq inch area, **Oo**.

Steel, 3.40 " " " " " " 1 sq inch area, **Oo**.

Timber per ft board measure, 4, **Aa**; creosoted, 5, **Oo**; oak, 4.5, **Cc**, **Oo**; other hard woods, 4.5, **Cc**; yellow pine, 4, **Oo**; spruce and white pine, 3.5, **Cc**; white pine and cedar, 3, **Oo**.

Concrete, etc., lbs per cu ft, 130, **Aa**; stone concrete, 125, **Oo**; cinder concrete, 100, **Oo**. Stone, 150, **Oo**, granite, 160, **Aa**.

Brick, 150, **Aa**; 125, **Oo**; sand, 100, **Oo**. Asphalt, 130, **Aa**; 90, **Oo**.

Rails, fastenings, splices and guard timbers, 100 lbs per lin ft of track, **Aa**.

Live Loads for Steam Railroad Bridges.

THEODORE COOPER'S STANDARD LOADING.*

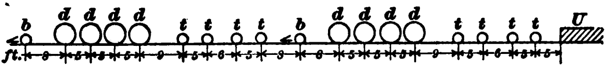


Fig. 1.

TWO CONSOLIDATION LOCOMOTIVES, WITH THEIR TENDERS AND TRAINS.

Class	Load in lbs on one pair of wheels for each track. Truck (bogie) b	Driver d	Tender t	Train load, lbs per lin ft. U
E 27	13,500	27,000	17,550	2,700
E 30	15,000	30,000	19,500	3,000
E 35	17,500	35,000	22,750	3,500
E 40	20,000	40,000	26,000	4,000
E 50	25,000	50,000	32,500	5,000

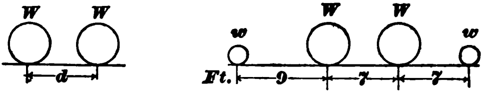
A, adopts Cooper's loading. See **C**, above.
B, Cooper's Class E 50, unless otherwise specified.
D, **E**, **P**, **R**, Loads and spaces differing slightly from Cooper's.
Y, Cooper's Class E 40.

---

* In Mr. Cooper's system of standard loading, the No. (27 to 50) following the letter E in the class designation gives the load, d, on one pair of drivers, in thousands of pounds. In each class, $d = 2\,b = \frac{40}{26}\,t = 10\,U$. Since these ratios are constant for all classes, the stresses due to any class are proportional to the number of the class. The cost and weight of metal, in bridges of all kinds, built under **C** specifications, will be, in each class, about 10 per cent greater than in the class next lighter.

ALTERNATIVE LOADINGS.

Use Fig. 1 or the alternative, whichever gives the greater stresses.



Figs. 2 and 3.

Load on one pair of wheels.

Fig. 2. d = 6 ft; W = W = 50,000 lbs, Above E 40, 60,000 lbs. **C.**
d = 7 ft; W = W = 65,000 lbs, **D.**
d = 7 ft; W = W = 60,000 lbs; L = 4,500, **Y.**

Fig. 3. d_1 = 9 ft; W = W = 66,000 lbs
$d_2 = d_3$ = 7 ft; w = w = 30,000 lbs } **R.**

Add 30 per cent in figuring floor beams, stringers, hangers, suspenders, and other floor connections. Add 0 to 30 per cent for spans from 100 ft down to 25 ft, **D.**

ON CURVES.

Distribution of live load between the two trusses.

$W = P \frac{m + b}{2 b}$; where W = proportion of live load borne by the outer truss; P = live load at panel considered; m = middle ordinate of entire curve on span; b = dist betw cens of trusses. Make both trusses alike, **B, Y.**

SPECIAL LOADINGS.

For rivets connecting upper flange angles with web in deck girders carrying the floor directly on the top flanges, and in deck spans with wooden floor beams, when distance between trusses exceeds 6 ft, 50,000 lbs on one pair of drivers, distributed equally over three ties or floor beams, **P.**

For floors, the load on a single pair of engine wheels distributed over 4 ties, **B;** over 3 ties, **C.** For trough floors, 60,000 lbs on one pair of wheels, distributed over two troughs, **Y.**

THREE-TRUSS BRIDGES.

In double-track deck spans, all three trusses of equal strength, **C.**

In plate girder bridges of more than one track, center girder figured for 0.75 × the live load, **E.**

FUTURE INCREASE OF LIVE LOADING.

Only 70 per cent (50 per cent, **R**) of the dead load shall be considered effective in counteracting live load stress, **A, R.** Use 1.5 × live load, **E.**

"That the heavier of these engines (see '**C,**' under 'Standard Loading,' above) is close to the possible maximum, considering the limitations of the permissible cross-section of existing railroads and the mechanical details of design and proportions, is not improbable. That the economical tendency toward heavier and heavier engines will in the near future reach the heavier class E 50 upon the most important roads is to be expected. The cars will also follow the same tendency for many kinds of traffic, as experience justifies the advance. There are now in use self-dumping coal cars of a nominal capacity of 100,000 pounds, which have, on four axles, a total load of 146,000 pounds (10 per cent increase over nominal capacity) on a wheel base, for two adjacent cars, of 17 ft, 2 ins. These cars on all ordinary bridges produce strains equivalent to those of E 33."—Theodore Cooper.

Members subject to reversal of stress must be so designed that a live load n per cent greater than that specified shall not increase their unit stresses more than n per cent. n = 25, **C;** 50, **B;** 100, **P.**

Live Loads for Highway and Electric Railroad Bridges.

Aa, (Am. Bridge Co.) and **Cc**, (Theo. Cooper.)	For the Floor and its Supports.				For the Trusses.			
	Concentrated.			Uniform (c).	Per lin ft of single track.		Per sq ft of remaining floor.	
					(Proportionally for intermediate spans.)			
Class.*	Wagon (a), tons		Car (b) on each track, tons	Per sq ft, lbs	Spans up to 100 ft lbs	Spans 200 ft and over lbs	Spans up to 100 ft lbs	Spans 200 ft and over lbs
A.........	24		..	100	1,800	1,200	100	80
B	12	or	24	100	1,800	1,200	80	60
C	12	or	18	100	1,200	1,000	80	60
							Up to 75 ft	
D	6		..	80	..	..	80	55
E1	..		24	..	1,800	1,200	..	..
E2.........	..		18	..	1,200	1,000	..	..

(a) On two axles, 10 ft cens (and, **Aa**, 5 ft gage); in classes A, B, and C, assumed to occupy a width of 12 ft in single line (or, **Cc**, 22 ft in double line) on any part of the roadway.

(b) On two axles, 10 ft centers.

(c) In classes A, B, and C, on remainder of floor, including footwalks. In class D, on total floor surface.

Oo. Osborn Engineering Co. Highway. May specify any combination of the following loadings, according to character of bridge and of load.

Uniform loads, lbs per sq ft. For spans up to 150 ft, 100 on roadway and 80 on sidewalks, or 80 on both. For spans over 150 ft, 80 or 60 on both.

A steam road roller; axles 11 ft apart, forward roll 4 ft face, two rear rolls 5 ft cens and each 20 ins face. 15,000 or 9,000 lbs on forward roll and 10,000 or 6,000 lbs on each rear roll;

A horse roller, 12,000 lbs on roll, 5 ft face;

A wagon load, 10,000 lbs on two axles, 8 ft apart, 5 ft gage;

Two electric cars on each track; Fig. a.

A train of electric cars on each track; Fig. b.

A train of coal cars of 60,000 lbs capacity; Fig. c.

Fig. a

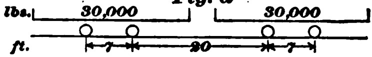


Fig. b

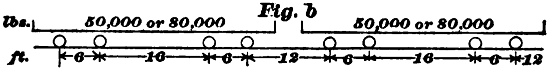
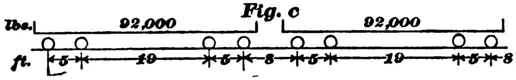


Fig. c



---

*Class A, city bridges.
" B, suburban or interurban.
" C, heavy country highway.
Class D, ordinary country highway.
" E1, heavy electric railway only.
" E2, light electric railway only.

FUTURE INCREASE OF LIVE LOADING. HIGHWAY.

In electric railroad bridges, Class E, only 70 per cent of dead load stress to be considered as effective in counteracting the live load stress, **Aa.** For bridges carrying electric or motor cars, counters so proportioned that a future increase of 25 per cent in the specified live load shall not increase the unit stress more than 25 per cent, **Cc.**

Impact.

$I = S \frac{300}{l + 300}$; where I = impact stress to be added to the live load stress; S = calculated max live load stress; l = length in feet of loaded distance which produces the maximum stress in the member, **A.**

$I = S\left(0.1 + \frac{56.25}{l \times 62.5}\right)$, Mr. G. Bouscaren, C. E.

In Highway Bridges; I = 25 per cent of live load stresses **Aa;** $I = L^2 + (L + D)$, where L and D = live and dead load stresses, **Oo.**

2. Horizontal Forces.

(Drag; Centrifugal and Wind.)

(a) Longitudinal.

Drag. In bridges for steam and electric railroads, provide for a longitudinal force, at the rails, = 0.2 of the max live load. In double track (**Y**), provide for trains moving either way.

(b) Transverse.

(1) Centrifugal Force.

F = centrifugal force; W = weight of train on bridge; d = degree of curvature = central angle subtended by a chord of 100 ft; v = velocity in miles per hour; c = a coefficient.

A, F = c d W. For d up to 5°, c = 0.03. Deduct from c 0.001 for each degree over 5°. Train on each track.

B, R, F = 0.02 of the live load for each deg of curvature. **B,** up to 5°. Deduct 0.001 for each degree over 5°.

C, F, computed for v = 60 — 3 d on steam railroads, = 40 on electric railroads; force acting 5 ft above base of rail.

D, v = 60.

E, F = force due to that uniform load which would produce the max specified live load bending moment on span; v = 60.

Y, $F = W v^2 d \div 85{,}666$. Up to d = 4°, v = 60. For d over 4°, v = 60 — 2 d. Max train load on each track.

(2) Wind.

(a) ON RAILROAD BRIDGES.

Wind pressure, in lbs per sq ft, = w; in lbs per lin ft = W.

w = either 30 lbs per sq ft on exposed surface of trusses and floor and on that of a train of 10 ft average height, beginning 30 ins above rail base; or 50 lbs per sq ft on exposed surface of trusses and floor; whichever gives the greater stresses, **A, P.**

In truss spans over 200 ft & in plate girders, w = 30 lbs per sq ft of exposed surf of 1 girder and floor, + W on train for lower chord, as below, **B.**

W = L + U. L = pressure in lbs per lin ft on loaded chord, U on unloaded chord. W includes both wind on bridge and wind on train.

Wind on bridge. L = U = 150, **B, C,* D, R;** = 200, **E.**

L = 200, on double track 300, acting 8 ft above rail top; } **Y.**
U = 150, on double track 225, acting at cen of chord, }

Wind on train. L = 300, **B, D, R,**† **Y;** = 450, **C,**‡ = 400, **E.**

---

*In spans over 300 ft, add to U 10 lbs to each additional 30 ft, **C.**

†Acting 7.5 ft above rail, **R.**

‡Acting 6 ft above rail base. Includes lateral vibrations of trains.

A. Wind stress, Sw, in any truss member, **C** (main truss member, **D**; chord or end post, **R**), need be considered only (1) when Sw exceeds 30 per cent, **C** (25 per cent, **D**, **R**), of max stress, S, due to dead and live loads. Then increase section to bring Sw within limit, **C**, **D**, **R**. (2) When Sw, alone or in combination with temperature stress, can balance or reverse S, **C**.

Anchorage. In determining the requisite anchorage for the loaded structure, the train is assumed to weigh 800 lbs per lineal foot, **A**, **B**, **C**, **P**; 600 lbs per lineal foot, **R**.

(b) ON HIGHWAY AND ELECTRIC RAILROAD BRIDGES.

Either 30 lbs per sq ft on the exposed surface of all trusses and floor, + 150 lbs (180, **Oo**) per lineal foot of a train covering the span; or 50 lbs per sq ft on the exposed surface of all trusses and floor; whichever gives the greater stresses, **Aa**, **Oo**.

On each chord, 150 lbs per lin ft, of span, due to bridge, and on the loaded chord 150 lbs per lin ft of span additional due to train. For spans exceeding 300 ft, add 10 lbs on each chord for each additional 30 ft, **Cc**.

Wind stresses (in truss members, **Cc**; in chords and end posts, **Oo**) to be provided for only when the wind stress exceeds 25 per cent of the max dead and live load stresses (of the sum of all other stresses, **Oo**), or when the wind stress (alone or in combination with temperature stress, **Cc**) can (neutralize or, **Cc**) reverse the stress in the member, **Cc**, **Oo**.

IV. STRESSES AND DIMENSIONS.

Effective Span and Depth.

In pin spans, span and depth are measured between centers of pins. In riveted trusses the span is measured between centers of end bearings and the depth between centers of gravity of chord sections. In plate girders the span is measured between centers of end bearings, and the depth between centers of gravity of flange areas or over backs of flange angles, whichever is the less. In floor beams the span is measured between centers of trusses, and in stringers between centers of floor beams, **G**.

Limiting Unit Stresses.

Tension.

Net section. The net section of any tension member or flange is determined by a plane cutting the member square across at any point. The greatest number of rivet holes which can be cut by the plane, or come within an inch of the plane, is deducted from the gross section, **B**. The rupture of a riveted tension member is considered equally probable, either through a transverse line of rivet holes, or through a diagonal line of rivet holes where the net section does not exceed by 30 per cent the net section along the transverse line, **C**, **Cc**.

In deducting rivet holes for net section, their diameter is taken at one-eighth of an inch greater than that of the cold rivet, **G**; for countersunk rivets (**Oo**), one-fourth of an inch greater.

Maximum permissible tensile stresses, in lbs per sq inch.

	Medium Steel	Soft Steel
A, Aa, Under vertical forces only or horizontal forces only	17,000	15,000
Under vertical and horizontal forces combined	21,000	19,000

B, For "Bridge" (soft) and Rivet Steel, same as Medium Steel under **A**.

D. For soft steel:	For dead load;	live load.
Eye-bars	14,000	9,000
Built sections	12,500	8,500
Counters		8,500

	For dead and live load.
Hip suspenders, floor beam hangers, members subject to sudden loading	7,500
Tension flanges of plate girders and rolled beams	9,000
Bracing	12,000

Main members of trusses, flanges and webs of girders and floor beams for double track, floors and girder flanges with ballast floor, add 10 per cent

For medium steel, add 10 per cent.

E. $8{,}000\left(1+\frac{\text{min stress}}{\text{max stress}}\right)$.

Oo. (Highway Bridges.) Medium steel, 22,000; soft steel, 20,000; wrought iron, 18,000.

P. M = max calculated stress in member
m = min " " " "

Let $r = \frac{m}{M}$; let $k = \frac{1 - r}{1 + r}$. Then M (1 + k) shall not exceed 15,000.

Long hip verticals must have 25 per cent excess strength; short floor beam hangers 50 per cent excess, **P.**

Y. Soft steel. Chords and web members of trusses, and flanges of plate girders, floor beams and stringers.

Dead load and drag	16,000
Live load and centrifugal force	8,000

MAXIMUM STRESSES IN TIMBER, LBS PER SQ INCH.

For Highway Bridges, Oo.	Transverse loading	End bearing	Short column *	Bearing across fibre	Shear along fibre
White oak	1,400	1,300	1,000	550	300
Long leaf pine	1,600	1,300	1,000	350	200
White pine	1,100	900	700	200	150
Hemlock	950	850	650	200	100

Extreme fibre stress, in floor beams, max, yellow pine and white oak, 1,200 lbs per sq inch; white pine and spruce, 1,000, **Aa, Cc.**

Compression.

p = permissible working stress in compression member, in lbs per sq inch.
f = generally the permissible stress in tension member, in lbs per sq inch.
a = a coefficient.
l = length of piece, in ins, between cens of connection.
r = least radius of gyration of cross-section of member, ins.

$$p = \frac{f}{1 + \frac{l^2}{r^2 a}}$$

		f	a
Aa.	In medium steel	17,000	11,000
	In soft steel	15.000	13,500
B.	In soft steel	17,000	11,000
C.	See below.		

	Dead load f	Dead load a	Live load f	Live load a
D.	12,000 to 12,500	18,000 to 24,000	8,000 to 8,500	18,000 to 24,000

E. $f = 8.000\left(1+\frac{\text{min stress}}{\text{max stress}}\right)$; a = 36,000 with both ends fixed; a = 24,000 with one end fixed; a = 18,000 with both ends hinged.

Oo. (Highway Bridges.) f = 22,000 for medium steel, 20,000 for soft steel, 18,000 for wrought iron; a as in **E**, above.

P. f = 15,000; a = 13,500.

R. $f = 6{,}500\left(1+\frac{\text{min stress}\,\dagger}{\text{max stress}}\right)$; f max = 8,000; a = 40,000 with flat ends; a = 20,000 with pin ends.

When one end is pinned, p = mean of values derived as above.
For angle iron struts, see below.

---

*Length not over 12 × least side. †Min stress = dead — live load stress.

Y. Soft steel in chords and web members:

	f	a
For dead load and drag	16,000	18,000
For live load and centrifugal force	8,000	18,000

C, Cc. $p = M - c\frac{l}{r}$.

For medium steel in stationary structures:

	Dead load		Live load	
	M	c	M	c
Chord segments, stiffeners.	20,000	90	10,000	45
For highway bridges.......	24,000	110	12,000	55
End and other posts	17,000 to 18,000	90 to 80	8,500 to 9,000	45 to 40
For highway bridges.......	20,000 to 22,000	90 to 80	10,000 to 11,000	45 to 40
Lateral struts, rigid bracing for railroad and highway bridges	13,000	60	8,666	40

For soft steel, deduct 15 per cent; for movable structures, deduct 25 per cent.

R. Angle iron struts.

With flat ends, $p = 9{,}000 - 30\frac{l}{r}$; with pin ends, $p = 9{,}000 - 34\frac{l}{r}$. In lateral and cross struts, add 30 per cent.

Length of compression members, max, = 40 to 45 diameters, or 100 to 120. In highway bridges, 120 to 140 r, **Aa**; 100 to 120 r, **Cc**; 125 to 150 r, **Oo**, where r = least radius of gyration.

Unsupported width (distance between rivets) of plates subject to compression, max = 45 × thickness, **Oo**; 30 × thickness, **C, Cc, D**; in cover plates of top chords and end posts, 40 × thickness, **C, Cc, D**; or, if a greater width is used, effective section shall be taken as 40 × thickness, **C, Cc.** Distance between supports in line of stress, max = 16 × thickness, **Oo.**

Timber columns, whose length exceeds 12 × their least sides, in highway bridges, **Oo.**

$$\text{Max unit stress} = \frac{C}{1 + \frac{l^2}{1{,}000\,d^2}}$$

where C = 1,000 lbs per sq inch for white oak and long leaf pine, 700 for white pine, 650 for hemlock; l = length of column, between supports, ins; d = least side, ins, **Oo.**

Alternating Stresses.

Total sectional area of member to be made = sum of areas required for both stresses, **A, B.**

Area sufficient to resist either stress plus 0.8 (0.6, **R**; 1.0, **Y**) × the lesser stress, **C, Cc, D, R, Y.**

Permissible working stress, in lbs per sq inch:

$$a = 8{,}000\left(1 + \frac{\text{max stress of lesser kind}}{2 \times \text{max stress of greater kind}}\right), \textbf{ E.}$$

M = max calculated stress of greater kind,
m = max calculated stress of lesser kind,
Let $r = \frac{m}{M}$. Let $k = \frac{2 + r}{2 - r}$. Then M (1 + k) shall not exceed 15,000 lbs per sq in, **P.**

IN BRIDGES FOR HIGHWAYS AND FOR ELECTRIC RAILROADS.

In Classes A, B, C, and D, members proportioned for that stress which requires the larger section. In Classes E 1 and E 2, make sectional area = sum of areas required for the two stresses, **Aa.** Members designed to resist either stress and given 25 per cent excess of strength in their joints and connections, **Oo.**

Shear and Bearing Stresses.

Shear in web plates, max, lbs per sq inch. 10,000, **B;** 4,000, **E;** 5,000, **R;** 13,000, **P;** in medium steel, 10,000, **A, Aa;** in soft steel, 9,000, **A, Aa;** across grain, 6,000, **D;** with grain, 5,000 (net section), **D;** dead load, 10,000, **Y;** live load, 5,000 (gross section), **Y.**

Shear and Bearing on Rivets, Bolts and Pins. Maximum, in lbs per sq inch.

	Shear		Bearing	
	Medium	Soft	Medium	Soft
A, Aa, B	12,000	11,000	24,000	22,000
C	9,000	9,000	15,000	15,000
Cc	10,000	10,000	18,000	18,000
Oo	10,000	10,000	22,000	20,000
D, R	7,500	7,500	12,000	12,000

Y, Shear = 0.75 S ; bearing = 1.50 S. S = permissible unit stress in tension.

In field riveting, increase number of rivets 25 per cent, **A, Aa, B, Oo, P;** if machine driven, 10 per cent, **A, Aa, P;** in stringers and floor beams, one-third, **P.** Take 0.66 to 0.80 × stress as above, **C, Cc, D, R, Y.**

In floor connections, use 0.8 × stresses as above, **C, Cc;** add 20 per cent to number of rivets, **Y.**

In wind and sway bracing, use 1.25 to 1.5 × stresses as above, **C, Cc, D, R.**

Rivets with countersunk heads taken at 0.75 × value of rivets with full heads, P.

Bearing, on phosphor bronze disks, 5,000 lbs per sq inch, **B.**

Bending Stresses.

Stress in extreme fibres, under bending moments, max, lbs per sq inch.

In pins and bolts, 25,000, **B;** 18,000, **C;** 20,000, **Cc;** 15,000, **D, R;** 16,000, **Y;** in pins, closely packed, 25,000, **Oo;** in medium steel, 25,000; in soft steel, 22,000, **A, Aa, P.** Centers of bearings of strained members taken as points of application of the stresses, **A, Aa, R.** Applied forces considered as uniformly distributed over the middle half of the bearing of each member, **C, Cc.** Bending calculated from distances between centers of bearing, **Oo.**

In rolled beams and channels, 14,000, **P.**

In wooden floor beams, 1,000, **A, B, C, P.**

Compound Stresses.

Compound (axial and bending), maximum, lbs per sq inch.

In end posts of through spans, dead + live + wind + bending, max = 15,000, **R.**

Proportion the member to resist sum of direct stress plus 0.75 bending stress, **A, Aa, B, P, R.** Max = $\dfrac{8{,}000}{1 + \dfrac{l^2}{40{,}000\ r^2}}$, where l = length, ins; r = least radius of gyration, ins.

If pins are out of neutral axis of section, max must include the additional stress due to the eccentricity, **R.**

Bending moment at panel points assumed equal and opposite to that at the center, **A, Aa.** If fibre stress due to weight of member alone exceeds 10 per cent of the allowed unit stress on such member, the excess must be considered in proportioning the areas, **C, Cc, R.**

Minimum Dimensions.

Minimum thickness of plates, in railroad bridges, three-eighths of an inch for main members, five-sixteenths of an inch for laterals; in highway and electric railroad bridges, five-sixteenths to one-fourth of an inch. Min diam of rod, three-fourths of an inch, **Oo.** Rods and bars, min section, 1 sq inch, **D, R;** counters 1.5 sq ins, **D, P.** Posts, in pin spans, min width 10 ins, **A.** In posts of through spans, channels min 10 ins, **B.** Angle, min, 3.5 × 3 × five-sixteenths, **B.**

G, gen'l; **Oo**, Osb'n; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, H'way.

V. PROTECTION.

At Shop. After removing loose scale and rust; 1 coat pure boiled linseed oil, **A, Aa, B, D, E, P, R**; raw linseed oil, **C, Cc**; with 10 per cent in weight of lampblack, **D**; standard red lead paint,* **Y**.

Inaccessible Parts. 2 coats iron ore paint in pure linseed oil, **A, Aa, B, C, Cc, E, R**; standard red lead paint,* **Y**; 1 coat, **D**; 1 heavy coat red lead in raw linseed oil, **P**; 2 coats, 18 lbs red lead in 1 gal boiled linseed oil, **Oo**.

Finished Surfaces. Coated with white lead and tallow. General.

Surfaces in Contact. Painted before joining, **A, Aa, B, C, Cc, R, Y**; with 2 heavy coats red lead in raw linseed oil on each surface, **Y**.

After Erection. 2 additional coats of paint in pure linseed oil, **A, Aa, B, C, Cc**; 2 coats of paint, of different colors, **R**; 2 heavy coats asphaltum varnish, **Y**.

At least 48 hours allowed for drying of each coat, **Y**.

Columns, etc., for 5 ft above surface of street, etc., 2 heavy coats asphaltum varnish; **under sides of bridges,** rest of columns, etc., 2 heavy coats standard white paint;* ballast side of trough floors, 1 part by weight refined Trinidad asphalt and 3 parts straight run coal tar pitch at 300° F, **Y**.

Wherever there is a tendency for water to collect, the spaces must be filled with a waterproof material, **C, Cc**.

First coat paint of graphite or carbon primer, **Oo**.

In highway bridges, upper surfaces of metal floor plates thoroughly coated with asphalt, **Oo**.

VI. ERECTION.

The Contractor is usually required

(1) to unload materials after delivery, to furnish falseworks and appliances, to remove the old bridge, to alter existing bridge seats;

(2) to drill and set anchor bolts, to erect and adjust the superstructure, and sometimes to furnish and place the wooden floor beams;

(3) to remove falseworks and appliances;

(4) to keep the road open for traffic and to avoid interference with any other thoroughfare by land or water and interference with other contractors; to furnish and pay watchmen; to keep material clean and in good order; and to assume all risks of damage to persons or property by reason of storms, floods or other casualties;

(5) to furnish pilot nuts for the protection of the ends of pins in driving.

(2) DIGEST OF SPECIFICATION FOR COMBINATION RAILROAD BRIDGES.†

By Baltimore and Ohio Railroad Co., 1901.

I. GENERAL DESIGN.

Type, Howe.

Rods of steel, with upset ends; standard nut and lock nut on each end.

Cast iron joint boxes and packing spools.

Steel gib plates from 1.25 ins thick for 1.25 inch rod, to 1.75 ins thick for 2.5 inch rod.

Splices in lower chord generally of steel construction.

II. MATERIAL.

Lumber, Georgia yellow pine, white oak or white pine.

Rolled steel. Open hearth. Ultimate strength 60,000 lbs per sq inch, permissible variation, 5,000 lbs; elastic limit, 30,000 lbs; elongation 25 per cent in 8 ins; to bend 180° flat upon itself.

---

* Standard red lead paint. 5 gals contain 100 lbs pure red lead, 4 gals pure raw linseed oil, one-half pint Japan, free from benzine, **Y**.

Standard white paint. 5 gals contain 42 lbs pure white lead in oil, 21 lbs white zinc in oil, 3 gals pure raw linseed oil, **Y**.

At least 48 hours between coats, and between final shop coat and loading, **Y**.

† To be used only for temporary purposes.

III. LOADS.

Dead Load.

Timber taken at 4.5 lbs per foot board measure. Track 100 lbs per lin ft.

Live Load.

Max intended load + 25 per cent, to provide for increase and impact.

IV. STRESSES AND DIMENSIONS.

Limiting Unit Stresses.

Timber, lbs per sq inch, max	Yellow pine	White pine	White oak
Bending or direct tension........	1,200	800	1,000
Columns under 17 diams in length	900	600	750
Columns over 17 diams in length..	1,200–18 n	800–12 n	1,000–15 n
where n = length ÷ least thickness; n max = 40.			
Shearing, along grain..........	150	100	200
Bearing, in direction of grain.....	1,500	1,000	1,250
Bearing, perpendicular to grain...	350	200	500

In columns made up of several sticks placed side by side, and bolted together at intervals, each stick treated as an independent column.

Steel rods, max unit stress = 12,000 lbs per sq inch.

Floor beams designed to carry the dead load and the heaviest engines in service without impact allowance. Reinforce for future increase of loads.

For loadings in excess of that used in designing, reduce speed from 60 to 15 miles per hour, as loads increase to limit of 25 per cent increase of load.

V. PROTECTION.

Steel rods, gibs, etc., 1 coat of paint in shop; 2 after erection.
Wood, at joints and at points of contact, to be painted.
Bolt and rod holes to be saturated with paint.

---

(3) DIGEST OF SPECIFICATION FOR ROOF TRUSSES, STEEL FRAMEWORK AND BUILDINGS.

By Baltimore and Ohio Railroad Co., 1901.

I. GENERAL DESIGN.

Made principally of shapes. No adjustable members, except in lateral bracing. Lateral bracing proportioned for a full wind pressure of 30 lbs per sq ft of exposed surface, acting in any direction. Tension members in bracing must in all cases pull directly against a stiff strut. If building is enclosed and the work is exposed to the action of gases, no open spaces less than 1 inch wide left between members, or open pockets inaccessible for painting.

II. MATERIAL.

Min thickness, 0.25 inch. When subject to the action of gases, five-sixteenths inch if building is open; 0.375 inch if enclosed.

III. LOADS.

Snow, 20 lbs per sq ft of horizontal projection of roof surface. Wind, 30 lbs per sq ft, horizontal, in any direction. Min total, 40 lbs per sq ft.

Covering. For roofs, and for sides unless otherwise ordered, corrugated sheets, No. 22 gage, 26 ins wide; corrugations, 2.5 ins; 3 ins for slope of 1 on 2; 6 ins for less slope. Purlins not more than 4 ft apart between centers.

IV. STRESSES.

Columns sustaining roof are considered as hinged at base, unless so anchored as to be absolutely fixed.

Unit stresses, if subject to no moving load other than wind, see **B**, in Digest of Specifications for Steel Bridges, and Digest (2) of B & O R R Specification for Combination Bridges. Stresses given in the latter to be increased 25 per cent.

V. PROTECTION.

Three coats of paint. If exposed to gases, use bridge paint (see **B** in Steel Bridge Specifications); if not, use standard building paints.

SUSPENSION BRIDGES.

Art. 1. Table of data required for calculating the main chains or cables of suspension bridges. Original.

Deflection in parts of the Chord.	Deflection in Decimals of the Chord.	Length of Main Chains between Suspension Piers, in parts of the Chord.	Tension on all the Main Chains at either Suspension Pier, in parts of the entire Suspended Wt. of the Bridge, and its Load.	Tension at the Center of all the Main Chains; in parts of the entire Suspended Wt. of the Bridge, and its Load.	Angle of Direction of the Chains at the Piers. Deg.	Min.	Natural Sine of the Angle of Direction of the Chains, at the Piers.	Natural Cosine of the Angle of Direction of the Chains at the Piers.
1-40	.025	1.002	5.03	5.00	5	43	.0995	.9950
1-35	.0286	1.002	4.40	4.37	6	31	.1135	.9935
1-30	.0333	1.003	3.78	3.75	7	36	.1322	.9912
1-25	.04	1.004	3.16	3.12	9	6	.1580	.9874
1-20	.05	1.006	2.55	2.51	11	19	.1961	.9806
1-19	.0526	1.007	2.43	2.38	11	53	.2060	.9786
1-18	.0555	1.008	2.30	2.25	12	32	.2169	.9762
1-17	.0588	1.009	2.18	2.12	13	14	.2290	.9734
1-16	.0625	1.010	2.06	2.00	14	2	.2425	.9701
1-15	.0667	1.012	1.94	1.87	14	55	.2573	.9663
1-14	.0714	1.013	1.82	1.74	15	57	.2747	.9615
1-13	.0769	1.016	1.70	1.62	17	6	.2941	.9558
1-12	.0833	1.018	1.57	1.49	18	33	.3162	.9480
1-11	.0919	1.022	1.46	1.37	19	59	.3418	.9398
1-10	.1	1.026	1.35	1.25	21	48	.3714	.9285
1-9	.1111	1.033	1.23	1.12	23	58	.4062	.9138
⅛	.125	1.041	1.12	1.00	26	33	.4471	.8945
1-7	.1429	1.053	1.01	.881	29	45	.4961	.8726
3-20	.15	1.058	.972	.833	30	58	.5145	.8574
⅙	.1667	1.070	.901	.750	33	41	.5547	.8320
1-5	.2	1.098	.800	.625	38	40	.6247	.7808
	·225	1.122	.747	.555	42	0	.6690	.7433
¼	.25	1.149	.707	.500	45	00	.7071	.7071
.3	.3	1.205	.651	.417	50	12	.7682	.6401
⅓	.3333	1.247	.625	.375	53	8	.8000	.6000
.4	.4	1.332	.589	.312	58	2	.8483	.5294
9-20	.45	1.408	.572	.278	60	57	.8742	.4855
½	.5	1.480	.559	.250	63	26	.8944	.4472

These calculations are based on the assumption that the curve formed by the main chains is a parabola; which is not strictly correct. In a finished bridge, the curve is between a parabola and a catenary; and is not susceptible of a rigorous determination. **It may save some trouble in making the drawings** of a suspension bridge, to remember that when the deflection does not exceed about $\frac{1}{10}$ of the span, a segment of a circle may be used instead of the true curve; inasmuch as the two then coincide very closely; and the more so as the deflection becomes less than $\frac{1}{10}$. The dimensions taken from the drawing of a segment will answer all the purposes of estimating the quantities of materials.

The deflection usually adopted by engineers for great spans is about $\frac{1}{12}$ to $\frac{1}{15}$ the span. As much as $\frac{1}{10}$ is generally confined to small spans. The bridge will be stronger, or will require less area of cable, if the deflection is greater; but it then undulates more readily; and as undulations tend to destroy the bridge by loosening the joints, and by increasing the momentum, they must be specially guarded against as much as possible. The usual mode of doing this is by trussing the hand-railing; which with this view may be made higher, and of stouter timbers than would otherwise be necessary. In large spans, indeed, it may be supplanted by regular bridge-trusses, sufficiently high to be braced together overhead, as in the Niagara Railroad bridge, where the trusses are 18 ft high; supporting a single-track railroad on top; and a common roadway of 19 ft clear width, below.*

---

* The writer believes himself to have been the first person to suggest the addition of very deep trusses braced together transversely, for large suspension bridges. Early in 1851, he designed such a bridge, with four spans of 1000 ft each; and two of 500; with wire cables; and trusses 20 ft high. It was intended for crossing the Delaware at Market Street. Philada. It was publicly exhibited for several months at the Franklin Institute, and at the Merchants' Exchange; and was finally stolen from the hall of the latter. Mr Roebling's Niagara bridge, of 800 ft span, with trusses 18 ft high, was not commenced until the latter part of 1852; or about 18 months after mine had been publicly exhibited.

Another very important aid is found in deep longitudinal floor timbers, firmly united where their ends meet each other. These assist by distributing among several suspender-rods, and by that means along a considerable length of main cable, the weight of heavy passing loads; and thus prevent the undue undulation that would take place if the load were concentrated upon only two opposite suspenders. With this view, the wooden stringers under the rails on the Niagara bridge are made virtually 4 ft deep. The same principle is evidently good for ordinary trussed bridges.

Another mode of relieving the main cables is by means of *cable-stays;* which are bars of iron, or wire ropes, extending like *c y*, Fig 1, from the saddles at the points of suspension *c*, *d*, obliquely down to the floor, or to some part of the truss. In the Niagara bridge are 64 such stays, of wire ropes of 1¼ inch diam; the longest of which reach more than quarter way across the span from each tower. They transfer much of the strain of the wt of the bridge and its load directly to the saddles at the top of the towers; thereby relieving every part of the main cable, and diminishing undulation. They end at *c* and *d*, where they are attached, not to the cables, but to the saddles. They of course do not relieve the *back stays*.

The greatest danger arises from the action of strong winds striking below the floor, and either lifting the whole platform, and letting it fall suddenly; or imparting to it violent wavelike undulations. The bridge of 1010 ft span across the Ohio at Wheeling, by Charles Ellet, Jr, was destroyed in this manner. It is said to have undulated 20 ft vertically before giving way. It had no effective guards against undulation; for although its hand-railing was trussed, it was too low and slight to be of much service in so great a span. Many other bridges have been either destroyed or injured in the same way. When the height of the roadway above the water admits of it, the precaution may be adopted of tie-rods, or anchor rods, under the floor at different points along the span, and carried from thence, inclining downward, to the abutments, to which they should be very strongly confined. In the Niagara Railroad bridge 56 such ties, made of wire ropes 1¼ inch diam, extend diagonally from the bottom of the bridge, to the rocks below. They, however, detract greatly from the dignity of a structure.

Mr Brunel, in some cases, for checking undulations from violent winds striking *beneath* the platform, used also *inverted* or *up-curving* cables under the floor. Their ends were strongly confined to the abuts several ft below the platform; and the cables were connected at intervals, with the platform, so as to hold it down.

Art. 2. The angle *a d g*, or *a c i*, Fig 1, which a tang *d g* or *c i* to the curve at either point of suspension *c* or *d*, forms with the *hor* line *c d* or chord, is called **the angle of direction of the main chains,** or cables, at those points. Frequently the ends *c h*, and *d r*, of the chains, called the **backstays,** are carried away from the suspension piers in straight lines; in which case the angles *l d r*, *e c h*, formed between the hor line *e l* and the chain itself, become the angles of direction of the backstays.

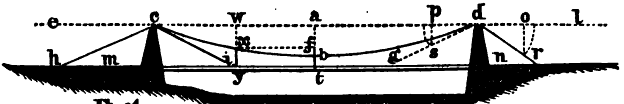


Fig 1

Sine of angle of direction $a\,d\,g$ $= \dfrac{\text{Twice the deflection } a\,b}{\sqrt{(\text{twice the deflection})^2 + (\text{Half the chord})^2}}$

NOTE 1. The direction of the tang *d g* or *c i*, can be laid down on a drawing, thus: Continue the line *a b*, making it twice as long as *a b*; then lines drawn from *d* and *c* to its lower end, will be tangs to the parabolic curve at the points of suspension.

NOTE 2. **If the chord *c d* be not hor,** as sometimes is the case, the angle must be measured from a hor line drawn for the purpose at each point of suspension; as the two angles will in that case be unequal, the piers being of unequal heights.

Tension on all the main chains or cables, together, at either one of the piers, *c* or *d*, Fig 1. $= \dfrac{\text{Half the entire suspended weight of the clear span and its load}}{\text{Sine of angle of direction } a\,d\,g}$

$$\text{or} = \frac{\sqrt{(\tfrac{1}{2}\text{ Span})^2 + (2\text{ Defl})^2}}{2\text{ Deflection}} \times \text{Half the entire suspended weight of the clear span and its load.}$$

Tension on all the main chains or cables, together, at the middle, *b*, of the span, Fig 1. $= \dfrac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Cosine of angle of direction } a\,d\,g}{\text{Sine of angle of direction } a\,d\,g}$

$$\text{or} = \frac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Half the span}}{\text{Twice the deflection}}$$

The diff between the tensions at the middle, and at the points of suspension, is so trifling with the proportion of chord and deflection commonly adopted in practice, viz, from about $\frac{1}{10}$ to $\frac{1}{15}$, that it is usually neglected; inasmuch as the saving in the weight of metal would be fully compensated for by the increased labor of manufacture in gradually reducing the dimensions of the chains from the points of suspension toward the middle; and in preparing fittings for parts of many different sizes. The reduction has, however, been made in some large bridges with wrought-iron main chains; but to none with wire cables.

Art. 2A. As it is sometimes convenient to form a rough idea at the moment, of the size of cables required for a bridge, we suggest the following rule for finding approximately the area in sq ins of *solid* iron in the wire required to sustain, with a safety of 3,* the weight of the bridge itself, together with an extraneous load of 1.205 tons per foot run of span; which corresponds to 100 lbs per sq ft of platform of 27 ft *clear available* width. This suffices for a double carriage-way, and two footways. The deflection is assumed at $\frac{1}{12}$ of the span; and the wire to have an ultimate strength of 36 tons per solid square inch,

For spans of 100 ft or more,

RULE. Mult the span in feet, by the square root of the span. Divide the prod by 100. To the quot add the sq rt of the span. Or, as a formula,

$$\textit{Area of SOLID metal of all the cables; in square ins; for spans over 100 feet} = \frac{\textit{span} \times \textit{sq rt of span}}{100} + \textit{sq rt of span.}$$

For spans less than 100 feet, proportion the area to that at 100 ft.

If a defl of $\frac{1}{10}$ is adopted instead of $\frac{1}{12}$, the area of the cables may be reduced very nearly $\frac{1}{7}$ part.

The following table is drawn up from this rule. The 3d col gives the united areas of all the actual wire cables, when made up, including voids. (Original.)

Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.
	Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.
1000	348	446	400	100	128	150	30.6	39.2
900	300	385	350	84	108	125	25.2	32.3
800	254	326	300	69	89	100	20	25.6
700	212	272	250	55	71	75	15	19.2
600	171	219	200	42	54	50	10	12.8
500	134	172	175	36.4	46.7	25	5	6.4

Having the areas of all the actual cables, we can readily find their diam. Thus, suppose with a span of 500 ft, we intend to use four cables. Then the area of each of them will be $\frac{172}{4} = 43$ sq ins; and from the table of circles, we see that the corresponding diam is full 7⅜ ins.

The above areas are supposed to allow for the increased wt of a depth of truss, and other additions necessary to secure the bridge from violent winds, and from undue vibrations from passing loads.

When these considerations are neglected, and a less maximum load assumed, the following descriptions of the Wheeling and Freyburg bridges show what reductions are practicable. Weight, sufficiently provided for, is of great service in reducing undulation.

We do not think that diagonal horizontal bracing should, as is usual, be omitted under the floor. It may readily be effected by iron rods.

All the cables need not be at the *sides* of the bridge. One or more of them may be over its axis; especially in a wide bridge. One wide footpath in the center may be used, instead of two narrow ones at the sides.

The platform or roadway should be slightly cambered, or curved upward, to the extent say of about $\frac{1}{100}$ of the span.

---

* The writer must not be understood to advocate a safety of 3 against 100 lbs per sq ft, in addition to the weight of the bridge, in all cases. He believes that limit to be about a sufficient one for a properly designed wire suspension bridge for ordinary travel; but for an important railroad bridge, he would (according to position, exposure, &c) adopt a safety of at least from 4 to 6 against the greatest possible load, added to the wt of the bridge. A train of cars opposes a great surface to the action of side winds; and trains must run during violent storms, as well as during calms; but a large open bridge for common travel is not likely to be densely crowded with people during a severe storm.

Art. 3. Tension on the back-stays, *c h* and *d r*, Fig 1. and strains on the piers, or towers, or pillars. If the angle of direction *a d g* and the angle *l d r*, between the back-stays and the horizontal, are equal to each other, the tension on the back-stays will be equal to that on the main cables at the tops of the piers; and the pressure on the piers will be vertical; but if the two angles are unequal, then these tensions and pressures will depend, to a very important extent, upon the manner in which the chains or cables are fixed to, or laid upon, the tops of the piers.

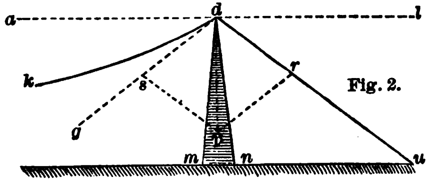


Fig. 2.

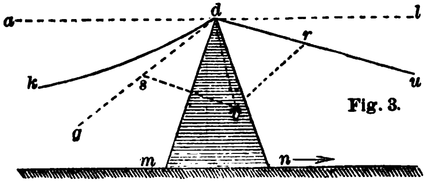


Fig. 3.

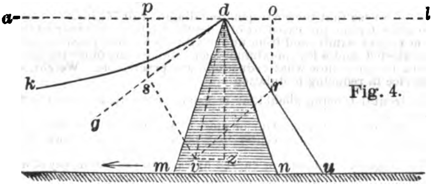


Fig. 4.

Art. 4. In Figs 2, 3 and 4, the piers *d n m* are supposed to be immovable; and the cables *k d u*, passing over them, rest immediately upon horizontal rollers, *which have no other motion than that of revolving about their horizontal axes; the frame to which they are attached being bolted to the top of the pier.* On these rollers the cables slide, when changes of loading or of temperature produce changes in their directions.

In this case the **tension on the back-stays** is equal to that on the main cable. See Funicular Machine.

To find the direction and amount of the **pressure on the pier;** from *d*, Fig 2, 3 or 4, lay off *d s* and *d r*, each equal, by scale, to the tension, in tons, on the main chain at *d*; and from *s* and *r* lay it off to *v*. In other words, draw the parallelogram *d s v r*, and its diagonal *d v*. Then will *d v* give the *direction* and *amount* of the pressure upon the pier.

When, as in Fig 2, the angles *a d g* and *l d u* are *equal*, the pressure *d v* will be vertical, and equal to the *entire* weight of the clear span and its load.

When, as in Figs 3 and 4, the angles *a d g* and *l d u* are *unequal*, the pressure *d v* will not be vertical, but will incline from *d* toward the smaller angle.

When, as in Fig 3, *a d g* exceeds *l d u*, the pressure *d v* will be *less* than the entire weight of the clear span and its load.

When, as in Fig 4, *l d u* exceeds *a d g*, the pressure *d v* will be *greater* than the entire weight of the clear span and its load.

If we suppose symmetrical piers, *d n m*, to be used in each case, the base *m n* of that in Fig 2, may be much narrower than in the other two figs; because, the direction of *d v* being vertical, the pressure has no tendency to overturn the pier. In Fig 2, the masonry of the pier should be laid in the usual horizontal courses, in order that its bed joints may be at right angles to the pressure upon them.

But, in Figs 3 and 4, if the bases were made as narrow as in Fig 2, the lines of direction *d v*, of the pressure, would fall outside of them; and the piers would consequently be in danger of overturning. Also, the stones of the masonry, if laid in horizontal courses, would have a tendency to slide on each other. To prevent this, the beds should be at right angles to *d v*.

In Fig 3, the obliquity of the pressure would tend to slide the base of the pier *outward* as shown by the arrow; but in Fig 4, *inward*. This tendency is produced by the *horizontal* component of the force *d v*. The amount of this may be found thus, in either fig: From *d* downward draw a vert line as in Fig 4; and from *v* a hor one, meeting it in *z*, then *v z*, measured by the same scale of tons as before, will give this horizontal force, and *d z* will give the vertical component of the pressure *d v*. The effect upon the pier, of the one pressure *d v* is precisely the same as would be produced upon it by one vertical force equal to *d z* and a horizontal one equal to *v z* acting at the same time, as explained under Composition and Resolution of Forces.

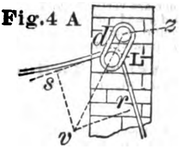
If, in either fig, we draw the vert lines *s p* and *r o*, see Fig 4, then *d o*, meas'd by the foregoing scale, will give the tons of horizontal pull, and *r o* the vertical pressure, produced on the pier by the back-stay; and *p d* and *p s* will, in like manner, give the corresponding forces produced by the main chain. If we add together *r o* and *p s* they will be found to be equal to *d z*, and if we subtract *d o* from *p d*, their difference will equal *v z*. It is this difference only that tends to slide, or to upset, the pier; the other portions of *d o* and *p d* neutralizing each other in that respect.

The foregoing strains may all be *calculated*, thus:

Horizontal pull inward by the main chain = Tension × Cosine of *a d g*
" " **outward by the back-stay** = Tension × Cosine of *l d u*.
Vertical pressure by main chain = Tension × Sine of *a d g*.
" " " **back-stay** = Tension × Sine of *l d u*.

Art. 5. If the cables pass freely over a loose pin, *d*, Fig 4 A, supported by a link L, hanging from the fixed pin *z*, and capable of moving freely about both of its pins; the tension in the back-stay will, as before, be equal to that in the main cable; and the direction and amount of the strain on the piers will be found in the same way as for Figs 2, 3 and 4; namely: lay off *d s* and *d r*, each equal to the tension, and draw the parallelogram *d s v r*. Then will *d v* give the amount and direction of the strain on the piers. This last will, of course, be transmitted through the pins and the link. The amount of tension on the link will be given by the length of *d v*; and the link (being free to move) will be in line with this tension. The shearing strain on each pin is also given by *d v*.

Fig.4 A



Art. 6. But if the ends of the cable and back-stay, Figs 4 B, 4 C and 4 D, at the top of the pier, be *made fast* to a truck or wagon which is supported by rollers on a smooth platform on top of the pier, the axles of the rollers being fixed in the truck; then the strain on the back-stay will not be the same as that on the cable, unless the angles *a d g* and *l d u* are equal, as in Fig 4 B.

If *a d g* exceeds *l d' u*, as in Fig 4 C, the strain on the back-stay will be less than that on the cable, and vice versa (Fig 4 D).

But, in either case, if the top of the pier is horizontal, as is usually the case, the horizontal components of the strains on the cables and on the back-stays, will be equal, and will thus counteract each other, and there will consequently be no horizontal or oblique strain on the pier. That is, the strain on the pier will be vertical.

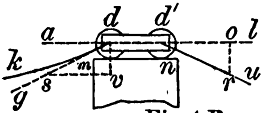


Fig.4 B

To find the amount of the tension on the back-stay, and of the pressure on the pier; on *dg* in either Fig. 4 B, 4 C or 4 D, lay off *d s*, equal, by scale, to the tension on the cable at *d*. Draw *d v* perpendicular to the surface *m n* on which the rollers rest. We assume that *m n* is horizontal, as is generally, *but not necessarily*, the case; and *d v*, therefore, vertical. Draw *s v* horizontal, or parallel to *m n*.*

Then *s v* will give the horizontal pull of the main cable on the wagon, and *d v* will give the vertical pressure of the wheel *d* on the tower (to which that of the wheel *d'* has yet to be added). From *d'* lay off *d' o* horizontal, and equal to *s v*; and draw *r o* vertically. Then *d' r* will give the amount of the pull on the back-stay; and *r o* will give the vertical pressure of the wheel *d'* on the pier; which must be added to *d v* for the *total* vertical pressure.

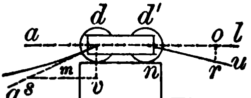


Fig.4 C

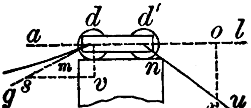


Fig.4 D

Or the various strains may be *calculated*, thus:

$$\left.\begin{array}{c}\textbf{Horizontal pull}\\ \textbf{\textit{s v} or \textit{d'o} at the}\\ \textbf{top of the pier}\end{array}\right\} = \begin{array}{c}\text{Horizontal pull at}\\ \text{middle of span}\end{array} = \begin{array}{c}\text{Tension } d\,s \text{ in}\\ \text{cable at } d\end{array} \times \text{Cosine of } a\,d\,g.$$

$$\left.\begin{array}{c}\textbf{Strain \textit{d'r} in back-}\\ \textbf{stay}\end{array}\right\} = \begin{array}{c}\text{Horizontal pull } s\,v \text{ or } d'o \text{ at top of}\\ \text{pier, or at middle of span}\end{array} \div \text{Cosine of } l\,d'\,u.$$

$$\left.\begin{array}{l}\textbf{Pres on pier,} \text{ perp}\\ \text{to surf on which the rol-}\\ \text{lers rest}\end{array}\right\} = d\,v + r\,o = \left(\begin{array}{c}\text{Tension } d\,s\\ \text{on main}\\ \text{cable at } d\end{array} \times \begin{array}{c}\text{Sine of}\\ a\,d\,g\end{array}\right) + \left(\begin{array}{c}\text{Tension } d'\,r\\ \text{on back-stay}\end{array} \times \begin{array}{c}\text{Sine of}\\ l\,d'\,u\end{array}\right)$$

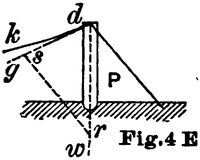


Fig.4 E

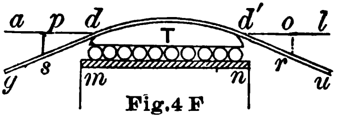


Fig.4 F

Art. 7. When, as is sometimes the case in light bridges, the piers are posts, P Fig 4 E, of wood or iron, hinged at the bottom, and having the cables and back-stays firmly fixed to their tops; from *d* draw *d s*, equal, by scale, to the tension on the main cable at *d*; and *d w* toward the foot of the post. From *s* draw *s r* parallel to the back-stay, and meeting *d w* in *r*. Then will *s r* give the strain in the back-stay, and *d r* will give the amount and direction of the pressure upon the post.

Art. 8. As in the Niagara bridge, the cables often merely rest upon movable trucks, or saddles, T Fig 4 F, curved on top to avoid sudden bends in the cables, and resting upon loose rollers which lie upon a thick horizontal iron plate bolted to the top of the pier, and are free to move horizontally. In such cases the angles *a d g* and *l d' u* are made equal; so that the pulls

---

* The lines *a l* and *s v* must be drawn *parallel to the surface* m n *on which the wagon rests*, whether said surface be horizontal or inclined.

d s and *d' r* are equal, as are also their horizontal components *p d* and *d' o* · and the pressures on the pier are vertical; and if changes of temperature or of loading produce slight changes in the angles *a d g* and *t d' u*, the truck will (by reason of the inequality thus brought about between the horizontal components) move far enough to restore the equality between the angles, and between the horizontal components, and consequently the pressure upon the pier will at all times be vertical.

Art. 9. To find, approximately, the length of a main chain *e b d*; **Fig. 1; having the span** *c d*, **and the middle defl** *a b*. **See preceding table, Art 1.**

$$\textit{Half length of main chain} = \sqrt{1\tfrac{1}{3}\,(\text{defl}^2) + (\tfrac{1}{2}\,\text{chord})^2}.$$

In Menai bridge the chord *c d* is 579.874 ft; and the defl is 43 ft.

According to the above formula, the entire length is 588.3 feet. By actual measurement the chain is precisely 590 feet. The approximate rule below gives 589.764 ft.

Note. The lengths obtained by this rule are only approximate, because the calculation is based upon the supposition that the chains form a parabolic curve; whereas, in fact, the curve of a finished bridge is neither precisely a parabola, nor a catenary, but intermediate of the two.

The following simple rule by the writer is quite as approximate as the foregoing tedious one, when, as is generally the case, the defl is not greater than $\frac{1}{12}$ of the chord, or span.

Length of main chain when defl does not exceed one-twelfth of the span = chord + .23 defl.

Art. 10. To find, approximately, the length of the vert suspending rods ***x y*, &c, Fig 1; assuming the curve to be a parabola.**

Let *x*, Fig 1, be any point whatever in the curve; and let *x w* be drawn perp to the chord *c d*; and *x f* perp to *a b*; then in any parabola, as $ac^2 : aw^2 :: ab : bf$. And *b f* thus found, added to *b t*, (which is supposed to be already known, being the length decided on for the middle suspending rod,) gives *x y*, the length of rod reqd at the point *x*; and so at any other point.

If *b f* thus found be taken from the middle deflection *a b*, it leaves *w x*; and thus any deflection *w x* of the main chain or cable, may be found when we know its hor dist, *a w*, from the center, *a*, of the span.

In the foregoing rule, the floor of the bridge is supposed to be straight; but generally it is raised toward the center; and in that case, the rods must first be calculated as if the floor were straight, and the requisite deductions be made afterward. When it rises in two straight lines meeting in the center, the method of doing this is obvious. When an arc of a circle is used, its ordinates may be calculated and deducted from the lengths obtained by this rule. Or, having drawn the curve by the rule for drawing a parabola, the dimensions can be approximated to by a scale. The adjustments to the *precise* lengths must be made during the actual construction of the bridge, by means of nuts on their lower screw-ends. The rods require, therefore, only to be made *long enough* at first.

The towers, piers, or pillars, which uphold the chains or cables, admit of an endless variety in design. According to circumstances, they may consist each of a single vertical piece of timber, or a pillar of cast or wrought iron; or of two or more such, placed obliquely, either with or without connecting pieces; like the bents of a trestle. Or they may be made (with any degree of ornamentation) of cast-iron plates; as in iron house-fronts. Or they may be of masonry, brick, or concrete; or of any of these combined.

Each of the suspending-rods, through which the floor of the bridge is upheld by the main chains, requires merely strength sufficient to support safely the greatest load that can come upon the interval between it and half-way to the nearest rod on each side of it; including the wt of the platform, &c, along the same interval.

In anchoring the backstays into the ground, it is necessary to secure for them a sufficiently safe resistance against a pull equal to the strain, upon the backstay.

As to the anchorage of the cables below the surface of the ground, natural rock of firm character is the most favorable material that can present itself. When it is not present, serious expense in masonry must be incurred in large spans, in order to secure the necessary weight to resist the pull of the cables. Our Figs 4½ give ideas of the modes most frequently adopted. For a very small bridge, such as a short foot-bridge, for instance, the backstays may simply be anchored to large stones, *t*, Fig A, buried to a sufficient depth. Or, if the pull is too great for so simple a precaution, the block of masonry, *m m*, may be added, enclosing the backstay. A close covering of the mortar or cement of the masonry has a protecting effect upon the iron.

To avoid the necessity for extending the backstays to so great a dist under ground, they are usually curved near where they descend below the surface, as shown at B, D, and E; so as sooner to reach the reqd depth. This curving, however, gives rise to a new strain, in the direction shown by the arrows in Figs B and D. The nature of this strain, and the mode of finding its amount, (knowing the pull on the backstay,) are very simple; and fully explained under the head of Funicular Machine. The masonry must be disposed with reference to resisting this strain, as well as that of the direct pull of the backstay. With this view, the blocks of stone on which the bend rests should be laid in the position shown in Fig D; or by the single block in Fig B. Sometimes the bend is made over a cast-iron chair or standard, as at *x*, Fig F, firmly bolted to the masonry.

Fig E shows the arrangement at the Niagara railway bridge of 821⅓ ft span. The *wire* backstays end at *c c*; and from there down to their anchors, they consist of heavy chains; each link of which is composed of (alternately) 7 or 8 parallel bars of flat iron, with eye ends, through which pass bolts Each of the 7 bars of each link is 1.4 ins thick, by 7 ins wide, near the lowest part of the chain; but they gradually increase from thence upward, until at *c*, *c*, where they unite with the wire cable, the sectional area of each *link* is 96 sq ins. These chain backstays pass in a curve through the massive approach walls. (28 ft high,) and descend vertically down shafts *s*, *s*, 25 ft deep in the solid rock. Here they pass through the cast-iron anchor-plates, to which they are confined below by a bolt 3½ ins diam. The anchor-plates are 6½ feet square, and 2½ ins thick; except for a space of about 20 ins by 26 ins, at the center where the chains pass through, where they are 1

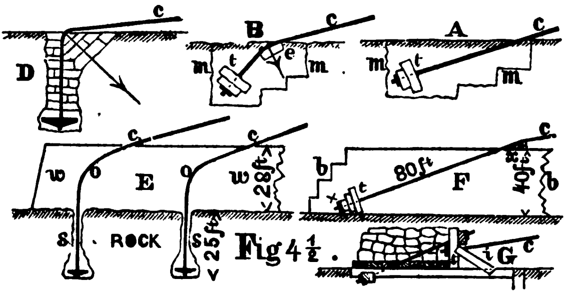


Fig 4½.

foot thick. Through this thick part is a separate opening for each bar composing the lowest link. From this part also radiate to the outer edges of the lower face of the plate, eight ribs, 2½ ins thick. The shafts *s*, *s*, have rough sides, as they were blasted; and average 8 ft by 7 ft across; except at the bottom, where they are 8 ft square. They are completely filled with cement masonry, with dressed beds, well in contact with the sides of the shafts; and thoroughly grouted, thus tightly enveloping the chains at every point; as does also the masonry of the approach wall *w w*; which extends 28 ft above ground; and is 6 ft thick at top, and 10½ ft thick at its base on the natural rock.

D, Figs 4½, shows a mode that may be used in most cases, for bridges of any span. The depth and the area of transverse section of the shaft, and consequently the quantity of masonry in it, will depend chiefly upon whether it is sunk through rock, or through earth. If through firm rock, then if its sides be made irregular, and the masonry made to fit securely into the irregularities, much reliance may be placed upon it to assist the weight of the masonry in resisting the pull on the backstays. Earth also assists materially in this respect.

F is the arrangement in the Chelsea bridge of 333 feet span, across the Thames, at London; Thos. Page, eng. The space from one wall *b b*, to the opposite one, is 45 feet; and is built up solid with brickwork and concrete; except a passage-way 4 ft wide, and 5 ft high, along the backstay; and a small chamber behind the anchor-plates. It rests chiefly on piles.

The arrangement by Mr Brunel, in the Charing Cross bridge, London,* is very similar. In it also the entire abutment rests on piles; and is 40 ft high, 80 ft thick, and solid, except a narrow passage-way along the chains. The backstays extend into it 60 ft. Span 676 feet. Defl 50 feet.

G is intended merely as a general hint, which, variously modified, may find its application in the case of a small temporary, or even permanent bridge; for the number of pieces, *t*, *t*, &c, may be increased to any necessary extent; and they may be made of iron or stone, instead of wood.

In order that the backstays may be accessible, they are frequently carried through openings left in the masonry for the purpose. Thus, the masses, *m m*, of masonry, at A and B, Figs 4½, instead of being made solid, may consist of two parallel walls, between which the backstay may pass; and the anchor-stones, or anchor-plates, will extend across the space between the walls, and have their bearings against the ends of the walls. In D, E, and F, the cable may be supposed either to be tightly surrounded by the masonry and grouted to it, or else to be surrounded by a cylindrical passage-way like a culvert, so as to be at all times accessible.

Soft friable stone must be carefully excluded from such parts of the anchorage as are most directly opposed to the pull of the backstays.

If blocks of stone large enough for securing good bond are not procurable, heavy T-rails, bars of iron, or I-beams, may be advantageously introduced for that purpose.

The masses must be founded at such a depth as not to slide by the yielding of the earth in front of them.

For safety, it is well to disregard the effect of friction in diminishing the tension on the backstay, and to regard that tension as continuing uniform throughout the backstay to its end, even when the backstay is curved and imbedded in the masonry, as at E, Figs 4½.

The side parapets should be high and stout, so as to act as stiffening trusses, and should not be restricted to service as mere hand-rails or guards. As a rule of thumb their depth may be made = $\frac{1}{2}\sqrt{\text{span}}$, provided the depth be not less than that required for a hand-rail. The parapets should be stoutly constructed, with special attention to the strength of their joints, for these are exposed, by the undulations and lateral motions of the bridge, to violent deranging forces in all directions.

---

* Removed to Clifton, England, in 1863, and replaced by an iron truss railway and foot bridge.

RIVETS AND RIVETING.

The **weights** in the **following table** of course include the head; but the **lengths,** as usual, are taken "under the head;" or are those of the shanks only. In practice, discrepancies of 5 or 6 per ct in wt may be expected.

Length of Shank. Ins.	Diameters of Rivets in inches.							
	⅜	½	⅝	¾	⅞	1	1⅛	1¼
	Weight of 100 Rivets, in pounds.							
½	3.0	8.5	....	....	....	....	....	.....
¾	3.8	9.9	17.3	....	....	....	....	.....
1	4.6	11.2	19.4	25.6	38.9	....	....	.....
¼	5.4	12.6	21.5	28.7	43.1	65.3	91.5	123
½	6.2	13.9	23.7	31.8	47.3	70.7	98.4	133
¾	6.9	15.3	25.8	34.9	51.4	76.2	105	142
2	7.7	16.6	27.9	37.9	55.6	81.6	112	150
¼	8.5	18.0	30.0	41.0	59.8	87.1	119	159
½	9.2	19.4	32.2	44.1	64.0	92.5	126	167
¾	10.0	20.7	34.3	47.1	68.1	98.0	133	176
3	10.8	22.1	36.4	50.2	72.3	103	140	184
¼	11.5	23.5	38.6	53.3	76.5	109	147	193
½	12.3	24.8	40.7	56.4	80.7	114	154	201
¾	13.1	26.2	42.8	59.4	84.8	120	161	210
4	13.8	27.5	45.0	62.5	89.0	125	167	218
¼	14.6	28.9	47.1	65.6	93.2	131	174	227
½	15.4	30.3	49.2	68.6	97.4	136	181	236
¾	16.2	31.6	51.4	71.7	102	142	188	244
5	16.9	33.0	53.5	74.8	106	147	195	253
¼	17.7	34.4	55.6	77.8	110	153	202	261
½	18.4	35.7	57.7	80.9	114	158	209	270
¾	19.2	37.1	59.9	84.0	118	163	216	278
6	20.0	38.5	62.0	87.0	122	169	223	287
½	21.5	41.2	66.3	93.2	131	180	236	304
7	23.0	43 9	70.5	99.3	139	191	250	321
½	24.6	46.6	74.8	106	147	202	264	338
8	26.1	49.4	79.0	112	156	213	278	355
9	29.2	54.8	87.6	124	173	234	306	389
10	32.2	60.3	96.1	136	189	256	333	423
11	35.3	65.7	105	148	206	278	361	457
12	38.4	71.2	113	161	223	300	388	491

The diam of rivets for bridge work is from ½ to 1 inch; usually ⅝ to ¾; and for plates more than .5 inch thick, it is about 1.5 times the thickness; and for thinner ones about twice; but these proportions are not closely adhered to. **The common form of rivets** as sold is shown at R, Figs 3, a **head** and the **shank** in one piece; and S shows the same when after being heated white hot it is inserted into its hole, and a second head (conical) formed on it by rapid hand-riveting as it cools. **When longer than about 6 ins** they are cooled near the middle before being inserted, lest their contraction in cooling should split off their heads. The hemispherical heads often seen, called **snap heads,** are formed by a machine. **The two heads alone require** about as much iron as 3 diams length of shank. **Length of a head** = about 1 diam of shank; and its **width** about 2 diams of shank.

Riveting of Steam and Water Tight Joints.

Joints for boilers and water-tight cisterns are usually proportioned about as per the following table by Fairbairn; and are made as shown either by Fig 1, or Fig 2. Fig 1 is called a **single-riveted,** and Fig 2 a **double-riveted lap-joint.** The dist *a a*, or *c c*, is the lap.

Mr Fairbairn considers the strength of the single-riveted lap-joint to be about .56; and that of the double-riveted, about .7 that of **one** of the full unholed

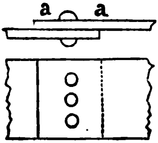


Fig 1.

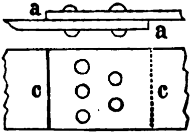


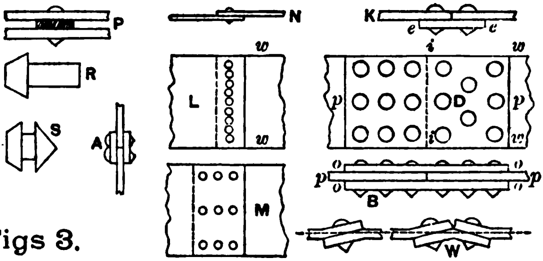
Fig 2.

plates, when both joints are proportioned as in his following table. But some later experimenters consider about .5 and .6 as nearer the correct average. Experiments on the subject are quite conflicting; and it is plain that no one set of proportions can precisely suit all the different qualities of plate and rivet iron. With fair qualities of both, there is every reason to rely upon .5 and .6 (or about one-seventh part less than Fairbairn's assumption) as safe for practice. These proportions include **friction** (Art 4), without which they would be *about* .4 and .5.

Fairbairn's table for proportioning the riveting for steam and water-tight lap-joints.

Thickness of each plate.	Diameter of rivets.	Length of shank before driving.	From center to center of rivets.	Lap in single riveting.	Lap in double riveting.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
3-16	3/8	7/8	1 1/4	1 1/4	2 1/16
1/4	1/2	1 1/8	1 1/2	1 1/2	2 1/2
5-16	5/8	1 3/8	1 5/8	1 7/8	3 1/16
3/8	3/4	1 5/8	1 3/4	2	3 5/8
1/2	13-16	2 1/4	2	2 1/4	3 3/4
5/8	15-16	2 3/4	2 1/2	2 3/4	4 5/8
3/4	1 1/8	3 1/4	3	3 1/4	5 1/2

Riveting of iron girders, bridges, &c.



Figs 3.

Art. 1. The subject of riveting is abstruse, and involved in much uncertainty; and experimental results are very discrepant. We here propose merely to confine ourselves to what is considered the best joint; and for safety we shall omit friction; see Art 4. In girder and bridge work the lap-joints above described are seldom used. Instead of them, the plates *p*, Figs 3, to be joined, are butted up square against each other, thus forming a **butt-joint,** *i i*, Fig D; and are united by either a single **covering-plate, cover, wrapper, fish-plate,** or **welt** *e e*, Fig K; or the best of all by two of them, as at A, or *o o*, *o o*, Fig B. In what follows, the term *plate* never includes the *covers.* The single cover, like the lap-joint, allows both plates and cover to bend under a strong pull, somewhat as at W, thus weakening them materially; whereas the double cover *o o*, *o o*, Fig B, keeps the pull directly along the axis of the plates, thus avoiding this bending tendency. It also brings the rivets into double shear, thus doubling their strength. When there is but one cover, it should be at least as thick as a plate; and when there are two, experience shows that each had better be about *two-thirds* as thick as a plate, although theory requires each to be but *half* as thick as a plate.

The length w w of covers across the joint is equal to that of the joint.

Butts require twice as many rivets as laps, because in the lap each rivet passes through both the joined plates; and in the butt through only one.

The rivets and plate on one side only (right or left) of the joint-line *i i* of any properly proportioned **butt-joint** D, represent the full strength of the joint, inasmuch as those on one side pull in one direction, against those on the other side, which pull in the opposite direction. Therefore in designing such joints we need keep in mind only those on one side, as is done in what follows. Thus a single, double, or triple-riveted **butt-joint** D implies one, two, or three rows of rivets **on each side** of the joint-line *i i*, and parallel to it. In a properly proportioned *lap* the strength is as **all** the rivets, because one-half of them do not pull against the other half, but one end of **every** rivet pulls in one direction, and its other end in the opposite direction.

The net iron, net plate, or net joint, is that which is left **between** the rivet holes, and **outside** of the two outer ones, all on a straight line drawn through the *centers* of the holes of *one* row. Its width and area are called the net ones of the joint. That between *other* rows does not increase the strength.

In Figs 3, N, and K, the rivets are in **single shear,** while those in A and B are in **double shear.**

Art. 2. Bridge-joints are not required to be steam or water-tight like those of boilers or cisterns; and, therefore, by increasing the breadth of the overlap, or the length of the covers, the rivets may be placed in several rows behind each other, as the 3 rows of 3 rivets each in M and D, instead of only one row of 9 rivets, as in L. By this means, without losing any of the strength of the 9 rivets, or of the **net** iron, we may narrow the width of the plate to an extent equal to the combined diams (6 in this case) of the holes thus dispensed with in the one row. Moreover, by using more than one row we lessen the weakening effect shown at W. This mode of placing the rivets directly behind each other in several rows, as at M, and at the left-hand half of Fig D, constitutes Mr Fairbairn's **chain riveting;** but the joint will be somewhat stronger if the rivets are placed in **zigzaging** order, as in the right-hand half of Fig D.

The dist apart of the rows from cen to cen should not be *less* than 2 diams. It is questionable to what extent this increase in the number of rows may be carried without an appreciable loss of strength in the rivets consequent upon the impossibility of quite equalizing the strains on the separate rows. But it is probable that if we do not exceed 2 or 3 rows in laps, or the same number *on each side* of the joint-line in butts, we may in practice assume that each row, and each rivet, is nearly equally strained.

Rivet-holes are usually of about one-sixteenth inch greater diam than the original rivet, so as to allow the hot rivet to be easily inserted. The subsequent hammering swells the diam of the rivet until it fills the hole. We may either take this increased diam of rivet into consideration, as we have done, in calculating its shearing and crippling strength, as explained farther on, or with reference to increased safety we may omit it. **Drilled rivet-holes** are said to be better than punched ones, as the drilling does not injure the iron around them; but on the other hand their sharper edges are said to shear the rivets more readily. Hence, such edges are sometimes reamed off. Both these points are, however, disputed; and both modes are in common use.

The dist from the edge of a hole to the end of a plate or cover should not be *less* than about 1.2 diams, to prevent the rivets from tearing out the end of the plate; nor nearer the side edge of a plate than half the clear dist between two holes as given by the Rule in Art 5. The first is rather more than Fairbairn directs.

Rivet holes weaken the net iron left between them, not only by the loss of the part cut out, but either by disturbing the iron around them, or perhaps by changing the shape of the net line of fracture, which may not then resist tension as well as while it was a continuous straight line. Some deny both cause and effect entirely, each party basing its opinion on experiments. But the mass of evidence seems to the writer to show that the net iron loses on an average about one-seventh of the strength due to the net width. With a view to safety, which we consider to be of paramount importance, we shall in what follows assume (until the question is definitely settled) that there **is** such a loss of strength in the net iron.

Riveted joints for resisting compression should depend, not as might be supposed upon their butting ends, but upon either the shearing or the crippling strength of the rivets; for contraction or bad work may throw the pressure on the rivets. **Machine riveting** is somewhat stronger than that done (as is assumed in our examples) by hand. **The thickness of plates** used in girders, tubular bridges, &c, is usually .25 to .5 inch; with thicker ones up to 1 inch sparingly in large ones. **A packing piece,** as the shaded piece in P, is one inserted between two plates to prevent their being bent or drawn together by the rivets.

Art. 3. A riveted joint may yield in three ways after being properly proportioned, namely, by the shearing of its rivets; or by the pulling apart of the net plate between the rivet holes; or by the **crippling** (a kind of compression, mashing, or crumpling) of the plates by the rivets when the two are too forcibly pulled against each other. It also compresses the rivets themselves transversely, **at a less strain than the shearing one;** and this partial yielding of both plates and rivets allows the joint to **stretch,** and may thus produce injurious unlooked-for strains in other parts of a structure, considerably before there is any danger of actual fracture. Or in steam and water joints it may cause leaks, without farther inconvenience, or danger. For a long time this crippling had entirely escaped notice, and it was supposed that the only important point in designing a riveted joint was that the tensile strength of the net plate, and the shearing strength of the rivets should be equal to each other.

The crippling strength of a joint is as the number of rivets, in a lap, or the number on one side of the joint-line in a butt × diam × thickness of joined plate. This product gives the crippled area of the joint. We shall here call the diam × thickness of plate, the **crippling area** of a rivet. If there are 2 or more plates (not covers) on top of each other at one joint, their united thickness is used for finding the crippling area. **The ultimate crippling unit,** by which the above product is to be multiplied for the actual ultimate crippling strength of the joint, may be safely taken at about 60000 lbs, or 26.8 tons, per sq inch.

The diam of a rivet in ins to resist safely a given single-shearing force is found thus: Mult the shearing force by the coef of safety, that is by the number, 3, 4, or 6, &c, denoting the required degree of safety. Call the product *g*. Mult the ultimate shearing strength per sq inch of the rivet-iron, by the decimal .7854. Call the product *b*. Divide *g* by *b*. Take the sq rt of the quotient. The shearing force and the shearing strength must *both* be in either lbs or tons.

Or by a formula,

$$\textbf{Diam in ins} = \sqrt{\frac{\text{Shearing force} \times \text{coef of safety}}{\text{Ult shearing strength per sq inch} \times .7854}}$$

If the rivet is to be double-sheared, first mult only half the shearing force by the coef of safety. Then proceed as before.

Or, near enough for practice, mult the diam in single shear by the decimal .7.

The ultimate shearing unit for average rivet-iron may be taken at about 45000 lbs, or 20.1 tons per sq inch of circular sheared section.

Table of ultimate single shearing strength of rivets.

(market sizes), in single shear; at 45000 lbs or 20.1 tons per sq inch.

This table is not to be used when as in our "Example," Art 5, the **crippling** strength of the rivet governs the strength of the joint.

If the rivet is in double shear it will have twice the strength in the table.

For the diam in double shear to equal the strength in the table, mult the diam in the table by the decimal .7; near enough for practice; strictly, .707.

Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam. Ins.	Diam. Ins.	lbs.	Tons.
⅛	.125	552	.246		.562	11188	4.99	1	1.000	35343	15.8
	.187	1242	.554	⅝	.625	13806	6.16		1.062	39899	17.8
¼	.250	2209	.986		.687	16705	7.46	1⅛	1.125	44731	20.0
	.312	3452	1.54	¾	.750	19880	8.88		1.187	49838	22.2
⅜	.375	4970	2.22		.812	23332	10.4	1¼	1.250	55224	24.6
	.437	6765	3.02	⅞	.875	27060	12.1		1.312	60885	27.2
½	.500	8836	3 94		.937	31064	13.9	1⅜	1.375	66820	29.8

The tensile strength of a properly proportioned joint is equally as either the sectional area of the **net** plate (not covers) across the centers of only **one** row of rivets; or as the shearing or the crippling (as the case may be) areas of **all** the rivets in a lap, or of all the rivets **on one side** of the joint-line in a butt. The tensile strength of fair quality of plate iron, before the rivet holes are made, averages about 45000 lbs, or 20.1 tons per sq inch; but we shall for safety assume, as stated in Art 2, that the making of the holes reduces the strength of the **net** iron that is left about one-seventh part, or to 38500 lbs, or 17.2 tons per sq inch.

Rem. Even this is considerably too great for laps, or for butts with one cover, owing to the weakening of the iron in such by the bending shown at W, Figs 3. But we are not speaking of such.

Art. 4. The friction between the plates in a lap, or between the plates and the covers in a butt, produced by their being pressed tightly together by the contraction of the rivets in cooling, adds much to the strength of a joint while new, perhaps as much as 1.5 to 3 tons per sq inch of circ section of all the rivets in a lap, or of all on one side of a single-cover butt; or 3 to 6 tons of all on one side of a double-cover butt. In quiet structures, this friction might continue to exist, either wholly or in part, for an indefinite period; but in bridges, &c, subject to incessant and violent jarring and tremor, it is probably soon diminished, or entirely dissipated. Hence good authorities recommend not to rely on it, and it is, therefore, omitted in what follows.

Art. 5. We now give rules for finding the number of rivets required for a **double cover butt-joint** (the only kind of which we shall treat), and their clear or net distance apart. This dist + one diam is the **pitch** of the rivets, or their dist from center to center. The principle of the rule will be explained further on, at Art 7.

First, select a diam of rivet either equal to or greater than .85 times the thickness of the plate. In practice they are generally 1.5 times for plates ½ inch or more thick; and 2 for thinner than ½ in.

Second, mult the greatest total pull in pounds that can come upon the entire joint by the coef (3, 4, or 6, &c) of safety, and call the product p.

Third, multiply the crippling area of the rivet (that is, its diam × the thickness of plate) by 60000. The prod is the ult crippling strength of a rivet. Call it m.

Fourth, divide p by m. The quotient will be the number of rivets to sustain the given pull with the reqd degree of safety.

Then, **the clear distance apart** will be

$$\frac{\text{Number of rows} \times \text{Diam} \times 60000}{38500}$$

Fifth. The clear dist from either end hole of a row to the side edge of the plate, should be not less than half the clear dist between two rivets in a row.

Example. A double-cover butt-joint in .5 inch thick plate is to bear an actual pull of 33750 lbs, with a safety of 4; or not to break with less than 33750 × 4 = 135000 lbs. How many rivets must it have; and how far apart must they be?

First, Here .85 times the thickness of the plate is .5 × .85 = .425 inch; therefore, our rivets must not be less than .425 inch in diam; but we will take .75 inch diam.

Second, The greatest pull × coef of safety = 33750 × 4 = 135000 lbs = p.

Third, The crippling area of a rivet × 60000 = .75 × .5 × 60000 = 22500 = m.

Fourth, $\frac{p}{m} = \frac{135000}{22500} = 6$ rivets required on each side of the joint-line.

And the **clear space** or net width between them will be, **if the 6 rivets are in one row;**

$$\frac{\text{Diam} \times 60000}{38500} = \frac{45000}{38500} = 1.1688 \text{ ins.}$$

And the **pitch** = net space + diam = 1.1688 + .75 = 1.9188 ins, $= \frac{1.9188}{.75}$ = 2.56 diams.

In practice, to avoid troublesome decimals, we might make the net space 1.2 ins; and the pitch 1.95; but to show farther on the working of the rule, we adhere to the more exact ones.

Fifth, The clear dist from each end hole to the side edge of the plate is half of 1.1688 = .5844 ins.

The entire width of net iron is equal to one clear space × number of rivets = 1.1688 × 6 = 7.0128 ins; and the entire width of plate is equal to one pitch × number of rivets, = 1.9188 × 6 = 11.5128 ins.

The area of cross section of unholed plate is 11.5128 × .5 = 5.7564 sq ins; its tensile strength **before the holes are made** is 5.7564 × 45000 = 259038 lbs. The strength of our joint, omitting friction, is therefore $\frac{135000}{259038}$ = .52 of that of the original unholed plate.

If the 6 rivets are in 2 rows of 3 rivets each, the **clear dist between two rivets** in one row will be twice as great as before, or twice 1.1688 = 2.3376 ins. **Pitch** = 2.3376 + .75 = 3.0876 ins = 3.0876 + .75 = 4.12 diams. **Clear dist from** end hole to side edge of plate = half of 2.3376 = 1.1688. **Entire width of net iron** = 2.3376 × 3 = 7.0128 ins. **Entire width of plate** = 3.0876 × 3 = 9.2628 ins. **Area of cross section of unholed plate**=9.2628×.5=4.6314 sq ins. **Ultimate tensile strength, unholed** = 4.6314 × 45000 = 208413 lbs. **Ult strength of riveted joint,** omitting friction = $\frac{135000}{208413}$ = .65 of that of the unholed plate.

Thus we see that the arrangement with two rows gives the same strength as one row, with a less total width and area of plate. It of course requires *longer covers*.

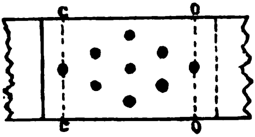
If the 6 rivets are in 3 rows of 2 rivets each, the **area of cross section of the unholed plate** is 4.2565 sq ins. Its **tensile strength**, 191542 lbs. Strength of riveted joint = $\frac{135000}{191542}$ = .7 of that of the unholed plate.

The entire width of net iron (7.0128 ins); its area (7.0128 × .5 = 3.5064 sq ins); and its ultimate tensile strength (3.5064 × 38500 = 135000 lbs), are the same in each case. The last is the required breaking strength of the joint, as in the beginning of our example; and is equal to the combined crippling strength of the six rivets.

Art. 6. The distance apart of the rows, from center to center of rivets, should not be less than two diameters of a rivet-hole.

Rem. 1. With our constants for tension, shearing, and compression, **the rivets will not yield first by shearing** in a double-cover butt (and of course in double shear), except when the diam is either equal to or **less** than .85 of the thickness of the plate, which will rarely happen. At .85 the crippling and shearing strength of a rivet are equal when using our assumed coeffs of crippling, shearing, and tension.

Rem. 2. Our example was chosen to illustrate the rule. It will rarely happen in practice that the rule will give a number of rivets without a fraction; or that may be divided by 2 and by 3 without a remainder. In case of a fraction, it is plainly best to call it a whole rivet; although the joint thereby becomes a trifle stronger than necessary. Or rivets of a slightly diff diam may be used. If the number of rivets comes out say 7 or 9, we may make 2 rows of 3 and 4, or of 4 and 5, &c. Moreover, the width of the plate is frequently fixed beforehand by some requirement of the structure, and we must arrange the rivets to suit, taking care in all cases to maintain the calculated area of net iron in one row, &c.



Rem. 3. We have (as we at first said we should do) confined ourselves to the simple butt-joint with 2 covers, and with the rivets in either 1, or in 2 or more parallel rows on each side of the joint-line; this being the strongest and the one in most common use in engineering structures. Necessity at times calls for less simple arrangements, for which we cannot afford space, and the strength of which is not so readily calculated. These sometimes yield results which appear strange to the uninitiated; thus, this lap-joint breaks across the net iron of one plate, along either *c c* or *o o*, *where there is most of it*, and where, therefore, it might be supposed to be the strongest.

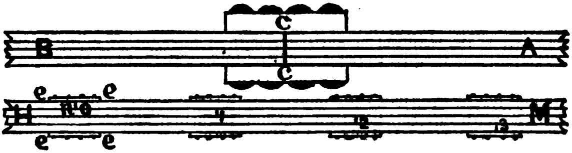
Rem. 4. The following table shows approximately the comparative strengths of the common forms of joints when properly proportioned; varying with quality of sheets, and of rivets:

	With friction.	Without friction.
The original unholed plate	1.00	1.00
Double-riveted butt with two covers	.80	.64
Double-riveted butt with one cover	.65	.52
Single-riveted butt with one cover	.50	.40
Double-riveted lap	.65	.52
Single-riveted lap	.50	.40

Rem. 5. The above tabular strengths for the **lap-joints** will be approximately attained by adopting the following proportions, according as the joint is double- or single-riveted.

	Double riv. zigzag.		Single riv.	
	In thicknesses.	In diams.	In thicknesses.	In diams.
Calling thickness of plate	1.	.6	1.	.6
Then make diam of rivet	1.67	1.0	1.67	1.0
" " breadth of lap	9.0	5.4	5.67	3.4
" " pitch from cen to cen	7.0	4.2	4.5	2.7
" " dist from *end* of plate to *edge* of holes	2.0	1.2	2.0	1.2
" " dist apart of rows from cen to cen	3.33	2.0		

Rem. 6. If two or more plates on top of each other, as the four in A B or M H, are to be jointed together so as to act as one plate of the thickness *c c*, the diams of the rivets, and the thickness of the covers *c c*, *e e* will depend upon whether the junctions of the plates are all in one line with each other as at *c c*, in A B, or whether they break joint with each other as at 0, 1, 2, 3 in M H.



It is plain that the two covers *c c* by means of their connecting rivets convey from A to B, across the joint *c c*, all the strength that partly compensates for the severance of the four plates at that joint; whereas the two covers *e e*, *e e*, and their rivets in like manner convey from *n* of one single plate, to *o* of the adjoining one, across the joint between those two letters, only the strength that partly compensates for the severance of that single plate; and so with the joints at 1, 2, and 3. Therefore the covers *c c*, and their rivets, must be four times as strong as those at any one of the four joints 0, 1, 2, 3. The first, *c c*, are to be regarded as joining two solid plates A and B, each of the fourfold thickness *c c*; and the others as joining two of the single thickness. The covers *c c* will, therefore, each be about two-thirds of the thickness *c c*; and the others each about two-thirds as thick as a single plate. Thus, suppose each of the 4 plates in A B or M H to be ¾ inch thick; making *c c* 3 ins. Then each cover, *c*, is ⅔ of 3 ins, or 2 ins thick; or the two covers, *c c*, together 4 ins, which is thus the effective thickness of the joint, *c c*. But each cover, *e e*, is only ⅔ of ¾ inch, or ½ inch thick; and the effective thickness of joint at either 0, 1, 2, or 3, is that of the 3 unbroken plates plus that of the 2 covers, or (3 × ¾) + (2 × ½) = 3¼ ins.

Art. 7. Principle of the Rule in Art 5. With our constants for shearing (45000 lbs per square inch) and for crippling (60000 lbs per square inch), and with diameter of rivet equal to, or greater than, .85 times the thickness of the plate, as by our rule, the *crippling* strength of a double cover butt joint will be equal to, or less than, its shearing strength. Therefore, to avoid waste of material, either in the plate or in the rivets, we must make

$$\begin{array}{c}\text{Tensile strength of net plate}\\\text{across one row of rivets}\end{array} = \text{Crippling strength of } \textit{all} \text{ the rivets. Or,}$$

$$\begin{array}{c}\text{Total net}\\\text{width of}\\\text{plate}\end{array} \times \begin{array}{c}\text{Thickness}\\\text{of plate}\end{array} \times \begin{array}{c}\text{Tension}\\\text{unit}\end{array} = \begin{array}{c}\text{Crippling area}\\\text{of one rivet}\end{array} \times \begin{array}{c}\text{Crippling}\\\text{unit}\end{array} \times \begin{array}{c}\text{Total number}\\\text{of rivets.}\end{array}$$

Now, by Art 3, the crippling area of a rivet is = diam of rivet × thickness of plate. We take the crippling unit at 60000 lbs; and the tension unit at 38500 lbs. Therefore (transposing) we must make

$$\begin{array}{c}\text{Total net width}\\\text{of plate}\end{array} = \frac{\begin{array}{c}\text{Diam of}\\\text{rivet}\end{array} \times \begin{array}{c}\text{Thickness}\\\text{of plate}\end{array} \times 60000 \times \begin{array}{c}\text{Total number}\\\text{of rivets}\end{array}}{\text{Thickness of plate} \times 38500}$$

By making the clear distance between each end rivet of a row and the side edge of the plate = half the clear distance between two rivets in a row; and calling the sum of the two end distances one space, we have

$$\text{Number of } \textit{spaces} \text{ in a row} = \text{Number of } \textit{rivets} \text{ in a row.}$$

So that

The clear distance between two rivets in a row,

which is

$$= \frac{\text{Total net width of plate}}{\text{Number of } \textit{spaces} \text{ in a row}}$$

is also

$$= \frac{\text{Total net width of plate}}{\text{Number of } \textit{rivets} \text{ in a row}}$$

$$= \frac{\text{Diam of rivet} \times \text{Thickness of plate} \times 60000 \times \text{Total number of rivets}}{\text{Thickness of plate} \times 38500 \times \text{Number of rivets in a row.}}$$

But

$$\frac{\text{Total number of rivets}}{\text{Number of rivets in a row}} = \text{Number of rows.}$$

Therefore, omitting "thickness of plate," common to both numerator and denominator, we have, as in rule in Art 5,

$$\textbf{Clear distance apart} = \frac{\text{Diam of rivet} \times 60000 \times \text{Number of rows}}{38500}$$

But if the diameter of the rivets is less than 0.85 times the thickness of the plates, the *shearing* strength of a double-cover butt joint (with our assumed constants for shearing and crippling) is less than its crippling strength. In such cases, for the clear distance between two rivets in a row, say

$$\textbf{Clear distance} = \frac{\text{Circular area of a rivet} \times \text{Shearing unit}}{\text{Thickness of plate} \times \text{Tension unit}} \times 2$$

Rem. 1. Butt joints in double shear, or with 2 covers, being the only ones here considered, and inasmuch as rivets may always be used with a diam greater than .85 of the thickness of the plate, we may in practice always use the Rule in Art 5 for such joints; and, therefore, we gave it alone.

Rem. 2. When using these rules for other kinds of joint, such as laps, or butts with *single* covers, remember that the rivets in such are in **single** shear; and, therefore, we can use Rule in Art 5 (for crippling) only when the diam is either **1.7 or more** times the thickness of plate. **If less,** use Rule above for **shearing;** all on the assumption that our foregoing coefs of crippling and shearing are used.

But the coef for tension must be changed for each kind of these other joints, to allow for the weakening effects of the bending shown at W, Figs 3, as deduced approximately from experiment. The writer believes that the following tension units will give safe approximate results without friction. **For double-cover butts,** double-riveted, 38500 lbs per sq inch, as adopted above. **For double-riveted** laps, or one-cover butts, 28000. **For single-riveted** laps, or one-cover butts, 24000. But, as before remarked, no great certainty is attainable in riveting.

Rem. 3. A joint may fail by crippling without the facts being known or even suspected, for it does not imply that anything **breaks,** but merely that the joint has **stretched;** and this might not be detected even on a *slight* inspection of it. Still it might, and probably often has sufficed to endanger, and even destroy both bridges and roofs by generating strains where none were provided for.

RAILROAD CURVES.

Definitions.

A circular railroad curve *a b c d*, Fig. 1, is an arc of a circle joining two straight lines, or **tangents,** *e t* and *i z*, in the survey.

The point of curve is the beginning *a* of the curve, or that end of it first reached by the survey in its progress.

The point of tangent is the other end *d* of the curve.

The point of intersection or **apex** is the point *i* where the two tangents *e t* and *i z* intersect.

P. C., P. T. and **P. I.** The stakes driven at the point of curve, point of tangent and point of intersection are marked P. C., P. T. and P. I. respectively, and the points and stakes are commonly referred to by those letters. The point of intersection, however, is not always located.

The apex distance* is the distance *a i* or *d i* measured along a tangent, from either end *a* or *d* of the curve, to the apex *i* or intersection of the two tangents.

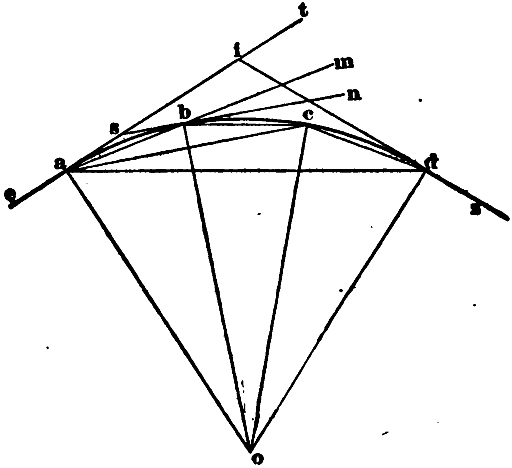


Fig. 1.

A curve may be located by setting up the transit at the point (as *a*) where the curve is to join either tangent, laying off equal angles *i a b*, *b a c*, *c a d*, and measuring off the equal chords (usually 100 feet) *a b*, *b c*, *c d*.† Inasmuch as these equal chords are usually laid off with the full length of a chain or steel tape, we shall call them **chains,** to distinguish them from other chords, such as *a c* or *a d*, etc., which may be drawn to the curve.

The total angle of the curve is the angle *t i z* between the two tangents. It is equal to the central angle *a o d* subtended by the curve.

The degree of curvature is the angle *a o b*, *b o c*, etc. subtended at the center by a chain. It is equal to the **deflection angle**‡ *m b c* formed between any chain *b c* and the extension *b m* of an adjoining chain *a b*, or to the angle *t s n* formed between the tangents *a t* and *b n* which touch the curve at the two ends *a* and *b* of a chain. It is therefore the angle through which the direction of the line *deflects* in that portion of the curve subtended by one chain. The sharper the curve, the greater the deflection angle and the shorter the **radius** *o a*, *o b*, etc.

---

*The apex distance is often, but unfortunately, called the "tangent," and sometimes the "tangent distance."

† But see Sub-chains.

‡ Many writers call *i a b*, *b a c*, etc. the *deflection* angle.

A one-degree, two-degree, three-degree, etc. curve is one whose deflection angle or degree of curvature is 1°, 2°, 3°, etc.

The tangential angle is the angle *i a b*, *b a c*, etc. used in laying off the curve. It is equal to one-half the deflection angle.

The deflection distance. Let any chain, as *a b*, Fig. 2, be extended to *m*, and *b m* made = *a b*. Then the distance *m c* from *m* to the end *c* of the next chain *b c* is called the *deflection distance* of the curve.

The tangential distance. Let any chain, as *b c*, be extended to *v*, and let *c v* be made = *b c*. Also let *c z* = *c v* be laid off upon a *tangent* to the curve at *c*. Then *v z* is called the *tangential distance* of the curve.

By means of the deflection and tangential distances given in the tables, pp. 784–786, a curve may be located without a transit by means of a chain (or 100 feet tape) for measuring *b m*, *c v*, etc., and a rod or tape, etc. for measuring *m c*, *v z*, etc.

An ordinate is any line drawn from a chord to the curve, at right angles to the chord, whether the chord be a "chain" or not.

The middle ordinate is the ordinate *e n*, Fig. 2, at the middle point *e* of a chord *b c*.

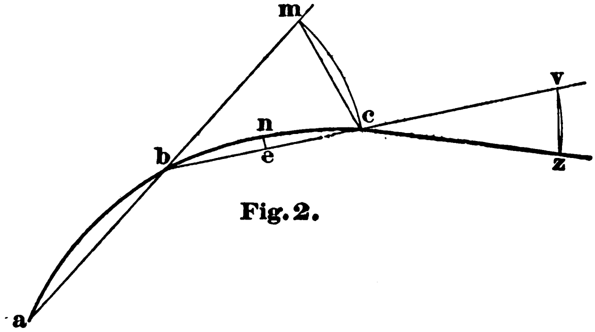


Fig. 2.

Sub-chains, etc. For facility of explanation we have hitherto treated of curves as being composed entirely of full chains; but such curves seldom occur in practice. Usually, after dividing a curve into as many full chains as possible, there is a fraction of a chain, or *sub*-chain, left over. Besides, the chances are that a curve will not begin or end at a full 100-feet station of the survey, but at a point between two such stations, as in Fig. 3; and, inasmuch as it is desirable to carry, throughout the curve, the same numbering of the stations as we have on the tangents, the P. C. and the P. T. are in these cases treated as *fractional* stations.

Thus, in Fig. 3, the P. C., at *a*, is supposed to be 41 feet beyond station 122. We therefore call the P. C., in this case, station 122 + 41, and station 123 thus falls in its proper place, at *b*, 100 feet in advance of station 122, as measured, first along the tangent *x a* 41 feet to the P. C., and then along the sub-chain *a b* of 59 feet.

Similarly, the P. T., in Fig. 3, happens at a point, *d*, on the tangent *d y*, 20.8 feet back from station 125, or 79.2 feet (the length of the sub-chain *c d*) beyond station 124 or *c*. The P. T. therefore becomes station 124 + 79.2.

Owing to the occurrence of the sign + (*plus*) in the number of the P. C. or P. T. of a curve beginning or ending with a sub-chain, such a station is commonly called a "**plus.**"

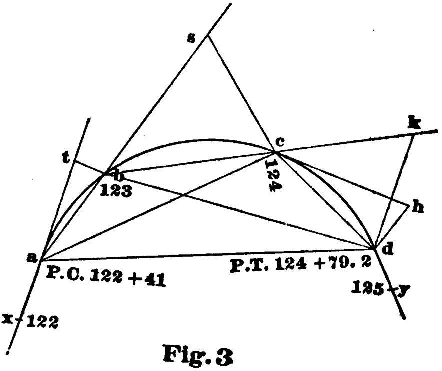


Fig. 3

The sub-tangential angle, Fig. 3, is the angle *b a t* (= *a d b*) or *d c h* (= *d a c*) subtended by a sub-chain; the vertex, *a*, *d* or *c*, of the angle lying in the curve itself. We shall give the names **initial** and **final** sub-tangential angles to the angles *b a t* and *d c h* subtended by the initial and final sub-chains *a b* and *c d* respectively. If *a t* be made = *a b*, and *c h* = *c d*, the chords *t b* and *h d* are called the **initial and final sub-tangential distances** respectively.

Sub-deflection angles, Fig. 3. The **initial** sub-deflection angle is the angle *s b c* formed between the first full chain *b c* and the extension *b s* of the initial sub-chain *a b*. The **final** sub-deflection angle is the angle *k c d* between the final sub-chain *c d* and the extension *c k* of the preceding full chain *b c*. If *b s* be made = *b c*, and *c k* = *c d*, the chords *s c* and *k d* are called the **initial** and **final sub-deflection distances** respectively.

A long chord is a chord, *a c* or *a d*, Figs. 1 and 3, subtending two or more chains or sub-chains.

A simple curve. Figs. 1 to 3, is one in which the radius remains of constant length throughout and in which the curvature is all in one direction. Such a curve is therefore an arc of but one circle.

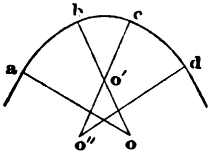


Fig. 4

A compound curve, as *a b c d*, Fig. 4, is one in which the curvature is all in one direction, but which consists of circular arcs described with two or more different radii, as ob, $o'b$, $o''c$, etc. The portions, as ab, bc, cd, described with the different radii, are called the **branches** of the curve.

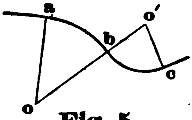


Fig. 5

A reverse curve, abc, Fig. 5, consists of two curves immediately adjoining one another (*i. e.*, without any straight track between them) and curving in *opposite* directions. The radii ob, $o'b$, of the two portions, or branches, of a reverse curve may be of equal or of unequal length, and the total angles, aob, $bo'c$, of the two branches, may be equal or unequal.

Table of Radii, Middle Ordinates, &c, of Curves. Chord 100 feet.
Contains no error as great as 1 in the last figure.

Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid. Ord.	Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid. Ord.
° ′					° ′				
1	343775	.029	.014	.004	1 36	3581	2.793	1.396	.349
2	171887	.058	.029	.007	38	3508	2.851	1.425	.356
3	114592	.087	.043	.011	40	3438	2.909	1.454	.364
4	85944	.116	.058	.014	42	3370	2.967	1.483	.371
5	68755	.145	.072	.018	44	3306	3.025	1.512	.378
6	57296	.175	.087	.022	46	3243	3.084	1.542	.385
7	49111	.204	.102	.025	48	3183	3.142	1.571	.393
8	42972	.233	.116	.029	50	3125	3.200	1.600	.400
9	38197	.262	.131	.033	52	3070	3.257	1.629	.407
10	34377	.291	.145	.036	54	3016	3.316	1.658	.414
11	31252	.320	.160	.040	56	2964	3.374	1.687	.422
12	28648	.349	.174	.043	58	2913	3.433	1.716	.429
13	26444	.378	.189	.047	2	2865	3.490	1.745	.436
14	24555	.407	.203	.051	2	2818	3.549	1.774	.443
15	22918	.436	.218	.054	4	2773	3.606	1.803	.451
16	21486	.465	.232	.058	6	2729	3.664	1.832	.458
17	20222	.494	.247	.062	8	2686	3.723	1.861	.465
18	19098	.524	.262	.065	10	2645	3.781	1.890	.473
19	18094	.553	.276	.069	12	2605	3.839	1.919	.480
20	17189	.582	.291	.073	14	2566	3.897	1.948	.487
21	16370	.611	.305	.076	16	2528	3.956	1.978	.495
22	15626	.640	.320	.080	18	2491	4.014	2.007	.502
23	14947	.669	.334	.083	20	2456	4.072	2.036	.509
24	14324	.698	.349	.087	22	2421	4.131	2.065	.516
25	13751	.727	.363	.091	24	2387	4.189	2.094	.523
26	13222	.756	.378	.095	26	2355	4.246	2.123	.531
27	12732	.785	.392	.098	28	2323	4.305	2.152	.538
28	12278	.814	.407	.102	30	2292	4.363	2.182	.545
29	11854	.844	.422	.105	32	2262	4.421	2.210	.552
30	11459	.873	.436	.109	34	2232	4.480	2.240	.560
31	11090	.902	.451	.113	36	2204	4.537	2.268	.567
32	10743	.931	.465	.116	38	2176	4.596	2.298	.574
33	10417	.960	.480	.120	40	2149	4.654	2.327	.582
34	10111	.989	.494	.123	42	2122	4.713	2.356	.589
35	9822	1.018	.509	.127	44	2096	4.771	2.385	.596
36	9549	1.047	.523	.131	46	2071	4.829	2.414	.603
37	9291	1.076	.538	.134	48	2046	4.888	2.444	.611
38	9047	1.105	.552	.138	50	2022	4.946	2.473	.618
39	8815	1.134	.567	.142	52	1999	5.003	2.501	.625
40	8594	1.164	.582	.145	54	1976	5.061	2.530	.632
41	8385	1.193	.596	.149	56	1953	5.120	2.560	.640
42	8185	1.222	.611	.153	58	1932	5.176	2.588	.647
43	7995	1.251	.625	.156	3	1910	5.235	2.618	.654
44	7813	1.280	.640	.160	2	1889	5.294	2.647	.662
45	7639	1.309	.654	.164	4	1869	5.350	2.675	.669
46	7473	1.338	.669	.167	6	1848	5.411	2.705	.676
47	7314	1.367	.683	.171	8	1829	5.467	2.734	.683
48	7162	1.396	.698	.174	10	1810	5.526	2.763	.691
49	7016	1.425	.712	.178	12	1791	5.583	2.792	.698
50	6876	1.454	.727	.182	14	1772	5.643	2.821	.705
51	6741	1.483	.741	.185	16	1754	5.707	2.850	.713
52	6611	1.513	.757	.189	18	1736	5.760	2.880	.720
53	6486	1.542	.771	.193	20	1719	5.817	2.908	.727
54	6366	1.571	.786	.197	22	1702	5.875	2.937	.734
55	6251	1.600	.800	.200	24	1685	5.935	2.967	.742
56	6139	1.629	.815	.204	26	1669	5.992	2.996	.749
57	6031	1.658	.829	.207	28	1653	6.050	3.025	.756
58	5927	1.687	.844	.211	30	1637	6.108	3.054	.764
59	5827	1.716	.858	.214	32	1622	6.166	3.083	.771
1	5730	1.745	.872	.218	34	1607	6.223	3.112	.778
2	5545	1.803	.902	.225	36	1592	6.281	3.140	.785
4	5372	1.862	.931	.233	38	1577	6.341	3.170	.793
6	5209	1.920	.960	.240	40	1563	6.398	3.199	.800
8	5056	1.978	.989	.247	42	1549	6.456	3.228	.807
10	4911	2.036	1.018	.255	44	1535	6.515	3.257	.814
12	4775	2.094	1.047	.262	46	1521	6.575	3.287	.822
14	4646	2.152	1.076	.269	48	1508	6.631	3.316	.829
16	4523	2.211	1.105	.276	50	1495	6.689	3.345	.836
18	4407	2.269	1.134	.284	52	1482	6.748	3.374	.843
20	4297	2.327	1.163	.291	54	1469	6.807	3.403	.851
22	4192	2.385	1.192	.298	56	1457	6.863	3.432	.858
24	4093	2.443	1.221	.305	58	1445	6.920	3.460	.865
26	3997	2.502	1.251	.313	4	1433	6.980	3.490	.873
28	3907	2.560	1.280	.320	5	1403	7.125	3.562	.891
30	3820	2.618	1.309	.327	10	1375	7.271	3.635	.909
32	3737	2.676	1.338	.334	15	1348	7.416	3.708	.927
34	3657	2.734	1.367	.342	20	1323	7.561	3.781	.945

Table of Radii, Middle Ordinates, &c, of Curves. Chord 100 feet.
(Continued.)

The Tangential Angle is always one-half of the Angle of Deflection.

Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid.* Ord.	Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid.* Ord.
° ′					° ′				
4 25	1298	7.707	3.854	.963	10 15	559.7	17.87	8.942	2.238
30	1274	7.852	3.927	.982	30	546.4	18.30	9.160	2.292
35	1250	7.997	3.999	1.000	45	533.8	18.73	9.378	2.347
40	1228	8.143	4.072	1.018	11	521.7	19.17	9.596	2.402
45	1207	8.288	4.145	1.036	15	510.1	19.60	9.814	2.456
50	1186	8.433	4.218	1.054	30	499.1	20.04	10.03	2.511
55	1166	8.579	4.290	1.072	45	488.5	20.47	10.25	2.566
5	1146	8.724	4.363	1.091	12	478.3	20.91	10.47	2.620
5	1128	8.869	4.436	1.109	15	468.6	21.34	10.69	2.675
10	1109	9.014	4.508	1.127	30	459.3	21.77	10.90	2.730
15	1092	9.160	4.581	1.145	45	450.3	22.21	11.12	2.785
20	1075	9.305	4.654	1.164	13	441.7	22.64	11.34	2.839
25	1058	9.450	4.727	1.182	15	433.4	23.07	11.56	2.894
30	1042	9.596	4.799	1.200	30	425.4	23.51	11.77	2.949
35	1027	9.741	4.872	1.218	45	417.7	23.94	11.99	3.003
40	1012	9.886	4.945	1 236	14	410.3	24.37	12.21	3.058
45	996.9	10.03	5.017	1.255	15	403.1	24.81	12.43	3.113
50	982.6	10.18	5.090	1.273	30	396.2	25.24	12.65	3.168
55	968.8	10.32	5.163	1.291	45	389.5	25.67	12.86	3.223
6	955.4	10.47	5.235	1.309	15	383.1	26.11	13.08	3.277
5	942.3	10.61	5.308	1.327	15	376.8	26.54	13.30	3.332
10	929.6	10.76	5.381	1.346	30	370.8	26.97	13.52	3.387
15	917.2	10.90	5.453	1.364	45	364.9	27.40	13.73	3.442
20	905.1	11.05	5.526	1.382	16	359.3	27.84	13.95	3.496
25	893.4	11.19	5.599	1.400	30	348.5	28.70	14.39	3.606
30	882.0	11.34	5.672	1.418	17	338.3	29.56	14.82	3.716
35	870.8	11.48	5.744	1.437	30	328.7	30.43	15.26	3.825
40	859.9	11.63	5.817	1.455	18	319.6	31.29	15.69	3.935
45	849.3	11.77	5.890	1.473	30	311.1	32.15	16.13	4.045
50	839.0	11.92	5.962	1.491	19	302.9	33.01	16.56	4.155
55	828.9	12.07	6.035	1.510	30	295.3	33.87	17.00	4.265
7	819.0	12.21	6.108	1.528	20	287.9	34.73	17.43	4.375
5	809.4	12.36	6.180	1.546	21	274.4	36.44	18.30	4.594
10	800.0	12.50	6.253	1.564	22	262.0	38.17	19.17	4.815
15	790.8	12.65	6.326	1.582	23	250.8	39.87	20.04	5.035
20	781.8	12.79	6.398	1.600	24	240.5	41.58	20.91	5.255
25	773.1	12.94	6.471	1.619	25	231.0	43.29	21.77	5.476
30	764.5	13.08	6.544	1.637	26	222.3	44.98	22.64	5.696
35	756.1	13.23	6.616	1.655	27	214.2	46.69	23.51	5.917
40	747.9	13.37	6.689	1.673	28	206.7	48.38	24.37	6.139
45	739.9	13.52	6.762	1.691	29	199.7	50.08	25.24	6.361
50	732.0	13.66	6.835	1.710	30	193.2	51.76	26.11	6.582
55	724.3	13.81	6.907	1.728	31	187.1	53.45	26.97	6.805
8	716.8	13.95	6.980	1.746	32	181.4	55.13	27.83	7.027
15	695.1	14.39	7.198	1.801	33	176.0	56.82	28.70	7.252
30	674.7	14.82	7.416	1.855	34	171.0	58.48	29.56	7.473
45	655.5	15.26	7.634	1.910	35	166.3	60.13	30.42	7.695
9	637.3	15.69	7.852	1.965	36	161.8	61.80	31.29	7.919
15	620.1	16.13	8.070	2.019	37	157.6	63.45	32.15	8.142
30	603.8	16.56	8.288	2.074	38	153.6	65.10	33.01	8.366
45	588.4	17.00	8.506	2.128	39	149.8	66.76	33.87	8.591
10	573.7	17.43	8.724	2.183	40	146.2	68.40	34.73	8.816

To find tangential and deflection angles for any given rad and chord. Divide *half* the chord by the rad. The quot will be nat sine of the tangl ang. Find this tangl ang in the table of nat sines; and mult it by 2 for the def ang.

To find the def dist for chords 100 ft long. Div 10000 by the rad in feet.

To find the def dist for equal chords of any given length. Div chord by rad. Mult quot by chord. Or div sq of chord by rad.

To find the tangl dist for equal chords of any given length. First find the tangl ang as above. Divide it by 2. Find in the table of nat sines the nat sine of the quot. Mult this nat sine by the given chord. Mult prod by 2.

To find the rad to any given def ang, for equal chords of any length. Divide the def ang by 2. Find nat sine of the quotient. Divide half the chord by this nat sine.

---

* **The middle ordinate** for a rad of 600 ft or more, (chord 100 ft,) may in practice be taken at one-fourth of the tang dist. Even in 400 ft rad it will be too short only 6 in the third decimal.

Radii, &c, of Curves; in metres. Chord, 20 metres = 2 dekametres.

The stakes, at the ends of the 2-dekametre chords, should be numbered 2, 4, 6, &c; meaning 2, 4, 6, &c. *dekametres*. The *tangential* angle in the table will then give the amount of deflection per unit (dekametre) of measurement.

Defl angle.	Tangential angle.	Radius. Metres.	Defl dist. Metres.	Tang'l dist. Metres.	Mid. ord. Metres.	Defl angle.	Tangential angle.	Radius. Metres.	Defl dist. Metres.	Tang'l dist. Metres.	Mid. ord. Metres.
0° 10'	0° 5'	6875.50	.058	.029	.007	8° 0'	4° 0'	143.36	2.790	1.396	.349
20	10	3437.75	.116	.058	.015	10	5	140.44	2.848	1.425	.356
30	15	2291.84	.175	.087	.022	20	10	137.63	2.906	1.454	.364
40	20	1718.88	.233	.116	.029	30	15	134.94	2.964	1.483	.371
50	25	1375.11	.291	.145	.036	40	20	132.35	3.022	1.512	.378
1° 0'	30	1145.93	.349	.175	.044	50	25	129.85	3.080	1.541	.386
10	35	982.23	.407	.204	.051	9° 0'	30	127.45	3.138	1.570	.393
20	40	859.46	.465	.233	.058	10	35	125.14	3.196	1.599	.400
30	45	763.97	.524	.262	.065	20	40	122.91	3.254	1.629	.407
40	50	687.57	.582	.291	.073	30	45	120.76	3.312	1.658	.415
50	55	625.07	.640	.320	.080	40	50	118.68	3.370	1.687	.422
2° 0'	1° 0'	572.99	.698	.349	.087	50	55	116.68	3.428	1.716	.429
10	5	528.92	.756	.378	.095	10° 0'	5° 0'	114.74	3.486	1.745	.437
20	10	491.14	.814	.407	.102	20	10	111.05	3.602	1.803	.451
30	15	458.40	.873	.436	.109	40	20	107.58	3.718	1.861	.466
40	20	429.76	.931	.465	.116	11° 0'	30	104.33	3.834	1.919	.480
50	25	404.48	.989	.494	.124	20	40	101.28	3.950	1.977	.495
3° 0'	30	382.02	1.047	.524	.131	40	50	98.39	4.065	2.035	.509
10	35	361.91	1.105	.553	.138	12° 0'	6° 0'	95.67	4.181	2.093	.524
20	40	343.82	1.163	.582	.145	20	10	93.09	4.297	2.152	.539
30	45	327.46	1.222	.611	.153	40	20	90.65	4.413	2.210	.553
40	50	312.58	1.280	.640	.160	13° 0'	30	88.34	4.528	2.268	.568
50	55	298.99	1.338	.669	.167	20	40	86.14	4.644	2.326	.582
4° 0'	2° 0'	286.54	1.396	.698	.175	40	50	84.05	4.759	2.384	.597
10	5	275.08	1.454	.727	.182	14° 0'	7° 0'	82.06	4.875	2.442	.612
20	10	264.51	1.512	.756	.189	20	10	80.16	4.990	2.500	.626
30	15	254.71	1.570	.785	.196	40	20	78.34	5.106	2.558	.641
40	20	245.62	1.629	.814	.204	15° 0'	30	76.61	5.221	2.616	.655
50	25	237.16	1.687	.844	.211	20	40	74.96	5.336	2.674	.670
5° 0'	30	229.26	1.745	.873	.218	40	50	73.37	5.452	2.732	.685
10	35	221.87	1.803	.902	.225	16° 0'	8° 0'	71.85	5.567	2.790	.699
20	40	214.94	1.861	.931	.233	20	10	70.40	5.682	2.848	.714
30	45	208.43	1.919	.960	.240	40	20	69.00	5.797	2.906	.729
40	50	202.30	1.977	.989	.247	17° 0'	30	67.65	5.912	2.964	.743
50	55	196.53	2.035	1.018	.255	20	40	66.36	6.027	3.022	.758
6° 0'	3° 0'	191.07	2.093	1.047	.262	40	50	65.12	6.142	3.080	.772
10	5	185.91	2.152	1.076	.269	18° 0'	9° 0'	63.92	6.257	3.138	.787
20	10	181.03	2.210	1.105	.276	20	10	62.77	6.372	3.196	.802
30	15	176.39	2.268	1.134	.284	40	20	61.66	6.487	3.254	.816
40	20	171.98	2.326	1.163	.291	19° 0'	30	60.59	6.602	3.312	.831
50	25	167.79	2.384	1.192	.298	20	40	59.55	6.717	3.370	.846
7° 0'	30	163.80	2.442	1.222	.306	40	50	58.55	6.831	3.428	.860
10	35	160.00	2.500	1.251	.313	20° 0'	10° 0'	57.59	6.946	3.486	.875
20	40	156.37	2.558	1.280	.320	21° 0'	30	54.87	7.289	3.660	.919
30	45	152.90	2.616	1.309	.327	22° 0'	11° 0'	52.41	7.632	3.834	.963
40	50	149.58	2.674	1.338	.335	23° 0'	30	50.16	7.975	4.008	1.007
50	55	146.40	2.732	1.367	.342	24° 0'	12° 0'	48.10	8.316	4.181	1.051
						25° 0'	30	46.20	8.658	4.355	1.095

Radius = $\frac{\text{Half the chord}}{\text{Sine of tangential angle}}$ = Half the chord × cosecant of tangential angle.

Deflection dist = $\frac{\text{Square of chord}}{\text{Radius}}$ = Twice the chord × sine of tangential angle.

Tangential dist = Twice the chord × sine of half the tangential angle.

Middle ord = Radius × (1 — cosine of tangential angle) = Half the chord × tangent of half the tangential angle.

For curves of 60 metres, or greater, radius, **the ordinate at 5 metres** from the end of the 20-metre chord, or midway between the end of the chord and the middle ordinate, may be taken at three-fourths of the middle ordinate.

Table of Long Chords.

Lengths of Chord in ft, required to subtend from 1 to 4 stations of 100 ft each.

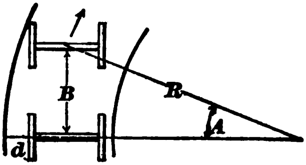
Ang. of Defl.	1 Sta.	2 Sta.	3 Sta.	4 Sta.	Ang. of Defl.	1 Sta.	2 Sta.	3 Sta.	4 Sta.
1°	100	200.0	300.0	400.0	¾	100	199.7	298.9	397.5
¼	100	200.0	300.0	399.9	6°	100	199.7	298.8	397.3
½	100	200.0	300.0	399.9	¼	100	199.7	298.7	397.0
¾	100	200.0	300.0	399.8	½	100	199.7	298.6	396.7
2°	100	200.0	299.9	399.7	¾	100	199.6	298.5	396.5
¼	100	200.0	299.9	399.6	7°	100	199.6	298.4	396.2
½	100	200.0	299.8	399.5	¼	100	199.6	298.3	396.0
¾	100	200.0	299.8	399.4	½	100	199.6	298.2	395.7
3°	100	200.0	299.7	399.3	¾	100	199.6	298.1	395.4
¼	100	200.0	299.7	399.2	8°	100	199.6	298.0	395.1
½	100	200.0	299.6	399.1	¼	100	199.5	297.9	394.8
¾	100	200.0	299.6	399.0	½	100	199.5	297.8	394.5
4°	100	199.9	299.6	398.9	¾	100	199.4	297.7	394.3
¼	100	199.9	299.5	398.7	9°	100	199.4	297.5	394.1
½	100	199.9	299.4	398.5	¼	100	199.4	297.4	393 7
¾	100	199 9	299.3	398.3	½	100	199.3	297.3	393.2
5°	100	199.9	299.2	398.0	¾	100	199.2	297.2	392.8
¼	100	199 8	299.1	397.8	10°	100	199.2	297.0	392.4
½	100	199.8	299.0	397.6					

Elevation of outer rail in curves theoretically is equal in ins to (square of vel in ft per sec × gauge in ins) ÷ (Rad of curve in ft × 32.2). Experience has shown that half an inch for each degree of def angle (100 ft chords) does very well for 4 ft 8.5 ins gauge up to 40 miles per hour. At 60 miles use 1 inch per deg. In dangerous places this may be increased for safety against high winds. Approaching the curve raise the outer rail at the rate of 1 inch in about 60 or 80 ft.

When the ends of a curve are tapered off by transition curves, the rise is made upon the latter.

Relation of radius to length of wheel-base.

Mr. A. M. Wellington* found by experiments with models that a rigid truck passing around a curve, whether alone or coupled with another truck, assumes the position shown in this Fig.,† *i.e.*, the flange of the outer front wheel presses against the outer rail, and the rear axle coincides with a radius to the curve. Then, for the angle A, between that radius and the radius R which passes through the center of the front axle:



$$\text{sine of A} = \frac{\text{wheel-base B}}{\text{radius R}};$$

and the space *d* between the flange of the outer hind wheel and the outer rail is,

$$d = \text{radius R} \times \text{versed sine of angle A, very nearly.}$$

For a given wheel-base B, we have, approximately,

$$d = (d \text{ for a } 1^\circ \text{ curve}) \times \text{degree of curvature};$$

and the inner hind wheel will touch the inner rail when *d* becomes equal to the total room for play left between the wheel-flanges and the rails, *i. e.*, when

$$\text{degree of curvature} = \frac{\text{total play}}{d \text{ for a } 1^\circ \text{ curve}}.$$

This commonly occurs on European railways, where the cars have rigid wheel-bases much longer than our pivoted trucks, and where *d* for a given radius is therefore much greater than with us. Hence the inner rails on curves are more generally worn there than here.

For a wheel-base 5 ft. long, "*d* for a 1° curve" is 0.0022 ft. It varies (nearly) as the square of the length of the wheel-base. *d* is independent of the gauge.

---

* The Economic Theory of Railway Location, New York, John Wiley & Sons, 1887.

† In our figure, necessarily much exaggerated, we omit, for simplicity, the treads of the wheels, all of which are supposed to rest on the rails, and show only so much of their flanges as extends below the top of the rail.

Table of Ordinates 5 ft apart. Chord 100 ft.

For Railroad Curves.

Ordinates for angles intermediate of those in the table can at once be found by simple proportion.

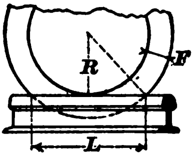
Distances of the Ordinates from the end of the 100 feet Chord.

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
0 ′										
4	.014	.014	.014	.013	.012	.010	.008	.008	.005	.003
8	.029	.029	.028	.026	.024	.022	.018	.015	.010	.005
12	.043	.043	.041	.038	.037	.033	.028	.022	.015	.008
16	.058	.058	.056	.052	.049	.044	.037	.030	.020	.011
20	.073	.072	.070	.066	.061	.055	.047	.037	.026	.014
24	.087	.086	.083	.077	.074	.066	.056	.045	.031	.017
28	.102	.101	.098	.092	.086	.077	.065	.052	.036	.019
32	.116	.115	.112	.106	.098	.088	.075	.058	.042	.022
36	.131	.130	.126	.119	.110	.099	.084	.066	.047	.024
40	.145	.144	.140	.133	.123	.110	.093	.074	.052	.027
44	.160	.158	.153	.145	.135	.121	.103	.081	.057	.030
48	.174	.172	.167	.158	.147	.132	.112	.088	.062	.033
52	.189	.187	.181	.171	.159	.143	.122	.096	.068	.035
56	.204	.202	.195	.185	.171	.154	.131	.103	.073	.038
1	.218	.216	.209	.198	.183	.164	.140	.111	.078	.041
4	.233	.231	.223	.211	.196	.175	.150	.118	.083	.043
8	.247	.245	.237	.224	.208	.186	.159	.125	.088	.046
12	.262	.260	.252	.237	.220	.196	.168	.133	.094	.049
16	.276	.274	.265	.251	.232	.207	.177	.140	.099	.052
20	.291	.288	.279	.264	.244	.218	.187	.148	.104	.055
24	.306	.303	.293	.277	.256	.229	.197	.155	.109	.057
28	.320	.317	.307	.291	.269	.240	.206	.163	.114	.060
32	.334	.331	.321	.304	.281	.251	.215	.171	.120	.063
36	.349	.345	.335	.317	.293	.262	.224	.178	.125	.066
40	.364	.360	.349	.330	.305	.273	.233	.185	.130	.069
44	.378	.374	.363	.343	.318	.284	.242	.192	.135	.072
48	.393	.389	.377	.356	.330	.295	.251	.200	.141	.075
52	.407	.403	.391	.370	.342	.305	.261	.208	.147	.077
56	.422	.418	.405	.383	.354	.316	.270	.215	.152	.080
2	.436	.432	.419	.397	.366	.327	.280	.222	.157	.083
4	.451	.446	.433	.409	.379	.338	.289	.230	.162	.085
8	.465	.461	.447	.425	.391	.349	.298	.237	.167	.088
12	.480	.475	.461	.437	.403	.360	.308	.245	.173	.090
16	.495	.490	.475	.450	.415	.371	.317	.252	.178	.093
20	.509	.504	.489	.463	.428	.382	.326	.260	.183	.096
24	.523	.518	.503	.476	.440	.393	.334	.267	.188	.099
28	.538	.533	.517	.489	.452	.404	.346	.275	.194	.102
32	.552	.547	.531	.503	.465	.415	.355	.282	.199	.104
36	.567	.562	.545	.516	.477	.425	.364	.289	.204	.107
40	.582	.576	.559	.529	.489	.436	.373	.297	.209	.110
44	.596	.590	.573	.542	.501	.447	.382	.304	.214	.113
48	.611	.605	.587	.555	.513	.458	.391	.312	.219	.116
52	.625	.619	.601	.569	.526	.469	.401	.319	.225	.118
56	.640	.634	.615	.582	.538	.480	.410	.326	.230	.121
3	.654	.648	.629	.596	.550	.491	.419	.334	.235	.124
4	.669	.662	.643	.608	.562	.502	.428	.341	.240	.127
8	.683	.677	.657	.621	.574	.512	.438	.349	.246	.130
12	.698	.691	.671	.635	.587	.523	.448	.357	.251	.132
16	.713	.705	.685	.649	.599	.534	.457	.364	.257	.135
20	.727	.720	.699	.662	.611	.545	.466	.371	.262	.138
24	.742	.734	.713	.675	.623	.556	.475	.378	.267	.141
28	.756	.749	.727	.688	.635	.567	.485	.386	.272	.144
32	.771	.763	.741	.702	.648	.578	.494	.394	.278	.146
36	.786	.777	.755	.715	.660	.589	.503	.401	.283	.149
40	.800	.792	.769	.728	.673	.600	.512	.408	.288	.152
44	.814	.806	.783	.741	.685	.611	.521	.415	.293	.155
48	.829	.821	.797	.754	.697	.621	.531	.423	.298	.158
52	.843	.835	.811	.768	.709	.632	.541	.431	.304	.160
56	.858	.850	.825	.781	.721	.643	.550	.438	.309	.163
4	.873	.864	.839	.794	.734	.655	.559	.445	.314	.166
10	.909	.900	.874	.827	.764	.682	.582	.464	.327	.173
20	.945	.936	.909	.860	.795	.709	.606	.482	.340	.179
30	.981	.972	.944	.893	.825	.736	.629	.501	.354	.186
40	1.017	1.008	.979	.926	.855	.764	.652	.519	.367	.193
50	1.054	1.044	1.014	.959	.886	.791	.676	.538	.380	.199
5	1.091	1.080	1.048	.993	.917	.818	.699	.557	.393	.207
10	1.127	1.116	1.083	1.026	.947	.845	.722	.576	.406	.214
20	1.164	1.152	1.118	1.058	.978	.872	.746	.594	.419	.220
30	1.200	1.188	1.153	1.092	1.009	.900	.769	.613	.432	.228

Table of Ordinates 5 ft apart.—(Continued.)

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
	Distances of the Ordinates from the end of the 100 feet Chord.									
° ′										
5 40	1.236	1.224	1.188	1.124	1.039	.927	.792	.631	.445	.235
50	1.273	1.260	1.223	1.157	1.070	.954	.816	.649	.458	.241
6	1.309	1.296	1.258	1.191	1.100	.982	.839	.668	.472	.248
10	1.345	1.332	1.293	1.224	1.130	1.009	.862	.686	.485	.255
20	1.382	1.368	1.328	1.256	1.161	1.036	.886	.705	.498	.262
30	1.419	1.404	1.362	1.290	1.192	1.064	.909	.724	.511	.269
40	1.455	1·440	1.397	1.323	1.222	1.091	.932	.742	.524	.276
50	1.491	1.476	1.432	1.355	1.253	1.118	.956	.761	.537	.283
7	1.528	1.512	1.467	1.389	1.284	1.146	.979	.779	.551	.290
10	1.564	1.548	1.502	1.422	1.314	1.173	1.002	.798	.564	.297
20	1.600	1.584	1.537	1.454	1.345	1.200	1.026	.816	.576	.304
30	1.637	1.620	1.572	1.488	1.375	1.228	1.048	.835	.590	.311
40	1.673	1.656	1.607	1.521	1.405	1.255	1.071	.854	.603	.318
50	1.710	1.692	1.641	1.553	1.436	1.282	1.095	.872	.616	.324
8	1.746	1.728	1.677	1.587	1.467	1.310	1.118	.891	.629	.332
30	1.855	1.836	1.782	1.687	1.559	1.392	1.188	.946	.669	.353
9	1.965	1.944	1.886	1.787	1.651	1.474	1.258	1.002	.708	.373
30	2.074	2.052	1.991	1.887	1.742	1.556	1.328	1.057	.748	.394
10	2.183	2.161	2.096	1.987	1.834	1.637	1.398	1.114	.787	.415
30	2.292	2.269	2.201	2.087	1.926	1.719	1.468	1.170	.827	.436
11	2.401	2.377	2.306	2.186	2.018	1.802	1.538	1.226	.866	.457
30	2.511	2.486	2.411	2.286	2.110	1.884	1.609	1.282	.906	.478
12	2.620	2.594	2.516	2.386	2.202	1.967	1.681	1.339	.946	.499
30	2.730	2.703	2.621	2.485	2.295	2.049	1.750	1.395	.985	.520
13	2.839	2.811	2.726	2.585	2.387	2.132	1.820	1.451	1.025	.541
30	2.949	2.920	2.832	2.685	2.479	2.214	1.891	1.507	1.065	.562
14	3.058	3.028	2.937	2.785	2.571	2.297	1.961	1.564	1.105	.583
30	3.168	3.136	3.042	2.884	2.664	2.379	2.031	1.620	1.144	.604
15	3.277	3.245	3.147	2.984	2.756	2.462	2.102	1.676	1.184	.625
30	3.387	3.354	3.252	3.084	2.848	2.544	2.172	1.732	1.224	.646
16	3.496	3.462	3.358	3.184	2.941	2.627	2.243	1.789	1.264	.667
17	3.716	3.680	3.569	3.384	3.125	2.792	2.384	1.902	1.344	.709
18	3.935	3.897	3.779	3.584	3.310	2.958	2.525	2.014	1.424	.751
19	4.155	4.115	3.990	3.784	3.495	3.123	2.666	2.127	1.504	.793
20	4.375	4.332	4.201	3.984	3.680	3.288	2.808	2.240	1.584	.836
22	4.815	4.768	4.624	4.386	4.050	3.620	3.093	2.467	1.744	.922
24	5.255	5.204	5.048	4.789	4.423	3.952	3.379	2.695	1.905	1.008
26	5.697	5.642	5.473	5.192	4.798	4.286	3.665	2.924	2.068	1.094
28	6.139	6.079	5.898	5.595	5.171	4.622	3.952	3.154	2.232	1.181
30	6.582	6.517	6.323	5.999	5.544	4.958	4.239	3.385	2.396	1.268
32	7·027	6.957	6.751	6.406	5.922	5.297	4.530	3.619	2.565	1.356
34	7.472	7.398	7.179	6.813	6.300	5.637	4.822	3.854	2.733	1.445
36	7.918	7.841	7.609	7.222	6.679	5.978	5.115	4.090	2.901	1.535
38	8.367	8.286	8.041	7.633	7.060	6.320	5.410	4.327	3.069	1.626
40	8.816	8.731	8.474	8.044	7.442	6.663	5.705	4.565	3.238	1.718

Gauge on curves. Let R = radius of wheel from center to tread, F = depth of flange of wheel, and L = the length of that portion of the wheel-flange which extends below the top of the rail, $= 2\sqrt{(R+F)^2 - R^2}$, all in inches. Then if, on the curve, we widen the gauge by a quantity, in *inches*, =



$$Q^* = L, \textit{ins.} \times \frac{L, \textit{feet} + \text{length of rigid wheel-base, ft.}}{\text{gauge, ft.} + 2 \times \text{rad. of the } \textit{curve}, \text{ft.}},$$

the wheels will have approximately the same play on the curve as on the tangent. For a rigid wheel-base 14 ft. long and drivers 4 ft. diam., with 1¼ inch flanges, Q is about 0.02 inch (= one-fiftieth of an in.) for each degree of curvature.

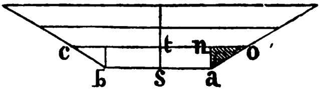
Many roads use the same gauge on curves as on tangents. Others widen the gauge on curves by from one thirty-second to one-eighth inch for each degree of curvature, seldom, however, exceeding 1 inch as a maximum. In Philadelphia the Pennsylvania Railroad has freight-car sidings of 60 feet radius; track gauge (same on curves as on tangents), 4 ft. 9 ins.; standard wheel-gauge, 4 ft. 8¼ ins.

---

* Except for very sharp curves and for very short wheel-bases with large wheels and deep flanges, it is amply approximate to say

$$Q \text{ in inches} = L \text{ in inches} \times \frac{\text{wheel-base in feet}}{2 \times \text{rad. of curve in ft.}}.$$

To prepare a Table, T, of Level Cuttings, for every $\frac{1}{10}$ of a foot of height, or depth.



Let the fig represent the cutting; or, if inverted, the filling; in which the horizontal lines are supposed to be $\frac{1}{10}$ foot apart. First calculate the area in square feet, of the layer *a b c o*, adjoining the roadway *a b*. Then find how many cubic yards that area gives in a distance of 100 feet. These cubic yards we will call Y; they form the first amount to be put into the Table T.

Next calculate the area in square feet of the triangle *a n o*. Multiply this area by 4. Find how many cubic yards this increased area gives in a distance of 100 feet. Or they will be found ready calculated below. We will call them *y*. This is all the preparation that is needed before commencing the table.

Exam.—Let the roadbed *a b* be 18 feet, and the side-slopes 1½ to 1. Then for the area of *a b c o*: since the side-slopes are 1½ to 1; and *s t* is .1 foot; *c o* must be 18.3 feet; and the mean length of *a b c o* must be 18.15 feet. Consequently, the area is 18.15 × .1 = 1.815 square feet; which, in a distance of 100 feet, gives 181.5 cubic feet; which is equal to $\frac{181.5}{27} = 6.7222$ cubic yards; or Y.

Next, as to the triangle *a n o*: its height *a n* being .1 foot, and its base *n o* .15 feet; its area $= \frac{.1 \times .15}{2} = \frac{.015}{2} = .0075$ square ft. This multiplied by 4, gives .03 square feet; which, in a distance of 100 feet, gives .03 × 100 = 3 cubic feet: which is equal to $\frac{3}{27} = .1111$ cubic yard; or *y*.

Having thus found Y and *y*, proceed to make out the table in the manner following, which is so plain as to require no explanation. The work should be tested about every 5 feet, by calculating the area of the full depth arrived at; multiply it by 100, and divide the product by 27 for the cubic yards The cubic yards thus found should agree with the table.

Y.......	6.7222	.... Y. 6.722	.1
y.......	.1111		
	6.8333	6.8333	
y.......	.1111		
		13.5555	.2
	6.9444	6.9444	
y.......	.1111		
		20 5000	.3
	7.0555	7.0555	
y.......	.1111		
		27.5555	.4
	7.1666	7.1666	
y.......	.1111		
		34.7222	.5
	7.2777	7.2777	
		42.0000	.6

TABLE T

Height. Feet.	Cub. Yds.
.1	6.72 Y.
.2	13.6
.3	20.5
.4	27.6
.5	34.7
.6	42.0
&c.	

The following table contains *y*, ready calculated for different side-slopes. It plainly remains the same for all widths of roadbed.

Side-slope.	*y*	Side-slope.	*y*
¼ to 1	.... .0185	1¾ to 1	.... .1296
½ to 1	.... .0370	2 to 1	.... .1482
¾ to 1	.... .0556	2¼ to 1	.... .1667
1 to 1	.... .0741	2½ to 1	.... .1852
1¼ to 1	.... .0926	3 to 1	.... .2222
1½ to 1	.... .1111	4 to 1	.... .2963

Table 1. Level Cuttings.[4]

Roadway 14 feet wide, side-slopes 1½ to 1.

For single-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		5.24	10.6	16.1	21.6	27.3	33.1	39.0	45.0	51.2
1	57.4	63.8	70.2	76.8	83.5	90.3	97.2	104.2	111.3	118.6
2	125.9	133.4	141.0	148.6	156.4	164.4	172.4	180.5	188.7	197.1
3	205.6	214.1	222.8	231.6	240.5	249.5	258.7	267.9	277.3	286.7
4	296.3	306.0	315.8	325.7	335.7	345.8	356.1	366.4	376.9	387.5
5	398.1	408.9	419.9	430.9	442.0	453.2	464.6	476.1	487.6	499.3
6	511.1	523.0	535.0	547.2	559.4	571.8	584.2	596.8	609.5	622.3
7	635.2	648.2	661.3	674.6	687.9	701.4	714.9	728.6	742.4	756.3
8	770.3	784.5	798.7	813.1	827.5	842.1	856.8	871.6	886.5	901.5
9	916.7	931.9	947.3	962.7	978.3	994.0	1010	1026	1042	1058
10	1074	1090	1107	1123	1140	1157	1174	1191	1208	1225
11	1243	1260	1278	1295	1313	1331	1349	1367	1385	1404
12	1422	1441	1459	1478	1497	1516	1535	1554	1574	1593
13	1613	1633	1652	1672	1692	1712	1733	1753	1773	1794
14	1815	1835	1856	1877	1898	1920	1941	1962	1984	2006
15	2028	2050	2072	2094	2116	2138	2161	2183	2206	2229
16	2252	2275	2298	2321	2344	2368	2391	2415	2439	2463
17	2487	2511	2535	2559	2584	2608	2633	2658	2683	2708
18	2733	2759	2784	2809	2835	2861	2886	2912	2938	2964
19	2991	3017	3044	3070	3097	3124	3151	3178	3205	3232
20	3259	3287	3314	3342	3370	3398	3426	3454	3482	3510
21	3539	3567	3596	3625	3654	3683	3712	3741	3771	3800
22	3830	3859	3889	3919	3949	3979	4009	4040	4070	4101
23	4132	4162	4193	4224	4255	4287	4318	4349	4381	4413
24	4444	4476	4508	4541	4573	4605	4638	4670	4703	4736
25	4769	4802	4835	4868	4901	4935	4968	5002	5036	5070
26	5104	5138	5172	5206	5241	5275	5310	5345	5380	5415
27	5450	5485	5521	5556	5592	5627	5663	5699	5735	5771
28	5807	5844	5880	5917	5953	5990	6027	6064	6101	6139
29	6176	6213	6251	6289	6326	6364	6402	6440	6479	6517
30	6556	6594	6633	6672	6711	6750	6789	6828	6867	6907
31	6946	6986	7026	7066	7106	7146	7186	7226	7267	7307
32	7348	7389	7430	7471	7512	7553	7595	7636	7678	7719
33	7761	7803	7845	7887	7929	7972	8014	8057	8099	8142
34	8185	8228	8271	8315	8358	8401	8445	8489	8532	8576
35	8620	8664	8709	8753	8798	8842	8887	8932	8976	9022
36	9067	9112	9157	9203	9248	9294	9340	9386	9432	9478
37	9524	9570	9617	9663	9710	9757	9804	9851	9898	9945
38	9993	10040	10088	10135	10183	10231	10279	10327	10375	10424
39	10472	10521	10569	10618	10667	10716	10765	10815	10864	10913
40	10963	11013	11062	11112	11162	11212	11263	11313	11364	11414
41	11465	11516	11567	11618	11669	11720	11771	11823	11874	11926
42	11978	12029	12081	12134	12186	12238	12291	12343	12396	12449
43	12502	12555	12608	12661	12715	12768	12822	12875	12929	12983
44	13037	13091	13145	13200	13254	13309	13363	13418	13473	13528
45	13583	13639	13694	13749	13805	13861	13916	13972	14028	14084
46	14141	14197	14254	14310	14367	14424	14480	14537	14595	14652
47	14709	14767	14824	14882	14940	14998	15056	15114	15172	15230
48	15289	15347	15406	15465	15524	15583	15642	15701	15761	15826
49	15880	15939	15999	16059	16119	16179	16239	16300	16360	16421
50	16481	16542	16603	16664	16725	16787	16848	16909	16971	17033
51	17094	17156	17218	17280	17343	17405	17467	17530	17593	17656
52	17719	17782	17845	17908	17971	18035	18098	18162	18226	18290
53	18354	18418	18482	18546	18611	18675	18740	18805	18870	18935
54	19000	19065	19131	19196	19262	19327	19393	19459	19525	19591
55	19657	19724	19790	19857	19923	19990	20057	20124	20191	20259
56	20326	20393	20461	20529	20596	20664	20732	20800	20869	20937
57	21005	21074	21143	21212	21280	21349	21419	21488	21557	21627
58	21696	21766	21836	21906	21976	22046	22116	22186	22257	22327
59	22398	22469	22540	22611	22682	22753	22825	22896	22968	23039
60	23111	23183	23255	23327	23399	23472	23544	23617	23689	23762

---

[4] From the Author's "Measurement and Cost of Earthwork."

Table 2. Level Cuttings.

Roadway 24 feet wide, side-slopes 1½ to 1.

For double-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		8.94	18.0	27.2	36.4	45.8	55.3	64.9	74.7	84.5
1	94.4	104.5	114.7	124.9	135.3	145.8	156.4	167.2	178.0	188.9
2	200.0	211.2	222.4	233.8	245.3	256.9	268.6	280.5	292.4	304.4
3	316.6	328.9	341.2	353.7	366.3	379.0	391.9	404.8	417.8	431.0
4	444.4	457.8	471.3	484.9	498.6	512.4	526.4	540.4	554.6	568.8
5	583.3	597.8	612.4	627.1	642.0	656.9	671.9	687.1	702.3	717.7
6	733.3	748.9	764.7	780.5	796.4	812.5	828.7	844.9	861.3	877.8
7	894.4	911.2	928.0	944.9	962.0	979.2	996.4	1014	1031	1049
8	1067	1085	1102	1121	1139	1157	1175	1194	1212	1231
9	1250	1269	1288	1307	1326	1346	1365	1385	1405	1425
10	1444	1465	1485	1505	1525	1546	1566	1587	1608	1629
11	1650	1671	1692	1714	1735	1757	1779	1800	1822	1845
12	1867	1889	1911	1934	1956	1979	2002	2025	2048	2071
13	2094	2118	2141	2165	2189	2213	2236	2261	2285	2309
14	2333	2358	2382	2407	2432	2457	2482	2507	2532	2558
15	2583	2609	2635	2661	2686	2713	2739	2765	2791	2818
16	2844	2871	2898	2925	2952	2979	3006	3034	3061	3088
17	3117	3145	3172	3201	3229	3257	3285	3314	3342	3371
18	3400	3429	3458	3487	3516	3546	3575	3605	3635	3665
19	3694	3725	3755	3785	3815	3846	3876	3907	3938	3969
20	4000	4031	4062	4094	4125	4157	4189	4221	4252	4285
21	4317	4349	4381	4414	4446	4479	4512	4545	4578	4611
22	4644	4678	4711	4745	4779	4813	4846	4881	4915	4949
23	4983	5018	5052	5087	5122	5157	5192	5227	5262	5298
24	5333	5369	5405	5441	5476	5513	5549	5585	5621	5658
25	5694	5731	5768	5805	5842	5879	5916	5954	5991	6029
26	6067	6105	6142	6181	6219	6257	6295	6334	6372	6411
27	6450	6489	6528	6567	6606	6646	6685	6725	6765	6805
28	6844	6885	6925	6965	7005	7046	7086	7127	7168	7209
29	7250	7291	7332	7374	7415	7457	7499	7541	7582	7625
30	7667	7709	7751	7794	7836	7879	7922	7965	8008	8051
31	8094	8138	8181	8225	8269	8313	8356	8401	8445	8489
32	8533	8578	8622	8667	8712	8757	8802	8847	8892	8938
33	8983	9029	9075	9121	9166	9212	9259	9305	9351	9398
34	9444	9491	9538	9585	9632	9679	9726	9774	9821	9869
35	9917	9965	10012	10061	10109	10157	10205	10254	10302	10351
36	10400	10449	10498	10547	10596	10646	10695	10745	10795	10845
37	10894	10945	10995	11045	11095	11146	11196	11247	11298	11349
38	11400	11451	11502	11554	11605	11657	11709	11761	11812	11865
39	11917	11969	12021	12074	12126	12179	12232	12285	12338	12391
40	12444	12498	12551	12605	12659	12713	12766	12821	12875	12929
41	12983	13038	13092	13147	13202	13257	13312	13367	13422	13478
42	13533	13589	13645	13701	13756	13813	13869	13925	13981	14038
43	14094	14151	14208	14265	14322	14379	14436	14494	14551	14609
44	14667	14725	14782	14840	14899	14957	15015	15074	15132	15191
45	15250	15309	15368	15427	15486	15546	15605	15665	15725	15785
46	15844	15905	15965	16025	16085	16146	16206	16267	16328	16389
47	16450	16511	16572	16634	16695	16757	16819	16881	16942	17005
48	17067	17129	17191	17254	17316	17379	17442	17505	17568	17631
49	17694	17758	17821	17885	17949	18013	18076	18141	18205	18269
50	18333	18398	18462	18527	18592	18657	18722	18787	18852	18918
51	18983	19049	19115	19181	19246	19313	19379	19445	19511	19578
52	19644	19711	19778	19845	19912	19979	20046	20114	20181	20249
53	20317	20385	20452	20521	20589	20657	20725	20794	20862	20931
54	21000	21069	21138	21207	21276	21346	21415	21485	21555	21625
55	21694	21765	21835	21905	21975	22046	22116	22187	22258	22329
56	22400	22471	22542	22614	22685	22757	22829	22901	22972	23045
57	23117	23189	23261	23334	23406	23479	23552	23625	23698	23771
58	23844	23918	23991	24065	24139	24213	24286	24361	24435	24509
59	24583	24658	24732	24807	24882	24957	25032	25107	25182	25258
60	25333	25409	25485	25561	25636	25713	25789	25865	25941	26018

For continuation to 100 feet, see TABLE 7.

Table 3. Level Cuttings.

Roadway 18 feet wide, side-slopes 1 to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		6.70	13.5	20.3	27.3	34.3	41.3	48.5	55.7	63.0
1	70.4	77.8	85.3	92.9	100.6	108.3	116.1	124.0	132.0	140.0
2	148.1	156.3	164.6	172.9	181.3	189.8	198.4	207.0	215.7	224.5
3	233.3	242.3	251.3	260.3	269.5	278.7	288.0	297.4	306.8	316.3
4	325.9	335.6	345.3	355.1	365.0	375.0	385.0	395.1	405.3	415.6
5	425.9	436.3	446.8	457.4	468.0	478.7	489.5	500.3	511.3	522.3
6	533.3	544.5	555.7	567.0	578.4	589.8	601.3	612.9	624.6	636.3
7	648.1	660.0	672.0	684.0	696.1	708.3	720.6	732.9	745.3	757.8
8	770.4	783.0	795.7	808.5	821.3	834.3	847.3	860.3	873.5	886.7
9	900.0	913.4	926.8	940.3	953.9	967.6	981.3	995.1	1009	1023
10	1037	1051	1065	1080	1094	1108	1123	1137	1152	1167
11	1181	1196	1211	1226	1241	1256	1272	1287	1302	1318
12	1333	1349	1365	1380	1396	1412	1428	1444	1460	1476
13	1493	1509	1525	1542	1558	1575	1592	1608	1625	1642
14	1659	1676	1693	1711	1728	1745	1763	1780	1798	1816
15	1833	1851	1869	1887	1905	1923	1941	1960	1978	1996
16	2015	2033	2052	2071	2089	2108	2127	2146	2165	2184
17	2204	2223	2242	2262	2281	2301	2321	2340	2360	2380
18	2400	2420	2440	2460	2481	2501	2521	2542	2562	2583
19	2604	2624	2645	2666	2687	2708	2729	2751	2772	2793
20	2815	2836	2858	2880	2901	2923	2945	2967	2989	3011
21	3033	3056	3078	3100	3123	3145	3168	3191	3213	3236
22	3259	3282	3305	3328	3352	3375	3398	3422	3445	3469
23	3493	3516	3540	3564	3588	3612	3636	3660	3685	3709
24	3733	3758	3782	3807	3832	3856	3881	3906	3931	3956
25	3981	4007	4032	4057	4083	4108	4134	4160	4185	4211
26	4237	4263	4289	4315	4341	4368	4394	4420	4447	4473
27	4500	4527	4553	4580	4607	4634	4661	4688	4716	4743
28	4770	4798	4825	4853	4881	4908	4936	4964	4992	5020
29	5048	5076	5105	5133	5161	5190	5218	5247	5276	5304
30	5333	5362	5391	5420	5449	5479	5508	5537	5567	5596
31	5626	5656	5685	5715	5745	5775	5805	5835	5865	5896
32	5926	5956	5987	6017	6048	6079	6109	6140	6171	6202
33	6233	6264	6296	6327	6358	6390	6421	6453	6485	6516
34	6548	6580	6612	6644	6676	6708	6741	6773	6805	6838
35	6870	6903	6936	6968	7001	7034	7067	7100	7133	7167
36	7200	7233	7267	7300	7334	7368	7401	7435	7469	7503
37	7537	7571	7605	7640	7674	7708	7743	7777	7812	7847
38	7881	7916	7951	7986	8021	8056	8092	8127	8162	8198
39	8233	8269	8305	8340	8376	8412	8448	8484	8520	8556
40	8593	8629	8665	8702	8738	8775	8812	8848	8885	8922
41	8959	8996	9033	9071	9108	9145	9183	9220	9258	9296
42	9333	9371	9409	9447	9485	9523	9561	9600	9638	9676
43	9715	9753	9792	9831	9869	9908	9947	9986	10025	10064
44	10104	10143	10182	10222	10261	10301	10341	10380	10420	10460
45	10500	10540	10580	10620	10661	10701	10741	10782	10822	10863
46	10904	10944	10985	11026	11067	11108	11149	11191	11232	11273
47	11315	11356	11398	11440	11481	11523	11565	11607	11649	11691
48	11733	11776	11818	11860	11903	11945	11988	12031	12073	12116
49	12159	12202	12245	12288	12332	12375	12418	12462	12505	12549
50	12593	12636	12680	12724	12768	12812	12856	12900	12945	12989
51	13033	13078	13122	13167	13212	13256	13301	13346	13391	13436
52	13481	13527	13572	13617	13663	13708	13754	13800	13845	13891
53	13937	13983	14029	14075	14121	14168	14214	14260	14307	14353
54	14400	14447	14493	14540	14587	14634	14681	14728	14776	14823
55	14870	14918	14965	15013	15061	15108	15156	15204	15252	15300
56	15348	15396	15445	15493	15541	15590	15638	15687	15736	15784
57	15833	15882	15931	15980	16029	16079	16128	17177	16227	16276
58	16326	16376	16425	16475	16525	16575	16625	16675	16725	16776
59	16826	16876	16927	16977	17028	17079	17129	17180	17231	17282
60	17333	17384	17436	17487	17538	17590	17641	17693	17745	17796

For continuation to 100 feet deep, see Table 7.

Table 4. Level Cuttings.

Roadway 18 feet, side-slopes 1½ to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		6.72	13.6	20.5	27.6	34.7	42.0	49.4	56.9	64.5
1	72.2	80.1	88.0	96.1	104.2	112.5	120.9	129.4	138.0	146.7
2	155.5	164.5	173.5	182.7	191.9	201.3	210.8	220.4	230.1	240.0
3	249.9	260.0	270.1	280.4	290.8	301.3	311.9	322.6	333.4	344.5
4	355.5	366.7	378.0	389.4	400.9	412.5	424.2	436.0	448.0	460.0
5	472.2	484.5	496.9	509.4	522.0	534.7	547.6	560.5	573.6	586.7
6	600.0	613.4	626.9	640.5	654.2	668.1	682.0	696.1	710.2	724.5
7	738.9	753.4	768.0	782.7	797.6	812.5	827.6	842.7	858.0	873.4
8	888.9	904.5	920.2	936.1	952.0	968.1	984.2	1001	1017	1033
9	1050	1067	1084	1101	1118	1135	1152	1169	1187	1205
10	1222	1240	1258	1276	1294	1313	1331	1349	1368	1387
11	1406	1425	1444	1463	1482	1501	1521	1541	1560	1580
12	1600	1620	1640	1661	1681	1701	1722	1743	1764	1785
13	1806	1827	1848	1869	1891	1913	1934	1956	1978	2000
14	2022	2045	2067	2089	2112	2135	2158	2181	2204	2227
15	2250	2273	2297	2321	2344	2368	2392	2416	2440	2465
16	2489	2513	2538	2563	2588	2613	2638	2663	2688	2713
17	2739	2765	2790	2816	2842	2868	2894	2921	2947	2973
18	3000	3027	3054	3081	3108	3135	3162	3189	3217	3245
19	3272	3300	3328	3356	3384	3413	3441	3469	3498	3527
20	3556	3585	3614	3643	3672	3701	3731	3761	3790	3820
21	3850	3880	3910	3941	3971	4001	4032	4063	4094	4125
22	4156	4187	4218	4249	4281	4313	4344	4376	4408	4440
23	4472	4505	4537	4569	4602	4635	4668	4701	4734	4767
24	4800	4833	4867	4901	4934	4968	5002	5036	5070	5105
25	5139	5173	5208	5243	5278	5313	5348	5383	5418	5453
26	5489	5525	5560	5596	5632	5668	5704	5741	5777	5813
27	5850	5887	5924	5961	5998	6035	6072	6109	6147	6185
28	6222	6260	6298	6336	6374	6413	6451	6489	6528	6567
29	6606	6645	6684	6723	6762	6801	6841	6881	6920	6960
30	7000	7040	7080	7121	7161	7201	7242	7283	7324	7365
31	7406	7447	7488	7529	7571	7613	7654	7696	7738	7780
32	7822	7865	7907	7949	7992	8035	8078	8121	8164	8207
33	8250	8293	8337	8381	8424	8468	8512	8556	8600	8645
34	8689	8733	8778	8823	8868	8913	8958	9003	9048	9093
35	9139	9185	9230	9276	9322	9368	9414	9461	9507	9553
36	9600	9647	9694	9741	9788	9835	9882	9929	9977	10025
37	10072	10120	10168	10216	10264	10313	10361	10409	10458	10507
38	10556	10605	10654	10703	10752	10801	10851	10901	10950	11000
39	11050	11100	11150	11200	11251	11301	11352	11403	11454	11505
40	11556	11607	11658	11709	11761	11813	11864	11916	11968	12020
41	12072	12125	12177	12229	12282	12335	12388	12441	12494	12547
42	12600	12653	12707	12761	12814	12868	12922	12976	13030	13085
43	13139	13193	13248	13303	13358	13413	13468	13523	13578	13633
44	13689	13745	13800	13856	13912	13968	14024	14081	14137	14193
45	14250	14307	14364	14421	14478	14535	14592	14649	14707	14765
46	14822	14880	14938	14996	15054	15113	15171	15229	15288	15347
47	15406	15465	15524	15583	15642	15701	15761	15821	15880	15940
48	16000	16060	16120	16181	16241	16301	16362	16423	16484	16545
49	16606	16667	16728	16789	16851	16913	16974	17036	17098	17160
50	17222	17285	17347	17409	17472	17535	17598	17661	17724	17787
51	17850	17913	17977	18041	18104	18168	18232	18296	18360	18425
52	18489	18553	18618	18683	18748	18813	18878	18943	19008	19073
53	19139	19205	19270	19336	19402	19468	19534	19601	19667	19733
54	19800	19867	19934	20000	20068	20135	20202	20269	20337	20405
55	20472	20540	20608	20676	20744	20813	20881	20949	21018	21087
56	21156	21225	21294	21363	21432	21501	21571	21641	21710	21780
57	21850	21920	21990	22061	22131	22201	22272	22343	22414	22485
58	22556	22627	22698	22769	22841	22913	22984	23056	23128	23200
59	23272	23345	23417	23489	23562	23635	23708	23781	23854	23927
60	24000	24073	24147	24221	24294	24368	24442	24516	24590	24665

For continuation to 100 feet deep, see Table 7.

Table 5. Level Cuttings.

Roadway 28 feet wide, side-slopes 1 to 1.

For double-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		10.4	20.9	31.4	42.1	52.8	63.6	74.4	85.3	96.3
1	107.4	118.6	129.8	141.1	152.4	163.9	175.4	187.0	198.7	210.4
2	222.2	234.1	246.1	258.1	270.2	282.4	294.7	307.0	319.4	331.9
3	344.4	357.1	369.8	382.6	395.4	408.3	421.3	434.4	447.6	460.8
4	474.1	487.4	500.9	514.4	528.0	541.7	555.4	569.2	583.1	597.1
5	611.1	625.2	639.4	653.7	668.0	682.4	696.9	711.4	726.1	740.8
6	755.6	770.4	785.4	800.4	815.5	830.6	845.8	861.1	876.5	891.9
7	907.5	923.0	938.7	954.5	970.3	986.2	1002	1018	1034	1050
8	1067	1083	1099	1116	1132	1149	1166	1182	1199	1216
9	1233	1250	1267	1285	1302	1319	1337	1354	1372	1390
10	1407	1425	1443	1461	1479	1497	1515	1534	1552	1570
11	1589	1607	1626	1645	1664	1682	1701	1720	1739	1759
12	1778	1797	1816	1836	1855	1875	1895	1914	1934	1954
13	1974	1994	2014	2034	2055	2075	2095	2116	2136	2157
14	2178	2199	2219	2240	2261	2282	2304	2325	2346	2367
15	2389	2410	2432	2454	2475	2497	2519	2541	2563	2585
16	2607	2630	2652	2674	2697	2719	2742	2765	2788	2810
17	2833	2856	2879	2903	2926	2949	2972	2996	3019	3043
18	3067	3090	3114	3138	3162	3186	3210	3234	3259	3283
19	3307	3332	3356	3381	3406	3431	3455	3480	3505	3530
20	3556	3581	3606	3631	3657	3682	3708	3734	3759	3785
21	3811	3837	3863	3889	3915	3942	3968	3994	4021	4047
22	4074	4101	4128	4154	4181	4208	4235	4263	4290	4317
23	4344	4372	4399	4427	4455	4482	4510	4538	4566	4594
24	4622	4650	4679	4707	4735	4764	4792	4821	4850	4879
25	4907	4936	4965	4994	5024	5053	5082	5111	5141	5170
26	5200	5230	5259	5289	5319	5349	5379	5409	5439	5470
27	5500	5530	5561	5591	5622	5653	5684	5714	5745	5776
28	5807	5839	5870	5901	5932	5964	5995	6027	6059	6090
29	6122	6154	6186	6218	6250	6282	6315	6347	6379	6412
30	6444	6477	6510	6543	6575	6608	6641	6674	6708	6741
31	6774	6807	6841	6874	6908	6942	6975	7009	7043	7077
32	7111	7145	7179	7214	7248	7282	7317	7351	7386	7421
33	7456	7490	7525	7560	7595	7631	7666	7701	7736	7772
34	7807	7843	7879	7914	7950	7986	8022	8058	8094	8130
35	8167	8203	8239	8276	8312	8349	8386	8423	8459	8496
36	8533	8570	8608	8645	8682	8719	8757	8794	8832	8870
37	8907	8945	8983	9021	9059	9097	9135	9174	9212	9250
38	9289	9327	9366	9405	9444	9482	9521	9560	9599	9639
39	9678	9717	9756	9796	9835	9875	9915	9954	9994	10034
40	10074	10114	10154	10194	10235	10275	10315	10356	10396	10437
41	10478	10519	10559	10600	10641	10682	10724	10765	10806	10847
42	10889	10930	10972	11014	11055	11097	11139	11181	11223	11265
43	11307	11350	11392	11434	11477	11519	11562	11605	11648	11690
44	11733	11776	11819	11863	11906	11949	11992	12036	12079	12123
45	12167	12210	12254	12298	12342	12386	12430	12474	12519	12563
46	12607	12652	12696	12741	12786	12831	12875	12920	12965	13010
47	13056	13101	13146	13191	13237	13282	13328	13374	13419	13465
48	13511	13587	13603	13649	13695	13742	13788	13834	13881	13927
49	13974	14021	14068	14114	14161	14208	14255	14303	14350	14397
50	14444	14492	14539	14587	14635	14682	14730	14778	14826	14874
51	14922	14970	15019	15067	15115	15164	15212	15261	15310	15359
52	15407	15456	15505	15554	15604	15653	15702	15751	15801	15850
53	15900	15950	15999	16049	16099	16149	16199	16249	16299	16350
54	16400	16450	16501	16551	16602	16653	16704	16754	16805	16856
55	16907	16959	17010	17061	17112	17164	17215	17267	17319	17370
56	17422	17474	17526	17578	17630	17682	17735	17787	17839	17892
57	17944	17997	18050	18103	18155	18208	18261	18314	18368	18421
58	18474	18527	18581	18634	18688	18742	18795	18849	18903	18957
59	19011	19065	19119	19174	19228	19282	19337	19391	19446	19501
60	19556	19610	19665	19720	19775	19831	19886	19941	19996	20052

For continuation to 100 feet. see Table 7.

Table 6. Level Cuttings.

Roadway 28 ft wide, side-slopes 1½ to 1.

For double-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.
0		10.4	21.0	31.6	42.4	53.2	64.2	75.3	86.5	97.9
1	109.3	120.8	132.5	144.3	156.1	168.1	180.2	192.4	204.8	217.2
2	229.6	242.3	255.0	267.9	280.9	294.0	307.2	320.5	334.0	347.5
3	361.2	374.9	388.8	402.8	416.9	431.1	445.4	459.9	474.4	489.1
4	503.7	518.6	533.6	548.6	563.9	579.3	594.7	610.2	625.8	641.6
5	657.5	673.4	689.5	705.7	722.1	738.5	755.0	771.7	788.4	805.3
6	822.2	839.3	856.5	873.8	891.2	908.8	926.4	944.2	962.0	980.0
7	998.1	1016	1035	1053	1072	1090	1109	1128	1147	1166
8	1185	1204	1224	1243	1263	1283	1303	1322	1343	1363
9	1383	1403	1424	1445	1465	1486	1507	1528	1549	1571
10	1592	1614	1635	1657	1679	1701	1723	1745	1767	1790
11	1812	1835	1858	1881	1904	1927	1950	1973	1997	2020
12	2044	2068	2092	2116	2140	2164	2189	2213	2238	2262
13	2287	2312	2337	2362	2387	2413	2438	2464	2489	2515
14	2541	2567	2593	2619	2645	2672	2698	2725	2752	2779
15	2806	2833	2860	2887	2915	2942	2970	2997	3025	3053
16	3081	3109	3138	3166	3195	3223	3252	3281	3310	3339
17	3368	3397	3427	3456	3486	3516	3546	3576	3606	3636
18	3667	3697	3728	3758	3789	3820	3851	3882	3913	3944
19	3976	4007	4039	4070	4102	4134	4166	4198	4231	4263
20	4296	4328	4361	4394	4427	4460	4493	4527	4560	4594
21	4627	4661	4695	4729	4763	4797	4832	4866	4900	4935
22	4970	5005	5040	5075	5111	5146	5181	5217	5253	5288
23	5324	5360	5396	5432	5469	5505	5542	5578	5615	5652
24	5689	5726	5763	5800	5838	5875	5913	5951	5989	6027
25	6065	6103	6141	6179	6218	6257	6295	6334	6373	6412
26	6451	6491	6530	6570	6609	6649	6689	6729	6769	6809
27	6850	6890	6931	6971	7012	7053	7094	7135	7176	7217
28	7259	7300	7342	7384	7426	7468	7510	7552	7594	7637
29	7680	7722	7765	7808	7851	7894	7937	7981	8024	8067
30	8111	8155	8199	8243	8287	8331	8375	8420	8464	8509
31	8554	8598	8643	8688	8734	8779	8824	8870	8915	8961
32	9007	9053	9099	9145	9191	9238	9284	9331	9378	9425
33	9472	9519	9566	9613	9661	9708	9756	9804	9851	9900
34	9948	9997	10045	10093	10142	10190	10239	10288	10337	10386
35	10435	10484	10534	10583	10633	10683	10732	10782	10832	10882
36	10933	10983	11034	11084	11135	11186	11237	11288	11339	11391
37	11443	11494	11546	11598	11649	11701	11753	11806	11858	11910
38	11963	12016	12068	12121	12174	12227	12281	12334	12387	12441
39	12494	12548	12602	12656	12710	12764	12819	12873	12928	12982
40	13037	13092	13147	13202	13257	13312	13368	13423	13479	13535
41	13591	13647	13703	13759	13815	13872	13928	13985	14042	14099
42	14156	14213	14270	14327	14385	14442	14500	14558	14615	14673
43	14731	14790	14848	14906	14965	15024	15082	15141	15200	15259
44	15318	15378	15437	15497	15556	15616	15676	15736	15796	15856
45	15917	15977	16038	16098	16159	16220	16281	16342	16403	16465
46	16526	16587	16649	16711	16773	16835	16897	16959	17021	17084
47	17146	17209	17272	17335	17398	17461	17524	17587	17651	17714
48	17778	17842	17905	17969	18033	18098	18162	18226	18291	18356
49	18420	18485	18550	18615	18680	18746	18811	18877	18942	19008
50	19074	19140	19206	19272	19339	19405	19472	19538	19605	19672
51	19739	19806	19873	19940	20008	20075	20143	20211	20279	20347
52	20415	20483	20551	20620	20688	20757	20826	20894	20963	21032
53	21102	21171	21241	21310	21380	21450	21519	21589	21659	21730
54	21800	21870	21941	22012	22082	22153	22224	22295	22366	22438
55	22509	22581	22652	22724	22796	22868	22940	23012	23085	23157
56	23230	23302	23375	23448	23521	23594	23667	23741	23814	23888
57	23961	24035	24109	24183	24257	24331	24405	24480	24554	24629
58	24704	24779	24854	24929	25004	25079	25155	25230	25306	25381
59	25457	25533	25609	25686	25762	25838	25915	25992	26068	26145
60	26222	26299	26376	26454	26531	26609	26686	26764	26842	26920

For continuation to 100 feet, see Table 7.

Table 7. Level Cuttings.

Continuation of the six foregoing Tables of Cubic Contents, to 100 feet of height or depth.

Height or Depth in Feet.	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
61	23835	26094	17848	24739	20107	26998
.5	24201	26479	18108	25113	20386	27390
62	24570	26867	18370	25489	20667	27785
.5	24942	27257	18634	25868	20949	28183
63	25317	27650	18900	26250	21233	28583
.5	25694	28046	19168	26635	21519	28986
64	26074	28444	19437	27022	21807	29393
.5	26457	28846	19708	27413	22097	29801
65	26843	29250	19981	27806	22389	30213
.5	27231	29657	20256	28201	22682	30627
66	27622	30067	20533	28600	22978	31044
.5	28016	30479	20812	29001	23275	31464
67	28413	30894	21093	29406	23574	31887
.5	28812	31313	21375	29813	23875	32312
68	29215	31733	21659	30222	24178	32741
.5	29620	32157	21945	30635	24482	33172
69	30028	32583	22233	31050	24789	33605
.5	30438	33013	22523	31468	25097	34042
70	30852	33444	22814	31889	25407	34481
.5	31268	33879	23108	33313	25719	34924
71	31687	34317	23404	32739	26033	35369
.5	32108	34757	23701	33168	26349	35816
72	32533	35200	24000	33600	26667	36267
.5	32960	35646	24301	34035	26986	36720
73	33390	36094	24604	34472	27307	37176
.5	33823	36546	24907	34913	27631	37635
74	34259	37000	25214	35356	27956	38096
.5	34697	37457	25522	35801	28282	38561
75	35139	37917	25832	36250	28611	39028
.5	35582	38379	26144	36701	28942	39498
76	36029	38844	26458	37156	29174	39970
.5	36479	39313	26774	37613	29608	40446
77	36931	39783	27092	38072	29944	40924
.5	37386	40257	27411	38535	30282	41405
78	37844	40733	27733	39000	30622	41889
.5	38305	41213	28056	39468	30964	42375
79	38768	41694	28381	39939	31307	42865
.5	39235	42179	28708	40413	31653	43357
80	39704	42667	29037	40889	32000	43852
81	40650	43650	29700	41850	32700	44850
82	41607	44644	30370	42822	33407	45859
83	42576	45650	31048	43806	34122	46880
84	43555	46667	31733	44800	34844	47911
85	44546	47694	32426	45806	35574	48954
86	45548	48733	33126	46832	36311	50008
87	46561	49783	33833	47850	37056	51072
88	47585	50844	34548	48889	37807	52148
89	48620	51917	35270	49939	38567	53235
90	49667	53000	36000	51000	39333	54333
91	50724	54094	36737	52072	40107	55443
92	51793	55200	37481	53156	40889	56563
93	52872	56317	38233	54250	41678	57694
94	53963	57444	38993	55356	42474	58837
95	55065	58583	39759	56472	43278	59990
96	56178	59733	40533	57600	44089	61155
97	57302	60894	41315	58739	44907	62331
98	58437	62067	42104	59889	45733	63518
99	59583	63250	42900	61050	46567	64716
100	60741	64444	43704	62222	47407	65926

Table 8,

Of Cubic Yards in a 100-foot station of level cutting or filling, to be added to, or subtracted from, the quantities in the preceding seven tables, in case the excavations or embankments should be increased or diminished 2 feet in width.

Cubic Yards in a length of 100 feet; breadth 2 feet; and of different depths.

Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.
.5	3.70	.5	152	.5	300	.5	448	.5	596
1	7.41	21	156	41	304	61	452	81	600
.5	11.1	.5	159	.5	307	.5	456	.5	604
2	14.8	22	163	42	311	62	459	82	607
.5	18.5	.5	167	.5	315	.5	463	.5	611
3	22.2	23	170	43	319	63	467	83	615
.5	25.9	.5	174	.5	322	.5	470	.5	619
4	29.6	24	178	44	326	64	474	84	622
.5	33.3	.5	181	.5	330	.5	478	.5	626
5	37.0	25	185	45	333	65	481	85	630
.5	40.7	.5	189	.5	337	.5	485	.5	633
6	44.4	26	193	46	341	66	489	86	637
.5	48.1	.5	196	.5	344	.5	493	.5	641
7	51.9	27	200	47	348	67	496	87	644
.5	55.6	.5	204	.5	352	.5	500	.5	648
8	59.3	28	207	48	356	68	504	88	652
.5	63.0	.5	211	.5	359	.5	507	.5	656
9	66.7	29	215	49	363	69	511	89	659
.5	70.4	.5	219	.5	367	.5	515	.5	663
10	74 1	30	222	50	370	70	519	90	667
.5	77.8	.5	226	.5	374	.5	522	.5	670
11	81.5	31	230	51	378	71	526	91	674
.5	85 2	.5	233	.5	381	.5	530	.5	678
12	88.9	32	237	52	385	72	533	92	681
.5	92.6	.5	241	.5	389	.5	537	.5	685
13	96.3	33	244	53	393	73	541	93	689
.5	100	.5	248	.5	396	.5	544	.5	693
14	104	34	252	54	400	74	548	94	696
.5	107	.5	256	.5	404	.5	552	.5	700
15	111	35	259	55	407	75	556	95	704
.5	115	.5	263	.5	411	.5	559	.5	707
16	119	36	267	56	415	76	563	96	711
.5	122	.5	270	.5	419	.5	567	.5	715
17	126	37	274	57	422	77	570	97	719
.5	130	.5	278	.5	426	.5	574	.5	722
18	133	38	281	58	430	78	578	98	726
.5	137	.5	285	.5	433	.5	581	.5	730
19	141	39	289	59	437	79	585	99	733
.5	144	.5	293	.5	441	.5	589	.5	737
20	148	40	296	60	444	80	593	100	741

REMARK. **The foregoing tables of level cuttings may also be used for widths of roadway greater than those at the heads of the tables.** Thus, suppose we wish to use Table 1, for a roadbed *m n*, 16 ft wide, instead of *c b*, which is only 14 ft, and for which the table was calculated. It is only necessary first to find the vert dist *s a*, between these two roadbeds; and to add it *mentally* to each height *t s*, of the given embkt, when taking out from the

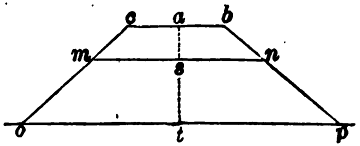


table the numbers of cub yds corresponding to the heights. By this means we obtain the contents of the embkt $cbop$, for any required dist. Next, from these contents subtract that corresponding to the height sa, for the same dist. The remainder will plainly be the embkt $mnop$.

In practice it will be sufficiently correct to take sa to the nearest tenth of a foot, which will save trouble in adding it mentally to the heights in the tables.

If the roadbed is narrower than the table, as, for instance, if mn be the width in the table, but we wish to find the contents for the width cb, then first find sa, and calculate the cubic yards in 100 feet length of $cbmn$. Then, in taking out the cubic yards from the table, first subtract sa mentally from each height; and to the cubic yards taken out for each 100 feet, opposite this *reduced* height, add the cubic yards in 100 feet of $cbmn$.

To avoid trouble with contractors about the measurement of rock cuts, stipulate in the contract, either that it shall conform with the theoretical cross section; or that an extra allowance of say about 2 feet of width of cut will be made, to cover the unavoidable irregularities of the sides.

Shrinkage of Embankment. Although earth, when first dug, and loosely thrown out, *swells* about ⅕ part, so that a cubic yard *in place* averages about 1⅕ or 1.2 cubic yards when dug; or 1 cubic yard dug is equal to ⅚, or to .8333 of a cubic yard in place; yet when made into embankment it gradually subsides, settles, or shrinks, into a less bulk than it occupied *before being dug.*

The following are approximate averages of the shrinkage; or, in other words, the earth measured in place in a cut, will, when made into embankment, occupy a bulk less than before by about the following proportions:

Gravel or sand	about	8 per ct;	or 1 in 12½ less.
Clay	"	10 per ct;	or 1 in 10 less.
Loam	"	12 per ct;	or 1 in 8⅓ less.
Loose vegetable surface soil	"	15 per ct;	or 1 in 6⅔ less.
Puddled clay	"	25 per ct;	or 1 in 4 less.

The writer thinks, from some trials of his own, that 1 cubic yard of any hard rock in place, will make from 1⅝ to 1¾ cubic yards of embankment; say on an average 1.7 cubic yards. Or that 1 cubic yard of rock embankment requires .5882 of a cubic yard in place. He found that a solid cubic yard when broken into fragments, made about as follows

	Cubic yards.	Of which there were	
		Solid	Voids
In loose heap	1.9	52.6 per cent.	47.4 per cent.
Carelessly piled	1.75	57 "	43 "
Carefully piled	1.6	63 "	37 "
Rubble, very carelessly scabbled	1.5	67 "	33 "
Rubble, somewhat carefully scabbled	1.25	80 "	20 "

COST OF EARTHWORK.

Art. 1. It is advisable to pay for this kind of work by the cubic yard of *excavation* only; instead of allowing separate prices for excavation and embankment. By this means we get rid of the difficulty of measurements, as well as the controversies and lawsuits which often attend the determination of the allowance to be made for the settlement or subsidence of the embankments.

It is, moreover, our opinion that justice to the contractor should lead to the **English practice of paying the laborers by the cubic yard**, instead of by the day. Experience fully proves that when laborers are scarce and wages high, men can scarcely be depended upon to do three-fourths of the work which they readily accomplish when wages are low, and when fresh hands are waiting to be hired in case any are discharged. The contractor is thus placed at the mercy of his men. The writer has known the most satisfactory results to attend a system of task-work, accompanied by liberal premiums for all overwork. By this means the interests of the laborers are identified with that of the contractor; and every man takes care that the others shall do their fair share of the task.

Ellwood Morris, C E, of Philadelphia, was, we believe, the first person who properly investigated the elements of cost of earthwork, and reduced them to such a form as to enable us to calculate the total with a considerable degree of accuracy. He published his results in the Journal of the Franklin Institute in 1841. His paper forms the basis on which, with some variations, we shall consider the matter: and on which we shall extend it to wheelbarrows, as well as to carts. Throughout this paper we speak of a cubic yard considered only as solid in its place, or before it is loosened for removal. It is scarcely necessary to add that the various items can of course only be regarded as tolerably close approximations, or averages. As before stated, the men do less work when wages are high; and more when they are low. A great deal besides depends on the skill, observation, and energy of the contractor and his superintendents. It is no unusual thing to see two contractors working at the same prices, in precisely similar material, where one is making money, and the other losing it, from a want of tact in the proper distribution of his forces, keeping his roads in order, having his carts and barrows well filled, &c, &c. Uncommonly long spells of wet weather may seriously affect the cost of executing earthwork, by making it more difficult to loosen, load, or empty; besides keeping the roads in bad order for hauling.

The aggregate cost of excavating and removing earth is made up by the following items, namely:

1st. *Loosening the earth ready for the shovellers.*
2d. *Loading it by shovels into the carts or barrows.*
3d. *Hauling, or wheeling it away, including emptying and returning.*
4th. *Spreading it out into successive layers on the embankment.*
5th. *Keeping the hauling-road for carts, or the plank gangways for barrows, in good order.*
6th. *Wear, sharpening, depreciation, and interest on cost of tools.*
7th. *Superintendence, and water-carriers.*
8th. *Profit to the contractor.*

We will consider these items a little in detail, basing our calculations on the assumption that common labor costs $1 per day, of 10 working hours. The results in our tables must therefore be increased or diminished in about the same proportion as common labor costs more or less than this.

Art. 2. Loosening the earth ready for the shovellers. This is generally done either by ploughs or by picks; more cheaply by the first. A plough with two horses, and two men to manage them, at $1 per day for labor, 75 cents per day for each horse, and 37 cents per day for plough, including harness, wear, repairs, &c, or a total of $3.87, will loosen, of strong heavy soils, from 200 to 300 cubic yards a day, at from 1.93 to 1.29 cents per yard; or of ordinary loam, from 400 to 600 cubic yards a day, at from .97 to .64 of a cent per yard. Therefore, as an ordinary average, we may assume the actual cost to the contractor for loosening by the plough, as follows: strong heavy soils, 1.6 cents; common loam, .8 cent; light sandy soils, .4 cent. Very stiff pure clay, or obstinate cemented gravel, may be set down at 2.5 cents; they require three or four horses.

By the pick, a fair day's work is about 14 yards of stiff pure clay, or of cemented gravel; 25 yards of strong heavy soils; 40 yards of common loam; 60 yards of light sandy soils — all measured in place; which, at $1 per day for labor, gives, for stiff clay, 7 cents; heavy soils, 4 cents; loam, 2.5 cents; light sandy soil, 1.666 cents. Pure sand requires but very little labor for loosening; .5 of a cent will cover it.

Art. 3. Shovelling the loosened earth into carts. The amount shovelled per day depends partly upon the weight of the material, but more upon so proportioning the number of pickers and of carts to that of shovellers, as not to keep the latter waiting for either material or carts. In fairly regulated gangs, the shovellers into carts are not actually engaged in shovelling for more than six-tenths of their time, thus being unoccupied but four-tenths of it; while, under bad management, they lose considerably more than one-half of it. A shoveller can readily load into a cart one-third of a cubic yard measured in place (and which is an average working cartload), of sandy soil, in five minutes; of loam, in six minutes: and of any of the heavy soils, in seven minutes. This would give, for a day of 10 working hours, 120 loads, or 40 cubic yards of light sandy soil; 100 loads, or 33⅓ cubic yards of loam; or 86 loads, or 28.7 yards of the heavy soils. But from these amounts we must deduct four-tenths for time necessarily lost; thus reducing the actual working quantities to 24 yards of light sandy soil, 20 yards of loam, 17.2 yards of the heavy soils. When the shovellers do less than this, there is some mismanagement.

Assuming these as fair quantities, then, at $1 per day for labor, the actual cost to the contractor for shovelling per cubic yard measured in place, will be, for sandy soils, 4.167 cents; loam, 5 cents; heavy soils, clays, &c, 5.81 cents.

In practice, the carts are not usually loaded to any less extent with the heavier soils than with the lighter ones. Nor, indeed, is there any necessity for so doing, inasmuch as the difference of weight of a cart and one-third of a cubic yard of the various soils is too slight to need any attention; especially when the cart-road is kept in good order, as it will be by any contractor who understands his own interest. Neither is it necessary to modify the load on account of any *slight inclinations* which may occur in the grading of roads. An earth-cart weighs by itself about ⅓ a ton.

Art. 4. Hauling away the earth; dumping, or emptying; and returning to reload. The average speed of horses in hauling is about 2¼ miles per hour, or 200 feet per minute; which is equal to 100 feet of trip each way; or to 100 feet of *lead*, as the distance to which *the earth is hauled* is technically called.* Besides this, there is a loss of about four minutes in every trip, whether long or short, in waiting to load, dumping, turning, &c. Hence, every trip will occupy as many minutes as there are lengths of 100 feet each in the lead; and four minutes besides. Therefore, to find the number of trips per day over any given average lead, we divide the number of minutes in a working day by the sum of 4 added to the number of 100 feet lengths contained in the distance to which the earth has to be removed; that is,

$$\frac{\textit{The number (600) of minutes in a working day}}{4+\textit{the number of 100-feet lengths in the lead}}=\begin{array}{l}\textit{the number of trips, or loads}\\ \textit{removed per day, per cart.}\end{array}$$

And since ⅓ of a cubic yard measured before being loosened, makes an average cart-load, the number of loads, divided by 3, will give the number of cubic yards removed per day by each cart; and the cubic yards divided into the total expense of a cart per day, will give the cost per cubic yard for hauling.

Remark. When removing loose *rock*, which requires more time for loading, say,

$$\frac{\textit{No. of minutes (600) in a working day}}{6+\textit{No. of 100-feet lengths of lead}}=\begin{array}{l}\textit{No. of loads removed,}\\ \textit{per day, per cart.}\end{array}$$

In leads of ordinary length one driver can attend to 4 carts; which, at $1 per day, is 25 cents per cart. When labor is $1 per day, the expense of a horse is usually about 75 cents; and that of the cart, including harness, tar, repairs, &c, 25 cents, making the total daily cost per cart $1.25. The expense of the horse is the same on Sundays and on rainy days, as when at work; and this consideration is included in the 75 cents. Some contractors employ a greater number of drivers, who also help to load the carts, so that the expense is about the same in either case.

Example. How many cubic yards of loam, measured in the cut, can be hauled by a horse and cart in a day of 10 working hours, (600 minutes,) the lead, or length of haul of earth being 1000 feet, (or 10 lengths of 100 feet,) and what will be the expense to the contractor for hauling, per cubic yard, assuming the total cost of cart, horse, and driver, at $1.25?

$$\textit{Here, }\frac{600\textit{ minutes}}{4+10\textit{ lengths of }100\textit{ feet,}}=\frac{600}{14}=43\textit{ loads.}\qquad\textit{And }\frac{43\textit{ loads}}{3}=14.3\textit{ cubic yards.}$$

$$\textit{And }\frac{125\textit{ cents}}{14.3\textit{ cub yds}}=8.74\textit{ cents per cubic yard.}$$

In this manner the 2d and 3d columns of the following tables have been calculated.

Art. 5. Spreading, or levelling off the earth into regular thin layers on the embankment. A bankman will spread from 50 to 100 cubic yards of either common loam, or any of the heavier soils, clays, &c, depending on their dryness. This, at $1 per day, is 1 to 2 cents per cubic yard; and we may assume 1½ cents as a fair average for such soils: while 1 cent will suffice for light sandy soils.

This expense for spreading is saved when the earth is either dumped over the end of the embankment, or is wasted; still, about ¼ cent per yard should be allowed in either case for keeping the dumping-places clear and in order.

Art. 6. Keeping the cart-road in good order for hauling. No ruts or puddles should be allowed to remain unfilled; rain should at once be led off by shallow ditches; and the road be carefully kept in good order; otherwise the labor of the horses, and the wear of carts, will be *very greatly* increased. It is usual to allow so much per cubic yard for road repairs; but we suggest so much per cubic yard, per 100 feet of lead; say $\frac{1}{10}$ of a cent.

Art. 7. Wear, sharpening, and depreciation of picks and shovels. Experience shows that about ¼ of a cent per cubic yard will cover this item.

Superintendence and water-carriers. These expenses will vary with local circumstances; but we agree with Mr. Morris, that 1¼ cents per cubic yard will, under ordinary circumstances, cover both of them. An allowance of about ¼ cent may in justice be added for extra trouble in digging the side-ditches; levelling off the bottom of the cut to grade; and general trimming up. In very light cuttings this may be increased to ½ cent per cubic yard.

At ¼ cent, all the items in this article amount to 2 cents per cubic yard of cut.

Art. 8. Profit to the contractor. This may generally be set down at from 6 to 15 per cent, according to the magnitude of the work, the risks incurred, and various incidental circumstances. Out of this item the contractor generally has to pay clerks, storekeepers, and other agents, as well as the expenses of shanties, &c; although these are in most cases repaid by the profits of the stores; and by the rates of boarding and lodging paid to the contractors by the laborers.

Art. 9. A knowledge of the foregoing items enables us to calculate with tolerable accuracy the cost of removing earth. For example, let it be required to ascertain the cost per cubic yard of excavating common loam, measured in place; and of removing it into embankment, with an average haul or lead of 1000 feet: the wages of laborers being $1 per day of 10 working hours; a horse 75 cts a day; and a cart 25 cts. One driver to four carts.

---

* When an entire cut is made into an embankment, the **mean haul** is the dist between centers of gravity of the cut and embkt.

		Cents.
Here we have cost of loosening, say by pick,	*Art* 2, *per cubic yard, say,*	2.50
Loading into carts,	*Art.* 3, " "	5.00
Hauling 1000 *feet, as calculated previously in example, Art.* 4, "		8.74
Spreading into layers,	*Art.* 5, "	1.50
Keeping cart-road in repair,	*Art.* 6, 10 *lengths of* 100 *ft*,	1.00
Various items in	*Art.* 7,	2.00
	Total cost to contractor,	20.74
	Add contractor's profit, say 10 *per cent,*	2.074
	Total cost per cubic yard to the company,	22.814

It is easy to construct a table like the following, of costs per cubic yard, for different lengths of lead. Columns 2 and 3 are first obtained by the Rule in Article 4; then to each amount in column 3 is added the *variable* quantity of $\frac{1}{10}$ of a cent for every 100 feet length of lead, for keeping the road in order; and the *constant* quantity (for any given kind of soil) composed of the prices per cubic yard. for loosening, loading, spreading, or wasting, &c, either taken from the preceding articles; or modified to suit particular circumstances. In this manner the tables have been prepared.

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	47.0	2.66	13.69	12 44	11.99	10.74	16.00	14.75	13.50	12.25
50	44.4	2.81	13.86	12.61	12.16	10.91	16.17	14.92	13.67	12.42
75	42.1	2.97	14.06	12.80	12.35	11.10	16.36	15.11	13.86	12.61
100	40.0	3.12	14.22	12.97	12.52	11.27	16.53	15.28	14.03	12.78
150	36.4	3.43	14.58	13.33	12.88	11.63	16.89	15.64	14.39	13.14
200	33.3	3.75	14.95	13.70	13.25	12.00	17.26	16.01	14.76	13.51
300	28.6	4.37	15.67	14.42	13.97	12.72	17.98	16.73	15.48	14.23
400	25 0	5.00	16.40	15.15	14.70	13.45	18.71	17.46	16.21	14.96
500	22.2	5.63	17.13	15 88	15.43	14.18	19.44	18.19	16.94	15.69
600	20.0	6.25	17.85	16.60	16.15	14.90	20.16	18.91	17.66	16.41
700	18.2	6.87	18.57	17.32	16.87	15.62	20.88	19.63	18.38	17.13
800	16.7	7.48	19.28	18.03	17.58	16.33	21.59	20.34	19.09	17.84
900	15.4	8.12	19.92	18.67	18.22	16.97	22.23	20.98	19.73	18.48
1000	14.3	8.74	20.74	19.49	19.04	17.79	23.05	21 80	20.55	19.30
1100	13.3	9.40	21.50	20.25	19.80	18.55	23.81	22.56	21.31	20.06
1200	12.5	10.0	22.20	20.95	20.50	19 25	24.51	23.26	22.01	20.76
1300	11.8	10.6	22.90	21.65	21.20	19.95	25.21	23.96	22.71	21.46
1400	11.1	11.2	23.60	22.35	21.90	20.65	25.91	24.66	23.41	22.16
1500	10.5	11.9	24.40	23.15	23.70	21.45	26.71	25.46	24.21	22.96
1600	10.0	12.5	25.10	23.85	23.40	22.15	27.41	26.16	24 91	23.66
1700	9.52	13.1	25.80	24.55	24.10	22.85	28.11	26.86	25 61	24.36
1800	9.09	13.7	26.50	25.25	24.80	23.55	28.81	27.56	26.31	25.06
1900	8.70	14.4	27.30	26.05	25.60	24.35	29.61	28.36	27.11	25.86
2000	8.33	15.0	28.00	26.75	26.30	25.05	30.31	29.06	27.81	26.56
2250	7.54	16.6	29.85	28.60	28.15	26.90	32.16	30.91	29.66	28.41
2500	6 90	18.1	31.60	30.35	29 90	28.65	33.91	32.66	31.41	30.16
½ mile	6.58	19.0	32.64	31.39	30.94	29.69	34.95	33.70	32.45	31.20
3000	5.88	21.2	35.20	33.95	33.50	32.25	37.51	36.26	35.01	33.76
3250	5.48	22.8	37.05	35.80	35.35	34.10	39.36	38.11	36.86	35.61
3500	5.13	24.3	38.80	37.55	37.10	35.85	41.11	39.86	38.61	37.36
3750	4.82	25.9	40.65	39.40	38.95	37.70	42.96	41.71	40.46	39.21
4000	4.54	27.5	42.50	41.25	40.80	39.55	44.81	43.56	42.31	41.06
4250	4.30	29.1	44.35	43.10	42.65	41.40	46.66	45.41	44.16	42.91
4500	4.08	30.6	46.10	44.85	44.40	43.15	48.41	47.16	45.91	44.66
4750	3.88	32.2	47.95	46.70	46.25	45.00	50.26	49.01	47.76	46.51
5000	3.70	33.8	49.80	48.55	48.10	46.85	52.11	50 86	49.61	48.36
1 mile	3.52	35.5	51.78	50.53	50.08	48.83	54.09	52.84	51.59	50.34
1¼ m.	2.86	43.8	61.40	60.15	59.70	58.45	63.71	62.46	61.21	59.96
1½ m.	2.40	52.1	71.02	69.77	69.32	68.07	73.33	72.08	70.83	69.58
1¾ m.	2.07	60.4	80.64	79.39	78.94	77.69	82.95	81.70	80.45	79.20
2 m.	1.82	68.7	90.26	89.01	88.56	87.31	92.57	91.32	90.07	88.82

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Pure stiff Clay, or cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu.Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	47.0	2.66	19.00	17.75	14.50	13.25	11.52	10.77	10.25	9.50
50	44.4	2.81	19.17	17.92	14.67	13.42	11.69	10.94	10.42	9.67
75	42.1	2.97	19.36	18.11	14.86	13.61	11.88	11.13	10.61	9.86
100	40.0	3.12	19.53	18.28	15.03	13.78	12.05	11.30	10.78	10.03
150	36.4	3.43	19.89	18.64	15.39	14.14	12.41	11.66	11.14	10.39
200	33.3	3.75	20.26	19.01	15.76	14.51	12.78	12.03	11.51	10.76
300	28.6	4.37	20.98	19.73	15.48	15.23	13.50	12.75	12.23	11.48
400	25.0	5.00	21.71	20.46	17.21	15.96	14.23	13.48	12.46	12.21
500	22.2	5.63	22.44	21.19	17.94	16.69	14.96	14.21	13.69	12.94
600	20.0	6.25	23.16	21.91	18.66	17.41	15.68	14.93	14.41	13.66
700	18.2	6.87	23.88	22.63	19.38	18.13	16.40	15.65	15.13	14.38
800	16.7	7.48	24.59	23.34	20.09	18.84	17.11	16.36	15.84	15.09
900	15.4	8.12	25.23	23.98	20.73	19.48	17.75	17.00	16.48	15.73
1000	14.3	8.74	26.05	24.80	21.55	20.30	18.57	17.82	17.30	16.55
1100	13.3	9.40	26.81	25.56	22.31	21.06	19.33	18.58	18.06	17.31
1200	12.5	10.0	27.51	26.26	23.01	21.76	20.03	19.28	18.76	18.01
1300	11.8	10.6	28.21	26.96	23.71	22.46	20.73	19.98	19.46	18.71
1400	11.1	11.2	28.91	27.66	24.41	23.16	21.43	20.68	20.16	19.41
1500	10.5	11.9	29.71	28.46	25.21	23.96	22.23	21.48	20.96	20.21
1600	10.0	12.5	30.41	29.16	25.91	24.66	22.93	22.18	21.66	20.91
1700	9.52	13.1	31.11	29.86	26.61	25.36	23.63	22.88	22.36	21.61
1800	9.09	13.7	31.81	30.56	27.31	26.06	24.33	23.58	23.06	22.31
1900	8.70	14.4	32.61	31.36	28.11	26.86	25.13	24.38	23.86	23.11
2000	8.33	15.0	33.31	32.06	28.81	27.56	25.83	25.08	24.56	23.81
2250	7.54	16.6	35.16	33.91	30.66	29.41	27.68	26.93	26.41	25.66
2500	6.90	18.1	36.91	35.66	32.41	31.16	29.43	28.68	28.16	27.41
½ mile	6.58	19.0	37.95	36.70	33.45	32.20	30.47	29.72	29.20	28.45
3000	5.88	21.2	40.51	39.26	36.01	34.76	33.03	32.28	31.76	31.01
3250	5.48	22.8	42.36	41.11	37.86	36.61	34.88	34.13	33.61	32.86
3500	5.13	24.3	44.11	42.86	39.61	38.36	36.63	35.88	35.36	34.61
3750	4.82	25.9	45.96	44.71	41.46	40.21	38.48	37.73	37.21	36.46
4000	4.54	27.5	47.81	46.56	43.31	42.06	40.33	39.58	39.06	38.31
4250	4.30	29.1	49.66	48.41	45.16	43.91	42.18	41.45	40.93	40.18
4500	4.08	30.6	51.41	50.16	46.91	45.66	43.93	43.18	42.66	41.91
4750	3.88	32.2	53.26	52.01	48.76	47.51	45.78	45.03	44.51	43.76
5000	3.70	33.8	55.11	53.86	50.61	49.36	47.63	46.88	46.36	45.61
1 mile	3.52	35.5	57.09	55.84	52.59	51.34	49.61	48.86	48.34	47.59
1¼ m.	2.86	43.8	66.91	65.46	62.21	60.96	59.23	58.48	57.96	57.21
1½ m.	2.40	52.1	76.33	75.08	71.83	70.58	68.85	68.10	67.58	66.83
1¾ m.	2.07	60.4	85.95	84.70	81.45	80.20	78.47	77.72	77.20	76.45
2 m.	1.82	68.7	95.57	94.32	91.07	89.82	88.09	87.34	86.82	86.07

Art. 10. By wheelbarrows. The cost by barrows may be estimated in the same manner as by carts. See Articles 1, &c. Men in wheeling move at about the same average rate as horses do in hauling, that is, 2¼ miles an hour, or 200 feet per minute, or 1 minute per every 100-feet length of lead. The time occupied in loading, emptying, &c (when, as is usual, the wheeler loads his own barrow,) is about 1.25 minutes, without regard to length of lead; besides which, the time lost in occasional short rests, in adjusting the wheeling plank, and in other incidental causes, amounts to about $\frac{1}{10}$ part of his whole time; so that we must in practice consider him as actually working but 9 hours out of his 10 working ones. Therefore

$$\frac{\textit{The number of minutes in a working day} \times .9}{1.25 + \textit{the number of } 100\textit{-feet lengths of lead}} = \textit{the number of trips or of loads removed per day per barrow.}$$

See Remark, next page.

The number of loads divided by 14 will give the number of cub yards, since a cub yard, measured in place, averages about 14 loads. And the cost of a wheeler and barrow per day, (say $1 per man, and 5 cents per barrow,) divided by the number of cub yards, will give the cost per yard for loading wheeling, and emptying.

Ex. How many cubic yards of common loam, measured in place, will one man load, wheel, and empty, per day of 10 working hours, (or 600 minutes;) the lead, or distance to which the earth is removed being 1000 feet, (or 10 lengths of 100 feet;) and what will be the expense per yard, supposing the laborer and barrow to cost $1.05 per day?

$$\textit{Here, } \frac{600 \textit{ minutes} \times .9}{1.25 + 10 \textit{ lengths}} = \frac{540}{11.25} = 48 \textit{ trips, or loads per day.}$$

$$\textit{And } \frac{48}{14} = 3.43 \textit{ cub yds per day.} \quad \textit{And } \frac{105 \textit{ cents}}{3.43 \textit{ cub yds}} = 30.6 \textit{ cents}$$

per cub yard for loading, wheeling away, emptying, and returning. This would be increased almost inappreciably by the cost of the shovel, which, in the following tables, however, is included in the cost of tools.

Rem. For rock, which requires more time for loading, say

$$\frac{\textit{No of minutes in a working day} \times .9}{1.6 + \textit{No of 100-feet lengths of lead}} = \begin{array}{l}\textit{No of loads removed}\\ \textit{per day, per barrow.}\end{array}$$

Art. 11. The following tables are calculated as in the case of carts, by first finding columns 2 and 3 by means of the Rule in Art 10, and then adding to each sum in column 3, the variable quantity of .1 of a cent per cubic yard per 100 feet of lead for keeping the wheeling-planks in order; and the prices of loosening, spreading, superintendence, water-carrying, &c, per cubic yard, as given in the preceding Articles 2 to 7.

By Wheelbarrows. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled, in feet.	Number of cubic yards in place, loaded, and wheeled per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Common Loam.				Strong, Heavy Soils.			
			TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	10.12	8.87	8.42	7.17	11.62	10.37	9.12	7.87
50	22.1	4.75	10.80	9.55	9.10	7.85	12.30	11.05	9.80	8.55
75	19.3	5.44	11.52	10.27	9.82	8.57	13.02	11.77	10.52	9.27
100	17.1	6.14	12.24	10.99	10.54	9.29	13.74	12.49	11.24	9.99
150	14.0	7.50	13.65	12.40	11.95	10.70	15.15	13.90	12.65	11.40
200	11.9	8.82	15.02	13.77	13.32	12.07	16.52	15.27	14.02	12.77
250	10.3	10.2	16.45	15.20	14.75	13.50	17.95	16.70	15.45	14.20
300	9.07	11.6	17.90	16.65	16.20	14.95	19.40	18.15	16.90	15.65
350	8.14	12.9	19.25	18.00	17.55	16.30	20.75	19.50	18.25	17.00
400	7.36	14.3	20.70	19.45	19.00	17.75	22.20	20.95	19.70	18.45
450	6.71	15.6	22.05	20.80	20.35	19.10	23.55	22.30	21.05	19.80
500	6.17	17.0	23.50	22.25	21.80	20.55	25.00	23.75	22.50	21.25
600	5.32	19.7	26.30	25.05	24.60	23.35	27.80	26.55	25.30	24.05
700	4.67	22.5	29.20	27.95	27.50	26.25	30.70	29.45	28.20	26.95
800	4.17	25.2	32.00	30.75	30.30	29.05	33.50	32.25	31.00	29.75
900	3.76	27.9	34.80	33.55	33.10	31.85	36.30	35.05	33.80	32.55
1000	3.43	30.6	37.60	36.35	35.90	34.65	39.10	37.85	36.60	35.35
1200	2.91	36.1	43.30	42.05	41.60	40.35	44.80	43.55	42.30	41.05
1400	2.53	41.5	48.90	47.65	47.20	45.95	50.40	49.15	47.90	46.65
1600	2.24	46.9	54.50	53.45	52.80	51.55	56.00	54.75	53.50	52.25
1800	2.00	52.5	60.30	59.05	58.60	57.35	61.80	60.55	59.30	58.05
2000	1.81	58.0	66.00	64.75	64.30	63.05	67.50	66.25	65.00	63.75
2200	1.66	63.3	71.50	70.25	69.80	68.55	73.00	71.75	70.50	69.25
2400	1.53	68.6	77.00	75.75	75.30	74.05	78.50	77.25	76.00	74.75
½ mile.	1.39	75.5	84.14	82.89	82.44	81.19	85.64	84.39	83.14	81.89

By Wheelbarrows. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled	Number of cubic yards in place, loaded, and wheeled, per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Pure Stiff Clay, or Cemented Gravel.				Light Sandy Soils.			
			TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	14.62	13.37	10.12	8.87	8.79	8.04	7.52	6.77
50	22.1	4.75	15.30	14.05	10.80	9.55	9.47	8.72	8.20	7.45
75	19.3	5.44	16.02	14.77	11.52	10.27	10.19	9.44	8.92	8.17
100	17.1	6.14	16.74	15.49	12.24	10.99	10.91	10.16	9.64	8.89
150	14.0	7.50	18.15	16.90	13.65	12.40	12.32	11.57	11.05	10.30
200	11.9	8.82	19.52	18.27	15.02	13.77	13.69	12.94	12.42	11.67
250	10.3	10.2	20.95	19.70	16.45	15.20	15.12	14.37	13.85	13.10
300	9.07	11.6	22.40	21.15	17.90	16.65	16.57	15.82	15.30	14.55
350	8.14	12.9	23.75	22.50	19.25	18.00	17.92	17.17	16.65	15.90
400	7.36	14.3	25.20	23.95	20.70	19.45	19.37	18.62	18.10	17.35
450	[illegible]	15.6	26.55	25.30	22.05	20.80	20.72	19.97	19.45	18.70
500	6.17	17.0	28.00	26.75	23.50	22.25	22.17	21.42	20.90	20.15
600	5.32	19.7	30.80	29.55	26.30	25.05	24.97	24.22	23.70	22.95
700	4.67	22.5	33.70	32.45	29.20	27.95	27.87	27.12	26.60	25.85
800	4.17	25.2	36.50	35.25	32.00	30.75	30.67	29.92	29.40	28.65
900	3.76	27.9	39.30	38.05	34.80	33.55	33.47	32.72	32.20	31.45
1000	3.43	30.6	42.10	40.85	37.60	36.35	36.27	35.52	35.00	34.25
1200	2.91	36.1	47.80	46.55	43.30	42.05	41.97	41.22	40.70	39.90
1400	2.53	41.5	53.40	52.15	48.90	47.65	47.57	46.82	46.30	45.55
1600	2.24	46.9	59.00	57.75	54.50	53.25	53.17	52.42	51.90	51.15
1800	2.00	52.5	64 80	63.55	60.30	59.05	58.97	58.22	57.70	56.95
2000	1.81	58.0	70.50	69.25	66.00	64.75	64.67	63.92	63.40	62.65
2200	1.66	63.3	76.00	74.75	71.50	70.25	70 17	69.42	68.90	68.15
2400	1.53	68.6	81.50	80.25	77.00	75.75	75.67	74.92	74.40	73.65
½ mile.	1.39	75.5	88.64	87.39	84.14	82.89	82.81	82.06	81.54	80.79

Art. 12. By wheeled scrapers and drag scrapers. The body of the wheeled scraper is a box of smooth sheet-steel about 3½ ft square by 15 ins deep, containing about ½ cubic yard of earth when "even full." The box is open in front (in some machines it is closed by an "end gate" when full), and can be raised and lowered, and revolved on a horizontal axis. To fill the box, it is lowered into, and held down in, the earth, while the team draws the machine forward. When full, it is raised to about a foot above ground; and, on reaching the dump, is unloaded by being overturned on its axis. All the movements of the box are made by means of levers, and without stopping the team, which thus travels constantly. The wheels have broad tires, to prevent them from cutting into the ground.

In the *drag* scraper the box, owing to the greater resistance to traction, is made much smaller. It contains about .15 to .25 cubic yard in place, and is always open in front. The operation of the drag scraper is similar to that of the wheeled scraper, except that the box, when filled, rests upon the ground and is dragged over it by the team.

Each scraper ("wheeled" or "drag") requires the constant use of a team of two horses with a driver. Besides, a number of men, depending on the shortness of the lead and the number of scrapers, are required in the pit and at the dump to load the scrapers (by holding the box down into the earth) and unload them (by tipping the box). Except in sand, or in very soft soil, it is economical to use a plow before scraping.

The severest work for the team is the filling of the box; and this occurs oftenest where the lead is shortest. Hence smaller scrapers are used on short than on long hauls. We base our calculations on the following loads:

For drag scrapers (used only on short hauls)	.2	cubic yard
For wheeled scrapers		
lead less than 100 feet	.33	"
" 100 to 300 feet	.4	"
" 400 to 500 feet	.5	"
" over 500 feet	.6	"

The daily expense per scraper, for driver's wages and the use of a 2-horse team, is about $3.50. For leads of 400 feet and over, we add 50 cts per day for use of "snatch team" to help load the larger scrapers then used. One snatch team generally serves a number of scrapers.

Owing to the fact that the teams are constantly in motion without rest, they travel somewhat more slowly than with carts. We take 150 ft per minute (or 75 ft of *lead* per minute) as an average.

In loading and unloading, the teams not only go out of their way in order to turn around, but travel more slowly than when simply hauling. To cover this we make an addition of [redacted] to each length of lead, whether long or short, ~~for wheeled scrapers~~; ~~and 15 feet for drag scrapers.~~

We add 1 cent per cubic yard for the cost of loading and dumping the scrapers; and estimate the approximate cost of the other items as follows:

Repairs of cart-road $\frac{1}{10}$ ct per cub yd in place for each 100 ft of lead

	Light Soils cts per cub yd in place	Heavy Soils cts per cub yd in place
Loosening		
by pick	*	5.
by shovel	*	2.
Spreading	1.	1.5
Superintendence, wear and tear etc	1.	1.

We repeat that our figures are to be regarded merely as tolerable approximations, and subject to great variations according to skill of contractor and superintendent, strength of teams, character of material moved, state of weather etc etc.

$$\text{No. of trips per day per } \textit{wheeled} \text{ scraper} = \frac{\text{No. (600) of mins in a working day}}{\text{No. of 75 ft lengths in (lead + [redacted] ft)}}$$

$$\text{No. of trips per day per } \textit{drag} \text{ scraper} = \frac{\text{No. (600) of mins in a working day}}{\text{No. of 75 ft lengths in (lead + [redacted] ft)}}$$

$$\text{No. of cub yds in place moved per day by each scraper} = \text{No. of trips per day per scraper} \times \text{No. of cub yds in place, per scraper per trip}$$

$$\text{Cost per cub yd in place, for loading, hauling, dumping and returning} = \frac{\text{Daily expense of one scraper}}{\text{No. of cub yds in place, moved per day by each scraper}} + \text{1 ct for loading and dumping}$$

$$\text{Total cost per cubic yard in place exclusive of contractor's profit} = \text{Cost per cub yd in place, for loading, hauling, dumping, and returning} + \text{.1 ct per cub yd in place, for each 100 ft of lead, for repairs of road} + \text{Cost, per cub yd in place, of loosening, spreading or wasting, and superintendence \&c.}$$

not to be used - see formulae above

By Wheeled Scrapers. Labor $1 per day of 10 working hours.

Length of lead, or dist to which earth is hauled	Quantity in place, hauled per day by each scraper	Cost per cub yd in place, for loading, hauling, dumping, and returning.	Total cost, per cubic yard in place, exclusive of contractor's profit.					
			Light Soils		Heavy Soils			
					Picked and		Plowed and	
			Spread	Wasted	Spread	Wasted	Spread	Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
50	200	2.8	4.9	3.9	10	8.5	7.3	5.8
100	140	3.4	5.5	4.5	11	9.5	8	6.5
150	105	4.3	6.5	5.5	12	11	9	7.5
200	80	5.4	7.6	6.6	13	12	10	8.5
300	56	7.3	9.6	8.6	15	14	12	11
400	50	8.5	11	10	16	15	13	12
600	43	10	13	12	18	17	15	14
800	33	13	16	15	21	20	18	17
1000	27	16	19	18	25	24	22	21

---

* Light soils can generally be advantageously loosened by the scrapers themselves in the act of loading.

By Drag Scrapers. Labor $1 per day of 10 working hours.

Length of lead, or dist to which earth is hauled	Quantity in place, hauled per day by each scraper	Cost, per cub yd in place, for loading, hauling, dumping and returning	Total cost, per cubic yard in place, exclusive of contractor's profit.					
			Light Soils		Heavy Soils			
					Picked and		Plowed and	
			Spread	Wasted	Spread	Wasted	Spread	Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
less than 40	220	2.6	4.6	3.6	10	8.5	7	5.5
50	140	3.5	5.5	4.5	11	9.5	8	6.5
75	100	4.5	6.6	5.6	12	11	9	8
100	80	5.4	7.5	6.5	13	12	10	9
150	54	7.5	9.6	8.6	15	14	12	11
200	42	9.3	12.	11	17	16	14	13

which is used in formula above

Art. 13. By cars and locomotive, on level track. We have based our calculations upon the following assumptions: Trains of 10 cars, each car containing 1½ cubic yards of earth measured in place. Average speed of trains, including starting and stopping, but not standing, 10 miles per hour, = 5 miles of *lead* per hour. Labor $1 per day of 10 working hours. Loosening, loading (by shovelers), spreading, wear &c of tools, superintendence, &c, the same as with carts, Arts 2, 3, 5, and 7. Loss of time in each trip for loading, unloading, &c, 9 minutes, = 0.15 hour. Therefore

$$\left.\begin{array}{l}\text{Number of trips per}\\ \text{day, per train}\end{array}\right\} = \frac{\text{The number (10) of hours in a working day}}{.15 + \text{the number of 5-mile lengths in the lead}}$$

$$\left.\begin{array}{l}\text{Number of cubic}\\ \text{yards in place, per}\\ \text{day per train}\end{array}\right\} = \begin{array}{l}\text{Number of}\\ \text{trips per day}\\ \text{per train}\end{array} \times \begin{array}{l}\text{Number (10)}\\ \text{of cars in a}\\ \text{train}\end{array} \times \begin{array}{l}\text{Number (1.5) of cubic}\\ \text{yards in place in each}\\ \text{car}\end{array}$$

$$\left.\begin{array}{l}\text{Cost per cubic yard, in place,}\\ \text{for hauling, dumping, and}\\ \text{returning}\end{array}\right\} = \frac{\text{One day's train expenses} + \text{1 day's cost of track}}{\text{Number of cubic yards in place per day per train}}$$

One day's train expenses:

Cost of 10 cars @ $100	$1000	
" locomotive	3000	
	$4000	
One day's interest at 6 per cent, on cost of train		$0.67
Wages of engine driver (who fires his own engine)		2.00
" foreman at dump		2.00
" 3 men at dump at $1		3.00
Fuel		2.00
Water		1.00
Repairs of locomotive and cars		2.33
Total daily expense of one train		$13.00

The daily expense of track, for interest and repairs, may be taken at $3 for each mile of lead.

Therefore,

Figures do not include depreciation of track, or of rolling stock or [illegible]

$$\left.\begin{array}{l}\text{Cost per cubic yard in place,}\\ \text{for hauling, dumping, and}\\ \text{returning}\end{array}\right\} = \frac{\$13 + (\$3 \text{ for each mile of lead})}{\begin{array}{l}\text{Number of}\\ \text{trips per day}\\ \text{per train}\end{array} \times \begin{array}{l}\text{Number (10)}\\ \text{of cars in a}\\ \text{train}\end{array} \times \begin{array}{l}\text{Number (1.5) of}\\ \text{cubic yards in}\\ \text{each car}\end{array}}$$

$$\left.\begin{array}{l}\text{Total cost per cubic}\\ \text{yard in place, ex-}\\ \text{clusive of contrac-}\\ \text{tor's profit}\end{array}\right\} = \begin{array}{l}\text{Cost per cub yd in}\\ \text{place for hauling,}\\ \text{dumping, and re-}\\ \text{turning}\end{array} + \begin{array}{l}\text{Cost per cubic yard, in place, for}\\ \text{loosening, loading, spreading or}\\ \text{wasting, and superintendence, \&c.}\\ \text{(Arts 2, 3, 5, and 7.)}\end{array}$$

By Cars and Locomotive. Labor $1 per day of 10 working hours.

see note page 807

Length of lead, or distance to which the earth is hauled.	Number of cubic yards, in place, hauled per day by each train.	Cost per cubic yard, in place, for hauling, dumping, and returning.	Total cost per cubic yard, in place, exclusive of contractor's profit.							
			Light Sandy Soils.				Strong Heavy Soils.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Miles.	Cu yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
0.125	855	1.56	10.4	9.6	9.1	8.4	14.9	13.6	11.4	11.1
0.250	750	1.83	10.7	9.9	9.4	8.6	15.1	13.9	12.6	11.4
0.375	660	2.14	11.0	10.2	9.7	9.0	15.4	14.2	12.9	11.7
0.500	600	2.42	11.3	10.5	10.0	9.2	15.7	14.5	13.2	12.0
0.750	495	3.08	11.9	11.2	10.6	9.9	16.4	15.1	13.9	12.6
1.000	420	3.81	12.6	11.9	11.4	10.6	17.1	15.9	14.6	13.4
2.000	270	7.04	15.9	15.1	14.6	13.9	20.3	19.1	17.8	16.6
3.000	195	11.30	20.1	19.4	18.9	18.1	24.6	23.4	22.1	20.9
4.000	150	16.70	25.5	24.8	24.3	23.5	30.0	28.8	27.5	26.3

Where large amounts of work are to be done, the **steam excavator, land dredge or steam shovel** generally economizes time and money. Where the depth of cutting is less than 10 ft, so much time is lost in moving from place to place that the excavators do not work to advantage. In stiff soils, cuttings may be made about from 17 to 20 ft deep without changing the level of the machine. For greater depths in such soils the work is done in two levels, since the bucket or dipper cannot reach so high. But in sand and loose gravel, much deeper cuts may be made from a single level.

The excavator resembles a dredging machine in its appearance and operation. A large plate-steel bucket, like a dredging bucket, with a flat hinged bottom, and provided with steel cutting teeth, is forced into and dragged through the earth by steam power. It dumps its load, by means of the hinged bottom, either into cars for transportation, or upon the waste bank, as desired.

Each machine is mounted on a car of standard gauge, which can be coupled in an ordinary freight train. The car is made of wood or iron, as desired, and is provided with a locomotive attachment, by which it can be moved from point to point as the work proceeds. The machines can be used as **wrecking or derrick cars.** Each machine has a water tank, holding from 300 to 550 gallons, for the supply of its boiler.

Before beginning to excavate, the end of the car nearest the work is lifted from the track by hydraulic or screw jacks, upon which it rests while working.

In stiff soils the excavator leaves the sides of the cut nearly vertical; and the desired slope is afterwards given by pick and shovel. When the soil is hard or much frozen, it may be loosened by blasting in advance of the excavator.

The excavator has to be moved forward (as the work advances) abt 8 ft at a time. As regularly made, it can dig at a distance of 17 ft, horizontally, from the center of the car in any direction, and can dump 12 ft above the track. In sand or gravel it takes out, while actually digging, 3 dipperfuls (= 4½ to 6 cub yards in the dipper, = 3.75 to 5 cubic yards in place) per minute; in stiff clay, 2 dipperfuls per minute (= 3 to 4 cub yards in the dipper, = 2.5 to 3.33 cubic yards in place). An average day's work (10 hours) for a "No 1" machine, including time lost in moving the machine, &c, is about 500 cubic yards in "hard-pan," and from 1200 to 1500 in sand and gravel. This allows for the usual and generally unavoidable delays in having cars ready for the excavator.

The excavators carry about 80 to 90 lbs of steam. They burn from 100 to 150 lbs of good hard or soft coal per hour; and require one engineer, one fireman, one cranesman, and 5 to 10 pitmen, including a boss. The pitmen are laborers, who attend to the jacks, lay track for the excavator and for the dump cars, assist in moving the latter, bring or pump water, &c, &c.

After reaching the site of the work, about 30 minutes are required for getting the excavator into working condition; and an equal length of time, after completion of the work, in getting it ready for transportation.

The following figures are taken from the records of work done by a No 1 machine, from May to Nov, 1883. The material was hard clay with pockets of sand. The expenses per day of 12 working hours, at $1.50 per such day for labor, were

Water (a very high allowance)	$ 5.00	
Coal, 1½ tons bituminous	10.00	
Wages of engineer	4.00	
" " fireman	1.50	
" " cranesman or dipper-tender	2.50	
" " pit boss	3.00	
" " 8 pitmen at $1.50	12.00	
Oil, waste, repairs, &c (estimated)	5.00	
Interest on cost ($7500) of machine	1.25	
		$44.25

Reduced to our standard of $1 for labor per day of 10 working hours, this would be say $30.00 per day. Reduced to the same standard, and allowing for the greater proportional loss of time in stopping at evening and starting in the morning: the average daily quantity excavated, measured in place, was, in shallow cutting, 530 cubic yards; in deep cutting, 1200 cubic yards; average of whole operation, 800 cubic yards. This would make the cost, per cubic yard measured in place, for loosening and loading into cars, 5.67 cts, 2.5 cts, and 3.75 cts respectively; while the cost by ploughing and shoveling, in strong heavy soils, by Arts 2 and 3, is 7.4 cts; and by picking and shoveling, say 10 cts.

Art. 14. Removing rock excavation by wheelbarrows. A cubic yard of hard rock, *in place*, or before being blasted, will weigh about 1.8 tons, if sandstone or conglomerate, (150 lbs per cubic foot;) or 2 tons if good compact granite, gneiss, limestone, or marble, (168 lbs per cubic foot.) So that, near enough for practice in the case before us, we may assume the weight of any of them to be about 1.9 tons, or 4256 lbs per cubic yard, in place; or 158 lbs per cubic foot.

Now, a solid **cubic yard, when broken up** by blasting for removal by wheelbarrows or carts, will occupy a space of about 1.8, or 1⅘ cubic yards; whereas average earth, when loosened, swells to but about 1.2, or 1⅕ of its original bulk in place; although, after being made into embankment, it eventually shrinks into less than its original bulk. In estimating for earth, it is assumed that $\frac{1}{14}$ cubic yard, in place, is a fair load for a wheelbarrow. Such a cubic yard will weigh on an average 2430 lbs, or 1.09 tons; therefore, $\frac{2430}{14} = 174$ lbs, is the weight of a barrow-load, of 2.31 cubic feet of loose earth. Assuming that a barrow of loose rock should weigh about the same as one of earth, we may take it at $\frac{1}{24}$ of a cubic yard; which gives $\frac{4256}{24} = 177$ lbs per load of loose rock, occupying 2 cubic feet of space.

In the following table, columns 2 and 3 are prepared on the same principle as for earth, as directed in Article 10. Column 4 is made up by adding to each amount in column 3, .2 of a cent for each 100 feet length of lead, for keeping the wheeling-planks in order; and 45 cents per cubic yard, in place, as the actual cost for loosening, including tools, drilling, powder, &c; as well as moderate drainage, and every ordinary contingency not embraced in column 3. Contractor's profits, of course, are not here included.

Ample experience shows that when labor is at $1 per day, the foregoing 45 cents per cubic yard, in place, is a sufficiently liberal allowance for loosening hard rock under all ordinary circumstances. In practice it will generally range between 30 and 60 cents; depending on the position of the strata, hardness, toughness, water, and other considerations. Soft shales, and other allied rocks, may frequently be loosened by pick and plough, as low as 15 to 20 cents; while, on the other hand, shallow cuttings of very tough rock, with an unfavorable position of strata, especially in the bottoms of excavations, may cost $1, or even considerably more. These, however, are exceptional cases, of comparatively rare occurrence. The quarrying of average hard rock requires about ¼ to ⅓ lb of powder per cubic yard, in place; but the nature of the rock, the position of the strata, &c, may increase it to ½ lb, or more. Soft rock frequently requires more powder than hard. A good churn-driller will drill 8 to 10 feet in depth, of holes about 2½ feet deep, and 2 inches diameter, per day, in average hard rock, at from 12 to 18 cents per foot. Drillers receive higher wages than common laborers.

Hard Rock, by Wheelbarrows.

Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor
Feet.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds	Cents.	Cents.
25	12.2	8.64	53.7	600	2.96	35.5	81.7
50	10.7	9.81	54.9	700	2.62	40.1	86.5
75	9.58	11.0	56.2	800	2.34	44.8	91.4
100	8.66	12.1	57.3	900	2.12	49.5	96.3
150	7.26	14.5	59.8	1000	1.94	54.1	101.1
200	6.25	16.8	62.2	1200	1.65	63.6	115.0
250	5.49	19.1	64.6	1400	1 44	72.9	120.7
300	4.89	21.5	67.1	1600	1.28	82.2	130.4
350	4.41	23.8	69.5	1800	1.15	91.5	140 1
400	4.02	26.1	71.9	2000	1.04	100.8	149.8
450	3.69	28.5	74.4	2200	.953	110.2	159.6
500	3.41	30.8	76.8	2400	.879	119.5	169.3

Art. 15. Removing rock excavation by carts. A cart-load of rock may be taken at ⅕ of a cubic yard, in place. This will weigh, on an average, 851 lbs; or but 41 lbs more than a cart-load of average soil. Since the cart itself will weigh about ½ a ton, the total loads are very nearly equal in both cases. Columns 2 and 3 of the following table are prepared on the same principle as for earth, as directed in Art. 4. Column 4 is made up by adding to each amount in column 3, the following items: For blasting, (and for everything except those in column 3; loading, and repairs of cart-road,) 45 cents per cubic yard, in place; for loading, 8 cents, per cubic yard, in place; and for repairs of road, .2, or ⅕ of a cent for each 100-feet length of lead. Contractor's profit not included.

Hard Rock, by Carts.

Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor
Feet.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents
25	19.2	6.51	59.6	1800	5.00	25.0	81.6
50	18.5	6.77	59.9	1900	4.80	26.0	82.8
75	17.8	7.08	60.2	2000	4.62	27.1	84.1
100	17.1	7.29	60.5	2250	4.21	29.7	87.2
150	16.0	7.81	61.1	2500	3.87	32.3	90.3
200	15.0	8.33	61.7	½ mile	3.70	33.7	92.0
300	13.3	9.37	63.0	3000	3.33	37.5	96.5
400	12.0	10.4	64.2	3250	3.12	40.1	99.6
500	10.9	11.5	65.5	3500	2.92	42.8	102.8
600	10.0	12.5	66.7	3750	2.76	45.3	105.8
700	9.23	13.6	68.0	4000	2.61	47.9	108.9
800	8.57	14.6	69.2	4250	2.47	50.6	112.1
900	8.00	15.6	70.4	4500	2.35	53.2	115.2
1000	7.50	16.7	71.7	4750	2.24	55.8	118.5
1100	7.06	17.7	72.9	5000	2.14	58.4	121.4
1200	6.67	18.7	74.1	1 mile	2.04	61.2	124.8
1300	6.32	19.8	75.4	1¼ "	1.67	75.0	141.2
1400	6.00	20.8	76.6	1½ "	1.41	88.8	157.6
1500	5.71	21.9	77.9	1¾ "	1.22	102.5	174.0
1600	5.45	22.9	79.1	2 "	1.08	116.3	190.4
1700	5.22	24.0	80.4	2¼ "	.962	130.0	206.8

"**Loose rock**" will cost about 30 cts per yd less; and even *solid* rock will *average* about 10 cts less than the tables.

Art. 16. Removing rock excavation by cars and locomotive, on level track. Our calculations are based upon the following assumptions: Trains of 10 cars, each car containing 1 cubic yard of rock measured in place. Average speed of trains, including starting and stopping, but not standing, 10 miles per hour = 5 miles of *lead* per hour. Labor $1 per day of 10 working hours. Loosening, 45 cts per cubic yard in place. Loading, 8 cts per cubic yard in place. Cost of track, for interest and repairs, $3 per day per mile of lead. The calculations are the same, in principle, as those in Art. 13.

Hard Rock, by Cars and Locomotive.

Labor $1 per day of 10 working hours.

Length of lead, or distance to which the rock is hauled ....miles	1	3	5	7	10
Number of cubic yards, in place, hauled per day by each train..	2900	1300	800	600	400
Cost, per cubic yard in place, for hauling, dumping, and returning........................cents	.6	1.7	3.5	5.7	10.8
Total cost, per cubic yard in place, exclusive of contractor's profit...............................cents	53.6	54.7	56.5	58.7	63.8

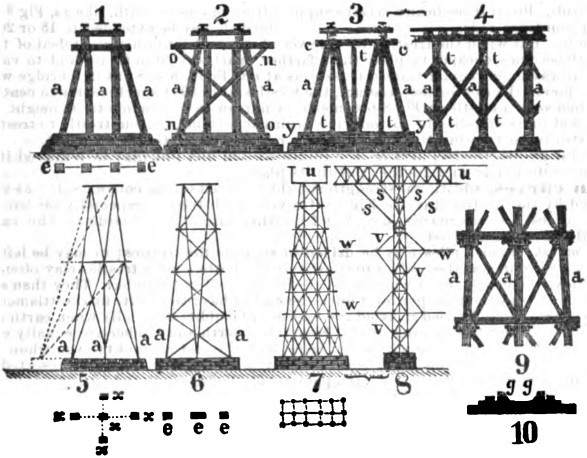
TUNNELS.

Tunnels for railroads should, if possible, be straight, especially when there is but a single track; inasmuch as collisions or other accidents in a tunnel would be peculiarly disastrous. A tunnel will rarely be expedient before the depth of cutting exceeds 60 feet. Firm rock of moderate hardness, and of a durable nature, is **the most favorable material** for a tunnel; especially if free from springs, and lying in horizontal strata. In soft rock, or in shales (even if hard and firm at first), or in earth, a lining of hard brick or masonry in cement, is necessary. A tunnel should have a **grade or inclination** in one direction, for ease of future drainage and ventilation. No special arrangement is essential for **ventilation** either during construction, or after, if the length does not exceed about 1000 feet; but beyond that, generally during construction either shafts are resorted to, or means provided for forcing air into the tunnel through pipes from its ends. But after the work is finished, except under peculiar circumstances, nothing of the kind is necessary. Shafts often draw air downwards; and frequently, even when aided by a steep, uniform grade, do not secure ventilation. The Mont Cenis tunnel under the Alps, completed in 1871, is 7½ miles long, and has no shafts, although it grades up from each end, which is the most unfavorable of all conditions for ventilation without shafts. It was made so for facilitating drainage. Its ventilation is maintained by air forced in from the ends. The Hoosac tunnel, Mass, 4½ miles long, has shafts; one of them 1080 feet deep; but they were for expediting the work. **Shafts generally cost** from 1½ to 3 times as much per cubic yard as the main tunnel, owing to the greater difficulty of excavating and removing the material, and getting rid of the water, all of which must be done by hoisting. When through earth, they must be lined as well as the tunnel; and the lining must usually be an under-pinning process. Or the lining may first be built *over* the intended shaft, and then sunk by undermining it gradually. Their sectional area commonly varies from about 40 to 100 square feet. They have the great advantage of expediting the work by increasing the number of points at which it can be carried on; but if placed too close together, their cost more than compensates for this. The air in some tunnels, while being constructed, is much more foul than in others; so that after the work has been commenced, shafts with forced air may be expedient where they were not anticipated. In excavating the tunnel itself, a **heading** or passage-way, 5 or 8 feet high, and 3 to 12 feet wide, is driven and maintained a short distance (10 to 100 feet, or more, according to the firmness of the material) in advance of the main work. In rock, the heading is just below the top of the tunnel, so that the men can conveniently drill holes in its floor for blasting; but in earth, the heading is driven along the bottom of the tunnel, that being the most convenient for enlarging the aperture to the full tunnel size, by undermining the earth, and letting it fall. In earth, the top and sides of the heading, as well as of the tunnel, must be carefully prevented from caving in before the lining is built; and this is done by means of rows of vertical rough timber props, and horizontal caps or overhead pieces, between which and the earth rough boards are placed to form temporary supporting sides and ceiling to the excavation. The props and caps are placed first; and the boards are then driven in between them and the earthen sides of the excavation. These are gradually removed as the lining is carried forward. **The lining,** when of brick, is usually from 2 to 3 bricks thick (17 to 26 inches) at bottom, and from 1½ to 2½ bricks thick at top; and when of rough rubble in cement, about half again as thick. It is important that the bricks or stone should be of excellent hard quality, and laid in good cement. The bricks should be moulded to the shape of the arch. As the lining is finished in short lengths, and before the centers are removed, any **cavities or voids** between it and the earth should be carefully and compactly filled up. Even in rock, if much fissured, or if not of durable character, as common shale, lining is necessary. **The cross-section** of a single-track railroad tunnel, in the clear of everything, and for cars of 11 feet extreme width, should not be *less* than about 15 feet wide, by 18 feet high; nor a double-track one, less than 27 feet wide, by 24 feet high; unless in the last case the material is firm rock, in which a high arch is not necessary for lining. The roof may then be much flatter, so that a height of 20 feet may answer. With cars of 10 feet extreme width, the width of the tunnel may be reduced to 25 feet; or with 9 feet cars, to 23 feet. Many have been made 22 feet. The Mont Cenis is 26 feet wide, by 25 high. **The rate of daily progress** from *each* face of a tunnel varies from 18 inches to 9 feet of length per 24 hours, with three relays of workmen. On the Mont Cenis the extremes were about 4 to 9 feet daily for a whole year, from each face. Drills worked by compressed air were employed in the headings, which were 12 feet wide by 8 feet high. Ordinarily, from 1½ to 3 feet may be taken as averages. The difference of rate of progress between a single and a double track tunnel is not so great as might be supposed; inasmuch as a larger force can be employed on the wider one. If the tunnel is in earth, the construction of the lining about makes up for the slower excavation of one in rock. In rock, with labor at $1 per day, **the cost** will usually vary with the character of the rock, from $2 to $5 per cubic yard for the main tunnel; and from $3 to $10 for the heading; while shafts will average about 50 per cent. more than heading. The cost of a single-track tunnel, when common labor is $1 per day, will generally range between $30 and $75 per foot of length. Tunnel work, however, is liable to serious contingencies which cannot be foreseen. Since the sides and roof are rough as blasted, the width and height should each be estimated to the contractor as about 18 inches or 2 feet greater than the established *clear* ones. At any rate, the mode of measurement should be clearly stated in the specifications for the work. When a tunnel is made with a uniform grade, the work generally progresses in a more satisfactory manner from the lower end, because the descent favors the drainage of the spring water that is usually met with; whereas, at the upper end, it must be removed by pumps or by bailing. The upper end has, however, the advantage of sooner getting rid of the smoke in blasting. Before commencing a tunnel, or even deciding upon one, **trial shafts** should be sunk to ascertain the nature of the material. In long ones, the greatest care and accuracy are necessary for preserving the line of direction, so that the work from both ends shall meet properly at the center.

In the heading of the Vosburg (Pa) tunnel of the Lehigh Valley R R, built 1884, cross-section 7½ feet × 26 feet, the **average progress** per working day of 24 hours with two shifts of 12 hours each, was as follows; by hand drilling 2.8 feet and 2.4 feet respectively from each end; by machine drills (two rival drills in competition) 5.6 feet and 7.8 feet. The material was hard gray sandstone. For the whole tunnel the rate was about 2 feet per day.

For further information respecting tunnels, the reader is referred to Mr. H. S. Drinker's very full treatise on the subject, published by the Messrs Wiley.

TRESTLES.



Figs 1, 2, 3, 5, 6, 7, are **elevations of trestles**; taken across the track or roadway. We may consider Fig 1 as adapted to a height of about 10 to 20 ft: Figs 2 and 3, to heights from 20 to 30 ft; Fig 5, from 30 to 40 ft; Fig 6, from 40 to 60 ft; as rough approximations merely. A single framework, such as that shown in each of these six figures, is called a "bent." These bents of course admit of many modifications. They are usually supported by bases of masonry, as in the figures. These preserve the lower timbers from contact with the earth, which would hasten their decay. It is advisable to make these bases high enough to prevent injury from cattle, or passing vehicles, &c. Up to heights of about 40 or 50 ft, a single row of posts or uprights, *a*, *a*, *a*, Figs 1 to 9, as shown at *e e* under Figs 1 and 6, will answer. But as the height becomes greater, more posts should be introduced, as shown at *x x* under Fig 5; or two entire rows of them; or three rows, as under Fig 7; and as also in Fig 8, which is an end view of Fig 7. Figs 7 and 8 bear much resemblance to the trestles 190 ft high, with masonry bases 30 ft high (S. Seymour, C. E.), which carried the Erie Rway (now the N Y, Lake Erie & West'n R R) over the **Genesee River at Portage, N Y.** There each bent had 21 posts 14 ins square, at its base; and 15 posts of 12 × 12, at its top. The other timbers were 6 × 12; many of them were in pairs, embracing the posts. This single-track viaduct was begun July 1, 1851, and completed Aug. 14, 1852. It contained 1,602,000 ft (B M) of timber, and 108,862 lbs of iron. In the foundations were 9200 cub yds of masonry. The entire cost was about $140,000. It was burned down in 1875, and was replaced, in less than 3 mos, with a single-track **viaduct of wrought-iron trestles,** containing, in all, 1,340,000 lbs of iron, and 130,600 ft (B M) of timber; and costing, complete, above the masonry, about $95,000. Frequently the posts of trestles are in pairs; and the other timbers pass between; all bolted together.

In Fig 4, the posts *a*, *a*, *a*, are end views of three trestles or bents, such as Fig 3; and *t t* are diag braces extending from trestle to trestle; the two outer ones inclining in one direction; and the central one crossing them. These may be placed either intermediate of the posts, as in Fig 3; with the heads of the two outer ones confined to the cap *c c* of one trestle; and their feet to the sill *y y* of the next one; or they may all be spiked or bolted to the posts themselves, as in Fig 4. The last is the best, as it serves also directly to stiffen the posts: as do also the braces *o o*, *n n*, Fig 2. Such bracing is too frequently omitted. During the passage of trains, the backward pressure of the steam, exerted through the driving wheels against the track, produces a serious strain lengthwise of the road, and tending to upset the trestles; and the sudden application of brakes to a moving train, produces a similar strain in the opposite direction. These strains become more dangerous as the ht increases. Hence the need for such braces. Usually the outer **posts may lean 1.5 to 2.5 ins to a ft.**

The posts should not be less than about 12 ins square, except in quite low trestles; and even then not less than about 10 × 10. The diag bracing may generally be about as wide as the posts; and half as thick. The dist apart of the bents, when the roadway is supported by simple longitudinal beams, should not exceed 10 or 12 ft, for railroads. But if these beams receive support from braces beneath, like *s s*, Fig 8: or from iron truss rods, the dist may be extended to 15 or 20 or more ft. But when the trestles become very high, and contain a great deal of timber, it becomes cheaper to place them farther apart, say 30 to 60 ft; and to carry the railway upon regular framed trusses, as at *u u*, Figs 7 and 8; as in a bridge with stone piers. In the Genesee viaduct, the trestles were 50 ft apart, center to center.

When such a trestle as Fig 8 becomes very narrow in proportion to its height, we may add to its stability by introducing beams *w*, extending from trestle to trestle; and still further by inserting diag braces *v v*, as in the old Genesee viaduct.

As far as practicable, arrange the pieces so that any one may be removed if it becomes decayed; and another put in its place.

On curves, additional strength should be given on the convex side; as suggested by the dotted lines in Fig 5. On very high trestles especially (as well as on bridges), wheel-guards, *g g*, Fig 10, either inside or outside of the rails, should never be omitted.

In marshy ground, piles may be driven to support the trestles; or may be left so far above ground, as themselves to constitute the posts. Such trestles may often be used advantageously, even when to be afterward filled in by embkt. They then sustain the rails at their proper level until the embkt has reached its final settlement.

They are generally used to avoid the expense of embkt; especially when earth can only be obtained from a great dist. Even when earth and timber are equally convenient, they will rarely much exceed about half the cost of embkt; even when but about 30 ft high; but owing to their liability to decay, they should be resorted to only in case of necessity; or as a temporary expedient.

BALLAST.

Table of cubic yards of ballast per mile of road.

Side-slope of the ballast 1 to 1. Width in clear between 2 tracks 6 ft. The ties and rails may be laid first, for carrying the ballast along the line; then raised a few ft of length at a time, and the ballast placed under them. **Deduct for ties,** as below.

Depth in Ins.	Top width, Single Track.			Top width, Double Track.		
	10 Ft.	11 Ft.	12 Ft.	21 Ft.	22 Ft.	23 Ft.
	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
12	2152	2347	2543	4308	4499	4695
18	3374	3667	3960	6600	6894	7188
24	4694	5085	5474	8996	9388	9780
30	6111	6600	7087	11490	11980	12470

A man can break 3 to 4 cubic yards per day. of hard quarried stone to a size suitable for ballast: say averaging cubes of 3 inches on an edge. Where other ballast cannot be had, hard-burnt clay is a good substitute. The slag from iron furnaces is excellent. The ties decay more rapidly when gravel or sand is used instead of broken stone, because these do not drain off the rain, but keep the ties damp longer.

TIES.

In the United States **the life of a tie** is about as follows:

	Years.	Average, Years.		Years.	Average, Years.
Chestnut,	5 to 12	7	White Oak,	5 to 12	7
Cedar,	6 to 15	9	Spruce Pine,	4 to 7	5
Hemlock,	3 to 8	5			

It will often, especially in the case of the softer and more perishable woods, be true economy to preserve ties by the injection of creosote. Creosote preserves the *spikes.*

The writer believes that most of the fault usually ascribed to cross-ties, as well as to rail-joints, is in reality due to imperfect drainage of the roadbed. Hence, he does not agree with those who advocate VERY LONG TIES; but considers that with good ballast, on a well-drained roadbed, 8½ ft is as good as more; and that 8½ ft, by 9 ins, by 7 ins; and 2½ ft apart from center to center, is sufficient for the heaviest traffic. On many important roads they are but 8 ft; and on some only 7½ ft long; track 4 ft 8½. On narrow-gauge roads the ties are generally from 6 to 7 ft long. The actual COST OF CUTTING DOWN THE TREES, lopping off the branches, and hewing the ties ready for hauling away to be laid, is about 6 to 9 cts per tie, at $1.75 per day per hewer.

The narrow bases of rails resting immediately on the cross-ties, without chairs, frequently produce in time such an amount of crushing in the ties as to injure them materially even before decay begins. Burnetised ties rust the spikes away rapidly. Creosoted ones preserve them.

Cross-ties of 8½ feet, by 9 inches, by 7 inches, contain 3.719 cubic feet each; and if placed 2½ feet apart from center to center, there will be 2112 of them per mile, amounting to 291 cubic yards. Therefore, if they are completely embedded in the ballast, they will diminish its quantity by that amount. At 2 feet apart there will be 2640 of them, occupying 364 cubic yards; and at 3 feet apart, 1760 of them; 243 cubic yards.

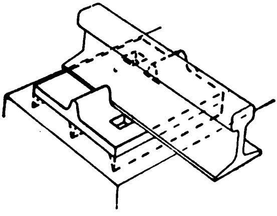
Cubic feet contained in cross-ties of different sizes.

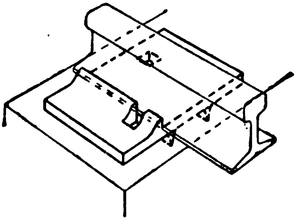
Dimensions.			Contents.	Dimensions.			Contents.
Ft.	Ins.	Ins.	Cub. Ft.	Ft.	Ins.	Ins.	Cub. Ft.
8	by 8	by 6	2.667	8½	by 10	by 7	4.132
8	9	6	3.000	8½	10	8	4.722
8	9	7	3.500	8½	12	8	5.667
8	10	6	3.333	9	8	6	3.000
8	10	7	3.889	9	9	6	3.375
8	10	8	4.444	9	9	7	3.938
8	12	8	5.333	9	10	6	3.750
8½	8	6	2.833	9	10	7	4.375
8½	9	6	3.188	9	10	8	5.000
8½	9	7	3.719	9	12	8	6.000
8½	10	6	3.542				

TIE PLATES.

Where the rails bear directly upon the ties, the great unit pressure of the narrow rail base, the churning action of the rail under passing wheels, and the hastening of decay by the bruising of the wood fibres, cause rapid wear of the tie immediately under the rail.

Among prominent forms of tie plates are the Servis, the Goldie, the Churchward, and the Wolhaupter.


Servis.


Goldie.

All consist essentially of a flat iron or steel plate, laid on the tie immediately under the rail. Spikes, holding the plate and the rail in place, are driven into the tie through holes in the plate. Nearly all successful forms have two or more ribs on the lower side. These ribs stiffen the plate, but their principal use is, by cutting into the upper face of the tie, to prevent motion of the plate and consequent abrasion of the tie. In some forms the ribs run *across* the fibres of the tie; but ribs running *with* the fibres, as in the Servis plate, are usually preferred, as being more easily imbedded, more difficult to displace, and less injurious to the fibres. Most forms have also a shoulder on the upper side, to assist the spikes in preventing spreading of the rails, and, in some cases, to act as a rail brace; but this shoulder is seldom considered essential.

Tie plates greatly lengthen the life of the tie. On curves and bridges the saving in a number of cases has been estimated at 50 per cent. in cost, and 60 to 75 per cent. in labor. The tie plate has often displaced small gangs of men whose sole duty it was to replace ties.

Tie plates cost from 5 to 15 cents each; and placing them costs from ½ to 1½ cents each.

Most roads use tie plates only under rail joints; on curves, heavy grades, bridges, and trestles; in tunnels where there is much dampness; at switches, at stations, and at street and road crossings; in yards; and where sand is much used by the locomotives.

Table of Middle Ordinates, to be used for the bending of rails of different lengths, so as to form portions of curves of different radii. Ordinates for lengths or radii intermediate of those in the table, may be found by simple proportion.

Def. Ang.	Radius.	LENGTHS OF RAILS.												
		30	28	26	24	22	20	18	16	14	12	10	8	6
Deg.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet
.5	11460.	.010	.008	.006	.005	.004	.004	.003	.002	.002	.001	.001	.000	.000
1.	5730.	.020	.016	.013	.011	.009	.008	.006	.005	.004	.003	.002	.001	.001
1.5	3820.	.029	.026	.021	.018	.016	.013	.010	.008	.006	.004	.003	.002	.001
2.	2865.	.038	.034	.029	.025	.021	.017	.014	.011	.008	.006	.004	.003	.001
2.5	2292.	.049	.043	.037	.031	.027	.022	.018	.014	.010	.007	.005	.003	.002
3.	1910.	.058	.051	.044	.037	.031	.026	.022	.017	.012	.009	.006	.004	.002
3.5	1637.	.070	.061	.052	.043	.037	.031	.025	.020	.015	.011	.008	.005	.003
4.	1433.	.079	.069	.060	.050	.042	.035	.029	.023	.018	.013	.009	.006	.003
4.5	1274.	.088	.077	.067	.056	.047	.039	.032	.026	.020	.015	.010	.007	.004
5.	1146.	.099	.086	.074	.063	.053	.044	.035	.029	.022	.016	.011	.007	.004
5.5	1042.	.108	.094	.082	.070	.059	.048	.039	.032	.024	.018	.012	.008	.004
6.	955.4	.117	.102	.088	.076	.064	.052	.042	.034	.026	.019	.013	.008	.005
6.5	882.	.128	.112	.097	.082	.069	.057	.046	.037	.028	.021	.014	.009	.005
7.	819.	.137	.120	.104	.088	.074	.061	.049	.039	.030	.022	.015	.010	.005
7.5	764.5	.146	.127	.111	.094	.079	.065	.053	.042	.032	.024	.016	.010	.006
8.	716.8	.158	.137	.119	.100	.085	.070	.056	.045	.034	.025	.017	.011	.006
8.5	674.6	.166	.145	.126	.106	.090	.074	.060	.048	.036	.027	.018	.012	.007
9.	637.3	.175	.153	.133	.112	.095	.078	.063	.050	.038	.029	.019	.012	.007
9.5	603.8	.187	.163	.141	.119	.101	.083	.067	.054	.042	.031	.021	.013	.008
10	573.7	.196	.171	.148	.125	.106	.087	.071	.057	.045	.032	.022	.014	.008
11	521.7	.216	.188	.163	.139	.117	.096	.078	.063	.049	.036	.024	.016	.009
12	478.3	.236	.206	.179	.151	.128	.105	.085	.069	.053	.039	.026	.017	.010
13	441.7	.254	.222	.192	.163	.138	.113	.092	.075	.057	.042	.028	.019	.010
14	410.3	.275	.239	.207	.175	.148	.122	.099	.080	.061	.045	.030	.020	.011
15	383.1	.295	.257	.223	.188	.159	.131	.106	.085	.065	.049	.033	.021	.012
16	359.3	.313	.273	.236	.200	.170	.139	.113	.091	.070	.052	.035	.023	.013
17	338.3	.333	.290	.252	.213	.180	.148	.120	.096	.074	.055	.037	.024	.014
18	319.6	.351	.306	.265	.225	.190	.156	.127	.102	.078	.058	.039	.025	.014
19	302.9	.371	.324	.280	.238	.201	.165	.134	.108	.082	.061	.041	.027	.015
20	287.9	.392	.341	.296	.250	.212	.174	.141	.114	.087	.066	.044	.028	.016
21	274.4	.410	.357	.309	.262	.222	.182	.148	.120	.091	.069	.046	.030	.017
22	262.	.430	.375	.325	.275	.233	.191	.155	.126	.096	.072	.048	.031	.018
23	250.8	.450	.390	.338	.287	.243	.199	.162	.131	.100	.075	.050	.033	.019
24	240.5	.469	.408	.354	.299	.253	.208	.169	.137	.104	.078	.052	.034	.019
25	231.	.486	.424	.367	.311	.263	.216	.176	.142	.108	.081	.054	.035	.020
26	222.3	.506	.441	.382	.323	.274	.225	.183	.148	.112	.084	.056	.037	.021
27	214.2	.524	.457	.396	.335	.284	.233	.190	.153	.116	.087	.058	.038	.022
28	206.7	.545	.475	.411	.348	.294	.242	.197	.158	.120	.090	.060	.039	.022
29	199.7	.564	.491	.424	.361	.303	.250	.203	.163	.124	.093	.062	.041	.023

RAILS.

$$\text{Tons (2240 lbs.) of rail per mile of single track} = \frac{11}{7} \times \text{weight of rail in lbs. per yard.} \quad \text{(exact)}$$

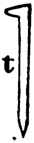
Every sq inch of sectional area of rail, corresponds to 10 lbs per yard of a single rail; or to 15.7143 tons per mile of single-track road. Consequently,

$$\frac{\text{Wt in lbs per yd of rail,}}{10} \text{ or } \frac{\text{Wt in tons per mile of single-track}}{15.7143} = \text{Area of rail in sq ins.}$$

Thus, a rail of 100 tons per mile of single track, will have a section of 6.364 sq ins; and will weigh 63.64 lbs per yd of single rail. Add for turnouts, sidings, road-crossings, and a trifle for waste in cutting. When the ties are in place, and the rails distributed in piles at short intervals, a gang of 6 men can lay ½ a mile of rails per day, of single track; or after the ballast is in place, a gang of 15 men will lay about one mile of complete single-track superstructure per week.

Steel rails last from 9 to 25 years; average 15 years.

RAILROAD SPIKES.



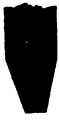
The hook-headed spikes *t*, commonly used for confining rails to the cross-ties, vary within the limits of the following table; the lightest ones for light rails on short local branches; and the heaviest ones for heavy rails on first-class roads. The spikes are sold in kegs usually of 150 lbs. For the weight of spikes of larger dimensions, we may near enough take that of a square bar of the same length. What is saved at the point suffices for the addition at the head.

Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per 100 lbs.	Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per 100 lbs.
4½ ×	7/16	526	350	5½ ×	½	350	233
4½ ×	½	400	266	5½ ×	9/16	289	193
5 ×	⅜	705	470	5½ ×	⅝	218	146
5 ×	7/16	488	325	6 ×	½	310	207
5 ×	½	390	260	6 ×	9/16	262	175
5 ×	9/16	295	197	6 ×	⅝	196	130
5 ×	⅝	257	171				

A mile of single-track road, with 2640 cross-ties, 2 feet apart from center to center; and with rails of the ordinary length of 30 feet, or fifteen ties to a rail; will have 352 rail-joints per mile; and, with 4 spikes to each tie, will require 10560 spikes, or nearly 37 kegs (5500 lbs.) of 5½ × 9/16, a size in very common use, which weighs a trifle more than ½ lb. per spike.

But an allowance must be made for rail-guards at road-crossings, which we may assume to be 30 feet wide, or the length of a rail. A guard will usually consist of 4 extra rails for protecting the track-rails, and spiked to the 15 ties by which said track-rails are sustained. Consequently such a crossing requires 15 × 8 = 120 spikes. For turnouts, sidings, loss, etc., we may roughly average 700* spikes more per mile; thus making in all (if we assume one road-crossing per mile) 10560 + 120 + 700 = 11380 spikes per mile; or say 6000 lbs. or 40 kegs of 150 lbs.

Adhesion of Spikes. Professor W. R. Johnson found that a plain spike .375, or ⅜ inch square, driven 3⅜ ins. into seasoned Jersey yellow pine or unseasoned chestnut, required about 2000 lbs. force to extract it; from seasoned white oak, about 4000; and from well-seasoned locust, about 6000 lbs. Bevan found that a 6-penny nail, driven one inch, required the following forces to extract it: Seasoned beech, 667 lbs; oak, 507; elm, 327; pine, 187.



Very careful experiments in Hanover, Germany, by Engineer Funk give from 2465 to 3940 lbs. (mean of many experiments, about 3000 lbs.), as the force necessary to extract a plain ½ inch square iron spike, 6 inches long, wedge-pointed for 1 inch (twice the thickness of the spike), and driven 4½ inches into *white or yellow pine*. When driven 5 inches, the force required was about 1/10 part greater. Similar spikes, 9/16 inch square, 7 inches long, driven 6 inches deep, required from 3700 to 6745 lbs. to extract them from pine; the mean of the results being 4873 lbs. In all cases *about twice as much force was required to extract them from oak*. The spikes were all driven *across* the grain of the wood. Experience shows that when driven *with* the grain, spikes or nails do not hold with much more than half as much force.

Jagged spikes, or twisted ones (like an auger), or those which were either swelled or diminished near the middle of their length, all proved inferior to plain, square ones. When the length of the wedge point was increased to 4 times the thickness of the spike, the resistance to drawing out was a trifle less. But see "Jag-spike" in Glossary.

When the length of the spike is fixed, there is probably no better shape than the plain square cross-section, with a wedge-point twice as long as the width of the spike, as per this fig.

---

* This allows that turnouts and sidings amount to about 1 mile of extra track on 15 miles of road.

RAIL-JOINTS.

Art. 1. A track, being weakest at the joints between the rails, where they are deprived of their vertical strength, has of course a greater tendency to bend at those points; and this bending produces an irregularity in the movement of the train, which is detrimental to both rolling-stock and track. Moreover, that end of a rail upon which a loaded wheel is moving, bends more than the adjacent unloaded end of the next rail; so that when the wheel arrives at said second rail, it imparts to its end a severe blow, which injures it. Thus, the ends of the rails are exposed to far more injury than its other portions. Numerous devices have been resorted to for strengthening the joints of the rails, with a view of preventing this bending entirely; or, at least, of causing the two adjacent rail-ends to bend equally, and together; so as to avoid the blows alluded to. None of these joint-fastenings, known as chairs, fish-plates, wooden blocks, &c, have proved entirely satisfactory.

Much of the deficiency ascribed to the fastenings, is, however, really due to want of stability in the cross-ties at the joints, and more attention must be directed to this latter consideration, before an efficient fastening can be obtained. Observation shows that when the joint-ties are very firmly bedded, almost any of the ordinary fastenings will (if the joint is placed *between* two ties, instead of resting *upon* a tie),* answer very well; whereas, when the cross-ties are so insecurely bedded as to play up and down for half an inch or more under the driving-wheels of the engines, the strongest and most effective fastenings soon become comparatively inoperative. **All the parts of the best of them will in that case become gradually loosened, warped, bent, or broken.**

Experience has established the superiority of suspended joints over supported ones. Long fastenings, perhaps, possess but little superiority over short ones, where the track is not kept in good repair; for the great bearing of the former, although imparting increased firmness on a good track, becomes converted into a powerful leverage, by which it accelerates its own destruction, in a bad one. An element in the injury of joints, is the omission of proper fastenings at the center of the rails. Each rail should be so firmly attached to the cross-ties at and near its center, as to compel the contraction and expansion to take place equally from that point, toward each end. It would probably be somewhat difficult to accomplish this perfectly. The attempts hitherto made have failed.

Under the extremes of temperature in the United States, **bar iron expands or contracts** about 1 part in 916; or 1 inch in 76⅓ feet; consequently, a rail 30 ft long will vary 7/16 inch; and one 20 ft long fully ¼ inch.

Beside this, the rails are very liable to move or creep bodily in the direction of the heaviest trade, especially when the grade descends in the same direction; and by this process also the joint-fastenings are exposed to additional strain and derangement.*

All rails appear to become elongated very slightly at their ends by use; and this renders a full allowance for contraction and expansion the more necessary.

Art. 2. Even joints and broken joints. If, in the two lines of rails forming a track, the joints are placed opposite to each other, they are called "even joints;" while "staggered" or "broken" joints are those where each joint in one of the lines of rails is opposite to the middle of a rail in the other line. In the latter case, the jar of passing from rail to rail is less severe, but of course more frequent, than where both wheels make that passage at the same time.

Art. 3. Beveled, or mitred joints. To lessen this jar, Mr. Sayre suggests cutting the rails so that the vertical plane forming the rail end shall make an angle of 45° to 60° with the longitudinal vert plane of the web of the rail, instead of the usual right angle. This would permit the use of longer rails than are now laid, as the great space (½ inch or more) between the ends of such long rails in cold weather, would not be so serious an objection when the ends were thus cut obliquely. This method of cutting the rails has been tried, with good results, but has not yet come into general use. It is claimed that a comparatively inexpensive change in the arrangement of the saws at the rolling mill, would permit the rails to be cut with ends at any angle, as readily as with square ends, and without further increase in the cost of sawing.

---

* In the first case the joint is called a **suspended** one; in the last a **supported** one.

Art. 4. Fish-plates, Fig. 1, and **Angle-plates,** Figs 2, 3, 4, and 5, have nearly supplanted all other forms of joint on the principal railroads of the U. S. They are rolled in long bars, and cut off in any desired length, generally about 2 ft; and are bolted together, and to the rails, by 4 or 6 bolts, 2 or 3 in each rail end.

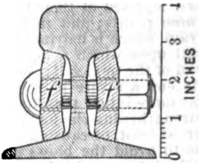


Fig.1.

Art. 5. The fish-plate joint was one of the earliest suggested. It was introduced upon the Newcastle and Frenchtown R R, in Delaware, by Robt. H. Barr, in 1843. The weight of a complete fish-plate joint, including bolts and nuts, is about 20 ℔s.

Fig 1, one-fifth of real size, represents a fish-plate joint with a steel rail weighing 50 ℔s per yard. The thickness of plate at *f* is $\frac{9}{16}$ inch. The plates weigh 4¼ ℔s per lineal foot of a single plate.

Art. 6. The principal advantage of the **angle-plates** is that the spikes, which are driven through slots in their flanges to confine them to the cross-ties, serve also to counteract the tendency of the rails to "creep,"* (See Art 1.) Where *fish*-plates are used, the flanges of the *rails* have to be slotted for this purpose.

A further advantage of the angle plate is that it transfers at least a part of the load directly to the ties. It thus supports the rail better than the fish-plate can do, and may continue to give some support even if the bolts should become somewhat loose, provided the spikes hold firm. Moreover, the spreading base of the angle joint adds greatly to the *lateral* strength of the rail at the joint.

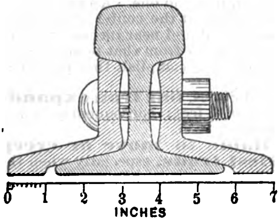


Fig.2.

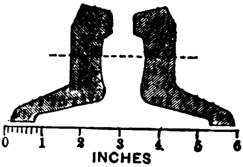


Fig. 2 A.

Art. 7. The following are usual dimensions of angle plates:

For rails.	Height.	Thickness on center line of bolt.	Weight of one bar.
60 ℔s & 70 ℔s	3⅛ ins.	¾ inch	26¼ ℔s
85 ℔s	3$\frac{9}{16}$ "	$\frac{25}{32}$ "	30¼ "

---

*On the St. Louis bridge (steel arches) and its eastern approach (plate girders on iron columns) the rails creep, in the direction of the traffic, *about a foot per day*, both up and down a grade of 80 feet per mile, and with such force that, although various fastenings were used, in order to prevent the creeping, none proved effectual, and the track was adjusted daily to accommodate the creeping. For a very interesting account of this case, see a paper by Prof. J. B. Johnson in the Journal of the Association of Engineering Societies, Vol. IV, No. 1, Nov. 1884, and abstract, with editorial discussion, in *Railroad Gazette*, Jan. 2d, 1885. Prof. Johnson attributes the creeping to a wave motion of the rail caused by the passage of trains.

Art. 8. The wheel-tread and 76 ℔ steel rail, shown (one-fifth of real size) in Fig 3, are those designed by **Robt. H. Sayre, C. E.,** and used by the **Lehigh Valley R R.** under very heavy traffic. **The angle-plate joint** was designed by **Mr. John Fritz,** Supt Bethlehem Iron Co, Bethlehem, Pa, and **Mr. Sayre.**

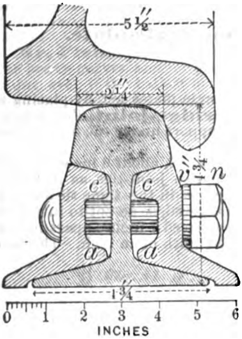


Fig.3.

These forms of wheel-tread, rail, and splice, are the result of careful study, and each detail has been modified from time to time as experience dictated. The stems of the two plates are placed wide apart, thus giving the joint greater lateral strength; at the same time adding to its vertical strength by the support given to the lower side of the rail-head by the upper enlargement *c*; while the lower one *a* secures a full bearing on the flange of the rail. The joint, for 76 ℔ rail, complete, 2 ft long, with 4 bolts ⅞ inch diam, weighs 40 to 48 ℔s, depending upon the thickness of the angle plate. The drilled bolt-holes in the stem of the *rail*, are 1 inch diam, to allow the rails to contract and expand.

Art. 9. Figs 4 and 5 (one-fifth of actual size) show an **angle-plate joint** made by Cambria Steel Co, and furnished with their **patent nut-lock,** which consists of a small piece, or "key," *p*, of Bessemer steel, semi-circular in cross-section at one end, and tapered to a horizontal edge at the other. After the nut has been screwed to its place, the key is driven close up to it, and then the pointed end of the key is bent up (as shown in Fig 5) by a special tool with a lever attached. The key is prevented from falling out sideways by the edge of the longitudinal groove, Fig 4, in the angle-plate, into which it fits.

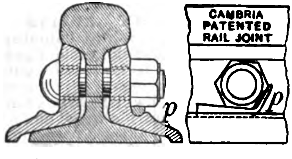


Fig.4. Fig.5.

Art. 10. Both fish- and angle-plates are apt to crack vertically about the middle of their length, or opposite to the joint in the rail. To obviate this, the "**Samson-bar**" (made either of fish or of angle form) was rolled about half inch thicker at the middle than at its ends. The thickened portion was about 8 ins long, extending say 4 ins each way from the joint; but the upper edge of the bar, upon which the head of the rail rested, and in *fish*-bars the lower edge also, were made of this increased thickness throughout their length.

Art. 11. Fish- and angle-plates, of all the patterns shown, and others, are **rolled to suit different sizes and shapes of rails.** The **bolt heads** are usually round, and the **shoulders** of the bolts, immediately under the heads, are therefore made of **oval** cross-section, fitting into corresponding oval holes in the fish- or angle-plate. The bolt is thus prevented from turning when the nut is screwed on, and afterwards. Many devices have been tried, with a view to **preventing the nuts from wearing loose**

The plates are frequently rolled with a **longitudinal groove,** as wide as the head or nut of the bolt, and about ⅛ inch deep, running their entire length. This groove receives either the head of the bolt, which in such cases is made square or oblong and inserted first, and the nut afterwards screwed on; or else the nut is first placed in the groove, and the bolt afterwards screwed into it. This is intended to prevent the unscrewing of the nut, but cannot be relied upon to do so.

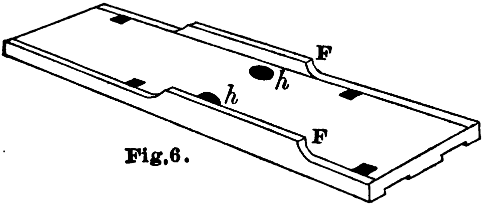
It is well to have the slots in the flanges of rails or of angle-bars so spaced that

the two spikes of a joint, driven into the same cross-tie, shall not be directly opposite to each other, but "staggered," so as to diminish the danger of splitting the tie.

Joints are frequently laid with **one fish- and one angle-plate.**

Art. 12. It will be noticed that both fish- and angle-plates act by placing a support under the *head* of the rail. **The Fisher bridge-joint,** Figs 6 to 9, made by Mr. Clark Fisher, Trenton, N J, applies the support under the *base* of the rail.

The principal feature of this joint is a flanged beam, Fig 6, about 6 ins wide and 22 ins long, which extends across, and is spiked to, the two joint-ties, as in Fig 7. The holes for the spikes are placed so that the two spikes in the same tie are not opposite to each other; and the flanges F F also are staggered, so as not to interfere with the driving of the spikes. The joint-ties T T are placed 7 inches apart in the clear. The beam has an upward camber of about one-eighth of an inch. The two rail-ends, forming the joint, rest upon the beam, and meet at the middle of its length. They are held down to it by a single U-shaped bolt B, of 1 inch diam, with a nut on each leg. These nuts bear directly upon the horizontal upper sides of the "fore-locks" L L, one of which is shown separately in Fig 9. The fore-locks are rolled to fit accurately to the rail-flanges. The legs of the U-bolt pass first through the circular holes *h h*, in the beam, Fig 6; next through rounded notches cut in the corners of the rail-flanges; then through the holes in the fore-locks; and lastly through the nuts. Between the U-bolt and the bottom of the beam is placed a small piece *s*, of spring steel, slightly cambered downward, and having two semi-circular notches for the legs of the U-bolt, which hold it in place. This is intended to keep the joint elastic, to take up any loose space produced by the wear of the surfaces in contact, to render less abrupt the strains on the bolt, and, by keeping the threads of the nut pressed against those of the bolt, to prevent the nuts from becoming loose. The joints are shipped from the factory complete, and with all the parts bolted together; the nuts being screwed down to within about two threads of their final places, so that the ends of the rail-flanges can be easily slid into place under the fore-locks.



Fig,6.

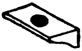


Fig.9.

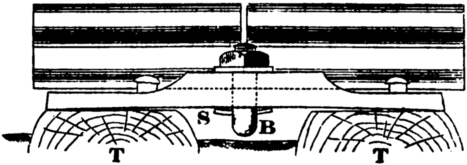


Fig. 7.

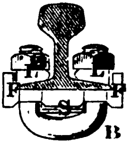
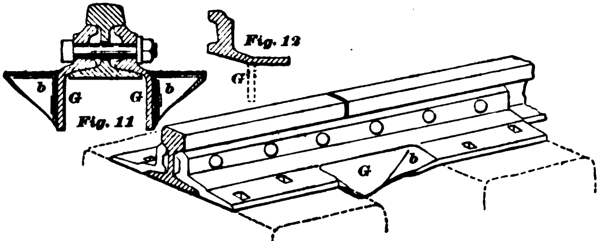


Fig. 8.

As an additional precaution against creeping of the rails, the rail-flanges may be slotted near their ends, as in cases where fish-plates are used, and spikes driven through these slots. For such cases the beams are punched, at the mill, with four additional square holes a little further from the edges of the beam than the others. Unlike the angle- and fish-plate joints, the Fisher may be used with any section of T-rail; and the head of the rail may be made stronger by being rolled pear-shaped, which is inadmissible with fish- and angle-joints, because these require a nearly horizontal bearing on the under side of the head. The "Fisher" requires no drilling or punching of the stem of the rail. It **costs** about 25 per cent more than a fish- or angle-joint for the same rail. Its **weight**, complete, for 65-℔ rail, is about 32 ℔s.

Mr. Fisher makes also an extra strong joint with three U-bolts, for heavy curves and for places liable to wash-outs. It is intended to support the joint, even if the ballast is removed from under the joint-ties. Either of the Fisher joints can be made of any desired weight. The "Fisher" is largely used on some of the principal eastern roads, and with very satisfactory results.



Art. 13. The Bonzano rail-joint, Figs. 10, 11. and 12, invented and patented by Mr. Adolphus Bonzano, C. E., of Philadelphia, is essentially an angle-plate joint (Art. 6), but. in the Bonzano joint, the horizontal flange of the angle bar is rolled about 3 inches wider than usual, and, after rolling, and cutting to the proper lengths, its middle portion is pressed downward by dies, forming a girder, G, which projects downward between the two joint ties.

The broad horizontal flange, being level with that of the rail, affords greatly increased bearing upon the ties, and adds to the lateral stiffness of the joint; the girder, G, between the joint ties increases the vertical strength of the joint; and the triangular gussets. *b*, Figs. 10 and 11, securely hold the horizontal flange and the downward-projecting girder in their relative positions.

The two splice-bars. in the Bonzano joint, have a combined cross-section about 1.2 times that of the rail, and, as shown by tests made by Prof. Henry T. Bovey, at McGill University, Montreal, equal strength with the rail, while the ordinary angle bar joint has but one-third that strength.

The joints are made either 30 inches long for 6 bolts, or from 24 to 26 inches long for 4 bolts. For 80 ℔. rails the splice-bars for the 30 inch and 24 inch joints weigh together about 59 ℔s. and 73 ℔s. respectively.

The price is about 0.3 cent per pound higher than that of the ordinary angle bar joint.

TURNOUTS.

Art. 1. To enable an engine and train to pass from one track, A B, Fig 1, to another, A D, a *turnout* is introduced. **This consists essentially** of a **switch,** *q m p s*, a **frog,** *f*, and two fixed **guard-rails,** *g* and *g'*. If a switch is made to serve for *two* turnouts, A D and A D', Fig 2, one on each side of the main track, A B, it is called a **three-throw switch.**

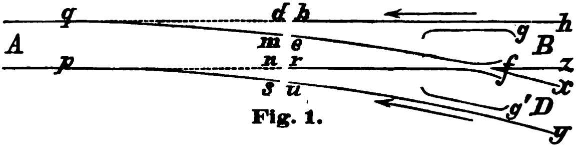


Fig. 1.

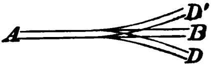


Fig. 2.

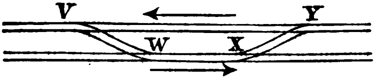


Fig. 3.

Art. 2. When a train approaches a switch in the direction of either arrow, Fig 1; or so that it passes the *frog* before reaching the *switch*, it is said to "**trail**" the switch. When it approaches in the *opposite* direction, passing the *switch* before reaching the *frog*, it is said to "**face**" the switch. Fig 3 represents a portion of a double-track road in which the trains keep to the right, as shown by the arrows. In this fig, V and W are "*trailing*" switches; and X and Y are "*facing*" switches. In order to leave the main track by a *trailing* switch, a train must move in a direction contrary to the proper one on said track.

Art. 3. Misplaced switches. A moving train, *facing* any switch, must plainly go as the switch is set, whether right or wrong. If wrong, serious accident may result. For instance, the train may run upon, and over the end of, a short trestle siding, or may collide with a train standing or moving upon the turnout. **Safety switches,** such as the Lorenz, Arts 13, &c, and Wharton, Arts 18, &c, are so arranged that trains *trailing* them can pass them safely, even if the switch is misplaced. But in the case of the plain **stub-switch,** Art 4, when misplaced, a trailing train will leave the rails at *b* and *r*, or *e* and *u*, Fig 4, and run upon the ties.

Stub-switches are frequently provided with "**safety-castings**" of iron, bolted to their sides, and reaching from their toes *m* and *s*, Fig 4, several feet toward *p* and *q*. These, in case of misplacement of the switch, receive the flanges of the wheels of a trailing train, and guide the wheels safely on to the switch-rails *q m* and *p s*. **The "Tyler" switch** is arranged in this way.

Art. 4. The common blunt-ended or stub-switch consists essentially of two rails, $q\,m$ and $p\,s$, Figs 1 and 4. The ends, q and p, of these rails, where they are fixed in line with the main track, form the "**heel**" of the switch. Their other ends, m and s, form the "**toe**," and are free to move from d and n (where they are in line with the main-track rails, $b\,h$ and $r\,z$) to m and s (where they are in line with the turnout-rails, $e\,x$ and $u\,y$).

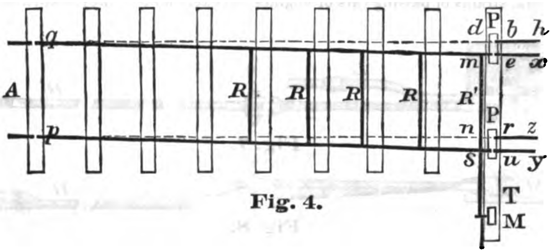


Fig. 4.

On main lines of road, the switch-rails are usually from 18 to 26 feet long from heel to toe. Formerly their heels, q and p, were fixed by being confined in the same chairs which held the adjoining ends of the main-line rails; or by being connected with said rails by short fish-plates. In either case they remained practically straight, even when set for the turnout. Now, however, they are generally made long enough to extend 4 to 6 feet back from their heels, toward A; and this additional length is spiked unyieldingly to the ties; so that, when set for the turnout, the switch-rails bend so as to form (at least approximately) a part of the turnout curve. Their toes, m and s, in any case, rest and slide upon iron "**head-plates,**" P. P., Fig. 4, shown in detail at Fig. 5, which represents a riveted steel plate, made by the Weir Frog Co. These head plates also receive the ends, $b\,e$ and $r\,u$, Fig. 4, of the main-track and turnout rails. In *three-throw* switches, the head-plates must of course be longer, to give room for the *three* rail-ends side by side.

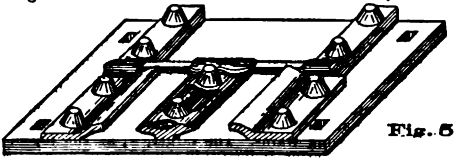


Fig. 5

Frequently a plain strip of iron, about 3 inches wide by half-inch thick, is fastened to the upper surface of each tie, under the base of the rail, for the latter to slide on.

The switch-rails are connected together by from 3 to 5 transverse wrought-iron **clamp-rods,** $R\,R\,R'$, Fig 4, full 1¼ ins diam. These are fastened to the rails in various ways; generally as shown in Fig 6. The clamp-bars should be placed, if possible, at least as low as the tops of the cross-ties, so as to avoid danger of their coming into contact with any portions of cars or engines that may become partially detached and drag on the track.

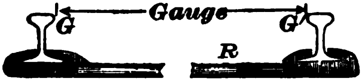


Fig. 6.

One of the clamp-bars, R', is near the toes, m and s, Fig 4. It projects beyond the track, and is jointed, as shown at Figs 7 and 8, and connected with the lever, L, by which the switch is moved. The tie, T, Fig 4, to which the head-plates are fastened, is made longer than the others, in order to give room for the switch-stand, M, Figs 4, 7, and 8, which is bolted to its upper surface. This tie should also be of larger cross-section than the others, perfectly sound, and well bedded; because upon it come severe strains due to the passage of cars and engines across the space between the rail-ends, m and e, s and u, etc, Fig 4.

Art. 5. The **switch-levers,** and the **switch-stands** to which they are attached, are made in a great variety of forms. See Figs 7, 8, 9, 14, 15, and 17. That shown in Figs 7 and 8 is the "**Tumbling-lever** stand" or "**Ground-lever** stand," and, in its numerous modifications, is very largely used. It is so arranged, that, whichever way the switch is set, the crank, C, is on the dead center, so that the lateral strains of passing cars or engines can exert no tendency to turn it.

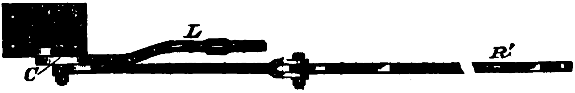


Fig. 7.

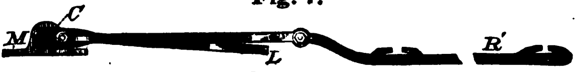


Fig. 8.

Tumbling switches are convenient because they occupy but little space. By means of a target or lantern, connected with the switch, they may be made to indicate to the engine driver the position of the switch.

When the switch is set either way, the lever is padlocked to a staple driven into the tie and passing up through the slot in the handle of the lever. The lever is frequently made with a weight of say 20 lbs on its free end, to aid in bringing it down to its proper position.

Art. 6. Fig 9 represents a common form of the **upright lever and stand.** The switch-rod, R' Figs 4, 7 and 8, is generally attached at the lower end, A, of the lever. The cast-iron frame, F, is fastened to the long tie, T, Fig 4, by large screws or spikes, which pass through its broad feet or flanges, B B. The top of the frame is provided with two notches, N N, and staples, to which the lever is secured by a padlock. When this stand is to be used for a *three*-throw switch, the frame has *three* notches and three staples. The upright stand may be used wherever it will not be in the way of passing trains. The target, T, at the top of the lever, by showing the position of the latter, indicates to the driver of an approaching engine which way the switch is set.

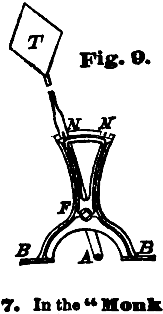
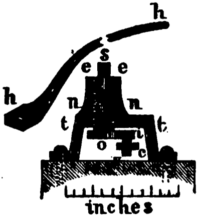


Fig. 9.



MONKEY SWITCH

Fig. 10.

Art. 7. In the "**Monkey-switch,**" Fig 10, the crank, *o i*, is moved horizontally through an arc of a circle by means of the lever, *h h*, about 3 ft long, which fits upon the square head, *s*, of the vertical spindle or pin, *s o*. The switch-rod, R' Figs 4, 7 and 8, is attached to the pin, *i v*.

Many modifications of the monkey-switch are in use. The spindle, *s o*, is frequently made long enough to bring the lever to about the level of the hand; and the lever is permanently attached to the stand, and hinged near the spindle so as to hang down, out of the way, when not in use. To the top of the spindle is frequently attached a vertical rod of any desired length, and carrying at its top a target which turns as the spindle does, and thus indicates the position of the switch.

Art. 8. All parts of the switch-stand, and the tie upon which it rests, should be perfectly rigid, because it is very important that they should hold the ends of the switch-rails exactly in line with those of the main line and turnout. They therefore, in view of the great strains to which they are subjected, must be strongly constructed, and frequently looked after.

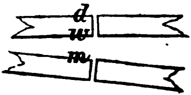


Fig. 11.

Art. 9. In Figs 1 and 4, *d q m* is called the **switch-angle.** The dist, *d m*, Figs 1, 4, and 11, required for the motion of the toes, is called the **throw** of the switch. It must be equal at least to the width, *d w*, Fig 11, of the top of the rail, in addition to a width, *w m*, sufficient to allow the flanges of the wheels to pass along readily between *b* and *e*, Fig 1, and between *r* and *s*. The tops of the rails are generally between 2 and 2½ ins wide; and about 1¾ to 2½ ins suffice for the flanges. The throw, *d m*, however, is commonly about 5 ins.

The **gauge,** Fig 6, of a railroad track, is the distance between the inner sides G G′ of the heads of its two rails. Hence these inner sides are called the **gauge sides** of the rails.

Art. 10. The stub-switch is cheaper in first cost than the improved safety switches, Arts 13, 18, etc, but is less economical in the long run.

As it is very essential that the toes of the switch-rails should never come into contact with the adjoining rail-ends, a space of about an inch must be allowed at the toes for expansion, and for "creeping" This renders the blows of passing trains very severe, and injurious to rolling stock, and to the rail-ends. The latter are worn away rapidly and must be frequently renewed. From the same cause the tie under the head-plate is apt to become loose in its bed.

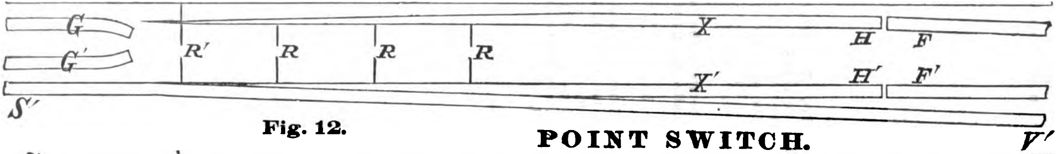


Fig. 12.

POINT SWITCH.

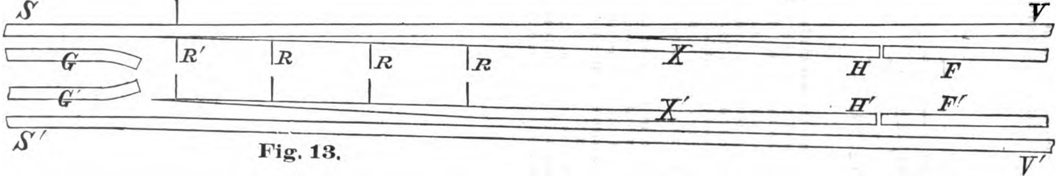


Fig. 13.

Art. 11. Split-, tongue-, or point-switches. Figs 12 and 13 represent these switches in their simplest form, and without the safety appliances described in Arts 12, 13, &c. They omit the *flanges* of the rails, and show only their *heads*. Fig 12 shows the switch set for the main line. Fig 13 shows it set for the turnout. The **"stock-rails,"** S V and S′ V′, are continuous, and are spiked to the ties throughout. Sometimes the **switch-rails,** X and X′, are made long enough to be spiked to the ties for some feet back from their heels, H H′, as is now generally done with stub-switches, see Art 4; but usually their heels are fastened by fish-plates to the ends of the rails, F and F′, which lead to the frog, and they are then not otherwise fastened. They are therefore free to move about their heels as fulcrums, without changing their shape. Iron plates are fastened to the ties under the feet of the switch-rails, to facilitate their sliding. It will be noticed that in the split-switch the **heels** are in the places occupied by the **toes** in the stub-switch, and *vice versa.* The switch-rails, X and X′, are straight. When the switch is set for the main line, as in Fig 12, the gauge side of X′ coincides with that of S′ and F′. **The toe of each switch-rail is pointed,** and so shaped as to fit close up to the head of the rail, S V or S′ V′, with which it is in contact. Hence the name "*point*"-switch. In Europe, where most of the switches are of this type, the name "points" is applied to *any* switch. For some distance back from the toes, where the switch-rails are thinnest, their tops are made a little *lower* than in the rest of their length, so that the weight of the train does not come upon said thinnest portion. The heels, H H′, are placed so that the heads of the switch-rails, X and X′, at those points, are about 2 ins, in the clear, from those of the stock-rails, S V and S′ V′, in order to allow room for the passage of the wheel-flanges; and, theoretically, the same clear distance from the stock-rail would be sufficient for the *points;* but *they* are given a throw of about 3½ or 4 ins, in order to avoid all possibility of a wheel-flange striking the point. As an additional precaution, short **guard-rails** are sometimes spiked to the ties about 2 ins from the stock-rails. Their ends are shown at G and G′. The switch-rails are connected with each other by means of **connecting-bars,** R R′, as in the stub-switch. As improved by the Penna Steel Co, and shown in Fig 16, these connecting-bars are jointed, so that, while *rigid vertically*, they are capable of sufficient *horizontal* motion about their joints, to render easy the moving of the switch. Point-switches may be moved by any of the ordinary **levers.** Arts 5, 6, 7, and 8, or by levers designed especially for them, Arts 12, 14.

Art. 12. The "Automatic" switch-stand is shown in horizontal section by Fig 14, and in vertical section by Fig 15. It is designed especially for split switches like those shown in Figs 12 and 13. It is operated by a tumbling-lever, L, Fig 14. To the horizontal axis, A, of the lever, is fastened the beveled pinion, P. This engages in the teeth of the quadrant, Q, and moves it horizontally through a quarter of a circle, when the lever L is thrown from its position L to that of L′, shown by the dotted lines. The rod, R′, from the switch, is attached at X, or at X′, to this quadrant; and the movement of the latter thus sets the switch, and at the same time turns the target, T, Fig 15, or lantern, fixed to the vertical spindle, S, thus indicating the position of the switch. The gearing is inclosed in a cast-iron case, as shown in Fig 15. If a train on the *turnout*, moving in the direction of the arrow, Fig. 1 (or "trailing"), approaches a split-switch provided with the automatic stand, and set (through oversight or otherwise) for the *main* track, as in Fig 12, the flange of the first wheel, pressing between rails X′ and S′ V′, will push the switch-rails into the proper position, Fig 18, for the *turnout;* at the same time necessarily throwing the weighted lever over into the reverse position, and turning the target so as to indicate that the switch is set for the turnout. A similar movement of the switch, in the opposite direction, takes place if a trailing train on the *main* track approaches the switch when set for the *turnout*, as in Fig 18. Hence the term "automatic," as applied to this stand. The switch is thus made a *safety* switch.

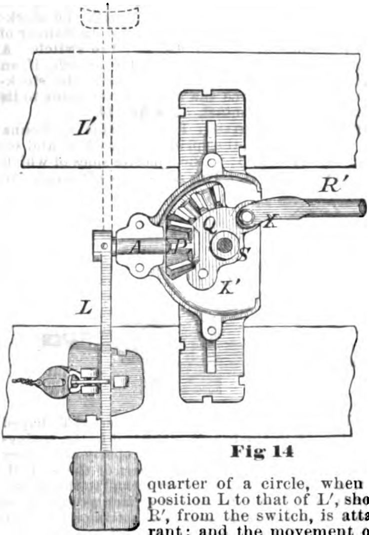


Fig 14

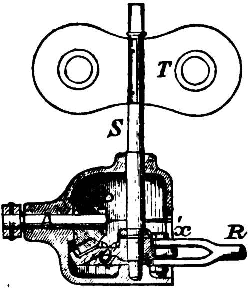


Fig 15

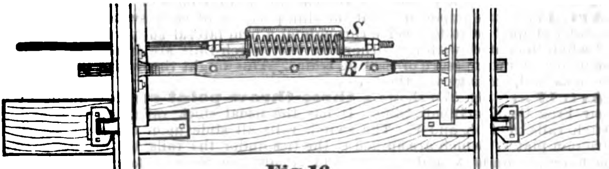
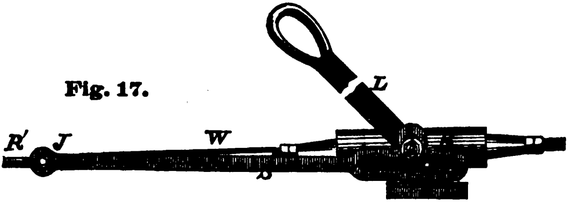


Fig 16

Art. 13. In the Lorenz safety-switch, the connecting-bar, R′, nearest to the toes, is provided with a **spring,** S, Fig 16, placed sometimes between the rails, as there shown; sometimes outside of the track. This spring permits the moving of the switch-rails by the wheels of a trailing train, as does the automatic switch-stand, Figs 14 and 15; but after the passage of each wheel, the spring returns the switch rails to their original position. The blow of the switch-rails against the stock-rails, thus occasioned, is injurious to both, and liable to break the former. On the other hand, the compression of the spring, during the passage of the train through the switch, sometimes impairs its elasticity, so that it then fails to return the switch-rail to its proper position in contact with the stock-rail, and allows it to remain half an inch or more away from it, and in danger of being struck by the wheel-flanges of approaching trains "facing" the switch. A similar accident may happen during the ordinary working of the switch, if an obstacle, as a small stone, becomes lodged between the switch-rail and the stock-rail; for the spring may permit the switchman to force the switch-lever home to its place without bringing the two rails properly into contact. See Art. 14.

Art. 14. De Vout's safety switch-stand, Fig 17, made by Penna Steel Co, is designed to remedy this. In this stand, the spring is placed in, and secured to, a semi-cylindrical iron spring-case or box, B; to the opposite sides of which are fixed two hor axles. One of these is shown at A. This axle passes through the

Fig. 17.



switch-lever, L, near its fulcrum, F. It also passes through the inverted T-shaped slot, H, in the rigid bar, S, which, together with the bar, W, attached to the spring-case, is jointed, at J, to the switch-rod, R′. When the switch is properly set, either for the main line or for the turnout, the axle, A, is in the hor part of the slot, H, and immediately under the vert part, so that there is no obstruction to the movement of the switch, and a trailing train will open a misplaced switch as explained in Arts 12 and 13. But when the lever is raised, for the purpose of setting the switch in the other position, the axle, A, rises into the vert part of the slot, as in the fig, lifting the spring-case with it. If now any obstruction prevents the switch-rail from being pressed home, the rigid bar, S, by means of the axle, A, prevents the lever, L, from moving farther.

Art. 15. Theory would require that **the lengths of the switch-rails,** in split-switches, should vary with the radius of the turnout curve, and formerly they were so made. Where this radius is such that a No 10 frog (see Art 26) is required, the switch-rails should, theoretically, be 28 ft long. But in practice a uniform length of 15 ft (just half the usual length of the steel rail from which the switch-rails are cut) for all turnouts, gives the best results, combining economy of manufacture with greater strength, and greater ease of handling, than are possible with much longer rails.

Art. 16. It will be noticed that in point-switches (as also in the Wharton switch, Arts 18, &c) there can be no such jar as that occasioned in the stub-switch by the long space between the toes of the switch-rails and the ends of the adjoining rails.

Art. 17. It is important that the thin portions of each switch-rail should be carefully shaped so as to receive throughout a firm lateral support from the stock-rail when in contact with it. Otherwise the switch-rails are in danger of bending under the lateral pressure of passing trains. This might throw the point out from the stock-rail, endangering the train.

Art. 17 a. Fig. 17 *a* shows a **three-throw point switch** made by The Weir Frog Co., Cincinnati, Ohio. It has the usual stock rails, C and Z, and four switch rails, A, B, X and Y. The switch rails all slide upon the same set of iron "friction plates," which are spiked to the ties under the rails, but are not shewn in the figure. Rails A and B, are held rigidly together by four connecting bars *a*, *a*, *a*, *a*, while X and Y are similarly connected by the other four connecting bars

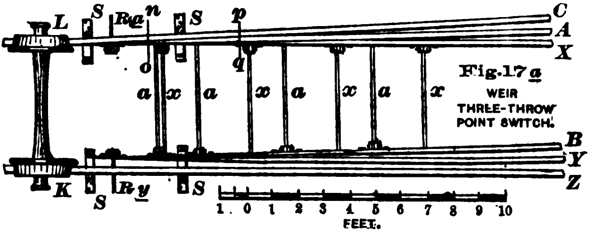


Fig. 17 *a*
WEIR THREE-THROW POINT SWITCH.

x, x, x, x. Each pair of switch rails, thus formed, moves independently of the other; the switch-rod R *a* operating the pair A-B, and R *y* operating the pair X-Y. Our figure shows the switch as arranged for two separate switch-stands, one on the near side operating the rod R *y* and the rails X-Y, and one on the farther side, operating the rod R *a* and the rails A-B; but it may be arranged, instead, so that both pairs of rails may be operated by means of a single stand, placed on one side of the switch.

The figure shows the switch set for the track X Z.

To set it for the track A Y, the rod R *y* is pulled, and draws rails X-Y over, bringing Y into contact with Z, so that wheel K may run upon rail Y, and leaving a space between rails X and A for the passage of the flange of wheel L, which wheel then runs upon rail A.

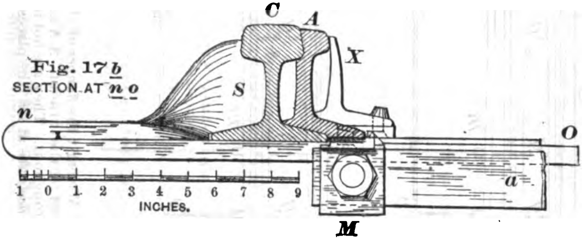


Fig. 17 *b*
SECTION AT *n o*

To set the switch for the track C B, the rod R *a* is then pushed, and throws rails A-B over, bringing B again into contact with Y, so that wheel K may run upon rail B, and leaving a space between rails C and A for the flange of wheel L, which wheel then runs upon rail C.

As in all point switches, it is impossible to spike the inner flanges of the stock rails C and Z to the ties along that portion of their length (some 12 feet from the switch point) where they come in contact with the switch-rails. As a substitute they are provided with special supporting blocks S, S, on the outer side. In the Weir switch, each of these is made of one piece of flat bar iron, bent over and twisted, and serving also as a sliding plate, as shown more clearly in the section *n o*, Fig. 17 *b*, which shows also the manner of fastening the end of the flat connecting bar *a* to the base of rail A by means of a malleable casting M, through which the end of the bar *a* passes, and to which it is bolted. The end shown in Fig. 17 *b* is that which passes under rail X and is therefore hidden by it in Fig. 17 *a*. The section at *p q*, (fig. 17 *c*) shows the attachment of one of the rods *x* to rail X. The arrangement here is similar to that at *n o*, except that in Fig. 17 *c* the malleable casting M is *bolted* to the *web* of the rail as shown, while in Fig. 17 *b* it is of different shape, and is *riveted* to the *flange* of the rail.

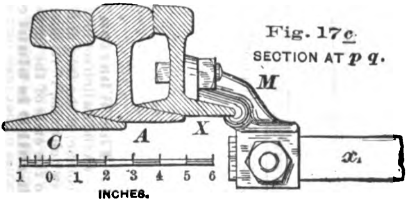


Fig. 17 *c*
SECTION AT *p q*.

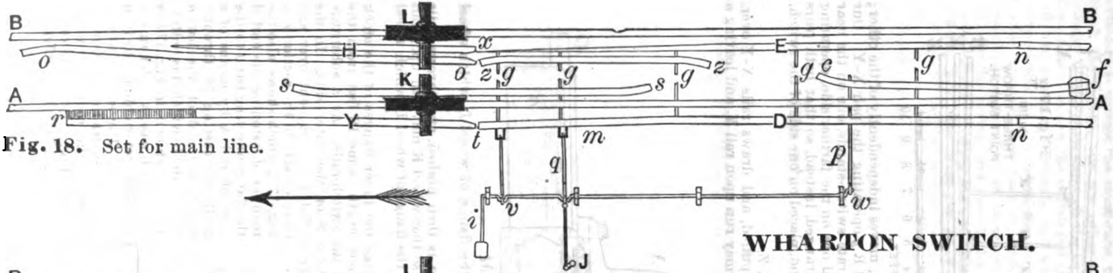


Fig. 18. Set for main line.

WHARTON SWITCH.

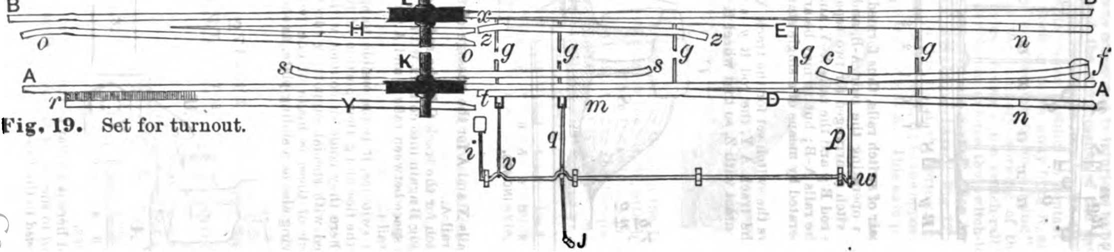


Fig. 19. Set for turnout.

Art. 18. The Wharton patent safety-switch. We omit the *flanges* of the rails, showing only their heads.

Fig. 18 shows the switch set for the main line; Fig. 19 shows it set for the turnout. A A and B B are the main-track rails. They are continuous, except, of course, at the frog, and are spiked to the cross-ties throughout. They are not moved or broken in the working of the switch. E and D are the switch-rails, 18 ft. long, or longer if desired. Their *heels* are at *n n* (the places of the *toes* in the *stub*-switch, Art. 4), where they are fastened to the ends of the turnout rails by fish-plate joints.

Switch-rail D is blunt ended. At its toe *t*, its top is level with that of A A; but from *t* it rises, until, at *m*, 3 ft. 6 ins. from the toe, its top is 1¾ inches higher than that of A A. This enables the flange of wheel, K, in passing to or from the turnout, to pass over the main rail, A A, without touching it. D retains this elevation from *m* to its heel, *n*. Beyond *n*, the turnout rail, continuous with D, slopes gently downward, until its top is again level with those of the main-track rails. The unbroken main rails, and this carrying of the flanges over the main rail, are special features of the Wharton switch. **The pointed switch-rail, E,** is elevated, like D, to prevent the lateral rocking of cars and engine which would otherwise result from the elevation of D.

The guard-rail, z z, is firmly bolted to the pointed switch-rail, E, and of course moves with it. Sufficient space is left between their heads to allow the passage of the wheel-flanges. Said guard-rail prevents the flange of wheel, K, from touching the head of rail, A A, in coming down the incline, *m t*, in Fig 19. As in other switches, the switch-rails, D and E, are connected together by **clamp-bars, g.** These pass underneath the main rail, A A. The switch-rails rest and slide upon iron plates fastened to the tops of the cross-ties.

Art. 19. The switch is operated by means of the **tumbling-lever, i,** and the cranks, *v w*. The lever, when the switch is set either way, lies so as to bring the crank *v* a little *below* its center, so that any lateral pressure on the switch-rails, instead of misplacing the switch, only presses the lever closer to the cross-tie to which the switch-stand is bolted. The weight on the end of the lever aids in bringing it down to its proper position. When the lever is moved from its position in Fig 18 to that in Fig 19, the switch-rails are pushed close up against the main rails. The point, *x*, of E, then fits close up against the stem, and under the head, of the main rail, B B; so that the wheel, L, moving in either direction, can pass the point, *x*, without striking it. **The fixed guard-rail, s s,** tends to keep L away from B B, and thus acts as an additional precaution against its striking *x*. At J is shown, in cross-section, a vertical iron rod, which carries, at its top, the **signal target** or lantern. This signal apparatus is operated automatically by the moving of the switch; the two being connected by the rod *q*.

Art. 20. Provision against accident in case of misplacement of the switch. Case 1. As explained in Art 3, trains moving in a direction *opposite* to the arrow, or "*facing*" the switch, must go as the switch is set, whether right or wrong. On this account it is usual, on **double-track** roads, to use, as far as possible, only *trailing* switches. See Art 2. For **single-track,** the Wharton may be so arranged that it will not remain set for the turnout unless actually held so by the switchman.

Case 2. If a *trailing* train comes from the *turnout* while the switch has, through oversight or otherwise, been left set for the *main line*, as in Fig 18; the wheels run upon the **safety-rails,** H and Y; and the **guide-rail,** *o o*, guides them safely on to the main-track rails, A A and B B. The guide-rail, *o o*, is firmly bolted to H, with room between for the passage of the flanges. In order that wheel, K, in this operation, may pass safely from the safety-rail, Y, to the main rail, A A, the former inclines gently upward from *t* toward *r*, and thus lifts wheel, K, so that its flange passes over rail A A without touching it. **The steel-plated casting, r,** keeps Y at its proper distance from A A, and prevents a wheel, K, with narrow tread, from dropping between the two when passing from one to the other in the operation just described.

Case 3. A *trailing* train moving along the *main track* while the switch is set for the *turnout*, as in Fig 19. **The cranks, v w,** are so arranged (see figs), that when the switch-rails are pushed up against the main-track rails, as in Fig 19, the end, *c*, of **the movable guard-rail, R** (moving in the opposite direction about its fulcrum in the chair, *f*), is also brought close up against the main rail, A A. In this case, the flange of the first wheel, K, of the train, pushes aside *c*, which, pulling the rod, *p*, attached to it, throws the crank, *w* (and through it the entire switch), into the proper position, Fig 18, for the main track. The crank, *w*, is placed on its shaft at such an angle as to be readily operated in this way.

Art. 21. Until recently, safety-*castings* were used in place of the safety-*rails*, H and Y. But when these castings were made long enough to avoid a too abrupt movement of the train they were found to be in danger of breaking. The switch-rail, E, and its guard-rail, *z z*, were heretofore rolled in one piece, called a **grooved rail.** The present arrangement was adopted in order to give these rails less lateral rigidity, and thus adapt them better to the passage of the heavy rolling stock now in use. In the earlier forms of this switch, the rail D rose from 2½ inches in 2 feet; but this was found to be too steep an incline, and the present one, of 1¾ ins in 3½ ft, was therefore substituted.

Art. 22. Frogs. The frog is a contrivance for allowing the flange of the wheel on the rail *e x*, Fig 1, to cross the rail *r z*; and that of the wheel on *r z*, to cross *e x*. **The first contrivance for this purpose was a bar,** approximately of the shape of the rail, pivoted at the point where the center lines of the rails *e x* and *r z* cross each other, and free to move horizontally about this pivot, so that it could form a portion of *e x* when the train was passing to or from the *turnout*, or a portion of *r z* when the train was using the *main track.* Sometimes the pivot passed through one *end* of the bar, as in Fig 20, and sometimes through its *center*, as in Fig 21. Such bars were generally moved by a rod (attached at *n*) and lever, similar to those used for switches; and they then, of course, required an attendant; but many attempts have been made to use such frogs by connecting them with the switch by means of rods, &c, so that the bar should move automatically when the switch was turned. Owing to the considerable distance (80 ft, more or less) between the frog and switch, it has been found difficult to secure simultaneous movements of the switch and frog, and the contrivances referred to have not come into extensive use. Such bars, while they avoid the jar produced by wheels passing across the throat of the frog (Art 35), labor under the same disadvantage as the stub-switch, Art 10, in requiring a liberal allowance of space between their ends and those of the adjoining rails, to avoid any possibility of their coming into contact.

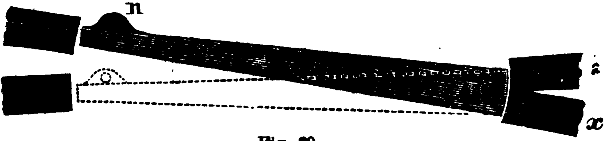


Fig. 20.

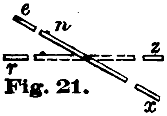


Fig. 21.

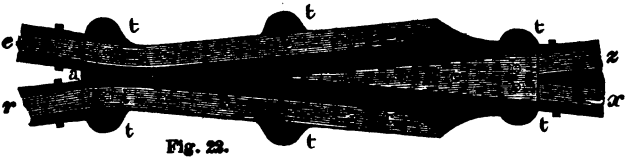


Fig. 22.

Art. 23. These bars were soon superseded by rigid **cast-iron frogs,** Fig 22 and 23. These were hardened by chilling, so as better to resist the action of passing wheels; but even with this precaution they wore out so much more rapidly than the rails, that the **wings,** *w m* and *i c*, and the **tongue,** P, were capped with steel from ½ inch to 1 inch thick, bolted or riveted to their upper surfaces.

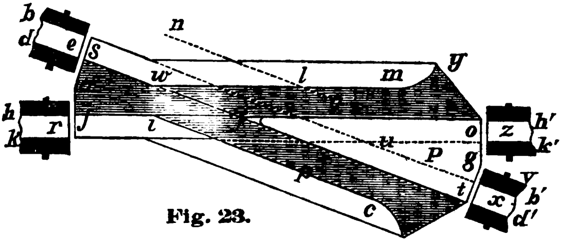


Fig. 23.

The triangle, P, called the tongue of the frog, is the meeting-point of the two rails, *f s* and *f x*, Fig 1; while the wings, *w m* and *i c*, are continuations of the rails *e* and *r*. The wings give support to the treads of the wheels in passing over the spaces between the point and *w* and *i*, which spaces are left for the passage of the flanges.

The channel is called the **mouth** of the frog at *a*, Figs 22 and 23; and its **throat** at the narrowest part, *w i*. That part of the tongue back of *u*, Fig 23, or between *u* and *g*, is called its **heel.**

The channel is made about 2 ins deep to prevent the flanges from touching its bottom.

The projections, *t t*, Fig 22, are for bolting the frog to the wooden cross ties.

Although one side of the frog forms a part of the turnout curve, its shortness warrants us in making both sides, *f o*, *s t*, Fig 23, straight.

Art. 24. Guide-rails, or guard-rails, *g g'*, Fig 1. Suppose wheels to be rolling from A toward B, Fig 1, on the main track; the switch-rails being in the dotted positions. On arriving opposite the frog, some irregularity of motion might cause the flanges of the wheels running along the rail, *r z*, to press laterally against said rail. Consequently, after passing the throat, *w i*, Fig 22, they would press against the wing, *i c*; and passing between *c* and P, they would leave the track; or strike the sharp end of P, breaking it, and endangering the train. To prevent this, the guard-rail, *g*, Fig 1, is placed so near the rail, *b h* (say 1¾ to 2 ins from it), that the flanges at *b h*, while passing between it and *g*, prevent those at the opposite rail from pressing against the wing, *i c*, Fig 22, and from striking the point; and guide them safely along their proper channel, *i m*. Similarly, if wheels be rolling from A toward D, Fig 1 (the switch-rails being in the positions, *q m*, *p s*), the centrifugal force due to the curve would cause the flanges to press against the rail, *e x*, and against the wing, *w m*, Fig 22, thus rendering the train liable to the same kind of accident as in the preceding case. This is prevented, in the same manner as before, by the guard-rail, *g'*, Fig 1, which keeps the flanges in their proper channel, *w c*, Fig 22.

The narrow flange-way between the guard-rail, *g*, Fig 1, and the rail, *b h*, **should extend** at least a foot each way from a point directly opposite the point, *f*, Fig 23, of the frog. In a distance of at least about 2 ft more at each of its ends the guard-rail should flare out to about 3 ins from the rail, *b h*, so as to guide the flanges into the narrow channel. The same with *g'*.

Guard-rails have to resist a strong side pressure, and should be very firmly secured to the wooden cross-ties. This is usually done by bolting against them two or more stout blocks of steel, or of wrought iron, which, in turn, are bolted to the ties.

Art. 25. The cast-iron frog, as first made, had **no provision for fastening it to the rails;** but was simply bolted to the cross-ties. It was afterwards provided with a recess at each end, of the exact shape and size of the end of the rail. The rail ends were inserted into these recesses, and the frog was thus kept in line with the rail. In frogs made of rails, the same purpose is served by fish- or angle-plates, by which the ends of the frog are secured to those of the rails.

Art. 26. The length, a g, Fig 23, of a cast-iron frog, usually varies from 4 to 8 ft; and depends upon the angle, *o f t*, at which the rails, *e x* and *r z*, Figs 1 and 23, cross each other. This is called the **frog-angle.** This angle may be expressed either in degs and mins, or in the number of times the width of the tongue on any line, as *o t*, Fig 23, is contained in the distance, *g f*, from the point, *f*, to the center, *g*, of that line. This number is called the **frog number.** Thus, if the angle, *o f t*, Fig 23, is such that the length, *g f*, is 3, 4, or 10, &c, times the width, *o t*, the frog is called a No 3, 4, or 10, &c, frog. Fig 23 is a No 8; Fig 22, No 5. Frogs are usually made of Nos 4 to 12; sometimes with half numbers, as 7½, 8½, &c.

Art. 27. Draw two parallel lines, *b b'*, *d d'*, for the top of rail, *e x*, Fig 23, and *h h'*, *k k'*, for that of rail, *r s*; crossing each other at the required angle. Then the intersection, *f*, of lines *d d'* and *h h'* is the theoretical **point of the frog.** As this point would be too narrow and weak for service, it is in practice rounded off where the tongue is about ½ inch wide, as shown. If the frog is to be simply abutted to the rail-ends, *s x*, Fig 23, as in some cast-iron frogs, the length, *f g*, need be only great enough to give a width, *t o*, sufficient to accommodate the rail-ends, *s* and *x*, and the heads of the two spikes at *v* which confine them to the ties. If desired, a portion of the flange of each rail-end may be cut away so that the rail-heads come together; thus diminishing the width necessary for *t o*, and, of course, the distance, *f g*. In the case of cast-iron frogs provided with recesses for holding the rail-ends, as in Art 25, the width, *o t*, and length, *f g*, must of course be greater.

In frogs made of rails, the length must be such that the rail-ends, *s* and *x*, Fig 23, are far enough apart to give room for fitting to their inner sides the splice-plates by which they are connected with the frog. Where *angle*-plates are used, this distance must be greater than in the case of *fish*-plates.

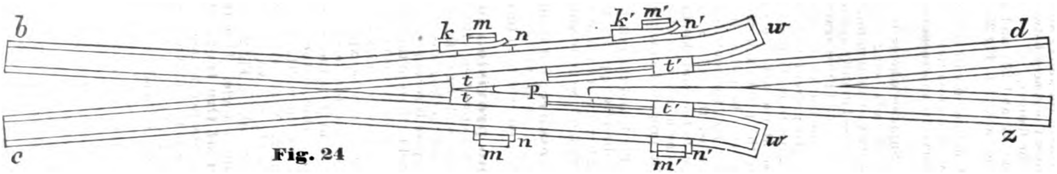


Fig. 24

Art. 28. Draw $w\,l$, Fig. 23, parallel to $f\,o$, and distant from it about 1¾ ins. or barely sufficient to allow the ready passage of the flanges of the wheels. Also, in like manner, draw $i\,p$, parallel to $f\,t$. Make $i\,j$ and $w\,s$ long enough to give a width, $s\,j$, sufficient to accommodate the width of the rail-ends (with or without provision for admitting them to recesses, or for splicing them to the ends of the frog, as the case may be); and to allow the corners at w and i to be rounded off.

Art. 29. Draw $n\,q$ parallel to $i\,p$ and distant from it about an inch more than the width of the widest tires (probably 6¼ ins.) used on the road. Then the wing should be made straight at least as far as from w to l. From l to y, the inner edge of the wing should be flared outward, as in the fig, so as to guide wheels entering that end of the frog.

Art. 30. Our Figs. 24 and 27 show **frogs made from steel rails.** The two rails, $d\,z$, composing the point are shaped by machinery and without being heated, and are fitted and dovetailed together, and then fastened to each other by several rivets passing through their webs. The webs of the four rail-ends, b, c, d, and z, are drilled for fish- or angle-plate splices, which confine them to the adjoining ends of the track-rails. They thus form, as it were, part of the track. Figs. 24 and 27 show the full lengths of the frogs.

Art. 31. Figs. 24, 25, and 26 show a "*keyed* **stiff frog,**" in which all the parts are immovable, as distinguished from the spring-rail frog, Fig. 27, in which one rail, s, is movable. In Figs. 24, 25, and 26 the wing rails, $w\,w$, and the point, p, are shown bound together by heavy wrought-iron **clamps,** $m\,m$, $m'\,m'$, which enclose their flanges, and in which they are secured and held at the proper distance apart, by solid iron **throat-pieces,** $t\,t$, $t'\,t'$, **blocks,** $n\,n$, $n'\,n'$, and tapering split **keys** or wedges, k, k'. After the keys are driven firmly home, their split ends are spread open to prevent the keys from falling out. The throat-pieces, t, extend beyond the point, p, thus bracing it against lateral strains. They are, of course, made low enough to clear the flanges of the wheels. The throat-pieces, t, and the brace-blocks, n, are **prevented from sliding longitudinally** from their proper positions by the heads of the rivets shown in Figs. 25 and 26. These rivets are driven through holes in the webs of the several rails; and their heads fit

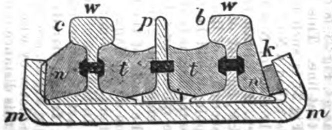


Fig. 25

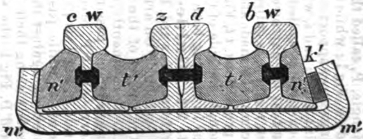


Fig. 26

into recesses let into the sides of the throat-pieces and blocks. **The clamps are prevented from sliding** by clips riveted on the flanges of the rails. Great care is taken to have all the **adjoining surfaces in full contact with each other,** throughout, so as to diminish the liability to wear.

Art. 32. In stiff frogs the parts are held together by **bolts** passing through the rails and the throat-pieces. In some there are no throat-pieces, and the four rails forming the frog are riveted to a wrought-iron **plate.** All of these frogs can be made of any desired length and to any desired angle.

Art. 33. The standard length of stiff frogs from *b c* to *d z*, for Nos 4 to 8, inclusive, is 8 ft; Nos 9 and 10, 9 ft; Nos 11 and 12, 10 ft; No 15, 12 ft.

Art. 34. In the **Weir frog** (Weir Frog Co., Cincinnati, Ohio), Figs. 26 A, 26 B and 26 C, the notching of the main or long point, *s*, to receive the short point *d*, is avoided by forging the latter, under hydraulic pressure, to fit the former which is used as the upper die in the forging process. The rail forming the long point *s* is thus preserved intact throughout the space where the two points are in contact, as shown in the section at *n o*, Fig. 26 B. The short point, *d*, is held tightly against, and under the head of, the long point *s*, by the tightening of the nuts on the bolts (which are provided with nut-locks), and thus gives it additional support.

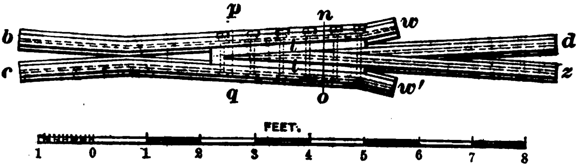


Fig. 26 A

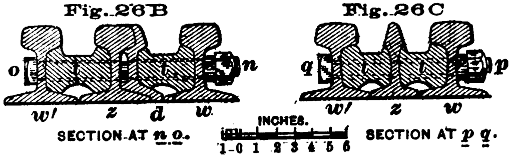


Fig. 26 B

Fig. 26 C

The long filling-blocks, *t t*, are forged in dies from wrought iron or steel. They are then planed to fit the point rails, drilled, and bolted as shown.

Art. 35. The object in reducing the width of the channels for some distance each way from the point, *f*, Fig 23, so as barely to admit the flanges freely, is to allow the treads of the wheels to have as much bearing as possible upon the wings and tongue while moving over the broadest part of the channel near *f*. In frogs shorter than about No 4, it is difficult to secure sufficient bearing for the treads, even with the utmost allowable contraction of the channel, when the width of the tires is, as usual, about 6 ins. In the earliest frogs this difficulty was partially overcome by gradually **raising the bottom of the channel** between the point and wings, so that the wheels, in traversing that part, ran upon their *flanges* instead of upon their *treads*. The jar occasioned by the tread, in striking against wing and point, was thus avoided. This arrangement is still used in *crossings*, where the tracks cross at a very obtuse angle, and where, consequently, the wings can give little or no support to the treads. The flanges, however, soon cut gutters in the bottoms of the channels, and thus increase their depths, so that the treads strike the wings and point, as in the ordinary frog.

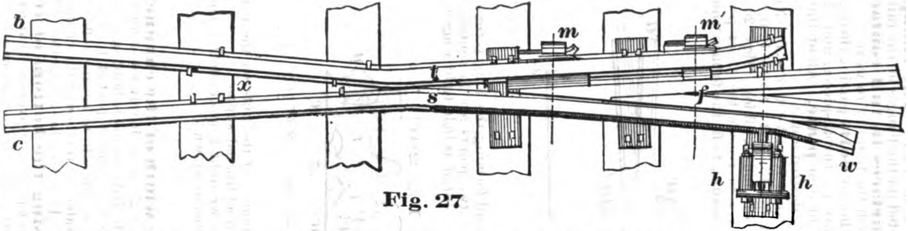


Fig. 27

Article 36. In spring-rail frogs, Figs. 27 and 28, **the wheel G, Fig. 28, moving in either direction** on the *main* track (as well as wheel E), has at all times a full bearing throughout the frog, and no throat to cross. The rail *t*, of the turnout, and the point *f*, are spiked to the ties at a proper distance apart (about 1¾ inches) to allow the passage of the flange of wheel G; and are secured to each other by clamps *m m′*, with iron throat pieces, blocks and keys. The rail *s* of the main track, called the spring-rail, is held in place by the rail-joint at its end *c*, and by the spikes on its *inner* side near *x*. Its other end, *w*, is movable, but is kept pressed against the point *f* by two stout spiral springs *h h*, so long as no trains are passing to or from the *turnout*. The flange of the spring-rail, on the side next to the point-rail, is removed in order to let the two heads come together.

In order that a wheel, J or L, Fig. 28, passing to or from the turnout, may cross the frog, the rail *s* must evidently be moved away from its position in contact with the point *f*, to allow the wheel-flanges to pass between the two. This is done as follows:

The flange of a wheel, L, moving from *x* to *e*, presses in at *w* between the point and the spring-rail, and thus pushes the latter aside.

When a wheel, J, moving from *e* to *x*, approaches the frog, the guard-rail, *g′*, pressing against wheel K, causes the opposite wheel, J, to press against the spring-rail, *s*, and push it away from the point.

In either case the springs *h h*, after the passage of each wheel, restore the spring-rail, *s*, to its position, close to the point *f*, as in the figures.

The free end *w* of the spring-rail *s* tends to rise when the weight on the rail is concentrated near its other end *c*. This is prevented by a stout T-shaped iron forging, the arms of which are riveted to the web of the spring-rail, and the stem of which slides in a malleable iron guide-box placed between the springs *h h* and riveted to the iron slide-plate, which passes under all the frog rails at that point. The guide-box also acts as a stop to prevent the free end of the spring-rail from moving out too far.

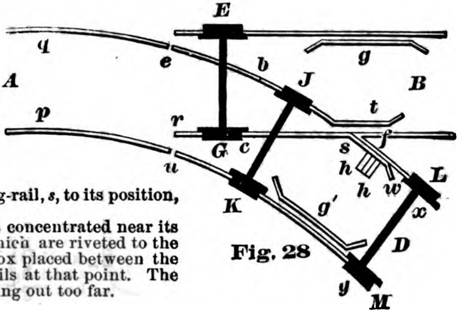


Fig. 28

If the bearing between the head of the spring-rail *s* and that of the point-rail *f*, is too short, a ridge worn on the outer edge of the tread of a wheel G passing from B toward A, may drop between the spring-rail and the point-rail when it reaches a place, near *f*, where the point is narrower than the gutter worn in the wheel-tread. The ridge is then liable to wedge the spring-rail *s* away from the point *f*. To avoid this, the wing *w* of the spring-rail, and its bearing against the point-rail, have been lengthened in the present pattern, thus giving a wider bearing and keeping the ridge of a worn wheel-tread on top of the rails until the frog has been passed. **The standard length** for either style of **spring-rail frog**, and for any angle, is 15 feet.

Since spring-rail frogs do not present a full bearing to wheels entering or leaving the *turnout*, but only to those passing to and fro on the *main track*, they are most useful where the greater part of the traffic moves on the main line, and where but few trains use the turnout.

Art. 37. In ordering frogs, give the frog angle or number, and the exact cross-section of the rail used on the road.

Art. 38. For spring-rail frogs, specify also whether the turnout is to the right or left hand. In the case of *stiff* frogs, this is not necessary.

Art. 39. The laying-out of Turnouts.

In each of the Figs 29, 30, and 31, *w p z* represents the main track. The **frog distance,** *p f*, is a straight line drawn from the theoretical *point of frog*, *f*, to the heel, *p*, of that switch-rail which, when opened, forms the *inner* rail of the *turnout*. Formerly, when the turnout curve was taken as starting at the *toe* of the switch, the frog dist was a straight line from the theoretical point of frog to the toe, *m*, Fig 4, of the outer switch-rail, *q m*, when opened.

Scrupulous accuracy is not necessary in these matters. Thus, a deviation, either way, of say 3 per cent in the length of the turnout radius from that given by the table or the formulas, will be almost inappreciable. So too, if a frog number should be used, intermediate of those in the first column of the table, the other dimensions may be found, approximately enough, by using quantities similarly intermediate. A rail almost always has to be cut in two in order to fill up the frog dist; and the exact length of the piece can be found by actual measurement at the time of cutting it.

Rem. When the turnout leaves a straight track, as in Fig 29, the **frog angle** is equal to the central angle, *f c o*. When the main track is curved, and the turnout curves in the **opposite** direction (Fig 30), it is equal to the **sum** (*v f n*) of the central angles, *f c o*, *f n o*; and when the two curve in **the same** direction (Fig 31), it is equal to the **diff** (*n f c*) of the central angles, *f c o*, *f n o*.

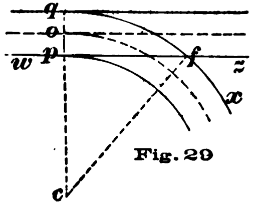


Fig. 29

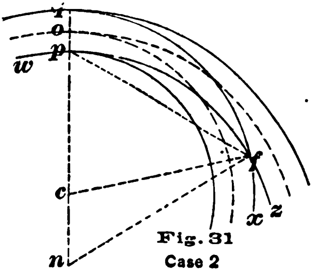


Fig. 31

Case 2

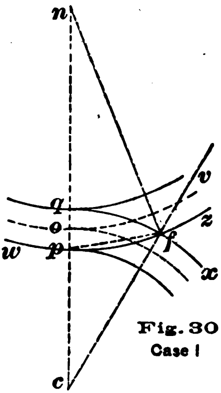


Fig. 30

Case 1

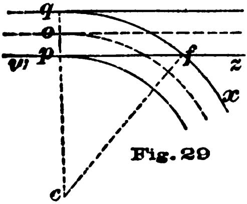


Fig. 29

Art. 40. To lay out a turnout, ***p x*****, Fig 29, from a straight track,** ***p z*****.** From the column of radii in the table below, select one, *c o*, suitable for the turnout; together with the corresponding frog number, frog dist, *p f*, and switch length. Place the frog so that the main-line side of its tongue shall be at *f z*, *precisely* in line with the inner edge of the rail, *w z*, and its theoretical point, *f*, at the tabular frog dist, *p f*, from the starting-point, *p*. Stretch a string from *q* (opposite *p*) to *f*; and from it lay off the three ordinates from the table; thus finding three points (in addition to *q* and *f*) in the outer curve. Do not, however, drive stakes at these points; but as each of them is found, measure off from it, inward, half the gauge of the track; and *there* drive stakes. Do the same from *q* and *f*. The five stakes will all then be in the dotted center line of the turnout, Fig 29; and will serve as guides to the work, without being liable to be displaced. The dimensions in the table below are found by **the following formulas,** the main track being straight:

Tangent of **half frog angle** = Gauge ÷ Frog dist.

Frog No............ = $\sqrt{\text{Radius } co \div \text{Twice the gauge.}}$

Or, **Frog No**............ = Half the cotangent of half the frog angle.

Radius c o......... = Twice the gauge × Square of frog number.

Or, **Radius c o**......... = (Frog dist *p f* ÷ Sine of frog angle) — half the gauge.

Or, **Radius c o**......... = (Gauge ÷ Versed sine of frog angle) — half the gauge.

Frog dist p f...... = Frog number × Twice the gauge.

Or, **Frog dist p f**...... = Gauge *p q* ÷ Tangent of half the frog angle.

Or, **Frog dist p f**...... = (Rad *c o* + half the gauge) × Sine of frog angle.

Middle ord........ = ¼ gauge, approx enough.

Each side ord... = ¾ mid ord = $\frac{3}{16}$ (or .188 of the) gauge, approx enough.

$$\textbf{Switch Length} = \sqrt{\frac{\text{Throw in ft} \times 10000}{\text{Tangential dist for chords of 100 ft, for rad } co \text{ of turnout curve.}}}$$
approx enough

TABLE OF TURNOUTS FROM A STRAIGHT TRACK. Fig 29.

Gauge 4 ft 8½ ins. Throw of switch 5 ins.

For any other gauge, the *frog angle* for any given frog number remains the same as in the table. The other items may be taken, approx enough, to vary directly as the gauge.

Frog Number	Frog Angle	Turnout Radius c o	Defl Ang of Turnout Curve	Frog Dist p f	Middle Ordinate	Side Ords	Stub Switch Length
	° ′	Feet.	° ′	Feet.	Feet.	Feet.	Feet.
12	4 46	1356	4 14	113.0	1.177	.883	34
11½	4 58	1245	4 36	108.3	1.177	.883	32
11	5 12	1139	5 2	103.6	1.177	.883	31
10½	5 28	1038	5 31	98.9	1.177	.883	29
10	5 44	942	6 5	94.2	1.177	.883	28
9½	6 2	850	6 45	89.5	1.177	.883	27
9	6 22	763	7 31	84.7	1.177	.883	25
8½	6 44	680	8 26	80.0	1.177	.883	24
8	7 10	603	9 31	75.3	1.177	.883	22
7½	7 38	530	10 50	70.6	1.177	.883	21
7	8 10	461	12 27	65.9	1.177	.883	20
6½	8 48	398	14 26	61.2	1.177	.883	18
6	9 32	339	16 58	56.5	1.177	.883	17
5½	10 24	285	20 13	51.8	1.177	.883	15
5	11 26	235	24 32	47.1	1.177	.883	14
4½	12 40	191	30 24	42.4	1.177	.883	13
4	14 14	151	38 46	37.7	1.177	.883	11

* Shortest length of stub switch that will at the same time form part of the turnout curve, and give 5 ins throw. Point switches require only half this throw. In practice, switches are frequently made much shorter than the table requires, thereby sharpening the beginning of the curve.

Art 41. Turnout from a curved main track. The following convenient approximate method was, we believe, first published by Mr. E. A. Gieseler. C. E., of New York, in 1878. There are two cases.

Case 1, Fig 30; when the two curves deflect in **opposite** directions.

Case 2, Fig 31; when the two curves deflect in **the same** direction.

Having determined approx upon a radius for the turnout curve, take from the table, p 784, its corresponding *deflection* angle, and that for the main curve. *In Case* 1, find the *sum* of these two angles. *In Case* 2, find their *difference*. In the table, p 840, find the *deflection* angle (not the frog angle) nearest to the sum or diff just found. The **frog number, switch length** and **frog distance** *p f*, in the table, opposite the deflection angle thus selected, are the proper ones for the turnout. *Theoretically* we should, in Case 1, *add* to the tabular frog distance *p f* about half an inch per 100 feet for each degree of deflection angle of the easier of the two curves; and, in Case 2, *deduct* it; but this refinement is unnecessary.

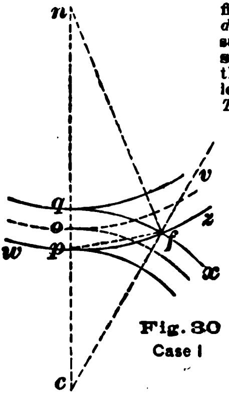


Fig. 30

Case 1

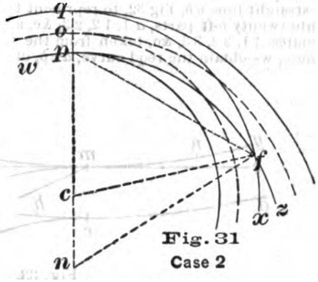


Fig. 31

Case 2

Deflection angle of turnout curve

(in Case 1) = Tabular deflection angle — deflection angle of main curve,
(in Case 2) = Tabular deflection angle + deflection angle of main curve.

Ex. Rad of main curve, 2865 ft. Rad selected approx for turnout, 716.8 ft. Here the defl angles are, respectively, 2° and 8°.

In Case 1; 8° + 2° = 10°. Nearest defl angles in table, p 840, 10° 50′ and 9° 31′. If we select 10° 50′, we have Frog No, 7½; Switch L'gth, 21 ft; Frog Dist = 70.6 ft + *twice* ½ in = say 70.7 ft; Defl Angle for turnout = 10° 50′ — 2° = 8° 50′; Rad *c o* 649 ft. If we select 9° 31′, we have Frog No, 8; Switch L'gth, 22 ft: Frog Dist = 75.3 ft + twice ½ in = say 75.4 ft; Defl Angle = 9° 31′ — 2° = 7° 31′; Rad *c o*, say 763 ft.

In Case 2; 8° — 2° = 6°. Tabular defl angles, p 840, 6° 5′ and 5° 31′. 6° 5′ gives Frog No, 10; Switch L'gth, 28 ft; Frog Dist = 94.2 ft — twice ½ in = say 94.1 ft; Defl Angle = 6° 5′ + 2° = 8° 5′; Rad, say 709 ft. 5° 31′ gives Frog No, 10½; Switch L'gth, 29 ft; Frog Dist = 98.9 ft — twice ½ in = say 98.8 ft; Defl Angle = 5° 31′ + 2° = 7° 31′; Rad, say 763 ft.

The frog dist *p f* may also be found thus:

$$\textbf{Tangent of half } f\,n\,o = \frac{\text{Gauge} \times \text{Frog No}}{\textbf{Radius } n\,o.}$$

$$\textbf{Frog Dist } p\,f = n\,p \times \text{twice the sine of half } f\,n\,o.$$

Place the frog with the main-line side of its tongue at *f z*, in line with the inner edge of the frog-rail, *p z*, of the main line, and with its theoretical point, *f*, at the dist, *p f* (found as above), from the heel, *p*, of the *inner* switch-rail. Stretch a string from *f* to the heel, *q*, of the *outer* switch-rail. Measure the dist, *q f*, divide it into four equal parts, and lay off three ordinates, found thus:

Middle ord = (Square of half *q f*) ÷ twice the rad of turnout curve. Each side ord = three-fourths of middle ord.

These three ords, and the points *q* and *f*, give us 5 points of the outer rail of the turnout curve; and from these we measure, inward, half the gauge, and drive 5 center guide-stakes, as in Art 40.

Art. 48. To find frog dists, &c, by means of a drawing to scale. The frog dist can generally be found near enough for practice, from a drawing on a scale of about ¼ or ⅛ inch to a foot. And so in the many cases where turnouts cross tracks in various directions, in and about stations, depots, &c.

Figs 32 and 33 are intended merely to furnish a few general hints in regard to such drawings. For instance, the curves of a main track, as well as those of a turnout, generally have radii too large to admit of being drawn on a scale of ¼ inch to a foot, by a pair of dividers or compasses. But they may be managed thus: Draw any straight line, ab, Fig 32, to represent by scale a 100-ft chord of the curve, divide it into twenty 5-ft parts, a 1, 1 2, 2 3, &c, and lay off by scale the 19 corresponding ordinates, 1 1, 2 2, 3 3, &c, taken from the table on page 730. By joining the ends of these, we obtain the reqd curve, acb, of the main track; and of course can draw the inner line, yt, distant from it by scale the width of track, say 4 ft 8½ ins. Now let acb and yt, Fig 33, be a curved main track so drawn; and let any point m be taken as the starting-point of the turnout, mv, &c. On each side of m measure off any two equidistant points, n and n, in the same curve; and through m draw sg, parallel to nn. Then is mg a tang to the curve, ymt, at m. Having determined on the rad of the turnout curve, mve, draw that curve by the same process as before; first laying off the angle, gmi, equal to the tangential angle of the curve, taken from the table, p 788. Then, beginning at m, lay off 5-feet dists along mi; and from them, as in Fig 32, draw the ords corresponding to the turnout curve. Through the ends of these ords draw the curve, mve, itself. Then the frog dist will be the straight dist from c to v, and can be measured by the scale, within a few inches; or near enough for practice. The middle ord of the arc, mv, cannot be found correctly by so small a scale as ¼ inch to a foot, but should be calculated thus: From the square of the rad take the square of half the chord, mv. Take the sq rt of the rem. Subtract this sq rt from the rad. If two other ords should be desired, half way between m and v and the center one, they may each be taken as ¾ of the center one. Make the switch-rail long enough to leave 2½ ins at its toe between mn and mu.

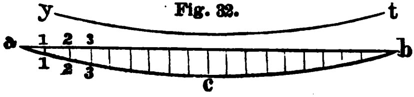


Fig. 32.

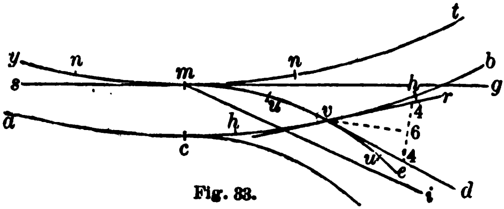


Fig. 33.

The frog angle at v will be equal to the angle, rvd, formed between the tang, vr, to the curve, acb; and the tang, vd, to the curve, mve. These tangs are found in the same way as mg; namely, for the tang, vr, lay off from v two equidistant points, h and h, on the curve, acb; and through v draw vr parallel to hh. Also, for vd, lay off from v any equidistant points, u and u, on the curve, mve, and through v draw vd parallel to them. This angle may be measured by a protractor. Or, if on the two tangs we make $v4$ and $v4$ equal to each other, and draw the dotted line 4 4; and from its center at 6 draw $6v$; then $6v$ divided by 4 4 will give the No of the frog. With care, and a little ingenuity, the young student will be able, by similar processes, to solve graphically any turnout case that may present itself. The method by a drawing has great advantages over the tedious and complicated calculations which otherwise become necessary in cases where curved and straight tracks intersect each other in various directions. The drawing serves as a check against serious errors, which would be detected at once by eye. None of the graphical measurements will be strictly accurate; but with care, none of the errors need be of practical importance.

Art. 43. An experienced track-layer, with a good eye, can place his own guide-stakes by trial on the ground; and by them lay his turnouts with an accuracy as practically useful as the most scrupulous calculations of the engineer can secure.

The following example, Fig 34, of a turnout from a straight track, Y Z, exhibits a common case, in which all the work may be performed on the ground, without previous calculation. Let *i v o* be the tongue of a frog, with which the assistant has been directed to make a turnout from Y Z; and that he has received no instructions more than that the turnout must start at *d*, and terminate in a track, W, to be laid parallel to Y Z, and distant from it *r x* or *r x*, equal to 6 ft.

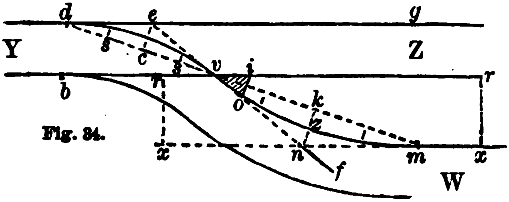


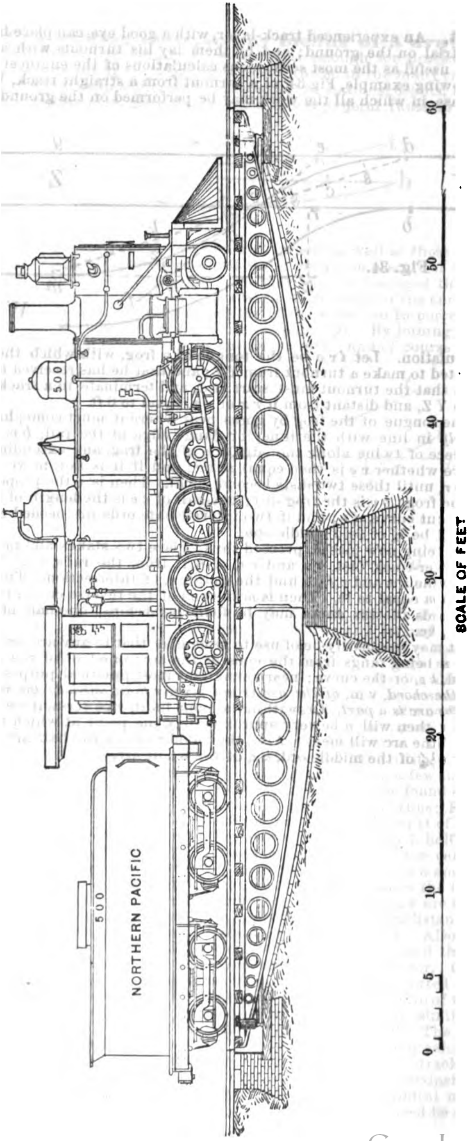
Fig. 34.

Place the tongue of the frog by guess near where it must come, having its edge, *v i*, *precisely* in line with the inner or flange edge of the rail, *b r*. Then stretch a piece of twine along the edge, *o v*, of the frog, and extending to *d g*. Try by measure whether *v e* is then equal to *e d*; and if it is not, move the frog along the line, *b r*, until those two dists become equal. Then is *v* the proper place for the point of the frog; *b v* is the frog dist; one-half of *c e* is the length of the middle ord of the turnout curve, *d v*; and if two intermediate ords are needed at *s* and *s*, each of them will be ¾ of said middle one.

The frog being now placed, proceed thus: Place two stakes and tacks, *x* and *x*, at the reqd inter-track dist, *r x* and *r x*, of 6 ft from the rails, *b r*. Then range by pieces of twine *x x* and *v f*, to find the point, *n*, of intersection. Then measure *n v*, and make *n m* equal to it. Then is *m* the end of the reverse curve, *v m*, of the turnout. The ords of this curve may be found as before; one-half of *n k* being the middle one, &c.

Rem. It may frequently be of use to remember that in any arc, as *v m*, of a circle; *v n* and *m n* being tangs from the ends of the arc; one-half of the dist, *k n*, is the middle ord, *k z*, of the curve; near enough for most practical purposes, *whenever the length of the chord*, v m, *of the arc is not greater than one-half the rad of the circle of which the arc is a part.* Or, within the same limit, vice versa, if we make *k n* equal to twice *k z*, then will *n* be very approximately the point at which two tangs from the ends of the arc will meet. Also, the middle ord of the half arc, *v z* or *z m*, may be taken as ¼ of the middle ord, *k z*, of the whole arc.

Fig. 1.



TURNTABLES.

Art. 1. A turntable is a platform, usually from 40 to 60 ft long and about 6 to 10 ft wide (see Fig 1,) upon which a locomotive and its tender may be run, and then be turned around hor through any portion of a circle; and thus be transferred from one track to another forming any angle with it. The table is supported by a pivot under its center; and by wheels or rollers under its two ends. Frequently other rollers are added between the center and ends. Beneath the platform is excavated a circular pit about 4 or 5 ft deep, having its circumf lined with a wall of masonry or brick about 2 ft thick, capped with either cut stone or wood. The diam of the pit in clear of this lining is about 2 ins greater than the length of the turntable. The lining is generally built with a step, as seen in Fig 1, for supporting the circular rail on which the end rollers travel; or, instead of this step, a detached support may be used for this circular rail, as at u, Fig 11. At the center of the pit is a solid well-founded mass of masonry or timber, for the pivot to rest on, as seen in Fig 1. This, as well as the step for the end rollers, should be very firm, and perfectly level; otherwise the platform will be hard to work. The platform is frequently floored across for a width of 6 to 10 ft to furnish a pathway across the pit, without stepping down into it; especially when under cover of a building. At first they were floored over so as to cover the entire circular pit; but this increased not only their cost but their wt, so as to make them difficult to turn; besides causing much expense for repairs; with greater trouble in making them. It is therefore rarely done at present, except where want of space sometimes renders it necessary in indoor turntables.

For the minimum length of a turntable, add from 1½ to 2 ft to the total wheel base of the longest locomotive and tender for which it is to be used; but a turntable should be several feet longer than is necessary for merely allowing the engine and tender to stand on it; for the increased length enables the engine-men to move them a little backward or forward, so as to balance them chiefly upon the central support; and thus relieve the end rollers. By this means the friction while turning is confined as much as possible to the center of motion; and is there-

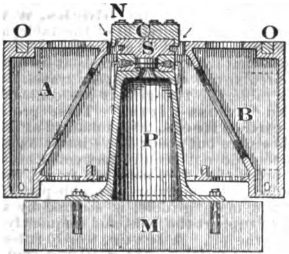


Fig. 2

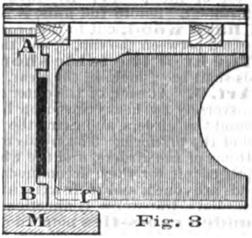


Fig. 3

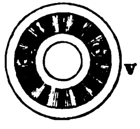


Fig. 5

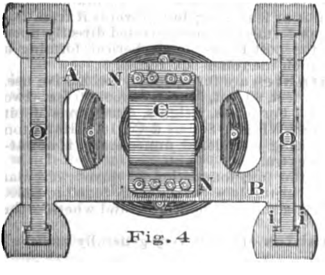


Fig. 4

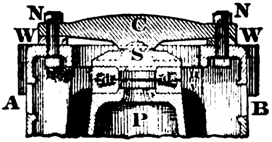


Fig. 6

fore more readily overcome than if it were allowed to act at the circumf. The engine-men soon learn, by feeling, the proper spot for stopping the engine so as thus to balance the platform.

Art. 2. Figs 1 to 6 represent the **Sellers cast-iron turntable** of Wm Sellers & Co., Phila. It consists of two cast-iron girders of about 1¾ ins average thickness, perforated by circular openings to save metal. One of these girders is shown in Fig 1; and parts of one in Figs 3 and 4. Each girder is in two separate pieces, which are fastened, as shown in Figs 1, 3, and 4, to a hollow cast-iron "center-box," A B, Figs 2, 3, 4, and 6, by means of 2¼ inch screw-bolts, at *f*, Fig 3; and by hor bars, *o o*, of rolled iron about 3¾ ins square, fitting into sunk recesses on top of the boxing, and tightened in place by wedges, *i i*, screw-bolted beneath.

Art. 3. The sides of the **center-box** are about 1¾ ins thick. It is suspended from the **steel cap,** C, by 8 screw-bolts 2 ins diam. On its lower side this cap has a semi-cylindrical groove extending across it, transversely of the track, as shown in Fig 6. This groove fits over a corresponding semi-cylindrical ridge on the top of the cast-iron **"socket,"** *s* (so called), on which the cap thus rests. The socket, in turn, rests upon the upper one, *u*, of two annular steel plates, *u* and *v*, which form a **circular box** containing 15 **steel conical anti-friction rollers,** *d d*, Figs 2, 5, and 6. These are about 3 ins in length, and in greatest diam. They have no axles, but merely lie loosely in the lower part, *v*, of the circular box; filling its circumf with the exception of about ½ inch left for play. In the direction of their axes they have ⅛ inch play. The lid, *u*, of the circular box, rests upon the tops of the rollers, which separate it from *v* by about ½ inch. *v* rests upon the top of the hollow **cast-iron post,** P, which, by means of its flanges, is bolted to the cap-stone, M, of the **foundation pier.**

In order to insure a perfect bearing of the revolving surfaces upon each other, and thus diminish the liability to abrasion, the rollers, *d d*, and the insides of the box in contact with them, are accurately finished, as are also the top and bottom of the roller-box, and the surfaces of the socket, *s*, and post, P, in contact with them. The rollers are oiled by means of the spaces shown by the arrows in Fig 2.

Art. 4. Adjustment of the height of the table. By turning the nuts, N N, of the 8 screw-bolts which support the table, the latter may be raised or lowered 1 or 2 ins; the cap, C, socket, *s*, and roller-box, *u v*, remaining stationary on top of the post, P. All turntables should have the means of making such adjustment. Before the nuts, N N, are finally tightened up, the **blocks, w w, of hard wood,** cut to the proper thickness for the desired ht of the table, are inserted between the cap, C, and the top of the center-box, A B, as shown.

The ht of the table should be such that each of the wheels at its outer ends shall be ¼ inch, in the clear, from the circular rail on which they travel.

Art. 5. At each of the outer ends of the table, the two girders are connected transversely by heavy cast-iron beams, called **"cross-girts."** These project beyond the girders, and carry the cast-iron **end-wheels,** 20 ins diam, 2 at each end of the platform. The treads of these wheels are but about 3 or 4 ins below the bottoms of the girders, and the wheels therefore do not require any considerable depth of pit for their accommodation. In order that they may roll freely, their treads are coned, and their axles are made radial to the circular turntable pit. Intermediate transverse connection between the main girders, is secured by the **wooden cross-ties** notched upon them to support the rails, and frequently 10 or 12 ft long, for giving a wide footway across the pit. **A lever,** 8 or 10 ft long, fitting into a staple, is used for turning the table, not on account of friction, but as a handle for the workmen.

Art. 6. The semi-cylindrical shape of the joint between the cap, C, and the socket, *s*, permits the slight longitudinal rocking motion of the turntable which takes place when a locomotive comes upon it or leaves it; but prevents it from tipping sideways, as it was apt to do, when, as formerly, the cap rested directly upon the lid *u* of the roller-box, and the top of the post P was hemispherical, forming a ball-and-socket joint with a casting upon which the roller-box rested.

Art. 7. Ten **sizes of these turntables** are furnished for locomotive use. The **diameters** are as follows: 70 feet, 65 feet, 60 feet, 56 feet, 54 feet, 50 feet (two patterns), 45 feet, 40 feet, 30 feet and 12 feet. For prices, weights, dimensions of pit and foundations, timber required, etc; address William Sellers & Co, 1600 Hamilton St., Philadelphia. Machinery for turning, being considered unnecessary, is not attached, unless specially ordered. Its cost is extra.

The entire cost of excavating and lining the pit; foundation for pivot; circular rail for end rollers, &c, complete for a 56-ft turntable will vary from $1200 to $2500 in addition, depending on the class of materials and workmanship; and whether the bottom of the pit is paved or not.

Art. 8. The girders of turntables are now very generally made of

rolled-iron plates, each girder being in one piece; and the two main girders are then connected with each other, near their centers, by two **cross-girders** of I or channel beams, or of plate-iron, one on each side of the central pivot. These cross-girders, and the sides of the main girders between them, form a sort of rectangular box, corresponding to the cast-iron center-box of the Sellers table, Arts 2 and 3. The arrangement of the center bearing apparatus, and the manner in which it is connected with the cross-girders, differ in the tables of the several makers.

Art. 9. In the plate-iron turntables made by the **Edge Moor** Bridge Works, Wilmington, Del., the cross-girders, G G, Fig. 8, are of plate-iron; and at their ends they have flanges, F F, of angle-iron, by which they are riveted to the main-girders, E. In the 54-ft. tables the cross-girders are 26 ins. apart in the clear.

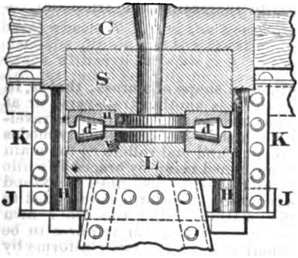


Fig. 7

Enlarged transverse section of roller-box &c, on center line of Fig 8.

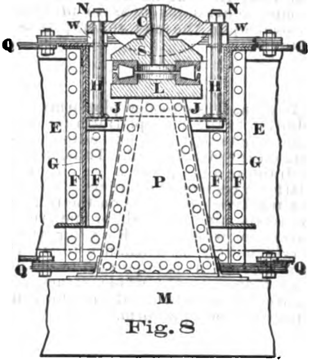


Fig. 8

Art. 10. The bolts, H, by which the table is suspended from the cap, C, are six in number, and are arranged in two rows of three bolts each, one row being on each side of the post, P, and between it and one end of the turntable. To each cross-girder is riveted a short hor bar of angle-iron, J J. Through the hor flange of each of these two bars pass the 3 bolts, H, of one row. The hor flange, bearing all the wt of its girder and load, rests upon the heads of these 3 bolts. To prevent the flange from yielding under its load, vert struts of angle-iron are riveted to the web of the cross-girder between the bolts. Two of these struts are shown at K K, Fig 7. Their ends abut against the upper flange of the cross-girder, and against J J.

The 6 bolts, H H, pass up through the flanges of the cap, C (three bolts through each flange), and their nuts rest upon its top.

Art. 11. The cap, C, is held in place on the socket, *s*, by means of flanges which extend down from it on both sides, as shown in Fig 7.

The **rollers,** *d d*, and the roller-box, *u v*, are those made by Wm. Sellers & Co, Phila, Art 3.

The ht of the table may be adjusted, within a range of 2 or 3 ins, by means of the nuts, N N, as in the Sellers table, Art 4.

The lower part, *v*, of the roller-box, instead of resting directly upon the post, P, as in the Sellers table, Art 3, rests upon an iron casting, L, which, in turn, rests upon the post, P, and is held in place upon it by a lug which projects down into it.

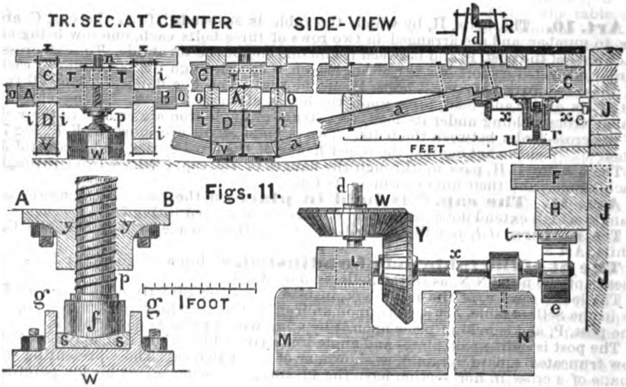
The post is built up of plate- and angle-irons riveted together, and may be a hollow truncated square pyramid, as shown, or of other shapes. Those presenting the shape of a cross in hor section have the advantage of being accessible for painting and inspection.

Art. 12. The main girders are braced by hor diag rods, whose ends are shown at Q, Fig 8. These rods are provided with sleeve-swivel turn-buckles, by which they may be tightened or loosened.

The remarks in Art 5 on the **end wheels** of the Sellers table apply also to those of the Edge Moor, except that in the latter the wheels are 25 ins in diam, and the cross-girts which carry them are of angle-iron. The wheels are fastened to their axles, which turn in brass bearings enclosed in cast-iron journal-boxes. Each box is provided with means for raising or lowering it; and the brasses are rounded on top, so as to insure a uniform bearing, no matter what inclination may be given to the axle by the vert adjustment of the journal-boxes.

Wooden turntables, with none but two common wheel rollers at each end of the platform, are sometimes resorted to from motives of original cost. They are, however, much harder to turn, generally requiring two men, aided by wheelwork; and are more liable to get out of order; and more expensive to repair. They are made of a great variety of patterns, both as regards the girders, and the central pivots, end rollers, &c. Frequently an addition is made of 8 to 12 small rollers travelling on a circular rail of 6 to 12 feet diameter, around the pivot as a center. These are intended to sustain the whole weight; the end rollers being so adjusted as to touch their rail only when the platform rocks or tilts as the engine enters or leaves it. Therefore, there is less resistance from friction than when, as in Figs 11, there are only the end rollers *r*. In this last case, the engine and tender cannot be balanced so precisely upon the slender central pivot, as to prevent a great part of the weight from being thrown upon the end rollers; thus materially increasing the frictional resistance.

In plan, these wooden platforms are sometimes in shape of a cross; that is, in addition to the main platform for the engine, there is another transverse or at right angles to it, also extending across the pit; and having end rollers travelling on the circular rail. Thus, in Figs 11, (which show one of the many modes of framing a table which has only a central pivot *f*, and end rollers *r*,) the main platform rests on the girders *c*, which are strengthened below by braces *a*; while the transverse one rests on the timbers *o*, *o*, which must be imagined to extend across the pit. One-half, or one arm, of this transverse platform is intended to carry the wheelwork R *x x*, for turning the platform; and the other arm serves merely as a balance to it: therefore, neither of them requires to be very strong. It is important to connect the four ends of the two platforms by four beams, as the whole structure is thereby materially stiffened. In the figures the wheelwork R *x x* is for convenience improperly shown as if it stood upon the main platform.



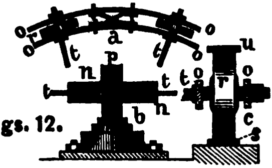
Figs. 11.

The figures need but little explanation. They represent an actual 45-foot platform, which has been in use for some years. The convex foot *f* of the central pivot, about 6 inches diameter, should be faced with steel: and should rest on a steel *step s s*. This should be kept well oiled; and protected from dust by a leather collar around *p*, and resting on *g g*. Its upper part, about 4 inches diameter, is cut into a screw with square threads about ¼ inch thick, for a distance of about 15 inches. It works in a female screw in the strong cast-iron nut *y y*; and serves for raising the whole platform when necessary. When not in use for this purpose, it is keyed tight to the platform, (by a key at its head *n*,) so as to revolve with it. Strong screw-bolts *i i* connect the several timbers at the center of the platform.

R is a light cast-iron stand supporting two bevel wheels about 1 foot diameter, which give motion by means of an axle *d*, 1¾ inches diameter to two similar ones below, shown more plainly at W and Y. These last give motion by the axle *x* to the pinion *e*, (6 inches diameter, and 2¼ inches face,) which turns the platform by working into a circular rack *t*, (teeth horizontal, 1 inch pitch; 3½ inches face,) which surrounds the entire pit. This rack is spiked to the under side of a continuous wooden curb H, which is upheld by pieces F, a few feet apart, which are let into the wall J J, which lines the pit. The short beam M N, (about 6 feet,) which carries the lower wheelwork, is suspended strongly from the beams of the transverse platform above it. Instead of the two lower bevel wheels W Y, and the horizontal axle *x*, a more simple arrangement is to place the pinion *e* at the lower end of the vertical axle *d*; and let it work into a rack with *vertical* teeth at *u*, on the inner face of the stone foundation of the circular rail. For this purpose the stand R should be directly over *u*. There are two cast-iron rollers *r*, 2 feet diameter, 3 inch face, under each end of the main platform; and one under each end of the secondary one.

Although this kind of platform necessarily has much friction, yet one man can generally turn a 45-foot one by means of the wheelwork, when loaded with a heavy engine and tender. Indeed, he may do it with some difficulty by hand only, while all is new and in perfect order; but when old, and the circular railway uneven and dirty, it requires two men at the winches to do it with entire ease.

As before remarked, the resistance to turning is diminished by employing a set of from 8 to 12 rollers or wheels *r*, Figs 12, about a foot to 15 inches in diameter, so arranged as to form a circle 8 to 12 feet diameter around the pivot. When this is done, the main girders of the platform are placed 8 to 12 feet apart; and long cross-ties are used for supporting the railway track. Also, the main girders are sometimes trussed by iron rods.



Figs. 12.

Fig 12 shows the arrangement of these rollers *r*, which revolve upon a circular track *s*; while the platform rests on their tops by the track *u*. The rollers *r* are held between two wrought-iron rings *o*, *o*, about 8 inches deep, ½ inch thick, which also are carried by the rollers. From each roller a radial tie-rod *t*, 1 inch diameter, extends to a ring *n n*, which surrounds the pivot *p*, closely, but not tightly, so as to revolve independently of it. These tie-rods keep the rings *o o* at their proper distance from the pivot, so that the rollers cannot leave the rails *s* and *u*. Between each two rollers, the rings *o o* should be strengthened by some arrangement like *a*, to prevent change of shape. The pivot *p* may be as in Figs 11. There must, of course, be the usual two rollers under each end of the platform, for sustaining the engine as it goes on or off; but during the act of turning the platform, the whole weight should rest on the central rollers. Such a platform of 50 feet length can, if carefully made, be turned, together with an engine and tender, by one man, by means of a wooden lever 12 to 15 feet long, inserted in a staple for that purpose; and therefore may dispense with the transverse platform for sustaining wheelwork.

Such rollers as have just been described, in connection with friction rollers, Fig 5, form perhaps **the best arrangement for a large turning bridge.** At least one end of a platform must be provided with a **catch** or **stop** for arresting its motion at the moment it has reached the proper spot.

A common mode is shown at Fig 13. It consists of a wrought-iron bar *m n*, 4 feet long, 3 inches wide, and ¾ thick; hinged at its end *m*, which is confined to the top of the platform. Its outer end *n* is formed into a ring *V* for lifting it. A strong casting *e e* (or in longitudinal section at *t t*,) about 15 inches long, 3 inches wide, and 1 inch thick, is also firmly bolted to the top of the platform; and the stop-bar *m n* rests in its recess *r*, while the platform is being turned. A similar casting *a a* is well bolted to the wooden or stone coping *c c*, which surrounds the top of the lining wall of the pit. When the stop-bar reaches this last casting, as the platform revolves, it rises up one of *its* little inclined planes *t t*, and falls into the recess of *a a*, bringing the platform to a stand. When the platform is to be started again, the bar is lifted out of its recess by the ring *V*, until it passes the casting; when it is again laid upon the coping *c c*, and moves with the platform; or, if required, the hinge at *m* allows it to be turned entirely over on its back. When there is a transverse platform, the proper place for the stop is at that end which carries the turning gear; as it is there handy to the men who do the turning. If there is only a main platform, the stop may be placed midway of the rails. Sometimes a **spring catch** is placed at each end of the platform; and each catch is loosened from its hold at the same instant by a long double-acting lever. All the details of a platform admit of much variety.

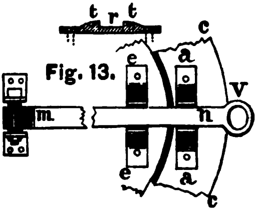


Fig. 13.

Instead of the friction *rollers*, Fig 5, friction *balls* 5 or 6 inches diameter, of polished steel, are sometimes used. The pivots also are made in many shapes.

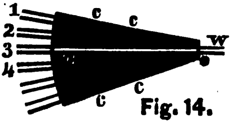


Fig. 14.

Platforms like *o a*, Fig 14, revolving around one end *o* as a center of motion, are sometimes useful. The shaded space is the pit. If an engine approaching along the track W, is intended to pass on to any one of the tracks 1, 2, 3, 4, the platform is first put into the required position, and the engine passes at once without detention. If the platform is long, it will be necessary to have roller-wheels not only under the moving end *a*, but at one or two other points, as indicated by the roller rails *c c*.

* * *

Engine houses, of brick, cost from $1000 to $1200 per engine stall, exclusive of the foundations.

The cost of a complete set of shops of brick, for the thorough repair of about 20 locomotives, and of the corresponding number of passenger and other cars; together with suitable smith shop, foundry, car shop, boiler shop, copper and brass shop, paint shop, store rooms, lumber shed, offices, &c; completely furnished with steam power, lathes, planing machines, scales, and all other necessary tools and appliances, will be about from $75000 to $100000 exclusive of ground. A large yard, of at least an acre, should adjoin the buildings. A moderate establishment, for the repairs of a few engines only, may be built and furnished for $25000.

WATER STATIONS.

Water stations are points along a railroad, at which the engines stop to take in water. **Their distance apart** varies (like that of the fuel stations, which accompany them,) from about 6 miles, on roads doing a very large business; to 15 or 20 miles on those which run but few trains. Much depends, however, upon where water can be had. It has at times to be conducted in pipes for 2 or 3 miles or more. The object in having them near together is to prevent delay from many engines being obliged to use the same station. To prevent interruption to travel, they are frequently placed upon a side track. A supply of water is kept on hand at the station, usually in large wooden tubs or tanks, enclosed in frame tank-houses. The tank-house stands near the track, leaving only about 2 to 4 feet clearance for the cars. It is two stories high; the tank being in the upper one; and having its bottom about 10 or 12 feet above the rails. In the lower story is usually the pump for pumping up the water into the tank; and a stove for preventing the water from freezing in winter.*

The tanks are usually circular; and a few inches greater in diameter at the bottom than at the top, so that the iron hoops may drive tight. **Their capacity** generally varies from 6000 to 40000 gallons, (rarely 80000 or more,) depending on the number of engines to be supplied. **A tender-tank holds** from 1000 to 3000 gallons; and **an engine evaporates** from 20 to 150 gallons per mile, depending on the class of engine; weight of train; steepness of grade, &c. Perhaps 40 gallons will be a tolerably full average for passenger, and 80 for freight engines. **The following are the contents of tanks** of different inner diameters, and depths of water. U. S. gallons of 231 cubic inches; or 7.4805 gallons to a cubic foot.

Diam.	Depth.	Contents.		Diam.	Depth.	Contents.	
Ft.	Ft.	Gallons.	Cub. Ft.	Ft.	Ft.	Gallons.	Cub. Ft.
12	8	6767	905	24	12	40607	5429
14	9	10363	1385	26	13	51628	6902
16	9	13535	1810	28	14	64481	8621
18	10	19034	2545	30	15	79310	10603
20	10	23499	3142	32	16	96258	12868
22	11	31277	4181	34	17	115451	15435

Cypress or any of the pines answer very well for tanks. The staves may be about 2½ inches thick for the smaller ones; to 4 or 5 inches for the largest. The bottoms may be the same. The staves should be planed by machinery to suit the curve precisely. Nothing is then needed between the staves to produce tightness. A single wooden dowel is inserted between each two near the top, merely to hold them in place while being put together. The bottom is dowelled together; and simply inserted into a groove very accurately cut, about an inch deep, around the inner circumference of the tub, at a few inches above the bottoms of the staves.

One of 20 feet diameter, and 12 feet deep, may have 9 hoops of good iron; placed several inches nearer together at the bottom of the tank than at the top. Their width 3 inches; the thickness of the lower two, ½ inch; thence gradually diminishing until the top one is but half as thick. The lower two are driven close together. These dimensions will allow for the rivet-holes for riveting together the overlapping ends; and for a moderate strain in driving the hoops firmly into place.† Three rivets of ½ inch diameter, and 3 inches apart, in line, are sufficient for a joint of a lower hoop. One of 34 feet diameter, 17 deep, may have 12 hoops; the lower ones 4 inches by ½; with three ¾-inch rivets to a lower hoop-joint.

The bottom planks of the tank must bear firmly upon their supporting joists, or bearers.

A tank must have an **inlet-pipe** by which the water may enter it; a **waste-pipe** for preventing overflow; and a **discharge or feed-pipe** 7 or 8 inches diameter, in or near the bottom; through which the water flows out to the tender. The inner end of the discharge-pipe is covered by a valve, to be opened at will by the engine man, by means of an outside cord and lever. To its outer end is generally attached a flexible canvas and gum-elastic hose about 7 or 8 inches diameter, and 8 or 10 feet long, through which the water enters the tender-tank. Or, instead of a hose, the feed-pipe may be prolonged by a metallic pipe, or nozzle, sufficiently long to reach the tender; and so jointed as, when not in use, to swing to one side, or to be raised to a vertical position, (in the last case it is called *a drop*,) so as not to be in the way of passing trains.

---

* A frame tank-house, 18 feet square, with stone foundations for both house and tank, will by itself **cost** $400 to $600. A brick or stone one somewhat more.

† Such a tank, put up in its place, will **cost** about $400.

The same tank may supply two engines on different tracks, at once. The tanks are very durable.

The patent frost-proof tank of John Burnham, Batavia, Illinois, is simply an ordinary tank, in which the water is prevented from freezing by means, 1st, of a circular roof which protects a ceiling of joists, between which is a layer of mortar; 2d, by an air-space obtained by a similar ceiling beneath the timbers on which the tank rests. Although the sides are entirely unprotected, no house is necessary; but merely strong posts and beams on a stone foundation, for the support of the tank.* The supply pipes are in boxes made of boards and tar-paper.

Tanks are frequently made rectangular, with vertical sides of posts lined with plank, and braced across in both directions by iron rods. They are more apt to leak than circular ones. They have been made of iron; but wood seems to be preferred.

The water for supplying the tanks, may be pumped by hand, steam, horse, wind, hydraulic ram, or otherwise, from a running stream; from a pond made by damming the stream if very small or irregular; from a cistern below the tank; or from a common well. Many roads doing a business of 10 or 12 engines daily in each direction, depend entirely upon wells; and pump by hand; generally two men to a pump. Those doing a very large business, when the supply cannot be obtained by gravity, mostly use steam. **The windmill is** the most economical power; and when well made, is very little liable to get out of order. Of course it will not work during a calm; but this objection may be obviated in most cases by having the tanks large enough to hold a supply for several days. Steam, however, is most reliable.

The following table will give some idea of the **power required in a steam engine for the pumping.** In ordering an engine, specify not its number of horse-powers, but the number of gallons it must raise in a given number of hours, to a given height; with a given steam pressure, (say about 60 to 80 lbs per square inch.) The pump should be sufficiently powerful not to have to work at night; and should be capable of performing at least 25 per cent. more than its required duty.

A fair average horse should pump in 8 hours the quantities contained in the first 3 columns; to the height in the 4th column; or sufficient to supply the number of locomotives in the 5th column, with about 2000 gallons each. Two men should do about one-third as much.

Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.	Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.
1600	100000	11968	100	6	4571	285714	34194	35	17
2000	125000	14960	80	7½	5833	333333	39893	30	20
2667	166666	19946	60	10	6400	400000	47872	25	24
3200	200000	23936	50	12	8000	500000	59840	20	30
3555	222222	26596	45	13¼	10667	666667	79787	15	40
4000	250000	29920	40	15	16000	1000000	119680	10	60

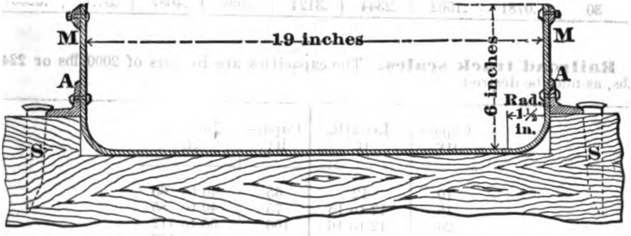
A reservoir, with a stand-pipe, or water column, is preferable to the ordinary tank, when the locality admits of it; being less liable than the pump to get out of order; and being cheaper in the end. The reservoir is supposed to be filled by water flowing into it by gravity; and to have its bottom at least about 8 feet above the rails; or at any greater height whatever that the ground and the height of the water may require. It may be excavated in the ground; lined with brick or masonry in cement; with a bottom of concrete; or it may be built above ground, according to the locality. It may be roofed and covered in, or not; and it may be near the tracks, or at a considerable distance from them, according to circumstances.* From its bottom, an iron pipe, from 8 to 12 inches diameter, is carried (generally underground,) to within a few feet of the track. At that point it turns vertically upward to about 8 or 10 feet above the track, forming a **stand-pipe, or water-column;** from the upper end of which the water flows (through either a hose or a jointed nozzle,) as in the case of a tank. Several such pipes, or one larger one, may be laid, for the supply of two or more engines at once, through as many stand-pipes. Where the pipe makes its bend, and becomes vertical, is a valve for opening and closing it; and which may be worked by a hand-wheel placed at such a height as to be easily reached by the engine man.

---

*The cost of windmill alone, for railway stations, varies from about $450 for 18 feet diameter; to $1500 for 36 feet diameter, at factory.

On some of the more important lines, the **tenders of fast trains scoop up water, while running,** from a long **trough,** or "**track tank**" laid between the rails. The tanks are about ¼ mile long. They must of course be level, and they therefore require a level track.

As originally introduced in England, by Ramsbottom, the trough was of cast-iron, in lengths of about 6 ft. These were bolted together by means of flanges at their ends. The ends were not in contact with each other, but were separated by a strip of vulcanized rubber.



Our figure shows a track tank of 3/16 inch rolled plate-iron, the sheets of which are 62 ins long. The lengths overlap each other 2 ins; leaving 5 ft as their *showing* length. The sheets are cut slightly tapering, so that at one end of each length the trough is 3/16 in deeper than at the other, and the tops are thus kept flush with each other throughout. The joints are double riveted with ⅜ inch rivets, about 1½ ins from center to center, and staggered. At each end of the trough, the bottom slopes upward, and in a length of 6 ft, comes to the level of the tops of the sides. The cross-ties are notched, as shown, to receive the trough, which is loosely held to them by two spikes, S and S, in each tie. The heads of the spikes fit over the horizontal flanges of the 1½ × 1½ inch angle bars, A and A. M and M are mouldings of 1⅛ × ½ inch bar-iron. The angles and the mouldings are in lengths of 15 ft, and are riveted to the sides of the trough continuously throughout its length.

The **scoop** on the tender is **lowered** into the trough, **and raised** from it, by means of a lever on the fireman's platform, and is not permitted to touch the bottom of the trough.

The **trough is supplied** with water by means of pipes leading from an adjacent tank. The supply is regulated by a man in charge.

To prevent the water from **freezing in winter,** steam is led to the trough from the boiler of the pumping engine, through iron pipes laid under ground alongside of the track. These pipes are provided with branches which introduce the steam to the trough at every 40 ft of its length. The steam-pipes are protected by wooden boxes, and are furnished with valves for regulating the supply of steam.

---

* An uncovered reservoir 50 ft diam by 12 ft deep, lined with brick or masonry, will usually cost from $2500 to $3500, according to circumstances.

Evaporation from Locomotives. The evaporation is usually from 6 to 7 lbs of water to 1 lb of fair coal. Hence if we take the average at 6½ lbs, or say .8 of a gallon of water to 1 lb of coal, and assume, as on page 800, that a passenger engine evaporates an average of 40 gallons per mile, and a freight engine 80 gallons, we shall have very nearly 2¼ tons of coal consumed per 100 miles by the former; and 4½ tons by the latter. The evaporation from a heavily tasked powerful engine may amount to 150 gallons or more per mile; but such is an exceptional case.

Theoretical thickness near bottom of sheet-iron water tanks, single riveted; safety 4; ultimate strength of the iron 40000 lbs per square inch, but reduced say one-half by punching the rivet holes. Although safe against the *pressure of the water*, some are plainly far too thin for *handling.*

Depth in Feet.	INNER DIAMETER IN FEET.							
	5	10	15	20	25	30	35	40
	THICKNESS IN INCHES.							
1	.0026	.0052	.0078	.0104	.0130	.0156	.0182	.0208
5	.0130	.0260	.0391	.0520	.0651	.0781	.0911	.1042
10	.0260	.0521	.0781	.1042	.1302	.1562	.1823	.2083
15	.0391	.0781	.1172	.1562	.1953	.2344	.2734	.3125
20	.0521	.1042	.1562	.2084	.2604	.3125	.3645	.4166
25	.0651	.1302	.1953	.2604	.3255	.3906	.4557	.5208
30	.0781	.1562	.2344	.3124	.3906	.4687	.5470	.6250

Railroad track scales. The capacities are in tons of 2000 lbs or 2240 lbs, as may be desired.

Capacity.	Length. ft.	Capacity.	Length. ft.
10	12	65	40 to 65
15	12 to 15	75	40 to 85
20	12 to 16	100	50 to 112
30	20 to 32	150	60 to 123
40	30 to 40	150	100 to 150
50	40 to 50		

Post-and-rail fences, in panels 8¼ ft long; 5 rails; usually cost between 40 to 100 cents per panel, including the putting up; or from $512 to $1280 per mile of road fenced on both sides, with 1280 panels.

Fence-posts are usually of chestnut, cedar, or white oak. They last about 10 years on an average. The usual size is 2 to 3 ins thick × 6 to 7 ins wide, 8 ft long, 5 ft above ground. Their cost varies greatly; say from 5½ to 25 cts each; average, 10 to 15.

Worm fences seven rails high, with two rails on end at each angle, cost about ¼th less. Labor $1.75 per day. The scarcity or abundance of timber chiefly influences the price; as is also the case with ties.

Barbed Steel wire fence costs. per mile of single row of fence, put up, including the wooden posts and all labor, from $150 to $250, depending on the height of fence, the varying market price of wire, labor, &c.

A way-station house, 30 × 60 feet, surrounded by a platform 12 feet wide, protected by projecting roof; for passengers and freight; will cost from $6000 to $10,000, according to finish and completeness, at eastern city prices.

Approximate average estimate for a mile of single-track railway. Labor $1.75 per day.

Grubbing and clearing, (*average of entire road*,) 3 *acres at* $50	$ 150
Grading; 20000 *cub yds of earth excavation, at* 35 *cts*	7000
" 2000 *cub yds of rock excavation*, at $1.00	2000
Masonry of culverts, drains, abutments of small bridges, retaining-walls, &c; 400 *cub yds, at* $8, *average*	3200
Ballast; 3000 *cub yds broken stone, at* $1.00	3000
Cross-ties; 2640, *at* 60 *cts, delivered*	1584
Rails; (60 *lbs to a yard*;) 96 *tons, at* $30, *delivered*	2880
Spikes	150
Rail-joints	300
Sub-delivery of materials along the line	300
Laying track	600
Fencing; (*average of entire road*,) *supposing only* ½ *of its length to be fenced*	450
Small wooden bridges, trestles, sidings, road-crossings, cattle guards, &c, &c	1000
Land damages	1000
Engineering, superintendence, officers of Co, stationery, instruments, rents, printing, law expenses, and other incidentals	2386
Total	$26000

Add for depots, shops, Engine-houses, Passenger and Freight Stations, Platforms, Wood Sheds, Water Stations with their tanks and pumps, Telegraph, Engines, Cars, Weigh Scales, Tools, &c, &c. Also for large bridges, tunnels, Turnouts, &c.

ROLLING STOCK.

LOCOMOTIVES.

Dimensions, Weights, &c.

Lists of some of the principal locomotives made by the Baldwin Locomotive Works, Philadelphia, 1902.

In the designation of the **class**, the first number (8, 10, &c) is the total number of wheels of the locomotive. The second (30, 32, &c) gives the diameter of the cylinders, thus: diam. in inches = $n/2 + 3$, where n = this second number. The *letter* (C, D, or E) indicates the number (4, 6, or 8, respectively) of *driving*-wheels.

The wheel-base is the distance from center to center of the front and back wheels. For minimum **length of turntable,** add 1½ to 2 ft to the *total* wheel-base of locomotive and tender, which is = wheel-base of locomotive + wheel-base of tender + dist. C to C of front tender wheel and hind engine wheel.

Under "**Service,**" **P** means *passenger*; **F,** *freight*; **M,** *mixed*; **S,** *switching*.

Since 1890, the **compound locomotive** has come into extensive use in many branches of service. The Baldwin (Vauclain) type has a high-pressure and a low-pressure cylinder on each side. Those of other makers usually have but two cylinders, the high-pressure cylinder being on one side and the low-pressure cylinder on the other. The dimensions and weights of compound engines do not differ materially from those of the corresponding classes of simple engines, as given below.

For gauge of 4 ft 8 1-2 ins.

Class.	Service.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels. No.	Driving Wheels. Diam.	Wheel-base. Locomotive. Driv's		Wheel-base. Locomotive. Total.		Wheel-base. Tender.		Wheel-base. Loco. and tender.		Extreme l'gth loco. and tender.		Extreme width.		Height of top of stack above rail.	
		Ins.	Ins.		Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.
8.30 C	P	18	24	4	66–72	7	6	21	8	15	0	49	8	65	3	9	0	14	6
8.34 C	"	20	24	4	72–78	7	6	21	11	15	6	50	0	66	0	10	0	14	6
10.34 D	P F	20	24	6	56–68	11	9	22	11	15	0	50	6	66	6	10	0	14	6
10.38 D	"	22	26	6	62–72	12	6	24	2	15	6	52	3	68	3	10	0	15	0
8.34 D	F	20	26	6	50–56	14	0	22	6	15	0	50	0	66	0	10	0	14	6
8.38 D	"	22	30	6	56–62	14	0	22	11	15	6	51	0	67	0	10	0	15	0
10.34 E	"	20	26	8	50–56	14	10	22	8	15	0	50	8	66	3	9	6	14	6
10.38 E	"	22	30	8	50–56	15	0	23	2	15	6	51	8	67	8	10	0	15	0
4.32 C	S	19	24	4	50	7	6	7	6	15	1	38	0	53	10	9	0	14	6
6.36 D	"	21	26	6	50	11	0	11	0	15	7	40	1	55	9	10	0	14	6

Class.	Service.	Type.	Weight in working order, in pounds. Locomotive. Greatest on 1 pair of driv. wh'ls	Weight in working order, in pounds. Locomotive. On all drivers.	Weight in working order, in pounds. Locomotive. Total.	Weight in working order, in pounds. Tender loaded.	Weight in working order, in pounds. Total of loco. and tender.	Capacity of tender. Coal, tons.	Capacity of tender. Water. Gals.	Capacity of tender. Water. lbs.
8.30 C	Pass'ger	Amer.	38000	74000	106000	66000	172000	5½	3300	27500
8.34 C	"	"	44000	86000	122000	80000	202000	7	4000	33333
10.34 D	P. & F.	10 Wh'l	36000	100000	136000	72000	208000	6	3600	30000
10.38 D	"	"	44000	124000	164000	80000	244000	7	4000	33333
8.34 D	Freight	Mogul	42000	120000	139000	72000	211000	6	3600	30000
8.38 D	"	"	52000	150000	171000	90000	261000	8	4500	37500
10.34 E	"	Consol.	34000	125000	140000	68000	208000	6	3400	28333
10.38 E	"	"	45000	170000	188000	90000	278000	8	4500	37500
4.32 C	Switch	....	53000	104000	104000	56000	160000	5	2800	23333
6.36 D	"	....	48000	140000	140000	76000	216000	6½	3800	31666

For gauge of 3 ft.

Class.	Service.	Type.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels. No.	Driving Wheels. Diam.	Wheel-base. Locomotive. Drivers.		Wheel-base. Locomotive. Total.		Wheel-base. Tender.		Wheel-base. Loco. and tender.		Extreme l'gth loco. and tender.		Extreme width.		Height of top of stack from rail.	
			Ins.	Ins.		Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.
8.22 C	P	A	14	18	4	41–45	8	2	20	1	13	4	39	5	48	5	8	3	13	6
10.24 D	P F	10 W	15	20	6	45	12	5	21	5	13	4	40	9	49	9	8	3	13	6
8.24 D	F	M	15	18	6	42	12	6	19	0	13	4	38	4	47	4	8	3	13	6
10.26 E	"	C	16	20	8	38	11	0	18	2	13	4	40	6	49	6	8	3	13	6
4.20 C	S	...	13	18	4	37	6	0	6	0	....		....		....		8	0	13	6
6.24 D	"	...	15	18	6	41	9	6	9	6	....		....		....		8	0	13	6

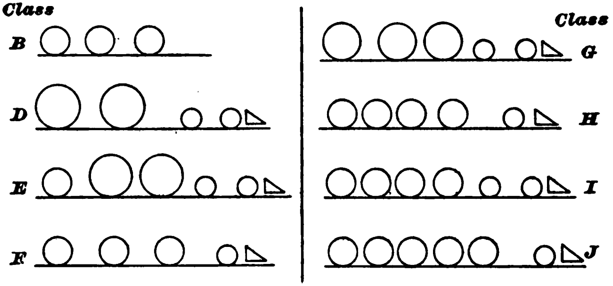
Class.	Service.	Type.	Weight in working order, in pounds. Locomotive. Greatest on 1 pair driv. wh'ls	Weight in working order, in pounds. Locomotive. On all drivers.	Weight in working order, in pounds. Locomotive. Total.	Weight in working order, in pounds. Tender loaded.	Weight in working order, in pounds. Total, loco. and tender.	Capacity of tender. Coal, tons.	Capacity of tender. Water. Gals.	Capacity of tender. Water. lbs.
8.22 C	Pass'ger	Amer.	17000	33000	52000	36000	88000	3½	1600	13333
10.24 D	P and F	10 Wh'l	17000	48000	64000	40000	104000	4	1800	15000
8.24 D	Freight	Mogul	17000	48000	57000	36000	93000	3½	1600	13333
10.26 E	"	Consol.	17000	64000	73000	40000	113000	4	1800	15000
4.20 C	Switch	....	23000	44000	44000	....	....	......	....	....
6.24 D	"	....	19000	56000	56000	....	....	......	....	....

Selected Penna. R. R. Standards, 1901. Gauge 4 ft 9 ins.

Class.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels No.	Driving Wheels Diam.	Wheel-base. Locomotive. Driv's		Wheel-base. Locomotive. Total.		Wheel-base. Tender.		Wheel-base. Loco. and tender.		Extreme l'gth loco. and tender.		Extreme width.	Height of top of chimney above top of rail.	
	Ins.	Ins.		Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ft.	Ins.
D 16, Passenger	18½	26	4	68	7	9	22	9½	16	1	50	3½	60	3½	10	14	11½
G 4, Passenger	20	28	6	72	13	10	25	1½	17	6	54	5½	66	¼	10	15	0
F 1, Freight	20	28	6	62	14	6	23	4	16	1	51	1¾	61	3½	10	14	8½
H 6, Freight	22	28	8	56	16	6½	24	9	17	6	55	4½	64	7½	10	14	11½

Class.	Weights in working order. Locomotive. Greatest on 1 pair of drivers.	Weights in working order. Locomotive. On all drivers.	Weights in working order. Locomotive. Total.	Weights in working order. Tender.	Weights in working order. Locomotive and tender.	Capacity of tender. Coal.	Capacity of tender. Water.	
	lbs.	lbs.	lbs.	lbs.	lbs.	Tons	Gals.	lbs.
D 16, Passenger	48300	93600	135300	82000	217300	6.7	3600	29900
G 4, Passenger	48500	140000	185000	134700	319700	8.9	7000	58300
F 1, Freight	44650	128100	146100	76800	222900	6.7	3600	29900
H 6, Freight	43800	166400	186500	111900	298400	9.8	6000	50000

Selected Erie Railroad Standards, 1902. Gauge 4 ft 8 1-2 ins.



Class.	Cylinders. Diam. Ins.	Cylinders. Stroke. Ins.	Driving Wheels. No.	Driving Wheels. Diam. Ins.	Wheel-base. Locomotive. Driv's Ft. Ins.	Wheel-base. Locomotive. Total. Ft. Ins.	Wheel-base. Tender. Ft. Ins.	Wheel-base. Loco. and tender. Ft. Ins.	Extreme lgth loco. and tender. Ft. Ins.	Extreme width. Ft. Ins.	Height of top of chim. above top of rail. Ft. Ins.
B— 2	19	26	6	52	10 10	10 10	18 0	42 10	57 11	10 2	15 4¼
D— 1	18	22	4	68	8 6	23 0½	16 1	46 4½	56 0⅞	8 10	13 1½
D— 2	18	22	4	68	8 6	22 11	16 1	45 11	55 8⅞	9 6	18 4¼
D—12	18	26	4	78	8 6	23 7½	18 0	51 7¾	62 10⅝	9 9	15 9½
D—13	19	26	4	72	8 6	23 6¾	16 11	49 8¼	60 3¾	9 11½	15 9¼
E— 1	13–22	26	4	76	6 7	24 9	18 0	52 9½	64 0⅞	9 11½	15 10½
F— 1	18	24	6	56	15 0	22 7¼	16 1	47 0¼	57 11	9 10	12 9⅞
F— 3	20	26	6	64	15 0	22 8½	16 11	48 4½	59 6⅜	9 11½	15 9½
G— 5	19	24	6	62	12 10	23 6	16 1	49 0	59 4⅞	9 11½	14 9
G— 8	21	26	6	62	12 0	22 10	16 11	51 10½	62 3⅜	9 11½	15 11
H— 1	20	24	8	50	14 0	22 10	16 1	47 2	58 0⅞	9 10	18 1½
H—11	21	28	8	57	15 9	24 0	16 9	52 3	63 3¼	10 1	15 4¼
J— 1	16–27	28	10	50	18 10	27 3	16 8	53 5½	64 8½	10 4½	15 6

Class.	Weight in working order, in pounds. Locomotive. Greatest on 1 pair of drivers.	Weight in working order, in pounds. Locomotive. On all drivers.	Weight in working order, in pounds. Locomotive. Total.	Weight in working order, in pounds. Tender loaded.	Weight in working order, in pounds. Total of loco. and tender.	Tractive force, per lb per sq. in. of theoretical cylinder pressure.	Capacity of tender. Coal, tons of 2000 lbs.	Capacity of tender. Water. Gals.	Capacity of tender. Water. lbs.
B— 2	52500	145100	145100	94050	239150	187.7	6	4500	37552
D— 1	26500	51500	80700	77000	157700	104.8	8	3600	30042
D— 2	28300	55400	84400	77000	161400	104.8	8	3600	30042
D—12	44000	87325	135525	116800	252325	108.0	12	6000	50070
D—13	51280	96060	136930	87900	224830	130.4	9	4500	37552
E— 1	44230	84290	151000	116800	267800	115.6	12	6000	50070
F— 1	26500	75300	88400	77000	165400	138.8	8	3600	30042
F— 3	43080	125330	142840	87900	230740	162.5	9	4500	37552
G— 5	34600	99800	122500	77000	199500	139.7	8	3600	30042
G— 8	42200	118500	144500	87900	232400	184.9	9	4500	37552
H— 1	25800	88700	103400	77000	180400	192.0	8	3600	30042
H—11	43500	170000	190000	131600	321600	216.6	12	6000	50070
J— 1	39300	173700	200550	90100	290550	197.6	10	4500	37552

The following are the principal **dimensions, &c,** of recent and **exceptionally large** locomotives:

	Decapod.* Largest built by Baldwin Works up to 1887.	Compound Consolidation. Largest built by Baldwin Works.	Consolidation. Pittsburg Locomotive Works. Heaviest engine ever built.	Consolidation, P. R. R. Class H 5.	Atlantic Type, P. R. R. Class E 1.	Compound Atlantic. Phila. & Reading Railw'y, No. 326 Baldwin Works, 1900.
Service	Freight.	Freight.	Freight.	Freight.	Passenger.	Passenger.
Gauge	4 ft. 8½ ins.	4 ft. 8½ ins.	4 ft. 8½ ins.	4 ft. 9 ins.	4 ft. 9 ins.	4 ft. 8½ ins.
Number of driving wheels	10	8	8	8	4	4
Diameter of driving wheels	45 ins.	55 ins.	54 ins.	56 ins.	80 ins.	84¼ ins.
Number of truck wheels	2	2	2	2	6	6
Wheel Base. Locomotive: Drivers	17 ft. 0 ins.	15 ft. 0 ins.	15 ft. 7 ins.	17 ft. 6 ins.	7 ft. 5 ins.	7 ft. 3 ins.
Total	24 ft. 4 ins.	23 ft. 10 ins.	24 ft. 4 ins.	25 ft. 11½ ins.	26 ft. 6½ ins.	26 ft. 7 ins.
Tender	14 ft. 5 ins.	18 ft. 0 ins.	17 ft. 10 ins.	17 ft. 6 ins.	12 ft. 6 ins.	16 ft. 9 ins.
Locomotive and tender	49 ft. 2 ins.	55 ft. 0½ in.	57 ft. 11¾ ins.	55 ft. 9 ins.	50 ft. 5 ins.	53 ft. 10½ ins.
Extreme Length:						
Locomotive and tender	59 ft. 9½ ins.	65 ft. 3½ ins.	68 ft. 0 ins.	63 ft. 11 ins.	62 ft. 6 ins.	66 ft. 5 ins.
Height—Top of rail to top of chimney	14 ft. 6 ins.	15 ft. 2 ins.	16 ft. 0 ins.	15 ft. 0 ins.	14 ft. 11½ ins.	14 ft. 2 ins.
Weight in working order, lbs.						
Locomotive—						
Greatest on 1 pair of drivers	27000	51700	59400	45600	51300	42900
On all drivers	134000	202232	225200	175700	101550	80890
Total, locomotive	148000	225082	250300	196500	173450	171860
Tender	82000	125000	141100	100600	90000	120000
Locomotive and tender	230000	350000	391400	297100	263450	291860
Capacity of tender—Coal: lbs.	16000	22400	31360	22000	12000	20160
Water: Gals. of 231 cubic ins.	3600	7000	7500	6000	4000	5950

From the foregoing lists it will be seen that **the weight of road locomotives per foot of their total wheel-base** varies from 2600 lbs. in light passenger engines for 3 ft. gauge to 10000 lbs. or more in very heavy freight engines for standard gauge.

---

* Built 1886 for Northern Pacific R. R., Nos. 500, 501; **Extreme width** 10 ft. The original "Decapod," built by same works in 1884 for Dom Pedro II R. R., Brazil (5 feet 3 inches gauge), and described in our 1886 edition, was a little lighter.

Performance of Locomotives.

Passenger engines usually carry **fuel and water sufficient for 40 or 50 miles;** some, 60 to 70. Freight trains, enough for 20 to 25 miles. Roads, or divisions, with steep grades require the fuel and water stations to be nearer together than where the grades are easy.

The following gives the **loads** (exclusive of locomotive and tender) which the above described **Baldwin engines** will haul, at their usual speeds, on a straight track and **on different grades** varying from a level to 3 ft. per 100 ft., or 158.4 ft. per mile. The loads are based upon the assumption that the so-called "**adhesion**" of the locomotive is nine-fortieths of the weight on all the drivers, and that the condition of road and cars is such that the **frictional resistance** of the cars does not exceed 7 lbs. per ton of 2240 lbs. of their weight. These are ordinarily favorable conditions. The adhesion is seldom less than one-fifth, or more than one-third, of the weight on the drivers.

Loads in tons of 2240 lbs. (exclusive of locomotive and tender).

Gauge 4 ft. 8 1-2 ins.

Class.*	Service.*	Type.	On a grade of 0 per ct. = 0 ft. per mile.	½ per cent. = 26.4 ft. per mile.	1 per cent. = 52.8 ft. per mile.	1½ per cent. = 79.2 ft. per mile.	2 per cent. = 105.6 ft per mile.	2½ per cent. = 132 ft. per mile.	3 per cent. = 154.4 ft per mile.
8—30—C	P	Amer.	1980	790	470	320	235	180	140
8—34—C	"	"	2300	920	545	375	275	210	165
10—34—D	P F	10 Wh'l	2675	1085	645	445	330	255	205
10—38—D	"	"	3325	1350	810	560	415	325	260
8—34—D	F	Mogul	3225	1315	795	555	415	325	260
8—38—D	"	"	4050	1650	995	695	520	410	330
10—34—E	"	Consol.	3375	1375	830	560	435	345	280
10—38—E	"	"	4600	1875	1135	795	600	470	380
4—32—C	S	....	2825	1150	700	490	370	295	235
6—36—D	"	....	3775	1550	940	660	495	395	320

Gauge 3 ft.

Class.*	Service.*	Type.	On a grade of 0 per ct. = 0 ft. per mile.	½ per cent. = 26.4 ft. per mile.	1 per cent. = 52.8 ft. per mile.	1½ per cent. = 79.2 ft. per mile.	2 per cent. = 105.6 ft per mile.	2½ per cent. = 132 ft. per mile.	3 per cent. = 154.4 ft per mile.
8—22—C	P	Amer.	610	265	160	105	75	65	50
10—24—D	P F	10 Wh'l	900	405	245	170	125	100	75
8—24—D	F	Mogul	915	410	255	175	135	105	80
10—26—E	"	Consol.	1225	555	345	240	185	145	115
4—20—C	S	....	800	365	230	165	125	100	80
6—24—D	"	....	1080	495	310	225	170	140	110

$$\text{\textbf{Mean tractive force} of a locomotive, in pounds} = \frac{\text{Square of diam. of 1 piston in ins.} \times \text{Single length of stroke in ins.} \times \text{Average steam pressure in the cylinders in lbs. per sq. inch.}}{\text{Diameter of driving-wheel in inches.}}$$

---

* See p. 856.

From the tractive force must be deducted 20 to 30 per cent. for internal friction, etc. The *effective* tractive force cannot exceed the adhesion, or say one-fourth the weight on the drivers.

The *initial* **steam pressure in the cylinders** is always less than the boiler pressure; and the disproportion increases with the speed. Thus, with the boiler pressure about 200 lbs. per square inch, the cylinder pressure, at 8 to 10 miles per hour, may be from 180 to 190 lbs., while at a speed of 30 or 40 miles, it may be only 160 to 170 lbs. The *average* cylinder pressure is ascertained by means of an indicator applied to the cylinder; and its proportion to the *initial* pressure depends upon how early in the stroke the supply of steam from boiler to cylinder is cut off; or, in other words, upon the extent to which the steam is used *expansively*.

The power and speed of locomotives, and their consumption of fuel and water, vary greatly with circumstances, such as grades and curvature; condition of track and rolling stock; number of cars in train; diameters, number and distance apart, of car wheels; manner of coupling the cars; skill of locomotive runner and fireman, &c, &c.

On the **Phila & Reading Ry** (Shamokin Division), between Catawissa and Lofty, 34 miles, **freight locomotives** are assigned train loads as below: (according to conditions of the engine and weather) in tons,* including weight of cars and loading, and 8.25 tons* for caboose, but exclusive of locomotive and tender. Each locomotive is assisted by a helper of Class I-5, or I-7. The grades and curves are as follows:

Grades.				Curvature.				
Feet per mile.		Miles.	Per cent. of total.	Degree of curve.	No. of curves	Feet.	Deflection. °	′
Less than	24.5	2.86	8.41	Less than 3°	24	8,203	376	5
Betw 30.62 and	35.05	30.72	90.27	" " 6°	59	29,694	1440	52
" 39.60 and	45.40	0.45	1.32	" " 9°	36	26,218	1343	56
				" " 13°	14	10,479	1099	51
	Total	34.03	100.00					
" 32.94 and	33.26	22.72	66.76	(14.13 miles)	133	74,594	4260	44

Engine.	Drivers.		No. of truck wh'ls.	Cylind's		Boiler pressure	Weight, tons.*			Train load, tons.*			
										Slow freight.†		Train No. 82.†	
Class.	No.	Diam. ins.		Diam. ins.	Stroke ins.	lbs. per sq. in.	On drivers.	Total loco.	Tender.	Max.	Min.	Max.	Min.
H—1	6	54	4	18	24	120	21.60	44.60	25.25	1553	1028		
L—1	6	61	4	20	24	145	47.00	60.50	34.50	1638	1128	1533	1023
I—1	8	50.5	2	20	24	120	48.25	55.75	27.50	1793	1198	1633	1098
I—2	8	50.5	2	20	24	145	52.67	60.72	28.00	1898	1268	1743	1163
I—5	8	50.5	2	22	28	145	63.00	70.50	32.75				
I—7	8	56	2	22	28	180	72.00	82.00	46.00	2193	1463	2088	1358

Special test runs. Catawissa to Lofty.	Class.		No. of cars.	Wt. of train. *	Time.		Speed miles per hour.	W'ter used gals.	Coal used tons.*
Date.	Loco.	Help.			H.	M.			
June 6, 1898................	I-1	I-5	45	1825	3	25	10.2	14500	8
" 7, "	I-2	I-5	45	1968	3	21	10.4	13575	7
March 8, 1901................	L-2	none.	20	578	2	2	17.0	7000	3.5

* Tons of 2000 pounds.

† Run from Newberry Junction to Tamaqua, 105 miles. Slow freights, 11 to 12 hours; train No. 82, 10 to 11 hours.

The greater the ratio of live or net load, to dead load or tare, the greater is the total tonnage (cars and load) that can be hauled by a given locomotive. In order to take account of this, a discrimination is made, on the Shamokin Division, in favor of cars having a capacity of not less than 80,000 lbs. Trains in which from 10 to 19 of the cars are of such capacity, are given 100 tons additional; with 20 or more such cars, 200 tons.

On the **Canadian Pacific Railway,**‡ a ratio of 2 tons of net load, or "contents," to 1 ton tare, is taken as standard; and it is found, on the eastern lines, where the controlling grades are generally about 1 per cent., that a train of empty cars offers about 30 per cent. greater resistance than a train of equal weight with net / tare = 2 / 1. A chart has therefore been prepared, by which the engines are loaded upon this basis. Heavier grades do not increase the *frictional* resistances. Hence the *total* resistance increases less rapidly than the grade, so that on grades steeper than about 1 per cent., an addition of 30 per cent. to the resistance of full trains, would give too high a resistance for empty trains of equal weight, and vice versa.

On the Canadian Pacific, the loading of freight trains is graded as follows, according to speed, weather conditions, &c.:

	Conditions.		
	Ordinary.	Temp. + 10° to —20° F. or bad rail.	Temp. colder than —20° Fahr.
	Reduce schedule by		
Ordinary trains..	0 per cent.	7 per cent.	12 per cent.
Fast trains.........	10 " "	12 " "	15 " "

In making deductions under this table, the probable conditions *on the ruling grade*, not those at the starting point, are considered. During snow or wind storms, loadings are determined by the conditions.

On the **Richmond, Fredericksburg & Potomac Railroad,** freight locomotives draw loads as follows:—exclusive of engine and tender.

Cylinders 18 × 26, 62 in. drivers, 90,000 lbs. on drivers, 630 tons.
" 19 × 26, " " 102,000 " " " 700 "

Maximum (and limiting) grade 1 per cent. on tangents.

Wood fuel. A ton (2240 lbs) of good anthracite or bituminous coal is about equal to 1⅓ cords of good dry, hard, mixed woods (chiefly white oak); or to 2 cords of such soft ones as hemlock, white, and common yellow pine. Much of the inferior bituminous coal of Illinois is hardly equal (per ton) to a cord of average wood.

A cord is 4 × 4 × 8 ft, or 128 cub ft. A cord of good dry, white oak (next to hickory, the best wood for fuel) weighs 3500 lbs or 1.563 tons. Dry hemlock, white, or common yellow pine (all of them inferior for fuel), about .9 ton. Perfectly green woods generally weigh about ⅕ to ¼ more than when partially dried for locomotive use; in other words, a cord of wood, in its partial drying, loses from ¼ to ⅓ ton of water, and still contains a large quantity of it. Since this water causes a great waste of heat, green wood should never be used as fuel. The values of woods as fuel are in nearly the same proportion as their weights per cord when perfectly dry.

When wood is used, about .2 cord; or when coal, about ⅓ cord of wood, must be used for kindling, and getting up steam ready for running; and this item is the same for a long run as for a short one; so that long roads have in this respect an advantage over short ones, in economy of fuel. Wood has the disadvantage of emitting more sparks; and is, moreover, nearly twice as heavy as coal, for the performance of equal duty; and is, therefore, more expensive to handle. It also occupies 4 or 5 times as much space as coal.

Up grades greatly increase the consumption of fuel. Thus, on a road 95 miles long, with grades mostly of less than 6 ft per mile, and with very few exceeding 14 ft per mile, with coal trains of 734 tons descending, and 291 tons (empty) ascending, at about 10 miles per hour each way, the coal consumption per 100 miles for each ton of total train (including engine and tender) was 14.5 lbs descending, and 36.6 lbs ascending.

---

‡ Paper by Mr. Thos. Tait, Manager, Canadian Pacific Railway Co., read before the New York Railroad Club, January 17, 1901. Proceedings, Vol. XI, No. 3.

On first-class roads **a passenger engine will average about 35000 miles per year,** or say 100 miles per day; a freight engine 25000 miles per year, or say 70 miles per day.

FAST RUNS.

Railroad.	Locomotive.	Cylinders.		Drivers diam. ins.	Weight of train, tons.	Run.			Time mins.	Aver. net sp'd miles per hour.	Coal burned lbs per mile.
		Diam. Inches.	Stroke. Inches.			From	To	Miles.			
Chicago, Burlington & Quincy, 1899.	Baldwin compound Atlantic type.	13½ & 23	26	84¼	200	Mendota	Chicago	83	77	64.7	
Lehigh Valley, 1898.	"	19 & 26		76	160	Sayre	Buffalo	177	179	59.7	56
Chicago, Mil. & St. P., 1898.	"	13 & 23	26	78	600§	Chicago	Milwaukee	74	82	54.1	
Central of N. Jersey, 1900.	"	14 & 24	26	84¼	235	Phila.	Jersey City	93	112 83	49.8 67.2	97
Wabash, 1898.	"	19 & 26	7	73	250	Tilton	Decatur	72	80	53.6	84
Phila. & Reading, 1900.	"	15 & 25		84	291 to 414§	Phila.	Atlan. City	55.5	50 44¼ 55¼	66.6* 75.2† 60.2‡	
Chicago, Burlington & Quincy, 1899.	Baldwin compound Class P.	13½ & 23	26	84	282§	Mendota	Riverside	72	62	69.7	
Chicago, Burlington & Quincy, 1899.	Rogers, Mogul	19	24	69	267§	Clyde	Burlington	197	184	67.3	
New York Central, 1891.	N. Y. C. & H. R. R., Class I. 60 tons.	19	24	78	230§	New York	Buffalo	436	425	61.6	

* Schedule average speed between termini.
‡ Minimum average speed between termini.
† Maximum average speed between termini.
§ Including locomotive and tender.

Locomotive expenses per 100 miles run, will average about as follows:

	Passenger.	Freight.
Fuel	$3.00	$6.00
Water	1.00	2.00
Oil, waste, &c.	.70	.90
Repairs	4.00	8.00
Engineer and fireman	5.00	6.00
Putting away, cleaning, and getting out	1.50	2.00
Locomotive superintendence	.30	.50
	$15.50	$25.40

An additional allowance of, say $2 per 100 miles, should be made annually for **depreciation of each engine.** An engine in active service, even under a judicious system of repairs, generally becomes worthless (except as old iron), in say 16 years on an average.

CARS.

Usual dimensions, weights and capacities. Gauge 4 ft 8½ ins.

	Length of body. ft.	Width. ft.	Height above rail. ft.	Weight, empty. lbs.	Nominal capacity, in passengers or lbs.
Passenger	50 to 60	10	14	50000 to 80000	50 to 64
Parlor	60 to 70	"	"	80000 to 100000	30 to 40
Sleeper	"	"	"	"	about 30
Baggage, mail, and express	50 to 60	"	"	40000 to 60000	
Box and cattle	28 to 36	8½ to 10	11 to 12½*	28000 to 35500	60000 to 80000
Gondola	"	"	6 to 7½*	20000 to 30000	"
Platform	32 to 36	8½ to 9	4 to 4¼†	"	"
Coal	29 to 36	8 to 9	8 to 9½*	25000 to 33000	"
Dump	10 to 18	7½ to 8½	6 to 7*	9000 to 11000	15000 to 20000

* Add 6 inches for brake shaft.
† Add 2½ to 3 feet for brake shaft.

On narrow gauge (3 ft and 3½ ft) roads there is but little uniformity in car building. Freight cars are usually from 25 to 32 ft long, 6½ to 8 ft wide; capacity, 30000 to 40000 lbs.

Pressed Steel Cars, built by the Pressed Steel Car Co., Pittsburg, Pa.

	Length over end sills.		Width over all.		Height over all.		Weight in pounds.	Capacity in pounds.	Height over Brake Mast.	
	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.			Ft.	Ins.
Hopper	31	6	10	0	10	0	36,000	100,000	10	5⅞
Flat	40	0	10	0	3	10⅞	28,800	80,000	6	5⅝
Box *	35	11¾	10	2	13	4 3/16	32,300	70,000	13	9
Furniture *	51	9¾	9	10¼	14	1½	43,800	60,000	14	8
Gondola	43	8	10	0	7	6¼	35,100	100,000	8	2⅝

* Wooden bodies with steel underframing: others all steel.

The **average life** of a passenger car is about 16 years. Average **annual repairs,** including painting, $300 to $700; for mail and express cars, $150 to $300; freight cars, $75 to $150.

Allowing 125 lbs per passenger, a *full* car-load of passengers (50 to 60 in number) would weigh but from 6250 to 7500 lbs, or say 3 tons; while the cars themselves weigh say 30 tons, or nearly **10 tons of dead load to 1 paying ton of passengers.** But, as a general rule, passenger trains are not *more* than half-filled; making the proportion about 20 to 1. The foregoing table shows that when **freight cars** are loaded to their nominal capacity, there is less than about **1-3 ton of dead load per ton of paying load;** or, with cars half loaded, 2 to 3.

The average cost, in the United States, of moving a passenger one mile is about double that of moving a ton of freight one mile, while the receipts per passenger-mile are nearly three times those per freight-ton-mile.

The resistance of cars to motion, on a level track, and with cars and track in fair order, is usually taken at about from 6 to 8 lbs per ton of 2240 lbs. With everything in perfect order, it may fall as low as 5, or even 4, lbs per ton. On the other hand, if the wheels are not truly round, and if the journals are not well lubricated, it may greatly exceed 10 or 12 lbs.

To estimate roughly the speed of a train in which one is riding; if, as usual, the rails are 30 feet long. By means of the sound of the trucks in passing the joints, count the number of rail-lengths passed in 20 seconds. This number is a very little less than the speed of the train in miles per hour.

For, let n = the number of 30-foot rail-lengths passed over in 20 seconds. Then:

$$\text{Speed in miles per hour} = \frac{60n \times 60 \times 30}{20 \times 5280} = 1.023\ n.$$

If the wheels are 28 inches in diameter, as is common in trolley cars and bicycles, the number of revolutions in 5 seconds (which may sometimes be counted by means of irregularities in a wheel) will give very closely the speed in miles per hour.

For, let n = the number of revolutions in 5 seconds. Then:

$$\text{Speed in miles per hour} = \frac{60 \times 60 \times 28 \times \pi}{5 \times 12 \times 5280} = 0.9995\ n.$$

RAILROAD STATISTICS.

Table 1. IN THE UNITED STATES.*

	1880.	1890.	1899.
Plant.			
Miles built in one year	7174	5498	3981
Miles in operation	87801	166817	190883
Rolling stock in operation.			
Number of locomotives	17412	33241	37245
" passenger cars	12330	22958	26184
" baggage, mail, and express cars	4475	7253	8121
" freight and other cars	455450	1061970	1328084
Cost of road and equipment,			
per mile, in dollars	51561	53783	54607
total, in millions of dollars	4530	8789	10254
Operation.			
For one year.			
Passengers carried one mile, per mile of road	65392	75062	79132
Tons of freight carried one mile, per mile of road	368514	474728	675743
Gross earnings,			
per mile of road, from passengers, dollars	1641	1732	1597
" " " freight "	4740	4686	4952
" " " mails, &c. "	230	528	612
" " total "	6611	6946	7161
per passenger-mile, from passengers, "	0.0251	0.0218	0.0200
" ton-mile, from freight "	0.0129	0.0093	0.0073
passenger earnings ÷ total earnings	0.2483	0.2519	0.2220
freight " ÷ " "	0.7169	0.6817	0.6900
mail, &c, " ÷ " "	0.0348	0.0664	0.0880
gross earnings ÷ total investment	0.1136	0.1015	0.1072
Expenses, (For details, see Table 3.)			
per mile of road dollars	6174	6792	4769
expenses ÷ gross earnings	0.6078	0.6854	0.6659
Net earnings,			
Net earnings ÷ total investment	0.0504	0.0340	0.0359

Table 2. UNITED STATES BY DIVISIONS, 1899.*

	Eastern States.	Central States.	Western States.	Pacific States.	Total, U. S.
Plant.					
Miles in operation	52735	64818	59562	10666	187781
Cost of road and equipment, per mile, dollars	68911	45648	48745	61696	54607
Operation.					
For one year.					
Gross earnings per mile, $	10131	6911	4697	6943	7161
Expenses per mile, $	6932	4803	3020	4575	4769
Expenses ÷ Gross earnings	0.6842	0.6950	0.6430	0.6589	0.6659

---

* Tables 1 and 2 are based chiefly upon Poor's Manual.

Table 3. Items of total annual expenses for maintenance and operation of all the railroads of the United States. Year ending June 30, 1899. From Report of **Interstate Commerce Commission, 1899.**

	Total $	$ per mile of road.	Per cent of total.
Repairs of roadway	87,307,140	457	10.72
Renewals of rails	10,767,381	57	1.32
" " ties	23,623,325	126	2.90
Repairs and renewals, bridges and culverts	19,335,860	103	2.38
" " " buildings, docks, wharves	19,832,218	105	2.43
" " " fences, crossings, &c	3,968,408	21	0.49
" " " telegraph	1,153,408	8	0.14
Sundries	3,837,314	20	0.47
Maintenance of way and structures	169,825,054	897	20.85
Repairs and renewals, locomotives	50,555,264	266	6.21
" " " passenger cars	17,623,134	93	2.16
" " " freight cars	57,320,521	303	7.04
Superintendence and sundries	17,795,526	94	2.18
Maintenance of equipment	143,294,445	756	17.59
Superintendence	14,392,691	76	1.77
Fuel, &c, for locomotives	86,586,108	454	10.64
Engine and roundhouse men	78,913,978	417	9.69
Train service, supplies, and expenses	74,196,282	392	9.11
Switchmen, flagmen, and watchmen	33,791,383	178	4.15
Telegraph	15,525,232	82	1.90
Station service and supplies	66,824,777	352	8.20
Balances. Switching charges, car mileage, &c	22,256,884	118	2.74
Yards, elevators, rentals	20,386,462	108	2.50
Loss, damage, accidents	14,290,195	75	1.76
Sundries	37,286,592	198	4.58
Total running expenses	464,450,584	2450	57.04
Unclassified	36,819,917	194	4.52
Aggregate annual expenses	814,390,000	4297	100.00

Each of these items is, however, subject to great variation, not only on diff roads, but on the same road, from year to year. A road with many bridges, deep cuts, high embkts, &c, to keep in repair, will have heavier maintenance of way than one which has but few; and this item may be but small one year, and twice as great the next. Fuel may be cheap on one road and dear on another; and this materially affects the item of motive power. And so with the other items. Sometimes maintenance of way exceeds motive power and cars together; at others, conducting transportation is fully half the total expense.

The total annual expenses on railroads in the United States usually range between 65 and 130 cents per train mile; that is, per mile actually run by trains. When a road does a very large business, and of such a character that the trains may be heavy, and the cars full (as in coal-carrying roads), the expense per train mile becomes large; but that per ton or passenger small; and vice versa, although on coal roads half the train miles are with empty cars.

Table 4. Gross annual earnings per mile, per passenger mile, and per ton mile, of some of the principal U. S. railroads in 1899. From Poor's Manual.

	Length, miles.	From passengers		From freight.	
		Per mile of road.	Per pass mile.	Per mile of road.	Per ton mile.
Pennsylvania	2780	$4690	$0.0194	$18877	$0.0047
New York Central & Hudson River,	2395	5730	0.0182	11495	0.0059
Baltimore & Ohio	2024	2791	0.0174	9909	0.0039
Chicago, Burlington & Quincy	7419	1276	0.0211	3945	0.0086
Philadelphia & Reading	915	4385	0.0162	19600	0.0078
Union Pacific	2421	1365	0.0279	5842	0.0154
Wabash	2278	1753	0.0189	4044	0.0055
Atchison, Topeka & Santa Fé	7029	1154	0.0228	4195	0.0102
Total and averages, United States	187781	1584	0.0200	4912	0.0073

Table 5. Annual earnings and expenses of some of the principal railroads in the United States in 1899. From Poor's Manual.

	Length, miles.	Gross earnings per mile of road.	Expenses per mile of road.	Expenses + gross earnings.
Pennsylvania	2780	$25787	$17670	0.6904
New York Central & Hudson River,	2395	19285	12163	0.6307
Baltimore & Ohio	2024	14160	10858	0.7667
Chicago, Burlington & Quincy	7419	5942	3662	0.6118
Philadelphia & Reading	915	24540	13422	0.5469
Union Pacific	2421	8181	4713	0.5760
Wabash	2278	6320	4571	0.7233
Atchison, Topeka & Santa Fé	7029	5763	3927	0.6814
Total and averages, United States	187781	7161	4769	0.6659

Operating expenses, Erie Railroad Company, 1900.

Entire system, comprising Erie and Ohio Divisions.

Coal used per mile,*	
per passenger locomotive, lbs.	86.9
" work " "	93.7
" switching " "	73.4
" pusher " "	147.7
" freight " "	152.2
" passenger car, "	18.3
" freight " "	6.4
" 100 tons† "	20.5
Oil and waste.	
Cylinder oil.	
Loco. mileage per quart	122.5
Lubricating oil.	
Loco. mileage, per quart	49.0
Waste, pounds per 100 miles	1.0

Cost per mile		per loco. mile.	per car mile.
Fuel	cents	7.29	0.55
Repairs and renewals,	"	7.45	0.57
Oil and waste	"	0.33	0.02
Water supply	"	0.42	0.03
Other supplies	"	0.14	0.01
Engineers and firemen	"	6.82	0.52
Roundhouse men	"	1.54	0.12
Total	"	23.99	1.82

Cost per 100 tons† per mile, 3.83 cents.

Cost of coal, per ton†, average of anthracite and bituminous $1.25

---

*1.5 cords wood taken as equivalent to 1 ton (2000 lbs) coal.

† Tons of 2000 lbs.

REQUIREMENTS FOR IRON AND STEEL.

(See also Bridge Specifications.)

Digest of Specifications adopted, subject to letter ballot, at 4th Annual Meeting of the **American Section** of the **International Association for Testing Materials,** June 29, 1901. Adopted by letter ballot, August, 1901, except wrought iron, on which action was deferred.

Process of Manufacture.

Wrought iron; puddled, charcoal hearth, or rolled from fagots or piles made from wrought iron scrap, alone or with muck bar added.

Steel castings. Open-hearth, crucible or Bessemer process.

Steel forgings. Open-hearth, crucible or Bessemer process.

Steel Rails. Bessemer or open-hearth. Ingots shall be kept vertical in pit-heating furnaces. No bled ingots shall be used. Sufficient material shall be discarded from the tops of the ingots to insure sound rails.

Steel Splice Bars. Bessemer or open-hearth.

Boiler Plate and Rivet Steel. Open-hearth.

Structural steel for bridges and ships. Open-hearth.

Structural steel for buildings. Open-hearth or Bessemer.

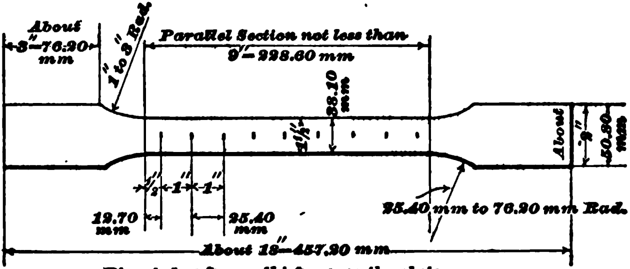
Test Pieces.

For flat plates, the specimen shown in Fig. J shall be used.

For large rounds, test specimen as shown in Fig. K. The center of the specimen shall be half way between the center and the outside of the round.

Whenever possible, iron shall be tested in full size, as rolled.

Test specimens shall be cut from bar as rolled.



Piece to be of same thickness as the plate.

Fig. J.

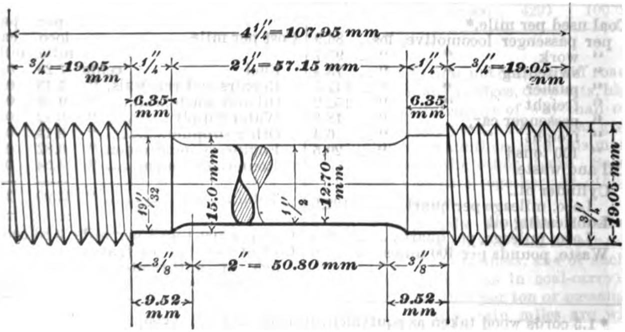


Fig. K.

Tests.

Nicking tests. The specimen shall be slightly and evenly nicked on one side, and bent back at this point through an angle of 180° by a succession of light blows.

Hot bending tests. Specimens shall be heated to a bright red, and bent by pressure or by a succession of light blows and without hammering on head.

Cold bending tests. Specimen to be bent by pressure or by a succession of light blows.

Yield point. The yield point shall be determined by careful observation of the drop of the beam or halt in the gage of the testing machine.

Drop tests. The drop testing machine for rails shall have a tup of 2000 pounds, the striking face of which shall have a radius of not more than 5 ins; and the test rail, not more than 6 feet long, shall be placed head upwards on solid supports 3 ft apart. The anvil block shall weigh at least 20,000 lbs, and the supports shall be a part of, or firmly secured to, the anvil. Height of drop from 15 ft for 45 lb rail to 19 ft for 85 lb and over. One test piece shall be selected from every fifth blow.

Homogeneity tests for fire box steel. A portion of the broken tensile test specimen is either nicked or grooved $\frac{1}{16}$ inch deep, in three places about 2 ins apart and on opposite sides. It is then clamped in a vise and broken off, by light hammer blows, bending away from each groove in succession. The specimen must not show any single seam or cavity more than ¼ inch long.

Notes to table, pp. 872 and 873.

(a) To be bent flat.

(b) Specimen to be bent about a bar of diameter equal to its own diameter or thickness.

(c) Specimen to be bent about a bar twice its diameter.

(d) Elongation, min, per cent, in sections less than 0.654 lbs. per linear ft, grade A, 19; B, 15; C, 12.

(e) Nicking test. Max per cent granular surface, grade A, 10; B, 10; C, 15.

(f) Hot bending test. Bar to be bent without cracking on outside of bend. To be bent flat in each grade; 180° in grades A and B, and sharply to 90° in C. Grade A, heated yellow and suddenly quenched in water between 80° and 90° F, to bend flat 180°. Also, heated bright red, split at end, and each part bent back 180°. Punched and drifted to hole at least 0.9 diam of rod or width of bar.

(g) Phosphorus, pieces for physical test, 0.05 for each grade.

(h) Sulphur, pieces for physical test, 0.05 for each grade.

(i) Bending. Specimen 1 inch × ½ inch to bend cold around a diam of 1 inch without fracture on outside of bent portion.

(j) Bending. Specimen 1 inch × ½ inch to bend cold, without fracture on outside of bent portion, around a diameter of ½ inch.

(k) Same, around diam of 1½ ins.

(l) Same, around diam of 1½ ins if not less than 20 ins diam; around a diam of 1 inch if less than 20 ins diam.

(m) Same, about a diameter of 1 inch.

(n) " " " " ½ "

(o) " " " " 1 "

(p) Deduct 1 per cent for each ⅛ inch in thickness above ¾ inch, and 2½ per cent for each $\frac{1}{16}$ inch below $\frac{5}{16}$ inch.

(q) Bending. Rivet rounds to be tested of full size, as rolled. Plate specimens shall be 1½ ins wide. For plates not over ¾ inch thick, the thickness shall be the same as that of the plate, and the specimen shall, where possible, have the natural rolled surface on two opposite sides. For plates thicker than ¾ inch, the specimen may be ½ inch thick. Shall be subjected to both cold and quenched bending tests. For the quenched test, the material is to be heated to a light cherry red (as seen in the dark) and quenched in water of temperature between 80° and 90° Fahr. Samples shall bend flat without fracture on the outside of bent portion. Bending may be done by pressure or by blows.

(r) For pins, the elongation shall be 5 per cent less. Center of test specimen 1 inch from surface.

(s) Eye-bars shall be of medium steel. Full-sized tests shall show 12½ per cent elongation in 15 ft of body. Min tensile strength, 55,000 lbs. per sq in. At least ⅔ of eye-bars tested shall break in the body.

(t) Same as (q) but omitting quenching test.

(u) See Homogeneity Test, in text, above.

See notes, p. 871. **Requirements for**

	Metal.	Allowable percentage of								
		Carbon.		Phosphorus.		Sulphur.	Manganese.		Nickel.	
1.	**WROUGHT IRON.**	Max.		Max.		Max.	Max.	Min.	Max.	Min.
	Grade A									
	Grade B									
	Grade C									
2.	**STEEL CASTINGS.**									
	Hard	0.40		0.08 g		0.05 h				
	Medium	0.40		0.08 g		0.05 h				
	Soft	0.40		0.08 g		0.05 h				
3.	**STEEL FORGINGS.**									
	Soft or low carbon			0.10		0.10				
	Carbon, not annealed			0.06		0.06				
	Carbon, annealed			0.04		0.04				
	Carbon, oil tempered			0.04		0.04				
	Nickel, annealed								4.00	3.00
	Nickel, oil tempered								4.00	3.00
4.	**STEEL RAILS.**	Max.	Min.							
	50 to 59 lbs. per yard	0.45	0.35	0.10		0.20	1.00	0.70		
	60 to 69 " " "	0.48	0.38	0.10		0.20	1.00	0.70		
	70 to 79 " " "	0.50	0.40	0.10		0.20	1.05	0.75		
	80 to 89 " " "	0.53	0.43	0.10		0.20	1.10	0.80		
	90 to 100 " " "	0.55	0.45	0.10		0.20	1.10	0.80		
5.	**STEEL SPLICE BARS**	Max. 0.15		0.10			0.60	0.30		
6.	**OPEN HEARTH BOILER PLATE AND RIVET STEEL.**			acid	basic					
	Flange or Boiler Steel			0.06	0.04	0.05	0.60	0.30		
	Fire Box Steel			0.04	0.03	0.04	0.50	0.30		
	Extra Soft Steel for Boiler Rivets			0.04	0.04	0.04	0.50	0.30		
7.	**STRUCTURAL STEEL FOR BRIDGES AND SHIPS.**			acid	basic					
	Rivet Steel			0.08	0.06	0.06				
	Soft Steel			0.08	0.06	0.06				
	Medium Steel			0.08	0.06	0.06				
8.	**STRUCTURAL STEEL FOR BUILDINGS.**			Max.						
	Rivet Steel			0.10						
	Medium Steel			0.10						

Iron and Steel.

See notes, p. 871.

Tensile Tests.					Cold Bending Tests.		Other Tests. Remarks.
Strength. lbs. per sq. in.		Elastic Limit and Yield Point. lbs. per sq. in.	Elongation. Percentage in 8 ins.	Contraction of Area. Percentage.	Angle of Bend.	How Bent.	
Max.	Min.	Yield Point. Min.	Min.	Min.	°		
....	50,000	25,000	25 d	....	180	a	e, f.
....	48,000	25,000	20 d	....	180	b	e, f.
....	48,000	25,000	20 d	....	180	c	e, f.
....	85,000	38,250	15	20	....	i	
....	70,000	31,500	18	25	90	i	
....	60,000	27,000	22	30	120	i	
	Average.	Average.	Avge.	Avge.			
....	58,000	29,000	28	35	180	j	
....	75,000	37,500	18	30	180	k	
....	75,000	37,500	23	32.5	180	l	
		Elastic Limit.					
....	85,000	....	21.5	42.5	180	m	
....	80,000	47,500	24.5	42.5	180	n	
....	90,000	60,000	22.5	47.5	180	o	
See Drop Test, in text, above.							
	Min.	Min.	Min.				
64,000	54,000	32,000	25	....	180	a	
		Max. and Min.					
65,000	55,000	½ Tensile Strength.	25 p	....	180	a, q	u
62,000	52,000		26 p	....	180	a, q	
55,000	45,000		28 p	....	180	a, q	
			r s.				
60,000	50,000	½ Tensile Strength.	26 p	....	180	a, t	
62,000	52,000		25 p	....	180	a, t	
70,000	60,000		22 p	....	180	b, t	
			r.				
60,000	50,000	½ Tensile Strength.	26 p	....	180	a, t	
70,000	60,000		22 p	....	180	b, t	

Iron is weakened by extreme cold.

The belief (originating with Styff of Sweden) is gaining ground that iron and steel are not rendered *more brittle by intense cold*, but that the great number of breakages of rails, wheels, axles, &c, in winter, is owing to the more severe blows incident to the frozen and unyielding nature of the earth at that period of the year. But Sandberg's experiments show conclusively that although these metals may perhaps bear as much *steady* force, *gradually* applied, in winter as in summer, yet their resistance to *impulse*, or *sudden force*, is not more than ½ or ⅓ as great in severe cold; which renders them less flexible and less stretchy. It is probable that this fact does not receive as much attention as it should, in proportioning iron bridges, &c.

Some experiments with good wrought iron showed that even at 23° Fah, or only 9° colder than freezing point, there was a loss of strength of from 2½ to 4 per cent.

Malleable Cast Iron. Experiments by Mr. D. L. Barnes, of Chicago, on a large number of samples of a single make of "malleable" cast iron, gave in most cases tensile strengths ranging from 24000 to 32000 lbs. per square inch, with an average of about 28000 lbs. The higher figures were obtained generally with the smallest bars (about 3 × ¼ inch) and the lower with the largest bars (about 3 × 1 inch). Pieces planed on all four sides averaged only about 24000 lbs. per square inch. This may explain the difference in favor of the smaller sections, in which the original "shell" forms a larger portion of the whole cross section.

CAST IRON.

Tensile strength	14,000 to 20,000	lbs* per sq inch
Compressive strength (average about 100,000)	90,000 to 130,000	" " "
Transverse strength, bar 1 in sq, 1 ft span, center load 2500 lbs. Deflection, minimum, 0.15 inch.		
Elastic limit	about 6,000	lbs per sq inch
Modulus of Elasticity	" 15,000,000	" " "

Specifications.

Tensile strength.

Bureau of Water, Philadelphia	16,000 to 20,000	lbs per sq inch
Water Department, St. Louis, Mo	18,000	" " "

Transverse strength.

Bureau of Water, Philadelphia.

1 in sq, 56 ins span, center load 500 lbs.

1 in sq, 36 ins span, " " 750 lbs. Deflection, minimum, 0.4 to 0.6 in.

Water Department, St. Louis, Mo.

3 in × ½ in (laid flat) 18 ins span, center load 1000 to 1250 lbs. Minimum deflection 0.3 to ⅝ inch.

Weight of Cast Iron.

Assuming 450 lbs per cub ft, specific gravity 7.2, a cub inch weighs 0.2604+ lbs; and a pound contains 3.83995+ cub ins.

Table, page 875: D = thickness or diameter, in inches.

Wt. of plate, 1 ft square, in lbs =	$37.5\ D$	(Exact)	$\text{Log } W = 1.574\ 0313 + \text{Log } D$
" " sq bar, 1 ft long, in lbs =	$3.125\ D^2$	(Exact)	$\text{Log } W = 0.494\ 8500 + 2\ \text{Log } D$
" " rd bar, 1 ft long, in lbs =	$2.45437\ D^2$		$\text{Log } W = 0.389\ 9400 + 2\ \text{Log } D$
" " ball, in lbs =	$0.136354\ D^8$		$\text{Log } W = \bar{1}.134\ 6551 + 3\ \text{Log } D$

Weight of a spherical shell = weight of ball having outer diam of shell *minus* weight of ball having its inner diam.

Weight of pattern. A casting weighs 20 × weight of pattern of perfectly dry white pine. If not perfectly dry, although well seasoned, for 20, substitute 19 or 18.

For lead, at 700 lbs per cub ft, multiply weight of cast iron by 1.555---;
For copper, at 550 lbs, multiply by 1.222---;
For brass, at 500 lbs, multiply by 1.111---;
For wrought iron, at 485 lbs, multiply by 1.0777---;
For tin, at 460 lbs, multiply by 1.022---;
Zinc, at 450 lbs = cast iron.

---

* High grade irons may reach 30,000 to 40,000 lbs per sq inch, tensile.

TABLE OF WEIGHT OF CAST IRON.

At 450 lbs per cubic foot; specific gravity, 7.2.

D = Thickness or diameter, in inches. For equivalents in feet, see p 221.

D	**Weights,** in pounds.				D	**Weights,** in pounds.			
	Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.		Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.
1/32	1.172	0.0031	0.0024		3⅛	117.2	30.52	23.97	4.161
1/16	2.344	0.0122	0.0096		¼	121.9	33.01	25.92	4.681
3/32	3.516	0.0275	0.0216	0.0001	⅜	126.6	35.60	27.96	5.242
1/8	4.688	0.0488	0.0383	0.0003	½	131.2	38.28	30.07	5.846
5/32	5.859	0.0763	0.0599	0.0005	⅝	135.9	41.06	32.25	6.495
3/16	7.081	0.1099	0.0863	0.0009	¾	140.6	43.94	34.51	7.191
7/32	8.203	0.1495	0.1174	0.0014	⅞	145.3	46.92	36.85	7.934
1/4	9.375	0.1953	0.1534	0.0021	4.	150.0	50.00	39.27	8.727
9/32	10.55	0.2472	0.1941	0.0030	⅛	154.7	53.17	41.76	9.571
5/16	11.72	0.3052	0.2397	0.0042	¼	159.4	56.45	44.33	10.47
11/32	12.89	0.3693	0.2900	0.0055	⅜	164.1	59.81	46.98	11.42
3/8	14.06	0.4394	0.3451	0.0072	½	168.8	63.28	49.70	12.43
13/32	15.23	0.5157	0.4051	0.0091	⅝	173.4	66.84	52.50	13.49
7/16	16.41	0.5982	0.4698	0.0114	¾	178.1	70.51	55.38	14.61
15/32	17.58	0.6866	0.5393	0.0140	⅞	182.8	74.27	58.33	15.80
1/2	18.75	0.7812	0.6136	0.0170	5.	187.5	78.12	61.36	17.04
9/16	21.09	0.9888	0.7766	0.0243	⅛	192.2	82.08	64.47	18.35
5/8	23.44	1.221	0.9587	0.0338	¼	196.9	86.13	67.65	19.73
11/16	25.78	1.477	1.160	0.0443	⅜	201.6	90.28	70.91	21.17
3/4	28.12	1.758	1.381	0.0575	½	206.2	94.53	74.24	22.69
13/16	30.47	2.063	1.620	0.0731	⅝	210.9	98.88	77.66	24.27
7/8	32.81	2.393	1.879	0.0913	¾	215.6	103.3	81.15	25.92
15/16	35.16	2.747	2.157	0.1124	⅞	220.3	107.9	84.71	27.65
1.	37.50	3.125	2.454	0.1363	6.	225.0	112.5	88.36	29.45
1/16	39.84	3.528	2.771	0.1636	¼	234.4	122.1	95.87	33.29
1/8	42.19	3.955	3.106	0.1941	½	243.8	132.0	103.7	37.45
3/16	44.53	4.407	3.461	0.2283	¾	253.1	142.4	111.8	41.94
1/4	46.88	4.883	3.835	0.2663	7.	262.5	153.1	120.3	46.77
5/16	49.22	5.383	4.228	0.3083	¼	271.9	164.3	129.0	51.96
3/8	51.56	5.908	4.640	0.3545	½	281.2	175.8	138.1	57.52
7/16	53.91	6.458	5.072	0.4050	¾	290.6	187.7	147.4	63.47
1/2	56.25	7.031	5.522	0.4602	8.	300.0	200.0	157.1	69.81
9/16	58.59	7.629	5.992	0.5202	¼	309.4	212.7	167.0	76.57
5/8	60.94	8.252	6.481	0.5851	½	318.8	225.8	177.3	83.74
11/16	63.28	8.899	6.989	0.6552	¾	328.1	239.3	187.9	91.35
3/4	65.62	9.570	7.517	0.7308	9.	337.5	253.1	198.8	99.40
13/16	67.97	10.27	8.063	0.8119	¼	346.9	267.4	210.0	107.9
7/8	70.31	10.99	8.629	0.8988	½	356.2	282.0	221.5	116.9
15/16	72.66	11.73	9.213	0.9917	¾	365.6	297.1	233.3	126.4
2.	75.00	12.50	9.818	1.091	10.	375.0	312.5	245.4	136.3
1/8	79.69	14.11	11.08	1.308	¼	384.4	328.3	257.9	146.8
1/4	84.38	15.82	12.43	1.553	½	393.8	344.5	270.6	157.9
3/8	89.06	17.63	13.84	1.827	¾	403.1	361.1	283.6	169.4
1/2	93.75	19.53	15.34	2.131	11.	412.5	378.1	297.0	181.5
5/8	98.44	21.53	16.91	2.466	¼	421.9	395.5	310.6	194.1
3/4	103.1	23.63	18.56	2.836	½	431.2	413.3	324.6	207.4
7/8	107.8	25.83	20.29	3.240	¾	440.6	431.5	338.9	221.2
3.	112.5	28.12	22.09	3.682	12.	450.0	450.0	353.4	235.6

WEIGHT OF CAST-IRON PIPES per running foot.

Assuming the weight of cast-iron at 450 lbs per cub ft, or .2604 lb per cub inch. No allowance is here made for the spigot and faucet-joints used in water-pipes. As these are now commonly made, they add to the weight of each length or section of pipe of any size, about as much as that of 8 inches in length of the plain pipe as given in the table.

For lead-pipe mult by 1.6; **copper,** mult by 1.2; **brass, add 1-7th; welded iron,** mult by 1.0667, or add one fifteenth part.

Inner diam, or bore, in Inches.	THICKNESS OF PIPE IN INCHES.												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.
1.	3.07	5.07	7.38	9.99	12.9	16.2	19.7	23.5	27.7	32.1	36.9	47.4	59.1
¼	3.69	6.00	8.61	11.5	14.8	18.3	22.2	26.3	30.8	35.5	40.6	51.7	64.0
½	4.30	6.92	9.84	13.1	16.6	20.5	24.6	29.1	33.8	38.9	44.3	56.0	68.9
¾	4.92	7.84	11.1	14.6	18.5	22.6	27.1	31.8	36.9	42.3	48.0	60.3	73.8
2.	5.53	8.76	12.3	16.2	20.3	24.8	29.5	34.6	40.0	45.7	51.7	64.6	78.7
¼	6.15	9.69	13.5	17.7	22.2	26.9	32.0	37.4	43.1	49.0	55.4	68.9	86.7
½	6.76	10.6	14.8	19.2	24.0	29.1	34.5	40.1	46.1	52.4	59.1	73.2	88.6
¾	7.37	11.5	16.0	20.8	25.9	31.2	36.9	42.9	49.2	55.8	62.7	77.5	93.5
3.	7.98	12.5	17.2	22.3	27.7	33.4	39.4	45.7	52.3	59.2	66.4	81.8	98.4
¼	8.60	13.4	18.5	23.8	29.5	35.5	41.8	48.4	55.4	62.3	70.1	86.1	103.
½	9.21	14.3	19.7	25.4	31.4	37.7	44.3	51.2	58.4	65.9	73.8	90.4	108.
¾	9.83	15.2	20.9	26.9	33.2	39.8	46.8	54.0	61.5	69.3	77.5	94.7	113.
4.	10.3	16.1	22.2	28.5	35.1	42.0	49.2	56.7	64.6	72.7	81.2	99.0	118.
¼	11.1	17.1	23.4	30.0	36.9	44.1	51.7	59.5	67.7	76.1	84.9	103.	123.
½	11.7	18.0	24.6	31.5	38.8	46.3	54.1	62.3	70.7	79.5	88.6	108.	128.
¾	12.3	18.9	25.8	33.1	40.6	48.5	56.6	65.0	73.8	83.9	92.3	112.	133.
5.	12.9	19.8	27.1	34.6	42.5	50.6	59.1	67.8	76.9	87.2	96.0	116.	138.
¼	13.5	20.8	28.3	36.1	44.3	52.8	61.5	70.6	80.0	90.6	99.6	121.	143.
½	14.2	21.7	29.5	37.7	46.1	54.9	64.0	73.3	83.0	94.0	103.	125.	148.
¾	14.8	22.6	30.8	39.2	48.0	57.1	66.4	76.1	86.1	97.4	107.	129.	153.
6.	15.4	23.5	32.0	40.8	49.8	59.2	68.9	78.9	89.2	99.8	111.	134.	158.
½	16.6	25.4	34.5	43.8	53.5	63.5	73.8	84.4	95.3	107.	118.	142.	167.
7.	17.8	27.2	36.9	46.9	57.2	67.8	78.7	89.4	102.	113.	126.	151.	177.
½	19.1	29.1	39.4	50.0	60.9	72.1	83.7	95.5	108.	120.	133.	159.	187.
8.	20.3	30.9	41.8	53.1	64.6	76.4	88.6	101.	114.	127.	140.	168.	197.
½	21.5	32.8	44.3	56.1	68.3	80.7	93.5	107.	120.	134.	148.	177.	207.
9.	22.8	34.6	46.8	59.2	72.0	85.1	98.4	112.	126.	140.	155.	185.	217.
½	24.0	36.4	49.2	62.3	75.7	89.3	103.	118.	132.	147.	163.	194.	226.
10.	25.1	38.3	51.7	65.3	79.4	93.6	108.	123.	138.	154.	170.	202.	235.
½	26.4	40.1	54.1	68.4	83.0	97.9	113.2	129.	145.	161.	177.	211.	245.
11.	27.6	42.0	56.6	71.5	86.7	102.	118.	134.	151.	168.	185.	220.	255.
½	28.8	43.8	59.1	74.6	90.4	107.	123.	140.	157.	174.	192.	228.	265.
12.	30.0	45.7	61.5	77.7	94.1	111.	128.	145.	163.	181.	199.	237.	275.
13.	32.5	49.4	66.4	83.8	102.	120.	138.	156.	175.	195.	214.	254.	294.
14.	35.0	53.1	71.4	89.4	109.	128.	148.	168.	188.	208.	229.	271.	314.
15.	37.4	56.7	76.3	96.1	116.	137.	158.	179.	200.	222.	244.	289.	334.
16.	39.1	60.4	81.2	102.	124.	145.	167.	190.	212.	235.	258.	306.	353.
17.	42.3	64.1	86.1	108.	131.	154.	177.	201.	225.	249.	273.	323.	373.
18.	44.8	67.8	91.0	115.	139.	163.	187.	212.	237.	262.	288.	340.	393.
19.	47.3	71.5	96.0	121.	146.	171.	197.	223.	249.	276.	303.	357.	412.
20.	49.7	75.2	101.	127.	153.	180.	207.	234.	261.	289.	317.	375.	432.
21.	52.2	78.9	106.	133.	161.	188.	217.	245.	274.	303.	332.	392.	452.
22.	54.6	82.6	111.	139.	168.	196.	227.	256.	286.	316.	347.	409.	471.
23.	57.1	86.3	116.	145.	175.	206.	236.	267.	298.	330.	362.	426.	491.
24.	59.6	89.9	121.	152.	183.	214.	246.	278.	311.	343.	375.	444.	511.
25.	62.0	93.6	126.	158.	190.	223.	256.	289.	323.	357.	391.	461.	531.
26.	64.5	97.3	131.	164.	198.	231.	266.	300.	335.	370.	406.	478.	550.
27.	66.9	101.	135.	170.	205.	240.	276.	311.	348.	384.	421.	495.	570.
28.	69.4	105.	140.	176.	212.	249.	286.	323.	360.	397.	436.	512.	590.
29.	71.8	109.	145.	182.	220.	257.	295.	334.	372.	411.	450.	530.	609.
30.	74.2	112.	150.	188.	227.	266.	305.	345.	384.	424.	465.	547.	629.
31.	76.7	116.	155.	195.	234.	275.	315.	356.	397.	438.	480.	564.	649.
32.	79.1	120.	160.	201.	242.	283.	325.	367.	409.	451.	495.	581.	668.
33.	81.6	123.	165.	207.	249.	292.	335.	378.	421.	465.	509.	598.	688.
34.	84.1	127.	170.	213.	257.	300.	345.	389.	434.	479.	524.	616.	708.
35.	86.5	131.	175.	219.	264.	309.	354.	400.	446.	492.	539.	633.	726.
36.	89.0	134.	180.	225.	271.	318.	364.	411.	458.	506.	554.	650.	746.
42.	104.	156.	210.	262.	315.	370.	423.	478.	532.	588.	644.	753.	864
48.	119.	178.	239.	298.	359.	422.	482.	544.	605.	669.	733.	856.	982.

For warming buildings by steam it usually suffices to allow 1 sq ft of cast or wrought pipe surface for each 120 cub ft of space to be warmed; and 1 cub ft of boiler for each 2000 cub ft of such space.

Table of Weight of WROUGHT IRON and STEEL.

At 485 lbs per cubic foot; specific gravity, 7.76. See page 879.

D = Thickness or diameter, in inches. For equivalents in feet, see p 221.

D	Weights, in pounds.				D	Weights, in pounds.			
	Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.		Plate 1 ft sq.	Square bar 1 ft long	Round bar 1 ft long	Ball.
1/32	1.263	0.0033	0.0026		3⅛	126.3	32.89	25.83	4.485
1/16	2.526	0.0132	0.0103		¼	131.4	35.57	27.94	5.045
3/32	3.789	0.0296	0.0232	0.0001	⅜	136.4	38.36	30.13	5.650
1/8	5.052	0.0526	0.0413	0.0003	½	141.5	41.26	32.40	6.301
5/32	6.315	0.0822	0.0646	0.0006	⅝	146.5	44.26	34.76	7.000
3/16	7.578	0.1184	0.0930	0.0010	¾	151.6	47.36	37.20	7.750
7/32	8.841	0.1612	0.1266	0.0015	⅞	156.6	50.57	39.72	8.551
1/4	10.10	0.2105	0.1653	0.0023	4.	161.7	53.89	42.32	9.406
9/32	11.37	0.2664	0.2092	0.0033	⅛	166.7	57.31	45.01	10.32
5/16	12.63	0.3289	0.2583	0.0045	¼	171.8	60.84	47.78	11.28
11/32	13.89	0.3980	0.3126	0.0060	⅜	176.8	64.47	50.63	12.31
3/8	15.16	0.4736	0.3720	0.0077	½	181.9	68.20	53.57	13.39
13/32	16.42	0.5558	0.4366	0.0099	⅝	186.9	72.04	56.58	14.54
7/16	17.68	0.6447	0.5063	0.0123	¾	192.0	75.99	59.68	15.75
15/32	18.95	0.7400	0.5812	0.0151	⅞	197.0	80.04	62.87	17.03
1/2	20 21	0.8420	0.6613	0.0184	5.	202.1	84.20	66.13	18.37
9/16	22.73	1.066	0.8370	0.0261	⅛	207.1	88.46	69.48	19.78
5/8	25.26	1.316	1.033	0.0359	¼	212.2	92.83	72.91	21.27
11/16	27.79	1.592	1.250	0.0478	⅜	217.2	97.31	76.42	22.82
3/4	30.31	1.895	1.488	0.0620	½	222.3	101.9	80.02	24.45
13/16	32.84	2.223	1.746	0.0788	⅝	227.3	106.6	83.70	26.16
7/8	35.36	2.579	2.025	0.0985	¾	232.4	111.4	87.46	27.94
15/16	37.89	2.960	2.325	0.1211	⅞	237.5	116.3	91.30	29.80
1.	40.42	3.368	2.645	0 1470	6.	242.5	121.3	95.23	31.74
1/16	42.94	3.802	2.986	0.1763	¼	252.6	131.6	103.3	35.88
1/8	45.47	4.263	3.348	0.2092	½	262.7	142.3	111.8	40.36
3/16	47.99	4.750	3.730	0.2461	¾	272.8	153.5	120.5	45.20
1/4	50.52	5.263	4.133	0.2870	7.	282.9	165.0	129.6	50.41
5/16	53.05	5.802	4.557	0.3323	¼	293.0	177.0	139.0	56.00
3/8	55.57	6.368	5.001	0.3820	½	303.1	189.5	148.8	62.00
7/16	58.10	6.960	5.466	0.4365	¾	313.2	202.3	158.9	68.41
1/2	60.63	7.578	5.952	0.4960	8.	323.3	215.6	169.3	75.24
9/16	63.15	8.223	6.458	0.5606	¼	333.4	229.2	180.0	82.52
5/8	65.68	8.894	6.985	0.6306	½	343.5	243.3	191.1	90.25
11/16	68.20	9.591	7.533	0.7062	¾	353.6	257.9	202.5	98.45
3/4	70.73	10.31	8.101	0.7876	9.	363.7	272.8	214.3	107.1
13/16	73.26	11.06	8.690	0.8750	¼	373.9	288.2	226.3	116.3
7/8	75.78	11.84	9.300	0.9687	½	384.0	304.0	238.7	126.0
15/16	78.31	12.64	9.930	1.069	¾	394.1	320.2	251.5	136.2
2.	80.83	13.47	10.58	1.176	10.	404.2	336.8	264.5	147.0
1/8	85.89	15.21	11.95	1.410	¼	414.3	353.9	277.9	158.3
1/4	90.94	17.05	13.39	1.674	½	424.4	371.3	291.6	170.1
3/8	95.99	19.00	14.92	1.969	¾	434.5	389.2	305.7	182.6
1/2	101.0	21.05	16.53	2.296	11.	444.6	407.5	320.1	195.6
5/8	106.1	23.21	18.23	2.658	¼	454.7	426.3	334.8	209.2
3/4	111.1	25.47	20.00	3.056	½	464.8	445.4	349.8	223.5
7/8	116.2	27.84	21.86	3.492	¾	474.9	465.0	365.2	238.4
3.	121.3	30.31	23.81	3.968	12.	485.0	485.0	380.9	253.9

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot. For cast iron, deduct $\frac{1}{16}$ part; for steel, add $\frac{1}{48}$; for copper, add $\frac{1}{7}$; for cast brass, add $\frac{1}{13}$; for lead, add ½; for zinc, deduct $\frac{1}{12}$.

Width in Ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
1.	.2083	.4166	.6250	.8333	1.042	1.250	1.458	1.666	2.083	2.500	2.916	3.333
⅛	.2344	.4688	.7033	.9375	1.172	1.406	1.640	1.875	2.344	2.812	3.280	3.75
¼	.2605	.5210	.7810	1.042	1.303	1.563	1.823	2.083	2.605	3.125	3.646	4.166
⅜	.2865	.5730	.8595	1.146	1.432	1.719	2.006	2.292	2.864	3.438	4.012	4.583
½	.3125	.6250	.9375	1.250	1.562	1.875	2.188	2.500	3.125	3.750	4.375	5.000
⅝	.3385	.6771	1.015	1.354	1.692	2.031	2.370	2.708	3.384	4.062	4.740	5.416
¾	.3646	.7292	1.094	1.458	1.823	2.188	2.550	2.916	3.646	4.375	5.105	5.833
⅞	.3906	.7812	1.172	1.562	1.953	2.344	2.735	3.125	3.906	4.688	5.470	6.25
2.	.4166	.8333	1.25	1.666	2.083	2.500	2.916	3.333	4.166	5.000	5.833	6.666
⅛	.4427	.8855	1.328	1.771	2.214	2.656	3.098	3.542	4.428	5.312	6.196	7.083
¼	.4688	.9375	1.406	1.875	2.344	2.812	3.281	3.750	4.688	5.624	6.562	7.500
⅜	.4948	.9895	1.484	1.979	2.474	2.968	3.463	3.958	4.948	5.936	6.926	7.916
½	.5210	1.042	1.562	2.083	2.605	3.125	3.646	4.166	5.210	6.250	7.291	8.333
⅝	.5470	1.094	1.641	2.187	2.735	3.282	3.829	4.375	5.470	6.564	7.658	8.750
¾	.5730	1.146	1.719	2.292	2.865	3.438	4.011	4.583	5.730	6.876	8.022	9.166
⅞	.5990	1.198	1.797	2.396	2.995	3.594	4.193	4.792	5.990	7.188	8.386	9.583
3.	.625	1.250	1.875	2.500	3.125	3.750	4.375	5.000	6.250	7.500	8.750	10.00
⅛	.6515	1.303	1.954	2.605	3.257	3.908	4.560	5.210	6.514	7.816	9.120	10.42
¼	.6770	1.354	2.031	2.708	3.385	4.062	4.739	5.416	6.770	8.124	9.478	10.83
⅜	.7031	1.406	2.109	2.812	3.516	4.218	4.921	5.625	7.032	8.436	9.842	11.25
½	.7291	1.458	2.188	2.916	3.646	4.375	5.105	5.833	7.291	8.750	10.21	11.66
⅝	.7555	1.511	2.266	3.021	3.777	4.533	5.288	6.042	7.554	9.066	10.58	12.08
¾	.7812	1.562	2.343	3.125	3.906	4.686	5.468	6.25	7.812	9.372	10.94	12.50
⅞	.8070	1.614	2.421	3.229	4.035	4.842	5.65	6.458	8.070	9.684	11.30	12.92
4.	.8333	1.666	2.500	3.333	4.166	5.000	5.833	6.666	8.333	10.00	11.66	13.33
⅛	.8595	1.719	2.578	3.438	4.297	5.156	6.016	6.875	8.594	10.31	12.03	13.75
¼	.8855	1.771	2.656	3.542	4.427	5.312	6.198	7.083	8.854	10.62	12.40	14.16
⅜	.9115	1.823	2.734	3.646	4.557	5.468	6.380	7.291	9.114	10.94	12.76	14.58
½	.9375	1.875	2.812	3.750	4.687	5.624	6.562	7.500	9.374	11.25	13.12	15.00
⅝	.9636	1.927	2.891	3.854	4.818	5.782	6.745	7.708	9.636	11.56	13.49	15.42
¾	.9895	1.979	2.968	3.958	4.947	5.936	6.926	7.917	9.894	11.87	13.85	15.83
⅞	1.016	2.031	3.048	4.062	5.080	6.096	7.112	8.125	10.16	12.19	14.22	16.25
5.	1.042	2.083	3.125	4.166	5.210	6.25	7.291	8.333	10.42	12.50	14.58	16.66
⅛	1.068	2.136	3.204	4.271	5.340	6.408	7.476	8.542	10.68	12.81	14.95	17.08
¼	1.094	2.188	3.282	4.375	5.470	6.564	7.658	8.750	10.94	13.13	15.31	17.50
⅜	1.120	2.240	3.360	4.479	5.600	6.720	7.840	8.958	11.20	13.44	15.68	17.92
½	1.146	2.292	3.438	4.584	5.730	6.876	8.022	9.167	11.46	13.75	16.04	18.33
⅝	1.172	2.344	3.516	4.687	5.860	7.032	8.204	9.375	11.72	14.06	16.40	18.75
¾	1.198	2.396	3.594	4.791	5.990	7.188	8.386	9.583	11.98	14.37	16.77	19.16
⅞	1.224	2.448	3.672	4.896	6.120	7.344	8.568	9.792	12.24	14.69	17.13	19.58
6.	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.00	12.50	15.00	17.50	20.00
⅛	1.276	2.552	3.828	5.104	6.380	7.656	8.932	10.21	12.76	15.31	17.86	20.42
¼	1.302	2.604	3.906	5.208	6.510	7.812	9.114	10.42	13.02	15.62	18.23	20.83
⅜	1.328	2.657	3.984	5.313	6.640	7.968	9.297	10.63	13.28	15.93	18.59	21.25
½	1.354	2.708	4.063	5.417	6.770	8.126	9.480	10.83	13.54	16.25	18.96	21.66
⅝	1.381	2.761	4.143	5.521	6.906	8.286	9.668	11.04	13.81	16.57	19.33	22.08
¾	1.406	2.813	4.218	5.625	7.030	8.436	9.843	11.25	14.06	16.87	19.69	22.50
⅞	1.432	2.864	4.296	5.729	7.160	8.592	10.02	11.46	14.32	17.18	20.04	22.92
7.	1.458	2.916	4.375	5.833	7.291	8.750	10.20	11.66	14.58	17.50	20.42	23.33
⅛	1.484	2.969	4.452	5.938	7.420	8.904	10.39	11.87	14.84	17.81	20.78	23.75
¼	1.511	3.021	4.533	6.042	7.555	9.066	10.58	12.08	15.11	18.13	21.16	24.16
⅜	1.536	3.073	4.608	6.146	7.680	9.216	10.75	12.29	15.36	18.43	21.50	24.58
½	1.562	3.125	4.686	6.250	7.810	9.372	10.93	12.50	15.62	18.74	21.86	25.00
⅝	1.588	3.177	4.764	6.354	7.940	9.528	11.12	12.71	15.88	19.05	22.24	25.42
¾	1.615	3.229	4.845	6.458	8.075	9.690	11.31	12.92	16.15	19.38	22.62	25.83
⅞	1.641	3.281	4.923	6.562	8.205	9.846	11.48	13.13	16.41	19.69	22.96	26.25
8.	1.666	3.333	5.000	6.666	8.333	10.00	11.66	13.33	16.66	20.00	23.33	26.66
⅛	1.693	3.386	5.079	6.771	8.455	10.15	11.85	13.54	16.91	20.30	23.70	27.08
¼	1.719	3.438	5.157	6.875	8.595	10.31	12.03	13.75	17.19	20.61	24.06	27.50
⅜	1.745	3.489	5.235	6.979	8.725	10.47	12.21	13.96	17.45	20.94	24.42	27.92
½	1.771	3.542	5.313	7.083	8.855	10.63	12.40	14.17	17.71	21.26	24.80	28.33
⅝	1.797	3.594	5.391	7.188	8.985	10.78	12.58	14.37	17.97	21.56	25.16	28.75
¾	1.823	3.646	5.469	7.292	9.115	10.94	12.76	14.58	18.23	21.88	25.52	29.17
⅞	1.849	3.698	5.547	7.396	9.245	11.09	12.94	14.79	18.49	22.18	25.88	29.58
9.	1.875	3.750	5.625	7.500	9.375	11.25	13.12	15.00	18.75	22.50	26.24	30.00
⅛	1.901	3.802	5.703	7.604	9.505	11.41	13.31	15.21	19.00	22.81	26.62	30.42
¼	1.927	3.854	5.781	7.708	9.635	11.56	13.49	15.42	19.27	23.12	26.98	30.83
⅜	1.953	3.906	5.859	7.812	9.765	11.72	13.67	15.62	19.53	23.44	27.34	31.25
½	1.979	3.958	5.937	7.916	9.895	11.87	13.85	15.84	19.79	23.74	27.70	31.67
⅝	2.005	4.010	6.015	8.021	10.02	12.03	14.04	16.04	20.04	24.06	28.08	32.08
¾	2.031	4.062	6.093	8.125	10.16	12.18	14.21	16.25	20.32	24.36	28.42	32.50
⅞	2.057	4.114	6.171	8.229	10.29	12.34	14.40	16.46	20.58	24.68	28.80	32.92
10.	2.083	4.166	6.250	8.333	10.41	12.50	14.58	16.66	20.82	25.00	29.16	33.33
⅛	2.109	4.219	6.327	8.438	10.55	12.65	14.76	16.87	21.10	25.30	29.52	33.75
¼	2.135	4.270	6.405	8.541	10.67	12.81	14.94	17.08	21.34	25.62	29.88	34.17

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot—(Continued.)

Width in ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
10⅜	2.162	4.323	6.486	8.646	10.81	12.97	15.13	17.29	21.62	25.94	30.26	34.58
½	2.188	4.375	6.564	8.750	10.94	13.13	15.31	17.50	21.88	26.26	30.62	35.00
⅝	2.214	4.427	6.642	8.854	11.07	13.28	15.50	17.71	22.14	26.56	31.00	35.42
¾	2.239	4.479	6.717	8.958	11.20	13.43	15.67	17.92	22.40	26.86	31.34	35.83
⅞	2.266	4.531	6.798	9.062	11.33	13.59	15.86	18.12	22.66	27.18	31.72	36.25
11.	2.291	4.583	6.873	9.166	11.46	13.75	16.04	18.33	22.90	27.50	32.08	36.66
⅛	2.318	4.636	6.954	9.271	11.59	13.91	16.22	18.54	23.18	27.82	32.44	37.08
¼	2.344	4.688	7.032	9.375	11.72	14.06	16.40	18.75	23.44	28.12	32.80	37.50
⅜	2.370	4.740	7.110	9.479	11.85	14.22	16.59	18.96	23.70	28.44	33.18	37.92
½	2.396	4.791	7.185	9.582	11.97	14.37	16.76	19.16	23.94	28.74	33.52	38.33
⅝	2.422	4.844	7.266	9.688	12.11	14.53	16.95	19.37	24.22	29.06	33.90	38.75
¾	2.448	4.896	7.344	9.792	12.24	14.68	17.13	19.58	24.48	29.36	34.26	39.16
⅞	2.474	4.948	7.422	9.896	12.37	14.84	17.32	19.79	24.74	29.68	34.64	39.58
12.	2.500	5.000	7.500	10.00	13.50	15.00	17.50	20.00	25.00	30.00	35.00	40.00

Weight of Wrought Iron and Steel.

Assuming 485 lbs. per cub ft,* specific gravity, 7.76; a cubic inch weighs 0.28067 lbs; and a pound contains 3.5629 cubic inches.

Table, page 875 : **D** = thickness or diameter, in inches.

Wt. of plate, 1 ft square, in lbs, = 40.4167 D; Log W = 1.606 5605 + Log D

" " sq bar, 1 ft long, in lbs, = 3.36806 D^2; Log W = 0.527 3792 + 2 Log D

" " rd bar, 1 ft long, in lbs, = 2.64527 D^2; Log W = 0.422 4693 + 2 Log D

" " ball, in lbs, = 0.146959 D^3; Log W = $\bar{1}$.167 1966 + 3 Log D

$$\textbf{Weight of a spherical shell} = \left\{\begin{matrix}\text{weight of ball having}\\ \text{outer diameter of shell}\end{matrix}\right\} - \left\{\begin{matrix}\text{weight of ball having}\\ \text{inner diameter of shell}\end{matrix}\right\}$$

Weights of equal masses.

For lead,	at 700 lbs per cub ft; weight	= 1.44 ×	weight of wrought iron
For copper,	" 550 " " " "	= 1.13 ×	
For brass,	" 500 " " " "	= 1.03 ×	
For tin,	" 460 " " " "	= 0.948 ×	
For zinc or cast iron,	" 450 " " " "	= 0.928 ×	

---

* Very pure soft wrought iron weighs from 488 to 492 lbs per cubic foot; average rolled iron about 480. At 480 lbs, a bar 1 inch square weighs exactly 10 lbs per yard = 3⅓ lbs per foot.

Weights per square foot of **galvanized sheet iron.** Standard list adopted by the American Galvanized Iron Ass'n, at Pittsburgh, April, 1884.

No.	Ounces avoir per sq ft	Sq ft per 2240 lbs.	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.
29	12	2987	24	17	2108	19	33	1086
28	13	2757	23	19	1886	18	38	943
27	14	2560	22	21	1706	17	43	833
26	15	2389	21	24	1493	16	48	746
25	16	2240	20	28	1280	14	60	597

The galvanizing is simply a thin film of zinc on both sides of the sheet, as in what is known as "tinned plates," or "tin;" which are in reality sheet iron similarly coated with tin. Zinc, like tin, resists corrosion from ordinary atmospheric influences, much better than iron; and hence the use of these metals as a protection to the iron. A well galvanized roof, of a good pitch, will suffer but little from 5 to 6 years' exposure without being painted. It will then take paint readily, and should be painted. It is better, however, always to paint tin ones at once.

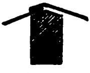
Paint does not adhere well to new zinc, and this is the principal reason why new galvanized roofs are not painted; but this may be remedied by first brushing the zinc over with the following: One part of chloride of copper, 1 part nitrate of copper, 1 part of salammoniac. Dissolve in 64 parts of water. Then add 1 part of commercial hydrochloric acid. When brushed with this solution, the zinc turns black; dries within 12 to 24 hours, and may then be painted.

Paint of some mineral oxide of a brown color is generally used; one coat being applied to both sides in the shop; and the other after being put on the roof. Repainting every 3 or 4 years will suffice afterward. Ungalvanized iron (called BLACK IRON, for distinction) is also very enduring for roofs, if well painted every 1 or 2 years. The chief advantage of galvanized roofing is that it does not require painting so often as the black. The galvanizing adds about ⅛ of a lb per square foot of surface, or about ¼ lb per sq ft of sheet as coated on both sides; without regard to the thickness of the sheet. Paint for roofs should not have much *dryer*. See Painting.

The sulphurous fumes from coal are very corrosive of EITHER GALVANIZED OR BLACK IRON; as may be seen in shops, railroad bridges, or engine houses, roofed with either; if efficient means are not provided for carrying off the smoke; and the same with other metals. THE ACID OF OAK TIMBER is said to destroy the zinc of galvanized iron.

Flat iron is usually nailed upon a sheeting of boards; but the strength of corrugated iron obviates the necessity for this, and enables it to stretch 5 or 6 ft from purlin to purlin, without intermediate support. The corrugated sheets are riveted together on the roof, by rivets of galvanized wire about one-eighth inch thick, 300 to a pound, well driven (so as to exclude rain) 3 or 4 inches apart, all around the edges. The rivet-holes are first punched by machinery, so as to insure coincidence in the several sheets; and the rivets are driven by two men, one above, and one beneath the roof. For black iron, ungalvanized nails, boiled in linseed oil as a partial preservative from rust, are commonly used; as also in shingling or slating. Galvanized ones, however, would be better in all these cases; or even copper ones for slating because good slate endures much longer than either shingles or iron, and therefore it becomes true economy to use durable metals for fastening it. In none of these cases, however, are the nails fully exposed to the weather.

The sheets of flat iron are put together by overlapping and FOLDING THE EDGES, much the same as shown by the fig page 916, head Tin; the joints which run up and down the roof being the same as at *s a*, and the horizontal ones as at *t t*; except that inasmuch as these are not soldered in the iron sheets, the joint is made about ¾ to 1 inch wide, instead of ⅜ inch, the better to provide against leaking. Cleats are used as in tin, with 2 nails to a cleat. The iron plates are best laid on sheeting boards; but in sheds, &c, are sometimes laid directly on rafters, not more than about 18 ins apart in the clear; the plates being allowed to sag a little between the rafters, so as to form shallow gutters. In such cases it is well to bevel off the tops of the rafters slightly, as in this fig.



A serious objection to iron as a roof covering, is its rapid condensation of atmospheric moisture; which falls from the iron in drops like rain, and may do injury to ceilings, floors, or articles in the apartments immediately beneath the roof. Painting does not appreciably diminish this; it may, however, be obviated by plastering.

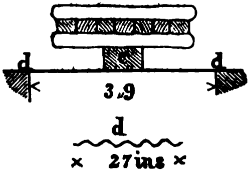
Corrugated sheet iron. The size of sheets generally used for corrugating, is 30 inches wide by 96 inches long. Corrugation reduces the width to 27½ inches. When the corrugated sheets are laid upon the roof, the overlapping of about 2½ inches along the sides, and of 4 inches along their ends, diminishes the area of roof covered by a sheet, to about seven-eighths of that of the entire corrugated sheet itself; or, the weight per square foot of roof covered, will be about one-seventh greater than that per square foot of the corrugated sheet; or, the weight of corrugated iron per square foot of roof covered is about one-fifth greater than that of the flat sheets from which it is made.

About 6 inches are usually allowed for the extension over the eaves.

The weights per square foot corresponding to the different numbers of the Birmingham wire gauge, vary somewhat with the different makers. The two styles of corrugation given in the table below, 5 x 1¼ and 2½ x ⅝, are those most frequently used.

No. Bmghm wire ga.	Thickness in ins.	Wt in lbs per sq ft of *sheets*.		Wt in lbs per sq ft of *roof*.	
Black	Black	Black	Lead cot'd or galv'd	Black	Lead cot'd or galv'd
20	.035	1.84	2.	2.12	2.3
22	.028	1.50	1.6	1.73	1.84
24	.022	1.20	1.25	1.38	1.44
26	.018	1.00	1.12	1.15	1.29

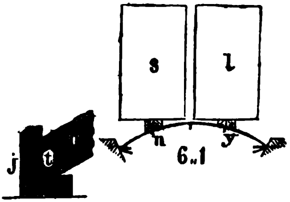
Strength of Corrugated Iron. Experiments by the author.



First. A sheet *d d*, of No. 16 iron, (about $\frac{1}{15}$ inch thick,) 27 ins wide, by 4 ft long, with five complete corrugations of 5 ins by 1 inch, was laid on supports 3 ft 9 ins apart. A block of wood c, 9 ins wide, by 7 ins thick, and 30 ins long, was placed across the center, and gradually loaded with castings weighing 1600 lbs.

This caused a deflection at the center of precisely ½ an inch. On the removal of the load after an hour, no permanent set was appreciable. The severity of the test was purposely increased by applying the several castings very roughly, jolting the whole as much as possible.* The suspended area of the sheet was 8.44 sq ft; and since the actual *center* load of 1600 lbs is about equivalent to 3000 lbs *equally distributed*, it amounts to $\frac{3000}{8.44}$=355 lbs per sq ft distributed. But 3000 lbs distributed would produce a deflection of but about full ¼ of an inch. Again, 355 lbs per sq ft is about 4 times the weight of the greatest crowd that could well congregate upon a floor. Consequently this iron, at 3' 9'' span, is safe in practice for any ordinary crowd. Moreover, such a crowd would produce a center deflection of only the ¼th part of ¼ of an inch; or $\frac{1}{16}$ of an inch; or $\frac{1}{720}$ of the clear span; which is but two-thirds of Tredgold's limit of $\frac{1}{480}$ of the span.

In one experiment the ends of the sheets rested upon supports dressed so as to present undulations corresponding tolerably closely with the shape of the corrugations; but in the other the supports were flat, and each end of the sheet rested only upon the lower points of the corrugations. No appreciable difference was observed in the results.



Second. An arch of No. 18 ($\frac{1}{20}$ inch) **iron,** corrugated like the foregoing, but the depth of corrugation increased to 1¼ ins by the process of arching the sheet; clear span 6 ft 1 inch; rise 10 ins; breadth 27 ins, (of which, however, only 25 ins bore against the abutments.)

Each foot *o* of the arch abutted upon a casting *j*, the inner portion *t* of which was undulated on top, to correspond with the corrugations of the arch, which rested upon it. At *y*, (one-fourth of the span,) two wooden blocks were placed, occupying a width of 9 inches, and extending across the arch; on them was piled a load, *l*, of castings, to the extent of 4480 lbs, or 2 tons. Under this load the arch descended about half an inch at *y*, becoming flatter on that side and slightly more curved upward along the unloaded side *n*. Two similar blocks were then placed at *n*, and two tons of load, *s*, were piled upon them, in addition to the 2 tons at *l*; making a total of 8960 lbs, or 4 tons. This brought the arch more nearly back to its original shape; but still slightly straightened at both *n* and *y*, and a little more curved in the center. The load was then increased to 10000 lbs, and left standing for several days. Two iron ties, each ½ by 1¾, which were used for preventing the abutment castings *j* from spreading, were found to have stretched nearly ¼ of an inch. Additional ones were inserted, and the load increased to a total of 6 tons, or 13440 lbs; parts of it on *s* and *l*, and part in the shape of long broad bars of iron at the center of the arch, below the loads *s* and *l*, and between *n* and *y*. So far as could be judged by eye, the shape of the arch was now almost perfect. *The loads* s *and* l *did not touch each other.* After standing more than a week, the load was accidentally overturned, crippling the arch. The load was equal to about 1000 lbs per sq ft of the arch. Such arches have since come into common use instead of brick, **for fireproof floors.**

Curved roofs of 25 to 30 ft span. rising about ¼ span, may be made of ordinary corrugated iron of Nos 16 to 13, riveted as usual; and having no accessories except tie-rods a few feet apart; continuous angle-iron skewbacks; and thin vertical rods to prevent the ties from sagging.

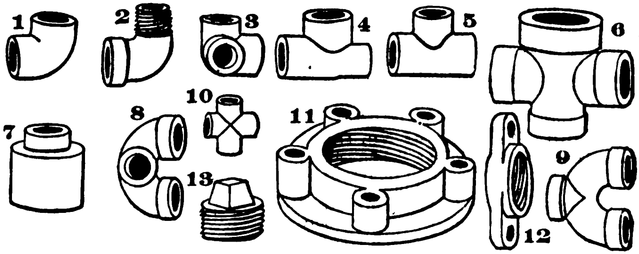
---

* Without letting the deflection exceed ½ inch; which was prevented by a stop under the sheet.

Welded wrought-iron pipes, for steam, gas, and water. Usually in lengths of about 18 feet. Standard sizes.

Inner Diam. Nominal.*	Inner Diam. Actual.	Thickness.	Wt. per foot. Nominal.	Threads per inch of screw.	List price per foot run.†	Inner Diam. Nominal.*	Inner Diam. Actual.	Thickness.	Wt. per foot. Nominal.	Threads per inch of screw.	List price per foot run.†
Ins.	Ins.	Ins.	lbs.		$	Ins.	Ins.	Ins.	lbs.		$
1/8	0.270	0.068	0.24	27	0.055	3½	3.548	0.226	9.00	8	0.95
1/4	0.364	0.088	0.42	18	"	4	4.026	0.237	10.66	"	1.08
3/8	0.494	0.091	0.56	"	"	4½	4.508	0.246	12.49	"	1.30
1/2	0.623	0.109	0.84	14	0.085	5	5.045	0.259	14.50	"	1.45
3/4	0.824	0.113	1.12	"	0.115	6	6.065	0.280	18.76	"	1.88
1	1.048	0.134	1.67	11.5	0.165	7	7.023	0.301	23.27	"	2.35
1¼	1.380	0.140	2.24	"	0.225	8	7.982	0.322	28.18	"	2.82
1½	1.611	0.145	2.68	"	0.27	9	9.001	0.344	33.70	"	3.40
2	2.067	0.154	3.61	"	0.36	10	10.019	0.366	40.00	"	4.25
2½	2.468	0.204	5.74	8	0.575	11	11.000	0.375	45.00	"	4.75
3	3.067	0.217	7.54	"	0.755	12	12.000	0.375	49.00	"	5.20

Fittings for Wrought-iron Pipes. 1, Elbow. 2, Service Elbow. 3, Elbow with side outlet. 4, Reducing T. 5, T. 6, Reducing Cross. 7, Reducing Coupling or Socket. 8, Return Bend with side outlet. 9, Return Bend with back outlet. 10, Cross. 11, Flange Union. 12, Oval Flange. 13, Plug.



Lap-welded charcoal-iron boiler tubes, in lengths up to 20 ft.

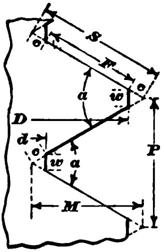
Outer Diam.*	Thickness.	Thickness.	Nom Wt. per ft.	Price per ft.†	Outer Diam.*	Thickness.	Thickness.	Nom Wt. per ft.	Price per ft.†	Outer Diam.*	Thickness.	Thickness.	Nom Wt. per ft.	Price per ft.†
Ins.	Ins.	B W G	lbs.	$	Ins.	Ins.	B W G	lbs.	$	Ins.	Ins.	B W G	lbs.	$
1	0.95	13	0.90	0.30	3	0.109	12	3.33	0.35	8	0.165	8	13.65	1.50
1¼	"	"	1.15	0.28	3¼	0.120	11	3.96	0.40	9	0.180	7	16.76	1.70
1½	"	"	1.40	0.27	3½	"	"	4.28	0.44	10	0.203	6	21.00	2.10
1¾	"	"	1.66	0.22	3¾	"	"	4.60	0.50	11	0.220	5	25.00	2.50
2	"	"	1.91	0.20	4	0.134	10	5.47	0.55	12	0.229	4½	28.50	2.90
2¼	"	"	2.16	0.24	4½	"	"	6.17	0.62	13	0.238	4	32.06	3.20
2½	0.109	12	2.75	0.28	5	0.148	9	7.58	0.75	14	0.248	3½	36.00	3.65
2¾	"	"	3.04	0.34	6	0.165	8	10.16	1.00	15	0.259	3	40.60	4.10
					7	"	"	11.90	1.20	16	0.270	2½	45.20	4.60

---

* For *pipes* give the "nominal" ***inner***, for ***boiler tubes*** the ***outer*** diam. See p 526.

† For discounts, see price list.

Screw Threads, Bolts, Nuts, and Washers.

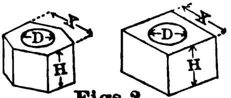


Screw threads. a = angle between two sides of a thread; P = pitch; w = width of flat top or bottom of each thread; all measured in a plane containing the axis of the screw; N = number of threads per inch, = 1/P. In the **Sellers or Franklin Institute Standard,** proposed by Mr. William Sellers and adopted by the Institute in 1864, $a = 60°$; $S = P$; $w = c = P/8$; $F = 0.75\ P$; $M = P \cos a/2 = 0.8660\ P$; D (diameter) $= d + 2 \times 0.866 \times 0.75\ P = d + 1.299\ P$. Under the name of **United States Standard,** the U. S. Navy Department in 1868 adopted the Sellers system, except for finished heads and nuts, which it made the same as for rough heads and nuts.

D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N
¼	.185	.0062	20	1	.837	.0156	8	2	1.712	.0277	4½	4	3.567	.0413	3
5-16	.240	.0074	18	1⅛	.940	.0178	7	2¼	1.962	.0277	4½	4¼	3.798	.0435	2⅞
⅜	.294	.0078	16	1¼	1.065	.0178	7	2½	2.176	.0312	4	4½	4.028	.0454	2¾
7-16	.344	.0089	14	1⅜	1.160	.0208	6	2¾	2.426	.0312	4	4¾	4.256	.0476	2⅝
½	.400	.0096	13	1½	1.284	.0208	6	3	2.629	.0357	3½	5	4.480	.0500	2½
9-16	.454	.0104	12	1⅝	1.389	.0227	5½	3¼	2.879	.0357	3½	5¼	4.730	.0500	2½
⅝	.507	.0113	11	1¾	1.491	.0250	5	3½	3.100	.0384	3¼	5½	4.953	.0526	2⅜
¾	.620	.0125	10	1⅞	1.616	.0250	5	3¾	3.317	.0413	3	5¾	5.203	.0526	2⅜
⅞	.731	.0138	9									6	5.423	.0555	2¼

Dimensions of Heads and Nuts.

	Rough.	**Finished.**
X	1½ D + ⅛ inch.	1½ D + 1-16 inch.
H (in *head*)	½ X.	D — 1-16 inch.
H (in *nut*)	D.	" "



Figs. 2

In the **Whitworth** (English) **standard** thread, the angle a, Fig 1, is 55°. The tops and bottoms of the threads are rounded, instead of flat as in the American standards. The number (N) of threads per inch is the same as above for diams of bolt up to three ins, except for D = ½ inch; where N = 12.

In the **International metric screw thread,** adopted at Zurich, October, 1898, the Sellers thread profile is used. The dimensions are as follows, all in millimeters:

Diam.	6	7	8	9	10	11	12	14	16	18	20	22	24	27	30	33	36	39	42	45	48	52	56	60	64	68	72	76	80
Pitch	1.0		1.25		1.5		1.75	2.0		2.5			3.0		3.5		4.0		4.5		5.0		5.5		6.0		6.5		7.0

Intermediate diameters are to be of an integral number of millimeters, and of the same pitch as the next smaller diameter in the table. Thus, for diam 65 or 69 mm; pitch = 6.0 mm.

Plate-iron washers. Standard sizes. Diameters of washers and bolt-holes in inches. Approximate thickness by Birmingham wire gauge. Approximate number in one ℔.

Diams.		Ths.	No.	Diams.		Ths.	No.	Diams.		Ths.	No.
½	¼	18	450	1¼	½	14	43	2¼	15-16	9	8.6
⅝	5-16	16	210	1⅜	9-16	12	26	2½	1 1-16	9	6.2
¾	5-16	16	139	1½	⅝	12	22.5	2¾	1 ¼	9	5.2
⅞	⅜	16	112	1¾	11-16	10	13.1	3	1 ⅜	9	4.
1	7-16	14	68	2	13-16	10	10.1	3½	1 ½	9	2.8

A *square* head and nut together, weigh about as much as a length of the bolt equal to 7 or 8 times D. *Hexagon*, 6 or 7.

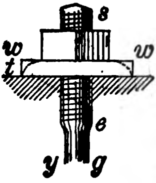


FIG 3.

With the above dimensions a bolt will generally fail by breaking off between the head and the nut, where the diameter is decreased by cutting the thread, rather than by stripping off its threads.

The diam D of the thread must of course be greater than that required to bear safely the proposed tensile strain, by an amount equal to *twice* the depth of the thread. The waste of iron, which would result from making the *entire bolt* of this greater diam, is frequently avoided by making the bolt from a bar of only sufficient dimensions to bear the strain safely, **and upsetting its ends** as in Fig 3, thus increasing their diam sufficiently to allow for the cutting of the threads.

In carpentry, as well as in ties for masonry, *washers*, *w w*, of either cast or wrought iron, are placed between the timber, or stone, and the head and nut; in order to distribute the pressure over a greater surface, and thus prevent crushing; especially in timber.

When much strained against wood, the side of a square wrought-iron washer; or the diam *w w* of a circular one, should not be less than 4 diams of the *screw*, as in the fig; and its thickness, *t w*, ½ diam *at least*.

Two such square washers will together weigh as much as 18 diams in length of a round rod of the same diam as the screw. Two round washers will weigh together as much as 14 diams of rod of same diam as screw. In either case, a square head and nut will weigh as much as 6 diameters. Cast-iron washers, being more apt to split under heavy strains, may be made about twice as thick as wrought ones. When the strain is very great, the diam of the washer may be 5 or 6 times that of the screw; and its thickness equal to diam; but 4 diams will suffice for most practical purposes, or even 2.5 when there is but little strain, and the thickness may then be but .1 or .2 diam of bolt.

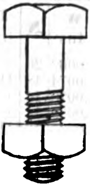


Fig. 4.

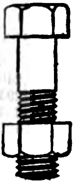
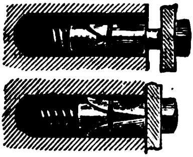


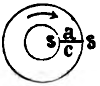
Fig. 5.

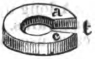
Table of machine and car bolts, with square and hexagon heads and nuts, Figs 4 and 5; made by Hoopes & Townsend, 1330 Buttonwood St, Phila. All their **bolts are cut with U. S. Standard threads,** as per first table on p 883, unless otherwise ordered. Discounts, see price list.

Diam D, Fig 1, of Bolt, ins.	Length, ins exclusive of head.		Weight, lbs of 100 bolts.		List price, $ per 100.	
	Min.	Max.	Min.	Max.	Min.	Max.
¼	1½	8	3.9	13.2	1.70	2.74
5-16	"	"	6.2	20.3	2.00	3.56
⅜	"	12	9.7	43.5	2.40	5.76
7-16	"	"	14.7	58.3	2.80	7.00
½	"	20	20.4	122.0	3.60	13.22
9-16	"	"	26	151.0	5.20	19.26
⅝	"	24	37	224.0	"	22.80
¾	"	"	58	330.0	7.20	29.70
⅞	2	"	97.7	470.0	11.20	42.00
1	"	"	145.0	625.0	16.00	55.60



Expansion bolts, for fastening plates, timbers, etc., to walls of brick or masonry. The wedge-shaped nut, traveling up the bolt, as the latter is turned, presses the wings against the sides of the hole, which, in practice, is drilled just large enough to admit the nut and wings, so as to prevent the former from turning with the bolt. If the hole is made larger, as shown, the nut must be held by a small wedge.





Lock-nut washers. When bolts are subjected to much rough jolting, as at rail-joints, &c, the *nuts are liable to wear loose, and unscrew themselves.* On railroads this is a source of great annoyance, and innumerable devices for preventing it have been tried. The **Verona** lock-nut washer* is a simple circular washer made of steel; with a slit *s s* cut through it, leaving sharp edges. On one side, *a*, of the slit, the metal is pressed upward about ⅛ inch; and that on the other side, *c*, downward, the same distance; so that a perspective view would be somewhat as at *t*. Now, when the nut is screwed down over the washer, in the direction of the arrow, the slit offers no obstruction; but if the nut afterward tends to unscrew itself, the sharp upper edge of the slit, along *a*, presents friction against the bottom of the nut, which tends to hold it in place. Besides, the washer, by its elasticity, tends to resume its original shape, and thus presses the threads of the nut against those of the bolt; and the additional friction thus produced also aids in holding the nut.

Another lock-nut washer consists of a long strip of steel, with *two* holes, each of which has its edges formed like those of a Verona washer, and through *each* of which passes one of the bolts of the rail-joint.

Another device is to cut, at the end of the screw, a few threads of a screw of less diameter than the main one, and in the opposite direction. The nut is then screwed upon the larger diameter; and after it the lock-nut is screwed in the other direction upon the smaller diam, until it comes into contact with the main nut. In the **Smith** lock-nut bolt, this second nut is only about ⅛ inch thick; and after being driven home, one of its corners is bent over the edge of the main nut.

The **Atwood** lock-nuts take advantage of elasticity in the nut itself, which is obtained either by slitting the nut, or by reducing its thickness near the bolt hole.

It is claimed that if the threads of an ordinary bolt and nut are carefully cut, so as to be in contact with each other throughout, no lock-nut contrivance is necessary, because the friction between the two threads is distributed over a larger surface, and *abrasion* does not take place so readily as if the threads touched each other at only a few points. The nuts are therefore less apt to wear loose under repeated jarring.

Owing to the difficulty of obtaining such perfect fitting bolts and nuts, due to the wear of the cutting tools used in their manufacture, bolts and nuts have been made in which the thread on the bolt differs slightly in shape from that in the nut. They also furnish nuts in which the thread, instead of being of uniform shape throughout, gradually becomes *deeper* and *thicker*, by having its side angle made more acute, and its top truncated. These nuts are used with bolts having the usual uniform thread. The bolt enters the nut upon the side where the thread is of the same shape as its own; but its thread encounters, and is forced into, the gradually narrowing and deepening path between the threads of the nut. In both devices, the enforced conformity between the two threads is relied upon to give the desired completeness of contact between them. The greater force required in screwing on the nut also increases the *friction* between the threads.

BUCKLED PLATES.

Buckled plates are usually of steel, ¼ to ½ in thick and 3 to 4 ft sq; sometimes in long plates having several buckles each. Buckle 2 to 3 ins. Flat rim or fillet, 2 to 4 ins. They are used for the floors of buildings and of highway bridges.

Total permissible load, lbs. on a single square buckled plate of any size and thickness.† Load = $4\,k\,t\,h$; where k = permissible unit stress in metal, lbs per sq in, say 6000; t = thickness of metal, ins, and h = depth of buckle, ins.

Buckled plates are stronger, and require less concrete, etc, for filling, when laid with convex side down. They weigh but little more than flat plates, or about 10 lbs per sq ft per ¼ in of thickness.

*** Invented by Mr. Thomas Shaw, M. E., of Philadelphia.**

† "Steel in Construction," by Pencoyd Iron Works, Philadelphia, 1900, p 147.

WEIGHT AND STRENGTH OF IRON BOLTS. (Original.)

Diameters, weights, and approximate breaking strains, **for round bolts**; breaking strain per square inch assumed as follows: Up to 1 inch square, or 1 inch diam, 20 tons, or 44800 lbs; from 1 to 2 ins sq or diam, 19 tons; 2 to 3 ins, 18 tons; 3 to 4 ins, 17 tons; 4 to 5 ins, 16 tons; 5 to 6 ins, 15 tons.

A long upset rod is no stronger than one not upset, against *slowly applied* loads or strains. Both will then break at about midlength, under equal pulls. In such cases use columns 5 and 6.

Square bars. Strength or wt = 1.273 × strength or weight of round bar.

Copper bars. { Strength = 0.8 × strength of similar iron bar.
{ Weight = 1.14 × weight " " " "

Ends enlarged, or upset.				Ends not enlarged.		Ends enlarged, or upset.				Ends not enlarged.	
Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.	Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.
Ins.	Pds.	Tons.	Pds.	Ins.	Pds.	Ins.	Pds.	Tons.	Pds.	Ins.	Pds.
1/8	.0414	.245	549			1 3/4	8.10	45.7	102368	2.14	12.0
3/16	.093	.553	1239			1 13/16	8.69	49.0	109760	2.22	12.9
1/4	.165	.983	2202	.35	.321	1 7/8	9.30	52.5	117600	2.30	13.8
5/16	.258	1.53	3427	.43	.452	1 15/16	9.93	56.0	125440	2.38	14.7
3/8	.372	2.21	4950	.50	.654	2.	10.6	59.7	133728	2.45	15.7
7/16	.506	3.00	6720	.58	.897	2 1/8	12.0	63.8	142912	2.59	17.5
1/2	.661	3.93	8803	.66	1.14	2 1/4	13.4	71.6	160384	2.73	19.5
9/16	.837	4.97	11133	.73	1.41	2 3/8	14.9	79.7	178528	2.88	21.6
5/8	1.03	6.14	13754	.80	1.67	2 1/2	16.5	88.4	198016	3.02	23.9
11/16	1.25	7.42	16621	.88	2.03	2 5/8	18.2	97.4	218176	3.16	26.1
3/4	1.49	8.83	19779	.96	2.41	2 3/4	20.0	106.9	239456	3.30	28.5
13/16	1.75	10.4	23296	1.04	2.81	2 7/8	21.9	116.8	261632	3.45	31.1
7/8	2.03	12.0	26880	1.12	3.26	3.	23.8	127.2	284928	3.60	33.9
15/16	2.33	13.8	30912	1.20	3.77	3 1/4	27.9	141.0	315840	3.86	39.1
1 in.	2.65	15.7	35168	1.27	4.27	3 1/2	32.4	163.6	366464	4.12	44.4
1 1/16	2.99	16.8	37632	1.35	4.77	3 3/4	37.2	187.7	420448	4.41	51.0
1 1/8	3.35	18.9	42336	1.42	5.28	4.	42.3	213.6	478464	4.70	57.8
1 3/16	3.73	21.1	47264	1.49	5.81	4 1/4	47.8	227.0	508480	4.98	65.2
1 1/4	4.13	23.3	52192	1.55	6.39	4 1/2	53.6	254.5	570080	5.25	72.9
1 5/16	4.56	25.7	57568	1.64	7.04	4 3/4	59.7	283.5	635040	5.53	80.5
1 3/8	5.00	28.2	63168	1.72	7.74	5.	66.1	314.2	703808	5.80	88.1
1 7/16	5.47	30.8	68992	1.80	8.48	5 1/4	72.9	324.7	727328	6.08	97.0
1 1/2	5.95	33.6	75264	1.87	9.20	5 1/2	80.0	356.4	798336	6.36	106.
1 9/16	6.46	36.4	81536	1.94	9.88	5 3/4	87.5	389.5	872480	6.63	116.
1 5/8	6.99	39.4	88256	2.00	10.6	6.	95.2	424.1	949984	6.90	126.
1 11/16	7.53	42.5	95200	2.07	11.3						

The Birmingham **wire gauge** is the one in most general use for iron. The new British w g went into effect March 1st 1884. In the "American" w g of Darling, Brown & Sharpe, Providence R. I., each diam or thick is = the next smaller one × 1.122932. We take the wt of wrot iron per cub ft at 485 lbs in the first two; and at 486 in the last. For the **wt of steel,** mult that of iron by 1.01. For **lead,** mult iron by 1.46. For **zinc,** mult iron by .9. For **brass** (approx), mult iron by 1.06. For **copper,** mult iron by 1.134.

No.	**Birmingham** W. Ga.			**New British** W. Ga.			**American** W. Ga.		
	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft
7–0	....	....	....	.500	.661	20.21			
6–0	....	....	....	.464	.569	18.75			
5–0	....	....	....	.432	.494	17.46			
4–0	.454	.546	18.35	.400	.423	16.17	.460000	.561	18.63
3–0	.425	.479	17.18	.372	.366	15.03	.409642	.445	16.58
2–0	.380	.383	15.36	.348	.320	14.06	.364796	.353	14.77
0	.340	.306	13.74	.324	.278	13.09	.324861	.280	13.15
1	.300	.238	12.13	.300	.238	12.13	.289297	.222	11.70
2	.284	.214	11.48	.276	.202	11.15	.257627	.176	10.43
3	.259	.178	10.47	.252	.168	10.19	.229423	.139	9.291
4	.238	.150	9.619	.232	.142	9.377	.204307	.111	8.273
5	.220	.128	8.892	.212	.119	8.568	.181940	.0877	7.366
6	.203	.109	8.205	.192	.0976	7.760	.162023	.0696	6.561
7	.180	.0859	7.275	.176	.0820	7.113	.144285	.0552	5.842
8	.165	.0721	6.669	.160	.0677	6.466	.128490	.0438	5.203
9	.148	.0580	5.981	.144	.0548	5.820	.114423	.0347	4.633
10	.134	.0476	5.416	.128	.0434	5.173	.101897	.0275	4.125
11	.120	.0382	4.850	.116	.0357	4.688	.090742	.0218	3.674
12	.109	.0315	4.405	.104	.0286	4.203	.080808	.0173	3.272
13	.095	.0239	3.840	.092	.0224	3.718	.071962	.0137	2.914
14	.083	.0183	3.355	.080	.0169	3.233	.064084	.0109	2.595
15	.072	.0137	2.910	.072	.0137	2.910	.057068	.00863	2.310
16	.065	.0112	2.627	.064	.0108	2.587	.050821	.00684	2.053
17	.058	.00891	2.344	.056	.00832	2.263	.045257	.00543	1.832
18	.049	.00636	1.980	.048	.00610	1.940	.040303	.00430	1.631
19	.042	.00467	1.697	.040	.00423	1.617	.035890	.00341	1.452
20	.035	.00325	1.415	.036	.00344	1.455	.031961	.00271	1.293
21	.032	.00271	1.293	.032	.00269	1.293	.028462	.00215	1.152
22	.028	.00208	1.132	.028	.00207	1.132	.025346	.00170	1.026
23	.025	.00166	1.010	.024	.00152	.9700	.022572	.00135	.913
24	.022	.00128	.8892	.022	.00128	.8891	.020101	.00107	.814
25	.020	.00106	.8083	.020	.00106	.8083	.017900	.000849	.724
26	.018	.000859	.7225	.018	.000857	.7275	.015941	.000673	.644
27	.016	.000678	.6467	.0164	.000712	.6628	.014195	.000534	.574
28	.014	.000519	.5658	.0148	.000579	.5982	.012641	.000423	.511
29	.013	.000448	.5254	.0136	.000489	.5497	.011257	.000336	.455
30	.012	.000382	.4850	.0124	.000408	.5012	.010025	.000266	.405
31	.010	.000265	.4042	.0116	.000357	.4688	.008928	.000211	.360
32	.009	.000215	.3638	.0108	.000309	.4365	.007950	.000167	.321
33	.008	.000170	.3233	.0100	.000265	.4042	.007080	.000133	.286
34	.007	.000130	.2829	.0092	.000224	.3718	.006305	.000105	.254
35	.005	.0000662	.2021	.0084	.000187	.3395	.005615	.0000837	.226
36	.004	.0000424	.1617	.0076	.000153	.3072	.005000	.0000662	.202
37	....	....	....	.0068	.000122	.2748	.004453	.0000525	.180
38	....	....	....	.0060	.0000952	.2425	.003965	.0000417	.159
39	....	....	....	.0052	.0000714	.2102	.003531	.0000330	.142
40	....	....	....	.0048	.0000608	.1940	.003144	.0000262	.127
41	....	....	....	.0044	.0000513	.1778			
42	....	....	....	.0040	.0000423	.1617			
43	....	....	....	.0036	.0000344	.1455			
44	....	....	....	.0032	.0000271	.1293			
45	....	....	....	.0028	.0000207	.1132			
46	....	....	....	.0024	.0000152	.0970			
47	....	....	....	.0020	.0000106	.0808			
48	....	....	....	.0016	.0000068	.0647			

American gauge for sheet and plate iron and steel (**1893**). We omit the columns of weight in kilograms per square foot and in pounds per square meter, and simplify the headings of the remaining columns.

An Act establishing a standard gauge for sheet and plate iron and steel.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That for the purpose of securing uniformity the following is established as the only standard gauge for sheet and plate iron and steel in the United States of America, namely:

No.	Approximate thickness.		Weight.		
	Inches.	Millimeters.	Per sq. foot, in avoirdupois ounces.	Per sq. foot, in avoirdupois pounds.	Per sq. meter, in kilograms.
7-0	1-2 =.5	12.7	320	20.00	97.65
6-0	15-32 =.46875	11.90625	300	18.75	91.55
5-0	7-16 =.4375	11.1125	280	17.50	85.44
4-0	13-32 =.40625	10.31875	260	16.25	79.33
3-0	3-8 =.375	9.525	240	15.	73.24
2-0	11-32 =.34375	8.73125	220	13.75	67.13
0	5-16 =.3125	7.9375	200	12.50	61.03
1	9-32 =.28125	7.14375	180	11.25	54.93
2	17-64 =.265625	6.746875	170	10.625	51.88
3	1-4 =.25	6.35	160	10.	48.82
4	15-64 =.234375	5.953125	150	9.375	45.77
5	7-32 =.21875	5.55625	140	8.75	42.72
6	13-64 =.203125	5.159375	130	8.125	39.67
7	3-16 =.1875	4.7625	120	7.5	36.62
8	11-64 =.171875	4.365625	110	6.875	33.57
9	5-32 =.15625	3.96875	100	6.25	30.52
10	9-64 =.140625	3.571875	90	5.625	27.46
11	1-8 =.125	3.175	80	5.	24.41
12	7-64 =.109375	2.778125	70	4.375	21.36
13	3-32 =.09375	2.38125	60	3.75	18.31
14	5-64 =.078125	1.984375	50	3.125	15.26
15	9-128 =.0703125	1.7859375	45	2.8125	13.73
16	1-16 =.0625	1.5875	40	2.5	12.21
17	9-160 =.05625	1.42875	36	2.25	10.99
18	1-20 =.05	1.27	32	2.	9.765
19	7-160 =.04375	1.11125	28	1.75	8.544
20	3-80 =.0375	.9525	24	1.50	7.324
21	11-320 =.034375	.873125	22	1.375	6.713
22	1-32 =.03125	.793750	20	1.25	6.103
23	9-320 =.028125	.714375	18	1.125	5.493
24	1-40 =.025	.635	16	1.	4.882
25	7-320 =.021875	.555625	14	.875	4.272
26	3-160 =.01875	.47625	12	.75	3.662
27	11-640 =.0171875	.4365625	11	.6875	3.357
28	1-64 =.015625	.396875	10	.625	3.052
29	9-640 =.0140625	.3571875	9	.5625	2.746
30	1-80 =.0125	.3175	8	.5	2.441
31	7-640 =.0109375	.2778125	7	.4375	2.136
32	13-1280=.01015625	.25796875	6½	.40625	1.983
33	3-320 =.009375	.238125	6	.375	1.831
34	11-1280=.00859375	.21828125	5½	.34375	1.678
35	5-640 =.0078125	.1984375	5	.3125	1.526
36	9-1280=.00703125	.17859375	4½	.28125	1.373
37	17-2560=.006640625	.168671875	4¼	.265625	1.297
38	1-160 =.00625	.15875	4	.25	1.221

And on and after July first, eighteen hundred and ninety-three, the same and no other shall be used in determining duties and taxes levied by the United States of America on sheet and plate iron and steel. But this act shall not be construed to increase duties upon any articles which may be imported.

Sec. 2. That the Secretary of the Treasury is **authorized and required to prepare suitable standards in accordance herewith.**

Sec. 3. That in the **practical use and application of the standard gauge hereby established a variation of two and one-half per cent. either way may be allowed.**

Approved March 3, 1893.

CIRCULAR MEASURE.

Used in comparing cross sections of wires, etc.

A circular unit is the area of a circle whose diameter is one linear unit. Thus, a circular inch is the area (=0.7854 square inch) of a circle whose diameter is one inch.

The following table is adapted, by permission, from Mr. Carl Hering's valuable Tables of Equivalents of Units of Measurement, New York, 1888. Inasmuch as we take 1 meter = 39.37 inches, instead of 39.37079 inches, our values differ slightly from his.

			Logarithm.
1 ○ mil *	=	0.78540 □ mil *	$\bar{1}$.895 0899
	=	0.00064516 ○ millimeter	$\bar{4}$.809 6692
	=	0.00050671 □ millimeter	$\bar{4}$.704 7591
1 □ mil *	=	1.2732 ○ mils *	0.104 9101
	=	0.00082145 ○ millimeter	$\bar{4}$.914 5793
1 ○ millimeter	=	1550.0 ○ mils *	3.190 3308
	=	1217.4 □ mils *	3.085 4207
	=	0.78540 □ millimeter	$\bar{1}$.895 0899
1 □ millimeter	=	1973.5 ○ mils *	3.295 2409
	=	1.2732 ○ millimeters	0.104 9101

EDISON STANDARD WIRE GAUGE.

Adopted by the Associated Edison Illuminating Companies.

In this table the gauge number is approximately equal to

$\frac{1}{1000}$ × area of cross section in circular mils *

= $\frac{1}{1000}$ × square of diameter in mils. *

No.	Diameter, in mils.	No.	Diameter, in mils.	No.	Diameter, in mils.
3	54.78	65	254.96	160	400.00
5	70.72	70	264.58	170	412.32
8	89.45	75	273.87	180	424.27
12	109.55	80	282.85	190	435.89
15	122.48	85	291.55	200	447.22
20	141.43	90	300.00	220	469.05
25	158.12	95	308.23	240	489.90
30	173.21	100	316.23	260	509.91
35	187.09	110	331.67	280	529.16
40	200.00	120	346.42	300	547.73
45	212.14	130	360.56	320	565.69
50	223.61	140	374.17	340	583.10
55	234.52	150	387.30	360	600.00
60	244.95				

* 1 mil = $\frac{1}{1000}$ inch.

No trade Stupidity is more thoroughly senseless than the adherence to the various Birmingham, Lancashire, &c, gauges; instead of at once denoting the thickness and diameter of sheets, wire, &c, by the parts of an inch; as has long been suggested. Thus, No. 1/8, or No. 1/20 wire, or sheet-metal of any kind, should be understood to mean 1/8 or 1/20 of an inch diam, or thickness. To avoid mistakes, which are very apt to occur from the number of gauges in use; and from the absurd practice of applying the same No. to different thicknesses of different metals, in different towns, it is best to ignore them all; and in giving orders, to define the diameter of wire, and the thickness of sheet-metal, by parts of an inch. Or the weight per hundred ft for wire; or per sq ft for sheets, may be employed. We *believe* that the *foregoing* Birmingham gauge applies to zinc, copper, brass, and lead; although it is generally stated to be for iron and steel only. Another Birmingham gauge is used for sheet-brass, gold, silver, and some other metals; but we have never seen it stated what those others are. There are different gauges even for wire to be used for different *purposes;* and various firms have gauges of their own; not even according among themselves.

As Mr. Stubs makes various English gauges, the term "**Stubs gauge**" *by itself* means nothing. Generally, however, in our machine shops, it applies to the Birmingham gauge of the preceding table.

Birmingham gauge for sheet Brass, Silver, Gold, and all metals except iron and steel?

No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.
	Inch		Inch		Inch		Inch		Inch		Inch
1	.004	7	.015	13	.036	19	.064	25	.095	31	.135
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.012	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

The mills rolling sheet iron in the United States generally use the following, which varies slightly from the Birmingham gauge:

No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft
1	12.50	8	6.86	15	2.81	22	1.25
2	12.00	9	6.24	16	2.50	23	1.12
3	11.00	10	5.62	17	2.18	24	1.00
4	10.00	11	5.00	18	1.86	25	.90
5	8.75	12	4.38	19	1.70	26	.80
6	8.12	13	3.75	20	1.54	27	.72
7	7.50	14	3.12	21	1.40	28	.64

When wire, sheet-metal, &c., are ordered by gauge number, and it is not specified what gauge is intended; dealers in the United States fill the order as follows:

Brass, bronze or German Silver in sheets, German Silver wire, brazed brass, bronze, zinc or copper tubing, by Brown & Sharpe's (or "American") gauge.

Copper in sheets; brass and copper wire; seamless brass, bronze or copper tubing; and small brass rods; by Stubs' (or Birmingham) gauge.

Unannealed or hard **brass wire** has about 3/4ths the strengths of the table p. 891, and about 1/9 more weight. If annealed, only full half the strength.

Hard copper wire may be taken at 2/3 of the tabular strengths, and full 1 more weight.

Table of Charcoal Iron Wire made by Trenton Iron Co., Trenton, N. J. The numbers in the first column are those of the **Trenton Iron Co's gauge.** The corresponding diameters in the second column will be seen to be somewhat less than those of the Birmingham gauge.

No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.
00000	.450	1.863	12598	11	.1175	27.340	1010	26	.018	1164.689
0000	.400	2.358	9955	12	.105	34.219	810	27	.017	1305.670
000	.360	2.911	8124	13	.0925	44.092	631	28	.016	1476.869
00	.330	3.465	6880	14	.080	58.916	474	29	.015	1676.989
0	.305	4.057	5926	15	.070	76.984	372	30	.014	1925.321
1	.285	4.645	5226	16	.061	101.488	292	31	.013	2232.653
2	.265	5.374	4570	17	.0525	137.174	222	32	.012	2620.607
3	.245	6.286	3948	18	.045	186.335	169	33	.011	3119.092
4	.225	7.454	3374	19	.040	235.084	137	34	.010	3773.584
5	.205	8.976	2839	20	.035	308.079	107	35	.0095	4182.508
6	.190	10.453	2476	21	.031	392.772	......	36	.009	4657.728
7	.175	12.322	2136	22	.028	481.234	......	37	.0085	5222.085
8	.160	14.736	1813	23	.025	603.863	......	38	.008	5896.147
9	.145	17.950	1307	24	.0225	745.710	......	39	.0075	6724.291
10	.130	22.333	1233	25	.020	943.396	......	40	.007	7698.253

The wire in this table is supposed to be hard, bright, or unannealed. The figures in the column of tensile strength are based upon tests made with good charcoal iron wire from Trenton blooms.

The tensile strength of wire made of	is about		
Good refined iron	15	per cent. less	than that of bright charcoal wire, given in the above table.
Swedish charcoal iron	10	“ “	
Mild Bessemer steel	10	“ more	
Ordinary crucible steel	25	“ “	
Special crucible steel	30 to 120	“ “	

Annealing renders wire more pliable and ductile, but less elastic; and reduces the tensile strength by from 20 to 25 per cent.

To find approximately the number of straight wires that can be got into a cable of given diameter.

Divide the diameter of the cable in inches, by the diameter of a wire in inches. Square the quotient. Multiply said square by the decimal .77. The result will be correct within about 4 or 5 per cent at most, in a *cylindrical* cable.

The solidity, or metal area of all the wires in a cable, will be to the area of the cable itself, about as 1 to 1.3. In other words, the area of the voids is nearly ¼ that of the cable; while that of the wires is fully ¾ that of the cable. All approximate.

CARNEGIE

L = span in ft

W = uniformly distributed safe load in lbs = $\frac{C}{L}$

M = moment of load, in ft-lbs = $\frac{WL}{8} = \frac{C}{8} = \frac{SX}{12}$

S = stress in extreme fibres, in lbs per sq in = $\frac{M}{X}$

I, i = moment of inertia; **I**, about XY; **i**, about A B

R, r = radius of gyration; **R**, " " **r**, " "

X = "section modulus" " " $X = \frac{12\,M}{S}$

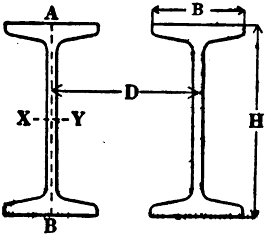
C = coefficient for uniformly distributed safe load = WL = 8 M = $\frac{8\,SX}{12}$

C_s, for static loads; S = 16,000 lbs. C_m, for moving loads; S = 12,500 lbs.

D = distance required to make $r = R$

Section index.	H. Depth ins.	Weight per ft lbs.	Area of section sq in.	Web thickness ins.	B. Flange width ins.
B 1	24	100.00	29.41	0.754	7.254
"	"	80.00	23.32	0.500	7.000
B 2	20	100.00	29.41	0.884	7.284
"	"	80.00	23.73	0.600	7.000
B 3	20	75.00	22.06	0.649	6.399
"	"	65.00	19.08	0.500	6.250
B 80	18	70.00	20.59	0.719	6.259
"	"	55.00	15.93	0.460	6.000
B 4	15	100.00	29.41	1.184	6.774
"	"	80.00	23.81	0.810	6.400
B 5	15	75.00	22.06	0.882	6.292
"	"	60.00	17.67	0.590	6.000
B 7	15	55.00	16.18	0.656	5.746
"	"	42.00	12.48	0.410	5.500
B 8	12	55.00	16.18	0.822	5.612
"	"	40.00	11.84	0.460	5.250
B 9	12	35.00	10.29	0.436	5.086
"	"	31.50	9.26	0.350	5.000
B 11	10	40.00	11.76	0.749	5.099
"	"	25.00	7.37	0.310	4.660
B 13	9	35.00	10.29	0.732	4.772
"	"	21.00	6.31	0.290	4.330
B 15	8	25.50	7.50	0.541	4.271
"	"	18.00	5.33	0.270	4.000
B 17	7	20.00	5.88	0.458	3.868
"	"	15.00	4.42	0.250	3.660
B 19	6	17.25	5.07	0.475	3.575
"	"	12.25	3.61	0.230	3.330
B 21	5	14.75	4.34	0.504	3.294
"	"	9.75	2.87	0.210	3.000
B 23	4	10.50	3.09	0.410	2.880
"	"	7.50	2.21	0.190	2.660
B 77	3	7.50	2.21	0.361	2.521
"	"	5.50	1.63	0.170	2.330

I-BEAMS.



The table gives the maximum and the minimum weight of each section. The **minimum weights** are standard. Others are special.

Caution.—With very short spans, the loads found by means of columns C_s and C_m, although safe against *bending*, may be so great as to endanger a *crushing* of the ends of the beam, or of the walls, etc., under them, unless the beam has, at its ends, a greater length of bearing than would otherwise be needed.

I	i	R ins.	r ins.	X	C_s lbs.	C_m lbs.	D ins.	Section index.
2380.3	48.56	9.00	1.28	198.4	2,115,800	1,653,000	17.82	B 1
2087.9	42.86	9.46	1.36	174.0	1,855,900	1,449,900	18.72	"
1655.8	52.65	7.50	1.34	165.6	1,766,100	1,379,800	14.76	B 2
1466.5	45.81	7.86	1.39	146.7	1,564,300	1,222,100	15.47	"
1268.9	30.25	7.58	1.17	126.9	1,353,500	1,057,400	14.98	B 3
1169.6	27.86	7.83	1.21	117.0	1,247,600	974,700	15.47	"
921.3	24.62	6.69	1.09	102.4	1,091,900	853,000	13.20	B 80
795.6	21.19	7.07	1.15	88.4	943,000	736,700	13.95	"
900.5	50.98	5.53	1.31	120.1	1,280,700	1,000,600	10.75	B 4
795.5	41.76	5.78	1.32	106.1	1,131,300	883,900	11.25	"
691.2	30.68	5.60	1.18	92.2	983,000	768,000	10.95	B 5
609.0	25.96	5.87	1.21	81.2	866,100	676,600	11.49	"
511.0	17.06	5.62	1.02	68.1	726,800	567,800	11.05	B 7
441.7	14.62	5.95	1.08	58.9	628,300	490,800	11.70	"
321.0	17.46	4.45	1.04	53.5	570,600	445,800	8.65	B 8
268.9	13.81	4.77	1.08	44.8	478,100	373,500	9.29	"
228.3	10.07	4.71	0.99	38.0	405.800	317,000	9.21	B 9
215.8	9.50	4.83	1.01	36.0	383,700	299,700	9.45	"
158.7	9.50	3.67	0.90	31.7	333,500	264,500	7.12	B 11
122.1	6.89	4.07	0.97	24.4	260,500	203,500	7.91	"
111.8	7.31	3.29	0.84	24.8	265,000	207,000	6.36	B 13
84.9	5.16	3.67	0.90	18.9	201,300	157,300	7.12	"
68.4	4.75	3.02	0.80	17.1	182,500	142,600	5.82	B 15
56.9	3.78	3.27	0.84	14.2	151,700	118,500	6.32	"
42.2	3.24	2.68	0.74	12.1	128,600	100,400	5.15	B 17
36.2	2.67	2.86	0.78	10.4	110,400	86,300	5.50	"
26.2	2.36	2.27	0.68	8.7	93,100	72,800	4.83	B 19
21.8	1.85	2.46	0.72	7.3	77,500	60,500	4.70	"
15.2	1.70	1.87	0.63	6.1	64,600	50,500	....	B 21
12.1	1.23	2.05	0.65	4.8	51,600	40,300	....	"
7.1	1.01	1.52	0.57	3.6	38,100	29,800	....	B 23
6.0	0.77	1.64	0.59	3.0	31,800	24,900	....	"
2.9	0.60	1.15	0.52	1.9	20,700	16,200	....	B 77
2.5	0.46	1.23	0.53	1.7	17,600	13,800	....	"

CARNEGIE

L = span in ft

W = uniformly distributed safe load in lbs = $\frac{C}{L}$

M = moment of load, in ft-lbs = $\frac{WL}{8} = \frac{C}{8} = \frac{SX}{12}$

S = stress in extreme fibres, in lbs per sq in = $\frac{M}{X}$

I, i = moment of inertia; **I**, about XY; **i**, about A B

R, r = radius of gyration; **R**, " " **r**, " "

X = "section modulus" " " $X = \frac{12\,M}{S}$

C = coefficient for uniformly distributed safe load = WL = 8 M = $\frac{8\,SX}{12}$

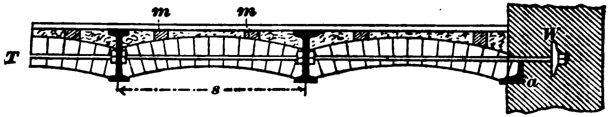
C_s, for static loads; S = 16,000 lbs. C_m, for moving loads; S = 12,500 lbs.

D = distance required to make $r = R$

Section index.	H. Depth ins.	Weight per ft lbs.	Area of section sq in.	Web thickness ins.	B. Flange width ins.
C 1	15	55.00	16.18	0.818	3.818
"	"	33.00	9.90	0.400	3.400
C 2	12	40.00	11.76	0.758	3.418
"	"	20.50	6.03	0.280	2.940
C 3	10	35.00	10.29	0.823	3.183
"	"	15.00	4.46	0.240	2.600
C 4	9	25.00	7.35	0.615	2.815
"	"	13.25	3.89	0.230	2.430
C 5	8	21.25	6.25	0.582	2.622
"	"	11.25	3.35	0.220	2.260
C 6	7	19.75	5.81	0.633	2.513
"	"	9.75	2.85	0.210	2.090
C 7	6	15.50	4.56	0.563	2.283
"	"	8.00	2.38	0.200	1.920
C 8	5	11.50	3.38	0.477	2.037
"	"	6.50	1.95	0.190	1.750
C 9	4	7.25	2.13	0.325	1.725
"	"	5.25	1.55	0.180	1.580
C 72	3	6 00	1.76	0.362	1.602
"	"	4.00	1.19	0.170	1.410

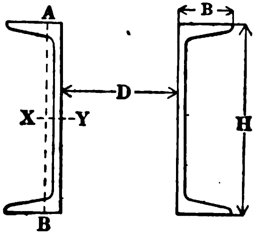
Fire-proof floors of I beams and brick-arches.

The arches are usually "four-inch"—or "half a brick" deep; span, *s*, from 4 to 6 feet; rise about one-twelfth to one-sixteenth of the span. Tie-rods, *T*, ¾ inch to 1 inch diameter, from 4 to 6 or 8 feet apart, and anchored into each wall



with a stout washer, *W*. At each wall an angle iron, *a*, or a tee iron is generally used instead of a beam. The spandrels are leveled up with concrete, enclosing wooden strips, *m m*, about 1 inch × 2 inches, two over each arch. To these strips the flooring is nailed.

CHANNELS.



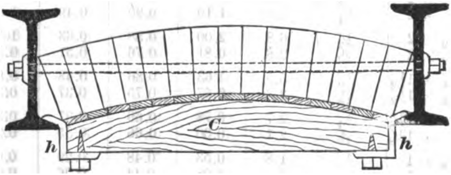
The table gives the maximum and the minimum weight of each section. The minimum weights are standard. Others are special.

Caution.—With very short spans, the loads found by means of columns C_s and C_m, although safe against *bending*, may be so great as to endanger a *crushing* of the ends of the beam, or of the walls, etc., under them, unless the beam has, at its ends, a greater length of bearing than would otherwise be needed.

I	i	R ins.	r ins.	X	C_s lbs.	C_m lbs.	D ins.	Section index.
430.2	12.19	5.16	0.868	57.4	611.900	478,000	8.53	C 1
312.6	8.23	5.62	0.912	41.7	444,500	347,300	9.50	"
197.0	6.63	4.09	0.751	32.8	350,200	273,600	6.60	C 2
128.1	3.91	4.61	0.805	21.4	227,800	178,000	7.67	"
115.5	4.66	3.35	0.672	23.1	246,400	192,500	5.17	C 3
66.9	2.30	3.87	0.718	13.4	142,700	111,500	6.33	"
70.7	2.98	3.10	0.637	15.7	167,600	130,900	4.84	C 4
47.3	1.77	3.49	0.674	10.5	112,200	87,600	5.63	"
47.8	2.25	2.77	0.600	11.9	127,400	99,500	4.23	C 5
32.3	1.33	3.11	0.630	8.1	86,100	67,300	4.94	"
33.2	1.85	2.39	0.565	9.5	101,100	79,000	3.48	C 6
21.1	0.98	2.72	0.586	6.0	66,800	52,200	4.22	"
19.5	1.28	2.07	0.529	6.5	69,500	54,300	2.91	C 7
13.0	0.70	2.34	0.542	4.3	46,200	36,100	3.52	"
10.4	0.82	1.75	0.493	4.2	44,400	34,700	2.34	C 8
7.4	0.48	1.95	0.498	3.0	31,600	24,700	2.79	"
4.6	0.44	1.46	0.455	2.3	24,400	19,000	1.85	C 9
3.8	0.32	1.56	0.453	1.9	20,200	15,800	2.06	"
2.1	0.31	1.08	0.421	1.4	14,700	11,500	1.07	C 72
1.6	0.20	1.17	0.409	1.1	11,600	9,100	1.31	"

The weight of a 4-inch arch, with its concrete filling and wooden flooring, but exclusive of the beams, is about 70 lbs. per square foot of floor.

A dense crowd of persons will hardly weigh more than 80 lbs. per square foot.



Each center, *C*, has fastened to it at each end a bent iron strap, *h*, forming a hook by which the center is suspended from the lower flanges of the beams.

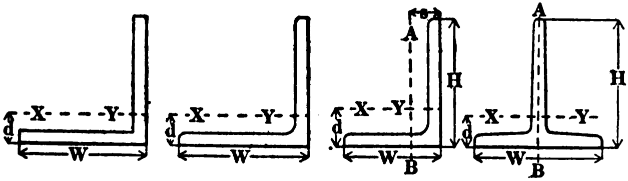
CARNEGIE ANGLES

d = distance between center of gravity and back of flange W
s = " " " " " " " " H
I, i = moment of inertia; **I**, about XY; **i**, about A B
X, x = least "section modulus"; **X**, " " **x**, " " $= \frac{12\,M}{s}$†
R, R′ = radius of gyration; **R**, " " **R′**, " "
r = least radius of gyration, about neutral axis forming acute angle a with each flange. In angles with equal legs, $a = 45°$
C = coefficient for uniformly distributed safe load: C_s, for static loads; fibre stress = 16,000 lbs. C_m, for moving loads; fibre stress = 12,000 lbs. } For T shapes only.

Section index.	Size H W ins. ins.	Thickness ins.	Weight per ft lbs.	Area of section sq ins.	d ins.	s ins.	I	i
Angles with Unequal Legs.								
* A 150	$7 \times 3\frac{1}{2}$	1	32.3	9.50	2.71	0.96	45.37	7.53
* A 159	$7 \times 3\frac{1}{2}$	$\frac{7}{16}$	15.0	4.40	2.50	0.75	22.56	3.95
A 89	6×4	1	30.6	9.00	2.17	1.17	30.75	10.75
A 168	6×4	$\frac{3}{8}$	12.3	3.61	1.94	0.94	13.47	4.90
A 92	$6 \times 3\frac{1}{2}$	1	28.9	8.50	2.26	1.01	29.24	7.21
A 177	$6 \times 3\frac{1}{2}$	$\frac{3}{8}$	11.7	3.42	2.04	0.79	12.86	3.34
* A 178	5×4	$\frac{7}{8}$	24.2	7.11	1.71	1.21	16.42	9.23
* A 186	5×4	$\frac{3}{8}$	11.0	3.23	1.53	1.03	8.14	4.67
A 187	$5 \times 3\frac{1}{2}$	$\frac{7}{8}$	22.7	6.67	1.79	1.04	15.67	6.21
A 96	$5 \times 3\frac{1}{2}$	$\frac{5}{16}$	8.7	2.56	1.59	0.84	6.60	2.72
A 196	5×3	$\frac{13}{16}$	19.9	5.84	1.86	0.86	13.98	3.71
A 280	5×3	$\frac{5}{16}$	8.2	2.40	1.68	0.68	6.26	1.75
* A 204	$4\frac{1}{2} \times 3$	$\frac{13}{16}$	18.5	5.43	1.65	0.90	10.33	3.60
* A 97	$4\frac{1}{2} \times 3$	$\frac{5}{16}$	7.7	2.25	1.47	0.72	4.69	1.78
* A 212	$4 \times 3\frac{1}{2}$	$\frac{13}{16}$	18.5	5.43	1.36	1.11	7.77	5.49
* A 98	$4 \times 3\frac{1}{2}$	$\frac{5}{16}$	7.7	2.25	1.18	0.93	3.56	2.59
A 220	4×3	$\frac{13}{16}$	17.1	5.03	1.44	0.94	7.34	3.47
A 228	4×3	$\frac{5}{16}$	7.1	2.09	1.26	0.76	3.38	1.65
A 229	$3\frac{1}{2} \times 3$	$\frac{13}{16}$	15.7	4.62	1.23	0.98	4.98	3.33
A 237	$3\frac{1}{2} \times 3$	$\frac{5}{16}$	6.6	1.93	1.06	0.81	2.33	1.58
A 238	$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{11}{16}$	12.4	3.65	1.27	0.77	4.13	1.72
A 245	$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{4}$	4.9	1.44	1.11	0.61	1.80	0.78
* A 246	$3\frac{1}{4} \times 2$	$\frac{9}{16}$	9.0	2.64	1.21	0.59	2.64	0.75
* A 251	$3\frac{1}{4} \times 2$	$\frac{1}{4}$	4.3	1.25	1.09	0.48	1.36	0.40
A 252	$3 \times 2\frac{1}{2}$	$\frac{9}{16}$	9.5	2.78	1.02	0.77	2.28	1.42
A 257	$3 \times 2\frac{1}{2}$	$\frac{1}{4}$	4.5	1.31	0.91	0.66	1.17	0.74
* A 258	3×2	$\frac{1}{2}$	7.7	2.25	1.08	0.58	1.92	0.67
* A 262	3×2	$\frac{1}{4}$	4.0	1.19	0.99	0.49	1.09	0.39
A 264	$2\frac{1}{2} \times 2$	$\frac{1}{2}$	6.8	2.00	0.88	0.63	1.14	0.64
A 269	$2\frac{1}{2} \times 2$	$\frac{3}{16}$	2.8	0.81	0.76	0.51	0.51	0.29
* A 270	$2\frac{1}{4} \times 1\frac{1}{2}$	$\frac{1}{2}$	5.5	1.63	0.86	0.48	0.82	0.26
* A 275	$2\frac{1}{4} \times 1\frac{1}{2}$	$\frac{3}{16}$	2.8	0.67	0.75	0.37	0.34	0.12
* A 276	$2 \times 1\frac{3}{8}$	$\frac{1}{4}$	2.7	0.78	0.69	0.37	0.37	0.12
* A 277	$2 \times 1\frac{3}{8}$	$\frac{3}{16}$	2.1	0.60	0.66	0.35	0.24	0.09
* A 278	$1\frac{3}{8} \times 1$	$\frac{1}{4}$	1.8	0.53	0.48	0.29	0.09	0.04
* A 279	$1\frac{3}{8} \times 1$	$\frac{1}{8}$	1.0	0.28	0.44	0.26	0.05	0.02

* Special sections. † For M and S see p. 892 or p. 894.

AND T SHAPES.



X	x	R	R′	r	C_s	C_m	Section Index.
Maximum and Minimum Weight of each Section.							
10.58	2.96	2.19	0.89	0.88	....	....	A 150*
5.01	1.47	2.26	0.95	0.89	....	....	A 159*
8.02	3.79	1.85	1.09	0.85	....	....	A 89
3.32	1.60	1.93	1.17	0.88	....	....	A 168
7.83	2.90	1.85	0.92	0.74	....	....	A 92
3.25	1.23	1.94	0.99	0.77	....	....	A 177
4.99	3.31	1.52	1.14	0.84	....	....	A 178*
2.34	1.57	1.59	1.20	0.86	....	....	A 186*
4.88	2.52	1.53	0.96	0.75	....	....	A 187
1.94	1.02	1.61	1.08	0.76	....	....	A 96
4.45	1.74	1.55	0.80	0.64	....	....	A 196
1.89	0.75	1.61	0.85	0.66	....	....	A 280
3.62	1.71	1.38	0.81	0.64	....	....	A 204*
1.54	0.76	1.44	0.88	0.66	....	....	A 97*
2.92	2.30	1.19	1.01	0.72	....	....	A 212*
1.26	1.01	1.26	1.07	0.73	....	....	A 98*
2.87	1.68	1.21	0.83	0.64	....	....	A 220
1.23	0.74	1.27	0.89	0.65	....	....	A 228
2.20	1.65	1.04	0.85	0.62	....	....	A 229
0.96	0.72	1.10	0.90	0.68	....	....	A 237
1.85	0.99	1.06	0.67	0.53	....	....	A 238
0.75	0.41	1.12	0.74	0.54	....	....	A 245
1.30	0.53	1.00	0.53	0.44	....	....	A 246*
0.68	0.26	1.04	0.57	0.45	....	....	A 251*
1.15	0.82	0.91	0.72	0.52	....	....	A 252
0.56	0.40	0.95	0.75	0.53	....	....	A 257
1.00	0.47	0.92	0.55	0.43	....	....	A 258*
0.54	0.25	0.95	0.57	0.43	....	....	A 262*
0.70	0.46	0.75	0.56	0.42	....	....	A 264
0.29	0.20	0.79	0.60	0.43	....	....	A 269
0.59	0.26	0.71	0.40	0.39	....	....	A 270*
0.23	0.11	0.72	0.43	0.40	....	....	A 275*
0.23	0.12	0.63	0.39	0.30	....	....	A 276*
0.18	0.09	0.63	0.40	0.31	....	....	A 277*
0.09	0.05	0.41	0.27	0.22	....	....	A 278*
0.06	0.03	0.44	0.29	0.22	....	....	A 279*

* Special sections.

CARNEGIE ANGLES

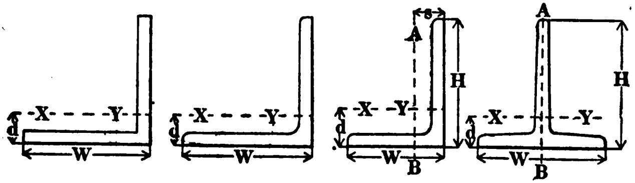
d = distance between center of gravity and back of flange W
s = " " " " " " " " H
I, i = moment of inertia; **I**, about XY; **i**, about A B
X, x = least "section modulus"; **X**, " " **x**, " " $= \frac{12\,M}{S}$†
R, R′ = radius of gyration; **R**, " " **R′**, " "
r = least radius of gyration, about neutral axis forming acute angle *a* with each flange. In angles with equal legs, *a* = 45°

C = coefficient for uniformly distributed safe load:
C_s, for static loads; fibre stress = 16,000 lbs. } For T shapes only.
C_m, for moving loads; fibre stress = 12,000 lbs. }

Section index.	Size H W ins. ins.	Thickness ins.	Weight per ft lbs.	Area of section sq ins.	**d** ins.	**s** ins.	**I**	**i**
								Angles with Equal Legs.
A 113	8 × 8	1⅛	56.9	16.73	2.41	....	97.97	....
A 103	8 × 8	½	26.4	7.75	2.19	....	48.68	....
A 86	6 × 6	1	37.4	11.00	1.86	....	35.46	....
A 88	6 × 6	3/8	14.8	4.36	1.64	....	15.39	....
*A 94	5 × 5	1	30.6	9.00	1.61	....	19.64	....
*A 17	5 × 5	3/8	12.3	3.61	1.39	....	8.74	....
A 18	4 × 4	13/16	19.9	5.84	1.29	....	8.14	....
A 90	4 × 4	5/16	8.2	2.40	1.12	....	3.71	....
A 26	3½ × 3½	13/16	17.1	5.03	1.17	....	5.25	....
A 99	3½ × 3½	5/16	7.1	2.09	0.99	....	2.45	....
A 34	3 × 3	5/8	11.4	3.36	0.98	....	2.62	....
A 40	3 × 3	¼	4.9	1.44	0.84	....	1.24	....
*A 41	2¾ × 2¾	½	8.5	2.50	0.87	....	1.67	....
*A 45	2¾ × 2¾	¼	4.5	1.31	0.78	....	0.93	....
A 46	2½ × 2½	½	7.7	2.25	0.81	....	1.23	....
A 100	2½ × 2½	3/16	3.1	0.90	0.69	....	0.55	....
*A 51	2¼ × 2¼	½	6.8	2.00	0.74	....	0.87	....
*A 101	2¼ × 2¼	3/16	2.8	0.81	0.68	....	0.39	....
A 56	2 × 2	7/16	5.3	1.56	0.66	....	0.54	....
A 60	2 × 2	3/16	2.5	0.72	0.57	....	0.28	....
A 61	1¾ × 1¾	7/16	4.6	1.30	0.59	....	0.35	....
A 65	1¾ × 1¾	3/16	2.1	0.62	0.51	....	0.18	....
A 66	1½ × 1½	3/8	3.4	0.99	0.51	....	0.19	....
A 102	1½ × 1½	1/8	1.2	0.36	0.42	....	0.08	....
A 70	1¼ × 1¼	5/16	2.4	0.69	0.42	....	0.09	....
A 73	1¼ × 1¼	1/8	1.0	0.30	0.35	....	0.044	....
A 78	1 × 1	¼	1.5	0.44	0.34	....	0.037	....
A 80	1 × 1	1/8	0.8	0.24	0.30	....	0.022	....
*A 81	7/8 × 7/8	3/16	1.0	0.29	0.29	....	0.019	....
A 82	7/8 × 7/8	1/8	0.7	0.21	0.26	....	0.014	....
A 83	¾ × ¾	3/16	0.8	0.25	0.26	....	0.012	....
A 84	¾ × ¾	1/8	0.6	0.17	0.23	....	0.009	....
								T Shapes.
T 50	5 × 3	....	13.6	3.99	0.75	....	2.6	5.6
T 57	4 × 5	....	15.6	4.56	1.56	....	10.7	2.8
T 61	4 × 3	....	9.3	2.73	0.78	....	2.0	2.1
T 3	3½ × 3½	....	11.7	3.45	1.06	....	3.7	1.89
T 72	3 × 4	....	11.8	3.48	1.32	....	5.2	1.21
T 77	3 × 3½	....	8.5	2.49	1.09	....	2.9	0.93
T 82	2½ × 3	....	7.2	2.10	0.97	....	1.8	0.54
T 12	2¼ × 2¼	....	4.9	1.44	0.69	....	0.66	0.33
T 16	1¾ × 1¾	....	3.1	0.90	0.54	....	0.23	0.12

* Special sections. † For M and S see p. 892 or p. 894.

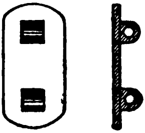
AND T SHAPES.—Continued.

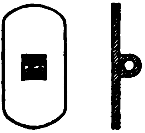


X	x	R	R′	r	C_s	C_m	Section Index.
Maximum and Minimum Weight of each Section.							
17.53	....	2.42	....	1.55	....	....	A 113
8.37	....	2.50	....	1.58	....	....	A 103
8.57	....	1.80	....	1.16	....	....	A 86
3.53	....	1.88	....	1.19	....	....	A 88
5.80	....	1.48	....	0.96	....	....	A 94*
2.42	....	1.56	....	0.99	....	....	A 17*
3.01	....	1.18	....	0.77	....	....	A 18
1.29	....	1.24	....	0.79	....	....	A 90
2.25	....	1.02	....	0.67	....	....	A 26
0.98	....	1.08	....	0.69	....	....	A 99
1.30	....	0.88	....	0.57	....	....	A 34
0.58	....	0.93	....	0.59	....	....	A 40
0.89	....	0.82	....	0.52	....	....	A 41*
0.48	....	0.85	....	0.55	....	....	A 45*
0.73	....	0.74	....	0.47	....	....	A 46
0.30	....	0.78	....	0.49	....	....	A 100
0.58	....	0.66	....	0.43	....	....	A 51*
0.24	....	0.70	....	0.44	....	....	A 101*
0.40	....	0.59	....	0.39	....	....	A 56
0.19	....	0.62	....	0.40	....	....	A 60
0.30	....	0.51	....	0.33	....	....	A 61
0.14	....	0.54	....	0.35	....	....	A 65
0.19	....	0.44	....	0.29	....	....	A 66
0 070	....	0.46	....	0.30	....	....	A 102
0.109	....	0.36	....	0.23	....	....	A 70
0.049	....	0.38	....	0.25	....	....	A 73
0.056	....	0.29	....	0.19	....	....	A 78
0.031	....	0.31	....	0.20	....	....	A 80
0.033	....	0.26	....	0.18	....	....	A 81*
0.023	....	0.26	....	0.19	....	....	A 82*
0.024	....	0.22	....	0.16	....	....	A 83
0.017	....	0.23	....	0.17	....	....	A 84
Selected Sections.							
1.18	2.22	0.82	1.19	....	12,550	9,410	T 50
3.10	1.41	1.54	0.79	....	33,070	24,800	T 57
0.88	1.05	0.86	0.88	....	9,430	7,070	T 61
1.52	1.08	1.04	0.74	....	16,210	12,160	T 8
1.94	0.81	1.23	0.59	....	20,650	15,480	T 72
1.21	0.62	1.09	0.61	....	12.910	9,680	T 77
0.87	0.43	0.92	0.51	....	9 280	6,960	T 82
0.42	0.30	0.68	0 48	....	4 480	3,360	T 12
0.19	0.14	0.51	0.37	....	2,050	1,540	T 16

* Special sections.

CARNEGIE STANDARD CAST IRON SEPARATORS FOR I BEAMS.





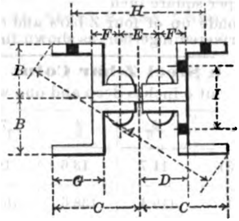
Separators for 18″, 20″ and 24″ beams are made of ⅝″ metal.
" " 6″ to 15″ beams are made of ½″ metal.
" " 5″ beams and under are made of ⅜″ metal.

Designation of Beam.			Distances.		Bolts.			Weights.			
Section Index.	Depth.	Weight.	Out to out of flanges of beams.	Center to center of beams.	Size.	Distance center to center.	Length.	Bolts and Nuts.	Increase in weight of separator bolts for 1 inch additional spread of beams.	Separator.	Increase in weight of separator for 1 inch additional spread of beams.
	Ins.	Lbs.	Ins.	Ins.	In.	Ins.	Ins.	Lbs.	Lbs.	Lbs	Lbs.
Separators with Two Bolts.											
B 1	24	80.	14¾	7¾	¾	12	9¼	3.41	.250	32.	5.50
B 2	20	80.	14¾	7¾	¾	10	9¼	3.41	"	28.	3.10
B 3	20	65.	13¼	7	¾	10	8½	3.23	"	25.	3.10
B 80	18	55.	12¾	6¾	¾	9	8¼	3.16	"	16.	2.75
B 4	15	80.	13⅝	7¼	¾	7½	9	3.35	"	15.	1.75
B 5	15	60.	12¾	6¾	¾	7½	8½	3.23	"	15.	1.75
B 7	15	42.	11¾	6¼	¾	7½	7½	2.98	"	15.	1.75
B 8	12	40.	11¼	6	¾	6	7½	2.98	"	11.	1.50
B 9	12	31.5	10¾	5¾	¾	6	7¼	2.92	"	11.	1.50
Separators with One Bolt.											
B 8	12	40.0	11¼	6	¾	....	7½	1.49	.125	10.	1.50
B 9	12	31.5	10¾	5¾	¾	....	7¼	1.46	"	10.	1.50
B 11	10	25.0	10⅛	5½	¾	....	6¾	1.40	"	8.	1.25
B 13	9	21.0	9 5/16	5	¾	....	6¼	1.34	"	7.	1.20
B 15	8	18.0	8½	4½	¾	....	5¾	1.28	"	6.	1.00
B 17	7	15.0	7⅞	4¼	¾	....	5½	1.25	"	4.	.75
B 19	6	12.25	7 5/16	4	¾	....	5¼	1.22	"	4.	.60
B 21	5	9.75	6½	3½	¾	....	4¾	1.16	"	3.	.50
B 23	4	7.50	5⅞	3¼	¾	....	4½	1.13	"	3.	.40
B 77	3	5.50	5 5/16	3	⅝	....	4¼	0.70	.09	2.	.25

CARNEGIE STEEL Z-BAR COLUMNS.

Table of dimensions, in inches.

Diameter of bolt or rivet = ¾ inch.



For area of section, weight per yard, least radius of gyration and safe load, see tables, pp. 902 and 903.

	Thickness of Metal.	See figure above.								
		A	B	C	D	E	F	G	H	I
6-inch columns.	$\frac{1}{4}$	$12\frac{5}{16}$	$3\frac{1}{8}$	$5\frac{5}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	$8\frac{1}{2}$	$3\frac{1}{4}$
	$\frac{5}{16}$	$12\frac{3}{8}$	$3\frac{7}{32}$	$5\frac{5}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$8\frac{3}{8}$	$3\frac{3}{8}$
	$\frac{3}{8}$	$12\frac{3}{16}$	$3\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	$8\frac{1}{4}$	$3\frac{3}{8}$
	$\frac{7}{16}$	$12\frac{1}{4}$	$3\frac{9}{32}$	$5\frac{3}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$8\frac{1}{8}$	$3\frac{1}{2}$
	$\frac{1}{2}$	12	$3\frac{1}{4}$	$5\frac{1}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{11}{16}$	8	$3\frac{1}{2}$
	$\frac{9}{16}$	$12\frac{1}{16}$	$3\frac{11}{32}$	$5\frac{1}{16}$	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{4}$	$7\frac{7}{8}$	$3\frac{5}{8}$
8-inch columns.	$\frac{1}{4}$	$14\frac{11}{16}$	$4\frac{1}{8}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$9\frac{1}{2}$	$4\frac{1}{4}$
	$\frac{5}{16}$	$14\frac{3}{4}$	$4\frac{7}{32}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	$9\frac{3}{8}$	$4\frac{3}{8}$
	$\frac{3}{8}$	$14\frac{13}{16}$	$4\frac{5}{16}$	$6\frac{1}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$9\frac{1}{4}$	$4\frac{1}{2}$
	$\frac{7}{16}$	$14\frac{1}{2}$	$4\frac{7}{32}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$9\frac{1}{8}$	$4\frac{7}{16}$
	$\frac{1}{2}$	$14\frac{9}{16}$	$4\frac{5}{16}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	9	$4\frac{9}{16}$
	$\frac{9}{16}$	$14\frac{5}{8}$	$4\frac{13}{32}$	$5\frac{7}{8}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$8\frac{7}{8}$	$4\frac{11}{16}$
	$\frac{5}{8}$	$14\frac{1}{4}$	$4\frac{5}{16}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{16}$	$8\frac{3}{4}$	$4\frac{5}{8}$
	$\frac{11}{16}$	$14\frac{5}{16}$	$4\frac{13}{32}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{1}{8}$	$8\frac{5}{8}$	$4\frac{3}{4}$
	$\frac{3}{4}$	$14\frac{3}{8}$	$4\frac{1}{2}$	$5\frac{11}{16}$	$3\frac{1}{4}$	3	$1\frac{3}{4}$	$3\frac{3}{16}$	$8\frac{1}{2}$	$4\frac{7}{8}$
10-inch columns.	$\frac{5}{16}$	$16\frac{1}{2}$	$5\frac{5}{32}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$10\frac{1}{8}$	$5\frac{5}{16}$
	$\frac{3}{8}$	$16\frac{9}{16}$	$5\frac{1}{4}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	10	$5\frac{7}{16}$
	$\frac{7}{16}$	$16\frac{5}{8}$	$5\frac{11}{32}$	$6\frac{7}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{7}{8}$	$5\frac{9}{16}$
	$\frac{1}{2}$	$16\frac{3}{8}$	$5\frac{1}{4}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$9\frac{3}{4}$	$5\frac{1}{2}$
	$\frac{9}{16}$	$16\frac{7}{16}$	$5\frac{11}{32}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	$9\frac{5}{8}$	$5\frac{5}{8}$
	$\frac{5}{8}$	$16\frac{1}{2}$	$5\frac{7}{16}$	$6\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{1}{2}$	$5\frac{3}{4}$
	$\frac{11}{16}$	$16\frac{3}{16}$	$5\frac{11}{32}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{1}{4}$	$9\frac{3}{8}$	$5\frac{11}{16}$
	$\frac{3}{4}$	$16\frac{1}{4}$	$5\frac{7}{16}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{5}{16}$	$9\frac{1}{4}$	$5\frac{13}{16}$
	$\frac{13}{16}$	$16\frac{5}{16}$	$5\frac{17}{32}$	$6\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{8}$	$3\frac{3}{8}$	$9\frac{1}{8}$	$5\frac{15}{16}$
12-inch columns.	$\frac{3}{8}$	$18\frac{7}{8}$	$6\frac{3}{16}$	$7\frac{1}{8}$	4	4	2	$3\frac{1}{2}$	$11\frac{1}{4}$	$6\frac{3}{8}$
	$\frac{7}{16}$	$18\frac{15}{16}$	$6\frac{9}{32}$	$7\frac{1}{8}$	4	4	2	$3\frac{9}{16}$	$11\frac{1}{8}$	$6\frac{5}{8}$
	$\frac{1}{2}$	19	$6\frac{3}{8}$	$7\frac{1}{8}$	4	4	2	$3\frac{5}{8}$	11	$6\frac{3}{4}$
	$\frac{9}{16}$	$18\frac{11}{16}$	$6\frac{9}{32}$	$6\frac{15}{16}$	4	4	2	$3\frac{1}{2}$	$10\frac{7}{8}$	$6\frac{9}{16}$
	$\frac{5}{8}$	$18\frac{3}{4}$	$6\frac{3}{8}$	$6\frac{15}{16}$	4	4	2	$3\frac{9}{16}$	$10\frac{3}{4}$	$6\frac{11}{16}$
	$\frac{11}{16}$	$18\frac{13}{16}$	$6\frac{15}{32}$	$6\frac{15}{16}$	4	4	2	$3\frac{5}{8}$	$10\frac{5}{8}$	$6\frac{13}{16}$
	$\frac{3}{4}$	$18\frac{9}{16}$	$6\frac{3}{8}$	$6\frac{3}{4}$	4	4	2	$3\frac{1}{2}$	$10\frac{1}{2}$	$6\frac{3}{4}$
	$\frac{13}{16}$	$18\frac{5}{8}$	$6\frac{15}{32}$	$6\frac{3}{4}$	4	4	2	$3\frac{9}{16}$	$10\frac{3}{8}$	$6\frac{7}{8}$
	$\frac{7}{8}$	$18\frac{11}{16}$	$6\frac{9}{16}$	$6\frac{3}{4}$	4	4	2	$3\frac{5}{8}$	$10\frac{1}{4}$	7

CARNEGIE STEEL Z-BAR COLUMNS.

Table of Safe Loads as given by Carnegie Steel Co., for columns with square ends. Safety factor = 4. The loads given are based upon the following allowed strains in pounds per square inch :

Each Z-bar column is made up of four Z-bars and one web-plate (all of uniform thickness) bolted or riveted together, as shown in the figure on page 901.

6-inch Steel Z-bar Columns.

Composed of four Z-bars about 3 inches deep and one web-plate $5\frac{3}{4}$ inches wide.

Thickness of metal, inch.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
Area of section, sq. ins.	9.31	11.7	13.6	16.0	17.6	20.0
Weight per yard, pounds.	95.1	119.4	138.6	162.9	179.7	203.7
Least rad of gyr, inches.	1.86	1.90	1.88	1.93	1.90	1.95

Length of column. Feet.	Safe load of column, in pounds.					
12 or less.	111800	140600	163200	191600	211400	239600
14	111400	140600	163200	191600	211400	239600
16	104600	133000	153200	182600	199800	229600
18	97600	124600	143400	171200	187200	215600
20	90800	116200	133400	159800	174400	201600
22	84000	107800	123600	148600	161800	187600
24	77200	99400	113800	137200	149200	173600
26	70400	91000	103800	126000	136400	159600
28	63400	82600	94000	114600	123800	145600
30	56600	74200	84000	103400	111000	131600

8-inch Z-bar Columns.

Composed of four Z-bars about 4 inches deep and one web-plate $6\frac{1}{2}$ inches wide.

Thickness of metal, inch.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
Area of section, sq. ins.	11.3	14.1	17.1	19.0	21.9	24.8	26.3	29.0	31.9
Weight per yard, pounds.	114.9	144.3	174.0	194.1	221.1	252.3	267.6	296.4	325.2
Least rad of gyr, inches.	2.47	2.52	2.57	2.49	2.55	2.60	2.52	2.58	2.63

Length of column. Feet.	Safe load of column, in pounds.								
18 or less.	135000	169600	204800	228400	262400	297000	315000	348600	382400
20	130000	165000	201000	221000	256400	292800	306600	342600	379200
22	123800	157400	191800	210600	244800	279800	292400	327000	362600
24	117600	149600	182600	200200	233000	266800	278200	311600	346000
26	111400	142000	173600	189600	221200	253800	264000	296200	329400
28	105200	134200	164600	179200	209400	240600	249600	280800	312800
30	98800	126600	155400	168800	197600	227600	235400	265400	296400
32	92600	119000	146400	158400	186000	214600	221200	250000	279800
34	86400	111200	137400	148000	174200	201600	207000	234600	263200
36	80200	103600	128200	137400	162400	188600	192800	219200	246600
38	74000	96000	119200	127000	150600	175600	178800	203800	230000
40	67800	88200	110000	116600	139000	162600	164400	188400	213400

CARNEGIE STEEL Z-BAR COLUMNS.

Table of Safe Loads (continued).

10-inch Steel Z-bar Columns.

Composed of four Z-bars about 5 inches deep and one web-plate 7 inches wide.

Thickness of metal, inch.	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$
Area of section, sq. ins.	15.8	19.0	22.3	24.5	27.7	30.9	32.7	35.8	39.0
Weight per yard, pounds.	161.1	194.1	227.4	249.9	282.6	315.6	330.0	368.4	397.8
Least rad of gyr, inches.	3.08	3.13	3.18	3.10	3.15	3.21	3.13	3.18	3.25
Length of column. Feet.	Safe load of column, in pounds.								
22 or less.	189400	228400	267800	294000	332400	371200	392000	429800	468000
24	185600	225200	266200	289200	329600	370600	387200	427800	468000
26	178600	217200	256600	278400	317400	357400	373000	412400	453200
28	171600	208800	247000	267600	305400	344200	358600	397000	436800
30	164600	200400	237400	256800	293400	331000	344400	381600	420400
32	157600	192200	227600	246000	281400	317800	330000	366200	404000
34	150600	183800	218200	235200	269400	304600	315800	350800	387600
36	143600	175600	208600	224400	257400	291400	301400	335600	371200
38	136600	167200	199000	213600	245400	278200	287200	320000	354800
40	129600	158800	189400	202800	233400	265000	273000	304600	338200
42	122600	150600	179800	192000	221200	251800	258800	289200	321800
44	115400	142200	170200	181200	209200	238600	244400	273800	305400
46	108400	134000	160600	170400	197200	225400	230200	258400	289000
48	101400	125600	151000	159600	185200	212200	215800	243000	272600
50	94400	117200	141400	148800	173200	199000	201600	227600	256200

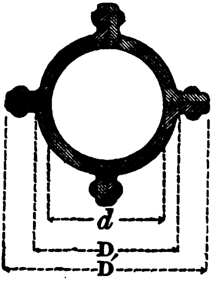
12-inch Steel Z-bar Columns.

Composed of four Z-bars about 6 inches deep and one web-plate 8 inches wide.

Thickness of metal, inch.	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
Area of section, sq. ins.	21.4	25.0	28.8	31.2	34.8	38.5	40.5	44.1	47.7
Weight per yard, pounds.	218.1	255.6	293.4	318.6	355.5	392.7	413.4	449.7	486.3
Least rad of gyr, inches.	3.67	3.72	3.77	3.70	3.75	3.73	3.68	3.66	3.64
Length of column. Feet.	Safe load of column, in pounds.								
26 or less.	256600	300600	345200	374600	418200	462000	486000	529000	572200
28	254000	299400	345000	372000	417800	460600	481600	522800	564200
30	246000	290200	335200	360400	405000	446600	466400	506400	546400
32	238000	281000	324800	349000	392200	432600	451400	490000	528400
34	230200	271800	314400	337400	379600	418400	436400	473400	510400
36	222200	262600	304000	325800	366800	404200	421200	456800	492600
38	214200	253400	293600	314200	354000	390200	406200	440400	474600
40	206200	244200	283000	302800	341400	376000	391200	423800	456600
42	198200	235000	272600	291000	328800	361800	376000	407400	438800
44	190200	225800	262200	279600	316000	347800	361000	391000	420800
46	182400	216600	252400	268000	303200	333600	345800	374400	402800
48	174400	207200	241400	256400	290600	319600	330800	358000	384800
50	166400	198200	231000	244800	277800	305400	315800	341400	367000

For loads greater than those given in the tables, the Z-bar columns may be re-enforced by additional plates, riveted to the flanges. The addition of such plates does not in any case diminish the least radius of gyration. Hence the same load *per square inch of cross-section* may be used.

Table of rolled-iron segment-columns of the Phœnix Iron Co, 410 Walnut St, Philada.



The **dimensions** given are subject to slight variations which are unavoidable in rolling iron shapes. The **weights** of columns given are those of the 4, 6, or 8 segments, of which they are composed. The *shanks* of the rivets used in joining them together, of course, merely make up the quantity of metal punched or drilled out, in making the holes; but the rivet-*heads* add from 2 to 5 per cent to the weights given. The **rivets are spaced** 3, 4, or 6 ins apart from cen to cen.

Any desired thickness between the minimum and maximum for any given size, can be furnished. We give the dimensions, weights, &c, corresponding to the principal thicknesses. G columns have 8 segments, E, 6 segments. All others, 4 segments.

Mark.	Thickn's, ins.	Diameters, ins.			One column.			Size of Rivets.
		d	D	D′	Area of cross sec, sq ins.	Wt per ft run, lbs.	Least rad of gyr, ins	
A	3/16	3 5/8	4	6 1/16	3.8	12.6	1.45	1 1/8 × 3/8
"	1/4	"	4 1/8	6 3/16	4.8	16.	1.50	1 1/4 × "
"	5/16	"	4 1/4	6 5/16	5.8	19.3	1.55	1 3/8 × "
"	3/8	"	4 3/8	6 7/16	6.8	22.6	1.59	1 1/2 × "
B^1	1/4	4 13/16	5 5/16	8 1/16	6.4	21.3	1.92	1 5/8 × 1/2
"	3/8	"	5 9/16	8 1/4	9.2	30.6	2.02	1 3/4 × "
"	1/2	"	5 13/16	8 7/16	12.	40.	2.11	1 7/8 × "
"	5/8	"	6 1/16	8 5/8	14.8	49.3	2.20	2 1/8 × "
B^2	1/4	5 15/16	6 7/16	9 1/8	7.4	24.6	2.34	1 5/8 × "
"	3/8	"	6 11/16	9 5/16	10.6	35.3	2.43	1 3/4 × "
"	1/2	"	6 15/16	9 1/2	13.8	46.	2.52	1 7/8 × "
"	5/8	"	7 3/16	9 11/16	17.	56.6	2.61	2 1/8 × "
C	1/4	7 3/16	7 11/16	11 9/16	10.	33.3	2.80	1 7/8 × 5/8
"	1/2	"	8 3/16	11 15/16	18.	60.	2.98	2 1/4 × "
"	3/4	"	8 11/16	12 3/16	25.2	84.	3.16	2 5/8 × 3/4
"	1	"	9 3/16	12 9/16	33.2	110.6	3.34	3 × "
"	1 1/4	"	9 11/16	12 15/16	41.2	137.3	3.52	3 1/4 × "
E	1/4	11	11 1/2	15 7/16	16.8	56.	4.18	2 × "
"	1/2	"	12	15 7/8	26.4	88.	4.36	2 3/8 × "
"	3/4	"	12 1/2	16 5/16	37.8	126.	4.55	2 3/4 × "
"	1	"	13	16 3/4	49.8	166.	4.73	3 × "
"	1 1/4	"	13 1/2	17 3/16	61.8	206.	4.91	3 1/4 × "
G	5/16	14 3/8	15	19 1/8	24.	80.	5.45	1 7/8 × 5/8
"	1/2	"	15 3/8	19 7/16	36.	120.	5.59	2 1/4 × "
"	3/4	"	15 7/8	19 7/8	52.	173.3	5.77	2 5/8 × 3/4
"	1	"	16 3/8	20 3/8	68.	226.6	5.95	3 × "
"	1 3/8	"	17 1/8	21	92.	306.6	6.23	3 3/8 × "

THE GRAY COLUMN.

The Gray Column, designed and patented by Mr. J. H. Gray, consists, in its original form, of angles, connected at intervals D (generally of 2 ft 6 ins) by transverse bent tie-plates T, usually 9 × 3/8 ins. This construction renders the parts of the column easily accessible, for painting, etc., but under transverse or buckling stresses the column must act somewhat like a rectangular frame without diagonals. To remedy this, a later form, the "twelve-angle" column, Fig 5, has been designed, having, in the square column, Figs 1 and 2, instead of the bent tie-plates T, four additional angles, running longitudinally like the others, and placed centrally, as shown. These angles supply the column with two webs, intersecting at right angles.

Figs 1 and 2 show the square column, used in the interior of buildings. It is used chiefly in the 14, 15, and 16 inch sizes. Fig 2 shows plates riveted to the outer flanges, as is done in some of the heaviest columns. Figs 3 and 4 show the wall and corner columns respectively. Fig 5 shows the "twelve-angle" column, and Fig 6 one of many forms of fireproofing, with plaster laid on terra cotta blocks. The rivets are usually 3/4 inch in diameter.

The safe load, in pounds per sq inch, of the ordinary column, is stated as $17{,}100 - 57\frac{L}{r}$, where L = length of column and r = its least rad of gyration.

The cost of the Gray column, at shop, is from 1 to 1.5 cents per lb plus the cost of the angles.

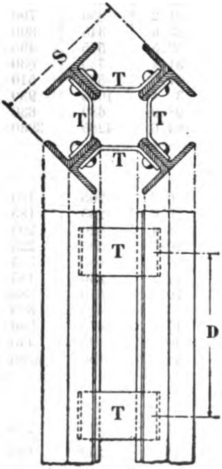


Fig. 1.

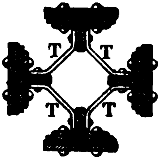


Fig. 2.

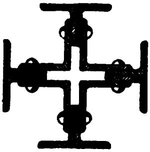


Fig. 5.

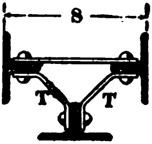


Fig. 3.

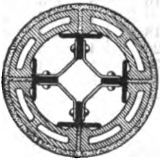


Fig. 6.

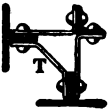


Fig. 4.

Gray Column. List of Selected Sizes.

Size S ins.	Angles ins.	Area sq in.	I Mom of in.	r Least rad. of gyr. ins.	r^2	Safe loads* for column lengths of	
						12 ft.	30 ft.
Square Columns. Figs. 1 and 2.							
9	2 × 2½ × ¼	8.48	64	2.7	7.3	115	80
"	2 × 3 × ½	18.00	119	2.6	6.8	250	165
10	2½ × 2½ × ¼	9.52	95	3.1	9.6	135	100
"	2½ × 3 × ½	20.00	179	3.0	9.0	285	205
12	3 × 3 × ⅜	16.88	241	3.8	14.4	250	195
"	3 × 5 × ½	30.00	327	3.3	10.9	435	325
13	3 × 3 × ⅜	16.88	285	4.1	16.8	255	205
"	3 × 5 × ¾	43.52	552	3.6	13.0	645	495
14	3 × 3 × ⅜	16.88	336	4.5	20.2	255	210
"	3 × 3½ × ⅝	29.36	526	4.3	18.5	445	360
"	3 × 4 × ½	26.00	444	4.2	17.6	390	315
"	3 × 5 × ½	30.00	463	4.0	16.0	450	355
"	3½ × 3½ × ⅝	31.92	597	4.4	19.4	485	395
"	3½ × 5 × ⅝	39.36	624	4.0	16.0	590	470
"	3½ × 6 × ½	36.00	526	3.8	14.4	535	420
15	3 × 3 × ½	22.00	496	4.8	23.0	335	280
"	3½ × 3½ × ⅝	31.92	693	4.7	22.1	490	405
"	3½ × 5 × ⅝	39.36	731	4.4	19.4	600	490
"	4 × 4 × ½	30.00	653	4.7	22.1	440	380
"	4 × 5 × ⅝	41.84	817	4.4	19.4	635	520
"	4 × 6 × ⅝	46.88	828	4.2	17.6	710	570
16	3 × 3 × ½	22.00	570	5.1	26.0	340	285
"	4 × 4 × ⅝	36.88	912	5.0	25.0	570	480
"	4 × 6 × ¾	55.52	1134	4.6	21.2	850	700
18	3 × 3 × ½	22.00	746	5.8	33.6	345	300
"	4 × 4 × ⅝	36.88	1182	5.7	32.5	575	495
"	5 × 5 × ⅝	46.88	1465	5.6	31.4	730	630
20	4 × 4 × ⅝	36.88	1485	6.4	41.0	580	510
"	6 × 6 × ¾	67.52	2588	6.2	38.4	1065	930
30	5 × 4 × ⅝	41.80	4147	9.9	98.0	680	630
"	6 × 6 × 1 †	264.40	22688	9.2	84.6	4285	3960
Wall Columns. Fig. 3.							
12	3 × 4 × ⅜	14.88	94	2.5	6.2	205	130
14	3 × 3½ × ½	18.00	160	3.0	9.0	255	185
"	3½ × 5 × ⅝	29.52	241	2.9	8.4	420	295
15	3½ × 4 × ½	21.00	217	3.3	10.9	305	225
"	4 × 5 × ⅝	31.38	317	3.2	10.2	455	335
16	3 × 3 × ½	16.50	200	3.5	12.2	240	185
"	4 × 6 × ⅝	35.16	375	3.3	10.9	510	380
18	4 × 4 × ⅝	27.66	434	4.0	16.0	415	330
20	4 × 5 × ⅝	31.38	562	4.2	17.6	475	380
30	5 × 4 × ⅝	31.38	1408	6.8	46.2	495	440
"	6 × 6 × 1 †	198.30	8937	6.7	44.9	3150	2785
Corner Columns. Fig. 4.							
10½	3½ × 3½ × 13/16	23.48	272	3.4	11.6	345	260
12½ 13	3½ × 3½ × ½	15.75	288	4.3	18.5	240	195
13 14	4 × 5 × ⅝	24.91	425	4.2	17.6	375	305

* In thousands of lbs, by formula: Safe load $= 17100 - 57 \frac{L}{r}$ in lbs per sq in.

† Three 1-in plates riveted to each pair of angles. Fig. 2.

TABLE OF BREAKING LOADS OF IRON PILLARS,

in tons per square inch of metal area. Deduced from Gordon. The ends are supposed to be planed to form perfectly true bearings; and all parts to be equally pressed. The last is rarely the case in practice. **If the pillar is rectangular** instead of square, use the least side for a measure of length. (Original.)

Hollow Round. Breakg loads per sq inch of metal area of transverse section.		Length meas'd in sides or diams.	**Hollow Square.** Breakg loads per sq inch of metal area of transverse section.		**Solid Round.** Breakg loads per sq inch of metal area of transverse section.		Length meas'd in sides or diams.	**Solid Square.** Breakg loads per sq inch of metal area of transverse section.	
Cast.	**Wrt.**		**Cast.**	**Wrt.**	**Cast.**	**Wrt.**		**Cast.**	**Wrt.**
Tons.	Tons.		Tons.	Tons.	Tons.	Tons.		Tons.	Tons.
35.7	16.1	1	35.7	16.0	35.6	16.1	1	35.6	16.1
35.5	16.1	2	35.5	16.0	35.2	16.1	2	35.4	16.0
35.2	16.1	3	35.3	16.0	34.6	16.0	3	34.9	16.0
34.8	16.0	4	35.0	16.0	33.9	15.9	4	34.3	16.0
34.3	16.0	5	34.6	16.0	33.0	15.9	5	33.6	16.0
33.7	16.0	6	34.2	16.0	31.9	15.8	6	32.8	15.9
33.0	15.9	7	33.7	16.0	30.6	16.7	7	31.8	15.8
32.3	15.8	8	33.1	15.9	29.3	15.6	8	30.8	15.7
31.5	15.8	9	32.4	15.9	28.0	15.5	9	29.7	15.7
30.6	15.7	10	31.7	15.8	26.7	15.4	10	28.6	15.6
29.7	15.6	11	31.0	15.8	25.4	15.2	11	27.4	15.5
28.8	15.5	12	30.3	15.7	24.1	15.1	12	26.3	15.3
27.9	15.5	13	29.5	15.7	22.8	14.9	13	25.1	15.2
27.0	15.4	14	28.7	15.6	21.6	14.8	14	24.0	15.1
26.0	15.3	15	27.9	15.5	20.5	14.6	15	22.9	15.0
25.1	15.2	16	27.1	15.5	19.4	14.4	16	21.8	14.8
24.2	15.1	17	26.3	15.4	18.3	14.2	17	20.7	14.7
23.3	15.0	18	25.4	15.3	17.2	14.0	18	19.7	14.5
22.3	14.9	19	24.6	15.2	16.2	13.8	19	18.8	14.3
21.4	14.8	20	23.8	15.1	15.3	13.6	20	17.9	14.2
20.6	14.7	21	23.0	15.0	14.5	13.4	21	17.0	14.0
19.8	14.6	22	22.2	14.9	13.7	13.2	22	16.2	13.8
19.0	14.4	23	21.5	14.8	12.9	13.0	23	15.4	13.6
18.3	14.3	24	20.8	14.7	12.2	12.8	24	14.6	13.5
17.5	14.2	25	20.1	14.6	11.6	12.6	25	13.9	13.3
16.8	14.0	26	19.4	14.5	11.0	12.4	26	13.3	13.1
16.2	13.9	27	18.7	14.4	10.4	12.1	27	12.7	12.9
15.5	13.7	28	18.0	14.3	9.90	11.9	28	12.1	12.7
14.9	13.5	29	17.4	14.2	9.41	11.7	29	11.5	12.6
14.3	13.4	30	16.8	14.0	8.93	11.5	30	11.0	12.4
13.8	13.2	31	16.3	13.9	8.50	11.3	31	10.5	12.2
13.2	13.1	32	15.7	13.8	8.10	11.1	32	10.0	12.0
12.7	12.9	33	15.2	13.6	7.70	10.8	33	9.59	11.8
12.3	12.8	34	14.6	13.5	7.36	10.6	34	9.18	11.6
11.7	12.6	35	14.1	13.3	7.04	10.4	35	8.79	11.4
11.3	12.5	36	13.6	13.2	6.71	10.2	36	8.42	11.2
10.9	12.3	37	13.2	13.1	6.41	10.0	37	8.07	11.0
10.5	12.2	38	12.7	13.0	6.11	9.79	38	7.75	10.8
10.1	12.0	39	12.3	12.9	5.85	9.59	39	7.44	10.7
9.75	11.9	40	11.9	12.7	5.62	9.39	40	7.14	10.5
9.40	11.7	41	11.5	12.6	5.40	9.20	41	6.86	10.3
9.07	11.6	42	11.1	12.4	5.19	9.00	42	6.60	10.1
8.76	11.4	43	10.8	12.3	4.99	8.81	43	6.35	9.96
8.46	11.3	44	10.4	12.2	4.79	8.64	44	6.11	9.77
8.16	11.1	45	10.1	12.0	4.59	8.46	45	5.89	9.59
7.88	10.9	46	9.80	11.9	4.42	8.27	46	5.68	9.42
7.61	10.8	47	9.50	11.7	4.26	8.10	47	5.48	9.25
7.36	10.6	48	9.20	11.6	4.11	7.94	48	5.28	9.09
7.13	10.4	49	8.94	11.4	3.97	7.76	49	5.10	8.92
6.91	10.3	50	8.66	11.3	3.84	7.60	50	4.92	8.77
5.90	9.61	55	7.47	10.7	3.22	6.86	55	4.16	8.00
5.10	8.92	60	6.49	10.0	2.75	6.18	60	3.57	7.30
4.44	8.29	65	5.68	9.43	2.37	5.59	65	3.09	6.67
3.90	7.69	70	5.01	8.84	2.06	5.06	70	2.70	6·10
3.44	7.14	75	4.44	8.30	1.80	4.59	75	2.37	5.59
3.08	6.64	80	3.97	7.77	1.59	4.18	80	2.10	5.13
2.46	5.73	90	3.21	6.88	1.27	3.49	90	1.68	4.34
2.02	4.99	100	2.65	6.02	1.04	2.95	100	1.37	3.71

Table 1. HOLLOW CYLIND CAST IRON PILLARS.

Breaking loads, flat ends, perfectly true, and firmly fixed; and the loads pressing equally on every part of the top. By Gordon's formula.

For diams or lengths intermediate of those in the table, the loads may be found near enough by simple proportion.

For thicknesses less than those in the table, the breaking loads may safely be *assumed* to diminish in the same proportion as the thickness, while the outer diam remains the same. But for *greater* thicknesses than those in the table, the loads do not increase as rapidly as the new thickness. Still, in practice, they may be assumed to do so approximately, **if the new thickness does not exceed about ⅙ part of the outer diam.**

Length in feet.	CAST IRON. THICKNESS ¼ INCH. (Original.)											Length in feet.
	Outer Diameter in inches.											
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	45.2	52.5	59.8	66.8	74.1	88.5	103.0	117.1	131.3	145.5	159.8	1
2	36.2	43.4	51.0	58.6	66.5	81.5	96.7	111.3	125.9	140.5	155.2	2
3	27.2	34.1	41.4	48.8	56.7	71.9	87.6	102.7	117.8	132.9	148.1	3
4	20.2	26.3	32.9	39.7	47.0	61.9	77.5	92.9	108.3	123.7	139.1	4
5	15.1	20.2	25.9	32.0	38.6	52.6	67.4	82.6	97.9	113.4	129.1	5
6	11.6	15.8	20.7	25.9	31.6	44.3	58.2	72.7	87.7	103.0	118.7	6
7	9.1	12.5	16.6	21.0	26.1	37.4	50.1	63.7	78.1	92.9	108.3	7
8	7.3	10.1	13.5	17.3	21.7	31.6	43.2	55.8	69.3	83.4	98.4	8
9	5.9	8.2	11.1	14.3	18.2	26.9	37.3	48.8	61.5	74.9	89.2	9
10	4.9	6.9	9.3	12.0	15.4	23.1	32.4	42.9	54.6	67.1	80.7	10
11	4.1	5.8	7.9	10.3	13.2	20.0	28.3	37.8	48.6	60.3	73.0	11
12	3.5	5.0	6.8	8.9	11.4	17.4	24.9	33.5	43.3	54.2	66.1	12
13	3.0	4.2	5.8	7.6	9.9	15.2	21.9	29.7	38.8	48.8	60.0	13
14	2.6	3.6	5.1	6.7	8.7	13.5	19.5	26.6	34.9	44.1	54.5	14
15	2.3	3.2	4.5	6.0	7.7	11.9	17.4	23.9	31.4	39.9	49.7	15
16	2.0	2.8	4.0	5.3	6.9	10 7	15.6	21.5	28.4	36.4	45.6	16
18	1.6	2.3	3.2	4.2	5.5	8.6	12.7	17.5	23.4	30.2	38.0	18
20	1.3	1.8	2.6	3.4	4.5	7.1	10.5	14.7	19.7	25.6	32.3	20
25	......	....	1.7	2.3	3.0	4.7	7.0	9.8	13.3	17.6	22.5	25
30	......	....	1.2	1.6	2.1	3.2	5·0	7.0	9.4	12.6	16.1	30
35	......	....	....	....	1.5	2.4	3.7	5.2	7.1	9.4	12.2	35
40	......	....	....	....	1.2	1.9	2.8	4.0	5.4	7.2	9.5	40
Weight of 1 foot of length of pillar in pounds.	4.31	4.91	5.53	6.13	6.75	7.97	9.22	10.4	11.7	12.9	14.1	
Area of ring of solid metal in square inches.	1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.52	

Length in feet.	CAST IRON. THICKNESS ½ INCH. (Original.)												Length in feet.
	Outer Diameter in inches.												
	5	5½	6	6½	7	7½	8	8½	9	10	11	12	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	239	268	297	325	354	383	412	440	469	526	583	640	2
4	205	235	266	296	326	356	387	416	445	504	563	622	4
6	166	196	227	257	288	319	350	380	410	471	532	598	6
8	131	159	188	218	248	279	310	340	371	432	494	557	8
10	103	127	154	182	210	240	270	300	330	391	454	517	10
12	82	103	126	151	177	205	233	262	292	351	418	475	12
14	66	84	104	126	149	174	200	227	255	313	372	434	14
16	54	69	87	106	126	149	178	198	224	277	334	394	16
18	45	58	73	90	108	128	149	172	196	246	300	357	18
20	37	48	62	77	93	111	130	151	173	219	270	323	20
22	32	41	53	66	80	96	113	132	151	194	241	292	22
25	25	34	43	54	65	79	93	109	126	164	206	252	25
30	18	24	31	39	48	59	70	83	96	126	160	199	30
35	13	18	24	30	36	44	53	63	73	96	126	159	35
40	10	14	18	23	28	35	42	50	59	78	102	129	40
45	8	11	15	19	23	28	34	41	48	64	84	107	45
50	6	9	12	15	19	23	28	33	39	53	70	89	50
60	4	6	9	11	14	17	20	24	28	38	50	65	60
70	3	4	6	8	10	12	15	18	21	29	38	50	70
80	3	4	5	6	8	9	11	13	16	22	30	38	80
Weight of 1 foot of length of pillar, in pounds.	22.1	24.5	27.0	29.4	31.9	34.4	36.9	39.4	41.9	46.6	51.6	56.6	
Area of ring of solid metal, in square inches.	7.07	7.85	8.64	9.43	10.2	11.0	11.8	12.6	13.4	14.9	16.5	18.1	

Table 1. HOLLOW CYLIND CAST IRON PILLARS.

BREAKING LOADS.—(Continued.) By Gordon's Rule.

CAST IRON. THICKNESS 1 INCH. (Original.)

Length in feet.	Outer Diameter in Inches.												Length in feet.
	12	13	14	15	16	18	20	22	24	27	30	36	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	1188	1301	1415	1530	1645	1874	2103	2330	2557	2896	3236	3915	4
6	1138	1253	1368	1484	1601	1833	2066	2295	2525	2866	3208	3890	6
8	1065	1184	1303	1423	1543	1779	2015	2247	2479	2824	3170	3860	8
10	989	1110	1231	1355	1475	1716	1957	2193	2430	2780	3128	3823	10
12	909	1030	1152	1275	1399	1644	1889	2129	2369	2723	3076	3778	12
14	829	949	1071	1195	1320	1566	1813	2056	2300	2659	3016	3726	14
16	756	878	992	1114	1237	1484	1733	1979	2226	2589	2951	3668	16
18	683	796	913	1034	1155	1401	1651	1899	2147	2515	2879	3604	18
20	618	727	840	958	1077	1320	1568	1817	2066	2437	2805	3536	20
22	559	663	772	887	1002	1241	1486	1734	1982	2356	2726	3464	22
24	508	606	709	818	929	1163	1404	1650	1899	2272	2644	3387	24
26	459	553	651	756	863	1090	1326	1570	1816	2188	2560	3308	26
28	418	506	598	697	800	1020	1250	1489	1733	2108	2475	3226	28
30	380	462	549	644	743	954	1178	1412	1653	2020	2392	3143	30
32	347	424	506	595	689	893	1110	1338	1575	1938	2308	3059	32
34	318	390	467	552	641	836	1046	1268	1500	1857	2225	2974	34
36	292	359	432	511	596	783	984	1199	1427	1779	2143	2889	36
38	268	331	400	475	556	734	928	1136	1361	1704	2063	2804	38
40	247	305	370	441	518	687	874	1076	1296	1630	1984	2720	40
42	229	283	344	411	484	645	825	1019	1232	1560	1909	2636	42
44	212	263	321	383	452	605	776	963	1169	1491	1834	2553	44
46	197	246	299	358	423	569	734	915	1114	1428	1765	2474	46
48	183	229	280	335	397	536	694	869	1060	1367	1696	2396	48
50	170	213	261	314	373	505	656	824	1008	1306	1627	2319	50
55	144	181	222	269	320	438	573	725	898	1172	1474	2135	55
60	124	157	192	233	278	383	503	641	795	1054	1333	1964	60
65	105	134	165	202	242	335	448	568	709	951	1211	1808	65
70	93	118	146	178	213	297	394	507	636	853	1099	1664	70
80	73	92	113	139	168	235	315	408	516	705	914	1414	80
90	58	73	91	112	136	191	257	335	426	586	768	1209	90
100	48	60	75	92	112	157	213	279	356	491	651	1040	100
Weight of one foot of length of pillar, in pounds.													
	108	118	128	138	147	167	187	206	226	255	285	344	
Area of ring of solid metal, in square inches.													
	34.6	37.7	40.8	44.0	47.1	53.4	59.7	66.0	72.2	81.7	91.1	110.0	

CAST IRON. THICKNESS 3 INCHES. (Original.)

Length in feet.	Outer Diameter in Feet.											Length in feet.
	3	3½	4	4½	5	5½	6	7	8	10	12	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
10	10806	12878	14908	16967	18986	21038	23052	27143	31196	39254	47375	10
15	10453	12564	14628	16712	18754	20825	22855	26972	31045	39183	47273	15
20	9996	12150	14250	16369	18440	20533	22585	26737	30837	38962	47131	20
30	8884	11102	13275	15459	17593	19743	21847	26084	30257	38486	46729	30
40	7688	9907	12113	14344	16531	18735	20891	25224	29476	37838	46175	40
50	6554	8702	10888	13126	15341	17580	19780	24107	28532	37037	45484	50
60	5553	7575	9690	11892	14100	16349	18570	23049	27460	36103	44666	60
70	4704	6570	8576	10702	12870	15097	17319	21824	26287	35058	43737	70
80	3998	5699	7570	9595	11692	13873	16070	20566	25048	33924	42712	80
90	3417	4953	6683	8588	10594	12706	14856	19304	23791	32725	41670	90
100	2940	4321	5909	7665	9588	11614	13700	18065	22521	31481	40438	100
110	2547	3788	5238	6888	8677	10606	12613	16869	21270	30213	39220	110
125	1960	2954	4400	5864	7483	9257	11132	14650	19448	27664	36692	125
150	1532	2350	3353	4547	5900	7417	9058	12701	16668	25182	34127	150
Weight of one foot of length of pillar, in pounds.												
	972	1150	1325	1503	1678	1856	2031	2388	2740	3444	4153	

Coefficients of safety for hollow cast iron pillars.

Mr. James B. Francis, of Lowell, Mass., a high authority, in his "Tables of Cast Iron Pillars." recommends that in order to allow for unequal loading, imperfect casting, bad end bearings, side blows, &c., we should not take the safe load at more than **one-fifteenth** of Hodgkinson's breaking load, if the pillars are roughly cast, and the ends not perfectly planed and adjusted; and **one-fifth** when they are so, and the loads about equally distributed.

HOLLOW CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, of breaking loads in tons of hollow cylindrical wrought iron pillars, with flat ends, perfectly true, and firmly fixed, and the loads pressing equally on every part of the top. Calculated by Gordon's formula.

(Original.)

Length in feet.	WROUGHT IRON. THICKNESS ⅛ INCH. Outer diameter in inches.										Length in feet.
	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	
	BREAKING LOAD.										
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	3.64	5.27	6.88	8.50	10.1	11.7	13.2	14.8	16.4	18.0	1
2	2.94	4.64	6.32	8.00	9.6	11.2	12.8	14.5	16.1	17.8	2
3	2.30	3.86	5.57	7.28	8.9	10.6	12.2	13.9	15.6	17.3	3
4	1.77	3.18	4.74	6.36	8.1	9.9	11.6	13.3	15.0	16.7	4
5	1.36	2.51	4.07	5.66	7.3	9.1	10.8	12.5	14.2	16.0	5
6	1.04	2.03	3.46	4.91	6.6	8.3	9.9	11.6	13.4	15.2	6
7	.81	1.65	2.91	4.24	5.7	7.4	9.1	10.8	12.6	14.4	7
8	.61	1.36	2.46	3.67	5.1	6.7	8.3	9.9	11.7	13.5	8
9	.50	1.05	2.08	3.18	4.5	6.0	7.5	9.1	10.8	12.6	9
10	.41	.95	1.75	2.77	4.0	5.4	6.9	8.4	10.1	11.8	10
11	.34	.81	1.52	2.41	3.6	4.8	6.2	7.7	9.3	11.0	11
12	.29	.70	1.34	2.14	3.2	4.3	5.6	7.0	8.6	10.2	12
13	.24	.60	1.16	1.88	2.8	3.9	5.2	6.5	8.0	9.5	13
14	.21	.53	1.08	1.69	2.5	3.5	4.7	6.0	7.4	8.9	14
15	.19	.47	.91	1.50	2.3	3.2	4.3	5.5	6.9	8.3	15
16	.18	.42	.84	1.38	2.1	2.9	4.0	5.1	6.4	7.7	16
18	.14	.33	.67	1.11	1.7	2.4	3.4	4.4	5.6	6.8	18
20	....	.27	.56	.91	1.4	2.0	2.8	3.7	4.7	5.8	20
25	....	....	....	....	.9	1.4	2.0	2.6	3.4	4.2	25

Weight of one foot of length of pillar, in pounds.

.820	1.15	1.47	1.80	2.13	2.45	2.78	3.11	3.43	3.77

Area of ring of solid metal, in square inches.

.246	.344	.442	.540	.638	.736	.835	.933	1.03	1.13

Length in feet.	WROUGHT IRON. THICKNESS ¼ INCH. Outer diameter in inches.											Length in feet.
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
	BREAKING LOAD.											
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	21.9	25.4	28.3	31.4	34.5	40	47	53	60	66	72	1
2	21.1	24.8	27.6	30.7	33.9	40	47	53	60	66	72	2
3	19.9	23.1	26.4	29.7	33.0	39	46	52	59	65	71	3
4	18.6	21.8	25.3	28.5	31.9	38	45	51	58	64	71	4
5	17.0	20.4	23.5	27.3	30.7	37	44	50	57	63	70	5
6	15.4	18.8	22.1	25.7	29.2	36	43	49	56	62	69	6
7	13.9	17.3	20.5	23.8	27.8	34	41	47	54	61	68	7
8	12.5	15.6	19.1	22.3	25.9	32	40	46	53	60	67	8
9	11.2	14.2	17.5	20.6	24.3	30	38	44	51	58	65	9
10	10.0	13.0	16.1	19.1	22.7	29	37	43	50	57	64	10
11	9.0	10.7	15.7	17.6	21.1	27	35	41	48	55	62	11
12	8.1	10.6	13.5	16.4	19.6	26	33	40	46	54	61	12
13	7.3	9.6	12.4	15.1	18.2	24	31	38	44	52	59	13
14	6.6	8.8	11.3	14.0	17.0	23	30	36	43	51	57	14
15	6.0	8.0	10.4	12.9	15.8	21	28	34	41	49	55	15
16	5.5	7.3	9.5	12.0	14.6	20	27	33	40	47	54	16
18	4.5	6.0	8.0	10.3	12.7	18	24	30	37	43	50	18
20	3.8	5.1	6.8	8.7	11.0	16	21	27	34	40	47	20
25	....	....	....	....	7.9	12	16	21	27	33	39	25
30	....	....	....	....	....	......	13	17	22	27	32	30
35	....	....	....	....	....	......	10	14	18	22	27	35
40	....	....	....	....	....	......	....	....	14	18	23	40
45	....	....	......	....	....	......	....	....	11	15	19	45
50	....	....	....	....	....	......	....	....	8	12	16	50

Weight of one foot of length of pillar, in pounds.

4.60	5.23	5.90	6.53	7.20	8.50	9.83	11.1	12.4	13.7	15.0

Area of ring of solid metal, in square inches.

1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.51

HOLLOW CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, (Continued.) (Original.)

WROUGHT IRON. THICKNESS ½ INCH.

Length in feet.	Outer diameter in inches. 5	5½	6	6½	7	7½	8	8½	9	10	11	12	Length in feet.
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	112	125	139	152	166	177	189	201	214	238	263	290	2
4	110	123	136	149	163	174	186	199	212	237	262	289	4
6	106	119	132	145	158	171	184	197	210	235	261	288	6
8	101	114	127	140	154	167	181	194	207	232	258	284	8
10	95	108	123	136	149	162	176	189	203	228	254	280	10
12	89	102	116	129	143	157	171	185	199	224	250	276	12
14	82	95	108	122	137	151	165	179	194	219	245	272	14
16	76	89	103	117	131	145	160	173	187	213	240	268	16
18	70	83	97	110	124	138	153	166	180	207	235	263	18
20	64	77	91	104	117	131	145	159	173	201	227	257	20
22	58	70	83	96	109	123	138	151	165	192	220	250	22
25	52	64	76	89	102	115	129	143	157	183	212	241	25
30	42	52	63	74	87	100	113	127	141	167	195	224	30
35	34	43	53	64	75	87	99	112	125	151	178	207	35
40	27	35	44	53	64	75	86	98	110	135	163	190	40
45	23	30	38	46	55	65	76	87	98	123	148	174	45
50	19	24	32	38	47	56	66	76	87	109	133	158	50
60	15	19	24	29	36	43	51	60	69	88	109	132	60
70	11	14	18	23	28	34	40	48	56	73	91	111	70
80	9	11	14	18	22	27	32	37	44	57	74	93	80
90	7	9	11	14	18	22	26	31	36	49	63	78	90
100	6	7	9	12	15	18	22	26	30	41	53	66	100

Weight of one foot of length of pillar, in pounds.

23.6 | 26.2 | 28.8 | 31.4 | 34.0 | 36.6 | 39.3 | 42.0 | 44.7 | 49.7 | 55.0 | 60.8

Area of ring of solid metal, in square inches.

7.07 | 7.85 | 8.64 | 9.43 | 10.2 | 11.0 | 11.8 | 12.6 | 13.4 | 14.9 | 16.5 | 18.1

Table 4, (Continued.) (Original.)

WROUGHT IRON. THICKNESS 1 INCH.

Length in feet.	Outer diameter in inches. 13	14	15	16	17	18	20	22	24	26	28	30	Length in feet.
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	603	653	704	753	805	854	955	1056	1157	1257	1357	1458	1
10	588	638	691	742	795	846	949	1049	1149	1248	1354	1457	10
20	543	595	651	702	759	810	913	1016	1120	1223	1327	1430	20
30	479	536	594	645	699	758	866	973	1077	1186	1289	1394	30
40	415	470	528	584	636	691	806	912	1027	1130	1237	1348	40
50	355	405	462	516	570	627	740	848	961	1067	1179	1294	50
60	300	348	400	452	505	559	669	781	891	1005	1115	1228	60
70	256	300	348	398	448	499	606	715	824	936	1046	1160	70
80	215	255	298	344	392	440	543	649	757	868	978	1094	80
90	185	222	261	303	347	392	489	590	694	800	910	1023	90
100	157	190	225	262	303	345	436	532	631	735	843	955	100
110	134	162	193	227	264	302	386	474	568	666	770	877	110
125	111	136	162	192	225	260	336	416	505	596	697	789	125
150	82	101	122	145	171	198	262	328	405	485	574	666	150
175	62	78	95	112	133	155	208	266	331	400	478	560	175
200	49	60	74	89	106	124	168	216	269	328	395	467	200

Weight of one foot of length of pillar, in pounds.

128 | 136 | 147 | 157 | 168 | 178 | 199 | 220 | 241 | 262 | 283 | 304

Area of ring of solid metal, in square inches.

37.7 | 40.8 | 44.0 | 47.1 | 50.3 | 53.4 | 59.7 | 66.0 | 72.3 | 78.5 | 84.8 | 91.1

The breaking loads for less thicknesses may safely be assumed to diminish at the same rate as the thickness.

PENCOYD FLOOR SECTIONS.

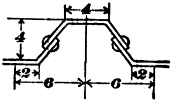
L = span, in feet.
C = coefficient.
W = distributed load, in lbs, per foot of floor width.

$$W = \frac{C}{L}.$$

Corrugated flooring, for bridges and buildings.
W = load producing fiber stress of 15,000 lbs per square inch.

SECTION 210 M.

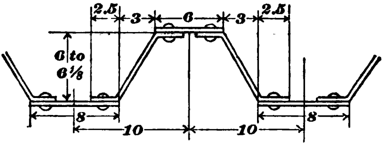
Dimensions in inches.



Thickness, inches. Web.	Flange.	Weight, lbs per sq ft.	C
3/16	1/4	14.8	44,000
15/64	5/16	18.4	55,000
9/32	3/8	21.9	66,000
21/64	7/16	25.5	77,400
3/8	1/2	29.1	88,800

SECTION 260 M.

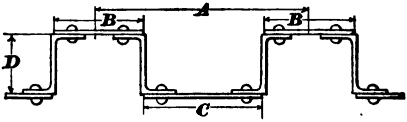
Dimensions in inches.



Thickness, inches. Web.	Flange.	Weight, lbs per sq ft.	C
1/4	1/4 to 5/8	20.0 to 30.7	105,000 to 186,000
5/16	3/8 to 3/4	26.5 to 37.2	148,000 to 224,000
3/8	3/8 to 3/4	29.4 to 40.1	153,000 to 237,000

Z Bar Flooring.

W = safe load.



Section No.	Dimensions, in inches. A	B	C	D	Thickness, ins. Z bars.	Plates.	Weight, lbs per sq ft.	C
1	15	6	9	4	1/4	1/4 to 1/2	25.9 to 36.1	93,400 to 147,400
					5/16		29.1 to 39.3	104,000 to 157,000
					3/8		32.3 to 42.5	114,400 to 167,000
2	18	8	10	5	5/16	5/16 to 9/16	32.1 to 42.3	143,000 to 209,400
					3/8		35.2 to 45.4	155,000 to 221,400
					7/16		38.4 to 48.6	166,400 to 233,000
3	21	9	12	6	3/8	3/8 to 5/8	39.3 to 49.5	203,400 to 281,000
					7/16		42.4 to 52.6	217,400 to 294,000
					1/2		45.5 to 55.7	231,000 to 307,200

WEIGHT AND STRENGTH OF IRON CHAINS.

Table of strength of chains.

Chains of superior iron will require ¼ to ⅓ more to break them. (Original.)

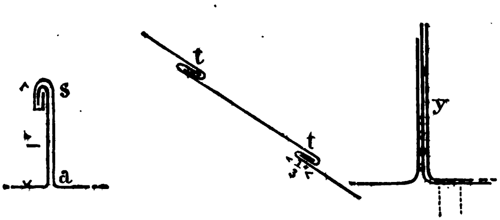
Diam of rod of which the links are made.	Weight of chain per ft run.	Breaking strain of the chain.		Diam of rod of which the links are made.	Weight of chain, per ft run.	Breaking strain of the chain.	
Ins.	Pds.	Pds.	Tons.	Ins.	Pds.	Pds.	Tons.
3-16	.5	1731	.773	1	10.7	49280	22.00
¼	.8	3069	1.37	1⅛	12.5	59226	26.44
5-16	1.	4794	2.14	1¼	16.	73114	32.64
⅜	1.7	6922	3.09	1⅜	18.3	88301	39.42
7-16	2.	9408	4.20	1½	21.7	105280	47.00
½	2.5	12320	5.50	1⅝	26.	123514	55.14
9-16	3.2	15590	6.96	1¾	28.	143293	63 97
⅝	4.3	19219	8.58	1⅞	32.	164505	73.44
11-16	5.	23274	10.39	2	38.	187152	83.55
¾	5.8	27687	12.36	2¼	54.	224448	100.2
13-16	6.7	32301	14.42	2½	71.	277088	123.7
⅞	8.	37632	16.80	2¾	88.	335328	149.7
15-16	9.	43277	19.32	3	105.	398944	178.1

The links of ordinary iron chains are usually made as short as is consistent with easy play, in order that they may not become bent when wound around drums, sheaves, &c.; and that they may be more easily handled in slinging large blocks of stone, &c. U. S. Government experiments, 1878, prove that studs weaken the links.

When so made, their weight per foot run is quite approximately 3⅓ times that of a single bar of the round iron of which they are composed. Since each link consists of two thicknesses of bar, it might be supposed that a chain would possess about double the strength of a single bar; but the strength of the bar becomes reduced about 30 per cent, by being formed into links; so that the chain has but about 70 per cent of the strength of two bars. As a thick bar will not sustain as heavy a unit stress as a thinner one, so of course, stout chains are proportionally weaker than slighter ones. In the foregoing table, 20 tons *per square inch*, is assumed as the average breaking strain of a single straight bar of ordinary rolled iron, 1 inch in diameter or 1 inch square; 19 tons, from 1 to 2 inches; and 18 tons, from 2 to 3 inches. Deducting 30 per cent from each, we have as the breaking strain of the two bars composing each link, as follows: 14 tons *per square inch*, up to 1 inch diameter; 13.3 tons, from 1 to 2 inches; and 12.6 tons, from 2 to 3 inches diameter; and upon these assumptions the table is based. The weights are approximate; depending upon the exactness of diameter of the iron, and shape of link.

TIN AND ZINC.

The pure metal is called block tin. When perfectly pure, (which it rarely is, being purposely adulterated, frequently to a large proportion, with the cheaper metals lead or zinc,) its sp grav is 7.29; and its weight per cub ft is 455 lbs. It is sufficiently malleable to be beaten into tin foil, only $\frac{1}{1000}$ of an inch thick. Its tensile strength is but about 4600 lbs per sq inch; or about 7000 lbs when made into wire. It melts at the moderate temperature of 442° Fah. Pure block tin is not used for common building purposes; but thin plates of sheet iron, covered with it on both sides, constitute the *tinned plates*, or, as they are called, the *tin*, used for covering roofs, rain pipes, and many domestic utensils. For roofs it is laid on boards.



The sheets of tin are united as shown in this fig. First, several sheets are joined together in the shop, end for end, as at *t t*; by being first bent over, then hammered flat, and then soldered. These are then formed into a roll to be carried to the roof; a roll being long enough to reach from the peak to the eaves. Different rolls being spread up and down the roof, are then united along their sides by simply being bent as at *a* and *s*, by a tool for that purpose. The roofers call the bending at *s* a *double groove*, or *double lock;* and the more simple ones at *t*, a *single groove*, or lock.

To hold the tin securely to the sheeting boards, pieces of the tin 3 or 4 ins long, by 2 ins wide, called cleats, are nailed to the boards at about every 18 ins along the joints of the rolls that are to be united, and are bent over with the double groove *s*. This will be understood from *y*, where the middle piece is the cleat, before being bent over. The nails should be 4-penny slating nails, which have broader heads than common ones. As they are not exposed to the weather, they may be of plain iron.

Much use is made of what is called **leaded tin, or ternes,** for roofing. It is simply sheet-iron coated with lead, instead of the more costly metal tin. It is not as durable as the tinned sheets, but is somewhat cheaper.

The best plates, both for tinning and for ternes, are made of charcoal iron; which, being tough, bears bending better. Coke is used for cheaper plates, but inferior as regards bending. In giving orders, it is important to specify whether charcoal plates or coke ones are required; also whether *tinned* plates, or *ternes*.

Tinned and leaded sheets of Bessemer and other cheap **steel,** are now much used. They are sold at about the price of charcoal tin and terne plates.

There are also in use for roofing, certain compound metals which resist tarnish better than either lead, tin, or zinc; but which are so fusible as to be liable to be melted by large burning cinders falling on the roof from a neighboring conflagration.

A roof covered with tin or other metal should, if possible, slope not much *less* than five degrees, or about an inch to a foot; and at the eaves there should be a sudden fall into the rain-gutter, to prevent rain from backing up so as to overtop the double-groove joint *s*, and thus cause leaks. Where coal is used for fuel, tin roofs should receive two coats of paint when first put up, and a coat at every 2 or 3 years after. Where wood only is used, this is not necessary; and a tin roof, with a good pitch, will last 20 or 30 years.

Two good workmen can put on, and paint outside, from 250 to 300 sq ft of tin roof, per day of 8 hours.

Tinned iron plates are sold by the box. These boxes, unlike glass, have *not* equal areas of contents. They may be designated or ordered either by their names or sizes. Many makers, however, have their private brands in addition; and some of these have a much higher reputation than others.

Table of Tinned and Terne Plates.

Caution.—Boxes often contain considerably less weight of tin plate than the table requires; the plates being rolled thin and plated thin, in order to enable mechanics to get pay for more material than they furnish.

The *marks* indicate the *thicknesses*, approximately as follows:

Mark.	Number Birmingham wire gauge.	Ins.	Lbs per sq ft.	Mark.	Number Birmingham wire gauge.	Ins.	Lbs per sq ft.
IC	30	.012	.48	DC	27	.016	.64
IX	28	.014	.56	DX	25	.020	.80
IXX	27	.016	.64	DXX	23	.025	1.00
IXXX	26	.018	.72	DXXX	22	.028	1.13
IXXXX	25	.020	.80	DXXXX	21	.032	1.29

Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.	Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.	Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.
9 × 18	IC	225	130	13 × 13	IXX	225	194	16 × 16	IC	225	205
"	IX	"	162	13 × 26	IC	112	135	"	IX	"	256
10 × 10	IC	"	80	"	IX	"	169	"	IXX	"	294
"	IX	"	100	"	IXX	"	194	16 × 19	IX	112	152
10 × 14	IC	"	112	14 × 14	IC	225	156	17 × 17	IX	"	141
"	IX	"	140	"	IX	"	196	"	IXX	"	166
"	IXX	"	161	"	IXX	"	225	17 × 25	DX	100	252
"	IXXX	"	182	"	IXXX	"	254	"	DXX	"	294
"	IXXXX	"	203	14 × 20	IC	112	112	18 × 18	IX	112	162
10 × 20	IC	"	160	"	IX	"	140	"	IXX	"	189
"	IX	"	200	"	IXX	"	161	"	IXXXX	"	235
11 × 11	IC	"	97	"	IXXX	"	182	20 × 20	IX	"	200
"	IX	"	121	"	IXXXX	"	203	"	IXX	"	230
11 × 22	IC	112	97	14 × 22	IX	"	154	"	IXXX	"	260
"	IX	"	121	"	IXX	"	177	"	IXXXX	"	290
"	IXX	"	139	14 × 24	IX	"	168	20 × 28	IC	"	224
12 × 12	IC	225	112	"	IXX	"	198	"	IX	"	280
"	IX	"	140	14 × 25	IC	"	140	"	IXX	"	322
"	IXX	"	161	"	IX	"	175	"	IXXX	"	364
12 × 24	IC	112	115	"	IXX	"	201	"	IXXXX	"	406
"	IX	"	144	14 × 26	IXXX	"	237				
"	IXX	"	166	14 × 28	IC	"	157				
"	IXXX	"	187	"	IX	"	196	**Terne Plates.**			
12½ × 17	DC	100	98	"	IXX	"	225				
"	DX	"	120	14 × 30	IXX	"	241	10 × 20	IC	112	80
"	DXX	"	147	14 × 31	IX	"	217	"	IX	"	100
"	DXXX	"	168	"	IXX	"	240	14 × 20	IC	"	112
"	DXXXX	"	189	15 × 15	IX	225	225	"	IX	"	140
13 × 13	IC	225	135	"	IXX	"	259	20 × 28	IC	"	224
"	IX	"	169	"	IXXX	"	326	"	IX	"	280

Sheets of larger size may be made to special order; those of tinned iron, in England; but leaded ternes are made in Philada also, and elsewhere.

A box of 225 sheets of 13¾ by 10. contains 214.84 sq ft; but, allowing for overlapping, it will cover but about 150 sq ft of roof; even without any allowance for the waste which occurs in cutting away portions in order to fit at angles, &c.

To find the area of roof covered by any sheet, first deduct 2 ins from its width, and 1 inch from its length.

Zinc, in sheets, and laid in the same manner as slates, is much used in some parts of Europe for roofing. By exposure to the weather, it soon becomes covered by a thin film of white oxide, which protects it from further injury, and renders the roof very durable. Corrugated sheet zinc is also used. See Galvanized Sheet Iron.

Zinc sheets are usually about 3 ft by 7 or 8 ft. The gauge differs from that of iron; thus No 13 is .032 of an inch thick, or 1.22 lbs per sq ft; No 14 = .035 inch, and 1.35 lbs; No 15 = .042 inch, and 1 49 lbs; No 16 = 049 inch, and 1.62 lbs per sq ft. Any of these numbers may be used on roofs, for which purpose it should be very pure.

Water kept in zinc vessels is said to become injurious to health; and recently an outcry has on that account arisen against galvanized-iron service-pipes in dwellings. Yet such have been in use for many years in New England, Philada, and elsewhere, without as yet any deleterious effects. This is possibly owing to the fact that service-pipes being short, the water is usually all drawn through them several times a day; and hence does not remain in contact with the zinc or lead long enough to acquire a poisonous character. In taking possession of a house in which the water has remained stagnant in the service-pipes for some considerable time, such water should all be run to waste; otherwise sickness may ensue from its use.

Roof copper is usually in sheets of 2½ feet × 5 feet; or 12½ square feet, weighing 10 to 14 ℔s per sheet; and is laid on boards.* No solder is used in the horizontal joints as it is in tin roofs; but both the horizontal and the sloping joints are formed by only overlapping and bending the sheets, much as shown by the figs on page 916; except that the horizontal joints are bent or locked together, and then flattened down close.

Sheet lead. List of **standard weights** in ℔s per square foot. Thicknesses in decimals of an inch.

Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.
2.5	.042	4	.068	6	.102	8	.136	10	.170	14	.237
3	.051	5	.085	7	.119	9	.153	12	.203	16	.271

Weight of Metal Balls.

W = Weight of ball, in pounds. **D** = Diameter of ball, in inches.

Lead = (700 ℔s per cub ft) $W = 0.212106\ D^3$; $\log W = \bar{1}.326\ 5529 + 3 \log D$

Copper = (550 ℔s per cub ft) $W = 0.166655\ D^3$; $\log W = \bar{1}.221\ 8176 + 3 \log D$

Brass = (500 ℔s per cub ft) $W = 0.151504\ D^3$; $\log W = \bar{1}.180\ 4249 + 3 \log D$

Steel and Wrought Iron = (485 ℔s per cub ft) $W = 0.146959\ D^3$; $\log W = \bar{1}.167\ 1966 + 3 \log D$

Cast Iron = (450 ℔s per cub ft) $W = 0.136354\ D^3$; $\log W = \bar{1}.134\ 6674 + 3 \log D$

For steel, wrought iron and cast iron balls, see also tables, pp 875 and 877.

Lead Pipe.
Standard Sizes.

Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wt in ℔s per ft.	Inner diam.	Thickness.	Wt in ℔s per ft.
Ins.	Ins.		Ins.	Ins.		Ins.	Ins.		Ins.	Ins.	
⅜	.06	7 ℔s R	¾	.08	16 ℔s R	1½	.14	3.5	3	3-16	9
"	.08	10 oz F	"	.10	1¼ ℔s F	"	.17	4.25	"	¼	12
"	.12	1 ℔ F	"	.12	1¾ ℔s F	"	.19	5.	"	5-16	16
"	.16	1¼ ℔s F	"	.16	2¼ ℔s F	"	.23	6.5	"	⅜	20
"	.19	1½ ℔s F	"	.20	3 ℔s F	"	.27	8	3½	3-16	9.5
½	.07	9 ℔s R	"	.23	3½ ℔s F	1¾	.13	4	"	¼	15
"	.09	¾ ℔ F	"	.30	4¾ ℔s F	"	.17	5	"	5-16	18.5
"	.11	1 ℔ F	1	.10	24¾ ℔s R	"	.21	6.5	"	⅜	22
"	.13	1¼ ℔s F	"	.11	2 ℔s F	"	.27	8.5	4	3-16	12.5
"	.16	1¾ ℔s F	"	.14	2½ ℔s F	2	.15	4.75	"	¼	16
"	.19	2 ℔s F	"	.17	3¼ ℔s F	"	.18	6	"	5-16	21
"	.25	3 ℔s F	"	.21	4 ℔s F	"	.22	7	"	⅜	25
⅝	.08	12 ℔s R	"	.24	4¾ ℔s F	"	.27	9	4½	3-16	14
"	.09	1 ℔ F	1¼	.10	2 ℔s F	2½	3-16	8	"	¼	18
"	.13	1½ ℔s F	"	.12	2½ ℔s F	"	¼	11	5	¼	20
"	.16	2 ℔s F	"	.14	3 ℔s F	"	5-16	14	"	⅜	31
"	.20	2½ ℔s F	"	.16	3¾ ℔s F	"	⅜	17			
"	.22	2¾ ℔s F	"	.19	4¾ ℔s F						
"	.25	3½ ℔s F	"	.25	6 ℔s F						

Lead service pipes for single dwellings in Philadelphia are usually of from ½ inch bore, wt 1 to 2½ ℔s; to ⅝ inch bore, wt 1½ to 3 ℔s per ft run, according to head. They **rarely burst** from sudden closing of stopcocks; but sometimes do so from the freezing of the contained water.

---

* **To which it is held by copper cleats;** as at Fig y, page 916.

ROLLED LEAD, COPPER, and BRASS: Sheets and Bars.

Thickness or Diameter, or side, in Inches.	LEAD.			COPPER.			BRASS.			Thickness or Diameter, or side, in Inches.
	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	
1-32	1.86	.005	.004	1.44	.004	.003	1.36	.004	.003	1-32
1-16	3.72	.019	.015	2.89	.015	.012	2.71	.014	.011	1-16
3-32	5.58	.044	.034	4.33	.034	.027	4.06	.032	.025	3-32
⅛	7.44	.078	.061	5.77	.060	.047	5.42	.056	.044	⅛
5-32	9.30	.121	.095	7.20	.094	.074	6.75	.088	.069	5-32
3-16	11.2	.174	.137	8.66	.135	.106	8.13	.127	.100	3-16
7-32	13.0	.237	.187	10.1	.184	.144	9.50	.173	.136	7-32
¼	14.9	.310	.244	11.5	.240	.189	10.8	.226	.177	¼
5-16	18.6	.485	.381	14.4	.376	.295	13.5	.353	.277	5-16
⅜	22.3	.698	.548	17.3	.541	.425	16.3	.508	.399	⅜
7-16	26.0	.950	.746	20.2	.736	.578	19.0	.691	.543	7-16
½	29.8	1.24	.974	23.1	.962	.755	21.7	.903	.709	½
9-16	33.5	1.57	1.23	26.0	1.22	.955	24.3	1.14	.900	9-16
⅝	37.2	1.94	1.52	28.9	1.50	1.18	27.1	1.41	1.11	⅝
11-16	40.9	2.34	1.84	31.7	1.82	1.43	29.8	1.70	1.34	11-16
¾	44.6	2.79	2.19	34.6	2.16	1.70	32.5	2.03	1.60	¾
13-16	48.3	3.27	2.57	37.5	2.55	1.99	35.2	2.38	1.87	13-16
⅞	52.1	3.80	2.98	40.4	2.94	2.31	37.9	2.76	2.17	⅞
15-16	56.0	4.37	3.42	43.3	3.38	2.65	40.6	3.18	2.49	15-16
1.	59.5	4.96	3.90	46.2	3.85	3.02	43.3	3.61	2.84	1.
1⅛	66.9	6.27	4.92	52.0	4.87	3.82	48.7	4.57	3.60	1⅛
1¼	74.4	7.75	6.09	57.7	6.01	4.72	54.2	5.64	4.43	1¼
1⅜	81.8	9.37	7.37	63.5	7.28	5.72	59.6	6.82	5.37	1⅜
1½	89.3	11.2	8.77	69.3	8.65	6.80	65.0	8.12	6.38	1½
1⅝	96.7	13.1	10.3	75.1	10.2	7.98	70.4	9.53	7.49	1⅝
1¾	104.	15.2	11.9	80.8	11.8	9.25	75.9	11.1	8.68	1¾
1⅞	112.	17.5	13.7	86.6	13.5	10.6	81.3	12.7	9.97	1⅞
2.	119.	19.8	15.6	92.3	15.4	12.1	86.7	14.4	11.3	2.

Seamless brass tubes. Principal sizes. Extras, in cents per pound, over base price. For base price, see price list.
Copper tubes, 3 cents per pound extra.

Thickness.		Outer Diameter, inches.										
Stubs gage.	Ins.	¼	½	1	1½	2	3	4	5	6	7	7¾
4	0.238	....	12	6	3	1	1	2	5	9	13	18
11	0.120	....	"	"	"	"	"	"	"	"	"	"
16	0.065	....	13	8	4	4	4	7	11	15	19	24
18	0.049	40	15	9	6	6	7	11	15	19	23	28
20	0.035	43	18	13	10	9	11	15	19	23	27	32
22	0.0295	50	21	16	15	13	16	20	....	....	....	....
25	0.0230	65	28	22	24	....	....	....	....	....	....	....

Average ultimate tensile strength of Metals.

The ultimate tensile or pulling load per square inch of any material is frequently called its *constant*, *coefficient*, or *modulus* of tension, or of tensile strength.	Pounds per sq. inch.	Tons per sq. in.
Antimony, cast	1000	.45
Bismuth, cast	3200	1.4
Brass, cast 3 to 13 tons, say 18000 to 29000 lbs	23500	10.5
" wire, unannealed or hard, 80000. Annealed	49000	22
Bronze, phosphor wire, hard, 150000. Annealed	63000	28.1
Copper, cast 18000 to 30000	24000	10.7
" sheet	30000	13.4
" bolts, 28000 to 38000	33000	14.7
" wire (annealed 16 tons); unannealed	60000	26.8
Gold, cast	20000	8.9
" wire, 25000 to 30000	27500	12.3
Gun metal of copper and tin, 23000 to 55000	39000	17.4
" " cast iron, U. S. ordnance, 36000 to 40000	38000	17
Iron, cast, English.........13400 to 22400	17900	8
" " ordinary pig..13000 to 16000	14500	6.47
American cast iron averages one-fourth more than the above.		
Average cast iron, when sound, stretches about .00018; or 1 part in 5555 of its length; or ⅛ inch in 57.9 ft. for every ton of tensile strain per sq inch, up to its elastic limit, which is at about ⅓ its break-strain. The extent of stretching, however, varies much with the quality of the iron; as in wrought-iron.		
Cast, malleable, annealed 18 to 25 tons.	48160	21.5
Iron and Steel, rolled.—See Digests of Specifications.		
Lead, cast, 1700 to 2400. by author	2050	0.92
" wire, 1200 to 1600. Pipe 1600 to 1700. " "	1650	0.74
Platinum wire, annealed, 32000. Unannealed	56000	25
Steel and Iron, rolled.—See Digests of Specifications.		
Silver, cast	41000	18.3
Tin, English block	4600	2.0
" wire	7000	3.1
Zinc, cast...3000 to 3700; (the last by author)	3350	1.5

Large bars of metal bear less per sq inch than small ones.

Iron bars re-rolled cold have tensile strength increased 25 to 50 per ct, with no increase of density. They are said to lose this strength if reheated.

Sheet lead is sometimes placed in the joints of stone columns, with a view to equalize the pressure, and thus increase the strength of the column. But experiments have proved that the effect is directly the reverse, and that the column is *materially weakened* thereby.

Average crushing load for Metals.

It must be remembered that these are the loads for pieces but two or three times their least side in height. As the height increases, the crushing load diminishes. See "Strength of Pillars."

	Pounds per sq. inch.	Tons per sq. inch.
Cast Iron, usually	85000 to 125000	38 to 56
It is usually assumed at 100000 lbs, or say 45 tons per sq inch. Its crushing strength is usually from 6 to 7 times as great as its tensile. Within its average elastic limit of about 15 tons per sq inch, average cast iron shortens about 1 part in 5555; or ⅛ inch in 58 ft under each ton per sq inch of load; or about twice as much as average wrought iron. Hence at 15 tons per sq inch it will shorten about 1 part in 370; or full ⅛ inch in 4 feet. Different cast irons may however vary 10 to 15 per ct either way from this.		
U. S. Ordnance, or gun metal; Some	175000	78.1
Wrought iron, within elastic limit	22400 to 35840	10 to 16
Its elastic limit under pressure averages about 13 tons per sq inch. It begins to shorten perceptibly under 8 to 10 tons, but recovers when the load is removed. With from 18 to 20 tons, it shortens *permanently*, about $\frac{1}{80}$th part of its length; and with from 27 to 30 tons, about $\frac{1}{16}$th part, as averages. The crushing weights therefore in the table are not those which absolutely mash wrought iron entirely out of shape, but merely those at which it yields too much for most practical building purposes. About 4 tons per sq inch is considered its average safe load, in pieces not more than 10 diams long; and will shorten it ⅛ inch in 30 ft. average.	29120	13
Brass, reduced $\frac{1}{10}$th part in length, by 51000; and ½ by	165000	73.6
Copper, (cast,) crumbles	117000	52.2
(wrought) reduced ⅛th part in length, by	103000	46.0
Tin, (cast,) reduced $\frac{1}{16}$th in length, by 8800; and ⅓ by	15500	6.92
Lead, (cast,) reduced ½ of its length, by 7000 to 7700	7350	3.28
" By writer. A piece 1 inch sq, 2 ins high, at 1200 lbs the compression was 1-200 of the ht, at 2000, 1-29; at 3000, 1-8; at 5000, 1-3; at 7000, 1-2 of the ht.		
Spelter or **Zinc,** (cast.) By writer. A piece 1 inch square, 4 ins high, at 2000 lbs was compressed 1-400 of its ht; at 4000, 1-200; at 6000, 1-100; at 10000, 1-38; at 20000, 1-15; at 40000 yielded rapidly, and broke into pieces.		
Steel, 224000 lbs or 100 tons shorten it from .2 to .4 part.		
" American. Black Diamond steel-works, Pittsburg, Penn. experiments by Lieut W. H. Shock, U. S. N., on pieces ½ in square; and 3½ ins, or 7 sides long.		
" Untempered. 100100 to 104000	102050	45.6
" Heated to light cherry red, then plunged into oil of 82° Fah, 173200 to 199200	186200	83.1
" Heated to light cherry red, then plunged into water of 79° Fah; then tempered on a heated plate, 325400 to 340800	333100	148.7
" Heated to light cherry red, then plunged into water of 79° Fah, 275600 to 400000	337800	150.8
" **Elastic limit,** 15 to 27 tons	47040	21
" **Compression,** within elas limit averages abt 1 part in 13300, or .1 of an inch in 111 ft per ton per sq inch; or .1 of an inch in 5.3 ft under 21 tons per sq inch.		
Best steel knife edges, of large R R weigh scales are considered safe with 7000 lbs pres per lineal inch of edge; and solid cylindrical steel **rollers** under bridges, and *rolling on steel*, safe with $\sqrt{\text{diam in ins}} \times 3\,100\,000$, in lbs per lineal inch of roller parallel to axis. **And per the same, for**		

Solid cast iron wheels rolling on wrought iron, $\sqrt{\text{Diam ins}} \times 352\,000.$

" " " " " " cast iron, $\sqrt{\text{Diam ins}} \times 222\,222.$

Solid steel " " " steel, $\sqrt{\text{Diam ins}} \times 1\,300\,000.$

" " " " " wrought iron, $\sqrt{\text{Diam ins}} \times 1\,024\,000.$

" " " " " cast iron, $\sqrt{\text{Diam ins}} \times 850\,000.$

From "Specifications for Iron Drawbridge at Milwaukee," by Don J. Whittemore, C. E.

Average ultimate tensile strengths of Stone, etc.

The strengths in all these tables may readily be one-third part more or less than our averages.	Pounds per sq. inch.	Tons per sq. ft.
Brick, 40 to 400	220	14.1
Caen stone, 100 to 200	150	9.7
Cement and concrete, see articles, Cement and Concrete.		
Glass, 2500 to 9000	5750	369.6
Glue holds wood together with from 300 to 800	550	35
Horn, ox	9000	579
Ivory	16000	1029
Leather belts, 1500 to 5000. Good	3000	193
Mortar, common, 6 mos old, 10 to 20	15	.96
Marble, strong, wh. Italy.*	1034	66.5
" Champlain, variegated *	1666	107.1
" Glenn's F'lls, N.Y. blk,* 750 to 1034	892	57.4
" Montg'y co, Pa, gray *	1175	75.6
" " white *	734	47.2
" Lee, Mass, white.*	875	56.3
" Manchester, Vt,* 550 to 800	675	43.4
" Tennessee, variegated *	1034	66.5
Oolites, 100 to 200	150	9.7
Plaster of Paris, well set.	70	4.5
Rope, Manilla, best	12000	771
" hemp, best	15000	965
Sandstone, Ohio *	106	6.75
" Pictou, N. S.*	434	27.9
" Conn, red.*	590	37.9
Slate, Lehigh *	2475	159.1
" Peach bot'm,* 3025 to 4600	3812	245.1
Stone, Ransome's artif.	300	19.3
Whalebone	7600	489

---

* **By the author's trials** with one of Riehlé's testing machines. Sections broken 1½ sq inches.

Ultimate average crushing loads in tons, per square foot, for stones, &c. The stones are supposed to be ON BED, and the heights of all to be from 1.5 to 2 times the least side. Stones generally begin to crack or split under about one-half of their crushing loads. In practice, neither stone nor brickwork should be trusted with more than $\frac{1}{6}$ to $\frac{1}{10}$th of the crushing load, according to circumstances. **When thoroughly wet** some absorbent sandstones lose fully half their strength.

	Tons per sq. ft.	Mean. Tons.
Granites and Syenites.	300 to 1200	750
Basalt		700
Limestones and Marbles *	250 to 1000	625
Oolites, good	100 to 250	175
Brownstone:		
Connecticut—		
"Building"	570 to 970	775
"Bridge"	400 to 630	535
Brick*	40 to 300	170
Brickwork, ordinary, cracks with*	20 to 30	25
Brickwork, good, in cement*	30 to 40	35
Brickwork, first-rate, in cement	50 to 70	60
Slate	400 to 800	600
Caen Stone	70 to 200	135
" " to crack		70
Chalk, hard,	20 to 30	25
Plaster of Paris, 1 day old		40

	Tons per sq. ft.	Mean. Tons.
Cement, Portland, neat, U. S. or foreign, 7 days in water	75 to 150	112.5
Common U.S. cements, neat, 7 days in water	15 to 30	22.5
Concrete of Port. cement, sand, and gravel or brok stone in the proper proportions, rammed 1 m old	12 to 18	15
6 months old	48 to 72	60
12 months old	74 to 120	97
With good **common hyd cements,** abt .2 to .25 as much		
Coignet beton, 3 months old	100 to 150	125
Rubble masonry, mortar, rough	15 to 35	25
Glass, green, crown and flint	1300to2300	1800
or 3 times that of granite		
Ice, firm †	12 to 18	15

Crushing height of Brick and Stone.

If we assume the wt of ordinary brickwork at 112 lbs per cub ft, and that it would **crush under** 30 tons per sq ft, then a vert uniform column of it 600 ft high, would crush at its base, under its own wt. Caen stone, weighing 130 lbs per cub ft, would require a column 1376 ft high to crush it. Average sandstones at 145 lbs per cub ft, would require one 4158 ft high; and average granites, at 165 lbs per cub ft, one of 8145 feet. But stones begin to crack and splinter at about half their ultimate crushing load; and in practice it is not considered expedient to trust them with more than $\frac{1}{6}$th to $\frac{1}{10}$th part of it, especially in important works; inasmuch as settlements, and imperfect workmanship, often cause undue strains to be thrown on certain parts.

The Merchants' shot-tower at Baltimore is 246 ft high; and its base sustains $6\frac{1}{2}$ tons per sq ft. The base of the granite pier of Saltash bridge, (by Brunel,) of solid masonry to the height of 96 ft, and supporting the ends of two iron spans of 455 ft each, sustains $9\frac{1}{2}$ tons per sq ft. The base of a brick chimney at Glasgow, Scotland, 468 ft high, bears 9 tons per sq ft; and Professor Rankine considers that in a high gale of wind, its leeward side may have to bear 15 tons. The highest pier of Rocquefavour stone aqueduct, Marseilles, is 305 ft, and sustains a pressure at base of $13\frac{1}{2}$ tons per sq ft.

---

* **Trials at St. Louis bridge,** by order of Capt James B. Eads, C. E., showed that some magnesian limestone did not yield under less than 1100 tons per sq ft. A column 8 ins high, 2 ins diam, shortened 0.0025 inch under pressure; and recovered when relieved. **Experiments made with the Govt testing machine at Watertown, Mass, 1882–3,** gave 1400 tons per sq ft ultimate crushg load for white and blue marble from Lee, Mass, 700 for blue marble from Montgomery Co, Pa, 960 for limestone from Conshohocken, Pa, 500 for limestone from Indiana, 840 for red sandstone from Hummelstown, Pa, 260 to 1000 for yellow Ohio sandstone; Phila bricks, flatwise; hard, machine-made, 350 to 700 tons; hand-made, 700 to 1300; pressed, machine-made, 450 to 580; Brickwork columns, 13 ins sq and 13 ins high; in lime, 100 tons; in cement, 150.

† Experiments by Col. Wm. Ludlow, U. S. A., with Govt testing machines, in 1882, gave from 21 to 64 tons per sq ft for pure, hard ice; and 16 to 59 tons for inferior grades. The specimens (6 and 12-inch cubes) compressed ¼ to 1 inch before crushing.

STONE BEAMS.

Table of safe quiescent extraneous loads for beams of good building granite one inch broad, supported at both ends, and loaded at the center; assuming the safe load to be one-tenth of the breaking one; and the latter to be 100 lbs for a beam 1 inch square, and 1 foot clear span. The half weight of the beams themselves is here already deducted at 170 lbs per cub ft.

Depth in ins.	CLEAR SPANS IN FEET.											
	1	2	3	4	5	6	7	8	10	12	15	20
	Safe center loads in pounds.											
1	10	5										
2	40	20	13	10								
3	90	45	29	21	17							
4	160	79	52	39	31	26	21					
5	250	124	82	61	48	40	34					
6	360	179	119	89	70	58	48	42	32			
7	490	244	162	120	96	79	67	58	45	36	27	16
8	639	319	212	158	126	104	88	76	59	47	36	22
10	999	499	331	248	197	163	139	120	94	76	58	38
12	1439	718	478	357	284	236	201	174	137	111	85	58
14	1959	978	650	487	388	322	274	238	188	153	118	81
16	2559	1278	850	636	507	421	359	312	246	201	157	109
18	3239	1618	1077	806	643	534	455	396	313	257	200	141
20	3999	1998	1329	995	794	660	563	490	388	319	249	176
22	4839	2417	1609	1205	961	800	682	594	470	387	303	216
24	5758	2877	1916	1434	1145	951	813	708	562	463	362	260
27	7288	3642	2425	1815	1450	1205	1030	898	713	588	462	332
30	8998	4496	2995	2243	1791	1489	1273	1110	882	728	573	415
33	10888	5441	3624	2714	2168	1803	1542	1345	1069	883	696	505
36	12958	6476	4314	3231	2581	2147	1836	1603	1275	1054	832	606

If uniformly distributed over the clear span, the safe extraneous loads will be twice as great as those in the table.

For good slate on bed the safe loads may be taken at about 3 times; **for good sandstone on bed** at about one-half; and for **good marble or limestone on bed** at about the same as those in the table.

MORTAR, BRICKS, &c.

LIME MORTAR.

Art. 1. Mortar. The proportion of 1 measure of quicklime, either in irregular lumps, or ground, and 5 measures of sand, is about the average used for common mortar, by good builders in our principal Atlantic cities; and if both materials are good, and well mixed (or *tempered*) with clean water, the mortar is certainly as good as can be desired for such ordinary purposes as require no addition of hydraulic cement. The bulk of the mixed mortar will usually exceed that of the dry loose sand alone about ⅛ part.

Quantity required. 20 cub ft, or 16 struck bushels of sand, and 4 cub ft, or 3.2 struck bushels of quicklime, the measures slightly shaken in both cases, will make abt 22½ cub ft of mortar; sufficient to lay 1000 bricks of the ordinary average size of 8¼ by 4 by 2 ins, with the coarse mortar joints usual in interior house-walls, varying say from ⅜ to ½ inch. With such joints, 1000 such bricks make 2 cubic yards of massive work. Nearly one-third of the mass is mortar. For outside or showing joints, where a whiter and neater looking mortar is required, house-builders increase the proportion of lime to 1 in 4, or 1 in 3. For mortar of fine screened gravel, for cellar-walls of stone rubble, or coarse brickwork, 1 measure of lime to 6 or 8 of gravel, is usual; and the mortar is good. In average rough massive rubble, as in the foregoing brickwork, about one-third the mass is mortar; consequently a cubic yard will require about as much as 500 such bricks; or 10 cubic feet, (8 struck bushels) of sand; and 2 cub ft, or 1.6 bushels of quicklime. Superior, well-scabbled rubble, carefully laid, will contain but about $\frac{1}{5}$ of its bulk of mortar; or 5½ cub ft sand, and 1.1 cub ft lime, per cub yard.

For public engineering works, especially in massive ones, or where exposed to *dampness*, an addition should be made in either of the foregoing mortars, of a quantity of good **hyd cement,** equal to about ⅓ of the lime; or still better, ⅓ of the lime should be omitted, and an equal measure of cement be substituted for it. **If exposed to water while quite new,** use little or no lime outside.

With bricks of 8¼ by 4 by 2 ins, the following are the **quantities of mortar and of bricks for a cubic yard of massive work.**

Thickness of Joints.	Proportion of Mortar in the whole mass.	No. of Bricks per cub yard.	No. of Bricks per cub foot.
1/8 inch	about 1/9	638	23.63
1/4 "	" 1/4	574	21.26
3/8 "	" 3/10	522	19.33
1/2 "	" 1/3	475	17.60
5/8 "	" 4/10	433	16.04

In estimating for bricks in massive work, **allow 2 or 3 per ct for waste;** and in common buildings, 5 per ct, or more. Much of the waste is incurred in cutting bricks to fit angles, &c. In Philadelphia a barrel of lump lime is allowed for 1000 bricks; or for 2 perches (25 cub ft each) of rough cellar-wall-rubble. Somewhat less mortar per 1000 is contained in thin walls, than in massive engineering structures; because the former have proportionally more outside face, which does not require to be covered with mortar; but thin walls involve more waste while building; so that both require about the same quantity of materials to be provided. Careful experiments show that mortar becomes harder, and more adhesive to brick or stone, if the proportion of lime is increased. Hence, on our public works the proportion of one measure of quicklime to 3 of sand, is usually specified, but probably never used.

Lime is usually sold in lump, by the barrel, of about 230 lbs net, or 250 lbs gross. **A heaped bushel** of lump lime averages about 75 lbs. **Ground quicklime,** loose, averages about 70 lbs per struck bushel; and 3 bushels loose just fill a common flour barrel; but from 3.5 to 3.75 bushels, or 245 to 280 lbs can readily be compacted into a barrel.

General remarks on mortar and lime. On too great a proportion of our public works, the common lime mortar may be seen to be rotten and useless, where it has been exposed to moisture; which will be carried by the capillary action of earth to several feet above the natural surface: or as far below the artificial surface of embankments deposited behind abutments, retaining-walls, &c. The same will frequently be seen in the soffits of arches under embankments. *Common lime mortar, thus exposed to constant moisture, will never harden properly.* Even when very old and hard, it absorbs water freely. *Cement also does so, but hardens.*

Brickdust, or burnt clay, improves common mortar; and makes it hydraulic. In localities where sand cannot be obtained, burnt clay, ground, may be substituted; and will generally give a better mortar.

Protection of quicklime from moisture, even that of the air, is absolutely essential, otherwise it undergoes the process of **air-slacking,** or spontaneous slacking, by which it becomes reduced to powder as when slacked by water as usual, but without heating, and with but little swelling. As this air slacking requires from a few months to a year or more, depending on quality and exposure, it gives the lime time to absorb sufficient carbonic acid from the air to injure or destroy its efficacy. But **quicklime will keep good for a long time** if first ground, and then well packed in air-tight barrels. The grinding also breaks down refractory particles found in all limes, and which injure the mortar by not slacking until it has been made and used. For the same reason it is better that lime should not be made into mortar as soon as it is slacked, but be allowed to remain slacked for a day or two (or even several) protected from rain, sun, and dust.

Lime slacked in great bulk may char or even set fire to wood.

Lime paste and mortar will keep for years, and improve, if well buried in the earth. Also for months if merely covered in heaps under shelter, with a thick layer of sand. The paste shrinks and cracks in drying; but the sand in mortar prevents this.

As approximate averages varying much according to the character and degree of burning of the limestone; and to the fineness or coarseness of the sand, one measure of good quicklime, either in lump, or ground; if wet with about ¼ a measure of water, will within less than an hour, slack to about 2 measures of dry powder. And if to this powder there be added about ¾ more measures of water, and 3 measures of dry sand, and the whole thoroughly mixed, the result will be about 3⅜ measures of mortar. Or the same slacked dry powder, with about 1 measure of water, and 5 measures of sand, will make about 5⅝ measures of mortar. In both cases the bulk of the mortar will be about ⅛ part greater than that of the dry sand alone. If ¾ of a measure of water be used for slacking, the result, instead of a dry powder, will be about 1⅛ measures of stiff paste; or with 1 whole measure of water for slacking, the result will be about 1¼ measures of thin paste, of about the proper consistence for mixing with the sand. Very pure, *fat* limes, slack quickly, and make about from 2 to 3 measures of powder; while poor, *meagre* ones, require more time, and swell less. Slow slacking, and small swelling, in case the lime has been properly burnt, are not in general bad properties; but on the contrary, usually indicate that it is to some extent hydraulic. In this case it makes a better mortar; especially for works exposed to moisture, or to the weather. Very pure limes are the worst of all for such exposures; or are bad *weather-limes;* and in important works, should never be used without cement.

Shell lime appears to be about the same as that from the purest limestones; but that from chalk is still more inferior, and will not bear more than about 1½ measures of sand; its mortar never becomes very hard. Madrepores (commonly called coral) appear to furnish a lime intermediate between those of chalk and limestone. They require to be but moderately burnt.

The average weight of common hardened mortar is about 105 to 115 lbs per cub ft.

Grout is merely common mortar made so thin as to flow almost like cream. It is intended to fill interstices left in the mortar-joints of rough masonry; but unless it contains a large amount of cement, it is probably entirely worthless; since the great quantity of water injures the properties of lime; and moreover, its ingredients separate from each other; the sand settling below the lime. Besides this, it will never harden thoroughly in the interior of thick masses of masonry; indeed, the same may probably be said of any common lime mortar. In such positions, it has been found to be perfectly soft, after the lapse of many years.

Both the sand and the water for lime mortar, should be **free from clay and salt.** The clay may be removed by thorough washing; but it is extremely difficult to get rid of the salt from seashore sand, even by repeated washings. Enough will generally remain to keep the work damp, and to produce efflorescences of nitre on the surface; whether with lime, or with cement mortar. Slacking by salt water gives less paste than fresh.

Mortar should not be mixed upon the surface of clayey ground; but a rough board, brick, or stone platform should be interposed. Pit sand sifted from decomposed gneiss, and other allied rocks, is excellent for mortar; its sharp angles making with the lime a more coherent mass than the rounded grains of river or sea sand. Mortar should be applied wetter in hot than in cold weather; especially in brickwork; otherwise the water is too much absorbed by the masonry, and the mortar is thereby injured.

The tenacity, or cohesive strength, that is, the resistance to a pull of good common lime mortar of the usual proportions of lime and sand, and 6 months old, is about from 15 to 30 lbs per sq inch; or .96 to 1.9 tons per sq ft. With less sand, or with greater age, it will be stronger.

The crushing strength of good common mortar 6 months old is from 150 to 300 lbs per sq inch, or 9.7 to 19.3 tons per sq foot.

The sliding resistance, or that which common mortar opposes to any force tending to make one course of masonry slide upon another, is stated by Roudelet, to be but 5 lbs per sq inch; or about one third of a ton per sq ft, in mortar 6 months old.

Transverse strength of good common mortar 6 months old. A bar 1 inch square and 12 ins clear span, breaks with a center load of 4 to 8 lbs.

The lime in mortar decays wood rapidly, especially in close, damp situations. Still the soaking of timber for a week or two in a solution of quicklime in water appears to act as a preservative. **Iron,** so completely embedded in mortar as to exclude air and moisture, has been found perfect after 1400 years; but if the mortar admits moisture the iron decays. So, probably, with other metals.

The adhesion to common bricks, or to rough rubble at any age will average about ¾ of the cohesive strength at the same age; or say 12 to 24 lbs per sq inch, or .75 to 1.5 ton per sq ft at 6 months old. If care be taken to exclude dust entirely, by dipping each brick into water before laying it, or by sprinkling the stone by a hose, &c, the adhesion will be increased. On the other hand, much dust may almost prevent any adhesion at all. The precaution of wetting is especially necessary in very hot weather, to prevent the warm bricks or stone from **killing the mortar** by the rapid absorption and evaporation of its water. **The adhesion to very smooth hard pressed bricks,** or to smoothly dressed or sawed stone is considerably less.

BRICKS.

Art. 2. Bricks, size, weight, &c. A brick 8.25 × 4 × 2 ins contains 66 cub ins; or 26.2 bricks to a cub ft; or 707 bricks to a cub yard.

In ordering a large number, a minimum limit of dimension should be specified, in order to prevent fraud. A brick ¼ inch less each way than the above, contains but 52.5 cub ins; thus requiring full 25 per cent more bricks to do the same work, and 25 per ct more cost for laying, which is generally paid by the 1000.

The weight of a good common brick, 8.25 × 4 × 2 ins, will average about 4.5 lbs; or 118 lbs per cub ft = 3186 lbs or 1.42 tons per cub yard; or 2.01 tons per 1000. A good pressed brick of the same size will average about 5 lbs, = 131 lbs per cub ft = 3537 lbs or 1.58 tons per cub yd; or 2.23 tons per 1000. Since the weight of hardened mortar averages but little less than that of good common brick, we may for ordinary calculations assume the **weight of brickwork,** with common bricks, at 1.4 tons per cub yard, or 116 lbs per cub ft; and, with pressed brick, at 1.56 tons per cub yd, or 129 lbs per cub ft.

In water, either brick will in a few minutes **absorb** from ½ to ¾ lb of water; or 0.1 to one-seventh of the weight of a pressed brick, or ¼ to one-third of its bulk.

Number of bricks 8¼ × 4 × 2, required per sq foot of wall, allowing for the usual waste in cutting bricks to fit corners, jambs, &c.:

Wall	8¼	ins, or	1	brick	.....14 bricks
"	12¾	" or	1½	"	.....21 "
"	17	" or	2	"	.....28 "
Wall	21½	ins, or	2½	brick	.....35 bricks
"	25¾	" or	3	"	.....42 "

Laying, per day. A bricklayer, with a laborer to keep him supplied with materials, will, in common house walls, lay on an average about 1500 bricks per day of 10 working hours. In the neater outer faces of back buildings, from 1000 to 1200; in good ordinary street fronts, 800 to 1000; or of the very finest lower story faces used in street fronts, from 150 to 300, depending on the number of angles, &c. In plain massive engineering work, he should average about 2000 per day, or 4 cub yds; and in large arches, about 1500, or 3 cub yds.

Since bricks shrink about $\frac{1}{12}$ part of each dimension in drying and burning, the moulds should be about $\frac{1}{12}$ part larger each way than the burnt brick is intended to be. Good well-burnt bricks will ring when two are struck together.

At the brick-yards about Philadelphia, a brick-moulder's work is 2333 bricks per day; or 14000 per week. He is assisted by two boys, one of whom supplies the prepared clay, moulding sand, and water; while the other carries away the bricks as they are moulded. A fourth person arranges them in rows for drying. About ¾ of a cord, or 96 cub ft of wood, is allowed per 1000 for burning. Where coal is used, the kilns are fired up with anthracite, and the finishing is done with bituminous. One ton of coal, in all, makes 4500 bricks.

For paving sidewalks the bricks are laid on a 6-inch layer of gravel, which should be free from clay, and well consolidated. With bricks of 8¼ × 4 × 2 ins, with joints from ⅛ to ¼ inch wide, a square yard requires, flatwise, 88 bricks; edgewise, 73; endwise, 149. An average workman, with a laborer to supply the bricks and gravel, will in 10 hours lay about 2000 bricks; or 53 sq yds flat, 27 edgewise, 13 endwise. When done, sand is brushed into the joints.

Art. 3. The crushing strength of bricks of course varies greatly. A rather soft one will crush under from 450 to 600 lbs per sq inch; or about 30 to 40 tons per sq ft; while a first-rate machine-pressed one will require about 200 to 400 tons per sq ft, or about the crushing limit of the best sandstone; two-thirds that of the best marbles or limestones; or ½ that of the best granites, or roofing slates. But masses of brickwork crush under much smaller loads than single bricks. In some English experiments, small cubical masses, only 9 inches on each edge, laid in cement, crushed under 27 to 40 tons per sq ft. Others, with piers 9 ins square, and 2 ft 3 ins high, in cement, only two days after being built, required 44 to 62 tons per sq ft to crush them. Another, of pressed brick, in best Portland cement, is said to have withstood 202 tons per sq ft; and with common lime mortar only ¼ as much.

It must, however, be remembered, that cracking and splitting usually commence under about one-half the crushing loads. To be safe, the load should not exceed ⅛ of the crushing one; and so with stone. Moreover, these experiments were made upon low masses; and the strength decreases with the proportion of the height to the thickness.

The pressure at the base of a brick shot-tower in Baltimore, 246 feet high, is estimated at 6½ tons per sq ft; and in a brick chimney at Glasgow, Scotland, 468 feet high, at 9 tons. Professor Rankine calculates that in heavy gales this is increased to 15 tons, on the leeward side.

With our present imperfect knowledge on this subject, it cannot be considered safe to expose even first-class pressed brickwork, *in cement*, to more than 12 or 15 tons per sq ft; or good hand-moulded, to more than two-thirds as much.

Tensile strength of brick, 40 to 400 lbs per sq inch; or 2.6 to 26 tons per sq ft.

The English rod of brickwork is 306 cub feet, or 11⅓ cub yards; and requires about 4500 bricks of the English standard size; with about 75 cub ft of mortar. The English *hundred of lime*, is a cub yd.

Frozen mortar. There is risk in using common mortar in cold weather. If the cold should continue long enough to allow the frozen mortar to set well, the work may remain safe; but if a warm day should occur between the freezing and the setting of the mortar, the sun shining on one side of the wall may melt the mortar on that side, while that on the other side may remain frozen hard. In that case, the wall will be apt to fall; or if it does not, it will at least always be weak; for mortar that has partially set while frozen, if then melted, will never regain its strength. By the writer's own trials hydraulic cements seemed not to be injured by freezing.

Experiments for rendering brick masonry impervious to water. Abstract of a paper read before the American Society of Civil Engineers, May 4, 1870, by William L. Dearborn, Civil Engineer, member of the Society.

The face walls of the Back Bays of the Gate-houses of the new Croton reservoir, located north of Eighty-sixth Street, in Central Park, were built of the best quality of hard-burnt brick; laid in mortar composed of hydraulic cement of New York, and sand mixed in the proportion of one measure of cement to two of sand. The space between the walls is 4 ft: and was filled with concrete. The face walls were laid up with great care, and every precaution was taken to have the joints well filled and insure good work. They are 12 ins thick, and 40 ft high; and the Bays when full generally have 36 ft of water in them.

When the reservoir was first filled, and the water was let into the Gate-houses, it was found to filter through these walls to a considerable amount. As soon as this was discovered, the water was drawn out of the Bays, with the intention of attempting to remedy or prevent this infiltration. After carefully considering several modes of accomplishing the object desired, I came to the conclusion to try "Sylvester's Process for Repelling Moisture from External Walls."

The process consists in using two washes or solutions for covering the surface of brick walls; one composed of Castile soap and water; and one of alum and water. The proportions are: three-quarters of a pound of soap to one gallon of water; and half a pound of alum to four gallons of water both substances to be perfectly dissolved in the water before being used.

The walls should be perfectly clean and dry; and the temperature of the air should not be below 50 degrees Fahrenheit, when the compositions are applied.

The first, or soap wash, should be laid on when at boiling heat, with a flat brush, taking care not to form a froth on the brickwork. This wash should remain twenty-four hours; so as to become dry and hard before the second or alum wash is applied; which should be done in the same manner as the first. The temperature of this wash when applied may be 60° or 70°; and it should also remain twenty-four hours before a second coat of the soap wash is put on; and these coats are to be repeated alternately until the walls are made impervious to water.

The alum and soap thus combined form an insoluble compound, filling the pores of the masonry, and entirely preventing the water from penetrating the walls.

Before applying these compositions to the walls of the Bays, some experiments were made to test the absorption of water by bricks under pressure after being covered with these washes, in order to determine how many coats the wall would require to render them impervious to water.

To do this, a strong wooden box was made, put together with screws, large enough to hold 2 bricks; and on the top was inserted an inch pipe forty feet long.

In this box were placed two bricks after being made perfectly dry, and then covered with a coat of each of the washes, as before directed, and weighed. They were then subjected to the pressure of a column of water 40 feet high; and, after remaining a sufficient length of time, they were taken out and weighed again, to ascertain the amount of water they had absorbed.

The bricks were then dried, and again coated with the washes and weighed, and subjected to pressure as before; and this operation was repeated until the bricks were found not to absorb any water. Four coatings rendered the bricks impenetrable under the pressure of 40 ft head.

The mean weight of the bricks (dry) before being coated, was 3½ lbs; the mean absorption was one-half pound of water. An hydrometer was used in testing the solutions.

As this experiment was made in the fall and winter, (1863,) after the temporary roofs were put on to the Gate-house, artificial heat had to be resorted to, to dry the walls and keep the air at a proper temperature. The cost was 10.06 cts per sq ft. As soon as the last coat had become hard, the water was let into the Bays, and the walls were found to be perfectly impervious to water, and they still remain so in 1870, after about 6½ years.

BRICK ARCH (FOOTWAY OF HIGH BRIDGE). The brick arch of the footway of High Bridge is the arc of a circle 29 ft 6 in radius; and is 12 in thick; the width on top is 17 ft; and the length covered was 1381 ft.

The first two courses of the brick of the arch are composed of the best hard-burnt brick, laid edgewise in mortar composed of one part, by measure, of hydraulic cement of New York, and two parts of sand. The top of these bricks, and the inside of the granite coping against which the two top courses of brick rest was, when they were perfectly dry, covered with a coat of asphalt one-half an inch thick, laid on when the asphalt was heated to a temperature of from 360° to 518° Fahrenheit.

On top of this was laid a course of brick flatwise, dipped in asphalt, and laid when the asphalt was hot; and the joints were run full of hot asphalt.

On top of this a course of pressed brick was laid flatwise in hydraulic cement mortar, forming the paving and floor of the bridge. This asphalt was the Trinidad variety; and was mixed with 10 per cent, by measure, of coal tar; and 25 per cent of sand. A few experiments for testing the strength of this asphalt, when used to cement bricks together, were made, and two of them are given below.

Six bricks, pressed together flatwise, with asphalt joints, were, after lying six months, broken. The distance between the supports was 12 ins; breaking weight, 900 lbs; area of single joint, 28½ sq ins. The asphalt adhered so strongly to the brick as to tear away the surface in many places.

Two bricks pressed together end to end, cemented with asphalt, were, after lying 6 months, broken. The distance between the supports was 10 ins; area of joint, 8½ sq ins; breaking weight, 150 lbs.

The area of the bridge covered with asphalted brick, was 23065 sq ft. There was used 94200 lbs of asphalt, 33 barrels of coal tar, 10 cub yds of sand, 93800 bricks.

The time occupied was 100 days of masons, and 148 days of laborers. Two masons and two laborers will melt and spread, of the first coat, 1650 sq ft per day. The total cost of this coat was 5.25 cents per sq ft, exclusive of duty on asphalt. There were three grooves, 2 ins wide by 4 ins deep, made entirely across the brick arch, and immediately under the first coat of asphalt, dividing the arch into four equal parts. These grooves were filled with elastic paint cement.

This arrangement was intended to guard against the evil effects of the contraction of the arch in winter; as it was expected to yield slightly at these points, and at no other point; and then the elastic cement would prevent any leakage there.

The entire experiment has proved a very successful one, and the arch has remained perfectly tight.

In proposing the above plan for working the asphalt with the brickwork, the object was to avoid depending on a large continued surface of asphalt, as is usual in covering arches, which very frequently cracks from the greater contraction of the asphalt than that of the masonry with which it is in contact; the extent of the asphalt on this work being only about one-quarter of an inch to each brick. This is deemed to be an essential element in the success of the impervious covering."

A cheap and effective process for preventing the percolation of water through the arches of aqueducts, and even of bridges, is a great desideratum. Many expensive trials with resinous compounds have proved failures. Hydraulic cement appears to merely diminish the evil. Much of the trouble is probably due to cracks produced by changes of temperature.

The **white efflorescence** so common on walls, especially on those of brick, is due to the presence of soluble salts in the bricks and mortar. These are dissolved, and carried to the face of the wall, by rain and other moisture. Sulphate of magnesia (Epsom Salt) appears to be the most frequent cause of the disfiguration. In many places mortar lime is made from dolomite, or *magnesian* limestone, which often contains 30 per cent or more of magnesia; which also occurs frequently in brick clay. Coal generally contains sulphur, most frequently in combination with iron, forming the well-known "iron pyrites". The combustion of the coal, as in burning the limestone or clay, in manufactures, in cooking etc, converts the sulphur into sulphurous acid gas, which, when in contact with magnesia and air, as in the lime or brick kiln, or in the finished wall or chimney, becomes sulphuric acid and unites with the magnesia, forming the soluble sulphate. We are not aware of any remedy that will prevent its appearance under such circumstances; but the formation of the sulphate may be prevented by the use of limestone and brick-clay free from magnesia.

CEMENT.

General Principles.

The elements chiefly concerned in the action of lime and cement mortars are—

$$\left.\begin{array}{lr}\text{Calcium,} & \text{Ca}\\ \text{Aluminum,} & \text{Al}\\ \text{Carbon,} & \text{C}\\ \text{Silicon,} & \text{Si}\\ \text{Hydrogen,} & \text{H}\end{array}\right\}\ \text{Oxygen, O.}$$

Oxygen combines with each of the others, forming oxides. Thus:
Calcium oxide, CaO, is lime;
Aluminum oxide, Al_2O_3,* is alumina;
Carbon dioxide, CO_2, is carbonic acid;
Silicon oxide, SiO_2, is silica, or silicic acid;†
Hydrogen oxide, H_2O, is water.

Limestone is a calcium carbonate, or combination of lime and carbonic acid, $CaO + CO_2$, or $CaCO_3$.

Clay (including argillaceous minerals in general) is an aluminum silicate, or combination of alumina and silicic acid, $Al_2O_3 + SiO_2$.

Lime. When limestone (without clay) is "burned," its CO_2 is driven off, and the remaining (**"quick"**) **lime** has a strong affinity for water, absorbing it with such avidity as to develop heat sufficient to produce steam, the generation of which disintegrates and swells the mass. Combining thus with the water, the lime forms calcium hydrate, $CaO.H_2O$, or CaH_2O_2. This process is called **slaking** or **slacking;** and lime which has satisfied its affinity for water is called slaked (or slack) lime. When slaked lime is used as mortar, it gradually absorbs carbonic acid from the air, forming calcium carbonate, the water being liberated and evaporated. Hardened lime mortar may thus be regarded as an artificial limestone.

Cement. When aluminum silicate, such as clay, in sufficient quantities, is burned with calcium carbonate, such as limestone, the burned product, called cement, is deficient in, or devoid of, the slacking property; but, on the other hand, when it is made into mortar, the combinations, formed between the elements of the lime, the alumina, the silica and the water, during the burning, and afterward in the mortar, are such that they readily proceed under water. Chemists differ as to the nature of these combinations. If free lime remains in the mortar, calcium carbonate is formed by reabsorption of carbonic acid from the air, as in the case of lime mortar.

Setting, or the loss of plasticity, usually occurs within a few hours (sometimes within a few minutes) after mixing cement with water; whereas **hardening** (which appears to result from a different set of chemical processes) often proceeds for months or even years.

The property of setting and hardening under water is called **hydraulicity;** and cements which do not slack, but which harden under water, are called **hydraulic cements;** or, more briefly, **cements.**

Hydraulic lime is a name given to cements (much used in Europe) which, while to some extent hydraulic, do not contain enough of the hydraulic elements to prevent slaking. The slaking, however, is slower, and the swelling less, than with lime proper.

The ratio of the weight of aluminum silicate to that of the lime, in a cement, is called its **hydraulic index.** Other things being equal, it may be used as an indication of the hydraulicity of the cement.

Natural and Portland Cements. Many natural limestones contain clay in such proportion that they afford cements when burned. Cements so made are called natural; while those which result from the burning of artificial mixtures of lime carbonates and aluminum silicates are called artificial, and, when certain refinements (see below) are observed, "Portland" cement.

In making **natural cement,** the material is burned in lumps; but for **Portland** cement the material is finely ground before burning, and the burning is done at a high temperature, producing incipient vitrifaction. In both natural and Portland cements, the burned product is ground to an impalpable powder.

---

* The subscripts indicate the combining ratios of the several elements. Thus, in alumina, Al_2O_3 means a compound of 2 atoms of aluminum with 3 oxygen.

† Quartz is silica; and most of the sand used in mortar is quartz sand.

In natural cement, the hydraulic index usually varies between 0.60 and 1.50; in Portland cements, between 0.40 and 0.60.

The higher cost of Portland cement is due to the more careful selection of the materials and to the more elaborate and expensive treatment given them, resulting in the ultimate attainment of much greater strength.

The name **Rosendale,** originally and properly restricted to natural cements made in Ulster County, New York, is often applied indiscriminately to American natural cements in general.

Limestones containing **magnesia** are called **Dolomitic Limestones** or **Dolomites.** The presence of more than 3 per cent. of magnesia, in the finished product, is usually considered objectionable.

Cement Mortar.

Cement mortar consists of cement and some inert granular material, as sand, fine gravel or ground cinder, mixed with water.

Owing to the cheapness with which cements are now manufactured, and the superiority of the mortars made from them, the latter have to a great extent superseded lime mortars, even in ordinary building operations.

Amount of Mortar Required for a Cubic Yard of Masonry.*

Description of Masonry.	Mortar. Cu. yd. Min.	Mortar. Cu. yd. Max.
Ashlar, 18″ courses and ¼″ joints,	0.03	0.04
" 12″ " " " "	0.06	0.08
Brickwork (bricks of standard size, 8¼ × 4 × 2¼ ins.):		
⅛″ joints,	0.10	0.15
⅜″ to ½″ joints,	0.25	0.35
⅝″ to ¾″ joints,	0.35	0.40
Rubble, of small, rough stones,	0.33	0.40
" " large stones, rough hammer-dressed,	0.20	0.30
Squared-stone masonry, 18″ courses and ½″ joints,	0.12	0.15
" " 12″ " " " "	0.20	0.25

Cement and Sand Required for 1 Cubic Yard of Mortar.*

Parts of Sand to 1 Part of Cement.	Mortar Proportioned by Weight.				Mortar Proportioned by Volumes of Packed Cement and Loose Sand.			
	Portland.		Natural.		Portland.		Natural.	
	Cement. Bbl.	Sand. Cu. Yd.	Cement. Bbl.	Sand. Cu. Yd.	Cement. Bbl.	Sand. Cu. Yd.	Cement. Bbl.	Sand. Cu. Yd.
0	7.40	0.00	7.91	0.00	7.40	0.00	7.91	0.00
1	4.05	0.57	4.92	0.51	4.17	0.57	4.58	0.58
2	2.80	0.78	3.43	0.72	2.91	0.78	3.04	0.76
3	2.00	0.85	2.54	0.80	2.08	0.85	2.24	0.81
4	1.60	0.89	2.04	0.84	1.66	0.89	1.70	0.86
5	1.30	0.91	1.64	0.86	1.35	0.91	1.39	0.88
6	1.10	0.93	1.40	0.88	1.14	0.93	1.28	0.89

---

* Taken, by permission, from "A Treatise on Masonry Construction," by Prof. Ira O. Baker. New York, John Wiley & Sons. 9th edition, 1899.

The effects of cold upon Portland cements, although it retards the setting, do not appear to be serious otherwise. Even if Portland cement mortar *freezes* almost as quickly as the masonry is laid with it, it does not seem to depreciate materially. We have found this to be the case also with lime mortar; even when, a few hours after freezing, the temperature became so high as to soften the frozen mortar again. But although the *mortar* of either lime or cement may not thereby be injured, the *work*, especially in thin brick walls, may be ruined and overthrown. Thus, if, soon after the mortar, through the entire thickness of such a wall, be frozen, the sun shines on one face of it, so as to soften the mortar of that face, while the mortar behind it remains hard, it is plain that the wall will be liable to settle at the heated face, and at least bend outward if it does not fall. Coatings of cement, applied to the backs of arches on the approach of winter, and left unprotected, have been found entirely broken up and worthless on resuming work the next spring.

Alternate freezing and thawing are apt to disintegrate both natural and Portland cement mortars.

The heating of sand and cement, in freezing weather, seems to be a bad practice, especially if they be placed in cold water. But for use out of water Mr. Maclay says they may be heated to 50° or 60°. **Cold water for mixing** is probably no farther injurious than that it retards the setting.

Strengths.

Factors Affecting Strength. The strength of samples, under test, is much affected by the temperature of the air and water, as also by the degree of force with which the cement is pressed into the molds; by the extent of setting before being put into the water, and of drying when taken out; and still more by the pressure under which it sets, which increases the strength materially. On this account, cements in actual masonry may, under ordinary circumstances, give better results than in tests of samples. The causes named, together with the degree of thoroughness of the mixing or gaging, the proportion of water used, and other considerations, may easily affect the results 100 per cent. or even much more. Hence the discrepancies in the reports of different experimenters. Specimens of the same cement, tested under apparently similar conditions, may give widely different results.

The Bureau of Surveys, Philadelphia, requires, 1901, the following tensile strengths, in lbs. per sq. inch; 1 day in air, remainder in water:

	7 days.	28 days.
Portland, neat,	500	600
" 3 parts sand, laboratory test,	170	240
" 3 " " mortar from mixing box,	125	175
Natural, neat,	200	300
" 2 parts sand, laboratory test,	120	200
" 2 " " mortar from mixing box,	50	125

See also diagrams, p. 933, and Requirements, p. 942.

Portland and Natural Cements. Effect of Age. The diagram * opposite illustrates approximately the strengths of average Portland and of average natural cements, neat and with proper doses of sand, up to an age of two years. Tests may readily vary 10 per cent. or more either way from the average.

Cements of the same class differ much in their **rapidity of hardening.** At the end of a month one may gain nearly one-half of what it will gain in a year, and another not more than one-sixth; yet at the end of the year both may have about the same strength. Hence, tests for 1 week or 1 month are by no means conclusive as to the final comparative merits of cements.

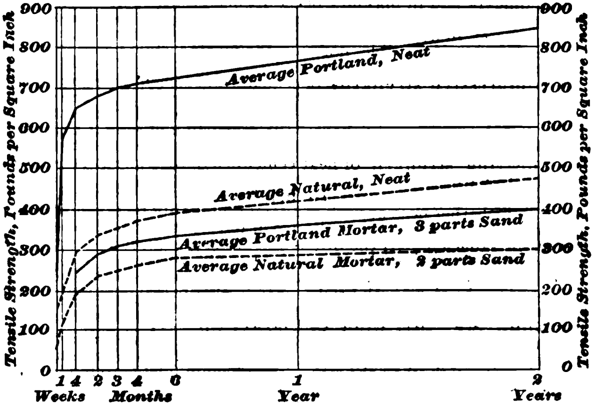
Many years are required to attain the greatest hardness; but after about a year the increase is usually very small and slow, especially with neat cement. Moreover, any subsequent increase is a matter of little importance, because generally by that time, and often much sooner, the work is completed and exposed to its maximum stresses.

There seems to be a period, occurring from a few weeks to several months after laying, during which cement and its mortars for a short time not only cease from hardening, but actually lose strength. They then recover, and the hardening goes on as before. This feature is not indicated in our diagram.

---

* See Richard L. Humphrey, in "Cement," Chicago, May, 1899.

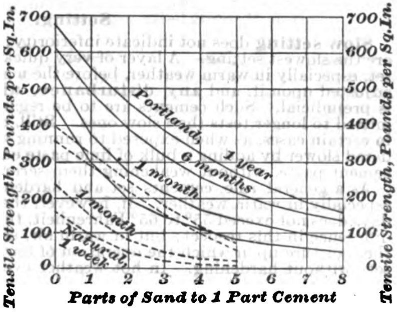
One advantage of strong cements is their economy, even at a higher cost, in allowing the use of a larger proportion of the cheaper ingredients, sand, gravel, and broken stone. Almost any common cement, if of good quality, will, with 1.5 or 2 measures of sand, give a mortar strong enough for most engineering purposes; but a good Portland will give one equally strong with 3 or 4 measures of sand; and will, therefore, be equally cheap at twice the price; besides requiring the handling, storing, and testing of only half the number of packages.



Any addition of sand weakens cement, especially as regards tension; as it does also lime mortar. But economy requires its use.

Although, with sand, the strength of the mortar may never attain to that of the neat paste, yet it increases with age in a greater proportion; so that a neat paste which at the end of a year would be but twice as strong as in 7 days, may with sand yield a mortar which at the end of a year will be 3, 4 or 5 times as strong as it was in 7 days. At the end of a year good Portlands neat usually have from 1.5 to 2 times their strength at the end of 7 days; and natural cements from 2.5 to 3.5 times; but, inasmuch as Portlands average about 5 or 6 times the strength of the others in 7 days, they still average about 2.5 to 3 times as strong in a year or longer.

Diagram * exhibiting, approximately, the effect of sand, in different proportions, upon the strengths of Portland and natural cements, at different ages from 1 week to 1 year. The four solid curves represent average Portland cements, and the four dotted curves represent average natural cements. For each kind of cement, the curves represent ages of 1 year, 6 months, 1 month and 1 week, respectively, beginning at the top. The curves for natural cement are carried only to 5 parts sand.



---

* Compiled, by permission, from Prof. Baker's "Masonry Construction."

Mr. Wm. W. Maclay, C. E.,[*] found that, in the *testing* of cements, the temperature of the air and water had far more influence than had before been suspected, but the *ultimate* effects of temperature, within certain limits, are fortunately not so important in actual practice as the first experiments might lead us to infer. Work must go on notwithstanding changes of temperature, but we must take care that our mortar shall at all times be strong enough, even under their most injurious influences. Cements in open air are certainly more or less injured by *drying* instead of *setting* when the temperature exceeds about 65° to 70°. But if mixed only in small quantities at a time, and quickly laid in masonry of dampened stone, so as to be sheltered from the air, the injury is much reduced. The sand and stone should both be *damp*, not wet, in hot weather, and a *little* more water may be used in the cement paste; also, if possible, not only the mortar while being mixed, but the masonry also, should then be shaded.

The compressive strengths of cements and cement mortars, in cubes, appear to be about 8 to 10 times their tensile strengths. The crushing strength, with sand, increases with age much more rapidly than the tensile strength, and the more so, the greater the proportion of sand. Cements are seldom tested in compression.

The shearing strength of neat cements averages about one-fourth of the tensile strength.

The adhesion of cements to bricks or rough rubble, at different ages, and whether neat or with sand, may probably be taken at an average of about three-fourths of the cohesive or tensile strength of the cement or mortar at the same age. If the bricks and stone are moist and entirely free from dust when laid, the adhesion is increased; whereas, if very dry and dusty, especially in hot weather, it may be reduced almost to nothing. The adhesion to very hard, smooth bricks, or to finely dressed or sawed masonry, is less than the adhesion to rough and porous surfaces.

Abrasion. See "**Mr. Eliot C. Clarke,**" p. 937.

Weight. See also Weight, p. 939, and (2) Specific Gravity, p. 940. Weight is an uncertain indication. A coarse-ground cement weighs heavier, but gives less strength, than the same cement more finely ground.

Color. See "Variations in Shade," p. 936.

Fineness. See "**Mr. Eliot C. Clarke,**" p. 936, "**Sieves,**" p. 938, "**Fineness,**" pp. 938, 940, and "**Requirements,**" p. 942.

Cement, when freshly ground, is not so good as when a few weeks old.

Precautions.

The engineer should reserve the right to take **a sample from each package,** and to reject every package of which the sample drawn out does not satisfy the stipulations. On works using large quantities, one person should be specially detailed to this duty.

Protection from moisture, even that of the air, is very essential for the preservation of cements, as well as of quicklime. With this precaution, the cement, although it may require more time to set, will not otherwise very appreciably deteriorate in many months.

Setting.

Slow setting does not indicate inferiority; for many of the best cements are the slowest setting. A layer of very quick-setting cement may partially set, especially in warm weather, before the masonry is properly lowered and adjusted upon it, and **any disturbance, after setting** has commenced, is prejudicial. Such cements are to be regarded with suspicion, and submitted to longer tests than slow ones. Still, quick-setting cements are best in certain cases, as when exposed to running water, etc. They may be rendered slower by adding a bulk of lime paste equal to 5 or 15 per cent. of the cement paste, without weakening them seriously.

As a general rule, cements set and harden better in water than in air, especially in warm weather. If, however, the temperature for the first few days does not exceed 55° to 65° Fahrenheit, there seems to be no appreciable difference in this respect; but in warm air, setting cement, in drying, loses the moisture upon which the operation of hardening depends. It therefore sets without hardening. In hot weather every precaution should be used against this.

---

[*] "Transactions American Society of Civil Engineers," Dec., 1877.

Sand Retards Setting. In our experiments with various hydraulic cements, of the consistence of mortar, even without sand, we have detected **no change of bulk in setting.** But Mr. Clarke (see p. 936) found an expansion of not more than 0.001 part in any dimension.

Sand.

The best sand is that with grains of very uneven sizes, and sharp. The more uneven the sizes, the smaller are the voids, and the heavier is the sand. It is generally considered that the sand should be well washed if it contains clay or mud. But see "**Adding Clay,**" p. 936. Mr. Clarke says, "the finer the sand, the less is the strength."

Proportion of Sand. As a general rule, with cements of good quality, we shall have mortars fit for most engineering purposes if we do not exceed from 1.5 to 2 measures of dry sand to 1 of the common cements; or from 2 to 3 of sand to 1 of Portland.

Voids in Sand. Since a cubic foot of pure quartz weighs 165 lbs., it follows that, if we weigh a cubic foot of pure dry sand, either loose or rammed; then, as 165 is to the weight found, so is 1 to the solid part of the sand. And, if this solid part be subtracted from 1, the remainder will be the voids, as below.

Wt., lbs. per cub. ft., dry,	80	85	90	95	100	105	110	115
Proportion of solid, ..	0.485	0.515	0.546	0.576	0.606	0.636	0.667	0.697
Proportion of voids, ..	0.515	0.485	0.454	0.424	0.394	0.364	0.333	0.303

But the sand, when wet in mortar, occupies about from 5 to 7 per cent. less space than when dry; the shrinkage averaging say 6 per cent.; thus making the voids 0.304 of the 105 lb. sand when wet; and 0.364 of the 95 lb.; the mean of which is 0.334. But, to allow for imperfect mixing, etc., it is better to assume the voids at 0.4 of the dry sand. Moreover, since the cements, as before stated, shrink more or less when mixed with water, and worked up into mortar, it would be as well to assume that, in order to make sufficient paste to fill the voids thoroughly, we should use, of dry common cement slightly shaken, not less than half the bulk of the dry sand; and not less than 45 per cent. if Portland.

To find the percentage of voids, pour into a graduated cylindrical measuring-glass 100 measures of dry sand. Pour this out, and fill the glass up to 60 measures with water. Into this *sprinkle slowly* the same 100 measures of dry sand. These will now be found to fill the glass only to say 94 measures, having shrunk say 6 per cent.; while the water will reach to say 121 measures; of which 121 — 94 = 27 measures will be above the sand; leaving 60 — 27 = 33 measures filling the voids in 94 measures of wet sand; showing the voids in the wet sand to be $\frac{33}{94}$ = 0.351 of the wet mass. **If the sand is poured into the water** hastily, air is carried in with it, the voids will not be filled, and the result will be quite different.

Compressibility of Sand. Careful experiments of our own, with ordinary pure sand from the seashore, both dry and moist (not wet), gave the following results. The dry sands were compacted by thorough shaking and jarring; the moist sands by ramming in thin layers. Sand B was of much finer grain than A. C consisted of the finest sifted grains from B.

	Perfectly dry.			Moist.		
	Lbs. per cubic ft.		Reduction of bulk,	Lbs. per cubic ft.		Reduction of bulk,
Sand.	Loose.	Shaken.	per cent.	Loose.	Rammed.	per cent.
A	97	112	13.4	86	107.5	20
B	88	101.6	13.4	69	107.5	33.3
C	82	98.5	16.8		103.5	

None of these sands, when dry and loose, if poured gently into water to a depth of 15 inches, settled more than about one-fifteenth part; the coarsest one, A, considerably less.

Water Required. See "Mr. Eliot C. Clarke," p. 936.

Cold water for mixing is probably no further injurious than that it retards setting.

Salt. See "Mr. Eliot C. Clarke," p. 936.

For pointing, the best Portland is none too good, and is best used neat, but it is often used with from 1 to 2 parts of sand. Mix under shelter, and in quantities of only 2 or 3 pints at a time, using very little water; so that the mortar, when ready for use, shall appear rather incoherent, and quite deficient in plasticity. The joints being previously scraped out to a depth of at least half an inch, the mortar is put in by trowel; a straight-edge being held just below the joint, if straight, as an auxiliary. The mortar is then to be well calked into the joint by a calking-iron and hammer; then more mortar is put in, and calked, until the joint is full. It is then rubbed and polished under as great pressure as the mason can exert. If the joints are very fine, they should be enlarged by a stonecutter, to about 1-4 inch, to receive the pointing. The wall should be well wet before the pointing is put in, and kept in such condition as neither to give water to, nor take it from, the mortar. In hot weather, the pointing should be kept sheltered for some days from the sun, so as not to dry too quickly.

Preservation of Metals.

We have found, by ten years' trial, that if, after setting, dampness is absolutely excluded, cements preserve iron, lead, zinc, copper and brass; and that plaster of Paris preserves all except iron, which it rusts somewhat unless the iron is galvanized. **Lime-mortar** probably preserves all of them, if kept free from damp.

Efflorescence.

Natural cements, when used as mortar for brickwork, often disfigure it, especially near sea-coasts, and in damp climates, by white efflorescence which sometimes spreads over the entire exposed face of the work, and also injures the bricks. This also occurs in stone masonry, but to a much less extent, and is confined to the mortar joints. It injures only porous stone. It is usually a hydrous carbonate of soda or of potash, or sulphate of lime (Epsom salts), often with other salts. As a preventive, General Gilmore recommends to add, to every 300 lbs. (1 barrel) of the cement powder, 100 lbs. of quicklime, and from 8 to 12 lbs. of any cheap animal fat; the fat to be well incorporated with the quicklime before slacking it, preparatory to adding it to the cement. This addition will retard the setting, and somewhat diminish the strength of the cement. It is said that linseed oil at the rate of 2 gallons to 300 lbs. of dry cement, either with or without lime, will in all exposures prevent efflorescence; but, like the fat, it greatly retards setting, and weakens the cement. See also Bricks.

Mr. Eliot C. Clarke has published * the results of a series of experiments made for the Boston Main Drainage Works. From his paper we condense as follows, by permission:

Variations in shade, *in a given kind of cement,* may indicate differences in the character of the rock or degree of burning. Thus, with Rosendale, a light color generally indicates an inferior or underburned rock. A coarse-ground cement, light in color and weight, would be viewed with suspicion.

The highest strength was obtained by the use of **just enough water** to dampen the cement thoroughly. An excess of water retards setting. Natural cements need more water than Portland; fine-ground more than coarse; quick-setting more than slow. Neat Rosendale, a year old, was strongest with 35 per cent. water. Neat Portland, same age, with 20 per cent.

The finer the sand, the less the strength.

Salt, either in the water used for mixing, or in that in which the cement is laid, retards setting somewhat, but has no important effect upon the strength.

Adding clay gives a much more dense, plastic, water-tight paste, useful for plaster or for stopping leaks. Half a part of clay did not seem to weaken mortar materially, except in the case of sample blocks exposed to the weather for 2½ years after a week's hardening in water.

A year's saturation in fresh or salt water, and in contact with **oak, hard pine, white pine, spruce or ash,** did not affect the mortars.

With sand, **fine-ground** cements make the strongest mortar; but when tested neat, **coarse-ground** cements are strongest. This is especially the case with Portlands.

---

* "Trans. Am. Soc. C. E.," April, 1885.

Good results were obtained from **mixing different cements.** A mortar of half a part each of Rosendale and Portland, and two parts sand, was stronger, at 1 week, 1 month, 6 months and 1 year, than the average of two mortars, one of 1 part Rosendale and one of 1 part Portland; each with 2 parts sand. Mixtures of Roman (quick-setting) and Portland (slow) set about as quickly as Roman alone, and were much stronger.

Portland **resisted abrasion** best when mixed with 2 parts sand; Rosendale with 1 part. A little more or less sand rapidly reduced the resistance in both cases.

Cements **expand in setting;** but not more than 1 part in 1000 of any given dimension.

Sand cement or **silica cement** is made by mixing cement with quartz sand (silica) and grinding the mixture. It is claimed that the cement, in the mixture, becomes much more finely ground, and that a mixture of 1 part cement and 3 parts sand can therefore carry, in mortar, nearly as much sand as could the pure cement alone before this treatment.

The fineness of cement and sand is indicated as follows, where the large numerals represent the sieve numbers; the small numeral, to the left of each sieve number, represents the percentage retained upon that sieve; and the final small numeral, to the right of the last sieve number, represents the percentage passed by the last sieve. The sum of the small numerals = 100. Thus, $^{5}20\ ^{15}30\ ^{35}40\ ^{45}$ means that 5 per cent. was retained on a No. 20 sieve, 15 per cent. on No. 30, and 35 per cent. on a No. 40, while the remaining 45 per cent. passed the No. 40 sieve.

Properties and tests of cement. Recommendations of American Society of Civil Engineers. Digest of Final Report of the Committee* on a Uniform System for Tests of Cement, Trans. Am. Soc. C. E., Vol. xiv, November, 1885.

The first tests of inexperienced, though intelligent and careful persons, are usually very contradictory and inaccurate, and no amount of experience can eliminate the variations introduced by the personal equations of the most conscientious observers. Many things, apparently of minor importance, exert so marked an influence upon the results, that it is only by the greatest care in every particular, aided by experience and intelligence, that trustworthy tests can be made.

Only a series of tests for a considerable period, and with a full dose of sand, will show the full value of any cement; and it would be safer to use a trustworthy brand without applying any tests whatever, than to accept a new article which had been tested only as neat cement and for but one day.

It is recommended that **tests** be confined to methods for determining (1) fineness, (2) liability to checking or cracking, and (3) tensile strength; and, for the latter, for tests of 7 days and upward, that a mixture of 1 part of cement to 1 part of sand for Natural † cements, and 3 parts of sand for Portland † cements, be used, in addition to trials of the neat cement. The quantities used in the mixture should be determined by weight.

The tests should be applied to the cements as offered for sale. If satisfactory results are obtained with a full dose of sand, the trials need go no further. If not, the coarser particles should first be excluded by using a No. 100 sieve, in order to determine approximately the grade the cement would take if ground fine, for fineness is always attainable, while inherent merit may not be.

The **amount of material** needed for making five briquettes of the standard size recommended is, for the neat cements, about 1.66 pounds, and for those with sand, in the proportion of 3 parts of sand to 1 of cement, about 1.25 pounds of sand and 6.66 ounces of cement.

---

* Q. A. Gillmore, Chairman, D. J. Whittemore, J. Herbert Shedd, Eliot C. Clarke, Alfred Noble, F. O. Norton, W. W. Maclay, Leonard F. Beckwith, Thomas C. McCollom.

† Where the word "natural" is used in this connection, it is to be understood as being applied to the lightly burned natural American or foreign cements, in contradistinction to the more heavily burned Portland cement, either natural or artificial.

Recommendations of Am. Soc. Civil Engrs. Continued.

Sampling. Usually, where cement has a good reputation, and is used in large masses, as in heavy concrete foundations, or in the backing or hearting of thick walls, the testing of every fifth barrel seems to be sufficient; but in very important work, where the strength of each barrel may in a great measure determine the strength of that portion of the work where it is used, or in the thin walls of sewers, etc., etc., every barrel should be tested, one briquette being made from each.

In selecting cement for experimental purposes, take the samples from the interior of the original packages, at sufficient depth to insure a fair exponent of the quality. Store the samples in tightly closed receptacles, impervious to light or dampness, until required for manipulation, when each sample of cement should be so thoroughly mixed, by sifting or otherwise, that it shall be uniform in character throughout its mass.

Sieves. For ascertaining the fineness of cement it will be convenient to use three sieves, viz.: (Sizes of wire by Stubs gage.)

No. 50— 50 meshes per linear inch (2,500 meshes per square inch), No. 35 wire.
No. 74— 74 " " " " (5,476 " " " "), No. 37 wire.
No. 100—100 " " " " (10,000 " " " "), No. 40 wire.

For sand, two sieves are recommended, viz.:

No. 20—20 meshes per linear inch (400 meshes per square inch), No. 28 wire.
No. 30—30 " " " " (900 " " " "), No. 31 wire.

Standard sand. Sands looking alike and sifted through the same sieves may give results varying within rather wide limits. The use of crushed quartz is recommended, the degree of fineness to be such that the sand will all pass a No. 20 sieve and be caught on a No. 30 sieve.

Mixing, etc. The proportions of cement, sand, and water should be carefully determined by weight, the sand and cement mixed dry, and all the water added at once. The mixing must be rapid and thorough; and the mortar, which should be stiff and plastic, should be firmly pressed into the molds with the trowel, without ramming, and struck off level; the molds in each instance, while being charged and manipulated, to be laid directly on glass, slate, or some other non-absorbent material. The molding must be completed before incipient setting begins. As soon as the briquettes are hard enough to bear it, they should be taken from the molds and be kept covered with a damp cloth until they are immersed. For the sake of uniformity, the briquettes, both of neat cement and those containing sand, should be immersed in water at the end of 24 hours, except in the case of 1 day tests.

Fresh, clean water, having a temperature between 60° and 70° F., should be used for the water of mixture and immersion of samples.

The proportion of water required varies with the fineness, age, or other conditions of the cement, and the temperature of the air, but is approximately as follows:

For neat cement: Portland, about 25 per cent.; natural, about 30 per cent.
For 1 cement, 1 sand: about 15 per cent. of total weight of sand and cement.
For 1 cement, 3 sand: about 12 per cent. of total weight of sand and cement.
The object is to produce the plasticity of rather stiff plasterer's mortar.

(1) **Fineness.** It is recommended that the tests be made with cement that has passed through a No. 100 sieve. See p 937.

(2) **Checking or cracking.** Make two cakes of neat cement, 2 or 3 inches in diameter, about $\frac{1}{2}$ inch thick, with thin edges. Note the time in minutes that these cakes, when mixed with water to the consistency of a stiff plastic mortar, take to set hard enough to stand the wire test recommended by Gen. Gillmore, viz., $\frac{1}{12}$ inch diameter wire loaded with $\frac{1}{4}$ pound for initial set, and $\frac{1}{24}$ inch loaded with 1 pound for final set. One of these cakes, when hard enough, should be put in water and examined from day to day to see whether it becomes contorted, and whether cracks show themselves at the edges. Such contortions or cracks indicate that the cement is unfit for use at that time. In some cases the tendency to crack, if caused by the presence of too much unslaked lime, will disappear with age. The remaining cake should be kept in

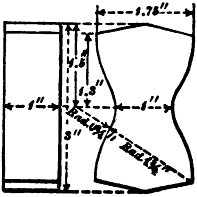
Recommendations of Am. Soc. Civil Engrs. Continued.

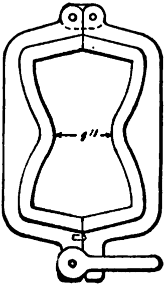
the air, and its color observed, which, for a good cement, should be uniform throughout; yellowish blotches indicating a poor quality; the Portland cements being of a bluish-gray, and the natural cements being light or dark, according to the character of the rock of which they are made. The color of the cements in air indicates the quality much better than when they are put in water.

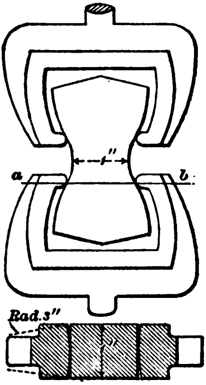
(3) **Tensile strength.** An average of 5 briquettes may be made for each test, only those breaking at the smallest section to be taken. The briquettes should always be broken immediately after being taken out of the water, and the temperature of the briquettes and of the testing room should be constant between 60° and 70° F.

The stress should be applied at a uniform rate of about 400 pounds per minute, starting at 0. With a weak mixture use half the speed.

The **molds** furnished are usually of iron or brass. Wooden molds, if well oiled to prevent absorption of water, answer a good purpose for temporary use, but speedily become unfit for accurate work. Our figures show the form of briquette and of metal mold recommended.







Section through a-b

The **clips** should be hung on pivots, so as to avoid, as far as possible, cross strain upon the briquettes.

Weight. The relation of the weight of cement to its tensile strength is an uncertain one. In practical work, if used alone, it is of little value as a test, while in connection with the other tests recommended it is unnecessary, except when the relative bulk of equal weights of cement is desired.

Setting. The rapidity of setting furnishes no indication of ultimate strength. It simply shows the initial hydraulic activity.

Quick-setting cements are those which set in less than half an hour; and **slow-setting** cements are those requiring half an hour or more to set. The cement must be adapted to the work required, as no one cement is equally good for all purposes. In submarine work a quick-setting cement is often imperatively demanded, and no other will answer, while for work above the water-line less hydraulic activity will usually be preferred. Each individual case demands special treatment. The slow-setting natural cements should not become warm while setting, but the quick-setting ones may, to a moderate extent, within the degree producing cracks. Cracks in Portland cement indicate too much carbonate of lime, and in the Vicat cements too much lime in the original mixture.

Properties and Tests of Cement. Report of Board U. S. A. Engineer Officers. Properties and tests of Portland, Natural and Puzzolan* cements. Digest of a Report of Majors W. L. Marshall and Smith S. Leach and Capt. Spencer Cosby, Board of Engineer Officers, on testing Hydraulic Cements. Professional Papers, No. 28, Corps of Engineers, U. S. A., 1901.

Unfortunately, tests for acceptance or rejection must be made on a product which has not reached its final stage. A cement, when incorporated in masonry, undergoes chemical changes for months, whereas it is seldom possible to continue tests for more than a few weeks at the most.

A few tests, carefully made, are more valuable than many, made with less care.

Cement which has been in **storage** for a long time should be **carefully tested** before use, in order to detect deterioration.

A cement should be rejected, without regard to the proportion of failures among samples tested, if the samples show dangerous variation in quality or lack of care in manufacture, and resulting lack of uniformity in the product.

The practice of offering a **bonus** for cement showing an abnormal strength is **objectionable,** as it leads to the production of cements with defects not easily detected.

For Portland or Puzzolan cement, **make tests for (1) fineness of grinding; (2)** specific gravity; (3) soundness, or constancy of volume in setting; (4) time of setting, and (5) tensile strength. For Natural cements omit tests (2) and (3).

(1) **Fineness.** Cementitious quality resides principally, if not wholly, in the very finely ground particles. Use a No. 100 sieve, woven from brass wire No. 40 Stubs gage; sift until cement ceases to pass through. The percentage that has passed through is determined by weighing the residue on the sieve. The screen should be frequently examined to see that no wires have been displaced. See p 937.

(2) **Specific gravity.** The specific gravity test is of value in determining whether a Portland cement is unadulterated. The higher the burning, short of vitrification, the better the cement and the higher the specific gravity. If underburned, the specific gravity of Portland cement may fall below 3; if overburned, it may reach 3.5. Natural cement has a specific gravity of about 2.5 to 2.8, and Puzzolan about 2.7 to 2.8.

The temperature may vary between 60° and 80° F. Any approved form of volumenometer or specific gravity bottle may be used, graduated to cubic centimeters with decimal subdivisions. Fill the instrument to zero of scale with benzine. Take 100 grams of sifted cement that has been previously dried by exposure on a metal plate for 20 minutes to a dry heat of 212° F., and allow it to pass slowly into the benzine, taking care that the powder does not stick to the sides of the graduated tube above the fluid, and that the funnel, through which it is introduced, does not touch the fluid. The approximate specific gravity will be represented by 100 divided by the displacement in cubic centimeters. The operation requires care.

(3) **Soundness,** and (4) **setting qualities.** The temperature should not vary more than 10° from 62° F. For Portland cement use 20, for Natural 30, and for Puzzolan 18 per cent. of water by weight. Mix thoroughly for 5 minutes. On glass plates make two cakes about 3 inches in diameter, ½ inch thick at the middle and drawn to thin edges, and cover them with a damp cloth. At the end of the minimum time specified for initial set, apply needle $\frac{1}{12}$ inch diameter, weighted to ¼ pound. If an indentation is made, the cement passes the requirement for initial setting. Otherwise the setting is too rapid. At the end of the maximum time specified for final set, apply the needle $\frac{1}{24}$ inch diameter, loaded to one pound. If no indentation is made, the cement passes the requirement for final set. Otherwise the setting is too slow.

Generally speaking, both periods of set are lengthened by increase of moisture, and shortened by increase of temperature.

---

*By **Portland** cement, in this report, is meant the product obtained by calcining intimate mixtures, either natural or artificial, of argillaceous and calcareous substances, up to incipient fusion. By **Natural** cement is meant one made by calcining natural rock at a heat below incipient fusion, and grinding the product to powder. By **Puzzolan** is meant the product obtained by grinding slag and slaked lime, without subsequent calcination.

Recommendations of Board of U. S. A. Engineer Officers. Continued.

In gaging Portland cement in damp weather, the samples should be thoroughly dried before adding water. This precaution is not deemed necessary with Natural cement. Sufficient uniformity of temperature will result if the testing room be comfortably warmed in winter, and if the specimens be kept out of the sun in a cool room in summer, and under a damp cloth until set. Temperatures may vary between 60° and 80° F., without affecting results more than the probable error in the observation.

Boiling test. Place the two cakes under a damp cloth for 24 hours. Place one of them, still attached to its plate, in water 28 days; immerse the other in water at about 70° F., and let it be in a rack above the bottom of the receptacle; heat the water gradually to the boiling point, maintain the heat for 6 hours and then let cool. The boiled cake should not warp or become detached from the plate, or show expansion cracks. If the cold-water cake shows evidences only of swelling, the cement may be used in ordinary work in air or fresh water for lean mixtures, but if distortion or expansion cracks appear in it, the cement should be rejected.

Accelerated tests are not generally recommended, but where a test must be made in a short time, the boiling test is considered about the best. It not only gives short-time indications, but at once directs attention to the presence of ingredients which might lead to disintegration. On the other hand, it may lead to the rejection of a cement which would behave satisfactorily in actual work and which would stand the test after air-slaking. Sulphate of lime, while enabling cements to pass the boiling tests, introduces an element of danger.

(5) **Tensile tests** are preferred to flexural or compressive tests. Sand tests are the more important and should always be made; and neat tests should be made if time permits.

A cement which tests moderately high at 7 days, and shows a substantial increase in strength in 28 days, is more likely to reach the maximum strength slowly and retain it indefinitely with a low modulus of elasticity, than a cement which tests abnormally high at 7 days with little or no increase at 28 days.

Use **briquettes** of the form recommended by the American Society of Civil Engineers,* measuring 1 inch square in cross-section at place of rupture, and held by close-fitting metal clips, without rubber or other yielding contacts. The tests should be made immediately after taking the briquettes from the water.

Neat tensile tests. Use unsifted cements. For Portland cement, use 20; for Natural, 30; and for Puzzolan, 18 per cent. water by weight. Place the cement on a smooth non-absorbent slab; in the middle make a crater sufficient to hold the water; add nearly all the water at once, the remainder as needed; mix thoroughly by turning with the trowel, and vigorously rub or work the cement for 5 minutes.

Place the briquette mold on a glass or slate slab. Fill the mold with consecutive layers of cement, each to be ¼ inch thick when rammed. Give each layer 30 taps with a soft brass or copper rammer weighing 1 pound, having a face ¾ inch diameter or 0.7 inch square, and falling about ½ inch.

After filling the mold and ramming the last layer, strike smooth with a trowel, tap mold lightly on side, to free cement from plate, remove the plate, and leave for 24 hours, covered with a damp cloth. Then remove the briquette from the mold and immerse it in fresh water, which should be renewed either continuously or twice in each week during the specified time.

Tensile tests with sand. For Portland and Puzzolan cements, use 1 part cement to 3 parts sand; for Natural or Rosendale, 1 to 1. Use crushed quartz sand, passing a No. 20 standard sieve, and being retained on a No. 30 standard sieve.

After weighing carefully, mix dry the cement and sand until the mixture is uniform, add the water as in neat mixtures, and mix for 5 minutes. The constituents should be well rubbed together.

For maximum strength in tested briquettes, Portland cements **require water** = 11 to 12½ per cent. by weight of constituent cement; Natural, 15 to 17; and Puzzolan, 9 to 10.

A machine which applies the **stress** automatically and at a **uniform rate**

---

* See page 939.

Recommendations of Board of U. S. A. Engineer Officers. Continued.

of increase is preferable to one controlled entirely by hand. The stress should be increased at the rate of about 400 lbs. per minute. A rate materially greater or less than this will give different results.

The highest tensile strength from each set of briquettes made at any one time is to be considered the governing test.

Field tests are recommended, whether or not the more elaborate tests above described have been made. In connection with tests of weight and fineness, and observations of texture and hardness in the work, field tests often suffice for well-known brands, showing whether the cement is genuine and whether it is reasonably sound and active. Pats and balls of neat cement from the storehouse, and of mortar from the mixing platform or machine, should be frequently made. Estimate roughly the setting and hardening qualities by pressure of the thumb-nail; hardness of set and strength by breaking with the hand and by dropping upon a hard surface. The boiling test may also be used. Should the simple tests give unsatisfactory or suspicious results, then a full series of tests should be carefully made.

A cement may be rejected if it fails to meet any of the following requirements.

Requirements.

	Portland. Slow.	Portland. Quick.	Natural.	Puzzolan.
Fineness. Percentage to pass through a No. 100 sieve as in (1)	87 to 92*		80	97
Specific gravity. Between	3.10	3.10	Not	2.7
and	3.25	3.25	given	2.8
Time of setting. Initial, not less than	45 m.	20 m.	20 m.	45 m.
nor more than	......	30 m.	......	......
Final, not less than	......	45 m.	......	......
nor more than	10 h.	2.5 h.	4 h.	10 h.
Tensile strength, neat, lbs. per sq. in. 7 days †	450	400	90	350
28 days †	540	480	200	500
Tensile strength. With sand, as in (5). lbs. per sq. in. 7 days †	140	120	60	140
28 days †	220	180	150	220

---

* 92 per cent. is quite commonly attained by high-grade American Portlands, but rarely by imported brands. For the latter, use 87.

† Reject any cement not showing an increase at 28 days over 7 days.

CONCRETE.

Cement concrete is an artificial stone, composed of cement mortar mixed with an "aggregate" consisting generally of broken stone, but often of gravel and sometimes of brick-bats, oyster shells, etc., or of all these together.

In concrete, as in mortar, it is advisable, on the score of strength, that all the voids be filled or overfilled.

Taking the voids, in broken stone, in gravel and in sand, at 50 per cent. of the mass, or equal to the solid portion, we have, **for 1 cubic yard of concrete:**

$$\left.\begin{array}{l}\text{1 cu. yd. broken stone}\\ \text{0.5 cu. yd. sand}\\ \text{0.25 cu. yd. cement}\end{array}\right\} = \left\{\begin{array}{l}\text{4 parts stone}\\ \text{2 parts sand}\\ \text{1 part cement}\end{array}\right.$$

or

$$\left.\begin{array}{l}\text{1 cu. yd. broken stone}\\ \text{0.5 cu. yd. gravel}\\ \text{0.25 cu. yd. sand,}\\ \text{0.125 cu. yd. cement}\end{array}\right\} = \left\{\begin{array}{l}\text{8 parts sandstone}\\ \text{4 parts gravel}\\ \text{2 parts sand}\\ \text{1 part cement.}\end{array}\right.$$

Caution. When a solid body is reduced to a mass consisting of broken pieces separated by voids, the increase in bulk is due solely to the voids, and is equal to the space occupied by them. Hence the ratio between the **increase of bulk,** or "**swelling,**" and the original bulk is that of the voids to the *original*, and not to the final bulk. Thus, if a solid cubic yard of stone, after being broken into pieces, occupies twice as much space as before, then the increase in bulk, or the space occupied by the voids, is = that occupied by solid pieces = *half* that occupied by the entire broken mass.

It is doubtful whether **hard rock,** when blasted and made into embankment, settles to less than 1.67 cubic yards for each original cubic yard. As the result of certain embankment in hard sandstone, Mr. Ellwood Morris gives 1.417 yards of embankment for each solid yard; but the rough sides of rock excavations make it difficult to measure them with accuracy, and this may have affected his result.

Stone Crushers. Principal sizes. From catalogue of Farrel Foundry and Machine Co. For prices, see Price-list..

Size of Stone, Ins.	Capacity per Day, Tons.	Capacity per Day, Size, Ins.	Horse-power Required.	Weight, Lbs.
7 × 10	50 15	2 ¾	8	8,000
16 × 10	120 60	2½ 1	15	16,400
24 × 13	300 150	3 1½	30	29,000
36 × 24	800 650	6 5	65	59,000

Concrete mixers are of various types. Some have a fixed trough, with a revolving worm; others a square box revolving about its diagonal axis, or a revolving drum. The gravity mixer is simply a steel trough, set at an angle of 18° to 22° with the vertical, and armed internally with fixed steel pins and deflectors. The materials for the concrete are fed in at the top of the trough, and, passing downward through it, are mixed by their contact with the pins and deflectors. The following figures, from the catalogue of the Drake Standard Machine Works, refer to the type first named.

Capacity, Cu. Yds. per Day.	Horse-power Required.	Weight, Lbs.
400	20	5,500
200	25	4,000
100	15	2,700
75	10	1,800
25	6	1,600

Compressive Strength.

The strength of concrete is affected by the quality of the broken stone, as well as by that of the cement, the degree of ramming, etc.

Ramming adds about 50 per cent. to the strength.

Slow-setting cements are best for concrete, especially when to be rammed.

Experiments on 12-inch concrete cubes, rammed in cast iron molds, by A. W. Dow, U. S. Inspector of Asphalt and Cement.* The cubes were thoroughly wet twice daily.

The results for one year are means of five cubes; the rest are means of two cubes. Deduct from 3 to 8 per cent. for friction of press.

The materials were as follows:

Cement.	Portland	Natural
Per cent. retained on sieve of 100 meshes per linear inch,	8.5	14
Time for initial set, minutes	190	20
" " hard " "	305	36

Tensile strength as follows, lbs. per square inch:

	1 Day.	7 Days.	1 Mo.	3 Mos.	6 Mos.	1 Year.
Portland, neat,	441	839				
" 3 parts standard broken quartz,		248	429	398	428	474
Natural, neat,	96	180				
" 2 parts standard broken quartz,		91	188	327	414	485

Sand used in concrete.

No residue on a No. 3 sieve; 0.5 per cent. passed No. 100. Voids 44 per cent., with 4.4 per cent. water.

Broken Stone. Gneiss. Of Nos. 6 and 12 (table below) 3 per cent. retained on 2.5 inch mesh; all on 1¼ inch. Others, 0 retained on 2.5 inch; nearly all on 0.1 inch. For voids, see table, below.

Gravel. Clean quartz, passing a 1¼-inch mesh, 2 per cent. passing a No. 10 mesh. Voids, 29 per cent.

Water. With Portland cement, 0.09 cu. ft. (= 5.7 lbs.) per cu. ft. of rammed concrete; with natural cement, 0.12 cu. ft. (= 7.5 lbs.).

Crushing Strength of 12 in. Concrete Cubes, in lbs. per sq. in.
Experiments by A. W. Dow, as above:
Parts by volume; cement, 1; sand, 2; aggregate, 6.

	No.	Aggregate		Voids in Aggregate.		Crushing Strength, lbs. per sq. in., after				
		Broken Stone, Parts.	Gravel, Parts.	Per Cent. of Vol.	Per Cent. of Voids Filled with Mortar.	10 Days.	45 Days.	3 Mos.	6 Mos.	1 Year.
Portland	7	6	..	45.3	83.9	908	1790	2260	2510	3060
	8	3	3	35.5	107.0	950	1850	..	2070	2750
	9	4	2	37.8	100.6	..	..	..	..	2840
	10	6	..	39.5	96.2	..	..	..	..	2700
	11	..	6	29.3	129.1	694	1630	2680	1840	2820
	12	6	..	45.7	83.9	..	..	1630	1530	1850
Natural	1	6	..	45.3	83.9	228	539	375	795	915
	2	3	3	35.5	107.0	108	364	593	632	841
	3	4	2	37.8	100.6	..	..	..	..	915
	4	6	..	39.5	96.2	..	..	..	..	800
	5	..	6	29.3	129.1	87	421	361	344	763
	6	6	..	45.7	83.9	..	..	596	..	829

To test the **effect of the quantity of water used,** Mr. Dow made four 12-inch cubes, of 1 part Portland cement, 2 parts sand, 6 parts coarse gravel; two so wet that water ran from the concrete when tamped in the mold; the other two with just sufficient water to make a plastic paste, as in the tests recorded in the table. At the end of a year the two wet cubes gave 2300 and 2500 lbs. per square inch respectively; average, 2400. The drier cubes gave 2513 and 3037 respectively; average, 2775.

---

* Report of the Operations of the Engineering Department of the District of Columbia for the year ending June 30, 1897.

In the floors of graving docks at Genoa, Italy, concrete of 1 part Portland cement, 2 parts sand, 3 parts small gravel, carries safely 107 lbs. per square inch; safety factor 15. In the concrete bridge over the Danube at Munderkingen, Germany, the maximum pressure is said to vary from 500 to 560 lbs. per square inch. Test pieces broke with from 2000 to 4650 lbs. per sq. inch, varying with their age.

Transverse Strength.

Concrete Beams. Tests by Boston Transit Commission, 1895–96. Beams 6 ins. square; spans 30 and 60 ins.; age from 29 to 37 days, 24 hours in air, remainder in water.

S = modulus of rupture = unit stress in extreme fibers at instant of rupture, in lbs. per sq. inch.

$W = \frac{S}{18}$ = extraneous center breaking load, in lbs., on a beam 1 inch square, 1 ft. span.

Mixture.				S.		
Portland cement.	Sand.	Stone.		Max.	Min.	Av'ge.
1	2	4½	40 tests	531	158	304
1	2½	4	38 tests	441	131	280

Effect of size of stone. Mixture, 1 — 2½ — 4; 24 hours in air, 29 days in wet ground. 6 tests for each size.

Size, Ins.	Wt., Lbs. per Cu. Ft.		Voids. Per Cent.	S.		
	Stone.	Concrete.		Max.	Min.	Av'ge.
2 to 1.5	95	156	48	328	263	298
1.5 to 1	95	156	45	349	272	311
1 to 0.5	99	150	40	305	219	263
0.5 to 0.25	100	150	40	312	229	261
0.25 and less	94	142	37	248	214	227
mixed	108	152	33½	349	230	293

When masonry is backed by concrete, the two are liable in time to crack apart, owing to unequal settlement, especially if the ramming has not been thorough.

In variable climates, **cast-iron cylinders, filled with concrete,** are frequently split horizontally by unequal expansion and contraction. In such structures it is safest to consider the cylinders as mere molds for the concrete; and to depend only upon the concrete for sustaining the load.

Molded blocks of Portland concrete, of even 50 tons weight, can generally be handled and removed to their places in from 1 to 2 weeks.

Ramming of concrete, when properly done, consolidates the mass about 5 or 6 per cent., rendering it less porous, and very materially stronger. The rammers, like those used in street paving, are of wood, about 4 ft. long, 6 to 8 ins. diameter at foot, with a lifting handle, and shod with iron; weight about 35 lbs. They are let fall 6 or 8 ins. The men using them, if standing on the concrete, should wear gum boots to preserve their feet from corrosion by the cement.

Ramming cannot be done under water, except partially, when the concrete is inclosed in bags. A rake may, however, be used gently for leveling concrete under water.

The size of the broken stone for concrete is generally specified not to exceed about 2 ins. on any edge; but if it is well freed from dust by screening or washing, all sizes, from 0.5 to 4 ins. on any edge, may be used, care being taken that the other ingredients completely fill the voids.

Concrete is used for **bringing up uneven foundations** to a level before starting the masonry. By this means the number of horizontal joints in the masonry is equalized, and unequal settlement is thereby prevented.

Concrete may readily be deposited under water in the usual way of lowering it, soon after it is mixed, in a V-shaped box of wood or plate iron, with a lid that may be closed while the box descends. The lid, however, is often omitted. This box is so arranged that, on reaching bottom, a pin may be drawn out by a string reaching to the surface, thus permitting one of the sloping sides to swing open below, and allow the concrete to fall out. The box is then raised to be refilled. In large works the box may contain a cubic yard or more, and should be suspended from a traveling crane, by which it can readily be brought over any required spot in the work. The concrete may if necessary be gently leveled by a rake soon after it leaves the box. Its consistency and strength will of course be impaired by falling through the water from the box; and moreover it cannot be rammed under water without still greater injury. Concrete has been safely deposited in the above-mentioned manner in depths of 50 ft.

The Tremie, sometimes used for depositing concrete under water, is a box of wood or of plate iron, round or square, open at top and bottom, and of a length suited to the depth of water. It may be about 18 ins. diam. Its top, which is always kept above water, is hopper-shaped, for receiving the concrete more readily. It is moved laterally and vertically by a traveling crane or other device suited to the case. Its lower end rests on the river bottom, or on the deposited concrete. In commencing operations, its lower end resting on the river bottom, it is first entirely filled with concrete, which (to prevent its being washed to pieces by falling through the water in the tremie) is lowered in a cylindrical tub, with a bottom somewhat like the box before described, which can be opened when it arrives at its proper place. When filled, the tremie is kept so by fresh concrete, thrown into the hopper to supply the place of that which gradually falls out below, as the tremie is lifted a little to allow it to do so. The weight of the filled tremie compacts the concrete as it is deposited. A tremie had better widen out downward to allow the concrete to fall out more readily.

The area upon which the concrete is deposited must previously be surrounded by some kind of inclosure, to prevent the concrete from spreading beyond its proper limits; and to serve as a mold to give it its intended shape. This inclosure must be so strong that its sides may not be bulged outward by the weight of the concrete. It is usually a close crib of timber or plate iron without a bottom; and will remain after the work is done. If of timber it may require an outer row of cells, to be filled with stone or gravel for sinking it into place. Care must be taken to prevent the escape of the concrete through open spaces under the sides of the crib or inclosure. To this end the crib may be scribed to suit the inequalities of the bottom when the latter cannot readily be leveled off. Or inside sheet piles will be better in some cases; or an outer or inner broad flap of tarpaulin may be fastened all around the lower edge of the crib, and be weighted with stone or gravel to keep it in place on the bottom. Broken stone or gravel or even earth (the last two where there is no current), heaped up outside of a weak crib, will prevent the bulging outward of its sides by the pressure of the concrete. After the concrete has been carried up to within some feet of low water, and leveled off, the masonry may be started upon it by means of a caisson, or by men in diving suits. Or, if the concrete reaches very nearly to low water, a first deep course of stone may be laid, and the work thus brought at once above low water without any such aids.

The concrete should extend out from 2 to 5 feet (according to the case) beyond the base of the masonry. All soft mud should be removed before depositing concrete.

Bags partly filled with concrete, and merely thrown into the water, are used in certain cases. If the texture of the bags is slightly open, a portion of the cement will ooze out, and bind the whole into a tolerably compact mass. Such bags, by the aid of divers, may be employed for stopping leaks, underpinning, and various other purposes, that may suggest themselves. Such bags may be rammed to some extent.

Tarpaulin may be spread over deep seams in rock to prevent the loss of concrete; and, in some cases, to prevent it from being washed away by springs.

"When concrete is deposited in water, especially in the sea, a pulpy gelatinous fluid exudes from the cement, and rises to the surface. This causes the water to assume a milky hue; hence the term **laitance**, which French engineers apply to this substance. As it sets very imperfectly, and, with some varieties of cements, scarcely at all, its interposition between the layers of concrete, even in moderate quantities, will have a tendency to lessen, more or less sensibly, the continuity and strength of the mass. It is usually removed from the inclosed space by pumps. Its proportion is greatly diminished by reducing the area of concrete exposed to the water, by using *large* boxes, say from 1 to 1½ cu. yds. capacity, for immersing the concrete."

MODERN EXPLOSIVES.

Art. 1. Most of the explosives, which, of late years, have been taking the place of gunpowder, **consist of** a powdered substance, partly saturated with nitro-glycerine, a fluid produced by mixing glycerine with nitric and sulphuric acids.

Art. 2. Pure **nitro-glycerine,** at 60° Fah, has a sp grav of 1.6. It is odorless, nearly or quite colorless, and has a sweetish, burning taste. It is poisonous, even in very small quantities. Handling it is apt to cause headaches. It is insoluble in water. At about 306° Fah it takes fire, and, if unconfined, burns harmlessly, unless it is in such quantity that a part of it, before coming in contact with air, becomes heated to the exploding point, which is about 380° Fah.

N-G, and the powders containing it, are always **exploded by means of** sharp percussion. See Arts 36, &c. After N-G is made, great care is required to wash it completely from the **surplus acids remaining in it** from the process of manufacture. Their presence, either in the liquid N-G, or in the powders containing it, renders the N-G liable to spontaneous decomposition, which, by raising the temperature, increases the danger of explosion.

Art. 3. N-G freezes at about 45° Fah. It is then very **difficult of explosion,** and must be thawed *gradually,* as by leaving it for a sufficient length of time in a comfortably warm room, or by placing the vessel containing it in a second vessel containing hot water, not over 100° Fah; but never by exposing it to intense heat, as in placing it before a fire, or setting it on a stove or boiler. Extra strong caps are made for exploding N-G and its powders when frozen.

Art. 4. N-G, owing to its incompressibility, is **liable to explosion through accidental percussion.** This, and its **liability to leakage,** render it inconvenient to transport and handle. Hence it is rarely used in the liquid state in ordinary quarrying and other blasting. In the oil regions of Penna, it is largely used in **oil wells,** in order to increase the flow. For this purpose it is confined in cylindrical tin casings, from 1 to 5 inches diam, called torpedo-shells. These are suspended from, and lowered into the well by means of, a cord or wire wound on a reel; and are destroyed when the charge is exploded. They are about 1 inch less in diam than the well, and contain usually from one to twenty quarts = 3 lbs, 5⅛ oz to 66. lbs, 6¼ oz of N-G. They are pointed at their lower ends, in order to facilitate their passage through the oil or water which may be in the well. When a greater charge than about 66⅛ lbs is required, two or more of these shells are placed in the well, one on top of another, the conical point on the lower end of each one fitting into the top of the one next below. In this case, the N-G is fired by means of a cap or series of caps placed in the top of the charge before it is lowered. When the charge is in place, the caps are exploded by electricity led to them by conducting wires, as in Art 37, or (as in the method more commonly practised) by letting a weight fall on them.

When a well has been repeatedly torpedoed, and a cavity has thus been formed in it so large that the space surrounding a torpedo would interfere too greatly with the effect of the explosion of the N-G on the walls of the well, the latter is placed directly in the well, by lowering a tin cylinder, filled with it, and provided with an automatic arrangement which allows the N-G to escape when at the bottom of the well. The N-G is then fired by a torpedo suspended on a line, and having caps placed in its top. These caps are exploded by a leaden or iron weight sliding down the line, or by electricity. When the rock is seamy, the N-G is confined in short cylindrical tin shells, lowered into the cavity, and fired by a torpedo. It is also used for increasing the flow of springs of *water.* It of course cannot be used in **hor or upward** holes, such as often occur in tunneling, &c.

Art. 5. N-G explodes so suddenly that **very little tamping is required.** Moist sand or earth, or even water, is sufficient. This, with the fact that N-G is unaffected by immersion in water, and is heavier than water, render it particularly **suitable for sub-aqueous work,** or for holes containing water, provided the rock has no seams which would permit the N-G to escape. **If the rock is seamy,** the N-G must be confined in a water-tight casing. Such casings, however, necessarily leave some spaces between the rock and the explosive, and these diminish considerably the effect of the latter.

Art. 6. The great **explosive force of N-G is due** partly to the very large volume of gas into which a small quantity of it is converted by explosion, and partly to the *suddenness* with which this conversion takes place, the gases being liberated almost instantaneously,* while with gunpowder their liberation requires a longer time. The suddenness of the explosion increases its effect, not only by applying all of its force practically at one instant, but also by greatly *heating* the gases produced, and thus still further increasing their *volume*.

Art. 7. The liquid condition of N-G is useful in causing it to **fill the drill-hole completely**, so that there are no vacant spaces in it to waste the force of the explosion. On the other hand, the liquid form is a disadvantage, because, when thus used without a containing vessel in seamy rock, portions of the N-G leak away and remain unexploded and unsuspected, and may cause accidental explosion at a future time.

Art. 8. N-G is stored in tin cans or earthenware jars. If properly washed from acid it does not injure tin. For transportation, these cans or jars are packed in boxes with sawdust, or in padded boxes, and loaded in *wagons*. The R R companies do not receive it.

Art. 9. When N-G and its compounds are *completely* exploded, **the gases given out** are not troublesome, but those resulting from *incomplete* explosion such as generally takes place, or from combustion, are very offensive.

Art. 10. For convenience, we apply the name "**dynamite**" to any explosive which contains nitro-glycerine mixed with a granular absorbent; "**true dynamite**" to those in which the absorbent of the N-G is "Kieselguhr,"† or some other *inert* powder which takes no part in the explosion; and "**false dynamite**" to those in which the absorbent itself contains explosive substances other than N-G.

Art. 11. The absorbent, by its granular and compressible condition, **acts as a cushion to the N-G**, and protects it from percussion, and from the consequent danger of accidental explosion.

N-G undergoes no change in composition by being absorbed; and it then freezes, burns, explodes, &c, under the same conditions as to pressure, temperature, &c, as when in the liquid form. The cushioning effect of the absorbent merely renders it more difficult to bring about sufficient percussive pressure to cause explosion. The absorption of the N-G in dyn enables the latter to be used in *hor* holes, or in holes drilled *upward*.

Art. 12. N-G and dyn **explode much more readily when rigidly confined**, as by a *metallic* vessel, or by the walls of a hole drilled in rock, than when confined by a *yielding* substance, as wood. Therefore the fact that dyn, not being liquid, can be packed in *wooden* boxes, renders it safer than N-G which has to be kept in stone or metal vessels.

Art. 13. True dynamites must contain at least about 50 per cent of N-G. Otherwise the latter will be too completely cushioned by the absorbent, and the powder will be too difficult to explode. False dynamites, on the contrary, may contain as small a percentage of N-G as may be desired; some containing as little as 15 per cent. The added explosive substances in the false dynamites generally contain large quantities of oxygen, which are liberated upon explosion, and aid in effecting the complete combustion of any noxious gases arising from the N-G.

Art. 14. Dynamites which contain large percentages of N-G explode (like the liquid N-G, Art 6) with great suddenness, tending to *shatter* the rock in their vicinity into small fragments. They are most useful in *very hard* rock. In such rock, **No 1 dynamite**, or that containing 75 per cent of N-G, is roughly estimated to have **about 6 times the force of an equal wt of gunpowder.**

For soft or decomposed rocks, sand, and earth, the **lower grades** of dynamite, or those containing a smaller percentage of N-G, are more suitable. They explode with less suddenness, and their tendency is rather to upheave large masses of rock, &c, than to splinter small masses of it. They thus more nearly resemble gunpowder in their action.

Judgment must be exercised as to the **grade and quantity of explosive** to be used in any given case. Where it is not objectionable to break the rock into small pieces, or where it is desired to do so for convenience of removal, the higher, *shattering* grades are useful. Where it is desired to get the rock out in large masses, as in quarrying, the lower grades are preferable.

For very difficult work in hard rock, and for submarine blasting, the highest grades, containing 70 to 75 per cent of N-G, are used. A small charge of these does the same execution as a larger charge of lower grade, and of course does not require the drilling of so large a hole. In submarine work their sharp explosion is not deadened by the water.

---

* Such sudden liberation of gas is called "**detonation.**"

† Kieselguhr is an earthy, silicious limestone, composed of the fossil remains of small shells. Each shell acts as a minute receptacle for nitro-glycerine. Kieselguhr is found in Hanover, Germany and in New Jersey.

For general railroad work, ordinary tunneling, mining of ores, &c, the **average grade**, containing 40 per cent of N-G, is used; **for quarrying**, 35 per cent; **for blasting stumps**, trees, piles, &c, 30 per cent; **for sand and earth**, 15 per cent.

Art. 15. Dynamite, like N-G, **can be readily exploded under water**, provided it is so immersed as not to be *scattered;* but **long exposure to water is injurious** to it. In the higher grades, the water, by its greater affinity for the absorbent, drives out the N-G. In the lower grades it is apt to wash away the salts used as additional explosives.

Art. 16. In dyns containing a large percentage of N-G, the latter is **liable to exude** in liquid form, or to "leak," especially in warm weather, and then to explode through accidental percussion. The same danger exists, even though the percentage of N-G be small, if the absorbent has but small absorbing power, and is, consequently, easily saturated.

Art. 17. **True dyn** resembles moist brown sugar. **Its properties** are generally those of the N-G contained in it. Thus, it takes fire at about 350° F, and burns freely. It freezes at 45° F, and is then difficult to explode. It is not exploded by friction, or by ordinary percussion, but requires, for general purposes, a strong cap, or exploder, containing fulminating powder, see Arts 36, 38, &c. It may, however, be exploded by a priming of gunpowder, tightly tamped, and fired by an ordinary safety-fuse.

Art. 18. **The charge should fill the cross section of the hole** as completely as possible. If water is not standing in the hole, the cartridge should be cut open before insertion, so that the powder may escape from it and fill the hole; or the powder may be simply emptied from the cartridge into the hole.

Art. 19. For **blasting ice** in place, holes are cut in it, and a number of dyn cartridges (one of which must contain an exploding cap) are tied together and lowered from 1 to 5 ft into the water. They are fired as soon as possible after immersion, to avoid the danger of freezing. Electrical exploders (Arts 37, &c,) are best for sub-aqueous work.

Art. 20. Dyn is useful for **breaking up pieces of metal**, such as old cannon, condemned machinery, "salamanders" (masses of hardened slag) in blast furnaces, &c. In cannon, the dyn is of course exploded in the bore. In other pieces, small holes are generally drilled to receive it; but plates, even of considerable thickness, may be broken by merely exploding dyn upon their surface.

Art. 21. For **blasting trees or stumps**, one or more cartridges are fired in a hole bored in the trunk or roots, or under the latter. This shatters both trunk and roots. **A tree may be felled neatly** by boring a number of small radial holes into it, at equal short dists in a hor line around its circumf, and, by means of an electric battery (Arts 37, &c), exploding simultaneously a small charge of dyn in each. Or a single long cartridge may be tied around the trunk of a small tree, and fired.

Art. 22. **Piles may be blasted** in the same way as trees; or a hole may be bored for the cartridge in the axis of the pile; or the cartridge may be simply tied to the side of the pile at any desired ht.

Art. 23. The higher grades of dyn, like N-G, **require but little tamping.** Use a *wooden* **tamping-bar**, *never a metallic* one, for any explosive. If a charge of dyn "**hangs fire**," it is dangerous to attempt to remove it. Remove the *tamping*, all but a few ins in depth, on top of which insert another cartridge, containing an exploder, and try again. See electrical exploders, Arts 37, &c. Dyn, like N-G, if frozen, must be thawed *gradually*, by leaving it in a warm room, far from the fire; or by placing it in a metallic vessel, which is then placed in another vessel containing hot water. The water should not be hotter than can be borne by the hand. Otherwise the N-G is liable to separate from the absorbent. The N-G in dyn may freeze without cementing together the particles of the absorbent; in which case the powder of course is still soft to the touch. **An overcharge** of N-G, or of dyn, is liable to be burned, and thus wasted, giving off offensive gases.

Art. 24. **Dyn is sold in cylindrical, paper-covered cartridges**, from ⅞ to 2 ins in diam, and 6 to 8 ins long, or longer. They are furnished to order of any required size, and are packed in boxes containing 25 ℔s or 50 ℔s each. The layers of cartridges are separated by sawdust.

Art. 25. Some of the **R R companies** decline to carry dyn or N-G in any shape. Others carry dyn under certain restrictions, based upon State laws; providing that it must be dry (*i e*, that no N-G shall be exuding from it); that boxes and cars containing it shall be plainly marked with some cautionary words, as "explosive," "dangerous," &c; that the cartridges shall be so packed in the boxes, and the boxes so loaded in the cars, that both shall lie upon their *sides;* and the boxes be in no danger of falling to the floor; that caps, &c, shall not be loaded in the same car with dyn, &c, &c.

Art. 26. A great many varieties of dyn are made. They differ (generally but slightly) in the composition of the absorbent, and in the method of manufacture. Each maker usually makes a number of grades, containing different percentages of N-G, &c, and gives to his powders some fanciful name.

Art. 27. The following **table of explosives,** made by the Repauno Chemical Co, Wilmington, Del, and known as "**Atlas**" **powders,** gives the percentage of N-G in each.

Brand.	Percentage of N-G.,	Brand.	Percentage of N-G.
A	75	D+	33
B+	60		
B	50	E+	27
C+	45	E	20
C	40		

The absorbents contain: in "A" brand, 18 per cent wood pulp and 7 per cent carbonate of magnesia; in "C" brand (the average grade), 46 per cent nitrate of soda (soda saltpetre), 11 per cent wood pulp, and 3 per cent carbonate of magnesia; in "E" brand, 62 per cent nitrate of soda, 16 per cent wood pulp, &c, and 2 per cent carbonate of magnesia.

Art. 28. "**Miner's Friend**" powder contains nitrate of soda, wood pulp, resin, and carbonate of magnesia. It freezes at 42°, and is then, like other dyn, difficult to explode. When used under water, the cartridges should not be broken, because the powder is injured by direct contact with water. Their "**Hecla**" **powder** is a lower grade. It is in granulated form, like ordinary blasting powder, but is said to be much stronger. It is intended as a substitute for it.

Art. 29. "**Giant**" **powder** is dyn proper, containing 75 per cent N-G, and 25 per cent Kieselguhr obtained near their works in New Jersey. The lowest grade, branded "M," contains 20 per cent N-G. The name "giant powder" was originally applied to dynamite in general.

Art. 30. Other brands are "**Hercules**" powder and "**Judson R R P powder,**" a substitute for ordinary blasting powder. It is put up in water-proof paper bags, of 6¼, 12½, and 25 lbs each, and these are packed in wooden boxes holding 50 lbs each. "**Judson F F F dynamite**" is a higher grade, in cartridges of the usual shape, packed in 50-lb boxes.

Art. 31. "**Rackarock**" cartridges are said to contain no N-G, and to be entirely inexplosive until immersed, for a few seconds, in an inexplosive liquid furnished by the same Co. They are then allowed to stand for 15 mins, after which they may be used at any time. They are fired in the same way as dyn, and can be used under water. The mfrs claim that they "approximate N-G in strength, and are stronger than dyn."

Art. 32. The following explosives are made and used in Europe, but have not yet been regularly imported into the U. S.

Compressed gun-cotton, is cotton dipped in a mixture of nitric and sulphuric acids, then reduced to a fine pulp, and made into discs 1 to 2 ins thick, and ⅞ to 2 ins diam, or larger. It is generally used wet, for the sake of greater safety. It then requires extra strong caps or primers. Roughly speaking, it is about as strong as dyn No 1, but is less *shattering* in its effect. Being lighter than dyn, it requires larger holes; and, owing to its rigidity, is less easily inserted, and does not fit the hole so completely. When dry, it is very inflammable, but, if not confined, it burns harmlessly. It contains no liquid, to freeze or to exude; and is safe to handle.

Art. 33. Tonite consists of finely divided gun-cotton mixed with nitrate of baryta. It is compressed into candle-shaped cartridges having, at one end, a recess for the reception of an exploder containing fulminate of mercury. The cartridges weigh about the same as dyn. They are generally made waterproof.

Art. 34. Forcite, Lithofracteur, and **Dualin** are foreign makes of nitro-glycerine explosives. In Dualin the absorbent is sawdust. It has greater bulk than dyn for a given wt, and requires larger holes.

Art. 35. Explosive gelatine is a transparent, pale yellow, elastic substance, and is composed of 90 per cent N-G and 10 per cent gun-cotton. It is less sensitive than dyn to percussion, friction, or pressure, and is not affected by water. Its specific gravity is 1.6. It burns in the open air. For complete detonation a special primer is required. The addition of a small proportion of camphor renders it still less sensitive, and increases its explosive force. The camphor evaporates to some extent.

In some experiments on the power of different explosives to increase the contents of a small cavity in a leaden block, explosive gelatine caused an increase 50 per cent greater than that caused by dyn No 1. In hard rock the diff would probably have been greater. The increase was 10 per cent less than that caused by N-G.

Art. 36. The cap or exploder, used with ordinary safety fuse for exploding N-G and dyn, is a hollow copper cylinder, about ¼ inch diam, and an inch or two in length. It contains from 15 to 20 per cent, or more, of fulminate of mercury, mixed with other ingredients into a cement, which fills the closed end of the cap. The cap is called "single-force," "triple-force," &c, according to the quantity of explosive it contains.

The end of the fuse, cut off square, is inserted into the open end of this cap, far enough to touch the fulminating mixture in it. In doing this, care must be taken not to roughly scratch the latter. The neck of the cap is then pinched, near its open end, so as to hold the fuse securely. The cap, with the fuse thus attached, is then inserted into the charge of N-G or dyn, care being taken not to let the fuse come into contact with the explosive, which would then be burned and wasted. If a dyn cartridge is used, the fuse, with cap, is first inserted into it. The neck of the cartridge is then tied around the fuse with a string, and the cartridge is then ready to be placed in the hole and fired.

Art. 37. The Siemens magneto-electric blasting apparatus, now in general use, consists of a wooden box about as large as a transit-box. Outside it has two metallic binding-posts with screws, for attaching the two wires leading to the exploder. From the top of the box projects a handle at the end of a vert bar. This bar, which is about as long as the box is high, is made so as to slide up and down in it, and is toothed, and gears with a small pinion inside the box. When a blast is to be fired, the bar is drawn up, by means of the handle, as far as it will come. It is then pressed quickly down to the bottom of the box. In its descent it puts into operation, by means of the pinion, a magneto-electric machine inside the box. This generates a current of electricity, which increases in force with the downward motion of the bar, but which is confined to a short circuit of wire *within the box*, until the foot of the bar strikes a spring near the bottom of the box, breaking the short circuit and forcing the electricity to travel through the two longer "leading wires," which lead it from the two binding-posts on the outside of the box to the cap or exploder placed in the charge.

Art. 38. The cap used with this machine is similar to that used with safety fuse (Art 36), except that its mouth is closed with a cork of sulphur cement, through which pass the two wires leading from the electric machine. The ends of these wires project into the fulminating mixture in the cap. They are ⅛ inch apart, but are connected by a platinum wire, which is so fine as to be heated to redness by the current from the battery. Its heat ignites the fulminate and thus explodes the cap.

Art. 39. Where a number of holes are to be fired simultaneously (thus increasing their effect), each hole has a platinum cap inserted into its charge, and one of the short wires attached to each cap is joined to one of those of the next cap, so that at each end of the series of caps there is one free end of a short wire. Each of these two ends is fastened to the end of one of the leading wires, placing the whole series "in one circuit." Where the holes are too far apart for the caps to be thus joined by the short wires attached to them, the ends of the latter are connected by cotton-covered "**connecting wires.**"

Art. 40. The magneto-electrical **machine weighs** about 16 ℔s. It can fire about 12 caps at once.

Caps for ordinary fuse and for electrical firing, fuses, wires, electrical machines, &c, are sold by most of the makers of, and dealers in, explosives, rock-drilling machines, &c.

Art. 41. Simultaneous firing of a number of holes can be conveniently accomplished only by electricity. Electric blasting apparatus is specially useful for blasting under water, where ordinary fuses are apt, especially at great depths, to become saturated and useless.

If an electrical machine fails to fire a charge, it is known that the charge cannot explode until the attempt is repeated. Therefore no time need be lost, and no risks run, on account of "hanging fire."

GUNPOWDER.

The explosive force of powder is about 40000 ℔s, or 18 tons, per square inch. Its **weight** averages about the same as that of water, or 62½ ℔s per cubic foot; hence, 1 ℔ = about 28 cubic inches. In ordinary quarrying, a cubic yard of solid rock in place, (or about 1.9 cubic yards piled up after being quarried,) requires from ¼ to ¾ ℔. In very refractory rock, lying badly for quarrying, a solid yard may require from 1 to 2 ℔s. In some of the most successful great blasts for stone for the Holyhead Breakwater, Wales (where several thousands of ℔s of powder were usually exploded by electricity at a single blast,) from 2 to 4 cubic yards solid were *loosened* per ℔; but in many instances not more than 1 to 1½ yards. Tunnels and shafts require 2 to 6 ℔s per solid yard; usually 3 to 5 ℔s. Soft, partially decomposed rock frequently requires more than harder ones. Usually sold in kegs of 25 ℔s.*

Weight of powder in one foot depth of hole.

Diameter of hole	1 in	1¼ ins	1½ ins	2 ins	2½ ins	3 ins
Weight of powder avoirdupois	0℔ 5oz	0℔ 8oz	0℔ 11oz	1℔ 4oz	2℔	2℔ 13oz
Diameter of hole	3½ in	4 ins	4½ ins	5 ins	5½ ins	6 ins
Weight of Powder avoirdupois	3℔ 14oz	5℔ 0oz	6℔ 6oz	7℔ 14oz	9℔ 8oz	11℔ 5oz

PRESERVATION OF TIMBER.

Art. 1. (a) The decay of timber is caused by the growth and activities of fungi. The minute spores of one of these fungi, germinating on a piece of wood, send out fine threads, which enter the wood cells and soon give off a complex compound called a ferment or enzyme, which dissolves certain parts of the wood fibre. The dissolved fibre serves as food for the fungus. The threads throw out branches and sub-branches, and soon the timber is permeated by a mass of such threads, the growing parts of which give off ferment. The action of the ferment changes the chemical and physical properties of the wood, rendering it, in some cases, like brown charcoal, in others white, soft and stringy, and the wood is said to be rotten or decayed. Eventually some of the threads grow out from the surface of the timber, and form toadstools and other excrescences. Under these are found cavities containing thousands of spores, which, when ripe, are blown off into the air and settle upon other timbers, where the process is repeated. Moisture and heat are favorable to the growth of the fungi, as are also the starches, sugars and oils found in the cells of the sapwood but wanting in the heartwood. If protected from the action of these fungi, wood will last indefinitely. Hence the accumulation of deadwood should be avoided.* If air is excluded, as when timber is kept constantly and entirely immersed in salt or fresh water, the fungi cannot thrive. Sap, confined in timber with air, ferments, producing **dry rot;** as where beams are enclosed air-tight in brickwork, etc.; and where green timber is painted or varnished, or treated with creosote, etc. The sap then not only prevents the thorough penetration of the oil, etc, but may cause the greater part of the wood to rot although its firm outer shell gives it a deceptive appearance of strength. (b) Sap should therefore be first removed by **seasoning;** *i e*, either by drying the wood in air at natural or higher temperatures, or by first steaming the wood under pres so as to vaporize the sap, and then removing the latter by means of a vacuum. Thorough seasoning of large timbers in dry air at ordinary temperatures may require years; and too rapid kiln-drying cracks and weakens the wood. But it is questionable whether steaming and vacuum remove sap as thoroughly as do the slower dry processes. (c) Alternate exposure to water and air is very destructive. It causes **wet rot.**

Art. 2. Sea-worms. The *limnoria terebrans* works from near high-water mark to a little below the surface of mud bottom; the *teredo navalis* within somewhat less limits. The teredo is said to be rendered less active by the presence of sewage in water.

Art. 3. (a) The best timber-preserving processes are practically **useless unless thoroughly well done.** If the gain in durability will not warrant the expenditure of time and money reqd for this, it is more economical to use the wood in its natural state. (b) **The woods best adapted to treatment** are those of an open or porous texture. They absorb the oil etc better than the denser woods; and their cheapness renders the use of the treatment more economical. (c) Most of the processes in common use seem to render wood less combustible. (d) After treatment by any process, the wood should be well dried before using.

Art. 4. (a) **Creosote oil, or dead oil,** is the best known preservative. Against sea-worms it is effective for 15 to 25 years, and is the only known protection. (b) As temporary expedients, piles are sometimes covered with sheet metal, or with broad-headed nails driven close together. These rust or wear away in a few years. Oak piles, cut in January, and driven with the bark on, have resisted the teredo for 4 or 5 years; and cypress piles, well charred, for 9 years. (c) For ordinary exposures on land, 8 to 10 lbs of creosote oil **per cub ft** are reqd = say 670 to 830 lbs per 1000 ft board measure = 30 to 40 lbs per cross tie of 4 cub ft. For protection against sea-worms 10 to 12 lbs per cub ft suffice in climates like those of Great Britain and the Northern U S; but in warmer waters where the teredo is very active, from 14 to 20 lbs per cub ft are used. Large timbers may not require saturation throughout, and thus may take less per cub ft. But see (i) and end of Art. 1 (a). (d) Creosote oil **weighs** about 8.8 lbs per U. S. gallon. (e) The sticks should be reduced to their intended final dimensions and framed (if framing is reqd) before treatment; especially if for exposure to teredo, which is sure to attack any spots which (as by subsequent cutting) are left unprotected. (f) Creosoted ties have remained sound after 22 years' exposure. The creosote protects the spikes from rusting. (g) Spruce and tamarack, owing to their irregular density, are unsuitable for creosoting. (h) Creosote renders wood stiffer and slightly more brittle. In hot weather it exudes to some extent and discolors the wood. Its smell excludes it from dwellings. (i) It does not wash out from the wood, but often fails to penetrate the heart-wood. Then, if any sap remains, decay begins at the center. See end of Art 1 (a). Burnettizing the cen of the stick (see Art 7) and using a coating of creosote outside, has long been suggested as the best possible method. It is the principal feature of the **Allardyce process.** This is cheaper than thorough creosoting. In the **Rütgers process,** which has been successfully employed for ties in Germany since 1874, the creosote and a solution of zinc chloride are injected simultaneously. (j) In the **creoresinate process** * the preservative fluid consists of creosote 38 per cent, formaldehyde 2 per cent, and melted resin 60 per cent.

---

* See paper by Dr. Hermann von Schrenk, read before the American Railway Engineering and Maintenance of Way Association, March, 1901.

Art. 5. (a) **Mineral solutions** are inferior to creosote, even on land; and useless in running water or against sea-worms; but they approximately double the life of inferior timber under ordinary land exposures; and their cheapness permits their use where that of creosote is too expensive. (b) They render wood harder; and brittle if the solution is too strong. They are liable to be washed out by rain, etc. Hence the outer wood decays first. See Art 4 (i) Art 8 (b) (c) (d). (c) A committee of the American Soc of Civ Engrs,† after collating a large number of experiments, recommended **Burnettizing** (Art 7) **for damp exposure,** as that of cross ties, damp floors, etc; and **Kyanizing** (Art 6) **for comparatively dry situations** with exposure to air and sun-light, as in bridge timbers, for which it is better suited than Burnettizing because it seems to weaken wood less. In such exposures it preserves wood sometimes for 20 to 30 years.

Art. 6. (a) **Kyanizing** consists in steeping the wood in a solution of 1 lb of bi-chloride of mercury (corrosive sublimate) in 100 lbs of water. (b) It is usual to allow the wood to soak a day for each inch of the thickness or least dimension of the piece, and one day in addition, whatever the size. (c) Gen'l Cram found the process very **unhealthy,** "salivating all the men"; but Mr. J. B. Francis, at Lowell, and Mr. H. Bissell, of the Eastern R R of Mass, had little or no trouble in this respect. The sublimate, however, which is very poisonous, is apt to effloresce, and the use of the timber is thus rendered dangerous. (d) The process is valuable for timber placed in moderately damp situations, but the salt is liable to be washed out by running water. Kyanized spruce fence posts, planted 4 ft in the ground, at Lowell, Mass, in 1850, were examined in 1891, and most of them were found very sound both above and below the surface of the ground.

Art. 7. (a) **Burnettizing** consists in immersing the wood for several hours in a solution of 2 lbs **chloride of zinc** in 100 lbs of water, under a pres of from 100 to 300 lbs per sq inch.

Art. 8. Other preventives. (a) Steeping in a solution of **sulphate of copper (blue vitriol)** has been extensively used, but does not seem to have been permanently successful. The blue vitriol washes out readily. **(b) In the Barschall or Hasselmann process,‡ introduced in Germany in 1887, in the U S in 1899, the wood is boiled, at a temperature from 212° to 284° Fahr. and under a pressure of from 15 to 45 lbs per sq inch, in a solution of iron, copper and aluminum sulphates and "Kainit," a sulphate of magnesia and potash, mined at Stassfurt, Germany. The solution is said to carry off the sap (timber being more readily treated by this process when green than when seasoned), while the copper destroys the fungi, and the iron forms an insoluble compound with the cellulose or woody fibre. It is claimed that the process greatly hardens the wood, especially the softer varieties, rendering them suitable for ties, without impairing their strength, elasticity or pliability. (c) The Wellhouse process injects first a solution of chloride of zinc with glue, and then one of tannin (both under pressure), in order to diminish the subsequent washing out of the chloride. In a later modification, the zinc, glue and tannin solutions are injected separately. Several millions of ties have been treated in this way. The**

*** See "A Proposed Method for the Preservation of Timber," by F. A. Kummer, Transactions, Am Soc C E, Vol XLIV, December, 1900.**

† See Transactions, Am Soc C E, July, Aug and Sept, 1885.

‡ Railroad Gazette, February 9, 1900.

process is not recommended for sub-aqueous use. (d) Processes in which the wood is treated by **painting or soaking** * are: Carbolineum Avenarius (Tar-oil, chlorine, etc), Ligni Salvor (Tar-oil, etc), Woodiline and Spirittine (chemical solutions) and a distillate of pine used by the Pennsylvania Railroad Co. for car work. (e) Fence-posts etc seem to be preserved, to some extent, by having only their lower ends dipped in **tar** well boiled to remove the ammonia, which last is destructive to wood. The upper end must be left untarred to let the sap evaporate. (f) Attempts at wood preservation by means of **vapor of creosote** etc have proved failures. (g) While wood remains thoroughly saturated with **petroleum** it does not decay. But unless the supply is kept up the oil evaporates and leaves the wood unprotected. (h) Cottonwood ties laid upon a **soil** containing about 2 per cent carbonate of lime, 1 per cent salt and 0.5 per cent each of potash and oxide of iron, on the Union Pacific R. R. in 1868, were found in 1882 "as sound and a good deal harder than when first laid," although such ties in other soils lasted but from 2 to 5 years. (i) The use of solutions of **lime** and of **salt**; and **charring** the surface; are sometimes found useful in damp situations.

---

* See Report by O. Chanute to the American Railway Engineering and Maintenance of Way Association, March, 1901.

Art. 4. Ultimate average tensile or cohesive strength of Timber,

Being the least weights in pounds which, if attached to the lower end of a vert rod one inch square, firmly upheld at its upper end, would break it by tearing it apart. For large timbers we recommend to reduce these constants ¼ to ⅓ part.

The strengths in all these tables may readily be one-third part more or less than our averages.	Lbs per sq. inch.		Lbs per sq. inch.
Alder	14000	**Mahogany, Honduras**	8000
Ash, English	16000	" **Spanish**	16000
" American (author) abt	16500	Mangrove, white, Bermuda	10000
Birch	15000	Mulberry	12000
" Amer'n black	7000	**Oak, Amer'n white**	10000
Bay-tree	12000	" " **basket**	
Beech, English	11500	" " **red**	
Bamboo	6000	" **Dantsic, seasoned**	
Box	20000	" **Riga**	
Cedar, Bermuda	7600	" **English**	
" Guadaloupe	9560	" live, Amer'n	
Chestnut	13000	**Pear**	10000
" horse	10000	Pine, Amer'n, white, red, and Pitch, Memel, Riga*.	10000
Cyprus	6000		
Elder	10000	**Plane**	11000
Elm	6000	**Plum**	11000
" **Canada**	13000	Poplar	7000
Fir, or Spruce	10000	Quince	7000
Hawthorn	10000	**Spruce, or Fir**	10000
Hazel	18000	**Sycamore**	12000
Holly	16000	**Teak**	15000
Hornbeam	20000	**Walnut**	8000
Hickory, Amer'n	11000	**Yew**	8000
Lignum Vitæ, Amer'n.	11000		
Lancewood	23000	**Across the grain,** Oak	2800
Larch, Scotch	7000	" " " Poplar	1800
Locust	18000	" " " Larch, 900 to	1700
Maple	10000	" " " Fir, & Pines	550

THESE ARE AVERAGES. The strengths vary much with the age of the tree; the locality of its growth; whether the piece is from the center, or from the outer portions of the tree; the degree of seasoning; straightness of grain; knots, &c, &c. Also, inasmuch as the constants are deduced from experiments with good specimens of small size, whereas large beams are almost invariably more or less defective from knots, crookedness of fibre, &c, it is advisable in practice to reduce these constants as recommended above.

---

* **Effect of Tapping Trees for Turpentine.** Preliminary experiments by the Forestry Division of the U. S. Department of Agriculture upon long-leaf pine from Alabama indicate that (contrary to the generally received impression) "turpentine timber," *i. e.* the timber of trees that have been "boxed" (robbed of their turpentine), while it has slightly less tensile and shearing strength, is from 20 to 30 per cent. stronger in compression (whether with or across the grain) and under transverse strain. In the "turpentine timber," however, the resin collects in spots, gumming the tools, and thus rendering the timber harder to work than that of trees which have not been deprived of their turpentine. The specimens tested were taken mostly at heights of from 7 to 33 feet above ground. (Circular No. 8. Issued 1892.) Boxed and unboxed timber are frequently called "bled" and "unbled" respectively.

Art. 1. Compressive strengths of American woods, whe *slowly and carefully seasoned.* Approximate averages deduced from many experiments made with the U S Govt testing machine at Watertown, Mass, by Mr. S. P. Sharples, for the census of 1880. **Seasoned woods** resist crushing much better than green ones; in many cases, twice as well. This must be allowed for when building bridges, &c, of timber recently cut. Different specimens of the same wood vary greatly; frequently as 5 to 8, 9, or more.

The strengths in all these tables may readily vary as much as one-third part more or less than our average.	**End-**wise.* lbs per sq in.	**Side-**wise.¶ lbs per sq in.	
		.01	.1
Ash, red and white	6800	1300	3000
Aspen	4400	800	1400
Beech	7000	1100	1900
Birch	8000	1300	2600
Buckeye	4400	600	1400
Butternut	5400	700	1600
Buttonwood (sycamore)	6000	1300	2600
Cedar, red	6000	700	1000
" white (arbor vitæ)	4400	500	900
Catalpa (Indian bean)	5000	700	1300
Cherry, wild	8000	1700	2600
Chestnut	5300	900	1600
Coffee tree, Kentucky	5200	1300	2600
Cypress, bald	6000	500	1200
Elm, Am'n or white	6800	1300	2600
" red	7700	1300	2600
Hemlock	5300	600	1100
Hickory	8000	2000	4000
Lignum vitæ	10000	1600	13000
Linden, American	5000	500	900
Locust, black and yellow	9800	1900	4400
" honey	7000	1600	2600
Mahogany	9000	1700	5300
Maple, broad-leafed, Oregon	5300	1400	2600
" sugar and black	8000	1900	4300
" white and red	6800	1300	2900
Oak, white, post (or iron) swamp white, red and black	7000	1600	4000
" scrub and basket	6000	1700	4200
" chestnut and live	7500	1600	4500
" pin	6500	1300	3000
Pine, white	5400	600	1200
" Red or Norway	6300	600	1400
" pitch and Jersey scrub	5000	1000	2000
" Georgia	8500	1300	2600
Poplar	5000	600	1100
Sassafras	5000	1300	2100
Spruce, black	5700	700	1300
" white	4500	600	1200
Sycamore (buttonwood)	6000	1300	2600
Walnut, black	8000	1300	2600
" white (butternut)	5400	700	1600
Willow	4400	700	1400

Hence it appears that *seasoned* white and yellow pines, spruce, and ordinary oaks, which are the woods most employed in the United States for bridges, roofs, etc., crush endwise with from 5000 to 7000 lbs per sq inch, ***in short blocks;*** average, 6000.

But it is well to bear in mind that in practice perfectly equable pressure is rarely secured. In a few trials on sidewise compression, with fairly seasoned white pine blocks, 6 ins high, 5 ins long, and 2 ins wide, we found that under an equally distributed pressure of 5000 lbs total or 500 lbs per sq inch, they compressed about from 1/8 to 1/4 inch; which is equal to from 1/4 to 1/2 inch per foot of height; or from 1/48 to 1/24 of the height; the mean being about 3/8 inch to a foot, or 1/32 of the height. Under 10000 lbs total, or 1000 lbs per sq inch, they split badly; and in some cases large pieces flew off.

The tensile or cohesive strengths of pine and oak average about 10000 lbs per sq inch, or 1/2 as much as average cast-iron, or nearly double their resistance to crushing. The tensile strength does not change with the length of the piece; so that in practice we may take its safe strain at from 1000 to 2000 lbs per sq inch, depending upon the character of the structure, &c., without regard to the length, except when this is so great that two or more pieces have to be spliced together to make it; thus weakening the piece very much.

* Specimens 4 centimetres (1.57 inch) square, 32 centimetres (12.6 ins) long. When the length exceeds 10 times the least side, see Wooden Pillars.

¶ Specimens 4 centimetres (1.57 inch) square, 16 centimetres (6.3 ins) long; laid upon platform of testing machine. Pressure applied at their mid-length, by means of an iron punch 4 centimetres square, or just covering the entire width of the specimen, and one-fourth of its length. The first column (headed ".01") gives the loads producing an indentation of .01 inch. The second column (headed ".1") gives those producing an indentation of .1 inch.

Table of safe quiescent loads for horizontal rectangular beams of white pine or spruce, one inch broad, supported at both ends, and loaded at the center; together with their deflections under said loads.

The safe load is here one-sixth of the breaking load.

For the neat loads, deduct ½ the wt of the beam itself. The *deflections*, however, are the *actual* ones; the wts of the beams having been introduced in calculating them, ~~by the rule in Art 27.~~

Loads applied suddenly will double the deflections in the table; as when, for instance, if a load is held by hand, just *touching* a beam, the hold should be *suddenly* loosed.

Caution. Inasmuch as this table was based upon well seasoned, straight grained pieces, free from knots, and other defects, we must not in practice take more than about two-thirds of the loads in the table for a safety of 6 in ordinary building timber of fair quality; and with these reduced loads should not reduce the deflections.

Observe also that our table is for safe **center** loads, but it is plain that in practice we cannot always apply the term in its utmost strictness; otherwise the load would have to be sustained by a mere knife-edge, at the *very* center of the beam. Now, in the instance Rem. p. 960, if we attempted to sustain the center load of 6075 lbs upon such a knife-edge, it would at once cut the beam in two. If we even applied it along 3 or 4 ins of the length, it would cut into it, and we should not have a safety of 6 against crushing the top of the beam until as in the case of the ends we distributed the load along full 46 ins of length, or about 32 ins for a safety of 4.

The safe load is here ⅙ of the breakg one; and the last at 450 lbs at the center of a beam 1 inch square, and 1 foot clear length between its supports. For mere temporary purposes, ½ part may be added to the loads in the table, thus making them equal to the ¼ of the breakg load. But in important structures, subject to vibration, ¼ part should be deducted from the tabular loads, thus reducing them to ⅛ of the breaking load. This is especially necessary if the timber is not well seasoned.

With the safe loads in this table a beam may bend too much for many practical purposes. When this is the case, we may, by reducing the loads, reduce the deflections in nearly the same proportion.

All the loads in the Table are superabundantly **safe against shearing. Against crushing at the ends,** &c, see "**Cautions**" below the Table.

Original.

Depth of beam. Ins.	Span 4 ft.		Span 6 ft.		Span 8 ft.		Span 10 ft		Span 12 ft		Span 14 ft		Span 16 ft		Wt. of 10 ft of beam. lbs.
	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	
1	19	.39	13	.92	10	1.8	8	3.0	6	4.4	......	......	......	......	2
2	75	.22	50	.45	38	.82	30	1.3	25	1.9	21	2.7	19	3.7	4
3	170	.13	114	.30	85	.53	67	.84	57	1.3	48	1.7	42	2.3	6
4	300	.10	200	.22	150	.39	120	.63	100	.92	86	1.3	75	1.7	8
5	469	.08	312	.18	234	.31	187	.50	156	.72	134	1.0	117	1.3	10
6	675	.06	450	.15	337	.26	270	.41	225	.60	193	.83	168	1.1	12
7	919	.06	612	.12	460	.22	367	.35	306	.51	262	.70	230	.93	14
8	1200	.05	800	.11	600	.19	480	.31	400	.45	343	.61	300	.81	16
9	1520	.04	1014	.10	760	.17	607	.27	507	.40	434	.54	380	.72	18
10	1875	.04	1250	.09	937	.16	750	.24	625	.35	536	.49	468	.64	20
11	2270	.04	1514	.08	1135	.14	907	.22	757	.32	648	.44	567	.58	22
12	2700	.03	1800	.07	1350	.13	1080	.20	900	.29	772	.40	675	.53	24
14	3675	.03	2450	.06	1837	.11	1470	.17	1225	.25	1050	.34	918	.45	28
16	4800	.02	3200	.05	2400	.10	1920	.15	1600	.22	1372	.30	1200	.40	32
18	6075	.02	4050	.05	3037	.09	2430	.14	2025	.20	1736	.27	1518	.35	36
20	7500	.02	5000	.04	3750	.08	3000	.12	2500	.18	2145	.24	1875	.31	40
22	9075	.02	6050	.04	4537	.07	3630	.11	3025	.16	2593	.22	2268	.29	44
24	10800	.02	7200	.04	5400	.06	4320	.10	3600	.15	3088	.20	2700	.26	48

(Continued on next page.)

Table, continued. (Original.)

Depth of beam.	Span 18 ft.		Span 20 ft.		Span 25 ft.		Span 30 ft.		Span 35 ft.		Span 40 ft.		Wt. of 10 ft of beam.
Ins.	load lbs.	def. ins.	load lbs.	def. ins.	load. lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	lbs.
6	150	1.4	135	1.8	108	2.9	90	4.5	77	6.5	67	9.2	12
7	204	1.2	184	1.5	147	2.5	122	3.9	105	5.8	92	7.6	14
8	267	1.0	240	1.3	192	2.1	160	3.2	137	4.6	120	6.4	16
9	338	.92	304	1.2	243	1.9	202	2.8	174	4.0	152	5.5	18
10	417	.82	375	1.0	300	1.7	250	2.5	214	3.5	188	4.9	20
11	505	.74	454	.93	363	1.5	302	2.2	259	3.2	227	4.3	22
12	600	.68	540	.85	432	1.4	360	2.0	308	2.9	270	3.9	24
14	817	.58	735	.72	588	1.2	490	1.7	420	2.4	367	3.2	28
16	1067	.50	960	.63	768	1.0	640	1.5	548	2.1	480	2.8	32
18	1350	.45	1215	.56	972	.90	810	1.3	694	1.8	607	2.5	36
20	1666	.40	1500	.50	1200	.79	1000	1.2	857	1.6	750	2.2	40
22	2017	.37	1815	.45	1452	.72	1210	1.1	1037	1.5	907	2.0	44
24	2400	.33	2160	.41	1728	.65	1440	.96	1234	1.3	1080	1.8	48
26	2817	.31	2526	.38	2018	.60	1684	.88	1449	1.2	1263	1.6	52
28	3267	.28	2940	.35	2352	.55	1960	.81	1680	1.1	1470	1.5	56
30	3750	.26	3375	.33	2700	.50	2250	.76	1928	1.1	1687	1.4	60
32	4267	.25	3840	.30	3072	.45	2560	.71	2194	1.0	1920	1.3	64
34	4817	.23	4335	.29	3468	.44	2890	.67	2477	.92	2167	1.2	68
36	5400	.22	4860	.27	3888	.43	3240	.63	2777	.86	2430	1.1	72

White oak, and best Southern pitch pine will bear loads ¼ greater.

For cast iron, mult the **loads** in the table by 4.5; and for **wrought** by 5.3. For these new loads, mult the **defs** by .4 for cast; and by .3 for wrought.

If the load is equally distributed over the span, it may be twice as great as the center one, and the defs will be 1¼ times those in the table. **If the loads in the table** be equally distributed along the whole beam, the defs will be but five-eighths as great as those in the table. When more accuracy is reqd, half the wt of the beam itself must be deducted from the center load; and the whole of it from an equally distributed load. The wt of the beam, in the last column, supposes the wood to be but moderately seasoned, and therefore to weigh 28.8 lbs per cub ft.

Uses of the foregoing table. Ex. 1. What must be the breadth of a hor rect beam of wh pine, 18 ins deep, supported at both ends, and of 20 ft clear length between its supports, to bear safely a load of 5 tons, or 11200 lbs at its center? Here, opposite the depth of 18 ins in the table, and in the column of 20 feet lengths, we find that a beam 1 inch thick will bear 1215 lbs; consequently, $\frac{11200}{1215} = 9.22$ ins, the reqd breadth; for the strength is in the same proportion as the breadth.

Ex. 2. What will be the safe load at the center of a joist of white pine, 18 ft long, 3 ins broad, and 12 ins deep? Here, in the col for 18 ft, and opposite 12 ins in depth, we find the safe load for a breadth of 1 inch to be 600 lbs; consequently, $600 \times 3 =$ 1800 lbs, the load reqd.

Rem. **Cautions in the use of the above table.** For instance, in placing very heavy loads upon short, but deep and strong beams, we must take care that the beams rest for a sufficient dist on their supports to prevent all danger from *crushing* at the ends. Thus, if we place a load of 6075 lbs at the center of a beam of 4 feet span, 18 ins deep, and only 1 inch thick, each end of the beam sustains a vert crushing force of $\frac{6075}{2} = 3037$ lbs, and that **sidewise of the grain,** in which position average white pine, spruce, and hemlock crush under about 800 lbs per sq inch, and do not have a safety of 6 until the pressure is reduced to about 133 lbs per sq inch. Therefore our beam, in order to have a safety of 6 against crushing at its ends, must rest on each support $3037 \div 133 = 23$ sq ins; or for a safety of 4 nearly 16 sq ins. When a pressure is equally distributed sidewise (that is, at right angles to the general direction of the fibres) over the entire pressed surface of a block or beam (to ensure which, the opposite surface must be supported throughout its entire length) the resulting compression might readily escape detection unless actually measured. But when a considerable pressure is applied to only a portion of the surface, as of caps and sills where in contact with the heads and feet of posts, or at the ends of loaded joists or girders, the compression becomes evident to the eye, because the pressed parts sink below the unpressed ones, in consequence of the bending or breaking of the adjacent fibres. What in the first case (especially if slight) would be called **compression,** would in the second be called **crushing** ; even when neither might be so great as to be unsafe.

Owing to the resistance which said adjacent fibres oppose to being bent or broken, it is plain that a given pressure **per sq inch,** or **per sq foot,** &c., will cause somewhat less compression or crushing when applied to only a part of a surface, than when to the whole of it.

The writer has seen 40 half seasoned hemlock posts, each 12 ins square, footing at intervals of 5 ft from center to center, upon similar 12 × 12 inch hemlock sills, to which they were tenoned, and which rested throughout their entire length on stone steps. Each post was gradually loaded with 32 tons, or equal to say 500 lbs per sq inch; and their feet all crushed into the sills from ¼ to ½ inch. Their heads crushed into the caps to the same extent. **In practice** the pressure at the heads and feet of posts is rarely, if ever, perfectly equable; and the same remark applies to the ends of loaded joists, girders, &c., in which a slight bending will throw an excess of pressure upon the inner edges of their supports

Art. 31. Table of greatest center loads for horizontal rectangular beams of white or yellow pine, or of spruce, 1 inch broad, supported at both ends, and required not to bend more than $\frac{1}{40}$ inch per foot of clear span, or $\frac{1}{480}$ part of the entire clear span. In practice, to allow for knots, &c, take only ⅔ rds. The loads in this table include the weight of the clear beam itself; .625 of which (or ⅝ of which) must be deducted from the tabular loads to get the neat load, when the beam is loaded at its center. When uniformly loaded, the loads will be 1.6 times as great as those in this table; but in that case the weight of the entire clear beam must be deducted. In practice this deduction need rarely be made.

Depths in Inches.	CLEAR SPANS IN FEET. (ORIGINAL.)																	Depth in Ins.	Wt. of 10 ft. Lgth of Beam.
	3	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40		
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.		lbs.
1	8.4	4.8	3.0	2.1	....	....	....	....	....	....	....	....	....	....	....	....	....	1	2
½	28.7	16.2	10.4	7.2	5.3	....	....	....	....	....	....	....	....	....	....	....	....	½	3
2	68.4	38.4	24.4	17.1	12.4	9.6	....	....	....	....	....	....	....	....	....	....	....	2	4
½		75	48	33	24	19	14	....	....	....	....	....	....	....	....	....	....	½	5
3	....	130	83	58	42	32	26	21	....	....	....	....	....	....	....	....	....	3	6
½	....		131	92	67	51	40	33	23	....	....	....	....	....	....	....	....	½	7
4	....	....	196	137	100	77	60	49	34	25	....	....	....	....	....	....	....	4	8
½	....	....		192	144	108	86	70	48	36	27	....	....	....	....	....	....	½	9
5	....	....	....	267	196	150	116	96	66	49	37	29	....	....	....	....	....	5	10
½	....	....	....		260	200	156	128	89	65	50	39	32	....	....	....	....	½	11
6	....	....	....	....	336	259	204	166	115	84	64	51	41	26	....	....	....	6	12
½	....	....	....	....		332	260	207	148	109	83	65	52	33	23	....	....	½	13
7	....	....	....	....	....	412	324	263	183	134	102	81	66	42	29	21	....	7	14
½	....	....	....	....	....	511	403	324	225	166	126	100	82	52	36	26	....	½	15
8	....	....	....	....	....		484	394	273	201	154	121	98	63	44	32	24	8	16
½	....	....	....	....	....	....	585	475	330	241	184	145	117	74	52	38	29	½	17
9	....	....	....	....	....	....	692	560	389	286	219	173	140	90	62	45	35	9	18
½	....	....	....	....	....	....		662	460	336	257	202	164	104	72	52	40	½	19
10	....	....	....	....	....	....	....	769	534	392	300	237	192	123	85	63	48	10	20
½	....	....	....	....	....	....	....		619	454	348	274	221	141	98	71	54	½	21
11	....	....	....	....	....	....	....	....	711	522	399	316	256	163	114	83	64	11	22
½	....	....	....	....	....	....	....	....	813	597	456	361	291	185	129	93	71	½	23
12	....	....	....	....	....	....	....	....	923	678	519	410	332	212	148	108	83	12	24
13	....	....	....	....	....	....	....	....		874	669	528	420	273	188	138	105	13	26
14	....	....	....	....	....	....	....	....	....	1075	823	651	527	337	234	172	132	14	28
15	....	....	....	....	....	....	....	....	....		1000	801	652	411	290	212	162	15	30
16	....	....	....	....	....	....	....	....	....	....	1230	971	787	504	350	257	197	16	32
17	....	....	....	....	....	....	....	....	....	....		1175	944	604	420	309	236	17	34
18	....	....	....	....	....	....	....	....	....	....	....	1385	1121	717	498	365	280	18	36
20	....	....	....	....	....	....	....	....	....	....	....		1538	964	684	502	384	20	40
22														1309	910	668	512	22	44
24														1701	1181	867	664	24	48
26															1503	1104	845	26	52
28															1876	1378	1055	28	56
30																1695	1297	30	60

On this side of the dark lines, the safe loads of table, p 969, would not bend the wooden beams as much as $\frac{1}{480}$ of their clear span.

Iron and Steel.

Average cast iron, with the same safe def will bear about 11½ as much as common yellow or white pine, or spruce; and wrought iron 19 times as much. The same proportion of the weight of the beam itself must, however, be deducted as stated above for wood. Average steel 29 times as much as pine.

WOODEN PILLARS.

The strengths of pillars, as well as of beams of timber, depend much on their **degree of seasoning.** Hodgkinson found that perfectly seasoned blocks, 2 diams long, required, in many cases, twice as great a load to crush them as when only moderately dry. This should be borne in mind when building with green timber.

In important practice, timber should not be trusted with more than $\frac{1}{8}$ to $\frac{1}{6}$ of its calculated crushing load; and for temporary purposes, not more than $\frac{1}{6}$ to $\frac{1}{4}$.

Mr. Charles Shaler Smith, C. E., of St. Louis, prepared the following formula for the breaking loads of either square or rectangular pillars or posts, of moderately seasoned white, and common yellow pine, with flat ends, firmly fixed, and equally loaded, based upon experiments by himself.

It is Gordon's formula adapted to those woods; and gives results considerably smaller than Hodgkinson's, It is therefore safer.

Call either side of the square, or the least side of the rectangle, the *breadth*. Then,

Rule.

$$\left.\begin{array}{c}\text{Breakg load in lbs, per}\\ \text{sq inch of area, of a}\\ \text{pillar of W or Y pine}\end{array}\right\} = \frac{5000\dagger}{1 + \left(\dfrac{\text{sq of length in ins}}{\text{sq of breadth in ins}} \times .004\right)}$$

Or in words, square the length in ins; square the breadth in ins; div the first square by the second one; mult the quot by .004; to the prod add 1; div 5000 by the sum.

Ex. Breakg load per sq inch, of a white pine pillar 12 ins square, and 30 ft, or 360 ins long. Here the sq of length in ins is $360^2 = 129600$. The square of the breadth is $12^2 = 144$; and $\frac{129600}{144} = 900$; and $900 \times .004 = 3.6$; and $3.6 + 1 = 4.6$. Finally, $\frac{5000}{4.6} = 1087$ lbs, the reqd breakg load per sq in. As the area of the pillar is 144 sq ins, the entire breakg load is $1087 \times 144 = 156528$ lbs, or 69.9 tons.

Recent experiments on wooden pillars 20 ft long, and 13 ins square, by Mr. Kirkaldy, of England, confirm the far greater reliability of Mr. Smith's formula. Hence we present the following new set of original tables based upon it.

For solid pillars of cast iron and of pine, whose heights range from 5 to 60 times their side or diam, we may say, near enough for practice, that a cast iron one is about $16\frac{1}{2}$ times as strong as a pine one; but no such approximate ratio holds good between wrought iron and pine, or between cast and wrought iron.

---

† The breaking load in lbs per sq inch in short blocks, by Mr. Smith.

Table of breaking loads in tons of square pillars of half seasoned white or common yellow pine firmly fixed and equally loaded. By C. Shaler Smith's formula. (Original.)

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	1.42	2.54	3.99	5.73	7.80	10.1	12.8	15.7	18.9	22.3	26.1	30.1	34.5	1
¼	1.17	2.22	3.59	5.26	7.25	9.6	12.2	15.1	18.3	21.7	25.4	29.2	33.7	¼
½	.97	1.93	3.19	4.80	6.74	9.0	11.6	14.5	17.6	21.0	24.7	28.6	33.0	½
¾	.81	1.66	2.81	4.35	6.19	8.4	10.9	13.7	16.8	20.2	23.9	27.8	32.1	¾
2	.68	1.44	2.48	3.92	5.66	7.8	10.2	12.9	15.9	19.3	23.0	26.9	31.2	2
¼	.57	1·24	2.19	3.53	5.17	7.2	9.6	12.3	15.2	18.5	22.0	25.8	30.1	¼
½	.49	1.07	1.93	3.16	4.70	6.7	8.9	11.5	14.3	17.6	21.1	24.9	29.1	½
¾	.42	.93	1.71	2.85	4.29	6.2	8.3	10.8	13.5	16.7	20.1	23.8	28.0	¾
3	.36	.82	1.52	2.55	3.89	5.6	7.6	10.0	12.7	15.8	19.2	22.9	27.0	3
½	.28	.63	1.21	2.07	3.23	4.8	6.6	8.8	11.3	14.2	17.4	20.9	24.8	½
4	.22	.50	.98	1.70	2.70	4.0	5.7	7.7	9.9	12.7	15.7	19.0	22.7	4
½	.18	·40	.81	1.42	2.28	3.4	4.9	6.7	8.8	11.4	14.1	17.2	20.7	½
5	.15	.34	.68	1.19	1.94	3.0	4.2	5.8	7.7	10.0	12.6	15.5	18.8	5
½	.12	.28	.57	1.02	1.67	2.6	3.7	5.1	6.8	8.9	11.3	14.0	17.1	½
6	.10	.24	.49	.86	1.44	2.3	3.3	4.6	6.1	8.0	10.2	12.7	15.6	6
½	.09	.21	.43	.74	1.26	2.0	2.9	4.1	5.4	7.2	9.2	11.6	14.2	½
7	.08	.18	.37	.66	1.11	1.8	2.6	3.6	4.9	6.5	8.3	10.5	12.9	7
½	.07	.16	.33	.59	.98	1.6	2.3	3.3	4.4	5.9	7.6	9.6	11.8	½
8	.06	.14	.29	.52	.87	1.4	2.0	2.9	3.9	5.2	6.8	8.7	10.8	8
½	.05	.12	.26	.47	.78	1.2	1.8	2.6	3.5	4.8	6.2	7.9	9.9	½
9	.05	.11	.23	.42	.71	1.1	1.6	2.3	3.2	4.3	5.6	7.2	9.1	9
½	......	.10	.21	.37	.64	1.0	1.5	2.1	2.9	3.9	5.1	6.6	8 4	½
10	......	.09	.19	.34	.58	.93	1.4	2.0	2.7	3.6	4.7	6.1	7.8	10
½	......	......	.17	.31	.53	.86	1.3	1.8	2.5	3.4	4.4	5.7	7.2	½
11	......	......	.16	.28	.48	.79	1.2	1.7	2.3	3.1	4.1	5.3	6.7	11
½	......	......	.14	.26	.44	.72	1.1	1.5	2.1	2.9	3.8	4.9	6.2	½
12	......	......	.13	.24	.41	.65	1.0	1.4	2.0	2.7	3.4	4.5	5.8	12
13	......	......	......	.21	.35	.55	.84	1.2	1.7	2.3	3.1	4.0	5.0	13
14	......	......	......	.18	.31	.46	.70	1.0	1.4	2.0	2.7	3.5	4.4	14
15	......	......	......	......	.27	.41	.63	.91	1.2	1.7	2.4	3.1	3.9	15
16	......	......	......	......	.24	.37	.57	.78	1.1	1.5	2.1	2.7	3.5	16
17	......	......	......	......	......	......	.50	.70	1.0	1.4	1.9	2.4	3.1	17
18	......	......	......	......	......	......	.45	.66	.92	1.3	1.7	2.2	2.8	18
20	......	......	......	......	......	......	......	......	.76	1.0	1.4	1.8	2.3	20

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	4¼	4½	4¾	5	5¼	5½	5¾	6	6¼	6½	6¾	7	7¼	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	35.8	40.6	45.7	51.1	56.8	62.8	69.0	75.5	82.3	89.4	96.8	104.5	112.4	2
3	31.4	36.1	41.1	46.2	51.8	57.7	63.8	70.2	76.9	84.0	91.3	98.9	106.7	3
4	26 8	31.2	35.9	40.8	46.1	51.7	57.6	63.8	70.4	77.3	84.5	92.1	99.9	4
5	22.6	26.5	30.8	35.4	40.5	45.8	51.5	57.4	63.7	70.3	77.2	84.5	92.1	5
6	18.9	22.5	26.4	30.5	35.2	40.1	45.4	51.0	57.0	63.3	69.9	76.8	84.1	6
7	15.8	19.0	22.6	26.2	30.5	35.0	39.9	45.0	50.5	56.5	62.8	69.4	76.3	7
8	13.3	16.1	19.2	22.5	26.3	30.4	34.9	39.7	45.9	50.4	56.2	62.4	69.0	8
9	11.3	13.7	16.5	19.5	22.9	26.6	30.7	35.0	39.9	44.8	50.2	56.0	62.1	9
10	9.7	11.8	14.2	16.9	19.9	23.2	26.9	30.9	35.2	39.9	44.9	50.3	56.0	10
11	8.3	10.2	12.4	14.8	17.5	20.4	23.8	27.4	31.3	35.6	40.2	45.1	50.4	11
12	7.2	8.8	10.7	12.9	15.4	18.0	21.1	24.3	27.9	31.8	36.0	40.6	45.5	12
13	6.2	7.7	9.4	11.4	13.6	16.0	18.8	21.7	24.9	28.5	32.4	36.6	41.1	13
14	5.5	6.8	8.3	10.1	12.1	14.2	16.7	19.4	22.4	25.7	29.2	33.1	37.3	14
15	4.8	6.0	7.4	9.0	10.8	12.7	15.0	17.5	20.2	23.2	26.4	30.0	33.9	15
16	4.4	5.4	6.7	8.1	9.8	11.5	13.6	15.8	18.3	21.0	24.0	27.3	30.8	16
17	4.0	4.9	6.1	7.3	8.8	10.4	12.3	14.3	16.6	19.1	21.9	24.9	28.3	17
18	3.6	4.4	5.5	6.6	8.0	9.4	11.2	13.0	15.1	17.4	19.9	22.7	25.8	18
19	3.3	4.0	5.0	6.0	7.3	8.6	10·2	11.9	13.8	16.0	18.3	20.9	23.7	19
20	3.0	3.7	4.6	5.5	6.6	7.8	9.3	10.9	12.6	14.6	16.8	19.2	21.8	20
22	2.5	3.0	3.8	4.6	5.6	6.6	7.9	9.2	10.7	12.4	14.3	16.3	18.6	22
24	2.1	2.6	3.2	3.9	4.7	5.6	6.7	7.9	9.1	10.6	12.2	14.1	16.0	24
26	1.8	2.2	2.8	3.4	4.1	4.9	5.8	6.8	7.9	9.2	10.6	12.2	13.9	26
28	1.5	1.9	2.4	2.9	3.5	4.2	5.1	5.9	6.9	8.0	9.3	10.7	12.2	28
30	1.3	1.7	2.1	2.6	3.1	3.7	4.4	5.2	6.1	7.1	8.2	9.4	10.8	30
32	1.2	1.5	1.9	2.3	2.7	3.2	3.9	4.6	5.4	6.3	7.3	8.4	9.6	32
34	1.1	1.3	1.7	2.0	2.4	2.9	3.5	4.1	4.8	5.6	6.5	7.5	8.6	34
36	1.0	1.2	1.5	1.8	2.2	2.6	3.1	3.7	4.3	5.0	5.8	6.7	7.7	36
38	.9	1.1	1.3	1.6	2.0	2.4	2.8	3.3	3.9	4.5	5.3	6.1	7.0	38
40	.8	1.0	1.2	1.5	1.8	2.1	2.6	3.0	3.5	4.1	4.8	5.5	6.3	40

Continued on next page.

Table of breaking loads in tons of square pine pillars, with flat ends firmly fixed, and equally loaded. (Continued.)

Original.

Height in feet.	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	10	10¼	10½	Height in feet.
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	120.6	129.1	137.9	147.0	156.3	165.9	175.8	186·0	196.4	207.2	218.2	229.5	241.0	2
4	107.9	116·3	125.0	133.9	143.0	152.6	162.4	172.5	182.8	193.4	204.2	215.3	226.6	4
6	91.7	99.7	108.0	116.5	125.3	134.3	143.5	153.0	162.8	173.5	184.4	195.6	207.0	6
8	75.9	83.2	90.8	98.7	106.8	115.5	124.4	133.6	143.0	153.0	163.2	173.7	184.4	8
10	62.0	68.5	75.3	82.4	89.7	97.7	105.8	114.3	123.0	132.3	141.8	151.6	161.6	10
12	50.7	56.5	62.5	68.7	75.1	82.2	89.6	97.2	105.0	113.5	122.2	131.2	140.5	12
14	41.8	46.7	51.9	57.3	62.9	69.2	75.7	82.4	89.4	97.1	105.0	113.2	121.6	14
16	34.7	38.9	43.4	48.1	53.0	58.5	64.3	70.3	76.5	83.3	90.4	97.7	105.3	16
18	29.1	32.7	36.6	40.7	45.0	49.8	54.9	60.1	65.6	71.7	78.0	84.6	91.4	18
20	24.6	27.7	31.1	34.7	38.5	42.7	47.2	51.8	56.7	62.0	67.6	73.5	79.6	20
23	19.6	22.1	24.8	27.7	30.9	34.4	38.0	41.9	46.0	50.5	55.2	60.2	65.4	23
26	15.8	17.8	20.1	22.5	25.2	28.1	31.1	34.4	37.9	41.6	45.6	49.8	54.3	26
29	13.1	14.8	16.7	18.7	20.9	23.4	25.9	28.6	31.6	34.9	38.2	41.9	45.6	29
32	10.9	12.3	13.9	15.7	17.6	19.7	21.8	24.2	26.7	29.5	32.3	35.5	38.7	32
35	9.3	10.6	11.9	13.4	15.0	16.8	18.7	20.7	22.8	25.2	27.7	30.5	33.3	35
38	8.0	9.1	10.2	11.5	12.9	14.6	16.3	18.0	19.7	21.8	23.9	26.3	28.8	38
41	6.9	7.9	8.9	10.0	11.2	12.6	14.1	15.6	17.2	19.1	21.0	23.1	25.2	41
44	6.0	6.9	7.8	8.8	9.8	11.0	12.3	13.6	15.0	16.7	18.4	20.2	22.1	44
50	4.7	5.4	6.1	6.9	7.7	8.8	10.0	11.3	12.6	13.7	14.9	16.2	17.5	50

Height in feet.	10¾	11	11¼	11½	11¾	12	10¾	11	11¼	11½	11¾	12	Height in feet.
	BREAKING LOAD.						BREAKING LOAD.						
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	238.9	251.0	263.2	275.9	288.8	302.1	26.5	28.6	31.1	33.8	36.7	39.9	42
6	218.8	230.6	242.7	255.2	267.9	281.0	24.2	26.4	28.7	31.2	33.9	36.8	44
8	195.5	207.0	218.5	230.4	242.5	255.1	22.4	24.3	26.4	28.7	31.2	33.9	46
10	172.2	183.0	194.1	205.6	217.3	229.5	20.7	22.6	24.5	26.7	28.9	31.4	48
12	150.8	160.3	170.7	181.5	192.5	204.0	19.2	20.9	22.7	24.7	26.8	29.2	50
14	130.5	139.8	149 4	159.4	169.6	180.2	17.8	19.5	21.1	23.0	25.0	27.2	52
16	113.2	121.7	130 4	139.6	149.0	158.8	16.6	18.2	19.7	21.5	23.3	25.4	54
18	98.7	106.2	114.1	122.5	131.0	140.0	15.5	17.0	18.4	20.1	21.8	23.7	56
20	86.2	92.9	100.0	107.6	115.4	123.6	14.5	15.9	17 2	18.8	20.4	22.2	58
22	75.6	81.7	88.1	95.0	102.0	109.5	13.6	14.9	16.2	17.7	19.2	20.9	60
24	66.7	72.2	77.9	84.1	90.5	97.3	11.8	12.9	13.9	15.2	16.5	18.0	65
26	59.1	64.0	69,2	74.9	80 6	86.8	10.1	11.1	12.0	13.2	14.3	15.6	70
28	52.6	57.1	61.8	66.9	72.1	77.7	8.9	9.8	10.6	11.6	12.5	13.7	75
30	47.0	51.1	55.3	60.0	64.7	69.9	7 8	8.6	9.4	10.2	11.1	12.1	80
32	42.1	46.0	49.9	54.0	58.4	63.0	7.0	7.7	8.4	9.1	9.9	10.7	85
34	38.2	41.5	45.0	48.8	52.8	57.1	6.2	6.8	7.4	8.1	8.8	9.6	90
36	34.6	37.7	40.9	44.3	48.0	51.8	5.6	6.1	6.7	7.3	8.0	8.7	96
38	31.5	34.2	37.2	40.4	43.9	47.4	5.1	5.6	6.1	6.6	7.2	7.8	100
40	28.8	31.3	34.0	37.0	40.1	43.5	3.5	3.9	4.2	4.6	5.0	5.5	120

Continued on next page.

Remarks. Mr Kirkaldy found for Riga and Dantzic firs, 20 ft long, and 13 ins square, (or 18½ sides high,) 148 and 138 tons total; or .876 and .817 tons, (1963 and 1829 lbs,) per sq inch. Mr Smith's rule gives for common pine, 160 tons total; or .947 ton, or 2121 lbs, per sq inch. Hodgkinson would give for Riga about 297 tons total.

Each of Mr Kirkaldy's 20-ft pillars shortened about .6 of an inch total; or .03 inch per ft; or ⅛ of an inch in 4 ft 2 ins, under a mean of 1900 lbs per sq inch.

The writer has known 8 unbraced **pillars of hemlock,** tolerably seasoned, 12 ins square, and 42 ft high, to be gradually loaded each with 32 tons, or 71680 lbs total; (or .2222 ton, or 498 lbs per sq inch) without appreciable yielding. Assuming their strength and stiffness to be about as for Mr Smith's pine, (as in all our tables,) they should by him yield at 39.9 tons total. With these same *data*, but with Hodgkinson's **formula,** they should yield at 69.3 tons; and with Hodgkinson's own data, for seasoned red deal, at 91.6 tons.

Table of breaking loads in tons of square pillars of half-seasoned white or common yellow pine, with flat ends firmly fixed, and equally loaded. By C. Shaler Smith's formula. (Continued.)

As this table was partly made by interpolation, the last figure is not always precisely correct.

Original.

Height in feet.	Side of square pine pillar in inches.												Height in feet.
	13	14	15	16	17	18	19	20	21	22	23	24	
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	358	418	482	552	625	703	786	872	964	1060	1161	1265	4
6	335	394	456	526	599	676	760	847	938	1033	1134	1236	6
8	308	367	429	500	572	649	732	818	910	1005	1106	1208	8
10	281	339	400	466	537	612	694	780	870	964	1064	1166	10
12	252	307	365	432	502	576	656	740	829	922	1022	1124	12
14	225	277	333	397	464	536	614	696	784	876	973	1074	14
16	201	250	303	363	428	497	573	652	739	829	925	1024	16
18	179	224	274	331	392	458	531	608	692	780	873	972	18
20	160	201	248	301	359	422	492	566	647	732	822	919	20
22	143	182	224	274	329	388	455	526	604	686	773	866	22
24	127	163	203	249	301	357	421	488	563	642	726	816	24
26	115	148	184	226	275	328	389	453	523	599	680	767	26
28	103	133	167	206	252	302	359	420	490	560	638	721	28
30	93	121	152	189	231	278	332	389	453	522	597	677	30
32	84	109	138	178	212	256	307	361	421	487	558	635	32
34	76	99	126	159	196	237	284	335	392	455	523	597	34
36	69	91	116	146	180	219	264	312	366	426	490	560	36
38	63	84	107	134	166	203	245	290	341	397	458	525	38
40	58	77	99	124	154	188	227	270	318	372	429	494	40
42	54	71	91	115	143	175	212	253	298	349	403	465	42
44	50	66	84	107	133	163	198	236	280	328	380	438	44
46	46	61	78	99	123	152	185	221	263	308	358	413	46
48	43	57	73	92	115	142	173	207	247	290	337	389	48
50	40	53	68	86	107	133	162	194	231	272	317	367	50
52	37	50	64	81	101	124	152	182	217	256	300	347	52
54	35	47	60	76	95	117	144	172	205	242	283	328	54
56	33	44	56	71	89	110	135	162	193	228	267	310	56
58	31	41	52	67	84	103	127	153	182	215	253	294	58
60	29	38	49	63	79	98	120	144	172	204	240	280	60
65	25	33	43	55	69	86	105	126	151	179	211	246	65
70	22	29	37	48	60	74	92	111	134	159	187	218	70
75	19	25	33	42	53	66	82	98	118	141	166	195	75
80	16	22	29	37	46	58	72	87	105	125	148	174	80
85	14	19	26	33	41	52	65	78	94	112	132	156	85
90	13	17	23	30	37	46	58	70	85	102	120	141	90
95	12	16	21	27	33	42	53	64	77	93	108	127	95
100	11	14	19	24	30	38	48	58	70	84	99	117	100
110	10	12	16	20	26	33	40	48	58	70	82	97	110
120	9	11	14	17	22	28	34	41	49	60	71	83	120
130	7	9	12	14	18	23	29	36	43	52	61	72	130
140	6	8	10	12	16	20	25	31	37	44	53	62	140
150	5	7	9	11	14	18	22	27	32	38	46	54	150
160	5	6	8	10	13	16	20	24	29	34	41	48	160
170	4	5	7	9	11	14	17	21	25	30	36	43	170
180	4	5	6	8	10	12	15	19	22	27	32	38	180
190	3	4	5	7	9	11	14	17	20	24	29	34	190
200	3	4	5	6	8	10	12	15	18	22	26	31	200

Breaking loads of half seasoned square pine pillars.

Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LD PER SQ IN.		Height in sides.	BR LD PER SQ IN.	
	Tons.	Lbs.		Tons.	Lbs.		Tons.	Lbs.		Tons.	Lbs.
1	2.2232	4980	26	.6027	1350	51	.1960	439	76	.0924	207
2	2.1969	4921	27	.5697	1276	52	.1888	423	77	.0902	202
3	2.1544	4826	28	.5398	1209	53	.1826	409	78	.0879	197
4	2.0978	4699	29	.5116	1146	54	.1763	395	79	.0862	193
5	2.0290	4545	30	.4853	1087	55	.1706	382	80	.0839	188
6	1.9513	4371	31	.4607	1032	56	.1647	369	81	.0821	184
7	1.8665	4181	32	.4379	981	57	.1598	358	82	.0799	179
8	1.7772	3981	33	.4165	933	58	.1545	346	83	.0781	175
9	1.6857	3776	34	.3969	889	59	.1496	335	84	.0763	171
10	1.5942	3571	35	.3781	847	60	.1451	325	85	.0746	167
11	1.5040	3369	36	.3599	809	61	.1406	315	86	.0728	163
12	1.4165	3173	37	.3447	772	62	.1362	305	87	.0714	160
13	1.3317	2983	38	.3296	738	63	.1321	296	88	.0696	156
14	1.2513	2803	39	.3152	706	64	.1286	288	89	.0683	153
15	1.1745	2631	40	.3018	676	65	.1250	280	90	.0670	150
16	1.1027	2470	41	.2889	647	66	.1210	271	91	.0656	147
17	1.0353	2319	42	.2772	621	67	.1179	264	92	.0638	143
18	.9723	2178	43	.2661	596	68	.1147	257	93	.0625	140
19	.9184	2046	44	.2554	572	69	.1112	249	94	.0616	138
20	.8585	1923	45	.2455	550	70	.1085	243	95	.0603	135
21	.8076	1809	46	.2357	528	71	.1054	236	96	.0589	132
22	.7603	1703	47	.2268	508	72	.1027	230	97	.0576	129
23	.7165	1605	48	.2183	489	73	.1000	224	98	.0567	127
24	.6755	1513	49	.2100	472	74	.0973	218	99	.0554	124
25	.6380	1429	50	.2031	455	75	.0951	213	100	.0545	122

PLASTERING.

The plastering of the inside walls of buildings, whether done on laths, bricks, or stone, generally consists of three separate coats of mortar. The first of these is called by workmen the *rough* or *scratch coat;* and consists of about 1 measure of quicklime, to 4 of sand; (which latter need not be of the purest kind;) and ⅛ measure of bullock or horse hair; the last of which is for making the mortar more cohesive, and less liable to split off in spots. This coat is about ⅜ to ½ inch thick; is put on roughly; and should be pressed by the trowel with sufficient force to enter perfectly between and behind the laths; which for facilitating this should not be nailed nearer together than ½ an inch. In rude buildings, or in cellars, &c, this is often the only coat used. When this first coat has been left for one or more days, according to the dryness of the air, to dry slightly, it is roughly *scored*, or *scratched*, (hence its name,) with a pointed stick, or a lath, nearly through its thickness, by lines running diagonally across each other, and about 2 to 4 ins apart. This gives a better hold to the second coat, which might otherwise peel off. If the first coat has become too dry, it is well also to dampen it slightly as the second one is put on.

The second coat is put on about ¼ to ⅜ inch thick, of the same hair mortar, or *course stuff.* Before it becomes hard, it is roughed over by a hickory broom, or some substitute, to make the third coat adhere to it better.

The third coat, about ⅛ inch thick, contains no hair; and for giving it a still whiter and neater appearance, more lime is used, say 1 of lime, to 2 of sand; and the purest sand is used. This mortar is by plasterers called **stucco;** a name also applied to mortar when used for plastering the outsides of buildings. Or instead of stucco, the third coat may be, and usually is, of *hard finish*, or *gauge stuff;* which consists of 1 measure of ground plaster of Paris, to about 2 of quicklime, without sand. Hard finish works easier; but is not as good as stucco, for walls intended to be painted in oil. The plaster of Paris is for hastening the hardening.

Either of these third coats is smoothed or polished to a greater or less extent, according to whether it is to show, or to be papered, painted, &c. The polishing tools are merely, the trowel; the hand-float, (a kind of wooden trowel;) and the water-brush, (a short-handled brush for wetting the surface part at a time with water, in order to polish more freely.) For finer polishing, a float made of cork is used. The smooth piece of board about 10 to 12 ins square, with a handle beneath, on which the plasterer holds his mortar until he puts it on to the wall with his trowel, is called a *hawk.*

The more thoroughly each coat is gone over with the water-brush and trowel, (which process is called *hand-floating*,) the firmer and stronger will it be. Frequently only two coats of plastering are put on in inferior rooms; or where great neatness of appearance is not needed. The first is of hair mortar, or coarse stuff; this is scratched with the broom, and then covered by the finishing coat of finer mortar, (stucco.) If this last is nearly all lime, or with but very little sand, to make it work easier, it is called a *slipped coat.* Without any sand it is called *fine stuff.* Neither is as good as stucco, if the wall is to be papered. When this is the case, the third coat also may have a little hair, to give it more strength; but this is not absolutely necessary.

A very good effect may be produced in station-houses, churches, &c, by only two coats of plaster in which fine clean screened gravel is used instead of sand. When lined into regular courses, it resembles a buff-colored sandstone, very agreeable to the eye.

In purchasing plastering hair, care must be taken that it has not been taken from salted hides, inasmuch as the salt will make the walls damp. For the same cause sea-shore sand should not be used. It is almost impossible to wash it entirely free from salt.

In brick walls intended to be plastered, the mortar joints should be left very rough, to let the plaster adhere. If it is put on smooth walls, without first raking out the mortar to the depth of nearly an inch, it is very apt to fall off; especially from outside walls; as can be seen daily in any of our cities. As this raking out of brick joints is tedious and expensive, it would generally be better to use paint rather than plaster. The walls should also be washed clean from all dust; and should be slightly dampened as the plaster is put on.

To imitate granite on outer walls: after the second or smooth coat of plaster is dry, it receives a coat of lime wash, slightly tinted by a little umber, or ochre, &c. After this is dry, in case it appears too dark, or too light, another may be applied with more or less of the coloring matter in it. Finally, a wash of lime and mineral-black is *sprinkled* on from a flat brush, to imitate the black specks of granite. By this simple means, a skilful workman can produce excellent imitations. The horizontal and vertical joints of the imitation masonry, may be ruled in by a small brush, using the same black wash, and a long straight-edge.

The rough surfaces of all walls are more or less warped, or out of line; and it is not possible for the plasterer to rectify this perfectly by eye, as may be seen in almost every house. Even in what are called first-class ones, a quick eye can generally detect unsightly undulations of the plastered surfaces.

To prevent this, the process of **screeding** is resorted to. Screeds are a kind of gauge or guide, formed by applying to the first rough coat, when partly dried, horizontal strips of the plastering mortar, about 8 ins wide, and from 2 to 4 ft apart all around the room. These are made to project from the first coat, out to the intended face of the second one; and while soft are carefully made perfectly straight, and out of wind with each other, by means of the plumb-line, straight-edge, &c. When they become dry, the second coat is put on, filling up the broad horizontal spaces between them; and is readily brought to a perfectly flat surface, corresponding with that of the screeds, by means of long straight-edges extending over two or more of the latter.

A day's work at plastering.

A plasterer, aided by one or two laborers to mix his mortar, and to keep his hawk supplied, can average from 100 to 200 square yards a day, of first coat; about ¾ as much of second; and half as much of third, which requires more care. The amount will depend upon the number of angles, size of rooms, whether on ceilings or on walls, &c, &c.

Gen Gillmore's estimate of cost of plastering 100 square yards with 2 or with 3 coats. Common labor $1 per day.

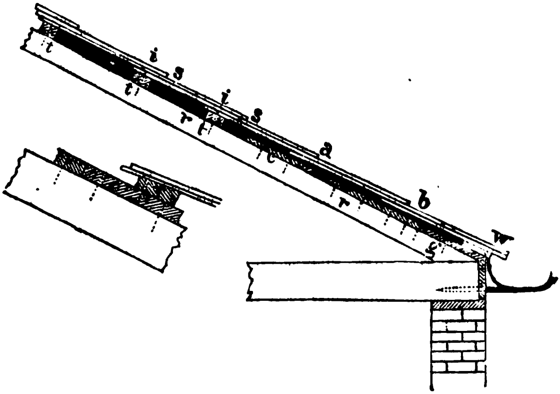
Materials.	Three Coats. Hard finished work.		Two Coats. Slipped coat finish.	
Quicklime	4 casks.	$4.00	3⅓ casks.	$3.33
" for fine stuff	⅔ "	.85		
Plaster of Paris	½ "	.70		
Laths	2000	4.00	2000.	4.00
Hair	4 bushels.	.80	3 bushels.	.60
Common Sand	7 loads.	2.00	6 loads.	1.80
White Sand	2½ bushels.	.25		
Nails	13 lbs.	.90	13 lbs.	.90
Mason's labor	4 days.	7.00	3½ days.	6.12
Laborer	3 days.	3.00	2 days.	2.00
Cartage	....	2.00	....	1.20
Cost of 100 square yards	....	$25.50	....	$19.95

This amounts to 25½ cts per sq yd for 3 coats; and say 20 cts for 2 coats.

Plastering laths are usually of split white or yellow pine, in lengths of about 3 to 4 feet; and hence called 3 or 4 ft laths. They are about 1½ ins wide, by ¼ inch thick. They are nailed up horizontally, about ⅜ inch apart. The upright studs of partitions are spaced at such distances apart, (generally about 15 ins from center to center,) that the ends of the laths may be nailed to them. Laths are sold by the bundle of 1000 each. A square foot of surface requires 1½ four feet laths; or 1000 such laths will cover 666 sq ft. Sawed laths may be had to order, of any required length. A carpenter can nail up the laths for from 40 to 60 sq yds of plastering in a day of 10 hours; depending on the number of angles in the rooms, &c.

SLATING.

ROOFING slates are usually from ⅛ to ¼ inch thick; about $\frac{3}{16}$ being a common average. They may be nailed either to a sheeting of rough boards (*c*, *g*, in the fig) from ¾ to 1¼ inch thick, (which should be, but rarely are, tongued and grooved,)



laid horizontally from rafter to rafter; or sloping, from purlin to purlin as the case may be; or to stout laths *t t t* about 2 to 3 ins wide, and from 1 to 1¼ thick, nailed to the rafters at distances apart to suit the gauges of the slates. Two nails are used to each slate; one near each upper corner. They may be either of copper, (which is the most durable, but most expensive,) of zinc, or of either galvanized or tinned iron. The last two are generally used; or in inferior work, merely plain iron ones, previously boiled in linseed oil, as a partial preservative from rust. Rust, however, sometimes weakens them so much that they break; and the slates are blown off in high winds, to the danger of passers by. Since good slate endures for a long series of years, it is true economy to use nails that are equally durable. In iron roofs, the slates, instead of being nailed to boards, are sometimes tied directly to the iron purlins, by wire. **A square** of slating, shingling, &c, is 100 sq ft.

In laboratories, chemical factories, &c, subject to **acid fumes**, it is difficult to provide a metal fastening that will not be eaten away. In such cases it is best to depend chiefly upon a layer of mortar between the slates. This will harden before the metal fastenings give way; and will hold the slates in place, while new fastenings are being inserted.

The **least pitch** considered advisable for a roof, to prevent rain or snow from being driven through the interstices between the slates, is about 26½°; or 1 vert to 2 hor; which corresponds to a rise of ¼ the span in a common double pitched roof. But even at steeper pitches, rain, and more particularly snow, will be forced through the roof by violent winds; especially if laths alone be used, or even boarding alone. To avoid this, a layer of mortar about ¼ inch thick, may be spread over the touching surfaces of the slates if on laths. If on boards, the same process may be adopted; or the more common one of first covering the boards with a layer of what is called *slating felt;* but which in reality is merely thick brown paper, soaked in tar. This is sold in long continuous rolls, 28 ins wide, and weighing from 40 to 50 lbs. A 50 lb roll will cover about 300 sq ft of roof. With proper precautions against the admission of rain and snow, a pitch as flat as 1 in 2½, or even 1 in 3, may be adopted.

The thickness of slate on a roof is double; except at the *laps t s*, *t s*, &c, where it is triple. The lap is measured from the nail hole (under *t*) of the lower slate, to the lower edge or *tail*, *s*, of the upper one; and is usually about 3 ins. In order that the showing lower edges of the slates shall, when laid, form regular straight lines along the roof, the nail holes are made at equal distances from said *lower* edges; so that any irregularity of length is concealed from view at the hidden *heads* of the slates. The slater estimates the length of his slate from the nail hole to the tail; discarding the narrow strip between the nail hole and the head. If from this reduced length the lap be deducted, then one-half of the remainder will be the *gauge*, *weathering*, or *margin*, of the slating; or, in other words, the *showing* or *exposed* width of the courses of slates. The gauge in ins multiplied by the width of a slate in ins, gives the area in sq ins of finished roof covered by a single slate; and if 144 (the sq ins in a sq foot) be divided by this area, the quotient will be the number of slates required per sq ft of roof. The upper side of a slate is called its *back;* the lower one, its *bed*.

Slating, like shingling, must evidently be commenced at the eaves, and extended upward. Since the beds of the slates are not exactly parallel to the boarding, and consequently do not rest flat upon it, those at the lower edge *w* would easily be broken. To prevent this, a *tilting strip* (a stout wide lath, with its upper side planed a little bevelling, to suit the slope of the slates) is first nailed around near the eaves, for the tails of the lowest course of slates to rest on. This is shown on a larger scale at T.

Slate of the best quality has a glistening semi-metallic appearance, somewhat like that of a surface of paper rubbed with black-lead pencil. That of a dull earthy aspect, is softer, more absorbent, and consequently more liable to yield to atmospheric influences, rain, frost, &c. Iron pyrites frequently occurs in slate; and since it always decomposes and leaves holes, should never be admitted on a roof. Of two qualities of slate, that which absorbs the least weight of water, when pieces of equal size are soaked for an hour or two, is generally the best; being least liable to split by frost, and become weather-worn. This test is easily applied.

In England the **different sizes** are distinguished by absurd names of no meaning. In the United States they are called 6 by 12's; 16 by 24's, &c, according to their measures in inches. They may be cut to order, of almost any prescribed dimensions, or shape. Those in common use vary from about 7 by 14, to 12 by 18. The first forms about 5 to 6 inch courses; and the last about 7 to 8 inch; depending upon how far from the head the nail holes are pierced. The farther this is, the firmer will the slating be.

Slate roofs, like iron ones, heat the rooms immediately below them very much. This is somewhat diminished when the slates are on boards, instead of laths; and still more by a coat of plaster beneath. They are also liable to break when walked on; less so when bedded in mortar.

Weight of slate roofs. Slate weighs about 175 lbs. per cub foot; therefore, a sq ft, ⅛ inch thick, weighs about 1.8 lbs; $\frac{3}{16}$, 2.7 lbs; and ¼ thick, 3.6 lbs. But owing to the overlapping, a square foot of roof requires about 2½ sq ft of slate of ordinary sizes; and if the slate is laid on boards an inch thick, the weight per sq ft of roof will be increased about 2¼ lbs; or with 1¼ inch boards, 2.8 lbs. Laths will weigh about ¼ lb per sq ft of roof.

Hence,

				Approx Weight of one sq ft of Slating, in lbs.
Slate	⅛	inch thick	on laths	4.75
"		"	on 1 inch boards	6.75
"		"	on 1¼ " "	7.30
"	3-16	"	on laths	7.00
"	"	"	on 1 inch boards	9.00
"	"	"	on 1¼ " "	9.55
"	¼	"	on laths	9.25
"	"	"	on 1 inch boards	11.25
"	"	"	on 1¼ " "	11.80

If slating felt is used, add ⅙ lb; or if the slates are bedded in ¼ inch of mortar, add 3 lbs.

For the total weight borne by the *roof trusses*, that of the purlins also must be added. This will not vary much from the limits of 1⅓ to 3 lbs per sq ft in roofs of moderate span. Add for wind and snow, say 20 lbs per sq ft; and finally add the weight of the truss itself.

For stopping the joints between slates (or shingles, &c) and chimneys, dormer windows, &c, a mixture of stiff white-lead paint, as sold by the keg, with sand enough to prevent it from running, is very good; especially if protected by a covering of strips of lead, or copper, tin, &c, nailed to the mortar-joints of the chimneys, after being bent so as to enter said joints; which should be scraped out for an inch in depth, and afterward refilled Mortar protected in the same way, or even unprotected, is often used for the purpose; but is not equal to the paint and sand. Mortar a few days old, (to allow refractory particles of lime to slack,) mixed with blacksmith's cinders and molasses, is much used for this purpose, and becomes very hard, and effective.

SHINGLES.

White cedar shingles are the best in use; and when of good quality will last 40 or 50 years in our Northern States. They are usually 27 ins long; by from 6 to 7 ins wide; about ¼ inch thick at upper end; and about ⅝ at lower end or butt; and are laid in courses about 8⅓ ins wide; so that not quite ⅓ of a shingle is exposed to the weather.

They are usually laid in three thicknesses; except for an inch or two at the upper ends, where there are four. They are nailed to sawed shingling-laths of oak or yellow pine; about 16 ft long; 2⅓ ins wide, and 1 inch thick; placed in horizontal rows about 8⅓ ins apart. These are nailed to the rafters, or purlins; which, for laths of the foregoing size, should not be more than 2 ft apart from center to center. Two nails are used to each shingle, near its upper end. They should not be of less size than 400 to a lb. Wrought nails being the strongest, are the best; cut ones are apt to break by the warping of the shingles. Two pounds of such nails will suffice for 100 sq ft of roof, including waste. An average shingle 7½ ins wide, in 8⅓ inch courses, exposes 63¾ sq ins; making 2¼ shingles to a sq ft of roof; but to allow for waste, and narrow shingles, it is better in practice to allow about 8 shingles to a sq ft.

Shingling, like slating, must plainly be begun at the eaves; and extended upward. For closing the joints between the shingles, and chimneys, dormer windows, &c, see at end of Slating.

Cypress and white pine are also much used for shingles, being much cheaper, but scarcely half as durable. All shingles wear quite thin in time by rain and exposure. In warm damp climates they all decay within 6 to 12 years.

PAINTING.

The principal material used in house-painting, is either white lead, or oxide of zinc, ground in raw (unboiled) linseed oil, by a mill, to the consistency of a thick paste. In this condition, it is sold by the manufacturers in kegs of 25, 50, and 100 lbs. To prepare it for actual use, merely requires the addition of more linseed oil, say 3 or 4 pints to 10 lbs of the keg paint, for thinning it sufficiently to flow readily under the brush.

Good painting requires 4 or 5 coats; but usually only 4 are used in principal rooms; and 3 in inferior ones. Each coat must be allowed to dry perfectly before the next one is put on. One lb of the keg paint will, after being thinned, cover about 2 sq yds of first coat; 3 yds of second; and 4 yds of each subsequent coat; or 1 sq yd of 3 coats will require in all, 1.08 lbs; of 4 coats, 1⅓ lbs; of 5 coats, 1.58 lbs. The reason why the first coats require so much more than the subsequent ones, is that the bare surface of the wood absorbs it more.

When, as is usual, *raw* or unboiled oil is used for thinning, *dryers* must be added to it; otherwise the paint might require several weeks to harden; whereas, with dryers, from 1 to 3 days, according to the weather, suffice for each coat to become hard enough to receive the next one. The dryers most commonly used, are powdered litharge, in the proportion of one heaped teaspoonful; or Japan varnish, 1 table-spoonful, to 10 lbs of the keg paint. Either sugar of lead, or sulphate of zinc, may also be used instead of litharge; and in the same proportion. Although both litharge and Japan varnish are dark-colored, yet the quantity is so small as not to appreciably affect the whiteness of the paint. If the varnish is used in excess, as is often done in the hurry to have work finished, it produces cracks all over the surface. No dryer is necessary if painters' boiled oil be used for thinning. Mere boiling will not cause oil to harden more rapidly; but that intended for painters, has litharge added to it previously to boiling; in the proportion of 1⅓ lbs to each 10 gallons of raw oil. In some works written for the use of house painters, it is asserted that boiling renders the oil too thick for any but coarse outdoor work. But this is entirely a mistake; for if the boiling be properly done, the oil will be quite thin enough for the best inside work; and will moreover be clearer than while raw; and will impart to the painted surface a more shining appearance. The heat should be barely sufficient to produce boiling; or about 600° Fah. The boiling should continue about 1½ hours; the oil being thoroughly stirred at short intervals, to prevent the litharge from settling at the bottom. The fire may then be allowed to subside; when the operation will be completed. A sediment will then form at the bottom; which must be left behind when the oil is poured off. Although no dryer is necessary with this oil, still a little litharge may be added when great expedition demands it. Painters rarely use this oil, on account of its trifling increase of cost.

Another substance much used with the thinning oil, (except for the first coat,) is spirits of turpentine; called "turp" by the workmen. The quantity of oil may be diminished, to the extent of the added turp. This being more fluid than oil, causes the paint to work more pleasantly under the brush. It moreover diminishes the tendency of the paint to become yellow; especially in rooms kept closed for some time. It is also much cheaper than oil. It should not be used, or but sparingly, for exposed outdoor work; inasmuch as its tendency is to impair the firmness of the paint; and although its effects are scarcely appreciable indoors, they are quite apparent when the work has to resist the weather. As the fashions change in house-painting, the surface is at times required to present a *shining* or *glossy* finish; at other times a *dead* one is in vogue. The glossy one is that which the paint will naturally have, provided that no more turp than oil be used in the thinning. The dead finish is obtained by using no oil, but turp alone, for the last coat; which in that case is called a *flatting coat*. Although turp is not properly a dryer, still, as it evaporates quickly, it facilitates the hardening of the paint.

In outdoor work it is usually advisable to use more dryer than inside, so that the paint may sooner become hard enough not to be injured by dust or rain. Otherwise less would be better.

When, instead of a white finish, one of some other color is required, the coloring ingredient is mixed with the white paint to be used in the last coat only; although two coloring coats are sometimes found to be necessary before a satisfactory effect is produced. The coloring ingredients may be indigo, lampblack, terra sienna, umber, ochre, chrome yellow, venetian red, red lead, &c, &c; which are ground in oil, ready for sale, by the manufacturers of the white-lead and zinc paints. They are simply well stirred into the white paint.

All surfaces to be painted, should first be thoroughly dry, and free from dust. If on wood, all plane-marks, and other slight irregularities, should first be smoothed off by sand-paper, when the neatest finish is required. Also, all heads of nails must be punched to about ⅛ inch below the surface. To prevent knots from showing through the finished work, (as those in white or yellow pine would do, on account of the contained turpentine,) they must first be *killed*, as it is termed. A usual and effective way of doing this, is by covering them with two coats of shellac varnish; which, when dry, should be smoothed by sand-paper. Another mode, not quite so certain, is by one or two coats of white lead mixed with thin glue-water, or *size*, as it is called.

After these preparations, the first, or *priming* coat, is put on; in which there should be no turp; because it would sink at once into the bare wood, leaving the white lead behind it, in a nearly dry friable condition. After this the nail holes, cracks, &c, must be filled with common glaziers' putty, made of whiting (fine clean washed chalk) and raw linseed oil; boiled oil will not answer; the putty would be friable. The putty would be apt to fall out, if put in before priming; because the wood would absorb the oil, and the putty would then shrink. After the first coat is perfectly dry, the second one is put on; and for it about 1 measure of turp may be mixed with 3 measures of the thinning oil. In the third, and any subsequent coats, equal measures of turp and oil, may be used for thinning, if the work is required to dry *with a gloss*; but if it is to finish *dead*, the last coat must be a *flatting* one; or one in which the thinning oil is entirely omitted, and turp alone substituted for it.

Painters generally clean their brushes by merely pressing out most of the paint with a knife; and then keep them in water until further use. If to be put away for some time, they may be thoroughly cleaned by turp; or by soap and water. To prevent a hard skin from forming on the top of their paint when not used for some days, they pour on a little oil.

The best paints for preserving iron exposed to the weather, appear to be pulverized oxides of iron, such as yellow and red iron ochres; or brown hematite iron ores finely ground; and simply mixed with linseed oil, and a dryer. White lead applied directly to the iron, requires incessant renewal: and indeed probably exerts a corrosive effect. It may, however, be applied over the more durable colors, when appearance requires it. Red lead is said to be very durable, when pure. An instance is recorded of pump-rods, in a well 200 ft deep, near London, which, having first been thus painted, were in use for 45 years; and at the expiration of that time, their weight was found to be precisely the same as when new; thus showing that rust had not affected them.

When the size of the exposed iron admits of it, its freedom from rust may be very much promoted by first heating it thoroughly; and then dipping it into, or washing it well with, hot linseed oil; which will then penetrate into the interior of the iron. For tinned iron exposed to the weather, on roofs, rain pipes, &c, Spanish brown is a very durable color. The tin is frequently found perfectly bright and protected, when this color has been used, after an exposure of 40 or 50 years. White paint washes off in a few years by rain.

Plastered walls should if possible be allowed to dry for at least a year, before being painted in oil; otherwise the paint will be liable to blister. They may, if preferred, be frescoed (water-colors mixed with size) to the desired tint during the interval.

The painting of unseasoned wood hastens its decay. If the surface to be painted is greasy, the grease must first be removed by water in which is dissolved some lime.

Washes for outside work. Downing, in his work on country houses, recommends the following: *For wood-work;* in a tight bushel, slack half a bushel of fresh lime, by pouring over it boiling water sufficient to cover it 4 or 5 ins deep; stirring it until slacked. Add 2 lbs of sulphate of zinc (white vitriol) dissolved in water. Add water enough to bring all to the consistence of thick whitewash. Apply with a whitewash brush. This wash is white; but it may be colored by adding powdered ochre Indian red, umber, &c. If lampblack is added to water-colors, it should first be thoroughly dissolved in alcohol. The sulphate of zinc causes the wash to become hard in a few weeks.

For brick, masonry, or rough-cast. Slack ½ a bushel of lime as before; then fill the barrel ¾ full of water, and add a bushel of hydraulic cement. Add 3 ℔s of sulphate of zinc, previously dissolved in water. The whole should be of the thickness of paint; and may be put on with a whitewash brush. The wash is improved by stirring in a peck of white sand, just before using it. It may be colored, if desired, like the preceding.

He also gives the following cheap oil-paint for outside work on wood, brick, stone, &c; and says it becomes far harder and more durable than common paint: One measure of *ground* fresh quicklime; add the same quantity of fine white sand, or fine coal ashes; and twice as much fresh wood ashes; all the foregoing to be passed through a fine sieve. Mix well together dry. Mix with as much raw linseed oil as will make the mixture as thin as paint. Apply with a painter's brush. It may be colored like the foregoing, taking care to mix the colors well with oil before adding them. It is best to put on two coats; the first thin, and the second thick.

Also, another, said to stand 15 to 20 years: 50 ℔s best white lead; 10 quarts raw linseed oil; ½ ℔ dryer; 50 ℔s finely sifted sharp clean sand; 2 ℔s raw umber. Add very little, say ½ pint of turpentine. Apply with a *large* brush.

Cement for stopping joints, such as around chimneys, &c, &c. White lead ground in oil, as sold by the keg; mixed with enough pure sand to make a stiff paste that will not run. It grows hard by exposure, and resists heat, cold, and water. Pieces of stone may be strongly cemented together by it, allowing a few months for proper hardening.

Whitewash for inside work, according to Mr. Downing, "is made more fixed and permanent, by adding 2 quarts of thin *size* to a pailful of the wash, just before using. The best size for this purpose is made of shreds of glove leather; but any clean size of good quality will answer," as thin glue-water. We will add, that the common practice of mixing salt with whitewash, should not be permitted. Paper pasted on a wall which has previously been covered with salt whitewash, is very apt to become wet, and loose, and to fall off during damp weather. The whitewash should be scraped off, and the wall or partition covered with a coat or two of thin size, to protect the paper from the effect of the salt that may still adhere to the plaster.

GLASS, AND GLAZING.

WINDOW glass is sold by the box. Whatever may be the size of the panes, a box contains as nearly 50 sq ft of glass as the dimensions of the panes will admit of.

Panes of any size may be made to order by the manufacturers. The sizes given in the following table, as well as many others, are generally to be had ready made. Ordinary window glass of all the sizes in the table, is about one-sixteenth of an inch thick; and this is the thickness supposed to be intended when a greater one is not specified. *Double-thick* glass is nearly ⅛ inch; and its price is 50 per ct more than the *single* thick. It is of course much stronger than the single.

The panes are confined to the sash by glaziers' putty, made of whiting (powdered chalk) and raw linseed oil; and by small triangular pieces of thin tin, about ¼ inch on a side, which uphold the glass while the putty is being put on; and are allowed to remain afterward, as a protection while the putty continues soft.

TABLE OF NUMBERS OF PANES IN A BOX.

Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.
6 × 8	150	12 × 36	17	16 × 42	11	24 × 24	12	30 × 66	4
7 × 9	115	13 × 14	40	48	9	26	12	70	3
8 × 10	90	16	35	54	8	30	10	32 × 34	7
12	75	18	31	60	8	36	9	36	6
9 × 12	67	20	28	18 × 20	20	42	7	42	6
14	57	24	23	22	18	48	6	48	5
16	50	32	17	24	17	54	6	60	4
18	45	14 × 16	32	30	14	60	5	66	3
10 × 12	60	18	29	36	11	66	5	34 × 36	6
14	52	20	26	42	10	26 × 28	10	44	5
16	45	24	22	50	8	32	9	48	5
18	40	30	17	60	7	36	8	54	4
20	36	36	14	20 × 22	17	42	7	60	4
24	30	42	12	24	15	48	6	66	3
30	24	46	11	30	12	54	5	36 × 40	5
11 × 12	55	15 × 16	30	38	10	60	5	44	5
14	47	18	27	42	9	28 × 30	9	48	4
16	41	20	24	48	8	36	7	54	4
18	37	24	20	54	7	42	6	60	3
20	33	30	16	64	6	56	5	70	3
24	27	36	13	22 × 24	14	66	4	38 × 44	4
12 × 14	43	40	12	30	11	30 × 34	7	52	4
16	38	16 × 18	25	36	9	36	7	40 × 46	4
18	34	20	23	42	8	42	6	54	3
20	30	24	19	48	7	48	5	72	3
24	25	30	15	56	6	54	4	44 × 50	3
28	22	36	13	60	5	60	4	56	3
30	20								

The best qualities of American glass made in the vicinity of Philadelphia, Boston, Pittsburg, &c, are for most purely *useful* purposes, as good as those from foreign countries; but when the highest degree of *beauty* is required, as in the lower front windows of first-class dwellings, fancy stores, &c, polished plate-glass of England, France, or Germany, must be used, although the price for moderate sized panes is from 5 to 8 times as great as that of the best quality single-thick American. Its perfectly *smooth* surface, free from distorted reflections, also makes it the best for covering pictures; still, if carefully selected American panes be used for this purpose, few except critics in glass will detect the difference.

A thick glass is made expressly for flooring, up to 1 inch thick, and up to 50 inches by 9 feet dimensions. Also, for skylights, from ¼ to ½ inch thick. This can be furnished to order of any size up to 40 inches by 8 or 10 feet. The smaller sizes can also be had *ground.* Grinding prevents the entrance of the full glare of the sun; and, moreover, diffuses the light over a much greater width of space below.

Strength of glass. Tensile 2500 to 9000 lbs per square inch. Boston rods by author, 3500 to 5200. Crushing strength, 6000 to 10000 lbs per square inch. Transversely, (by the writer's trials,) flooring glass, 1 inch square, and 1 foot between the end supports, breaks under a center load of about 170 lbs; consequently, it is considerably stronger than granite, except as regards crushing; in which the two are about equal.

REMARK. Window and other glass which contains an excess of potash or of soda is very liable to become dull in time, owing to the decomposition of those ingredients by atmospheric influences.

ROPE.

The strength of rope varies greatly. Pieces from the same coil may vary 25 per cent. The table below supposes an average quality **Manila.** Good **Italian hemp** is considerably stronger. The **tarring** of ropes is said to lessen their strength; and, when exposed to weather, their durability also. We believe that its use in standing rigging is partly to diminish contraction and expansion by alternate wet and drying weather. A few months of **exposed work** weakens ropes 20 to 50 per cent.

Table of Manilla rope.

Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load.		Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load.	
			Tons.	lbs.				Tons.	lbs.
.239	¾	.019	.25	560	1.91	6	1.19	11.4	25536
.318	1	.033	.35	784	2.07	6½	1.39	13.0	29120
.477	1½	.074	.70	1568	2.23	7	1.62	14.6	32704
.636	2	.132	1.21	2733	2.39	7½	1.86	16.2	36288
.795	2½	.206	1.91	4278	2.55	8	2.11	17.8	39872
.955	3	.297	2.73	6115	2.86	9	2.67	21.0	47040
1.11	3½	.404	3.81	8534	3.18	10	3.30	24.2	54208
1.27	4	.528	5.16	11558	3.50	11	3.99	27.4	61376
1.43	4½	.668	6.60	14784	3.82	12	4.75	30.6	68544
1.59	5	.825	8.20	18368	4.14	13	5.58	33.8	75712
1.75	5½	.998	9.80	21952	4.45	14	6.47	37.0	82880

Working loads. For manila ropes from 1 to 1¾ ins diam, running at different speeds over sheaves of the diams stated, Mr. C. W. Hunt (Trans Am Soc Mechl Engrs, Vol. XXIII, 1901) gives a table embodying approximately the following results of experience. Working load = C × ultimate strength of new rope. D = minimum diam of sheave, in ins.

Speed	ft per min	as for work on	C	1″ rope D	1¾″ rope D
Slow	50 to 100	derrick, crane, quarry	0.140	8	14
Medium	150 to 300	wharf, cargo	0.056	12	18
Rapid	400 to 800		0.028	40	70

Such ropes wear out rapidly. A rope 1½ ins diam wears out in lifting from 7,000 to 10,000 tons of coal. On the other hand, 1½ inch transmission ropes, running 5000 ft per min and carrying 1000 H. P. over sheaves 5 ft and 17 ft in diam, last for years.

Mr. Hunt's figures for ultimate strength, based upon tests of full-sized specimens of manila rope made by three independent rope-walks and purchased in open market, are practically identical with those given in our table above, as are also those of Prof. B. Kirsch, of the Imperial Royal Technological Industrial Museum, Vienna, quoted by Mr. Hunt.

WEIGHTS AND STRENGTHS OF WIRE ROPES.

Wire Rope manufactured by **John A. Roebling's Sons Co., Trenton, N. J.** The prices and weights given are for ropes with *hemp* centers. When made with *wire* centers, the prices per foot are 10 per cent. higher, and the weights 10 per cent. greater.

Trade No.	Diam. in ins.	Approx. circum. in ins.	Wt. per ft. in lbs.	Approx. breaking strength* in tons of 2000 lbs.		Minimum diam. of drum in feet.		Price in cents per foot.†	
				Iron.	C. steel.	Iron.	C. steel.	Iron.	C. steel.
Standard Hoisting Rope, with 6 strands of 19 wires each.									
1	2¼	7⅛	8.00	78	156	13	8½	117	142
2	2	6¼	6.30	62	124	12	8	92	111
3	1¾	5⅛	4.85	48	96	10	7¼	80	93
4	1⅝	5	4.15	42	84	8½	6¼	63	74
5	1½	4¾	3.55	36	72	7½	5¾	57	66
5½	1⅜	4¼	3.00	31	62	7	5½	48	56
6	1¼	4	2.45	25	50	6½	5	40	46
7	1⅛	3½	2.00	21	42	6	4½	33	38
8	1	3	1.58	17	34	5¼	4	26	30
9	⅞	2¾	1.20	13	26	4½	3½	20	23
10	¾	2¼	0.89	9.7	19.4	4	3	16	18
10¼	⅝	2	0.62	6.8	13.6	3½	2¼	12	14
10½	9/16	1¾	0.50	5.5	11.0	2¾	1¾	10	12
10¾	½	1½	0.39	4.4	8.8	2¼	1½	8	11
10*a*	7/16	1¼	0.30	3.4	6.8	2	1¼	7½	10
10*b*	⅜	1⅛	0.22	2.5	5.0	1½	1	7	9½
Transmission or Haulage Rope, with 6 strands of 7 wires each.									
11	1½	4¾	3.55	34	68	13	8½	51	60
12	1⅜	4¼	3.00	29	58	12	8	43	51
13	1¼	4	2.45	24	48	10¾	7¼	36	43
14	1⅛	3½	2.00	20	40	9½	6¼	29	36
15	1	3	1.58	16	32	8½	5¾	23	28
16	⅞	2¾	1.20	12	24	7½	5	17½	22
17	¾	2¼	0.89	9.3	18.6	6¾	4½	14	16
18	11/16	2⅛	0.75	7.9	15.8	6	4	12	13½
19	⅝	2	0.62	6.6	13.2	5¼	3½	10	11
20	9/16	1¾	0.50	5.3	10.6	4½	3	8	9
21	½	1½	0.39	4.2	8.4	4	2½	6½	7½
22	7/16	1¼	0.30	3.3	6.6	3¼	2¼	5½	6½
23	⅜	1⅛	0.22	2.4	4.8	2¾	2	4½	5½
24	5/16	1	0.15	1.7	3.4	2½	1¾	3¾	4½
25	9/32	⅞	0.125	1.4	2.8	2¼	1½	3¼	4

Notes on the Use of Wire Rope, by the Roebling's Company.

The ropes with 19 wires per strand are the more pliable, and therefore best adapted for **hoisting** and **running** rope. The others are stiffer and better adapted for **guys,** &c. Ropes of iron or steel, up to 3 inches diameter, made to order. Hemp center rope is more pliable than wire center. Wire rope **must not be coiled or uncoiled** like hemp rope. When on a reel, the reel should be mounted on a spindle or flat turn-table in order to pay off the rope. When forwarded in a small coil without a reel, roll the coil on the ground like a wheel, and thus run off the rope. Avoid untwisting and short bends. **To preserve wire rope,** apply raw linseed oil (which may be mixed with an equal quantity of Spanish brown or lamp-black) with a piece of sheepskin, keeping the wool against the rope. **If for use in water or underground,** add 1 bushel of fresh-slacked lime and some sawdust to 1 barrel of tar. Boil the mixture well and saturate the rope with it while hot.

---

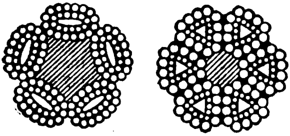
* **For the safe working load,** take one-fifth to one-seventh of the breaking load, according to speed.

† Discounts, 1901: bright rope, 30 per cent. and 7½ per cent.; galvanized, 25 per cent. and 7½ per cent.

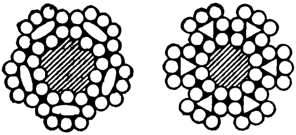
Galvanized wire rope for rigging is cheaper and more durable than hemp rope; and does not stretch permanently under great strains. Its bulk is one-sixth and its weight one-half that of hemp rope. Roebling's wire rope has been made the standard by the United States Navy Department. Shackles, sockets, swivel-hooks, and fastenings, &c., furnished and put on and splices made. Pulley-wheels furnished. Also galvanized steel cables for suspension bridges. Crucible cast-steel wire ropes are much more durable than iron ones. They should be kept well lubricated.

Patent Flattened Strand Wire Rope.

Manufactured by A. Leschen & Sons Rope Co., St. Louis, Mo.



Hoisting Ropes.



Haulage and Transmission Ropes.

Diameter inches.	Average weight, per ft., lbs.	Breaking* strength in tons of 2000 lbs.			Minimum diam. of drum in feet.			List price† per foot, in cents.			Diameter inches.
		"Hercules."‡	Crucible cast steel.	Swedes iron.	"Hercules."‡	Crucible cast steel.	Swedes iron.	"Hercules."‡	Crucible cast steel.	Swedes iron.	
Hoisting Rope.											
2¼	8.50	260	176	75	"Can be used over the ordinary size drums and sheaves."	9	10	257	182	152	2¼
2	6.50	211	140	66		8	9	202	144	120	2
1¾	5.00	168	109	54		7.25	7.5	173	121	104	1¾
1½	3.71	124	81	40		5.75	6.5	123	86	74	1½
1¼	2.50	84	56	28		5	5	82	59.5	52	1¼
1⅛	2.15	67	47	21		4.5	4.5	68	50	43	1⅛
1	1.70	56	38	17		4	4	56.5	39.5	34	1
⅞	1.25	40	29	13		3.5	3.5	45	30	26	⅞
¾	0.96	32	21	9		3	3	35	24	21	¾
⅝	0.67	22	15	6		2	2.5	26	18.25	15.5	⅝
½	0.44	13	9	4		1.5	1.75	19.5	14.5	10.5	½
Haulage and Transmission Rope.											
1¼	2.40	80	54		....	....	....	81	54	45	1¼
1⅛	2.00	64	45		....	....	....	67	45	36.5	1⅛
1	1.64	53	36		....	....	....	53.5	35	29	1
⅞	1.20	38	27		....	....	....	42	27.5	22	⅞
¾	0.93	30	20		....	....	....	34	20.5	17.5	¾
⅝	0.68	21	14		....	....	....	24	14	12.5	⅝
½	0.40	13	9		....	....	....	15.5	10	8.25	½

* Working load = 0.2 × breaking strength.

† Discount, 1901, 30 per cent. and 7½ per cent.

‡ "Hercules." "Made from a specially drawn and patent tempered steel, which is solely made for this brand of rope."

PAPER.

24 sheets 1 quire. 20 quires 1 ream.

Sizes of drawing papers.

	Ins. Ins.		Ins. Ins.
Antiquarian	31 × 52	Super Royal	19 × 27
Double Elephant	26 × 40	Royal	19 × 24
Atlas	26 × 34	Medium	17 × 22
Imperial	21 × 30	Demy	15 × 20
		Cap	13 × 17

The English drawing-papers are stronger and superior to the American. Those by Whatman have a high reputation; they are, however, of different qualities. When paper is pasted on muslin, the difference in quality is not so important. Of paper in rolls, the German makes are the best. There is but little of other makes imported.

Both white and tinted papers, for the use of engineers, are made in continuous rolls, without seams. Widths 36, 42, 54, 58, and 62 ins; usual lengths 40 yds; but can be had to order to 400 yds or more. These may also be had mounted on muslin, in rolls 10 to 40 yds long.

Cartridge or **pattern** paper is furnished in long rolls, of same lengths as white paper, mounted or not; widths up to 54 ins. Color, a light buff.

Tracing paper. Most of that sold, whether domestic or foreign, tears so readily as to be of comparatively little service. Parchment paper, 37 and 38 ins wide, rolls of 20 and 33 yds, is better, but does not take ink perfectly.

Tracing cloth, usually called *tracing muslin*, and sometimes *vellum cloth*, is altogether preferable to tracing paper, on account of its great strength. Widths 18, 30, 36, and 42 ins; lengths to 24 yds.

Profile paper is made in widths of 9 ins and 20 ins, and in single sheets or in long, continuous rolls.

Cross section paper, mounted or unmounted, tracing paper and cloth, are furnished in sheets and in rolls, ruled in quarters, fifths, eighths, tenths, twelfths, and sixteenths of an inch, or in millimeters.

Colors. Since the introduction of blue printing, tinted drawings are seldom made, except for architectural effect; but colors may be used to advantage on black-line prints from tracings, p 432 *d*. A good draughtsman needs but few colors; say India ink, Prussian blue, lake, or carmine, light red, burnt umber, burnt sienna, raw sienna, gamboge, Roman ochre, sap green. Winsor & Newton's colors are among the best in use. Purchase none but the very best India ink. Cakes of colors should always be wiped dry on paper, after being rubbed in water; and but little water should be used while rubbing; more being added afterward.

Lead pencils. *Genuine* A. W. Faber's Nos. 2, 3, and 4, are very good. The hardness increases with the number. Nos. 3 and 4 are good for field-book use: which to prefer, will depend on the character of the paper; No. 3 for smooth, and No. 4 for the coarser or more granular papers. His *lettered* pencils are of a higher grade and better suited for draughting. "H" stands for "hard," "B" for "soft." The degree of hardness or of softness is indicated by the number of H's or of B's. "F" (intermediate) corresponds with No. 3. Dixon's American pencils are good. The office draughtsman should have a flat file, or a piece of fine emery paper glued to a strip of wood, upon which to rub his lead to a fine point readily, after using the knife.

BLUE-PRINTS, ETC.*

Art. 1. (a) In order to obtain the best results, all **unnecessary exposure,** either of the sensitized paper or of the solutions, to sunlight or to other white light should be avoided.

(b) **Cleanliness** is of the first importance. The vessels in which the solutions are made and mixed must be scrupulously clean, and, if washed with soap, must be carefully rinsed with clean water. They should be left full of water when not in use. The presence of free alkali of any kind is fatal to good results, and immediately destroys the blue color of a finished print. See Art. 19 (b). The solutions must not be allowed to come in contact with iron.

Art. 2. (a) **The Solution** used in sensitizing the paper for blue-prints is usually that of *ferri*cyanide of potassium (*red* prussiate of potash) † and ammoniocitrate of iron (citrate of iron and ammonia) in water.

(b) The two salts are usually **dissolved separately** and the two solutions then mixed. The potassium salt should be broken up fine. The iron salt is usually quite pure and dissolves very rapidly. It may be kept indefinitely in a solid state if perfectly dry, but it readily absorbs moisture, and then becomes sticky and unfit for use; and the solution is apt to become mouldy after a few days, either alone or when mixed with the potassium solution. Hence, it should be prepared (in a dark room) in small quantities as required.

Art. 3. (a) The following is an average of several **recipes** that give excellent results:

Solution A. 1 ounce of red prussiate of potash to 6 ounces of water, or 2½ ounces of the salt to a pint of water.

Dissolve thoroughly and filter. The solutions may be sufficiently filtered through raw cotton, and much more rapidly than through paper.

Solution B. 1½ ounces of ammoniocitrate of iron to 6 ounces of water, or 4 ounces of the salt to 1 pint of water.

Dissolve thoroughly. Filter, unless the solution is perfectly clear.

(b) Keep the two solutions in separate glass-stoppered bottles in a dark place until they are to be used. Then mix them in equal parts, and filter the mixture. Take care that no undissolved particles of the red prussiate get into the double solution. It must be rejected when its brown color changes to bluish green.

(c) The combined solution will **cost** amateurs from 1 to 2 cents per ounce to make. About 4 ounces **will suffice for** coating 100 square feet of paper.

(d) If a few drops of strong ammonia solution be added to the citrate solution, B, until the odor is quite perceptible, the addition of a saturated solution of **oxalic acid** in water to the double solution **will hasten the printing in cloudy weather.** 10 per cent. of the oxalic-acid solution will increase the rapidity of printing about 2½ times; 20 per cent., 5 times; 30 per cent., 10 times; but with more than 20 per cent., it is difficult to get clear white lines. In sunlight the difference is much less marked. (*Engineering News*, Dec. 15, 1892.)

Art. 4. (a) Where fine work is not essential, any well-sized **paper,** sufficiently tough to bear the washing, will answer. For important work use paper of fine uniform texture and smooth hard surface, free from injurious chemical substances. If the solution penetrates below the surface, a portion of the chemicals may remain in the paper in spite of the washing, and damage the result. Many papers are made especially for this purpose. The Saxe (German) and Rives (French) papers are considered among the best. Johannot and Steinbach papers give good prints, but are not very strong. Weston's and Scotch linen papers are stronger, and the latter gives excellent prints. Before sensitizing a large quantity of paper of a new kind, try a small sheet of it. **Linen** for sensitizing is also sold by dealers in photographic material and engineers' supplies.

Art. 5. (a) **The solution is applied** (in the dark room of course) to one side only of the paper. This is sometimes done by "**floating**" the paper upon the solution, taking care that none gets upon the *back* of the sheet.

---

* See "Modern Heliographic Processes," by Ernst Leitze; D. Van Nostrand Co., New York, $3.00; a work to which we are indebted for many valuable suggestions.

"Modern Reproductive Graphic Processes," by Lieut. J. S. Pettit, D. Van Nostrand Co., Science Series, No. 76, 50 cents, deals chiefly with artistic photography, lithography, etc.

See also paper by Benj. H. Thwaite, Proc's, Inst'n Civ. Eng'rs, Vol. lxxxvi, p. 312, reprinted in Engineering News, Nov. 27, 1886.

† *Not* the *ferro*cyanide or *yellow* prussiate.

The paper is held by two diagonally opposite corners, and the diagonal joining the other two corners is then allowed to touch the surface of the liquid. Then the two corners held in the hand are dropped, first one and then the other. The paper should then be lifted, one half at a time, to see whether any air bubbles have been formed under it. If so, they may be removed by drawing over the solution that half of the sheet under which they occur, while the other half is held up from the liquid. One or two minutes suffice for floating, and the paper is then drawn out over an edge of the bath, draining off the surplus liquid. This process requires a tray larger than the sheet, and the inner surface of the tray must be not only water-proof, but also proof against chemical action from the solution. Considerable care is required in the manipulation.

Art. 6. (a) The solution is usually applied by means of a soft wide **brush** (such, for instance, as those used for wetting the leaves in letter-copying books) or a large soft **sponge** entirely free from sand or other grit.

Art. 7. (a) In applying the solution, the paper may be laid upon a board covered with soft smooth **oil-cloth,** which, after each sheet is sensitized, should be wiped off, to avoid smearing the back of the next sheet.

(b) **The operation** must be quickly performed, so that no portion of a sheet may become dry before its entire surface has been coated. For very large sheets it may be necessary, for this reason, to employ two persons. First cover the sheet by strokes of the wet sponge or brush, moved in the direction of the length of the paper, and then, immediately, by light strokes at right angles to these and with the sponge or brush squeezed out, so that the solution may be uniformly and thinly distributed over the entire surface. Wash out the sponge immediately in the dark room.

Art. 8. (a) The paper is then hung up **to dry** in the dark room, by means of clips, of any convenient form and free from iron. Small sheets may be hung by one corner; larger sheets by two adjacent corners, or by three or more places (according to size) along one edge, taking care to buckle this edge slightly, so that the paper may not be stretched in drying. If the sheets are hung over a rod or rail the solution will dry unevenly at the bend. In order that the whites in the print may be clear, the air should be warm, so that the paper may dry quickly and the solution be thus prevented from penetrating it deeply.

Art. 9. (a) Make sure that the paper is **perfectly dry** before it is used or put away, and see that it is kept both *dry* and *dark* until it is wanted for use. If carefully prepared and preserved it will retain sensitiveness for a long time, but the best results are obtained with fresh paper, and it is best not to keep it more than a month or two.

Art. 10. (a) **The tracing paper** or tracing cloth should be of a bluish cast (a yellow paper delays printing), thin (see Art. 15,), and as nearly transparent as possible. It should be preserved, both before and after drawing, from long exposure to light, which tends to render it opaque.

(b) Both before and after drawing, it should be kept either flat or rolled, and not folded, because folds render it difficult to bring the drawing into perfect contact with the sensitive paper in printing.

Art. 11. (a) **The drawing** or tracing should be made with the best India ink, rubbed very black. The addition of a little gamboge or chrome yellow increases the opacity. Lines drawn in chrome yellow and in gamboge print well; but Prussian blue or carmine should be rendered more opaque by the addition of a little Chinese white or flake white. Hold the tracing up to a strong light, in order to detect any weak places in the lines.

Art. 12. (a) **Printing** consists in exposing the sensitive paper to the action of light, the drawing being placed between the light and the sensitive surface. The arc **electric light** prints more slowly than direct sunlight, but has the advantage of constancy in all weathers and at all hours, and of fixedness of position. See Art. 16 (a).

(b) Place the frame with its face perpendicular to the rays of light, as nearly as may be, and see that no shadows, as of trees, buildings, etc., are allowed to fall upon a portion of the drawing.

(c) All handling of the paper, such as cutting it to size or placing it in the frame, should be done in a weak light.

Art. 13. (a) To secure close contact between the tracing and the sensitive paper (see Art. 15,) they are usually placed in a **printing-frame.** The essential parts of an ordinary frame are: the frame proper, a plate of clear glass for the passage of the light, and a padded back, which, by means of clamps and springs, presses the two sheets closely together and against the glass.

(b) The tracing is laid in the frame, with its drawn side next to the glass (but see Art. 16 b), and then the sensitive paper, with the sensitive side next to the tracing. Finally, the padded back is placed in the frame.

(c) The back is often made in two halves, hinged together and each provided with a spring, so that one half may be raised to permit examination of the progress of the exposure, while the other half, remaining clamped, holds the tracing and the sensitive paper in position.

(d) By using a frame left open at both ends long strips of sensitive paper may be used, a part at a time, the rest being rolled up at the ends of the frame and wrapped for protection from light.

(e) In any frame it is important that the glass be sufficiently thick to withstand the pressure required in order to secure close contact between the two papers (see Art. 15, below), of excellent quality, and free from defects which would obstruct or unequally refract the light. The glass should be carefully cleaned before printing.

(f) Improved forms of printing-frames have rubber air-cushions in place of flannel pads. In others the necessary pressure is secured by means of a vacuum produced between the tracing and the glass by means of a pump.

(g) Printing-frames are supplied by dealers. The prices, including glass, vary from about $2 for frames 10 x 12 inches, to $30 or $45 for frames 36 x 60 inches. Frames running on rollers, with fittings for exposing them outside of windows, are also furnished, at prices varying with the dimensions and the requirements.

(h) For large blue-prints, Prof. E. C. Cleaves, of Cornell University, uses, instead of a frame, a wooden cylinder covered with felt and revolving on its axis. Upon this cylinder the tracing and sensitive paper are stretched by means of a suitable clamping device, and the cylinder is then revolved in the sunlight. This method dispenses with the use of glass. It of course requires a longer exposure than the ordinary method. (*Trans. Am. Soc. Mech. Eng.*, vol. viii, p. 722.)

(i) For still larger prints, Prof. R. H. Thurston stretches the two papers upon a thin board, which is then sprung into a curve and held in that shape, keeping the papers in tension upon the convex side. This method also dispenses with the use of glass, and, the curvature of the board and the papers being but slight, the whole of the paper is exposed to the light at one and the same time. (*Trans. Am. Soc. Mech. Eng.*, vol. ix, p. 696.)

Art. 14. **(a)** **The time required for exposure** varies with the color, directness, and intensity of the light, with the thickness and opacity of the tracing paper, with the blackness of the drawing, with the materials and the care used in sensitizing the paper, and with the freshness of the latter, from two or three minutes to hours or even days. Roughly, we may say that in full sunlight, in Philadelphia, about three minutes ordinarily suffice from noon to 2 P. M., and ten minutes at 10 A. M. or 4 P. M.; in the shade, thirty to forty-five minutes at noon; but no fixed rules can be given. Experience must decide in each case. A preliminary experiment may be made with a small frame. If the back of the frame is in two or more pieces, the process may be inspected from time to time.

(b) If perfectly opaque ink be properly used, the blue background may be printed very dark without spoiling the lines, but over-exposure in printing renders the background first blackish and then of a dingy shade. See Art. 17 (c) and (d). Drawings in pale ink must be printed very lightly, in order that the lines may remain white, and it is best to use with them a weak light, or to protect them by tissue paper or ground glass. See Art. 18 (a).

Art. 15. **(a)** To obtain perfectly sharp impressions, the side of the tracing upon which the drawing is made should be in **immediate contact** with the sensitized surface of the blue-print paper, especially if, as with sunlight, the direction of the light is variable; for, if any appreciable distance intervenes between the two, as in printing through cardboard (see Art. 16, below), the shadows cast by the lines of the tracing will move over the sensitized surface as the direction of the light changes, and thus give a blurred impression. In most cases, however, it is practically out of the question to place the two surfaces in this way, because that position gives a reversed impression as regards right and left.* Hence a thin tracing paper or linen is recommended in Art. 10 (a). For the same reason it is imperative that the two papers be firmly and evenly pressed against the glass.

Art. 16. **(a)** By using a light which is constant in position, relatively to the surface of the tracing, such as an arc electric light, it is possible, by prolonging the exposure for hours or even days, to obtain blue-prints **from drawings made upon stout drawing paper** or even upon **bristol board.**

(b) With sunlight the same object may be accomplished, either by placing the original with its back to the glass, and the sensitive paper (which should be very thin) with its back to the sunlight, or by placing the printing frame in the bottom of a deep and narrow box, so that the light can shine directly upon the frame only when approximately parallel with the long sides of the box. To print rapidly, the sunlight must be kept full upon the frame by frequently moving the box.

---

* A print, thus reversed in position, may of course be easily read by means of a mirror. This is commonly done with Patent Office drawings.

Art. 17. (a) The print, when sufficiently exposed, is taken from the frame, and both its face and back are **washed thoroughly** in clean water until the characteristic blue color is perfectly developed.

(b) The washing should be done in a tray with a flat bottom larger than the largest print to be washed, and care should be taken not to injure the surface of the prints by hard rubbing or by sharp bending, or otherwise. It is better to have a circulation of water in the tray, not only to keep the water clean, but also to bring about the necessary agitation of the prints without handling them.

(c) The washing may be hastened, and dark or "over-exposed" prints may be lightened somewhat, by having the water warm, say at 90° or 100° Fahrenheit.

(d) Over-exposed prints may also be lightened by immersing them in water rendered slightly alkaline by ammonia. In this bath they at once assume a purple tint, which soon becomes weaker. At the proper moment, which must be learned by experience, the alkaline action must be stopped by drawing the print rapidly through a solution of 1 part of hydrochloric ("muriatic") acid (H. Cl.) in 100 parts of water.

(e) Continue washing until the water has for some time come off perfectly clear. Then hang the prints up smoothly to dry.

Art. 18. (a) After washing, the application of a solution of from 1 to 5 per cent. of hydrochloric acid, or of oxalic acid, in water, intensifies the blue color, and is therefore useful in bringing out pale or "under-exposed" prints; but the prints must then be afterward washed again in pure water. Hydrochloric acid applied *before* washing, or to imperfectly washed prints, will make the *lines* show blue.

Art. 19. (a) **To erase** a (white) line on a blue-print, go over the line with the sensitizing solution applied with a clean brush or *quill* pen. This should be done in a weak light. Then expose the entire print and re-wash.

(b) **White lines are added** to blue prints, usually in Chinese white; but the blue color may be removed, showing the white paper beneath, by applying a saturated solution of concentrated lye (caustic soda or potash) or of carbonate of soda* or carbonate of potash, with a fine clean pen nearly dry. If laid on too freely, it spreads rapidly. Even if the pen is perfectly clean, the surface thus produced has a yellowish cast as compared with the white of the paper. The carbonate solutions act more slowly than the lye, but not less surely, and they are not injurious to the skin, whereas the lye burns badly. The ordinary lime-water sold by druggists makes little or no impression upon the blue color. **If red, instead of white,** lines are desired, mix with the soda or potash solution ordinary carmine writing-ink, in such quantity (to be ascertained by trial) as will give the desired color.

Art. 20. (a) Blue prints which are to be subjected to much handling should be **mounted** upon cloth, or the prints may be made, in the first place, upon sensitized tracing linen.

Art. 21. (a) **Processes giving a white ground.** with either blue or black lines, are usually so complicated as to be beyond the reach of most engineers. Their results, also, are generally uncertain, even when applied by experts; the background often lacking in whiteness.

(b) **Vandyke paper** (Eugene Dietzgen Co., Chicago) and **Maduro paper** give excellent dark **brown lines** on good, smooth, hard paper. The "Nigrosine" and other so-called black-line prints, furnished by dealers, usually give perishable purple lines on a gray and somewhat glossy ground, and on brittle, unserviceable paper.

(c) **Francis LeClère,** 21 North 18th Street, Philadelphia, furnishes excellent **black-line** prints to order at 10 cents per square foot. The lines are perfectly black and permanent, and the prints are made on good drawing paper, the color and durability of which are not affected by the process. He also furnishes fine **blue-line** prints, on similar paper, at 5 cents per square foot.

---

*Either carbonate ("washing-soda") or bicarbonate ("baking-soda") will answer.

PRICE LIST AND BUSINESS DIRECTORY.

For a work of this kind, any attempt to present a list of exact or even of closely approximate prices would be useless. We aim merely to give indications of the average costs or of the ranges of cost. For actual quotations, apply to those named in the business directory, following the list of prices. See the numbers given in the line or lines immediately following each title in the price list, and referring to said names.

In selecting names for the business directory the aim has been merely to furnish a useful (though by no means exhaustive) list of representative names. No other consideration has been entertained.

Abbreviated Outline of Classification.

For principle of classification, see Bibliography, p. 1008.

1.0 Materials and Elementary Shapes.
1.1 Chemicals, etc. 1.13, Preservatives; Paints, Impregnating, etc. 1.14, Explosives.
1.2 Wood, Lumber, Poles, Posts, and Piles.
1.3 Stone, Concrete, Asphalt, etc. 1.34, Cement. 1.35, Brick, Tile, Glass, etc.
1.4 Iron and Steel. 1.45, Nails, Rivets, Screws, Bolts, etc., Chains. 1.46, Tubes. 1.47, Wire, etc.
1.5 Other Metals and Alloys.
1.6 Paper. 1.7, Ropes, etc. 1.8, Packings, Gaskets, Belting, Lagging, etc.

2.0 Constructions.
2.1 Earthwork. 2.12, Dredging. 2.13, Foundations.
2.2 Masonry. 2.21, Brick. 2.22, Stone. 2.23, Concrete.
2.3 Metal Structures. 2.31, Bridges. 2.32, Turntables. 2.33, Tanks, Stacks, etc. 2.34, Boilers. 2.35, Fireproofing, Concrete Metal Construction.
2.4 Paving. 2.5, Sewers. 2.6, Chimneys. 2.7, Wharves, Docks, Harbor Improvement.

3.0 Machinery.
3.1 Electrical Machinery.
3.2 Tools. 3.22, Machine Tools.
3.3 Engines, Locomotives, Cars. 3.35, Water Engines and Motors, Turbines. 3.36, Cars. 3.37, Wagons.
3.4 Blowing and Pumping Machinery. 3.44, Wind Mills. 3.45, Hydraulic Rams. 3.46, Pumps.
3.5 Hoisting and Conveying Machinery. 3.51, Power Transmission.
3.6 Excavators, Dredges, Machinery for Road and General Construction. 3.65, Diving Apparatus. 3.66, Pile Drivers. 3.67, Wells and Well Driving Machinery. 3.68, Road Making Machinery.
3.7 Heating, Ventilating, and Refrigerating.

4.0 Engineering, Surveying, and Scientific Instruments and Supplies. 4.1, Testing Machines. 4.2, Surveying Instruments. 4.3, Computing Instruments. 4.4, Drawing Insts and Materials. 4.5, Heliography. 4.8, Testing Laboratories.

9.0 Miscellaneous Supplies (Arranged according to class of work).
9.1 Railroad Supplies.
9.2 Hydraulic Supplies. 9.22, Filters. 9.24, Water Meters. 9.25, Pipe and Hose. 9.26, Hydrants and Valves.

PRICE LIST.

1.0 Materials and Elementary Shapes.

1.1 Chemicals, etc.

1.13 Preservatives.

1.131 Coatings, Paints.

35, 76, 198, 274, 289, 327, 361, 433, 502, 564, 586, 635.
Paints, in oil, $1 to $1.50 per gal.
In cts per lb:
Lead: White, foreign, 8 to 10; American, 7. Red, foreign, 8; American, 6.
Zinc: American, 5; Paris, 9 to 10; Antwerp, 7 to 8.
Lampblack, 12 to 14.
Blue, Chinese, 40; Prussian, 35; ultramarine, 15; brown, Vandyke, 10 to 13. Green, chrome, 10 to 12. Sienna, burnt and raw, 10 to 13. Umber, burnt and raw, 10 to 12.
Metal coatings, $1.50 to $2.50 per gal.
Preservatives, fillers, oils, etc., 25 to 50 cts per gal.
Graphite pipe-joint compound, 13 to 20 cts per lb.
Linseed oil, 60 to 70 cts per gal. Turpentine, 40 cts per gal.
Plain varnish, 30 cts per gal.
Carbolineum avenarius, 80 cts per gal. Woodiline or spirittine, 25 cts per gal. Creosote oil, 1 to 1¼ cts per lb.

1.132 Creosoting, Impregnating, etc.

48, 63, 133, 149.5, 229, 319, 373.5, 442, 453, 578, 612.5, 634, 663.
Creo-resinate and creosote process, 13 to 19 cts per cu ft.
Creosoting, 20 to 60 cts per cu ft of material treated, depending chiefly on degree of saturation, and exclusive of cost of timber; = $16 to $50 per 1000 ft B M.
Kyanizing (mercury bichloride process), 8 to 9½ cts per cu ft.
Barschall or Hasselmann process, 8 cts per cu ft.
Wellhouse (zinc-tannin process), 12 to 19 cts per tie.
Burnettizing (zinc chloride process), 8 to 18 cts per tie.
The treatment of ties is usually cheaper per cu ft than that of larger lumber.

1.14 Explosives.

214, 311, 346, 452, 492, 517.
Gunpowder, 16 cts per lb.
Smokeless powder, 60 cts per lb.
Rackarock, 18 to 25 cts per lb.
Dynamite, 13 to 21 cts per lb for different grades, varying between 20% and 75% nitroglycerine.
Percussion caps, 30 to 60 cts per M.
Blasting machinery, see 3.231.
Drills, see 3.23.

1.2 Wood, Lumber, Timber.

16.5, 268, 271, 330, 599, 636.
Lumber, in dollars per 1000 ft board measure (B M):
Yellow pine, short leaf, 12 to 13; flooring, 20 to 35; long leaf, 19 to 20; flooring, 22 to 25.
Walnut, 110 to 130. Poplar, 25 to 40. Ash, 60 to 65.
Oak, culls, 20; common, 28; plain sawed, 40; boards, 60 to 70; 10-inch and wider, 100 to 125; plank, 40.
Hemlock joists and boards, 15 to 20.
Spruce, 30 to 40.
Shingles, cypress, per 1000, 8 to 11.
Studding, joists, rafters, etc., hemlock, 15 to 18.
Clearing and grubbing, see 2.11.
Wood pipe, see 9.254. Ties, 9.14. Piles, 1.23.

1.23 Poles, Posts, Piles.

408, 494, 599.
Piles, 15 to 25 cts per linear ft of pile.
Piling, round or sheet, 30 to 50 cts per linear ft of pile.

1.3 Stone, Concrete, Asphalt, etc.

Earthwork, dredging, foundations, see 2.1.

1.32 Stone.

92, 368, 388, 393, 620.
Sand and gravel (within 100 miles of seashore), $1 to $2 per cu yd.
Broken stone, 75 cts to $2 per cu yd.
Rip-rap, $1 to $3 per cu yd.
Trap rock, 70 cts per ton of 2000 lbs.
Ordinary building stones, $1 to $5 per cu yd.
Granite, $15 to $45 per cu yd
Slate roofing, 12 to 25 cts per sq ft.

1.33 Asphalt.

17, 51, 61, 272, 437, 444, 449, 559, 649.
Paving, see 2.4.

1.34 Cement.

22, 23, 30, 54, 86, 97, 101, 106, 149, 170, 177, 179, 227, 252, 258, 310, 326, 327, 336, 352, 358, 360, 366, 418, 445, 456, 569, 588, 616, 626, 642, 657.
Portland (artificial) cements, per bbl of about 400 lbs gross: German, $2.25 to $3.00; American, $1.10 to $1.60.
Rosendale (natural) cements, per bbl of about 300 lbs net: From Rosendale Township and vicinity, Ulster Co., N. Y., 95 cts to $1.10; other Rosendales, 75 to 85 cts.
About $1 to $2 worth of cement mortar required per cu yd of masonry in buildings, $1.50 to $3 per 1000 bricks.
Lime, 60 to 90 cts per bbl of about 250 lbs.
About 60 cts worth required per cu yd of masonry in buildings. $1 to $1.50 per 1000 bricks.
Plaster, $1.50 to $2 per bbl of varying weight.
Concrete construction, see 2.35.

1.35 Brick, Tile, Glass, etc.

Sewer pipe, see 9.255.

1.351 Brick.

217, 232, 291, 299, 383, 448, 468, 505, 515.
Paving, see 2.4.
Building bricks, per 1000: Salmon, $5 to $7; hard, $7 to $9; stretchers, $9 to $14; pressed, $17 to $20; colored, $20 to $30; iron spots, $30; Pompeiian, $35.
Fire-brick, $20 to $24 per M.
Vitrified paving brick, $15 to $25 per M.
Sewer pipe, see 9.255. Paving, 2.4.

1.352 Tiling.

Floors and walls, 35 and 40 cts per sq ft and upward.
Tile, 380, 0.13 to 0.8 ct per cu in of material.
Roofing tile, $7 to $30 or more per square. 1 square = 100 sq ft.

1.353 Glass.

557.

American window. $ per box of about 50 sq ft. Discount, 80% to 85%.

" United inches "	25	50	80	100
Single AA	32	38	49	
Double AA	43	56	68	88
Single A	27	32	45	
Double A	38	50	62	80
Single B	26	30	39	
Double B	36	46	56	75

Or, say, single thick, $\frac{1}{16}$ to $\frac{1}{8}$ ct per united in: double thick, $\frac{1}{8}$ to $\frac{1}{4}$ ct per united in: where the number of united ins equals the sum of the two dimensions. Thus, a sheet of glass 24 × 36 contains 60 united ins.

1.4 Iron and Steel.

98, 123, 128, 131, 134, 176, 189, 195, 233, 242, 283, 301, 310, 329, 526, 535, 560, 612, 640, 662, 670.

Scrap iron and steel, $12 to $19 per ton of 2240 lbs.

1.41 Cast Iron and Steel.

71, 145, 151, 185, 189, 192, 218, 249, 280, 281, 329, 372, 377, 463.2, 612, 624, 652.

Cast-iron pipe, see 9.251.

Pig iron, per ton of 2240 lbs: Foundry, $13 to $15; Bessemer, $16; gray forge, $14; Lake Superior charcoal, $17.

1.42 Forged Iron and Steel.

16, 29, 69, 79, 128, 132, 137, 147, 153, 176, 219, 255, 269, 273, 312, 329, 341, 343, 463.2, 463.6, 550, 553, 580, 597, 622, 662, 665, 668, 677.

1.43 Rolled and Structural Iron and Steel.

16, 28, 29, 34, 41, 45, 73, 84, 137, 176, 248, 263, 324, 329, 349, 377, 434, 435, 466, 547, 560, 571, 606.

Iron and steel, cts per lb:

Refined iron bars and steel bars, ordinary sizes. Angles, ordinary sizes, T shapes. Beams and channels, structural shapes. Tank plates, structural plates. Bessemer machinery steel.	1¼ to 2¼.

Steel rails, $28 per ton of 2240 lbs. Old, $15.

1.431 Sheet and Plate Iron and Steel.

38, 41, 44, 73, 84, 189, 263, 324, 329, 382, 395, 435, 463.2, 471.5, 673, 675, 681.

Galvanized iron sheets:

Discount, 60% to 80%.

Gage,	14 to 17	22	25	28	29	30
Cts per lb,	12 to 13	14	16	17	19	21

Extra, for additional widths, 36 to 48 in, 1 to 4 cts per lb.

Black iron, gage 16, 3 cts per lb; gage 28, about 4 cts.

1.44 Bar Steel.

45, 84, 98, 189, 283, 301, 324, 329, 331, 342, 429, 435, 466, 518, 560, 612, 662.

1.45 Fastenings.

28, 309.

1.451 Nails and Spikes.

40, 44, 45, 170.6, 202, 283, 329, 356, 434, 518, 621, 640.

Nails, etc., cts per lb: Cut, 2 to 2¼; wire, 2½.

Spikes, railway, 1¾ to 2 cts per lb.

1.452 Rivets.

28, 40, 81, 123, 154, 170.6, 262, 305, 310, 329, 356, 491, 529, 621.

1.453 Screws.

40, 81, 518, 529.

1.454 Bolts and Nuts.

28, 34, 40, 152, 237, 305, 309, 329, 356, 431, 518, 519, 529.

Bolts and nuts for machines, price per 100, square or button heads. Length under head, 2 ins. Discount, 70% to 75%. See list, p. 884.

Diameter, ins.	¼	½	¾	1
Price per hundred	$1.78	$3.86	$7.70	$16.00
Extra per in over 2 ins.	0.16	0.52	1.00	1.80

1.455 Turnbuckles.

157, 409, 518.

Open, price each. Discount, 67%.

Rod, ins.	½	1	2
With ends	$0.80	$1.60	$5.35
Without ends	0.60	1.10	3.10

With upset ends, 30% extra.

1.456 Washers.

163, 305, 356, 529.

1.457 Chains.

28, 96, 100, 150, 262, 329, 398, 462, 585.

American coil chain:

Inch	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$ to $1\frac{1}{2}$
Cts per lb	8	6	4	3.5

1.46 Tubes.

10, 321, 407, 424, 434, 556, 562. See also 9.25, etc.

1.47 Wire, Wire Rope, and Fencing.

44, 332, 342, 355, 369, 384, 407, 530.5, 621, 623, 646.

Hoisting and conveying machinery, see 3.5, etc.

1.471 Wire.

44, 342, 407, 621, 623.

Cts per lb:

	IRON.	TINNED.	CAST STEEL.
Nos. 0 to 9	2.5	3.7	8.0
No. 18	4.0	4.6	13.0

1.472 Wire Fencing.

332, 355.

For the simpler patterns, 30 cts to $1 per rod ($16\frac{1}{2}$ ft), according to style and finish.

1.473 Wire Rope.

36, 44, 369, 384, 646. See pages 976, 977. Discount, 30% and $7\frac{1}{2}$%; galvanized, 25% and $7\frac{1}{2}$%.

1.5 Other Metals and Alloys.

15, 46, 77, 407, 459, 471.5, 483, 609.

Roll and sheet brass, random lengths. Discount, 20% to 30%.

Width in ins	2 to 18	18 to 24	24 to 32	32 to 40
Cts per lb.	22 to 30	29 to 39	36 to 49	50 to 75

Extra quality, 4 to 7 cts per lb extra. Bronze metal, 7 cts per lb extra.

Soft sheet copper, 20 to 23 cts per lb net for 16 ozs to the sq ft in thickness, base sizes.

Lead, pig, 4.3 to 4.4 cts per lb; sheet, 6 cts. Sheet zinc, 6 to 7 cts per lb. Spelter, 3.75 to 4 cts per lb. Antimony, 9 to 11 cts per lb. Nickel, 55 to 60 cts per lb. Mercury, about $1.50 per lb in large lots.

Tin plates, per box of 112 lbs:

American charcoal plates, $6 to $8.

American coke plates, Bessemer, $5 to $7.

American terne plates, per box of 224 lbs, $10 to $12.

1.6 Paper.

Tar, 2 cts per lb, 50 to 70 cts per roll of 108 sq ft.

Tarred felt, $2\frac{1}{2}$ cts per lb. Straw paper, $1\frac{1}{4}$ cts per lb.

Rosin sized sheathing, 30 to 60 cts per roll of 500 sq ft.

1.7 Ropes, etc.

36, 246.

Rope, cts per lb: Manila, plain or tarred, 10 to 12; Sisal, 7 to 9; hay, 7 to 8; cotton, 9 to 14; jute, 6 to 7.

Twine, 7 to 10. Oakum, best, 6 to 7.

1.8 Packing, Gasket, Belting, Lagging, etc.

25, 90, 256, 325, 334, 459, 632.

Belting, rubber, cts per in of width, per ft of length: 2-ply, 7.5; 5-ply, 13.5; 8-ply, 21.

Leather:

Width, ins	1	6	12	24	72
Per ft	$0.12	$0.92	$1.88	$4.20	$15.00

Pipe jointing supplies, see 3.26.

2.0 Constructions.

Water-works supplies, see 9.2. Railroad supplies, 9.1.

2.1 Earthwork, Dredging, Foundations.

2.11 Excavation and Embankment.

See also pp. 800, etc.
Clearing and grubbing, $50 to $150 per acre.
Earth excavation, 20 cts to $1 per cu yd. In trenches, $1 to $5, according to depth.
Pipe trenching, see also pp. 658, 659.
Embankment, 50 cts to $1.50 per cu yd. Rolled, $1 to $2.
Cut and fill, 10 to 25 cts per cu yd.
Sodding, 20 to 40 cts per sq yd.
Rock excavation, $1 to $6 per cu yd, depending on hardness of rock, etc.
Rock filling, $1 to $4 per cu yd.

2.12 Dredging.

10 cts to $1 per cu yd, according to length of haul, etc.
See also pp. 631, 632.

2.13 Foundations.

402, 419, 539, 600.
Piles, see 1.23.

2.2 Masonry.

2.21 Brick Masonry.

$10 to $20 per cu yd. $5 to $10 per 1000 + cost of bricks.
Bricks, see 1.351.

2.22 Stone Masonry.

Dollars per cu yd: Rubble, dry, 2 to 5; in cement, 3 to 6.
Ashlar, 7 to 30. Granite coping, 30 to 45.
Plain cellar work, 3 to 4.
Stone, see 1.32.

2.23 Concrete and Cement Masonry.

In place, $4 to $10 per cu yd.
Cement, see 1.34. Floors and sidewalks, 2.4.

2.24 Plastering.

Three-coat work, 25 to 50 cts per sq yd.

2.3 Metal Structures.

146, 215, 600, 603.

2.31 Bridges.

28, 70, 75, 91, 143, 146, 160, 161, 167, 190, 215, 216, 218, 328, 339, 344, 378, 391, 394, 402, 419, 426, 439, 465, 466, 471, 472, 474, 482, 526, 539, 542, 544, 598, 600, 603, 638, 639, 642.5.
Riveters, see 3.27. Foundations, 2.13.

2.32 Turntables.

28, 91, 167, 190, 344, 394, 419, 466, 474, 482, 617, 639, 672.

2.33 Tanks, Stacks, etc.

64, 126, 146, 228, 250, 266, 302, 335, 463.4, 525, 542, 554, 615, 617, 633, 648.
Pipes, see 9.25. Wind mills, 3.44.
Stand-pipe, 22 × 60 ft, including foundations, $4000 to $7000.

2.34 Boilers.

21, 56, 180, 257, 266, 288, 294, 302, 335, 464, 467, 525, 545, 579, 617, 638.
Boilers. Upright tubular. Price with base and fixtures:

H P	4	12	30	50
Dollars	150	225	375	550

Riveters, see 3.27. Engines, 3.3.

2.35 Fire-proofing, Concrete-metal Construction.

50, 142, 223, 406, 410, 411, 530, 583.

Concrete-metal construction: Concrete and wire, flooring, 15 to 25 cts per sq ft, exclusive of floor beams. For covering columns and girders, 10 cts per sq ft; if concreted, 15 cts. Walls, 70 cts to $1.40 per sq yd. Wall furring, 40 to 50 cts per sq yd. Wire lathing, 12 to 20 cts per sq yd.

2.36 Wharves, Docks, Harbor Improvement.

212, 330, 402, 533.

Excavators, etc., see 3.6. Cement, 1.34.

Dredging, 2.12.

2.4 Paving.

411, 583.

Per sq yd. Macadam, 65 cts to $1; brick, $1 to $2; Belgian block, $2 to $3; asphalt, $2 to $3.

Cellar floors, $1.25 to $2; sidewalks, $3 to $5.50. Cement, see 1.34.

2.5 Sewers.

From 6 to 18 ins diameter, dollars per ft run:

Depth, ft ...	5	10	13	15	17	20
	0.30 to 0.90	0.65 to 1.00	0.90 to 1.20	1.30 to 1.40	1.50	2.00

Laying sewer pipe, cts per ft, exclusive of excavations: 15 in, 30 to 60; 4 in, 20 to 30.

Brickwork in sewers, $8 to $12 per cu yd.

2.6 Chimneys.

181.

2.7 Roofing.

Slate, 7 cts and upward per sq ft. Slag, 4 cts. Tin, 6 to 8 cts. Shingle, 10 cts.

Skylights, complete and erected, 50 cts per sq ft and upward.

3.0 Machinery.

Testing machines, see 4.1. Surveying instruments, 4.0. Miscellaneous, 9.0.

3.1 Electrical Machinery.

253, 267, 454, 591, 660.

Power transmission, mechanical, see 3.51. Blasting apparatus. See 3.231.

3.2 Tools, Machine Tools.

Road-making machinery, see 3.68. Pile drivers, 3.66

3.21 Small and Wood Tools.

53, 240, 498, 589.

Shovels, see 3.62.

3.22 Machine Tools.

103.5, 375, 451, 498, 552.5, 674.

3.23 Drills.

32, 121, 148, 159, 194, 306, 315, 346, 400, 478, 481, 510, 531, 596, 605.

Air compressors, see 3.43. Explosives, 1.14.

Rock drills, percussion:

Diam cylinder, ins	$1\frac{7}{8}$	$3\frac{1}{8}$	$4\frac{1}{2}$
Length stroke, ins	$3\frac{3}{4}$	$6\frac{1}{4}$	$7\frac{1}{2}$
Depth hole, ft	$1\frac{1}{2}$	10 to 15	20 to 30
Bottom diam hole, ins	1	$1\frac{1}{4}$	$2\frac{1}{4}$
Boiler H P required	3	10	15
Price, complete,	$150	$300	$450

Rock drills, diamond (rotary). Discount, 10%.

Depth hole, ft	4000	1500	1000	600	400
Diam hole, ins	$2\frac{9}{16}$	$2\frac{1}{16}$	$2\frac{1}{16}$	$1\frac{9}{16}$	$1\frac{9}{16}$
Diam core, ins	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	1
Boiler H P required	25	15	12 to 15	10	hand
Card price, dollars	4000	2500	1900	1400	425
Pump, extra, dollars		3400	2800	1900	

3.231 Blasting Machinery.

346, 492.

Blasting machine:

To fire 20 to 30 holes	$25
To fire 75 to 100 holes	75

Connecting wire, 40 cts per lb.
Leading wire, 1½ cts per ft.

3.24 Presses.

172, 242.5, 645.

3.25 Punches, Shears.

74, 375.

3.26 Pipe-cutting, Tapping, and Jointing Machinery.

3.261 Cutters.

404, 541.

3.262 Tappers.

224, 466.5, 563.

For dry pipe, $20 to $30 each. For pipes under pressure, $100 to $200 each.

3.263 Jointers.

643.

2-in, $2 each; 12-in, $9; 36-in, $16; 72-in, $44.

3.27 Riveters.

12, 16, 292, 478, 674.

3.3 Engines, Locomotives, Cars.

Pumps, see 3.4. Road rollers, 3.681. Boilers, 2.34.

Portable engines:

H P	Cylinder.	Boiler Diam.	Price.
10	7 × 8 ins	29 ins	$600
25	9 × 10 "	35 "	850
50	13 × 15 "	44 "	1300

3.31 Stationary and Hoisting Engines.

33, 125, 136, 171.4, 200, 204, 231, 234, 241, 251, 257, 259, 333, 354, 370, 397, 427, 512, 536, 545, 572, 602.

Hoisting engines:

Single cylinder, friction drum, but no foot brake, 4 to 6 H P, $250 to $400.

Double cylinder:

	12 H P	20 H P	50 H P
Friction drum and foot brake, or reversible	$600	$1100	$1300
Same, with boiler	1000	2000	3000
2-winch engine	600	100	1300
6-winch engine	900	1400	

Single cylinder stationary engines:

H P	12	30	50	100	200	300
From	$300	$450	$650	$1100	$2000	$3100
To	450	600	800	1400	2000	3400

With base, about 10% extra.

Portable engines on wheels, complete, 6 to 15 H P, $700 to $1500.

3.32 Locomotives.

102, 103, 124, 171.6, 199, 371, 387, 486, 493, 520, 524, 543.

3.33 Gas and Gasoline Engines.

47, 57, 172.2, 184, 247, 415, 440, 463.4, 581, 584, 593, 595, 601, 661, 674.

3.34 Oil Engines.

500.

3.35 Water Engines and Motors, Turbines.

18, 57, 365, 469, 514, 522, 565, 590, 610, 633, 674.

Water-works supplies, see 9.2. Pumps, 3.4.

3.36 Cars.

29, 53.5, 102, 108, 278, 351, 405, 486, 499, 509, 571, 672.

3.37 Wagons.

160, 644.

3.4 Blowing and Pumping Machinery.

3.41 Blowers, Forges.

572, 604.

3.42 Mechanical Draft.

604.

3.43 Air Compressors.

47, 156, 170.2, 315, 346, 353, 400, 443, 457, 510, 552, 567, 594, 605.

3.44 Wind Mills.

250, 633.

3.45 Hydraulic Rams.

207, 265, 497.

Drive pipe:

Diam, ins	1	2	4	6
Dollars, net	5 to 7	10 to 12	35 to 40	60 to 70

Certain manufacturers make much heavier and more elaborate machines at several times these prices.

3.46 Pumps.

20, 58, 59, 62, 82, 83, 129, 182, 183, 186, 187, 188, 200, 208, 221, 222, 231, 261, 279, 284, 296, 297.5, 303, 319.5, 320.5, 345, 348, 353, 359, 396, 423, 503, 504, 507, 508, 522, 532, 538, 551, 555, 567, 568, 572, 594, 674, 676.

Prices in dollars each:

Capacity in gals per min	5	20	100	500	2500	10,000
Single cylinder:						
Boiler feed	50	200	400			
Tank and low lift		150	350	600		
Duplex:						
High pressure			250	750		
Low pressure			150	500		
Centrifugal			50 to 75	80 to 175	200 to 450	800 to 1300

3.5 Hoisting and Conveying Machinery.

234, 257, 344, 370, 373, 522.

Electrical, see 3.1. Excavating machinery, 3.6. Hoisting engines, 3.31. Cars, 3.36. Wire rope, 1.473.

3.51 Power Transmission.

173, 205, 373.

3.52 Cranes, Derricks.

33, 67, 104, 136, 140, 148, 171.4, 204, 293, 314, 333, 344, 674.

3.53 Pulley Blocks and Trucks.

89, 333.

3.54 Elevators, Hoists.

65, 88, 148, 171.4, 322, 370, 373, 390, 441.

Hoisting crabs on winches, ½ to 2½ tons capacity, $35 to $100.

Differential hoists, ½ to 3 tons capacity, $10 to $15 to $40 each.

3.55 Jacks.

148, 211, 455, 645.

3.56 Tramways, Conveyors.

65, 88, 104, 127, 171, 251, 307, 322, 333, 344, 351, 354, 369, 370, 373, 405, 441, 527.

Track, see 9.1. Cars, 3.36. Wire rope, 1.473.

3.6 Excavators, Dredges, Road and General Construction Machinery, etc.

Hoisting and conveying, see 3.5. Wagons, 3.37. Hoisting engines, 3.31. Drills, 3.23. Explosives, 1.14. Excavation and embankment, 2.11.

3.61 Excavators.

67, 107, 333, 389, 405, 613, 618, 678.

3.62 Trenching Machinery.

67, 139, 333, 389, 421, 496.

3.63 Scrapers, Plows, etc.

127, 160, 333, 558, 607, 658.
Wheeled scrapers. Discount, 20%. $40 to $60 each.
Drag scrapers. Discount, 50% to 60%.
Ordinary, $10 to $15 each. Fresno or back, $35 to $40 each.
Plows. Discount 20%.

Horses	2	4	6	8	10	12
Each, dollars,	15 to 30	30	35	40	45	55

Hardpan plow, $70.

3.64 Dredging Machinery.

52, 67, 94, 107, 222, 241, 293, 389, 447, 463, 539, 566.
Pumps, see 3.4.

3.65 Diving and Diving Apparatus.

236, 425, 546.
Air pumps. Discount 10%. Each, $125 to $400, according to depth.
Helmets, $100. Rubber suits, 40. Weights, underwear, line, tubing, repair materials, etc., per outfit, $110 to $145.
Complete outfit: Deep sea, $700 to $750; moderate depths, $550 to $600; shallow water, $350 to $400.

3.66 Pile-driving Machinery.

136, 314, 333, 641.

3.67 Wells and Well-driving Machinery.

47, 171.8, 282, 480, 484, 666.
Drills, see 3.23.

3.68 Road-making Machinery, etc.

13, 39, 160, 545.
Rock drills, etc., see 3.23.

3.681 Road Rollers.

110, 230, 290, 295, 320, 333, 337, 545.

3.682 Concrete Mixers.

164, 171.2, 209, 230, 245, 260, 320, 333, 511, 545.
Each, $225, $500 and upward to about $1250.

3.683 Rock and Ore Crushers.

55, 105, 155, 160, 194, 238, 264, 275, 333, 346, 390, 513, 658, 664, 680.

RECEIVING CAPACITY, INCHES.	CAPACITY, TONS PER HOUR.	H P REQUIRED.	PRICE, DOLLARS.
1½ × 3	——	Hand	30
7 × 10	4 to 6	8 to 10	500
10 × 20	12 to 15	18 to 20	1000
13 × 60	40 to 50	40 to 50	4000
24 × 72	160 to 200	150 to 180	13000

3.7 Heating, Ventilating, and Refrigerating.

26, 318, 604.

4.0 Engineering, Surveying, and Scientific Instruments and Supplies.

14, 68, 72, 99, 120, 196, 201, 239, 243, 285, 297, 304, 340, 347, 350, 364, 367, 386, 473, 501, 506, 523, 528, 569.5, 582, 625, 667, 679.
Prices, see below.

4.1 Testing Machines.

235, 460, 475, 521.

4.2 Surveying Instruments.

Dealers, see 4.0. Prices, see below.

4.21 Transits, Plane Tables, Compasses, etc.

Transits, plain, $150 to $200. Engineers', complete, $200 to $300. Mining, $200 to $300 and $700. Mountain, $150 to $300.

Theodolites and portable alt-azimuth astronomical instruments, $500 to $1000.

Solar attachments, $50 to $70. Sextants, $60 to $150. Pocket sextants, $50.

Plane tables, complete, $150 to $300. Compasses, pocket, $10 to $25.

4.22 Levels.

Engineers' levels, $100 to $200. Hand levels, $5 to $18; usually about $9.

4.23 Rods, Tapes, etc.

381, 450.

Leveling rods, $13 to $16 each. Range poles, $2 to $5.

Tapes, from 5 to 10 cts per ft, depending upon graduation and length.

Chains, 8 to 12 cts per ft.

4.29 Miscellaneous.

Current meters, $65 to $100. Direction meters, $200 to $250. Velocity register and timepiece, $50 to $60.

Hook gages, $15 to $60.

4.3 Computing Instruments.

317.

Planimeters, $25 to $125.

Slide rules, plain Mannheim, $1 to $5; other forms, $1 to $50.

Computing machines, $100 to $300.

4.4 Drawing Instruments and Materials.

24, 201, 243, 340, 386, 506, 569.5, 667.

Drawing instruments, $10 to $30 per set, very elaborate sets as high as $60 and even $100. Drawing pens, $1 to $3 each. Compasses (drawing), $3 to $9. Dividers, $2 to $4.50.

Triangular boxwood scales, ordinary, 12 ins long, $1 to $3.

Metal straight edges, 36 ins long, $4 to $5; shorter, as low as $1.50.

T-squares, 36 ins, with celluloid edges, about $2.25; wood, $1 to $2.40; steel, $7 to $10, usually with adjustable angle.

Triangles, celluloid, 6 ins, 50 cts; 12 ins, $1.10. French curves, celluloid, 50 cts to $1.50.

Protractors, German silver, 4 ins, 50 cts to $1.50; 6 ins, about $3; with arms, 6 ins, $9; 8 ins, $20.

Drawing inks, 25 cts per bottle.

4.5 Heliography.

363, 438, 569.5, 573.

Black-line prints, Le Clère's, 10 cts per sq ft. Nigrosine, 8 cts.

Blue prints, 3 cts per sq ft.

Blue-print paper, 30 ins wide, 10 to 20 cts per yd.

Blue-print frames, hand, from $2 for 10 × 12 ins to $40 for 36 × 60 ins.

4.6 Tents, etc.

428, 637.

Wall tents. Discount, 55%.

SIZE, FT.	HEIGHT, FT.	8-OZ. DUCK.	15-OZ. DUCK.	
9 × 9	7½	$11	$25	
12 × 12	8	15	35	
18 × 24	11	44	95	
24 × 28	13	105	205	roped.
30 × 70	15	265	515	

U S folding canvas cot, $1.50 to $2.

4.7 Drawing Tables.

19, 226, 357.

4.8 Testing Laboratories.

308, 399, 488.

9.0 Miscellaneous Supplies.

9.1 Railroad Supplies.

108, 158, 202, 300, 351, 405, 479, 508.5, 655, 664.5, 672.

Bridges, see 2.31. Earthwork, 2.1. Locomotives, 3.32. Cars, 3.36. Tramways, 3.56.

9.11 Rails.

45, 329, 434, 472, 495.
Steel rails, $28 per ton. Old, $15.

9.12 Joints.

87, 128, 170.8, 329, 461, 653.
1.5 to 2.5 cts per lb.

9.13 Switches, Frogs, and Signals.

60, 158, 225, 286, 472, 509, 587, 611, 628, 655, 664.5, 672.

9.14 Ties.

66, 494, 599, 636.
White oak, 50 to 70 cts each; chestnut, 35 to 45; yellow pine, 45 to 65.
Tie plates, 5 to 15 cts each.

9.15 Spikes.

202, 283, 329, 356, 518, 640. 1¾ to 2 cts per lb.

9.2 Hydraulic Supplies.

Gaskets, see 1.8. Stand pipes, tanks, 2.33. Pipe-cutting, tapping, and jointing machines, 3.26.

9.21 Water-softening Plants.

313, 654.

9.22 Filters.

193, 288, 376, 385, 446, 534, 542.

9.23 Water-wheel Governors.

374.
Water-wheels, see 3.35.

9.24 Water Meters.

109, 122, 244, 298, 432, 436, 487, 590, 614, 629, 676.
All types (except piston; see below):

Nom'l diam, pipe, ins.	⅝ or ¾	1	3	6	12
Dollars each, from	9	18	95	300	1000
" " to	12	26	125	500	1500

Reciprocating piston meters, about 50% more.
Venturi meters, see pp. 532, etc. Current meters, 4.29.

9.25 Pipe.

37, 95, 169, 206, 362, 490, 514.
Pipe-cutting machinery, etc., see 3.26.
Pipe laying, see pp. 658, 659.
Lead- and tin-lined iron pipe:

	Lead-lined, 5 Cts per Lb.	Tin-lined, 10 Cts per Lb.
Discount,	25%	10%
Size in Ins.	Price per Ft.	Price per Ft.
1	$0.30	$0.55
2	0.65	1.35
4	1.72	3.00
6	3.28	4.25

Block tin pipe, 35 cts per lb.
Lead pipe, 5 cts per lb.
Boiler tubes. Discount, 40% to 50%. Prices, see p. 882.
Seamless brass tubes, base price, 20 to 25 cts per lb. Extras, see p. 919.
Spiral riveted pipe, in dollars per linear ft. Discount, 40%.

Diam, Ins.	Black.	Asphalted.	Galvanized.
	Thickness, 0.022 in.		
3	0.20	0.23	0.30
6	0.33	0.39	0.50
12	0.68	0.80	1.05
	Thickness, 0.049 in.		
3	0.34	0.37	0.46
6	0.57	0.63	0.85
12	1.15	1.27	1.65
	Thickness, 0.109 in.		
6	1.25	1.31	1.90
12	2.50	2.62	3.25
24	4.70	4.94	6.25

9.251 Cast-iron Pipe.

29, 162, 203, 254, 270, 392, 416, 630, 640, 647, 674.

Cast-iron water pipe, $24 per ton. Pipe and laying, $30 to $35 per ton. See also pp. 658, 659, 875, 876.

9.252 Steel Pipe.

11, 42, 138, 270, 403, 469, 542.

Steel pipe, 4 cts per lb at works.

9.253 Wrought-iron Pipe.

424, 434.

Wrought-iron pipe, per ft. Discount, 1½-in and smaller, 50%; 2-in and larger, 60%. Prices, see p. 882.

9.254 Wood Pipe.

Prices in cts per ft:

Inner diam, ins.	1	3	6	10	16	30	84
Plain, square	5	15	40				
Plain, round	5	20	45				
Strengthened for							
40 lbs per sq in.	12	25	50				
160 " " " ".	18	30	70				
Woodstave pipe,							
40 lbs per sq in.			50	90	200	350	1200
160 " " " ".			70	140	270		

9.255 Sewer Pipe, etc.

31, 80, 130, 175, 197, 232, 291, 416, 420, 448, 608, 619, 631.

Sewer pipe. Discount, 70 to 80%. Prices, see p. 575.

9.256 Hose.

90.

Water hose, price per in of internal diameter, per ft of length:

2-PLY	4-PLY	6-PLY
$0.333	$0.50	$0.75

Other plies at nearly proportional rates.

Air, hot water, and steam hose, price per in of internal diam, per ft of length:

4-PLY	6-PLY	8-PLY
$0.83	$1.24	$1.66

Other plies at nearly proportional rates.

9.26 Hydrants and Valves.

93, 141, 144, 162, 165, 166, 172.8, 210, 220, 254, 323, 338, 379, 401, 412, 458, 489, 490, 514, 516, 563, 615, 627, 669.

Gate valves, iron body, bronze mounted, bell or spigot ends. Discount, 40%.

Diam, ins.	4	6	12	18	24	36	48
Dollars each	20	30	90	200	350	900	2000

Single gate, iron body, low pressure steam and water. Discount, 40%.

Diam, ins.	4	6	12	18	24	48
Screw ends	$15	$25	$85			
Flange ends	20	30	80	$160	$280	$1600

Double gate, iron body, brass mountings, steam or water. Discount, 55%.

Diam, ins.	2	4	6	12
Screw or flange	$10	$20	$30	$100

Double gate, iron body, composition or bronze mounted, for heavy pressures (about 500 lbs per sq in):

Diam, ins	2	4	6	12	
From	$10	$20	$35	$90	Discount, 75%
To	15	30	60	150	Discount, 60%

Double gate valves, all bronze. Discount, 50%.

Screw ends	$1.40	$2.35	$6.25	$34.00	$76.00
Flange ends	$3.40	4.15	11.00	43.00	88.00

Fire hydrants. Discount, 30%. 5½-in stand pipe, $38; 6½-in, $47.

Compression fire hydrants. Discount, 20%. Length from pavement to bottom of connections, 5 ft. 1 to 2½-in nozzles, $28; 2 to 2½-in, $33. Add or deduct $1 for each 6 ins difference in height from 5 ft. 2½-in nozzles, $2. Frost case, $5.

9.27 Anti-bursting Devices.

48.5.

BUSINESS DIRECTORY.

10 Abbé, Max F.— Mfg. Co., 220 Broadway, New York.
11 Abendroth & Root Mfg. Co., 99 John St., New York.
12 Acme Machinery Co., Bolt and Rivet Headers, Cleveland, Ohio.
13 Acme Road Machinery Co., Frankfort, N. Y.
14 Ainsworth, Wm.— & Sons, Pocket Transit, Denver, Col.
15 Ajax Metal Co., Bearings, Ingots, Philadelphia.
16 Albree, Chester B.— Iron Works, 14 to 30 Market St., Allegheny, Pa
16.5 Albro, E. D.— Co., Hardwood Lumber, Cincinnati, Ohio.
17 Alcatraz Co., Alcatraz Asphalt, Crocker Bldg., San Francisco, Cal.
18 Alcott's High Duty Turbine Water-wheel, Mount Holly, N. J.
19 Alexander, J. S.— Mfg. Co., Grand Rapids, Mich.
19.5 Allis-Chalmers Co., Milwaukee, Wis.
20 Allis, Edward P.— Co., Milwaukee, Wis. See 19.5.
21 Almy Water-tube Boiler Co., 178 to 184 Allens Ave., Providence, R. I.
22 Alpha Portland Cement Co., Easton, Pa.
23 Alsen's German Portland Cement, 143 Liberty St., New York.
24 Alteneder, T.— & Sons, Philadelphia.
25 American Belting and Packing Co., 35 Warren St., New York.
26 American Blower Co., Detroit, Mich.
27 American Brake Shoe Co., Chicago.
28 American Bridge Co., 100 Broadway, New York.
29 American Car and Foundry Co., St. Louis, Mo.
30 American Cement Co., 22 S. Fifteenth St., Philadelphia.
31 American Clay Mfg. Co., Akron, Ohio.
32 American Diamond Rock Drill Co., 120 Liberty St., New York.
33 American Hoist and Derrick Co., St. Paul, Minn.
34 American Iron and Steel Mfg. Co., Lebanon, Pa.
34.5 American Locomotive Co., 25 Broad St., New York.
35 American Lucol Co., 44 Broadway, New York.
36 American Mfg. Co., 65 Wall St., New York.
37 American Pipe Mfg. Co., 112 N. Broad St., Philadelphia.
38 American Pressed Steel Co., 333 Walnut St., Philadelphia.
39 American Road Machine Co., Kennett Square, Pa.
40 American Screw Co., Providence, R. I.
41 American Sheet Steel Co., Battery Park Bldg., New York.
42 American Spiral Pipe Works, 64–66 Wabash Ave., Chicago.
43 American Steam Gage Co., Jamaica Plain, Boston.
44 American Steel and Wire Co., Chicago and New York.
45 American Steel Hoop Co., Battery Park Bldg., New York.
46 American Tin Plate Co., Battery Park Bldg., New York.
47 American Well Works, Aurora, Ill.
48 American Wood Preserving Co., 806 Chestnut St., Philadelphia.
48.5 Anti-bursting Pipe Co., Pneumatic Dome to Prevent Bursting from Freezing, Standard Bldg., Pittsburg, Pa.
49 Ashton Valve Co., Pop Valves, etc., Boston.
50 Associated Expanded Metal Cos., New York, Phila., Chicago, etc.
51 Assyrian Asphalt Co., Chicago.
52 Atlantic Gulf and Pacific Co., Park Row Bldg., New York.
53 Atlantic Works, Incorporated, Wood-working Machinery, Twenty-third and Arch Sts., Philadelphia.
53.5 Atlas Car and Mfg. Co., Cleveland, Ohio.
54 Atlas Portland Cement Co., 143 Liberty St., New York.
55 Austin, F. C.— Mfg. Co., Harvey, Ill.

56 Babcock & Wilcox Co., 29 Cortlandt St., New York.
57 Backus Water Motor Co., Newark, N. J.
58 Bacon Air Lift Co., 100 Broadway, New York.
59 Baldwinsville Centrifugal Pump Works, Syracuse, N. Y.
60 Baltimore Enamel and Novelty Co., Signals, Baltimore, Md.
61 Barber Asphalt Paving Co., 11 Broadway, New York.
62 Barr Pumping Engine Co., Germantown Junction, Philadelphia.
63 Barschall Impregnating Co., 31 Nassau St., New York.
64 Bartlett, Hayward & Co., Baltimore, Md.
65 Bartlett, C. O.— & Co., Cleveland, Ohio.
66 Baxter, G. S.— & Co., 18 Wall St., New York.
67 Beatty, M.— & Sons, Welland, Ontario.
68 Beckmann, L.—, Toledo, Ohio.
69 Belden Machine Co., New Haven, Conn.

70 Bellefontaine Bridge and Iron Co., Bellefontaine, Ohio.
71 Belle City Malleable Iron Co., Racine, Wis.
72 Berger, C. L.— & Sons, Boston.
73 Berger Mfg. Co., Sheet Flooring, Canton, Ohio.
74 Berlin-Erfurt Machine Works, 66 and 68 Broad St., New York.
75 Berlin Iron Bridge Co., East Berlin, Conn.
76 Berry Bros., Limited, Varnishes, Detroit, Mich.
77 Bethlehem Steel Co., South Bethlehem, Pa.
79 Billings & Spencer Co., Hartford, Conn.
80 Blackmer & Post Pipe Co., St. Louis, Mo.
81 Blake & Johnson, Waterbury, Conn.
82 Blake, George F.— Co., 91 Liberty St., New York.
83 Bliss, E. W.— Co., Brooklyn, N. Y.
84 Boker, Hermann — & Co., 103 Duane St., New York.
86 Bonneville Portland Cement Co., 1307 Real Estate Trust Bldg., Phila.
87 Bonzano Rail Joint Co., 22 S. Fifteenth St., Philadelphia.
88 Borden & Selleck Co., Chicago.
89 Boston and Lockport Block Co., 146 Commercial St., Boston.
90 Boston Belting Co., 256 Devonshire St., Boston.
91 Boston Bridge Works, 70 Kilby St., Boston.
92 Bound Brook Stone Crushing Co., Trap Rock, Bound Brook, N. J.
93 Bourbon Copper and Brass Works, 618 and 620 E. Front St., Cin., O.
94 Bowers, A. B.—, Hydraulic Dredging, First Nat. Bank Bldg., San Francisco, Cal.
95 Bowes & Co., 23 Lake St., Chicago.
96 Bradlee & Co., Philadelphia.
97 Bradley Pulverizer Co., Boston.
98 Braeburn Steel Co., Braeburn, Pa.
99 Brandis, F. E.— Sons & Co., 812 Gates Ave., Brooklyn, N. Y.
100 Bridgeport Chain Co., Bridgeport, Conn.
101 Brier Hill Iron and Coal Co., Youngstown, Ohio.
102 Brill, J. G.— Co., Philadelphia.
103 Brooks Locomotive Works, Dunkirk, N. Y. See 34.5.
103.5 Brown & Sharpe Co., Providence, R. I.
104 Brown Hoisting Machinery Co., Incorporated, Cleveland, Ohio.
105 Buchanan, C. G.—, 141 Liberty St., New York.
106 Buckeye Portland Cement Co. Bellefontaine, Ohio.
107 Bucyrus Co., South Milwaukee, Wis.
108 Buda Foundry and Machine Co., Hand Cars, Crossing Gates, Harvey, Ill.
109 Buffalo Meter Co., 363 Washington St., New York.
110 Buffalo Pitts Co., Buffalo, N. Y.
120 Buff & Buff, Transits and Levels, Boston.
121 Bullock, M. C.— Mfg. Co., Diamond Drills, Chicago.
122 Builders' Iron Foundry, Venturi Meter, Providence, R. I.
123 Burden Iron Co., Troy, N. Y.
124 Burnham, Williams & Co., Baldwin Locomotive Works, Philadelphia.
125 Byers, John F.— Machine Co., Ravenna, Ohio.

126 Caldwell, W. E.— Co., Louisville, Ky.
127 California Wire Works, 9 Fremont St., San Francisco, Cal.
128 Cambria Steel Co., Philadelphia.
129 Cameron Steam Pump Works, New York.
130 Camp, H. B.— Co., Akron, Ohio.
131 Canton Steel Co., Tool Steel, etc.. Canton, Ohio.
132 Cape Ann Tool Co., Pigeon Cove, Mass.
133 Carbolineum Wood Preserving Co., 13 Park Row, New York.
134 Carbon Steel Co., Pittsburg, Pa.
135 Carborundum Co., Niagara Falls, N. Y.
136 Carlin's, Thomas— Sons Co., Allegheny, Pa.
137 Carnegie Steel Co., Carnegie Bldg., Pittsburg, Pa.
138 Carroll-Porter Boiler and Tank Co., Pittsburg, Pa.
139 Carson Trench Machine Co., Boston.
140 Case Mfg. Co., Columbus, Ohio.
141 Cayuta Wheel and Foundry Co., Sayre, Pa.
142 Central Fire-proofing Co., Fire-proofing System, 874 Broadway, New York.
143 Champion Bridge Co., Wilmington, Ohio.
144 Chapman Valve Mfg. Co., Indian Orchard, Mass.

145 Chester Steel Castings Co., 407 Library St., Philadelphia.
146 Chicago Bridge and Iron Co., Chicago.
147 Chicago Drop Forging and Foundry Co., Kensington, Ill.
148 Chicago Pneumatic Tool Co., Painting Machines, Casting Cleaners, Chicago.
149 Chicago Portland Cement Co., 513 Stock Exchange Bldg., Chicago.
149.5 Chicago Tie Preserving Co., Chicago.
150 Chillcott-Evans Chain Co., Allegheny, Pa.
151 Chrome Steel Works, Crucible Steel Castings, Brooklyn, N. Y.
152 Church, Isaac—, Expansion Bolts, Toledo, Ohio.
153 Clapp, E. D.— Mfg. Co., Auburn, N. Y.
154 Clark & Cowles, Plainville, Conn.
155 Clark, J. C.—, Atlanta, Ga.
156 Clayton Air Compressor Works, 26 Cortlandt St., New York.
157 Cleveland City Forge and Iron Co., Cleveland, Ohio.
158 Cleveland Frog and Crossing Co., Cleveland, Ohio.
159 Cleveland Twist Drill Co., Lake and Kirkland Sts., Cleveland, Ohio.
160 Climax Road Machine Co., Marathon, N. Y.
161 Clinton Bridge and Iron Works, Clinton, Iowa.
162 Clow, James B.— & Sons, Pipe and Specials, Lake and Franklin Sts., Chicago.
163 Cobb & Drew, Washers, etc., Plymouth, Mass., and Rock Falls, Ill.
164 Cockburn Barrow and Machine Co., 240 Eleventh St., Jersey City, N. J.
165 Coffin Valve Co., Boston.
166 Coldwell-Wilcox Co., Newburgh, N. Y.
167 Columbia Bridge Co., Pittsburg, Pa.
168 Columbian Powder Co., Hamilton Bldg., Pittsburg, Pa.
169 Colwell Lead Co., 63 Center St., New York.
170 Commercial Wood and Cement Co., 156 Fifth Ave., New York.
170.2 Compressed Air Co., 621 Broadway, New York.
170.4 Continental Compressed Air Power Co., Bourse Bldg., Philadelphia.
170.6 Continental Iron Co., Niles, Ohio.
170.8 Continuous Rail Joint Co. of America, 908 Lawyers' Bldg., Newark, N. J.
171 Consolidated Telpherage Co., 20–22 Broad St., New York.
171.2 Contractors' Plant Co., 172 Federal St., Boston.
171.4 Contractors' Plant Mfg. Co., Buffalo, N. Y.
171.6 Cooke Locomotive and Machine Co., Paterson, N. J. See 34.5.
171.8 Cook Well Co., System of Wells, St. Louis, Mo.
172 Cornell, J. B. & J. M.—, Hydraulic Presses, etc., Twenty-sixth St. and Eleventh Ave., New York.
172.2 Cornell Machine Co., 176 W. Superior St., Chicago.
172.5 Corning Brake Shoe Co., Corning, N. Y.
172.8 Crane Co., Chicago.
173 Cresson, George V.— Co., Eighteenth St. and Allegheny Ave., Phila.
174 Crosby Steam Gage and Valve Co., Boston.
175 Crown Fire Clay Co., Akron, Ohio.
176 Crucible Steel Co. of America, Empire Bldg., Pittsburg, Pa.
177 Cumberland Hydraulic Cement and Mfg. Co., Cumberland, Md.
178 Cummer, F. D.— & Son Co., Driers, Cleveland, Ohio.
179 Cummings Cement Co., Ellicott Square Bldg., Buffalo, N. Y.
180 Cunningham Iron Co., Summer, B, and Fargo Sts., Boston.
181 Custodis, Alphons — Chimney Construction Co., Bennett Bldg., New York.

182 D'Auria Pumping Engine Co., Philadelphia.
183 Davidson, M. T.—, 133 Liberty St., New York.
184 Dayton Globe Iron Works Co., Dayton, Ohio.
185 Dayton Malleable Iron Co., Dayton, Ohio.
186 Dean Bros. Steam Pump Works, Indianapolis, Ind.
187 Deane Steam Pump Co., Holyoke, Mass.
188 Deming Co., The—, Pumps, Salem, Ohio.
189 Denman & Davis, Steel Castings, 85 John St., New York.
190 Detroit Bridge and Iron Works, Detroit, Mich.
191 Detroit Lubricator Co., Detroit, Mich.
192 Detroit Steel and Spring Co., Steel Castings, etc., Detroit, Mich.
193 Deutsch, Wm. M.—, New York.
194 Diamond Drill and Machine Co., Birdsboro, Pa.

195 Diamond State Steel Co., Wilmington, Del.
196 Dibble, F. J.—, Weir Gages, Peabody, Mass.
197 Dickey, W. S.— Clay Mfg. Co., Kansas City, Mo.
198 Dixon, Joseph— Crucible Co., Graphite Pipe Joint Compound, etc., Jersey City, N. J.
199 Dickson Locomotive Works, Narrow Gage Locomotives, Scranton, Pa. See 34.5.
200 Dickson Mfg. Co., Scranton, Pa. See 19.5.
201 Dietzgen, Eugene— Co., 149, 151 Fifth Ave., New York.
202 Dilworth, Porter & Co., Limited, Spikes, Rail Braces, etc., Pittsburg.
203 Dimmick Pipe Co., Birmingham, Ala.
204 Dobbie Foundry and Machine Co., 138 Dey St., New York.
205 Dodge Mfg. Co., Mishawaka, Ind.
206 Donaldson Iron Co., Emaus, Lehigh Co., Pa.
207 Douglas, W. & B.—, Middletown, Conn.
208 Downie Pump Co., Deep-well and other Pumps, Downieville, Pa.
209 Drake Standard Machine Works, 298 W. Jackson Boulevard, Chicago.
210 Drummond, M. J.— & Co., 192 Broadway, New York.
211 Duff Mfg. Co., Allegheny, Pa.
212 Dutton Pneumatic Lock and Engineering Co., Yonkers, N. Y.
214 Dupont, E. I.— & Co., Wilmington, Del.

215 Eastern Bridge and Structural Co., Worcester, Mass.
216 Eastern Construction Co., Brooklyn, N. Y.
217 Eastern Paving Brick Co., Catskills, N. Y.
218 Easton Foundry and Machine Co., Easton, Pa.
219 Eccles, Richard—, Auburn, N. Y.
220 Eddy Valve Co., Waterford, N. Y.
221 Edson Mfg. Co., 132 Commercial St., Boston.
222 Edwards, Joseph— & Co., Centrifugal Pumps and Hydraulic Dredging Machinery, 414 Water St., New York.
223 Electric Fire-proofing Co., Fire-proofing System, 119 W. Twenty-third St., New York.
224 Eley, Philip N.—, Bayonne, N. J.
225 Elliot Frog and Switch Co., East St. Louis, Ill.
226 Emerson, F. W.— Mfg. Co., Rochester, N. Y.
227 Empire Portland Cement Co., Warners, N. Y.
228 Enterprise Boiler Co., Youngstown, Ohio.
229 Eppinger & Russell Co., Creosote and Creosoting, 66 Broad St., N. Y.
230 Erie Machine Shops, Asphalt Rollers and Mixers, etc., Erie, Pa.
231 Erie Pump and Engine Co., Erie, Pa.
232 Evens & Howard Fire Brick Co., St. Louis, Mo.
233 Ewald Iron Co., Staybolt Iron, St. Louis, Mo.
234 Exeter Machine Works, Pittston, Pa.

235 Fairbanks, The — Co., Philadelphia.
236 Falcon, Jos. G.—, Submarine Work, Flexible Joint, Evanston, Ill.
237 Falls Hollow Staybolt Co., Cuyahoga Falls, Ohio.
238 Farrel Foundry and Machine Co., Havemeyer Bldg., New York.
239 Fauth & Co., 108 Second St., S. W., Washington, D. C.
240 Fay, J. J.— & Egan Co., Wood Tools, 202 W. Front St., Cin., Ohio.
241 Featherstone Foundry and Machine Co., 1215 Monadnock Block, Chicago.
242 Federal Steel Co., Empire Bldg., New York. See Illinois and Lorain Steel Cos.
242.5 Ferracute Machine Co., Bridgeton, N. J.
243 Ferrari, Guido—, S. E. Cor. Seventh and Chestnut Streets, Phila.
244 Ferris, Walter—, Ferris-Pitot Water Meter, Drexel Building, Phila.
245 Fisher & Saxton, 123 G St., N. E., Washington, D. C.
246 Fitler, Edwin H.— & Co., Rope, Philadelphia.
247 Foos Gas Engine Co., Station F, Springfield, Ohio.
248 Fort Pitt Bridge Works, Pittsburg, Pa.
249 Flagg, Stanley G.— & Co., Small Steel Castings, Philadelphia.
250 Flint & Walling Mfg. Co., Kendallville, Ind.
251 Flory, S.— Mfg. Co., Cableways, Bangor, Pa.
252 Fort Scott Cement Association, Kansas City, Mo.
253 Fort Wayne Electric Works, Fort Wayne, Ind.
254 Fox, John— & Co., Special Castings, Water, Gas, and Flange Pipes, New York.

255 Frankford Steel and Forging Co., Shafting, Ellwood City, Pa.
256 Franklin Mfg. Co., Franklin, Pa.
257 Fraser & Chalmers, Chicago. See 19.5.
258 French, Sam'l H.— & Co., Cements, Plaster, etc., York Ave. and Callowhill St., Philadelphia.
259 Fulton Iron and Engine Works, Detroit, Mich.

260 Garden City Sand Co., 188 Madison St., Chicago.
261 Gardner Governor Co., Quincy, Ill.
262 Garland Chain Co., Rankin Station, Pa.
263 Garry Iron and Steel Roofing Co., 202 Mervin St., Cleveland, Ohio.
264 Gates Iron Works, 650 Elston Ave., Chicago. See 19.5.
265 Gawthrop, Allen— Jr., Wilmington, Del.
266 Gem City Boiler Co., Dayton, Ohio.
267 General Electric Co., Schenectady, N. Y.
268 Georgia Car and Mfg. Co., Logging, etc., Savannah, Ga.
269 General Forging Co., Boonton, N. J.
270 Gillespie, T. A.— & Co., Westinghouse Bldg., Pittsburg, Pa.
271 Gillingham, Garrison & Co., Philadelphia.
272 Gilson Asphaltum Co., Buena Vista Asphalt, Wainwright Bldg., St. Louis, Mo.
273 Glasgow Iron Co., Pressed Shapes, Pottstown, Pa.
274 Goheen Mfg. Co., Canton, Ohio.
275 Good Roads Machinery Co., Kennett Square, Pa.
278 Goodwin Car Co., Dumping Cars Leased, New York and Chicago.
279 Goulds Mfg. Co., Seneca Falls, N. Y.
280 Grand Rapids Malleable Co., Grand Rapids, Mich.
281 Graphite Metal Co., Graphitic Steel Castings, 15 Cortlandt St., New York.
282 Gregory, Elisha—, Artesian Wells, 60 Liberty St., New York.
283 Green Ridge Iron and Spike Works, Spikes, Scranton, Pa.
284 Guild & Garrison, Brooklyn, N. Y.
285 Gurley, W. & L. E.—, Troy, N. Y.

286 Hall Signal Co., 44 Broad St., New York.
288 Hammond Iron Works, Warren, Pa.
289 Harrison Bros. & Co., Incorporated, Thirty-fifth and Gray's Ferry Road, Philadelphia.
290 Harrisburg Foundry and Machine Works, Harrisburg, Pa.
291 Harvey, Moland & Co., Conshohocken, Pa.
292 Havana Bridge Works, Montour Falls, N. Y.
293 Hayward Co., 97 Cedar St., New York.
294 Hazleton Boiler Co., 120 Liberty St., New York.
295 Heisler, Chas. L.—, Erie, Pa.
296 Heisler Pumping Engine Co., Erie, Pa.
297 Heller & Brightly, Spring Garden St. and Ridge Ave., Philadelphia.
297.5 Henion & Hubbell, Deep-well Pumps, 61 N. Jefferson St., Chicago.
298 Hersey Mfg. Co., South Boston.
299 Higley, Keplinger & Co., Canton, Ohio.
300 Hildenbrand, W.—, Abt System of Rack Railways, 1 Broadway, New York.
301 Hobson, Houghton & Co., Drill Rods, 98 John St., New York.
302 Hodge, E.— & Co., East Boston.
303 Holly Mfg. Co., Lockport, N. Y.
304 Holmes, J. W.—, Batavia, N. Y.
305 Hoopes & Townsend, 1330 Buttonwood St., Philadelphia.
306 Howells Mining Drill Co., Plymouth, Pa.
307 Hunt, C. W.— Co., West New Brighton, N. Y.
308 Hunt, The Robert W.— Co., 1121 The Rookery, Chicago.
309 Hurd & Co., Wall Ties, 570 West Broadway, New York.

310 Illinois Steel Co. Cement Department, Steel Portland Cement, The Rookery, Chicago.
311 Independent Powder Co., Terre Haute, Ind.
312 Indianapolis Drop Forging Co., Indianapolis, Ind.
313 Industrial Water Co., 15 Water St., New York.
314 Industrial Works, Bay City, Mich.
315 Ingersoll-Sergeant Drill Co., Park Place, New York.
317 International Arithmachine Co., 141 La Salle St., Chicago.
318 International Cooling Co., 32 Pine St., New York.

319 International Creosoting and Construction Co., Galveston, Texas.
319.5 International Steam Pump Co. (Includes Nos. 82, 187, 303, 348, 353, 568, 676.) 114–118 Liberty St., New York.
320 Iroquois Iron Works, Asphalt Machinery, etc., Buffalo, N. Y.

320.5 Jackson, Byron— Machine Co., Centrif. Pumps, San Francisco, Cal.
321 Janney, Steinmetz & Co., Drexel Bldg., Philadelphia.
322 Jeffrey Mfg. Co., Columbus, Ohio.
323 Jenkins Bros., New York, Philadelphia, Chicago, Boston.
324 Jessop, Wm.— & Sons, 91 John St., New York.
325 Johns, H. W.— Mfg. Co., Packings, 100 William St., New York.
326 Johnson & Wilson, 150 Nassau St., New York.
327 Johnson Cement Coating Co., Coating for Walls, 156 Fifth Ave., New York.
328 Joliet Bridge and Iron Co., Joliet, Ill.
329 Jones & Laughlins, Limited, Third Ave. and Ross St., Pittsburg, Pa.
330 Jones, B. F.—, 147 E. One Hundred and Twenty-fifth St., New York.
331 Jones, B. M.— & Co., 81 Milk St., Boston.
332 Jones National Fence Co., Columbus, Ohio.

333 Kaltenbach & Griess, 1618 Williamson Bldg., Cleveland, Ohio.
334 Keasby & Mattison Co., Lagging, Ambler, Pa.
335 Keeler, E.— Co., Williamsport, Pa.
336 Kelley Island Lime and Transport Co., Lägerdorfer Portland Cement, Cleveland, Ohio.
337 Kelly, O. S.— Co., Springfield, Ohio.
338 Kennedy Valve Mfg. Co., 50 Beekman St., New York.
339 Kenwood Bridge Co., Chicago.
340 Keuffel & Esser Co., 127 Fulton St., New York.
341 Keystone Drop Forge Works; office, Harrison Building, Philadelphia.
342 Kidd Bros. & Burgher Steel Wire Co., McKee's Rocks, Pa.
343 Kilborn & Bishop Co., New Haven, Conn.
344 King Bridge Co., Cleveland, Ohio.
345 Kingsford Foundry and Machine Works, Oswego, N. Y.
346 Kirk, Arthur— & Son, Pittsburg, Pa.
347 Knight, F. C.— & Co., 400 Locust St., Philadelphia.
348 Knowles Steam Pump Works, 91 Liberty St., New York.
349 Koken Iron Works, Koken Bldg., 715 Locust St., St. Louis, Mo.
350 Kolesch & Co., 138 Fulton St., New York.
351 Koppel, Arthur—, Narrow Gage Railway Materials, 68 Broad St., New York.
352 Krause, Wm.— & Sons Cement Co., Portland Cement, 1623 N. Fifth St., Philadelphia.

353 Laidlaw-Dunn-Gordon Co., Cincinnati, Ohio.
354 Lambert Hoisting Engine Co., Newark, N. J.
355 Lamb Fence Co., Adrian, Mich.
356 Lanz, M.— & Sons, Pittsburg, Pa.
357 Laughlin-Hough Co., 30 Broad St., New York.
358 Lawrence Cement Co., 1 Broadway, New York.
359 Lawrence Machine Co., Lawrence, Mass.
360 Lawrenceville Cement Co., 26 Cortlandt St., New York.
361 Lawrence, W. W.— & Co., Pittsburg, Pa.
362 Lead-lined Iron Pipe Co., Lead and Tin-lined Pipe, Wakefield, Mass.
363 Le Clère, F.—, Black-line Prints from Tracings, 21 N. Thirteenth St., Philadelphia.
364 Ledder, G. G.—, 302 Washington St., Boston.
365 Leffel, James— & Co., Automatic Engines, Springfield, Ohio.
366 Lehigh Portland Cement Co., Allentown, Pa.
367 Leitz, A.— Co., 422 Sacramento St., San Francisco, Cal.
368 Leopold, J.— & Co., Granite and Trap Rock, 18 Broadway, New York.
369 Leschen, A.— & Sons Co., 920–922 N. Main St., St. Louis, Mo.
370 Lidgerwood Mfg. Co., 96 Liberty St., New York.
371 Lima Locomotive and Machine Co., Lima, Ohio.
372 Lima Steel Casting Co., Lima, Ohio.
373 Link Belt Engineering Co., Nicetown, Philadelphia.
373.5 Locks and Canals Co., Timber Preservation, Lowell, Mass.
374 Lombard Water-wheel Governor Co., 61 Hampshire St., Boston.

375 Long & Allstatter Co., Hamilton, Ohio.
376 Loomis-Manning Filter Co., 420 Chestnut St., Philadelphia.
377 Lorain Steel Co., Steel Castings, Lorain, Ohio.
378 Louisville Bridge and Iron Co., Incorporated, Louisville, Ky.
379 Ludlow Valve Mfg. Co., Troy, N. Y.
380 Ludowici Roofing Tile Co., 508 Chamber of Commerce Bldg., Chicago.
381 Lufkin Rule Co., Measuring Tapes, Saginaw, Mich.
382 Lukens Iron and Steel Co., Coatesville, Pa.

383 Mack Mfg. Co., New Cumberland, W. Va.
384 Macomber & Whyte Rope Co., 21 South Canal St., Chicago.
385 Maignen Filtration Co., 1310 Arch St., Philadelphia.
386 Manasse, L.— Co., 88 Madison St., Chicago.
387 Manchester Locomotive Works, Manchester, N. H. See 34.5.
388 Manhattan Trap Rock Co., 11 Broadway, New York.
389 Marion Steam Shovel Co., Marion, Ohio.
390 Martin, Wm. R.—, Iron Works, Screens, Lancaster, Pa.
391 Massillon Bridge Co., Massillon, Ohio.
392 Massillon Iron and Steel Co., Massillon, Ohio.
393 McClenahan & Bros., Granite, Port Deposit, Md.
394 McClintic-Marshall Construction Co., Pottstown and Pittsburg, Pa.
395 McCullough Iron Co., Sheet Iron and Steel, Wilmington, Del.
396 McGowen, John H.— Co., Cincinnati, Ohio.
397 McIntosh, Seymour & Co., Auburn, N. Y.
398 McKay, Jas.— & Co., Pittsburg, Pa.
399 McKenna, Chas. F.—, 221 Pearl St., New York.
400 McKiernan Drill Co., 120 Liberty St., New York.
401 McLean, John—, 298 Monroe St., New York.
402 McMullen, Arthur— & Co., 13 Park Row, New York.
403 McNeil, James— & Bro., Twenty-ninth and Railroad Sts., Pittsburg.
404 McWilliams & McConnell, Shamokin, Pa.
405 Mead, John A.— Mfg. Co., 7 Broadway, New York.
406 Melan Arch Construction Co., 13 Park Row, New York.
407 Merchant & Co., Metals, Seamless Tubes, etc., Philadelphia.
408 Meredith, J. P.— Cedar Co., Memphis, Tenn.
409 Merrill Bros., 467 Kent Ave., Brooklyn, N. Y.
410 Merritt & Co., Expanded Metal Construction, Philadelphia.
411 Metalloid Sidewalk Co., Concrete and Expanded Metal Cement Walks, 606 Century Bldg., St. Louis, Mo.
412 Michigan Brass and Iron Works, Detroit, Mich.
413 Michigan Lubricator Co., Detroit, Mich.
414 Michigan Pipe Co., Stand Pipe, Bay City, Mich.
415 Mietz, A.—, 128–138 Mott St., New York.
416 Millar, Charles— & Son Co., Utica Pipe Foundry Co., Utica, N. Y.
418 Milwaukee Cement Co., Milwaukee, Wis.
419 Missouri Valley Bridge and Iron Works, Roofs, Leavenworth, Kan.
420 Monmouth Mining and Mfg. Co., Monmouth, Ill.
421 Moore Mfg. Co., Syracuse, N. Y.
422 Moran Flexible Steam Joint Co., Incorporated, Louisville, Ky.
423 Morris Machine Works, Centrifugal Pumps, Baldwinsville, N. Y.
424 Morris, Tasker & Co., Incorporated, Philadelphia.
425 Morse, A. J.— & Son, Diving Outfits, 140 Congress St., Boston.
426 Mount Vernon Bridge Works, Mt. Vernon, Ohio.
427 Mundy, J. S.—, Newark, N. J.
428 Murray & Co., Tents, 329 S. Canal St., Chicago.

429 Nash, Geo.— & Co., 15 Platt St., New York.
430 Nathan Mfg. Co., 92 Liberty St., New York.
431 National Elastic Nut Co., Milwaukee, Wis.
432 National Meter Co., New York.
433 National Paint Works, Williamsport, Pa.
434 National Tube Co., Pittsburg, Pa., New York, Philadelphia, Boston, Chicago.
435 National Steel Co., Battery Park Bldg., New York.
435.5 National Wood Pipe Co., Los Angeles, Cal.
436 Neptune Meter Co., Jackson Ave. and Crane St., New York.
437 Neuchatel Asphalt Co., Limited, Val de Travers Rock Asphalt, 265 Broadway, New York.
438 New Blue on White Process Co., Cincinnati, Ohio.
439 New England Structural Co., Boston.

440 New Era Iron Works, Gas Engines, Dayton, Ohio.
441 New Jersey Foundry and Machine Co., Overhead Trackage Systems, 26 Cortlandt St., New York.
442 New Orleans Wood Preserving Works, Creosote and Creosoting, New Orleans, La.
443 New York Air Compressor Co., 95 Liberty St., New York.
444 New York and Bermudez Co., Bowling Green Bldg., New York.
445 New York and Rosendale Cement Co., 200 Broadway, New York.
446 New York Continental Jewell Filtration Co., 15 Broad St., New York.
447 New York Dredging Co., Park Row Bldg., New York.
448 New York State Drain Tile and Pipe Works, Albany, N. Y.
449 New York Mastic Works, Seyssel Rock Asphalt, 11 Broadway, New York.
450 Nichols Engineering and Contracting Co., Steel Measuring Tape, etc., 1538 Monadnock Block, Chicago.
451 Niles Tool Works Co., Hamilton, Ohio.
452 Nitro-powder Co., Kingston, N. Y.
453 Norfolk Creosoting Co., Norfolk, Va.
454 Northern Electrical Mfg. Co., Madison, Wis.
455 Norton, A. O.—, 167 Oliver St., Boston.
456 Norton, F. O.— Cement Co., 92 Broadway, New York.
457 Norwalk Iron Works Co., South Norwalk, Conn.
458 Norwood Engineering Co., Florence, Mass.

459 Ogden, J. Edward— Co., Yarn, Pig Lead, 147 Cedar St., New York.
460 Olsen, Tinius— & Co., 500 N. Twelfth St., Philadelphia.
461 100 Per Cent. Splice Co., 803 Land Title Bldg., Philadelphia.
462 Oneida Community, Limited, Niagara Falls, N. Y.
463 Osgood Dredge Co., Albany, N. Y.
463.2 Otis Steel Co., Cleveland, Ohio.
463.4 Otto Gas Engine Works, Philadelphia and Chicago.

463.6 Page-Storms Drop Forge Co., Chicopee Falls, Mass.,
464 Parker Engine Co. of California, 1041 Drexel Bldg., Philadelphia.
465 Parker, Thatcher A.—, Terre Haute, Ind.
466 Passaic Rolling Mill Co., Paterson, N. J.
466.5 Payne, Walter S.— & Co., Fostoria, O.
467 Peck, C. C.—, Rochester, N. Y.
468 Peerless Brick Co., Philadelphia.
469 Pelton Water-wheel Co., 143 Liberty St., New York.
470 Penberthy Injector Co., Detroit, Mich.
471 Penn Bridge Co., Beaver Falls, Pa.
471.5 Pennsylvania Galvanizing Co., Limited, Philadelphia.
472 Pennsylvania Steel Co., Rails, Switches, etc., Steelton, Pa.
473 Pfister, Herm.—, 428 Plum St., Cincinnati, Ohio.
474 Philadelphia Bridge Works, Pottstown, Pa.
475 Philadelphia Machine Tool Co., 443–449 N. Darien St., Philadelphia.
478 Philadelphia Pneumatic Tool Co., Stephen Girard Bldg., Philadelphia.
479 Philadelphia Railway Track Equipment Co., Philadelphia.
480 Phillips & Worthington, 11 Broadway, New York.
481 Phillips Rock Drill Co., 40 N. Seventh St., Philadelphia.
482 Phœnix Bridge Co., 410 Walnut St., Philadelphia.
483 Phosphor-Bronze Smelting Co., Limited, 2208 Washington Ave., Philadelphia.
484 Pierce Well Engineering and Supply Co., 136 Liberty St., New York.
485 Pittsburg Gage and Supply Co., Steam Specialties, Pittsburg and Philadelphia.
486 Pittsburg Locomotive and Car Works, Pittsburg, Pa. See 34.5.
487 Pittsburg Meter Co., East Pittsburg, Pa.
488 Pittsburg Testing Laboratory, Limited, 325 Water St., Pittsburg, Pa.
489 Pittsburg Valve, Foundry, and Construction Co., Empire Bldg., Pittsburg, Pa.
490 Pleuger & Henger Mfg. Co., St. Louis, Mo.
491 Plymouth Mills, Washers, Plymouth, Mass.
492 Pollard, J. G.—, Firing Tools, 141 Raymond St., Brooklyn, N. Y.
493 Porter, H. K.— Co., Light and Compressed Air Locomotives, 531 Wood St., Pittsburg, Pa.
494 Porter Morse Co., Saginaw, Mich.
495 Potomac Steel Co., Light Steel Rails, Times Bldg., Pittsburg, Pa.
496 Potter Mfg. Co., Sewer Machines, Indianapolis, Ind.

497 Power Specialty Co., 126 Liberty St., New York.
498 Pratt & Whitney Co., Hartford, Conn.
499 Pressed Steel Car Co., Pittsburg, Pa.
500 Priestman & Co., Incorporated, Bourse Building, Philadelphia.
501 Prince, L. M.—, 108 W. Fourth St., Cincinnati, Ohio.
502 Prince Mfg. Co., 71 Maiden Lane, New York.
503 Prindle Engineering Co., Centrifugal Pumps, 120 Liberty St., New York.
504 Pulsometer Steam Pump Co., 135 Greenwich St., New York.
505 Purington Paving Brick Co., Galesburg, Ill.

505.5 Q. & C. Co., see 508.5, Railroad Supply Co.
506 Queen & Co., 1010 Chestnut St., Philadelphia.
507 Quimby, Wm. E.—, Screw Pumps, 86 Liberty St., New York.
508 Quimby Engineering Co., Ridge Ave., Philadelphia.

508.5 Railroad Supply Co., Bedford Bldg., Chicago.
509 Ramapo Iron Works, Hillburn, Rockland Co., N. Y.
510 Rand Drill Co., 128 Broadway, New York.
511 Ransome & Smith Co., Brooklyn, N. Y.
512 Rawson & Morrison Mfg. Co., Cambridgeport (Boston), Mass.
513 Raymond Bros. Impact Pulverizer Co., 1402 Monadnock Block, Chicago.
514 Reading Foundry Co., Limited, Reading, Pa.
515 Reese-Hammond Fire Brick Co., 912 Park Bldg., Pittsburg, Pa.
516 Rensselaer Mfg. Co., 1108-9 Monadnock Bldg., Chicago.
517 Repauno Chemical Co., Wilmington, Del.
518 Republic Iron and Steel Co., Chicago.
519 Rhode Island Tool Co., Providence, R. I.
520 Richmond Locomotive Works, Richmond, Va.
521 Riehlé Bros., 1424 N. Ninth St., Philadelphia.
522 Risdon Iron Works, San Francisco, Cal.
523 Ritchie, E. S.— & Sons, Current Meters, Brookline, Mass.
524 Richmond Locomotive and Machine Works, Richmond, Va. See 34.5.
525 Riter-Conley Mfg. Co., 55 Water St., Pittsburg, Pa.
526 Roberts, A & P.— Co., Pencoyd, Pa., 261 S. Fourth St., Philadelphia.
527 Robins Conveying Belt Co., Park Row Bldg., New York.
528 Robinson, Prof. S. W.—, Pitot Meter, Ohio State College, Columbus, Ohio.
529 Rockford Bolt Works, Rockford, Ill.
530 Roebling Construction Co., 121 Liberty St., New York.
530.5 Roebling's Sons Co., John A.—. Trenton, N. J.
531 Rogers, John M.—. Gloucester, N. J.
532 Roots, P. H. & F. M.— Co., Connersville, Ind.
533 Ross, Sanford P.—, Incorporated, 277 Washington St., Jersey City, N. J.
534 Ross Valve Co., Feed Water Filter, Troy, N. Y.
535 Rowland, Wm. & Harvey—, Norway Iron, Frankford, Philadelphia.
536 Ruger, J. W.— Mfg. Co., 300 Chicago St., Buffalo, N. Y.
538 Rumsey & Co., Limited, Centrifugal, Hand, and Power Pumps, Seneca Falls, N. Y.

539 San Francisco Bridge Co., 220 Market St., San Francisco, Cal.
540 Sargent Co., Chicago.
541 Saunders, D.— Sons, Pipe Threading Machinery, Yonkers, N. Y.
542 Scaife, Wm. B.— & Sons, Pittsburg, Pa.
543 Schenectady Locomotive Works, Schenectady, N. Y. See 34.5.
544 Scherzer Rolling Lift Bridge Co., Monadnock Block, Chicago.
545 Scholl, Julian— & Co., 126 Liberty St., New York.
546 Schrader's Son, A.—, Diving Apparatus, 32 Rose St., New York.
547 Schreiber, L.— & Sons Co., Iron Work, Cincinnati, Ohio.
549 Scott, Charles— Spring Co., Philadelphia.
550 Scranton Forging Co., Scranton, Pa.
551 Scranton Steam Pump Co., Scranton, Pa.
552 Sedgwick-Fisher Co., Chicago.
552.5 Sellers, Wm.— & Co., Incorporated, 1600 Hamilton St., Phila.
553 Seward, M.— & Son, New Haven, Conn.
554 Sharon Boiler Works, Sharon, Pa.
555 Shaw, Thomas— (Quimby Engineering Co., Successor), Compound Propeller Pump, 915 Ridge Ave., Philadelphia.

556 **Shelby Steel Tube Co., Cleveland, Ohio.**
557 **Shoemaker, Benj. H.—, Window Glass, etc., 205 N. 4th St., Phila.**
558 **Shuart Grader Co., Oberlin, Ohio.**
559 **Sicilian Asphalt Paving Co., Trinidad Rock Asphalt, Times Bldg., New York.**
560 Singer, Nimick & Co., Incorporated, Pittsburg, Pa.
561 Sipe, James B.— & Co., Allegheny, Pa.
562 Smidth, F. L.— & Co., 66 Maiden Lane, New York.
563 Smith, A. P.— Mfg. Co., Newark, N. J.
564 Smith, Edward— & Co., 45 Broadway, New York.
565 Smith, S. Morgan— Co., York, Pa.
566 Smith, Theo.— & Sons Co., Jersey City, N. J.
566.5 Smith-Vaile Co. See 594.
567 Snider-Hughes Co., Cleveland, Ohio.
568 Snow Steam Pump Works, Buffalo, N. Y.
569 Snyder, Hiram— & Co., 229 Broadway, New York.
569.5 Soltmann, E. G., 125 E. 42d St., New York.
570 Souther, John— & Co., 12 Post Office Square, Boston.
571 Southern Car and Foundry Co., Gadsden, Ala.
572 Southwark Foundry and Machine Co., Philadelphia.
573 Spaulding Print Paper Co., Blue Print Cloth, 44 Federal St., **Boston.**
578 Spirittine Chemical Co., Wilmington, N. C.
579 Springfield Boiler and Mfg. Co., Springfield, Ill.
580 Springfield Drop Forging Co., Springfield, Mass.
581 Springfield Gas Engine Co., Springfield, Ohio.
582 Stackpole & Bro., 39 Fulton St., New York.
583 Stamsen & Blome, Unity Bldg., Chicago.
584 Standard Automatic Gas Engine Co., Oil City, Pa.
585 Standard Chain Co., Pittsburg, Pa.
586 Standard Paint Co., 77 John St., New York.
587 Standard Railroad Signal Co., Troy, N. Y.
588 Standard Silica Cement Co., 66 Maiden Lane, New York.
589 Standard Tool Co., Cleveland, Ohio.
590 Standard Water Meter Co., 22–26 Reade St., New York.
591 Stanley Electric Mfg. Co., Pittsfield, Mass.
592 Starr Brass Mfg. Co., Boston.
593 Stickney, Charles A.— Co., St. Paul, Minn.
594 Stilwell-Bierce & Smith-Vaile Co., Dayton, Ohio.
595 Stover Engine Works, Freeport, Ill.
596 Stow Flexible Shaft Co., Twenty-sixth and Callowhill Sts., **Phila.**
597 Strieby & Foote Co., Newark, N. J.
598 Strobel, C. L.—, 1744 Monadnock Block, Chicago.
599 Strock, S. C.—, 11 Broadway, New York.
600 Structural Engineering Co., 41 Cortlandt St., New York.
601 Struthers, Wells & Co., Warren, Pa.
602 Stuebner, G. L.—, Long Island City, N. Y.
603 Stupp Bros. Bridge and Iron Co., St. Louis, Mo.
604 Sturtevant, B. F.— Co., Boston.
605 Sullivan Machinery Co., 135 Adams St., Chicago.
606 Susquehanna Iron and Steel Co., Columbia, Pa.
607 Syracuse Chilled Plow Co., Syracuse, N. Y.
608 Syracuse Stoneware Co., Syracuse, N. Y.

609 **Tatham & Bros., Fifth and Locust Sts., Philadelphia.**
610 **Taylor, John W.— Co., Mt. Holly, N. J.**
611 **Taylor Signal Co., Buffalo, N. Y.**
612 **Tennessee Coal, Iron, and Railroad Co., Basic Open-hearth Steel, Birmingham, Ala.**
612.5 **Texas Tie and Lumber Co., Timber Preservation, Somerville, Tex.**
613 **Thew Automatic Shovel Co., Lorain, Ohio.**
614 **Thomson Meter Co., 79 Washington St., Brooklyn, N. Y.**
615 **Thompson, J.— & Co., Union Hydraulic Works, Van Horn and Sophia Sts., Philadelphia.**
616 **Thorn Cement Co., 112 Church St., Buffalo, N. Y.**
617 **Tippett & Wood, Philipsburg, N. J.**
618 **Toledo Foundry and Machine Co., Toledo, Ohio.**
619 **Tomkins Brothers, 257 Broadway, New York.**
620 **Tomkins, Calvin—, Broken Stone, 120 Liberty St., New York.**
621 **Townsend, C. C. & E. P.—, Wire, New Brighton, Pa.**
622 **Transue & Williams Co., Alliance, Ohio.**

623 Trenton Iron Co., Trenton, N. J.
624 Troy Malleable Iron Co., Troy, N. Y.

625 Ulmer & Hoff, 224 Champlain St., Cleveland, Ohio.
626 Union Akron Cement Co., 141 Erie St., Buffalo, N. Y.
627 Union Hydraulic Works (J. Thompson & Co.), Philadelphia.
628 Union Switch and Signal Co., Swissvale, Pa.
629 Union Water Meter Co., Worcester, Mass.
630 United States Cast-iron Pipe and Foundry Co., 217 La Salle St. Chicago.
631 United States Clay Mfg. Co., Empire Bldg., Pittsburg, Pa.
632 United States Mineral Wool Co., 143 Liberty St., New York.
633 United States Wind Engine and Pump Co., Batavia, Ill.
634 United States Wood Preserving Co., Creo-resinate and Creosote Process, 29 Broadway, New York.

635 Valentine & Co., Varnishes, New York.
636 Vanderbilt & Hopkins, Lumber, Timber Preservation, 120 Liberty St., New York.
637 Vanderherchen, M. F.— & Co., Vine and Water Sts., Philadelphia.
638 Variety Iron Works Co., Cleveland, Ohio.
639 Virginia Bridge and Iron Co., Roanoke, Va.
640 Virginia Iron, Coal, and Coke Co., Spikes, Pig Iron, Bristol, Tenn.
641 Vulcan Iron Works, Chicago.
642 Vulcanite Portland Cement Co., Vulcanite Bldg., 1710 Market St., Philadelphia.

642.5 Wabash Bridge and Iron Works, Wabash, Ind.
643 Watkins, Thomas—, Johnstown, Pa.
644 Watson Wagon Co., Canastota, N. Y.
645 Watson-Stillman Co., 453 Rookery, Chicago.
646 Waterbury Rope Co., 69 South St., New York.
647 Warren Foundry and Machine Co., Phillipsburg, N. J., 160 Broadway, New York.
648 Warren City Boiler Works, Warren, Ohio.
649 Warner-Quinlan Asphalt Co., Trinidad Asphalt, 4 Warner Bldg., Syracuse, N. Y.
652 Weaver-Hirsh Co., Gray Iron Castings, Allentown, Pa.
653 Weber Railway Joint Mfg. Co., Empire Bldg., 71 Broadway, New York.
654 Wefugo Co., Cincinnati, Ohio.
655 Weir Frog Co., Cincinnati, Ohio.
656 Wells Light Mfg. Co., Wells Light, 44 Washington St., New York.
657 Western Cement Co., Louisville, Ky.
658 Western Wheeled Scraper Co., Aurora, Ill.
659 Westinghouse Air Brake Co., Pittsburg, Pa.
660 Westinghouse Electric and Manufacturing Co., Pittsburg, Pa.
661 Westinghouse Machine Co., Pittsburg, Pa.
662 Westmoreland Steel and Mfg. Co., Tool Steels, Pittsburg, Pa.
663 West Pascagoula Creosote Works, West Pascagoula, Miss.
664 West Pulverising Machine Co., 220 Broadway, New York.
664.5 Wharton Railroad Switch Co., Philadelphia.
665 Wilcox, D.— Mfg. Co., Mechanicsburg, Pa.
666 Williams Bros., Machinists, Ithaca, N. Y.
667 William, Brown & Earle, 918 Chestnut St., Philadelphia.
668 Williams, J. H.— & Co., Brooklyn, N. Y.
669 Williamsport Valve and Hydrant Co., Williamsport, Pa.
670 Wilmot & Hobbs Mfg. Co., Steel, Strip-steel, Crucible-steel, Bridgeport, Conn.
672 Wonham & Magor, Switches, 29 Broadway, New York.
673 Wood, Alan— Co., Sheet Iron and Steel, 519 Arch St., Philadelphia.
674 Wood, R. D.— & Co., 400 Chestnut St., Philadelphia.
675 Worth Bros. Co., Coatesville, Pa.
676 Worthington, Henry R.—, 120 Liberty St., New York.
677 Wyman & Gordon, Worcester, Mass.
678 Wyoming Shovel Works, Wyoming, Pa.

679 Young & Sons, 43 North Seventh St., Philadelphia.
680 Young-Brennan Crusher Co., 111 Hancock St., Brooklyn, N. Y.
681 Youngstown Iron and Steel Roofing Co., Trough Floor, Youngstown, Ohio.

BIBLIOGRAPHY.

The following list of books makes no pretensions to completeness. It aims simply to be usefully suggestive to the general civil engineer. A few works, believed to be specially useful, are designated by single and double stars. The list is arranged according to the Decimal Classification of Melville Dewey. In this classification (see outline below) all subjects are arranged under ten general heads, and to each of these heads is assigned a number in the hundreds place, as Natural Science, 500; Useful Arts, 600; etc. Then each general head is divided into ten sub-heads, to each of which a number in the tens place is assigned. Thus, Natural Science (500) is subdivided into Mathematics, 510; Astronomy, 520; Physics, 530; etc. Again, each of these is subdivided, and decimally numbered, and this successive subdivision and decimal enumeration may be continued indefinitely. To find a subject, it is best to inquire, first, under which of the ten general heads it belongs, then under which sub-head, and so on. Thus, Plane Geometry is seen to belong (1) under Natural Science, 500, (2) under Mathematics, 510, and (3) under Geometry, 513. However, matter on a special subject is often contained in books on a more general subject which embraces the special one. Thus, matter pertaining to Geometry (513) is found in many works which would be classified under Mathematics (510). Conversely, in looking for books on Mathematics, the sub-heads, Algebra, Geometry, etc., should be consulted as well. In general, it is advisable to look under all heads that may contain the information wanted. Thus, information on Locomotives may be found under Mechanical Engineering, 621, or under Railroads, 625. To avoid duplication in such cases, however, one such head has usually been selected, and reference to it made under the other.

Outline of Classification.

500	Natural Science
510	Mathematics
510.8	Tables Math. Insts.
512	Algebra
513	Geometry
514	Trigonometry
515	Descriptive Geometry
516	Analytical Geometry
517	Calculus
519	Least Squares
520	Astronomy
526	Geodesy
526.9	Surveying
530	Physics
531	Mechanics
531.2	Statics
531.3	Dynamics. Kinetics
531.4	Work. Friction.
531.6	Energy
531.7	Power
532	Hydraulics
533	Pneumatics
535–6	Light and Heat
537–8	Elec'y and Magnetism
540	Chemistry
550	Geology
551.5	Meteorology
600	Useful Arts
620	Engineering
620.c	Civil Engineering
620.i	Indexes
620.1	Strength of Materials
621	Mechanical Engin'g
621.1	Steam Engineering
621.13	Locomotives
621.18	Steam Generation
621.2	Water Eng'& Motors
621.3	Electrical Engin'g
621.4	MiscellaneousMotors
621.6	Pumps & Blowers
621.8	TransmissionM'ch'm
622	Mining
624	Bridges and Roofs
624.0s	Specifications
624.a	Trestles. Viaducts
624.2	Girders. Beams
624.3	Trusses
624.6	Arches
624.7	Compound Bridges
624.8	Draw Bridges
625	Roads and Railroads
625.1	Route. Track
625.1a	R.R. Surveying
625.1ac	R.R. Curves
625.1ae	R.R. E'rthwork
625.2	Trains. Cars
625.8	Roads & Pavements
626–7	Hydraulic Engineering
627.8	Dams
628	Sanitary Engineering
628.1	Water Works
628.12–13	Reservoirs
628.15	Pipes
628.16	Purification
628.17	Meters.
660	Chem. Tech'y.Explosives
670	Manufactures. Iron & St'l
690	Building
691	Materials
691.7	Iron and Steel
693	Masonry
694	Carpentry
697	Heating and Vent'g
700	Fine Arts
720	Architecture
721	Arch'l Construction
721.1	Foundations
725	Public Buildings
740	Drawing
770	Photography

Abbreviations.

A	D. Appleton & Co., 1, 3 & 5 Bond St., New York, N. Y.
B	Henry Carey Baird & Co., 810 Walnut St., Philadelphia, Pa.
C G	Charles Griffin & Co., Limited, Exeter Street, Strand, London.
C H	Chapman & Hall, 11 Henrietta St., Covent Garden, London, W. C.
C L	Crosby Lockwood & Son, 7 Stationers' Hall Court, London, E. C.
E N	Engineering News Publishing Co., 220 Broadway, New York, N. Y.
L	J. B. Lippincott Co., Philadelphia, Pa.
L G	Longmans, Green & Co., 91 & 93 Fifth Ave., New York, N. Y.
M	The Macmillan Company, 66 Fifth Ave., New York, N. Y.
R G	Railroad Gazette, 83 Fulton St., New York, N.Y.
S C	Spon and Chamberlain, 12 Cortlandt St., New York, N. Y.
S L	Sampson, Low, Marston & Co., London.
V N	D. Van Nostrand Co., 23 Murray St., New York, N. Y.
W	John Wiley & Sons, 43 & 45 E. 19th St., New York, N. Y.

500 Natural Science.

510 Mathematics.

**Bledsoe, A. T.—. The Philosophy of Mathematics. L.

Hutton, Charles—. Mathematics. 1818.

Merriman, Mansfield— and Robert C. Woodward. Higher Mathematics. 1 vol. 8vo. Cloth. $5.00. W.

510.8 Tables and Mathematical Instruments.

Babbage, Chas.—. Logarithms of Nos. from 1 to 108,000. $3.00. S C.

Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, Reciprocals, to 10,000. $2.50. S C.

Buchanan, E. E.—. Tables of Squares. $1.00. S C.

*Chambers's Mathematical Tables. Logarithms of Numbers from 1 to 108,000, Trig., Nautical and other Tables. 8vo. Cloth. $1.75. V N.

Compton, Alfred G.—. Manual of Logarithmic Computations. 3d ed. 12mo. Cloth. $1.50. W.

Hall, John L.—. Tables of Squares. $2.00. E N.

*Hering, Carl—. Tables of Equivalents of Units of Measurement. W. J. Johnston, New York.

Holman, Silas W.—. Computation Rules and Logarithms. 8vo. Cloth. $1.00. M.

*Hutton, Charles—. Mathematical Tables. London. 1822.

Johnson, J. B.—. Three-Place Logarithmic Tables, Numbers and Trigonometric Functions. 15c. each, $5.00 per 100. W.

Jones, G. W.—. Logarithmic Tables. 8vo. $1.00. M.

Ludlow, Lt., H. H.— and Edgar W. Bass. Logarithmic, Trigonometric, and other Mathematical Tables. 8vo. Cloth. $2.00. W.

Osborn, Frank C.—. Tables of Moments of Inertia and Squares of Radii of Gyration. 175 pp. $3.00. V N and E N.

Pickworth, Charles N.—. The Slide Rule. 12mo. Flexible cloth. $0.80. V N.

Rankine, W. J. M.—. Rules and Tables. C G.

Skidmore and Vidal. Table of Tangents. 4to. Cloth. $2.00. V N.

Unwin, W. C.—. Short Logarithmic and other Tables. Small 4to. Cloth. $1.40. S C.

Tables. See also under subject in question.

Surveying Instruments. See 526.91.

512 Algebra.

Chrystal, G.—. Algebra. Part I. 8vo. $3.75. Part II. 8vo. $4.00. M.

Lübsen, H. B.—. Mathematics Self Taught. Arithmetic and Algebra. Translated and published by H. H. Suplee, 120 Liberty St., New York.

Smith, Charles—. Algebra. $1.90. M.

Todhunter, I.—. A Treatise on the Theory of Equations. $1.80. M.

Todhunter, I.—. Algebra. 16mo. $0.75. M.

Wentworth, G. A.—. Algebra. G.

513 Geometry.

*Chauvenet, William—. Geometry. L.

Halstead, Geo. Bruce—. Elements of Geometry. 8vo. Cloth. $1.75. W.

Wentworth, G. A.—. Geometry. G.

Analytical Geometry. See 516.

Descriptive Geometry. See 515.

---

* ** Believed to be specially useful.

514 Trigonometry.

Todhunter, I.—. Trigonometry. Plane—. M.

Todhunter, I.—. Trigonometry. Spherical—. M.

515 Descriptive Geometry.

See 744. Drawing.

516 Analytical Geometry.

Johnson, W. W.—. Curve Tracing in Cartesian Co-ordinates. 12mo. Cloth. $1.00. W.

Todhunter, I.—. Treatise on Plane Co-ordinate Geometry. $1.80. M.

Todhunter, I.—. Examples of Anal. Geom. of Three Dimensions. $1. M.

517 Calculus.

*Barker, Arthur H.—. Graphical Calculus. 197 pp. 8vo. $1.50. L G.

Greenhill, A. G.—. Differential and Integral Calculus. $2.60. M.

Johnson, W. W.—. An Elementary Treatise on the Integral Calculus. Small 8vo. 243 pp. Cloth. $1.50. W.

Rice, J. M.— and Johnson, W. W.—. An Elementary Treatise on the Differential Calculus. Small 8vo. 485 pp. Cloth. $3.00. Same, abridged, 208 pp. $1.50. W.

Todhunter, I.—. Calculus, Differential—Integral. $2.60 each. M.

Wansbrough, Wm. D.—. The A. B. C. of the Differential Calculus. 12mo. Cloth. $1.50. V N.

519 Probabilities. Least Squares.

Johnson, W. W.—. The Theory of Errors and the Method of Least Squares. 12mo. 182 pp. Cloth. $1.50. W.

Merriman, Mansfield—. The Method of Least Squares. 7th ed. 8vo. Cloth. $2.00. W.

520 Astronomy.

Doolittle, C. L.—. A Treatise on Practical Astronomy. 8vo. 652 pp. Cloth. $4.00. W.

Loomis, Elias—. Practical Astronomy. Harper & Bros., New York.

Young, C. A.—. Astronomy. G.

526 Geodesy.

Comstock, George C.—. A Text-Book of Field Astronomy for Engineers. 8vo. 213 pp. Cloth. $2.50. W.

Gore, J. H.—. Elements of Geodesy. 8vo. Cloth. $2.50. W.

Hayford, John F.—. Geodetic Astronomy. 8vo. Cloth. $3.00. W.

Merriman, Mansfield—. The Elements of Precise Surveying and Geodesy. 8vo. Cloth. $2.50. W.

Least Squares. See 519.

526.9 Surveying.

Burt, W. A.—. Key to the Solar Compass, and Surveyor's Companion. 5th ed. Pocket-book form. $2.50. V N.

Clevenger, S. R.—. A Treatise on the Method of Government Surveying as prescribed by the U. S. Congress and Commissioner of the General Land Office. 16mo. Morocco. $2.50. V N.

Gillespie, W. M.— LL.D. Treatise on Land-Surveying. A.

Gribble, Theodore Graham—. Preliminary Survey and Estimates. 8vo. Cloth. 415 pp. L G.

*Johnson, J. B.—, The Theory and Practice of Surveying. Revised and enlarged. 900 pp. 15th ed. Small 8vo. Cloth. $4.00. W.

Merriman, Mansfield— and John P. Brooks. Hand-Book for Surveyors. 2d ed. 16mo. Morocco. $2.00. W.

Nugent, Paul C.—. Plane Surveying. 8vo. 593 pp. 320 figures. Cloth. $3.50. W.

Plympton, Geo. W.—. The Aneroid Barometer: Its Construction and Use. 18mo. $0.50. V N.

*Williamson, R. S.—. On the Use of the Barometer on Surveys and Reconnoissances. 4to. Cloth. $15.00. V N.

Winslow, Arthur—. Stadia Surveying. 18mo. $0.50. V N.

Railroad Surveying. See also 625.1a.

526.91 Instruments.

Baker, Ira O.—. Engineers' Surveying Instruments. 2d ed. 400 pp. 12mo. Cloth. $3.00. W.

*United States Coast and Geodetic Survey. The Plane Table: Its Uses in Topographical Surveying. 8vo. Cloth. $2.00. V N.

---

* ** Believed to be specially useful.

Webb, W. L.—. Problems in the Use and Adjustment of Engineering Instruments. Revised and enlarged. 16mo. Morocco. $1.25. W.

526.94 Leveling.

Baker, Ira O.—. Leveling: Barometric, Trigonometric, and Spirit. $0.50. V N.

526.98 Topography.

*Haupt, Lewis M.—. The Topographer: His Instruments and Methods. J. M. Stoddard, Phila., 1883.

Reed, Lt., Henry A.—. Topographical Drawing and Sketching. Including Photography Applied to Surveying. Illus. 4th ed. 4to. Cloth. $5.00. W.

Specht, George J.—, A. S. Hardy, John B. McMaster, H. F. Walling. Topographical Surveying. 18mo. $0.50. V N.

Wilson, Herbert M.—. Topographic Surveying, including Geographic, Exploratory and Military Mapping, with Hints on Camping, Emergency Surgery and Photography. 8vo. Cloth. $3.50. W.

530 Physics.

*Barker, George F.—. Physics, Advanced Course.

*Deschanel, A. Privat—. Elementary Treatise on Natural Philosophy. Translated by J. D. Everott. A.

*Ganot. Physics. Translated by E. Atkinson, Ph.D. Wm. Wood & Co., New York.

Tait, P. G.—. Properties of Matter. 3d ed. $2.25. M.

Meteorology. See 551.5.

531 Mechanics.

Ball, Robert Stawell—. Experimental Mechanics. M.

Church, I. P.—. Mechanics of Engineering. 8vo. Cloth. $6.00. W.

Church, I. P.—. Notes and Examples in Mechanics. 135 pp. 8vo. Cloth. $2.00. W.

DuBois, A. Jay—. Mechanics. Vol. I, Kinematics, $3.50; Vol. II, Statics, $4.00; Vol. III, Kinetics, $3.50. 8vo. Cloth. W.

DuBois, A. Jay—. The Mechanics of Engineering. Small 4to. Cloth. Vol. I, 669 pp. $7.50. Vol. II, 632 pp. $10.00. W.

Goodeve, T. M.—. Principles of Mechanics. L G.

Greene, Chas. E.—. Structural Mechanics. 271 pp. $3.00. E N.

*Lanza, Prof. G.—. Applied Mechanics and Resistance of Materials. 7th ed. 8vo. Cloth. $7.50. W.

Mach, Dr. Ernst—. The Science of Mechanics. Translated by Thos. J. McCormack. 12mo. 534 pp. $2.50. Open Court Publishing Co.

*Maxwell, J. Clerk—. Matter and Motion. 18mo. $0.50. V N.

*Merriman, Mansfield—. Text-Book on Mechanics of Materials. 8th ed. 8vo. Cloth. $4.00. W.

Michie, Peter S.—. Elements of Analytical Mechanics. 4th ed. 8vo. Cloth. $4.00. W.

Morin, A.—. Fundamental Ideas of Mechanics and Experimental Data. Revised and translated by Joseph Bennett. 1860. A.

Moseley, Henry—. The Mechanical Principles of Engineering and Architecture. With additions by D. H. Mahan. W.

Nystrom, John W.—. A New Treatise on Elements of Mechanics. 352 pp. 8vo. Cloth. $3.00. B.

Perry, John—. Practical Mechanics. Edited by W. R. Ayrton. 3½ shillings. V N.

Rankine, W. J. M.—. Mechanical Text-Book. 9 shillings. C G.

Rankine, W. J. M.—. Applied Mechanics. 8vo. Cloth. $5.00. C G.

Weisbach, Julius—. Mechanics of Engineering and Machinery. Translated by J. F. Klein. 2d ed. 8vo. Cloth. $5.00. W.

Weisbach, Julius—. Mechanics of Engineering. Translated by Eckley B. Coxe. 1 vol. Large 8vo. 1112 pp. 902 illus. $10.00. V N.

Weisbach, Julius—. A Manual of Theoretical Mechanics. Translated by Eckley B. Coxe. 1100 pp. 8vo. Cloth. $10.00. V N.

Wood, De Volson—. The Elements of Analytical Mechanics. 6th ed. 500 pp. 8vo. Cloth. $3.00. W.

Ziwet, Alexander—. An Elementary Treatise on Theoretical Mechanics—Kinetics—Statics—Dynamics. $2.25 each. In one vol., $5.00. M.

---

* ** Believed to be specially useful.

Hydromechanics. See 532.
Machinery and Applied Mechanics. See also 621.8.
Strength of Materials. See 620.1.

531.2 Statics.

Johnson, L. J.—. Statics by Algebraic and Graphic Methods. 8vo. 141 pp. Cloth. $2.00. W.
*Lock, J. B.—. Elementary Statics. M.
Todhunter, I.—. A Treatise on Analytical Statics. $2.60. M.
Arches. See 624.6.
Testing and Strength of Materials. See 620.1.

531.3 Dynamics. Kinetics.

Garnett, William—. A Treatise on Elementary Dynamics. 6 shillings. G B.
*Lock, J. B.—. Dynamics for Beginners. M.
Tait, P. G.—. Dynamics. $2.50. M.

531.4 Work. Friction.

*Thurston, Robt. H.—. Treatise on Friction and Lost Work in Machinery and Mill Work. 5th ed. 8vo. Cloth. $3.00. W.

531.6 Energy.

Stewart, Balfour—. The Conservation of Energy. 12mo. Cloth. $1.50. A.

531.7 Power.

Flather, J. J.—. Dynamometers, and the Measurement of Power. 394 pp. 12mo. Cloth. $3.00. W.

532 Hydraulics.

Bovey, Henry T.—. A Treatise on Hydraulics. 8vo. Cloth. $5.00. W.
**Coffin, Freeman C.—. The Graphical Solution of Hydraulic Problems. 80 pp. 16mo. Morocco. $2.50. W.
Fidler, T. Claxton—. Calculations in Hydraulic Engineering. 167 pp. 8vo. $2.50. L G.
*Merriman, Mansfield—. A Treatise on Hydraulics. 8vo. Cloth. $5.00. W.
*Unwin, W. C.—. Hydromechanics. Encyclopedia Britannica.
Hydraulic Engineering. See 626 and 627.
Hydraulic Motors. See 621.2.
Waterworks. See 628.1.
See also under 531.

532.5 Fluids in Motion.

Bazin, H.—. Experiments upon the Contraction of the Liquid Vein Issuing from an Orifice. Translated by John C. Trautwine, Jr. 8vo. Cloth. $2.00. W.
*Flynn, P. J.—. Flow of Water in Open Channels, Pipes, Conduits, Sewers, etc. With Tables. 18mo. $0.50. V N.
**Francis, Jas. B.—. Lowell Hydraulic Experiments. 4to. Cloth. $15.00. V N.
Ganguillet, E.— and W. R. Kutter. A General Formula for the Uniform Flow of Water in Rivers and Other Channels. Translated by Rudolph Hering and John C. Trautwine, Jr. 2d ed. 8vo. Cloth. $4.00. W.
*Herschel, Clemens—. One Hundred and Fifteen Experiments on the Carrying Capacity of Large, Riveted, Metal Conduits. 8vo. Cloth. $2.00. W.
Moore, C. S.—. New Tables for the Complete Solution of Ganguillet and Kutter's Formula. 6″ × 9″, 231 pp. 15 shillings. B.
*Weston, Edmund B.—. Tables Showing Loss of Head Due to Friction of Water in Pipes. 170 pp. $1.50. V N and E N.
See also under 628.1, Waterworks.

533 Pneumatics.

Blowers, Pumps. See 621.6.
Meteorology. See 551.5.
See also under 531 and 532.

---

* ** Believed to be specially useful.

535 Light.

Tait, P. G.—. Light. $2.00. M.

536 Heat.

Barr, Wm. M.—. A Practical Treatise on the Combustion of Coal. 307 pp. 8vo. Cloth. $2.50. B.
Garnett, William—. An Elementary Treatise on Heat. 3 s. 6 d. G B.
Maxwell, J. Clerk—. Theory of Heat. 357 pp. 12mo. $1.50. L G.
Tait, P. G.—. Heat. $2.50. M.
*Tyndall, John—. Heat Considered as a Mode of Motion.
Steam. See 621.1.
Heating and Ventilating. See 697.

537-8 Electricity and Magnetism.

Jenkin, Fleeming—. Electricity and Magnetism. 415 pp. 12mo. $1.25. L G.
Electrical Machines and Instruments. See 621.3.

540 Chemistry.

Phillips, Joshua—. Engineering Chemistry. 2d ed. 8vo. Cloth. $4.00. V N.
Remsen, Ira—. Chemistry. Henry Holt & Co., New York.
Sexton, A. H.—. Chemistry of the Materials of Engineering. 12mo. Cloth. $2.50. V N.
Potable Waters. See 628.1.

550 Geology.

Merrill, G. P.—. Rocks, Rock Weathering, and Soils. 8vo. $4.00. M.
Stockbridge, Horace Edward—. Rocks and Soils. Their Origin, Composition, and Characteristics; Chemical, Geological, and Agricultural. With 15 full-page plates. 2d ed. 8vo. Cloth. $2.50. W.
Tarr, R. S.—. Economic Geology of the United States. 8vo. Cloth. $3.50. M.
Tillman, S. E.—. A Text-Book of Important Minerals and Rocks. 8vo. Cloth. $2.00. W.
Building Stones. See 691.2.
Mining. See 622.

551.5 Meteorology.

Ferrel, William—. A Popular Treatise on the Winds. 2d ed. 8vo. Cloth. $4.00. W.
*Russell, Thomas—. Meteorology. 8vo. Cloth. $4.00. M.
Williamson, R. S.—. Practical Tables in Meteorology and Hypsometry in Connection with the Use of the Barometer. 4to. Cloth. $2.50. V N.

600 Useful Arts.

620 Engineering.

Brooks, Robt. C.—. A Bibliography of Municipal Problems and City Conditions. 346 pp. Cloth. $1.50. Reform Club.
Carpenter, R. C.—. Experimental Engineering. 5th ed. Revised 1897. 8vo. Cloth. $6.00. W.
Crehore, Wm. W.—. Tables and Diagrams for Engineers and Architects. $0.25 to $0.50 each. Set, $7.50. E N.
Cross, C. S.—. Engineers' Field Book. 4th ed. 166 pp. $1.00. E N.
Dawson, Philip—. The "Engineering" and Electric Traction Pocket-Book. 1056 pages. 1300 illus. 16mo. Morocco flap. $5.00. W.
Goodhue, W. F.—. Municipal Improvements. 3d ed. 12mo. Cloth. $1.75. W.
Haupt, L. M.—. Engineering Specifications and Contracts. J. M. Stoddard & Co., Phila.
Hurst. Tables and Memoranda for Engineers. Vest-pocket edition. 64mo. Roan. $0.50. S C.
**Johnson, J. B.—. Engineering Contracts and Specifications. $3.00. E N.
Kempe, H. R.—. The Engineer's Year-Book. 670 pp. Crown 8vo. Leather. $3.00. C L.

---

* ** Believed to be specially useful.

*Molesworth and Hurst. The Pocket Book of Pocket Books. 32mo. Russia. $5.00. S C.
Philbrick, P. H.—. Field Manual for Engineers. 16mo. Morocco. $3.00. W.
Rankine, W. J. M.—. Useful Rules and Tables for Engineers and Others. 8vo. Cloth. $4.00. V N.
*Shunk, W. F.—. The Field Engineer. 11th ed. 12mo. Morocco, tucks. $2.50. V N.
Smart, Richard A.—. A Hand-Book of Engineering Laboratory Practice. 290 pp. 12mo. Cloth. $2.50. W.
Wait, John Cassan—. Engineering and Architectural Jurisprudence. 985 pp. 8vo. Cloth, $6.00; sheep, $6.50. W.
Wait, John Cassan—. The Law of Operations Preliminary to Construction in Engineering and Architecture. 720 pp. 8vo. Cloth, $5.00; sheep, $5.50. W.
Weale, John—. A Dictionary of Terms Used in Architecture, Building, Engineering, Mining, Metallurgy, Archæology, the Fine Arts, etc. 5th ed. 12mo. Cloth. $2.50. V N.
Retaining Walls, etc. See 721.1.
River Embankments. See 627.
Railroad Earthwork. See 625.1ac.

620.c Civil Engineering.

Butts, Edward—. The Civil Engineers' Field Book. 16mo. Morocco. $2.50. W.
Mahan, D. H.—. A Treatise on Civil Engineering. Revised by De Volson Wood. New chapter on River Improvements by F. A. Mahan. 8vo. Cloth. $5.00. W.
*Patton, W. M.—. A Treatise on Civil Engineering. 8vo. Half leather. $7.50. W.
Rankine, W. J. M.—. A Manual of Civil Engineering. 20th ed. 8vo. Cloth. $6.50. C G.
Trautwine, John C.—. The Civil Engineer's Pocket-Book. Revised and enlarged by John C. Trautwine, Jr., and John C. Trautwine 3d. 18th ed., 70th thousand. 16mo. Morocco, gilt edges. $5.00. W, C H.
Wheeler, J. B.—. An Elementary Course of Civil Engineering. 8vo. Cloth. $4.00. W.

620.i Indexes.

*Assn. Eng. Soc., John C. Trautwine, Jr., Sec'y. Descriptive Index of Current Engineering Literature. Vol. I, 1884–1891. $5.00. 257 S. 4th St., Phila.
Engineering Magazine. Descriptive Index of Current Engineering Literature. *Vol. II, 1892–1895, $5.00. **Vol. III, 1896–1900. 120 Liberty St., New York.
Galloupe, F. E.—. Index to Engineering Periodicals. 1888–1892. $3.00. R G.

620.1 Strength of Materials.

Baker, B.—. On the Strengths of Beams, Columns, and Arches. V N.
Bovey, Henry T.—. Theory of Structure and Strength of Materials. 3d ed. 830 pp. 8vo. Cloth. $7.50. W.
Burr, W. H.—. Elasticity and Resistance of Materials of Engineering. 6th ed., Re-written. 772 pp. 8vo. Cloth. $7.50. W.
Johnson, J. B.—. The Materials of Construction. Large 8vo. 810 pp. Cloth. $6.00. W.
Martens, Adolph—. Hand-Book on Testing Materials. Part I: Methods, Machines and Auxiliary Apparatus. Translated by Gus. C. Henning, M.E. 2 vols. 8vo. Cloth. $7.50. W.
*Merriman, Mansfield—. The Strength of Materials. 2d ed. 12mo. Cloth. $1.00. W.
Moore and Kidwell. Tables of Safe Loads for Wooden Beams and Columns. 57 pp. $0.50. E N.
Spangenburg, Ludwig—. The Fatigue of Metals under Repeated Strains. Translated. 18mo. $0.50. V N.

---

* ** Believed to be specially useful.

Thurston, Robert H.—. A Text-Book of the Materials of Construction 8vo. 785 pp. Cloth. $5.00. W.

Unwin. W. C.—. The Testing of Materials of Construction. 500 pp. 8vo. $6.00. L G.

Wood, De Volson—. A Treatise on the Resistance of Materials, and an Appendix on the Preservation of Timber. 7th ed. 8vo. Cloth. $2.00. W.

See also 721, Architectural Construction; 624, Bridges; 691, etc.; 532, Mechanics; 532.2, Statics.

621 Mechanical Engineering.

Clark, D. Kinnear—. The Mechanical Engineer's Pocket-Book of Tables, Formulæ, Rules, and Data. 3d ed. 700 pp. 8vo. Leather. 6 shillings. C L.

Clark, D. Kinnear—. A Manual of Rules, Tables, and Data for Mechanical Engineers. 1012 pp. 6th ed. 8vo. Cloth. $5.00. V N.

Kennedy, Alex. B. W.—. The Mechanics of Machinery. M.

**Kent, William—. The Mechanical Engineer's Pocket-Book. 1100 pp. 6th ed. 16mo. Morocco. $5.00. W.

Lockwood's Dictionary of Terms Used in the Practice of Mechanical Engineering. Edited by Joseph G. Horner. 6000 definitions. 8vo. Cloth. 7 s. 6 d. C L.

Rankine, W. J. M.—. A Manual of Machinery and Millwork. 580 pp. C G.

**Reuleaux, F.—. The Constructor. Translated by H. H. Suplee. $7.50. 312 pp. 1200 illustrations. Eng. Mag., 120 Liberty St., New York.

*Unwin, W. C.—. Elements of Machine Design. Part I, General Principles, 476 pp. $2.00. Part II, Engine Details, 306 pp. $1.50. L G.

Weisbach, Dr. Julius— and Prof. Gustav Herrmann. The Mechanics of Hoisting Machinery, including Accumulators, Excavators and Pile Drivers. Translated by Karl P. Dahlstrom. 8vo. Cloth. 332 pp. 177 illus. $3.75. M.

621.1 Steam Engineering.

*Holmes, Geo. C. V.—. The Steam Engine. 12mo. $2.00. L G.

*Pray, Thomas— Jr. Steam Tables and Engine Constants. Compiled from Regnault, Rankine, and Dixon. 8vo. Cloth. $2.00. V N.

Rankine, W. J. M.—. A Manual of the Steam Engine and other Prime Movers. C G.

Spangler, H. W.—; Greene, W. M.—; Marshall, S. M.—. The Elements of Steam Engineering. 8vo. Cloth. 273 pp. $3.00. W.

Thurston, Robert H.—. Handy Tables. From the "Steam-Engine Manual." 8vo. Cloth. $1.50. W.

Thurston, Robert H.—. A Manual of the Steam-Engine. Part I, Structure and Theory; Part II, Design, Construction, Operation. Each part, 8vo, 1000 pp., $6.00. Two parts, $10.00. W.

Whitham, Jay M.—. Steam-Engine Design. 8vo. Cloth. $5.00. W.

621.13 Locomotives.

**Forney, Mathias N.—. Catechism of the Locomotive. $3.50. V N & R G.

Meyer, J. G. A.—. Modern Locomotive Construction. 4to. Cloth. $10.00. W.

Reagan, H. C.—. Locomotives: Simple, Compound and Electric. 12mo. 617 pp. Cloth. $2.50. W.

"Railroad Gazette." Modern Locomotives. $7.00. R G.

621.18 Steam Generation.

Barr, Wm. M.—. A Practical Treatise on High Pressure Steam Boilers. 456 pp. 8vo. Cloth. $3.00. B.

*Heine Safety Boiler Co. "Helios." (Tables.) Published by author.

Peabody, Cecil H.— and Miller, Edward F.—. Steam Boilers. 8vo. 384 pp. Cloth. $4.00. W.

Thurston, R. H.—. A Manual of Steam-Boilers: Their Designs, Construction, and Operation. 5th ed. 8vo. Cloth. $5.00. W.

*Wilson, Robert—. A Treatise on Steam-Boilers. Enlarged and Illustrated by J. J. Flather. 3d ed. 12mo. Cloth. $2.50. W.

Steam Heating. See 697.

---

* ** Believed to be specially useful.

621.2 Water Engines and Motors.

**Francis, Jas. B.—. Lowell Hydraulic Experiments. 4to. Cloth. $15.00. V N.
*Frizell, J. P.—. Water Power. 584 pp. 8vo. Cloth. $5.00. W.
Weisbach, Julius—. Hydraulics and Hydraulic Motors. Translated by A. J. Du Bois. 2d ed. Illus. 8vo. Cloth. $5.00. W.
Wood, De Volson—. Turbines. Old ed., 8vo, cloth, $1.00; 2d ed., revised and enlarged, 8vo, cloth, $2.50. W.
See also 532. Pumps, 621.6.

621.3 Electrical Engineering.

*Foster, H. A.—. Electrical Engineer's Pocket-Book. V N.
Haupt, Herman—. Street Railway Motors. 213 pp. 12mo. Cloth. $1.75. B.
Rosenberg, E.—, Haldane Gee, W. W.—, Kinzbrunner, Carl—. Electrical Engineering. 8vo. 275 pp. Cloth. $1.50 net. W.
See also 537.

621.4 Air, Gas, and Other Motors.

Goldingham, A. H.—. The Design and Construction of Oil Engines. 196 pp. $2.00. S C.
*Kennedy, A. B. W.— and W. C. Unwin. Transmission by Air-Power. 18mo. $0.50. V N.
Richards, Frank—. Compressed Air. 12mo. Cloth. $1.50. W.
Saunders, W. L.—. Compressed Air Production. 58 pp. $1.00. E N.
Wolff, A. R.—. The Windmill as a Prime Mover. 2d ed. 8vo. Cloth. $3.00. W.

621.6 Blowing and Pumping Engines.

Barr, Wm. H.—. Pumps. L.
Weisbach, Julius— and Gustave Hermann. The Mechanics of Pumping Machinery. Translated by K. P. Dahlstrom. 8vo. $3.75. M.

621.8 Transmission Mechanism.

Cooper, John H.—. A Treatise on the Use of Belting for the Transmission of Power. 8vo. Cloth. $3.50. B.
Flather, J. J.—. Rope Driving. 12mo. Cloth. $2.00. W.
Kerr, E. W.—. Power and Power Transmission. 8vo. 368 pp. Cloth. $2.00. W.
Stahl, Albert W.—. Transmission of Power by Wire Ropes. 18mo. $0.50. V N.
Principles of Mechanism. See also 531, Mechanics. See also 621, Mechanical Engineering.

622 Mining.

Bowie, Aug. J.— Jr. A Practical Treatise on Hydraulic Mining in California. 5th ed. Small 4to. Cloth. $5.00. V N.
*Drinker, Henry S.—. Tunneling, Explosive Compounds, and Rock Drills. 1000 Illus. 3d ed. 4to. Half-bound. $25.00. W.
Hermann, E. A.—. Steam Shovels and Steam Shovel Work. 60 pp. $1.00. E N.
Ihlseng, M. C.—. A Manual of Mining. 8vo. 585 pp. Cloth. $4.00. W.
Prelini, Charles—. Tunneling. 311 pp. 6" × 9¼". $3.00. V N.
Simms, W. F.—. Practical Tunneling. 4th ed. 8vo. Cloth. $12.00. V N.
Wilson, E. B.—. Hydraulic and Placer Mining. 12mo. Cloth. $2.00. W.
Retaining Walls, etc. See 721.1.
Explosives. See also 660.

624 Bridges and Roofs.

Baker, B.—. Long-Span Railway Bridges. 97 pp. 12mo. Cloth. $1.00. B.
Bender, Charles E.—. Proportions of Pins Used in Bridges. 18mo. $0.50. V N.
Boller, A. P.—. Practical Treatise on the Construction of Iron Highway Bridges. (Written in popular language.) 4th ed. 8vo. Cloth. $2.00. W.

---

* ** Believed to be specially useful.

Boller, A. P.—. The Thames River Bridge. Limited edition. Illus. 4to. Paper. $5.00. W.

Burr, W. H.—. Stresses in Bridges and Roof Trusses, Arched Ribs, and Suspension Bridges. 9th ed. Revised. Plates. 8vo. Cloth. $3.50. W.

Chanute, O.— and George S. Morison. The Kansas City Bridge. 4to. Cloth. $6.00. V N.

*Fidler, T. Claxton—. A Practical Treatise on Bridge-Construction. 2d ed. 8vo. 30 shillings. C G.

*Green, Chas. E.—. Graphics for Engineers, Architects, and Builders. Part I: Roof Trusses. Diagrams. New revised ed. 8vo. Cloth. $1.25. Part II: Bridge Trusses. New revised ed. 8vo. Cloth. $2.50. Part III: Arches in Wood, Iron, and Stone. 3d ed. 8vo. Cloth. $2.50. W.

Johnson, J. B.—, Bryan, C. W.—, Turneaure, F. E.—. The Theory and Practice of Modern Framed Structures. Small 4to. 538 pp. Cloth. $10.00. W.

McMaster, John B.—. Bridge and Tunnel Centers. 18mo. $0.50. V N.

**Merriman, Mansfield— and Henry S. Jacoby. A Text-Book on Roofs and Bridges. Part I: Stresses in Simple Trusses. 5th ed., revised and enlarged. 8vo. Cloth. $2.50. Part II: Graphic Statics. 3d ed., enlarged. With 5 folding plates. 8vo. Cloth. $2.50. Part III: Bridge Design. 3d ed. 8vo. Cloth. $2.50. Part IV: Cantilever, Continuous, Draw, Suspension and Arch Bridges. 2d ed. 8vo. Cloth. $2.50. W.

Morison, George S.—. The Memphis Bridge. $10.00. W.

Ritter, August—. Iron Bridges and Roofs. Translated by H. R. Sankey. S C.

*Waddell, J. A. L.—. De Pontibus. A Pocket-Book for Bridge Engineers. 416 pp. 16mo. Morocco. $3.00. W.

Whipple, S.—. An Elementary and Practical Treatise on Bridge Building. 8vo. Cloth. $3.00. V N.

Wood, De Volson—. A Treatise on the Theory of the Construction of Bridges and Roofs. Illus. 6th ed. Revised and corrected. 8vo. Cloth. $2.00. W.

Retaining Walls, Foundations, etc. See 721.1. See also 531.2, Statics. Architectural Construction. Strength of Materials, see 620.1.

624.0s Specifications for Bridges.

*Bouscaren, G.—. Specifications for Railway Bridges and Viaducts of Iron and Steel. 9 pp. $0.25. E N.

*Cooper, Theodore—. Specifications for Steel Highway Bridges. 25 pp. $0.25. E N.

*Cooper, Theodore—. Specifications for Steel Railroad Bridges. 24 pp. $0.25. E N.

*Osborn Co. Specifications for Metal Highway Bridge Superstructure 12 pp. $0.25. E N.

*Osborn Co. General Specifications for Railway Bridges. 10 pp. $0.25. E N.

*Thacher, Edwin—. General Specifications for Highway Bridges. 8 pp. $0.25. E N.

*Thomson, G. H.—. Standard Specifications for Structural Steel for Modern Railroad Bridges. $0.10. E N.

*Waddell, J. A. L.—. Specifications for Steel Bridges (from "De Pontibus"). 12mo. Cloth. $1.25. W.

624.a Trestles. Viaducts.

*Foster, Wolcott C.—. A Treatise on Wooden Trestle Bridges. 4to. Cloth. 271 pp. $5.00. W.

Katte, W.—. Specifications for Standard Pile and Timber Trestle Bridges. $0.05. E N.

624.2 Girders.

Birkmire, Wm. H.—. Compound Riveted Girders as Applied in Buildings. 8vo. Cloth. $2.00. W.

---

* ** Believed to be specially useful.

Philbrick, P. H.—. Beams and Girders. Practical Formulas for Their Resistance. 18mo. $0.50. V N.
*Stoney, Bindon B.—. The Theory of Stresses in Girders and Similar Structures. 777 pp. 8vo. $12.50. V N.

624.3 Trusses.

Ricker, N. C.—. Elementary Graphic Statics and the Construction of Trussed Roofs. 4th ed. 8vo. Cloth. $2.00. E N.

624.6 Arches.

Buck, G. W.—. Oblique Bridges (Arches). Revised by J. H. W. Buck. With plates. C L.
Cain, William—. Voussoir Arches Applied to Stone Bridges, Tunnels, Culverts, and Domes. 18mo. $0.50. V N.
Howe, Malverd A.—. A Treatise on Arches. 371 pp. 8vo. Cloth. $4.00. W.
Woodbury, D. P.—. Stability of Arches. V N.

624.7 Compound Bridges.

Bender, Charles—. Practical Treatise on the Properties of Continuous Bridges. 18mo. $0.50. V N.

624.8 Draw Bridges.

Wright, Chas. H.—. The Designing of Draw-Spans. Part I: Plate Girder Draws. Part II: Riveted-Truss and Pin-Connected Long-Span Draws. Illus. 8vo. Cloth. $3.50. W.

625 Roads and Railroads.

Berg, Walter G.—. Building and Structures of American Railroads. Large 4to. Cloth. $5.00. V N.
Cleemann, Thos. M.—. The Railroad Engineer's Practice. 4th ed. 12mo. Cloth. $1.50. V N.
Dredge, James—. History of the Pennsylvania Railroad. Engravings, Map, Plates, etc. Folio. Half-morocco, $10.00. Paper, $5.00. W.
Godwin, H. C.—. Railroad Engineer's Field Book. (An Explorer's Guide.) 2d ed. 16mo. Morocco. $2.50. W.
*Henck, John B.—. Field Book for Railroad Engineers. 1896. $2.50. A.
Nagle, J. C.—. A Field Manual for Railroad Engineers. 2d ed. 16mo. Morocco. $3.00. W.
Paine, Charles—. The Elements of Railroading. $1.00. R G.
*Paine, George H.—. The New Roadmaster's Assistant. $1.50. R G.
Vose, G. L.—. Manual for Railroad Engineers. Two vols., text and plates. Lee & Shepard, New York, 1873.
Bridges. See 624.
Tunnels. See 622.
Electric Railways. See 621.3.
Locomotives. See 621.13.

625.1 Route, Track. Fixed Equipment.

Katte, W.—. General Specifications for Cross Ties. $0.05. E N.
Katte, W.—. Specifications for Track-laying. $0.10. E N.
*Parsons, W. B.— Jr. Track. A Complete Manual of Maintenance of Way. 8vo. Cloth. $2.00. E N.
Pratt, Mason D.— and C. A. Alden. Street-Railway Roadbed. 8vo. Cloth. $2.00. W.
Railroad Gazette. Block Signaling. $2.00. R G.
*Tratman, E. E. R.—. Railway Track and Track Work. 502 pp. 200 illus. $3.00. 1901. E N.
*Tratman, E. E. Russell—. Metal Railroad Ties. Report on the Use of. Preservative Processes and Metal Tie-Plates for Wooden Ties. Published by U. S. Department of Agriculture.
Webb, Walter Loring—. Railroad Construction. 16mo. 691 pp. Morocco. $5.00. W.
Railroad Stations. See 725.
Bridges. See 624.

---

* ** Believed to be specially useful.

625.1a R. R. Surveying.

Allen, C. Frank—. Railroad Curves and Earthwork. 194 pp. Pocket-book Form. $2.00. S C.
Brooks, John P.—. Hand-Book of Street Railroad Location. 16mo. Morocco. $1.50. W.
*Gribble, T. G.—. Preliminary Survey and Estimates. 480 pp. 12mo. $2.50. L G.
**Searles, Wm. H.—. Field Engineering. Railway Surveying, Location, and Construction. 16th ed. 16mo. Morocco. $3.00. W.
*Shunk, William F.—. A Practical Treatise on Railway Curves and Location, for Young Engineers. 12mo. Cloth, tucks. $2.00. B.
*Wellington, A. M.—. Economic Theory of Railway Location. 980 pp. $5.00. W & E N.
See also 526.9.

625.1ac R. R. Curves.

Clark, Jacob M.—. A New System of Laying Out Railway Turnouts Instantly, by Inspection from Tables. 12mo. Leatherette. $1.00. V N.
*Crandall, Chas. L.—. The Transition Curve. Revised and enlarged. 16mo. Morocco. $1.50. W.
Fox, Walter G.—. Transition Curves. 18mo. $0.50. V N.
Gieseler, E. A.—. Scales for Turnouts. Stiff cardboard. $0.25. R G.
Howard, Conway R.—. The Transition Curve Field Book. 16mo. Morocco. $1.50. W.
**Searles, Wm. H.—. The Railroad Spiral. The Theory of the Compound Transition Curve Reduced to Practical Formulas and Rules for Application in Field Work. 6th ed. 16mo. Morocco. $1.50. W.
Torrey, A.—. Switch Layouts and Curve Easements. $1.00. R G.
Trautwine, John C.—. The Field Practice of Laying out Circular Curves for Railroads. Revised by John C. Trautwine, Jr. 13th ed. 12mo. Morocco. $2.50. W, C H.

625.1ae R. R. Earthwork.

Allen, C. F.—. Tables for Earthwork Computation. 8vo. Cloth. $1.50. V N.
*Crandall, Chas. L.—. Railway and Other Earthwork Tables. 8vo. Cloth. $1.50. W.
*Hudson, J. R.—. Tables for Calculating the Cubic Contents of Excavations and Embankments by an Improved Method of Diagonals and Side Triangles. New edition with additional tables. 8vo. Cloth. $1.00. W.
Johnson, J. B.—. Stadia and Earthwork Tables. 8vo. Cloth. $1.25. W.
Katte, W.—. Specifications for Grading and Masonry. 16 pp. $0.25. E N.
Taylor, Thomas V.—. Prismoidal Formulæ and Earthwork. 8vo. Cloth. $1.50. W.
Trautwine, John C.—. A Method of Calculating the Cubic Contents of Excavations and Embankments by the Aid of Diagrams. Revised and enlarged by John C. Trautwine, Jr. 9th ed. 8vo. Cloth. $2.00. W.
Trautwine, John C.— Jr. Cross-Section Sheet. To be Used with Trautwine's Excavations. Sheet form. $0.25. W.
Trautwine, John C.— Jr., and Woodson, D. Meade—. Cross-Section Sheet. $0.50. Williams, Brown & Earle, Phila.
Earth Handling. See 622.
Foundations. See 721.1.

625.2 Trains. Rolling Equipment.

*Railroad Gazette. Car-Builder's Dictionary. $5.00. R G.
Locomotives. See 621.13.

625.8 Roads and Pavements.

Aitken, Thomas—. Road Making and Maintenance. Cloth, 7″ x 9″. 440 pp. 139 illus. $6.00. L.

---

* ** Believed to be specially useful.

Baker, Ira O.—. A Treatise on Roads and Pavements. 8vo. 663 pp. Cloth. $5.00. W.
Byrne, Austin T.—. Highway Construction. 8vo. Cloth. $5.00. W.
Gillmore, Q. A.—. Practical Treatise on the Construction of Roads, Streets, and Pavements. 12mo. Cloth. $2.00. V N.
Herschel, Clemens— and E. P. North. Road Making and Maintenance. 156 pp. $0.50. E N.
Spalding, Fred. P.—. A Text-Book on Roads and Pavements. 12mo. Cloth. $2.00. W.
Stone, Gen. Roy—. New Roads and Road Laws in the United States. 200 pp. 12mo. Cloth. $1.00. V N.
Tillson, George W.—. Street Pavements and Paving Materials. 500 pp. 8vo. Cloth. W.

626-7 Hydraulic Engineering.

Flynn, P. J.—. Irrigation Canals, and Other Irrigation Works, etc. Author, San Francisco, Cal.
Hewson, Wm.—. Principles and Practice of Embanking Lands from River Floods, as Applied to the Levees of the Mississippi. 8vo. Cloth. $2.00. V N.
Hill, C. S.—. Chicago Main Drainage Channel. 129 pp. $1.50. E N.
Newell, Frederick Haynes—. Irrigation in the United States. $2.00. Thos. Y. Crowell & Co., New York.
*Starling, Wm.—. Floods of the Mississippi River. 57 pp. $0.50. E N.
*U. S. Geological Survey. Water Supply and Irrigation Papers. About 50 pamphlets have been issued, and more are to follow. 6″ x 9″. U. S. Geol. Surv., Wash., D. C.
Wilson, Herbert M.—. Manual of Irrigation Engineering. 3d ed. Small 8vo. Cloth. $4.00. W.
Retaining Walls, etc. See 721.1.

627.8 Dams.

Gould, E. Sherman—. Specifications for Dams and Reservoirs. 11 pp. $0.25. E N.
Gould, E. Sherman—. High Masonry Dams. 18mo. $0.50. V N.
Leffel, James— & Co. The Construction of Mill Dams. 312 pp. 8vo. Cloth. $2.50. B.
*Wegmann, Edward—. The Design and Construction of Dams. 4th ed., revised and enlarged. 4to. Cloth. $5.00. W.
Reservoirs. See 628.13.

628 Sanitary Engineering.

*Adams, J. W.—. Sewers and Drains for Populous Districts. 5th ed. 8vo. Cloth. $2.50. V N.
*Baker, M. N.—. Sewerage and Sewage Purification. 18mo. $0.50. V N.
*Baumeister, R.—. The Cleaning and Sewerage of Cities. Adapted from the German by J. M. Goodell. 2d ed. 291 pp. 8vo. Cloth. $2.00. V N.
*Folwell, A. Prescott—. Sewerage. The Designing, Construction, and Maintenance of Sewerage Systems. 445 pp. 8vo. Cloth. $3.00. W.
*Kiersted, Wynkoop—. Sewage Disposal. 12mo. Cloth. $1.25. W.
*Merriman, Mansfield—. Elements of Sanitary Engineering. 2d ed. 8vo. Cloth. $2.00. W.
*Ogden, H. N.—. Sewer Design. 234 pp. 12mo. Cloth. $2.00. W.
*Rafter, G. W.— and M. N. Baker. Sewage Disposal in the United States. 598 pp. $6.00. V N & E N.
Rideal, Samuel—. Sewage and the Bacterial Purification of Sewage. 8vo. Cloth. $3.50. W.
Sedgwick, William T.—. Sanitary Science and the Public Health. Cloth. 8vo. $3.00. M.
Swaab, S. M.—. Tables and Diagrams for Making Estimates for Sewerage Work. 20 pp. $0.50. E N.
Waring, Geo. E.— Jr. Modern Methods of Sewage Disposal for Towns, Public Institutions, and Isolated Houses. 2d ed. 260 pp. Cloth. $2.00. V N.
Waring, Geo. E.— Jr. Sewerage and Land Drainage. 3d ed. Quarto. Cloth. $6.00. V N.
Ventilation and Heating. See 697.

---

* ** Believed to be specially useful.

628.1 **Water Works.**

**Baker, M. N.—. Manual of American Water-Works. (Descriptive list of works, with names of officers.) 700 pp. $3.00. E N.

*Baker, M. N.—. Potable Water and Methods of Detecting Impurities. 18mo. $0.50. V N.

Croes, J. J. R.—. Statistical Tables of American Water Works. $2.00. E N.

Fanning, J. T.—. A Practical Treatise on Hydraulic and Water-Supply Engineering. 14th ed. 8vo. Cloth. $5.00. V N.

*Folwell, A. Prescott—. Water-Supply Engineering. Water-Supply Systems. 562 pp. 8vo. Cloth. $4.00. W.

*Fuertes, James H.—. Water and Public Health. 70 figures. 12mo. Cloth. $1.50. W.

*Fuertes, James H.—. Water Filtration Works. 300 pp. Cloth. $2.50. W.

Goodell, John—. Water Works for Small Cities and Towns. 300 pp. Engineering Record.

Gould, E. Sherman—. The Elements of Water Supply Engineering. 168 pp. $2.00. E N.

Mason, William P.—. Water Supply. With special reference to healthfulness of. 8vo. Cloth. $5.00. W.

McPherson, J. A.—. Water Works Distribution. 6" x 8". 154 pp. Illus. Cloth. $2.50. V N.

Nichols, Wm. Ripley—. Water Supply. Considered mainly from a chemical and sanitary standpoint. With plates. 4th ed. 8vo. Cloth. $2.50. W.

*Turneaure, F. E.— and Russell, H. L.—. Public Water Supplies. 8vo. 760 pp. $5.00. W.

Turner, J. H. Tudsbery— and A. W. Brightmore. The Principles of Waterworks Engineering. Large 8vo. Cloth. $10.00. S & C.

Wegmann, Edward— Jr. The Water Supply of the City of New York from 1658–1895. 4to. Cloth. $10.00. W.

Pumps. See 621.6.

Dams. See 627.8.

Flow in Pipes, Channels, etc. See 532.

628.12–13 **Stand Pipes. Tanks. Reservoirs.**

Hazlehurst, J. N.—. Towers and Tanks for Water-Works. 8vo. Cloth. $2.50. W.

Jacob, Arthur—. On the Designing and Construction of Storage Reservoirs. 18mo. $0.50. V N.

*Pence, W. G.—. Standpipe Accidents and Failures. 195 pp. $1.00. E N.

**Schuyler, James Dix—. Reservoirs for Irrigation, Water-Power and Domestic Water Supply. Revised, 1901. 432 pp. Large octavo. Cloth. $5.00. W.

Retaining Walls, etc. 721.1.

See also 627.8, Dams.

628.15 **Pipes.**

Barstow, C. D.—. Cost of Laying Water Pipe. 16 pp. $0.10. E N.

Weston, Edmund B.—. Tables for Estimating the Cost of Laying Cast-Iron Water Pipe. 12 pp. $0.25. E N.

Flow in Pipes. See 532.

628.16 **Purification.**

*Fuertes, James H.—. Water Filtration Works. Cloth. 5" x 8". Illus. $2.50. W.

**Hazen, Allen—. The Filtration of Public Water Supplies. 333 pp. 8vo. Cloth. $3.00. W.

*Hill, John W.—. The Purification of Public Water Supplies. 304 pp. 8vo. Cloth. $3.00. V N.

*Kirkwood, Jas. P.—. Report on the Filtration of River Waters for the Supply of Cities, as Practised in Europe, made to the Board of Water Commissioners of the city of St. Louis. 4to. Cloth. $7.50. V N.

Rideal, Samuel—. Water and Water Purification. Crown 8vo. 7s. 6d. C L.

---

* ** Believed to be specially useful.

628.17 **Use and Waste. Meters.**

Browne, Ross E.—. Water Meters: Comparative Tests of Accuracy, Delivery, etc. 18mo. $0.50. V N.

Kent, W. G.—. The Water Meter. 8vo. Cloth. S C.

630 **Agriculture, Forestry.**

Green, Samuel B.—. Principles of American Forestry. 12mo. Cloth. $1.50. W.

Pinchot, Gifford—. A Primer of Forestry. Bulletin 24, Div. Forestry, U. S. Dept. Agr.

660 **Chemical Technology. Explosives.**

*Eissler, Manuel—. The Modern High Explosives—Nitro-glycerin and Dynamite. 3d ed. Plates. 8vo. Cloth. $4.00. W.

Sanford, P. Gerald—. Nitro-Explosives. 270 pp. 8vo. Cloth. $3.00. V N.

Wisser, John P.—. Explosive Materials. 18mo. $0.50. V N.

Metallurgy. See 670, Manufactures.
See 691, Building Materials.
Explosives. See also 622, Mining.

670 **Manufactures. Iron and Steel.**

*Bauerman, H.—. A Treatise on the Metallurgy of Iron. 515 pp. 12mo. Cloth. $2.00. B.

Bolland, Simpson—. The Encyclopedia of Founding and Dictionary of Foundry Terms Used in the Practice of Moulding. 12mo. Cloth. $3.00. W.

Bolland, Simpson—. "The Iron Founder." Supplement. 400 pp. 12mo. Cloth. $2.50. W.

Campbell, H. H.—. Manufacture and Properties of Structural Steel. The Scientific Pub. Co., N. Y. and London.

Overman, Frederick—. The Manufacture of Steel. 285 pp. 12mo. Cloth. $1.50. B.

West, Thomas D.—. American Foundry Practice. 10th ed. 12mo. Cloth. $2.50. W.

West, Thos. D.—. Moulder's Text-Book, Being Part II of American Foundry Practice. 7th ed. 12mo. Cloth. $2.50. W.

Iron and Steel. See also 691.7, Building Materials.

690 **Building.**

See 721, Architectural Construction.

691 **Materials and Preservatives.**

Byrne, Austin T.—. Inspection of the Materials and Workmanship Employed in Construction. 556 pp. 16mo. Cloth. $3.00. W.

*Johnson, J. B.—. The Materials of Construction. 795 pp. 3d ed. 8vo. Cloth. $6.00. W.

Terry, George—. Pigments, Paint, and Painting. 12mo. Cloth. $3.00. S C.

*Thurston, Robt. H.—. Materials of Construction. 6th ed. 8vo. Cloth. $5.00. W.

Strength of Materials. See 620.1.
See also Tratman, under 625.1.

691.1 **Wood.**

Boulton, S. B.—. The Preservation of Timber by the Use of Antiseptics. 18mo. $0.50. V N.

Snow, Chas. H.—. The Principal Species of Wood. Their Characteristic Properties. Large 8vo. 214 pp. Cloth. $3.50. W.

691.2 **Natural Stone.**

*Merrill, George P.—. Stones for Building and Decoration. Illus. 2d ed. 8vo. Cloth. $5.00. W.

See also 693, Masonry.

---

* ** Believed to be specially useful.

691.3-5 Artificial Stone, Concrete, Cement.

Butler, D. B.—. Portland Cement. 360 pp. $6.00. E N.
*Gillmore, Gen. Q. A.—. Treatise on Limes, Hydraulic Cements, and Mortars. 8vo. Cloth. $4.00. V N.
*Jameson, Charles D.—. Portland Cement. 8vo. Cloth. $1.50. V N.
*Newman, John—. Notes on Concrete and Works in Concrete. 12mo. Cloth. $2.50. S C.
Spalding, Frederick P.—. Hydraulic Cement. 12mo. Cloth. $2.00. W.

691.7 Iron and Steel.

Birkmire, Wm. H.—. Architectural Iron and Steel. 3d ed. 8vo. Cloth. $3.50. W.
Davies, James—. Galvanized Iron: Its Manufacture and Use. 8vo. Cloth. $2.00. S C.
Greenwood, W. H.—. Steel and Iron. 536 pp. 12mo. Cloth. $1.75. B.
Keep, William J.—. Cast Iron. 8vo. 238 pp. Cloth. $2.50. W.
*Metcalf, William—. Steel. 12mo. Cloth. $2.00. W.
Thurston, Robt. H.—. Iron and Steel. 6th ed. 8vo. Cloth. $3.50. W.
See also 670, Manufactures.

693 Masonry.

*Baker, Ira O.—. A Treatise on Masonry Construction. 9th ed. 8vo. Cloth. $5.00. W.
Maginnis, Owen B.—. Bricklaying. 8vo. Cloth. $2.00. V N.
Siebert, Jno. S.— and F. C. Biggin. Modern Stone Cutting and Masonry. 8vo. Cloth. $1.50. W.
Dams. See 627.8.
See also 691.2, Stone.

697 Heating and Ventilation.

*Carpenter, Rolla C.—. The Heating and Ventilating of Buildings. 400 pp. 8vo. Cloth. 4th ed. $4.00. W.

700 Fine Arts.

720 Architecture.

721 Architectural Construction.

Birkmire, Wm. H.—. Skeleton Construction in Buildings. 2d ed. 8vo. Cloth. $3.00. W.
Birkmire, Wm. H.—. The Planning and Construction of High Office Buildings. 8vo. Cloth. $3.50. W.
Black, W. M.—. The United States Public Works. Summary of Methods of Construction, Materials, and Plants under War and Treasury Departments. Ob. 4to. Cloth. $5.00. W.
Bovey, Henry T.—. Theory of Structures and Strength of Materials. 8vo. Cloth. 830 pp. $7.50. W.
Christie, W. Wallace—. Chimney Design and Theory. 12mo. Cloth. $3.00. V N.
Fowler, Chas. E.—. General Specifications for Steel Roofs and Buildings. 12 pp. $0.25. E N.
Freitag, Joseph K.—. Architectural Engineering. High Building Construction. 8vo. Cloth. 2d ed. Re-written. $3.50. W.
Freitag, Joseph K.—. The Fireproofing of Steel Buildings. 8vo. Cloth. $2.50. W.
**Johnson, J. B.—, W. H. Bryan and F. E. Turneaure. The Theory and Practice of Modern Framed Structures. 7th ed. Revised and enlarged. Small 4to. Cloth. $10.00. W.
*Kidder, F. E.—. The Architect's and Builder's Pocket-Book. 1030 pp. 500 engravings. 13th ed. Revised and greatly enlarged. 16mo. Morocco. $4.00. W.
Weyrauch, J. J.—. Strength and Calculations of Dimensions of Iron and Steel Construction, with reference to the Latest Experiments. 12mo. Cloth. $1.00. V N.
Winslow, Benj. E.—. Diagrams for Calculating the Strength of Wood, Steel, and Cast-Iron Beams and Columns. 19 plates. 11¾″ x 9½″. $2.00. E N.
Bridges, 624.
Strength of Materials, 620.1.

---

* ** Believed to be specially useful.

721.1 Foundations.

Baker, Benjamin—. The Actual Lateral Pressure of Earth-Work. 18mo. $0.50. V N.
Fowler, Charles Evan—. The Coffer-Dam Process for Piers. 173 pp. 100 illus. 8vo. Cloth. $2.50. W.
Howe, Malverd A.—. Retaining Walls for Earth. 3d ed., rewritten and enlarged. 12mo. Cloth. $1.25. W.
Jacob, Arthur—. Practical Designing of Retaining-Walls. 2d ed. 18mo. $0.50. V N.
Newman, John—. Earthwork Slips and Subsidences Upon Public Works. 234 pp. 12mo. Cloth. $3.00. S C.
Osborn Co. General Specifications for Bridge Substructure. 10 pp. $0.25. E N.
*Patton, W. M.—. Practical Treatise on Foundations. 429 pp. 8vo. Cloth. $5.00. W.
Wellington, A. M.—. Piles and Pile-Driving. $1.00. E N.
Embankments. See 627.

725 Public Buildings.

Dillenbeck, Clark—. Standard Specifications for Railroad Structures, Brick Passenger Stations, Brick Freight Houses, Frame Passenger Stations, Frame Freight Houses. $0.40 each. E N.

740 Drawing.

*Jacoby, H. S.—. Text-Book on Plain Lettering. 82 pp. $3.00. E N.
Mahan, D. H.—. Industrial Drawing. Revised and enlarged by D. F. Thompson. 30 plates. 8vo. Cloth. $3.50. W.
*Reinhardt, C. W.—. Lettering for Draughtsmen, Engineers, and Students. 32 pp. $1.00. E N.
*Reinhardt, C. W.—. Technic of Mechanical Drafting. 36 pp. $1.00. E N.
Smith, R. S.—. Manual of Topographical Drawing. Revised and enlarged by Chas. McMillan. 12 folding plates. 3d ed. 8vo. Cloth. $2.50. W.
Warren, S. Edward—. Drafting Instruments and Operations. Thoroughly revised, with additions. 12mo. Cloth. $1.25. W.
Warren, S. Edward—. Elements of Plane and Solid Free Hand Geometrical Drawing. Plates and wood-cuts. 12mo. Cloth. $1.00. W.
Descriptive Geometry, 515.
Blue Printing, etc. See 770.
Topographical Drawing. See also 526.98.

770 Photography.

*Leitze, Ernest—. Modern Heliographic Processes. 2d ed. 8vo. Cloth. V N.
Pettit, James S.—. Modern Reproductive Graphic Processes. 18mo. $0.50. V N.
Photographic Surveying. See 526.9.

---

* ** Believed to be specially useful.

GLOSSARY OF TERMS.

Abacus; the flat square member on top of a column.

Abscis or *abscissa;* any portion of the axis of a curve, from the vertex to any point from which a line leaves the axis at right angles, and extends to meet the curve itself; said line being called an *ordinate*. An abscis and ordinate together are called *co-ordinates*.

Acclivity; an upward slope, or ascent of ground, &c.

Adit; a horizontal passage into a mine, &c.

Adze; a well-known curved cutting instrument, for dressing or chipping horizontal surfaces.

Alternating motion; up and down, or backward and forward, instead of revolving, &c.

Angle-bead, or plaster bead: a bead nailed to projecting angles in rooms, to protect the plaster on their edges from injury.

Angle-block; a triangular block against which the ends of the braces and counters abut in a Howe bridge.

Anneal; to toughen some of the metals, glass, &c, by first heating them, and then causing them to cool very slowly. This process however lessens the tensile strength.

Anticlinal axis; in geology; a line from which the strata of rocks slope away downward in opposite directions, like the slates on the roof of a house; the ridge of the roof representing the axis.

Apex; a point in either chord of a truss, where two web members meet.

Apron; a covering of timber, stone, or metal, to protect a surface against the action of water flowing over it. Has many other meanings.

Arbor. See Journal.

Architrave; that part of an entablature which is next above the columns. Applies also when there are no columns. Also, the mouldings around the sides and tops of doors and windows, attached to either the inner or outer face of the wall.

Arris; a sharp edge formed by any two surfaces which meet at an angle. The edges of a brick are arrises.

Ashler; a facing of cut stone, applied to a backing of rubble or rough masonry, or brickwork.

Astragal; a small moulding, about semi-circular or semi-elliptic, and either plain or ornamented by carving.

Axis; an imaginary line passing through a body, which may be supposed to revolve around it; as the diam of a sphere. Any piece that passes through and supports a body which revolves; in which case it is called an axle, or shaft.

Axle-box. See Journal-box.

Axletree; an axle which remains fixed while the wheel revolves around it, as in wagons, &c.

Azimuth. The azimuth of a body is that arc of the horizon that is included between the meridian circle at the given place, and another great circle passing through the body.

Backing; the rough masonry of a wall faced with finer work. Earth deposited behind a retaining-wall, &c.

Balance-beams; the long top beams of lock-gates, by which they are pushed open or shut.

Balk; a large beam of timber.

Ballast; broken stone, sand or gravel, &c, on which railroad cross-ties are laid.

Ball-cock; a cistern valve at one end of a lever, at the other end of which is a floating ball. The ball rises and falls with the water in the cistern; and thus opens or shuts the valve.

Ball-valve. See Valve.

Bargeboards; boards nailed against the outer face of a wall, along the slopes of a gable end of a house, to hide the rafters, &c; and to make a neat finish.

Bascule bridge; a hinged lift-bridge furnished with a counterpoise.

Batter, (sometimes affectedly *battir*,) or *talus;* the sloping backward of a face of masonry.

Bay; on bridges, &c, sometimes a panel; sometimes a span.

Bead; an ornament either composed of a straight cylindrical rod; or carved or cast in that shape on any surface.

Bearing; the course by a compass. The span or length in the clear between the points of support of a beam, &c. The points of support themselves of a beam, shaft, axle, pivot, &c.

Bed-mouldings; ornamental mouldings on the lower face of a projecting cornice, &c.

Bed-plate; a large plate of iron laid as a foundation for something to rest on.

Beetle; a heavy wooden rammer, such as pavers use.

Bell-crank. See Crank.

Bench-mark; a level mark cut at the foot of a tree for future reference, as being more permanent than a stake.

Berm, or *berme;* a horizontal surface, as if for a pathway, and forming a kind of step along the face of sloping ground. In canals, the level top of the embankment opposite and corresponding to the towpath is called the berm.

Bessemer steel is formed by forcing air into a mass of melted cast iron; by which means the excess of carbon in the iron is separated from it, until only enough remains to constitute cast steel. The carbon is *chemically* united with the steel, but *mechanically* with the iron.

Beton; concrete of hydraulic cement, with broken stone and bricks, gravel, &c.

Bevel; the slope formed by trimming away a sharp edge, as of a board, &c. Edges of common drawing rulers and scales are usually bevelled.

Bevel gear; cog-wheels with teeth so formed that the wheels can work into each other at an angle.

Bilge; the nearly flat part of the bottom of a ship on each side of the keel. Also, the swelled part of a barrel, &c. To bilge is to spring a leak in the bilge, or to be broken there.

Bitts; the small boring points used with a brace.

Blast-pipes; in a locomotive; those through which the waste steam passes from the cylinder into the smoke-pipe, and thus creates an artificial draft in the chimney, or smoke-pipe.

Boasting; dressing stone with a broad chisel called a boaster, and mallet. The boaster gives a smoother surface after the use of the point, or the narrow chisel called a tool.

Body; the thickness of a lubricant or other liquid. Also, the *measure* of that thickness, expressed in the number of seconds in which a given quantity of the oil, at a given temperature, flows through a given aperture.

Bolster; a timber, or a thick iron plate placed between the end of a bridge and its seat on the abutment.

Bond; the disposing of the blocks of stone or brickwork so as to form the whole into a firm structure, by a judicious overlapping of each other, so as to break joint. Applies also to timber, &c, in various ways.

Bonnet; a cap over the end of a pipe, &c. A cast-iron plate bolted down as a covering over an aperture.

Bore; inner diameter of a hollow cylinder.

Borrow-pit; a pit dug in order to obtain material for an embankment.

Boss; an increase of the diameter at any part of a shaft for any purpose. A projection in shape of a segment of a sphere, or somewhat so, whether for use or for ornament; often carved, or cast.

Box-drain; a square or rectangular drain of masonry or timber, under a railroad, &c.

Brace; a kind of curved handle used for boring holes with bitts. The head of the brace remains stationary, being pressed against by the body of the person using it, while the other part with the bitt is turned round by his hand. Also, an inclined beam, bar, or strut, for sustaining compression.

Bracket; a projecting piece of board, &c, frequently triangular, the vertical leg attached to the face of a wall, and the horizontal one supporting a shelf, &c. Often made in ornamental shapes for supporting busts, clocks, &c. Also, the supports for shafting; as pendent, wall, and pedestal brackets.

Brake; an arrangement for preventing or diminishing motion by means of friction. The friction is usually applied at the circumference of a revolving wheel, by means of levers. On railroads, the car-brakes should be worked by steam, as those of Loughridge, Westinghouse, and Creamer. Also, such a handle as that of a common pump.

Brass is composed of copper and zinc.

Brasses; fittings of brass in many plummer-blocks, and in other positions, for diminishing the friction of revolving journals which rest upon them.

Braze; to unite pieces of iron, copper, or brass, by means of a hard solder, called spelter solder, and composed, like brass, of copper and zinc, but in other proportions.

Break joint; to so overlap pieces that the joints shall not occur at the same place, and thus produce a bad bond.

Breast-summer; a beam of wood, iron, or stone, supporting a wall over a door or other opening; a kind of lintel.

Breast-wall; one built to prevent the falling of a vertical face cut into the *natural* soil; in distinction to a retaining-wall or revetment, which is built to sustain earth *deposited* behind it.

Breech; the hind part of a cannon, &c.

Bridge, or *bridge-piece*, or *bridge-bar;* a narrow strip placed across an opening, for supporting something without closing too much of the opening.

Bronze is composed of copper and tin.

Bulkhead; on ships, &c, the timber partitions across them. Also, a long face of wharf parallel to the stream.

Buoy; a floating body, fastened by a chain or rope to some sunk body, as a guide for finding the latter. Sometimes also used to indicate channels, shoals, rocks, &c.

Burnish; to polish by rubbing; chiefly applies to metals.

Bush; to line a circular hole by a ring of metal, to prevent the hole from wearing larger. Also, when a piece is cut out, and another piece neatly inserted into the cavity, the last piece is sometimes said to be bushed in; sometimes it is called a plug.

Butt-joint; one in which the ends of the two pieces abut together without overlapping, and are joined by one or more separate pieces called covers or welts, which reach across the joint and are fastened to both pieces.

Buttress; a vertical projecting piece of brickwork or masonry, built in front of a wall to strengthen it.

Caisson; a large wooden box with sides that may be detached and floated away.

Caliber; the inner diameter, or bore.

Calipers; compasses or dividers with curved legs, for measuring outside and inside diameters.

Calk, or *caulk;* to fill seams or joints with something to prevent leaking.

Calking iron, a tool for forcing calking into a joint.

Camb, or *cam*, or *wiper;* a piece fixed upon a revolving shaft in such a manner as to produce an alternating or reciprocating motion in something in contact with the cam. An eccentric.

Camber; a slight upward curve given to a beam or truss, to allow for settling.

Camel; a kind of barges or hollow floating vessels, which, when filled with water, are fastened to the sides of a ship; and the water being then pumped out, they rise by their buoyancy; and lift the ship so that she can float in shallower water.

Cantilevers; projecting pieces for supporting an upper balcony, &c.

Cants, *rims*, or *shroudings;* the pieces forming the ends of the buckets of water-wheels, to prevent the water from spilling endwise.

Capstan; a long hollow rope-drum surrounding a strong vertical pivot, upon the head of which it rests, and around which it turns. Its top is a thick projecting circular piece, having holes around its outer edge or circumference, for the insertion of the ends of levers; or capstan-bars. It is a kind of vertical windlass.

Case-harden; to convert the outer surface of wrought iron into steel, by heating it while in contact with charcoal.

Casemate; in fortification; the small apartment in which a cannon stands.

Castors; rollers usually combined with swivels; as those used under heavy furniture, &c.

Causeway; a raised footway or roadway.

Cavetto; a moulding consisting of a receding quadrant of a circle.

Cementation; the process of converting wrought iron into steel, by heating it in contact with charcoal. This process produces blisters on the steel bars; hence *blister* steel. These are removed, and the steel compacted, by reheating it, and then subjecting it to a tilt-hammer. It is then *tilted* steel, or *shear* steel. Or if the blister steel is broken up, remelted in a crucible, and then run into ingots or blocks, it is called *crucible*, *cast*, or *ingot steel;* which is harder and closer-grained than tilted steel. It may be softened, and thus become less brittle, by annealing. The ingots may be converted into bars by either rolling or hammering, the same as shear and blister steel.

Center; the supports of an arch while being built.

Center of percussion, in a moving body, is that point which would strike an opposing body with greater force than any other point would. If the opposing body is immovable, it will receive *all* the force of a rigid moving body which strikes with its center of percussion. See Pendulum, page 365.

Cesspool; a shallow well for receiving waste water, filth, &c.

Chamfer; means much the same as bevel; but applies more especially when two edges are cut away so as to form either a chamfer-groove, (see 14, p 735, of Trusses,) or a projecting sharp edge.

Cheeks; two flat parallel pieces confining something between them.

Chilling, chill-hardening, or *chill-casting;* giving great hardness to the outside of cast-iron, by pouring it into a mould made of iron instead of wood. The iron mould causes the outside or skin of the casting to cool very rapidly; and this for some unknown reason increases its hardness. This process is frequently confounded with case-hardening.

Chock; any piece used for filling up a chance hole, or vacancy.

Chuck; the arrangement attached to the revolving shaft, arbor, or mandril of a lathe, for holding the thing to be turned.

Churn-drill; a long iron bar, with a cutting end of steel; much used in quarrying, and worked by raising it and letting it fall. When worked by blows of a hammer or sledge it is called a jumper.

Cima, or *cyma;* a moulding nearly in shape of an S. When the upper part is concave, it is called a cima recta; when convex, a cima reversa.

Clack valve. See Valves.

Clamp; a piece fastened by tongue and groove, transversely along the end of others, to keep them from warping. A kind of open collar, which, being closed by a clamp-screw, holds tight what it surrounds. See Cramp.

Clap boards; short thin boards, shingle-shaped, and used instead of shingles.

Claw, a split provided at the end of an iron bar, or of a hammer, &c, to take hold of the heads of nails or spikes for drawing them out; as in a common claw-hammer.

Cleat; a piece merely bolted to another to serve as a support for something else; as at 7, 8, 10, &c, p. 735, of Trusses. Often used on shipboard for fastening ropes to, as at 11. Also a piece of board nailed across two or more other boards, for holding them together, as is often done in temporary doors, &c.

Clevis. See Shackle.

Click. See Ratchet.

Clip; a fastening like that on the tops of the Y's of a spirit level; being a kind of half collar opening by a hinge.

Clutch; applied to various arrangements at the ends of separate shafts, and which by clutching or catching into each other cause both shafts to revolve together. A kind of coupling.

Cock; a kind of valve for the discharge of liquids, air, steam, &c.

Coefficient; or a *Constant* of friction, safety, or strength, &c, may usually be taken to be a number which shows the proportion (or rather the ratio) which friction, safety, tensile strength, &c, bear to a certain something else which is not generally expressed at the time, but is well understood. Thus, when we say that the coeff of friction of one body upon another is $\frac{1}{10}$, &c, it is understood that the friction is in the proportion of $\frac{1}{10}$th of the *pressure* which produces it. A coeff of safety of 3, means that the safety has a proportion or ratio of 3 to 1 to the *theoretical breaking load.* A coeff of 500 ℔s, or of 20 tons, &c, of tensile strength of any material, denotes that said strength is in the proportion of 500 ℔s, or of 20 tons, &c, to each *square inch of transverse section,* &c. Same as *Modulus.*

Coffer-dam; an enclosure built in the water, and then pumped dry, so as to permit masonry or other work to be carried on inside of it.

Cog; the tooth of a cog-wheel.

Collar; a flat ring surrounding anything closely.

Collar-beam; a horizontal timber stretching from one to another of two rafters which meet at top; but above the main tie-beam.

Concrete; artificial stone formed by mixing broken stone, gravel, &c, with common lime. When hydraulic cement is used instead of lime, the mixture is called beton. The terms "lime concrete" and "cement concrete" would be convenient.

Connecting-rod; a piece which connects a crank with something which moves it, or to which it gives motion.

Console; a kind of ornamental bracket, somewhat in shape of an S; much used in cornices, &c, for supporting ornamental mouldings above it.

Coping; flat plates of stone, iron, &c, placed on the tops of walls exposed to the weather.

Corbel; a horizontal projecting piece which assists in supporting one resting upon it which projects still farther.

Core; anything serving as a mould for anything else to be formed around. A term much used in foundries.

Cornice; the ornamental projection at the eaves of a building, or at the top of a pier, or of any other structure.

Cotter-bolt, or *key-bolt;* a bolt which, instead of a screw and nut at one end, has a slot cut through it near that end, for the insertion of a wedge-shaped key or cotter, for keeping it in its place. Sometimes the ends of these keys are split, so as to spread open after being inserted, so as not to be jolted out of place.

Counterfort; vertical projections of masonry or brickwork built at intervals along the *back* of a wall to strengthen it; and generally of very little use.

Counter-shaft; a secondary shaft or axle which receives motion from the principal one.

Countersunk. See Ream.

Counter-weight; or counter-balance; any weight used to balance another.

Couplings; a term of very general application to arrangements for connecting two shafts so that they shall revolve together.

Cover; see "butt-joint."

Cover; in re-rolling iron and steel from piles of small pieces, a large bar or slab, called a cover, of the same width and length as the pile, is employed to form the bottom of the pile, and a similar slab for the top. The covers serve to hold the pile together; and, after rolling, they form unbroken top and bottom surfaces of the finished plate, bar, rail, I beam, &c.

Crab; a short shaft or axle, which serves as a rope-drum in raising weights; and is revolved either by cog-wheels, a winch, or by levers or handspikes, inserted in holes around its circumference like a windlass, or capstan, of which it is a variety. It may be either vertical or horizontal. It is often set in a frame, to be carried from place to place. Also the *whole machine* is called a crab.

Cradle; applied to various kinds of timber supports, which partly enclose the mass sustained.

Cramp; a short bar of metal, having its two ends bent downward at right angles for insertion into two adjoining pieces of stone, wood, &c, to hold them together. Much used at the ends of coping-stones. Also a similar bent piece, with a set-screw passing through one of the bent ends, for holding things tight between it and the other end. This last is also called a clamp.

Crane; a hoisting machine consisting of a revolving vertical *post* or *stalk;* a projecting *jib;* and a *stay* for sustaining the outer end of the jib. The stay may be either a strut or a tie. There are also cog-wheels, a rope drum or barrel, with a winch, ropes, pulleys, &c. In a crane the post, jib, and stay do not change their relative positions, as they do in a derrick.

Crank; a double bend at right angles, somewhat like a Z, at the end of a shaft or axle, and forming a kind of handle by which the axle may be made to revolve. Sometimes, as in common grindstones, this crank is formed of a separate piece removable at pleasure. That part of this piece which has the square opening in it for fitting it to the square end of the axle, is called the *crank-arm;* and the other part the *crank-handle.* A *bell-crank* consists of 4 bends at right angles at the center of an axle, forming in it a kind of U. A *double crank* consists of two bell cranks arranged thus, U∩. The bend in the U forms the *crank-wrist.* The term bell-crank is applied also to those used in fixing common dwelling house bells: and to larger ones on the same principle. A *crank-pin* is a pin projecting from a revolving *wheel,* disk, or other body, and serving as a crank-handle. A *crank-shaft* is a shaft which has a crank in it, or at its end. A *cranked* shaft has it *in* it only. A ship or other vessel is said to be crank when its breadth is so small in proportion to its depth as to make it liable to upset easily; or when the same liability is caused by want of sufficient ballast.

Crest; that top part of a dam over which the water pours.

Cross-cut saw; a large horizontal saw worked by two men, one at each end.

Cross-head; a piece attached across the end (or near it) of another piece, and at right angles to it, so as to form a kind of T or cross. Often seen on piston rods, which they serve to keep in place by resting on the slides, or guides.

Crowbar; a bar of iron used as a lever for various purposes; often pointed at one end.

Crown, or *contrate wheel;* a cog-wheel in which the teeth stand not upon its outer circumference as usual, but upon the plane of its circle.

Curb; a broad flat circular ring of wood, iron, or stone, placed under the bottoms of circular walls, as in a well, or shaft, to prevent unequal settlement; or built into the walls at intervals, for the same purpose. Has many other meanings.

Cut-off; an arrangement for cutting off the steam from a cylinder before the piston has made its full stroke. Also a channel cut through a narrow neck of land, to straighten the course of a river.

Cutwater, or starling; the projecting ends of a bridge pier, &c, usually so shaped as to allow water, ice, &c, to strike them with but little injury.

Damper; a door or valve to regulate the admission of air to a furnace, stove, &c.

Dead load; the cars, engine, &c, in a train; non-paying load.

Dead-load; in a bridge, the weight of the bridge itself, with flooring, roof, &c; as distinguished from the *live* load of passing trains, vehicles, pedestrians, &c.

Dead points; those two points in the revolution of a crank, where the crank arm is parallel with the rod which connects it with the moving power; and at which said rod exerts no tendency to turn the crank.

Declination, of the sun, or of a star, is its angle north or south of the earth's equator at the time of observation.

Declivity; a downward slope or descent of ground, &c.

Dentils; blocks constituting ornaments in a cornice; placed at short intervals apart, they resemble teeth. When, instead of mere blocks, they are handsomely carved in various shapes, they are called modillions.

Derrick; a kind of crane, differing from common ones, chiefly in the fact that the rope or chain which forms the stay may be let out or hauled in at pleasure, thus raising or lowering the inclination of a jib; thereby enabling the raised load to be placed vertically at the required spot. This cannot be done with a crane, which, therefore, is not as well adapted for laying heavy masonry, especially at great heights.

Diaphram; a thin plate or partition placed across a tube or other hollow body.

Die; that part of a stamp that gives the impression. Dies are also two flat plates of hardened steel, on an edge of each of which is hollowed out a semicircular half of a short female screw. When these plates are put in contact they form a complete female screw, like that in a nut; and being strongly held together by an iron boxing called the die-stocks, which have long handles for revolving them, they constitute a mould or cutter for forming threads on a male screw. Also the main body of a pedestal.

Dip; in geology, either the angle which the slope of a stratum forms with a horizontal; or the direction by compass, toward which it slopes. In surveying, the inclination at which an unbalanced compass-needle rests on its pivot after being magnetized.

Disk; a flat circular piece.

Dock; an artificial enclosure, either partial or total, in which ships and other vessels are placed for being loaded or unloaded, or repaired. The first is a *wet* dock; the last a *dry* one.

Dog-iron; a short bar of iron, forming a kind of cramp, with its ends bent down at right angles and pointed, so as to hold together two pieces into which they are driven. Often used for temporary purposes. It is also called a dog-iron when only one end is bent down and pointed for driving, the other end being formed into an eye or a handle by which the piece into which the other end is driven may be hauled or towed away.

Donkey-engine; a small steam engine attached to a large one, and fed from the same boiler. It is used for pumping water into the boiler.

Double crank. See Crank.

Dovetail; a joint like 20. page 735; it is a poor one for timber when there is much strain, being then apt to draw out more or less.

Dowel; a straight pin of wood or metal, inserted part way into each of two faces which it unites.

Draft; the depth to which a floating vessel sinks in the water; in other words the water it draws.

Draught; a drawing. A narrow level stripe which a stonecutter first cuts around the edges of a rough stone, to guide him in dressing off the face thus enclosed by the draught.

Draw-plate; a plate of very hard steel, pierced with small circular holes of different diameters, through which in succession rods of iron are drawn, and thus lengthened out into wire. Sometimes the holes are drilled through diamond or ruby, &c, instead of steel.

Drift; a horizontal or inclined passage-way, or small tunnel, in mines, &c. To float away with a current. Trees, &c, carried along by freshets.

Drip; a small channel cut under the lower projecting edge of coping, &c, so that rain when it reaches that point will drip or fall off, instead of finding its way horizontally beneath to the wall, which it would make damp.

Drop; short pieces of nearly complete cylinders, placed at small distances apart, in a row like teeth, as an ornament to cornices, &c.

Drum; a revolving cylinder around which ropes or belts either travel or are wound. When narrow and used with belts they are called pulleys.

Dry-rot; decay in such portions of the timber of houses, bridges, &c, as are exposed to dampness, especially in confined warm situations. The timber in cellars and basement stories is more liable to it than in other parts, owing to the greater dampness absorbed by the brickwork from the ground. Contact with lime or mortar hastens dry rot. The ends of girders, joists, &c, resting on damp walls, may be partially protected by placing pieces of slate or sheet iron under them. The painting or tarring of *unseasoned* timber expedites internal dry rot. A thorough soaking of timber in a solution of 28 grains of quicklime to 1 gallon of water is said to be a preventive of dry-rot; but the best process for that purpose is saturation with creosote or carbolic acid.

Dyke; mounds of earth, &c, built to prevent overflow from rivers or the sea. A kind of geological irregularity or disturbance, consisting of a stratum of rock injected as it were by volcanic action, between or across strata of rocks of another kind. A levee.

Eccentric; a circular plate or pulley, surrounded by a loose ring, and attached to a revolving shaft, and moving around with it, but not having the same center; for producing an alternate motion. Often used instead of a crank, as they do not weaken the axle by requiring it to be bent. There are many modifications.

Escarpment; a nearly vertical natural face of rock or soil.

Escutcheon; the little outside movable plate that protects the keyhole of a lock from dust.

Eye; a circular hole in a flat bar, &c, for receiving a pin, or for other purposes.

Eye and strap; a hinge common for outside shutters, &c, one part consisting of an iron strap one end of which is forged into a pin at right angles to it; and the other part, of a spike with an eye, through which the pin passes. When the eye is on the strap, and the pin on the spike, it is called a hook and strap. Such hinges are sometimes called "backflaps."

Eye-bolt; a bolt which has an eye at one end.

Face-wall; one built to sustain a face cut into natural earth, in distinction to a retaining-wall, which supports earth deposited behind it.

Fall; the rope used with pulleys in hoisting.

False-works; the scaffold, center, or other temporary supports for a structure while it is being built. In very swift streams it is sometimes necessary to sink cribs filled with stone, as a base for false-works to foot upon.

Fascines; bundles of twigs and small branches, for forming foundations on soft ground.

Fatigue; of materials; the increase of weakness produced by frequent bending; or by sustaining heavy loads for a long time.

Faucet; a short tube for emptying liquids from a cask, &c; the flow is stopped by a spigot. The wider end of a common cast-iron water or gas pipe.

Feather; a slightly projecting narrow rib lengthwise of a shaft, and which, catching into a corresponding groove in anything that surrounds and slides along the shaft, will hold it fast at any required part of the length of the feather. Has other applications.

Feather-edge; when one edge of a board, &c, is thinner than the other.

Felloe, or *felly;* the circular rim of a wheel, into which the outer ends of the spokes fit; and which is often surrounded by a tire.

Felt; a kind of coarse fabric or cloth made of fibres of hair, wool, coarse paper, &c, by pressure, and not by weaving.

Fender; a piece for protecting one thing from being broken or injured by blows from another; frequently vertical timbers along the outer faces of wharves, to prevent injury from the rubbing of vessels.

Fender-piles; piles driven to ward off accidental floating bodies.

Ferrule; a broad metallic ring or thimble put around anything to keep it from splitting or breaking. A small sleeve.

Fillet; a plain narrow flat moulding in a cornice, &c. See Platband.

Fish; to join two beams, &c, by fastening other long pieces to their sides.

Flags; broad flat stones for paving.

Flange; a projecting ledge or rim.

Flashings; broad strips of sheet lead, copper, tin, &c, with one edge inserted into the joints of brickwork or masonry an inch or two above a roof, &c; and projecting out several inches, so as to be flattened down close to the roof, to prevent rain from leaking through the joint between the roof and the brick chimney, &c, which projects above it.

Flasks; upper and lower; the two parts of the box which contains the mould into which melted iron is poured for castings.

Flatting; causing painting to have a dead or dull, instead of a glossy finish, by using turpentine instead of oil in the last coat.

Fliers; a straight flight of steps in a stairway.

Floodgate; a gate to let off excess of water in floods, or at other times.

Flume; a ditch, trough, or other channel of moderate size for conducting water. The ditches or culverts through which surplus water passes from an upper to a lower reach of a canal.

Flush; forming an even continuous line or surface. To clean out a line of pipes, sewers, gutters, &c. by letting on a sudden rush of water. The splitting of the edges of stones under pressure.

Fluxes; various substances used to prevent the instantaneous formation of rust when welding two pieces of hot metal together. Such rust would cause a weak weld. Borax is used for wrought iron; a mixture of borax and sal ammoniac for steel; chloride of zinc for zinc; sal ammoniac for copper or brass; tallow or resin for lead.

Fly-wheel; a heavy revolving wheel for equalizing the motion of machinery.

Foaming; an undue amount of boiling, caused by grease or dirt in a boiler.

Follower; any cog-wheel that is driven by another; that other is the *leader.*

Forceps; any tools for holding things, as by pincers, or pliers.

Forebay, or *penstock;* the reservoir from which the water passes immediately to a water-wheel.

Forge; to work wrought iron into shape by first softening it by heat, and then hammering it into the required form.

Forge-hammer; a heavy hammer for forging large pieces; and worked by machinery.

Foxtail; a thin wedge inserted into a slit at the lower end of a pin, so that as the pin is driven down, the wedge enters it and causes it to swell, and hold more firmly.

Frame; to put together pieces of timber or metal so as to form a truss, door, or other structure. The thing so framed.

Friction-rollers; hard cylinders placed under a body, that it may be moved more readily than by sliding.

Friction-wheels; wheels so placed that the journals of a shaft may rest upon their rims, and thus be enabled to revolve with diminished friction.

Frieze; in architecture, the portion between the architrave and cornice. The term is often applied when there is no architrave.

Fulcrum; the point about which a lever turns.

Furrings; pieces placed upon others which are too low, merely to bring their upper surfaces up to a required level; as is often done with joists, when one or more are too low; a kind of chock.

Fuze, or *fuse;* to melt. A slow match, which, by burning for some time before the fire reaches the powder, gives the men engaged in blasting, time to get out of the way of flying fragments of stone.

Gasket; rope-yarn or hemp, used for stuffing at the joints of water-pipes, &c.

Gearing; a train of cog-wheels. Now much supplanted by belts.

Gib; the piece of metal somewhat of this shape, └─┘, often used in the same hole with a wedge-shaped key for confining pieces together. In common use for fastening the strap to the stub-end of the connecting-rod of an engine.

Gin; a revolving vertical axis, usually furnished with a rope-drum, and having one or more long arms or levers, by means of which it is worked by horses walking in a circle around it. Used for hoisting. Cotton-gin, a machine for separating cotton from its seeds.

Girder; a beam larger than a common joist, and used for a similar purpose.

Glacis; in fortification, an easy slope of earth.

Gland. See Stuffing-box. Also, a kind of coupling for shafts.

Glue; a cement for wood, prepared chiefly from the gelatine furnished by boiling the parings of hides. Good glue will hold two pieces of wood together with a force of from 400 to 750 lbs per sq in.

Governor; two balls so attached to an upright revolving axis as to fly outward by their centrifugal force, and thus regulate a valve.

Grapnel; a kind of compound hook with several curved points, for finding things in deep water.

Grillage; a kind of network of timbers laid crossing each other at right angles; frequently placed on the heads of piles, for supporting piers of bridges, and other masonry.

Groin; an arch formed by two segmental arches or vaults intersecting each other at right angles. Also, a kind of pier built from the shore outward, to intercept shingle or gravel.

Groove; a small channel. A triangular one is called a chamfered groove.

Ground-swell; waves which continue after a storm has ceased; or caused by storms at a distance.

Grout; thin mortar, to be poured into the interstices between stones or bricks.

Gudgeons; the metal journals of a horizontal shaft, such as that of a water-wheel. For moderate speeds

$$\left.\begin{array}{r}\text{Diam, ins}\\\text{if of cast-iron}\end{array}\right\} = \frac{\sqrt{\text{Weight in lbs on one gudgeon}}}{20}$$

For wrought-iron, add one-twentieth.

Gun-metal, or bronze; a compound of copper and tin, sometimes used for cannon. Also, a quality of cast iron fit for the same purpose.

Gussets; plain triangular pieces of plate iron, riveted by their vertical and horizontal legs to the sides, tops, and bottoms of box-girders, tubular bridges, &c, inside, for strengthening their angles.

Guys; ropes or chains used to prevent anything from swinging or moving about.

Gyrate; to revolve around a central axis, or point.

Halving; to notch together two timbers which cross each other, so deeply that the joint thickness shall equal only that of one whole timber.

Hammer dress; to dress the face of a stone by slight blows of a hammer with a cutting edge. The *patent* hammer for such purposes has several such edges placed parallel to each other, each of which may be removed and replaced at pleasure.

Hand-lever; in an engine, a lever to be worked by hand instead of by steam.

Handspike; a wooden lever for working a capstan or windlass; or other purposes.

Hand-wheel; a wheel used instead of a spanner, wrench, winch, or lever of any kind, for screwing nuts, or for raising weights, or for steering with a rudder, &c.

Hangers, or *pendent brackets;* fixtures projecting below a ceiling, to support the journals of long lines of shafting; and for other purpose. Should be "self-adjusting."

Hasp; a piece of metal with an opening for folding it over a staple.

Hatchway; a horizontal opening or doorway in a floor, or in the deck of a vessel.

Haunches; the parts of an arch from the keystone to the skewback.

Head-block; a block on which a pillow-block rests.

Header; a stone or brick laid lengthwise at right angles to the face of the masonry.

Heading; in tunnelling, a small driftway or passage excavated in advance of the main body of the tunnel, but forming part of it; for facilitating the work.

Headway; the clear height overhead. Progress.

Heel-post; that on which a lock gate turns on its pivot.

Helve; the handle of an axe.

Hinge; those commonly used on the doors of dwellings are called butts, or butt hinges. (Eye and Strap, .) *Rising hinges* are such as cause the door to rise a little as it is opened, and thus cause the door to shut itself.

Hip roof, or *hipped roof;* one that slopes four ways; thus forming angles called hips.

Hoarding; a temporary close fence of boards, placed around a work in progress, to exclude stragglers.

Holding-plates, or *anchors;* strong broad plates of iron sunk into the ground, and generally surrounded by masonry; for resisting the pull of the cables of suspension bridges; and for other similar purposes.

Hook and strap. See Eye and strap.

Horses; the sloping timbers which carry the steps in a staircase.

Housings; in rolling mills, &c, the vertical supports for the boxes in which the journals revolve.

Hub, or *nave;* the central part of a wheel, through which the axletree passes, and from which the spokes radiate.

Impost; the upper part of a pier from which an arch springs.

Ingot; a lump of cast metal, generally somewhat wedge-shaped. A pig of cast iron is an ingot.

Invert; an inverted arch frequently built under openings, in order to distribute the pressure more evenly over the foundation.

Jack; a raising instrument, consisting of an iron rack, in connection with a short stout timber which supports it, and worked by cog-wheels and a winch. A *screw-jack* is a large screw working in a strong frame, the base of which serves for it to stand on; and which is caused to revolve and rise, carrying the load on top of it, by turning a nut, or otherwise.

Jack-rafters, or common rafters; small rafters laid on the purlins of a roof, for supporting the shingling laths, &c.

Jag-spike; a spike whose sides are jagged or notched, with the mistaken idea that its holding power is thereby much increased. If a spike or bolt is first put into its place loosely, and then has melted lead run around it, the jagging does assist; but not when it is driven into wood.

Jambs; the sides of an opening through a wall, &c; as door, window, and fireplace jambs.

Jamb-linings; the facing of woodwork with which jambs are covered and hidden.

Jaw; an opening, often V-shaped, the inner edges of which are for holding something in place.

Jettie, or *jetty;* a pier, mound, or mole projecting into the water; as a wharf-pier, &c.

Jib; the upper projecting member or arm of a crane, supported by the stay.

Jig-saw; a very narrow thin saw worked vertically by machinery, and used for sawing curved ornaments in boards.

Joggle; a joint like that at 3 or 4, &c, p 735, of Trusses, for receiving the pressure of a strut at right angles or nearly so. Also applied to squared blocks of stone sometimes inserted between courses of masonry to prevent sliding, &c.

Joist; binding joists are girders for sustaining common joists. The common ones are then called *bridging* joists. *Ceiling* joists are small ones under roof trusses, or under girders, and for sustaining merely the plastered ceiling.

Journal-box; a fixture upon which a journal rests and revolves, instead of a plummer-block.

Journals; the cylindrical supporting ends of a horizontal revolving shaft. Their length is usually about 1 to 1½ times their diam. In lines of shafting 4 diams. To find the diam, see Gudgeon.

Jumper; a drill used for boring holes in stone by aid of blows of a sledge-hammer.

Kedge; a small anchor.

Keepers; the pieces of metal or wood which keep a sliding bolt in its place, and guide it in sliding.

Kerf; the opening or narrow slit made in sawing.

Key-bolt. See Cotter-bolt.

Keystone; the center stone of an arch.

Kibble; the bucket used for raising earth, stone, &c, from shafts or mines.

King-post, king-rod; the center post, vertical piece, or rod, in a truss; all those on each side of it are queen-posts, or queen-rods. Frequently called simply kings and queens.

Knee; a piece of metal or wood bent at an angle; to serve as a bracket, or as a means of uniting two surfaces which form with each other a similar angle.

Lagging, or *sheeting;* a covering of loose plank; as that placed upon centers, and supporting the archstones. Also, an outer wooden casing to locomotive boilers and others.

Landing; the resting-place at the end of a flight of stairs.

Lantern wheel. See Trundle.

Lap: to place one piece upon another, with the edge of one reaching beyond that of the other.

Lap-welding; welding together pieces that have first been lapped; in distinction to butt-welding

Lead, (pronounced *leed*;) in steam-engines, a certain amount of opening of the port-valve before each stroke of the piston begins. The distance to which earth is hauled or wheeled.

Leader; a cog-wheel that gives motion to the next one or follower.

Leading-beam; leading-pile; one placed as a guide for placing others.

Leading-wheels; in a locomotive, those frequently placed in front of the driving-wheels.

Leaves; the cogs of pinions.

Ledge; a part projecting over like a shelf; a rock so projecting. A narrow strip of board nailed across other boards, to hold them together, as in temporary ledge-doors.

Lewis; an arrangement composed of 2 or 3 pieces of metal let into a wedge-shaped hole in a block of stone, by which to raise the block.

Lighter; a scow, raft, or other vessel, used for unloading vessels out from the shore.

Linch-pin; a pin near the end of an axle, to hold the wheel on.

Link; one of the divisions of a chain; or a piece shaped like one.

Link-motion; a device for regulating the movement of the main or port valve in a steam-engine.

Lintel; a horizontal beam across an opening in a wall, as seen in windows, doors, &c. When of wide span, and supporting heavy brickwork or masonry, it is called a breast-summer, or bressummer.

Lock; those common door locks which are entirely concealed within the thickness of the door, are called mortice locks; those which are screwed against the face of a door, rim locks. It must be remembered that locks are "right and left."

Louvre; a kind of vertical window, frequently at the tops of roofs of depots, &c, provided with horizontal slats, which permit ventilation, and exclude rain.

Lozenge; the shape of a rhomb; often called diamond-shaped.

Lug; in casting, small projections from the general surface, and for various purposes, such as for lifting the body; or for a flange for joining it to another; or for a support for something else.

Mallet; the wooden hammer used by stonecutters.

Mandrel; an iron rod used as a core around which a flat piece may be beat into a cylindrical shape. Also the shaft that carries the chuck of a lathe.

Manhole; an opening by which a man can enter a boiler, culvert, &c. to clean or repair it.

Mattock; a kind of pick with broad edges for digging.

Maul; a heavy wooden hammer.

Mean, arithmetical; half the sum of two numbers.

" , *geometrical;* the sq rt of the product of two numbers.

Mean-proportional; the same as the geometrical mean.

Meridian; a north and south line. Noon.

Mitre-joint; a joint formed along the diagonal line where the ends of two pieces are united at an angle with each other.

Mitre-sill; the sill against which the lock gates of a canal shut.

Modulus; a datum serving as a means of comparison. Same as *constant* or *coefficient.*

Moment; tendency of force acting with leverage.

Moment of rupture, or *of bending;* the tendency which any load or force exerts to break or bend a body by the aid of leverage. Its amount is found in foot-pounds by multiplying the force in lbs, by the length of leverage in feet between it and that part of the body upon which the tendency is exerted.

Monkey; the hammer or ram of a pile-driver.

Monkey-wrench, or *screw-wrench;* a spanner, the gripping end of which can be adjusted by means of a screw to fit objects of different sizes.

Moorings; fixtures to which ships, &c, can make fast.

Mortise; a hole cut in one piece, for receiving the tenon which projects from another piece.

Muck; soft surface soil containing much vegetable matter.

Muntins, or *mullions;* the vertical pieces which separate the panes in a window-sash.

Nailing-blocks; blocks of wood inserted in walls of stone or brick, for nailing washboards, &c, to.

Nave; the main body of a building, having connecting wings or aisles on each side of it. The hub of a wheel.

Newel; the open space surrounded by a stairway.

Newel-post; a vertical post sometimes used for sustaining the outer ends of steps. Also the large baluster often placed at the foot of a stairway.

Nippers; pincers. An arrangement of two curved arms for catching hold of anything.

Normal; perpendicular to. According to rule, or to correct principles.

Nosing; the slight projection often given to the front edge of the tread of a step; usually rounded.

Nut, or *burr;* the short piece with a central female screw, used on the end of a screw-bolt, &c, for keeping it in place.

Ogee; a moulding in shape of an S, the same as a cima.

Ordinate; a line drawn at right angles from the axis of a curve, and extending to the curve.

Oscillate; to swing backward and forward like a pendulum.

Out of wind, pronounced *wynd;* perfectly straight or flat.

Ovolo; a projecting convex moulding of quarter of a circle; when it is concave it is a cavetto, or hollow.

Packing; the material placed in a stuffing-box, &c, to prevent leaks.

Packing-pieces; short pieces inserted between two others which are to be riveted or bolted together to prevent their coming in contact with each other.

Pall, or *pawl.* See Ratchet.

Parapet; a wall or any kind of fence or railing to prevent persons from falling off.

Parcel; to wrap canvas or rags round a rope.

Parge; to make the inside of a flue smooth by plastering it.

Patent hammer; a hammer with several parallel sharp edges for dressing stone.

Pay. To cover a surface with tar, pitch, &c. A ship word.

Pay out. To slacken, or let out rope.

Pediment; the triangular space in the face of a wall that is included between the two sloping sides of the roof and a line joining the eaves.

Penstock. See Forebay.

Pier; the support of two adjacent arches. The wall space between windows, &c. A structure built out into the water.

Pierre-perdue; lost stone; *random stone,* or rough stones thrown into the water, and let find their own slope.

Pilaster; a thin flat projection from the face of a wall, as a kind of ornamental substitute for a column.

Pile-planks; planks driven like piles.

Pillow-block, or *plummer-block;* a kind of metal chair or support, upon which the journals of horizontal shafts are generally made to rest, and on which they revolve.

Pinion; a small cog-wheel which gives motion to a larger one.

Pintle; a vertical projecting pin like that often placed at the tops of crane-posts, and over which the holding rings at the tops of the wooden guys fit. Also, such as is used for the hinges of rudders, or of window-shutters to turn around.

Pitch; the slope of a roof, &c. The distance from center to center of the teeth of a cog wheel, or the threads of a screw. Boiled tar. Also the dist apart of rivets, &c.

Pitman; a connecting-rod for transmitting motion from a prime mover to machinery at a distance, moved by it.

Pit-saw; a large saw worked vertically by two men, one of whom (the pitman) stands in a pit.

Pivot; the lower end of a vertical revolving shaft, whether a part of the shaft itself, or attached to it. It should be flat; and both it and the step or socket upon which it rests should be of hard steel. If a steel pivot has to revolve rapidly and continuously, it is well to proportion its diam, so as not to have to sustain *more* than 250 lbs per sq inch; otherwise it will wear quickly. Dust and grit should for the same reason be carefully guarded against. Pivots which revolve but seldom, and slowly, as those of a railroad turntable, may be trusted with half a ton, or even a whole ton per sq inch. As a rude rule, cast-iron pivots should not be loaded with more than half as much as steel ones. A steel one may be welded to the foot of its cast-iron shaft; or may be inserted part way into it; and the whole strengthened by iron bands shrunk on.

Planish; to polish metals by rubbing with a hard smooth tool.

Plant; the outfit of machinery, &c, necessary for carrying on any kind of work.

Plaster-bead; a small vertical strip of iron or wood nailed along projecting angles in rooms, to protect the plaster at those parts.

Platband; a plain, flat, wide, slightly projecting strip, generally for ornament. When narrow it is called a fillet.

Pliers; a kind of pincers.

Plinth; the square lowest member of the base of a column or pillar.

Plug; a piece inserted to stop a hole. *Screw-plug*, a plug that is screwed into a hole.

Plumb; vertical.

Plummet, or *plumb-bob;* a weight at the lower end of a string, for testing verticality.

Plunger; a kind of solid piston, or one without a valve.

Point; a kind of pointed chisel for dressing stone. To put a finish to masonry by touching up the outer mortar joints. To dress stone with a point and mallet.

Pole-plate; a longitudinal timber resting on the ends of tie-beams of roofs; and for supporting the feet of the common or jack rafters, when such are used.

Port; the opening or passage controlled by a valve.

Prime; to put on the first coat of paint. Priming also is when water passes into a steam cylinder along with the steam.

Projection. If parallel straight lines be imagined to be drawn in any one given direction, from every point in any surface *s*, whether flat, curved, or irregular, then if all these lines be supposed to be intersected or cut by a plane, either at right angles to their direction, or obliquely, the figure which their cross-section thus made would form upon said plane, is called the projection of the surface *s*. If such lines be supposed to be drawn from a person's face, in a direction in front of him, and to be cut by a plane at right angles to their direction, their projection on the plane would be the person's full-face portrait. If the lines be drawn *sideways* from his face, the projection will be his profile. The projection of a globe upon a flat plane, will evidently be a circle if the plane cuts the lines at right angles; and an ellipse if it cuts them obliquely. Shadows cast by the sun are projections.

Puddle; earth well rammed into a trench, &c, to prevent leaking. A process for converting cast iron into wrought by a puddling furnace.

Pug-mill; a mill for tempering clay for bricks or pottery, &c.

Pulley; a circular hoop which carries a belt in machinery.

Puppet; in machinery, a small short pedestal or stand. *Puppet-valve*. See Valves.

Purlins; the horizontal pieces placed on rafters, for supporting the roof covering.

Put-logs, or *put-locks;* horizontal pieces supporting the floor of a scaffold; one end being inserted into *put-log holes* left for that purpose in the masonry.

Quay; a wharf.

Quoin; the hollow into which a quoin-post of a canal lock-gate fits. Stones, usually dressed, placed along the vertical angles of buildings, chiefly for ornament.

Quoin-post; the vertical post on which a lock-gate turns. The heel-post.

Rabbet, or *rebate;* a half groove along the edge of a board, &c. See 16, p 785, of Trusses, where two rabbets are shown overlapping each other.

Race; the channel which conducts water either to or from a water-wheel: the first is a head race; the last a tail race. The waves produced by the meeting of strong opposing currents; also, a rapid tideway, or roost.

Rack and pinion; the rack is a straight row of cogs on a bar, and called a rack-bar; and the pinion is a small cog-wheel working into it.

Radius of gyration. See Center of gyration.

Rag-bolt. See Jag-spike.

Rag-wheel, or *Sprocket-wheel;* one with teeth or pins which catch into the links of a chain.

Ram; the hammer of a pile-driver.

Random stone; rip rap, or rough stones thrown promiscuously into the water, to form a foundation, &c.

Rasp; a coarse file.

Ratchet and pall; the former is sometimes a straight bar, at others a wheel: in either case it is furnished with teeth between which the pall drops and prevents backward motion. Used for safety in hoisting machinery, &c. The pall is sometimes called a click.

Ream; to enlarge the diameter of a hole by scraping away the material forming its circumference.

Rebate. See Rabbet, above.

Reciprocal of a number is the quotient found by dividing 1 by that number.

Reciprocating motion. See Alternating motion.

Re-entering angle; an angle or corner projecting inward. See Salient, below.

Revetment; steep facing of stone to the sides of a ditch or parapet in fortification. A retaining-wall.

Rib; the curved pieces which form the arches of iron or wooden bridges, &c. Also, those to which the outer planking of a sailing vessel, &c, are fastened.

Ridge of a roof; its peak, or the sharp edge along its very top. Has various similar applications.

Ridge-pole, ridge-piece, or *ridge-plate;* the highest horizontal timber in a roof, extending from top to top of the several pairs of rafters of the trusses; for supporting the heads of the jack-rafters.

Right and left; a lock which in its proper position suits one flap of a pair of folding doors, will not suit if fastened to the other flap; nor even to the same flap if required to open to the right instead of to the left, or vice versa, according to whether it is a right or a left-hand lock. And so with many other things, as, for instance, certain arrangements for working railway switches, &c. Right and left boots and shoes are a familiar illustration; also, right and left screws. Therefore, in ordering several of anything, it is necessary to consider whether they may all be of the same pattern, or whether some must be right-hand, and others left-hand ones.

Right shore of a river; that which is on the right hand when descending the river.

Right-solid body; one which has its axis at right angles to its base; when not so, it is *oblique.*

Ring-bolt; a bolt with an eye and a ring at one end.

Rip-rap. See Random stone.

Roadstead; anchorage at some distance from shore.

Rock-shaft; a shaft which only rocks or makes part of a revolution each way, instead of revolving entirely around.

Rockwork; squared masonry in which the face is left rough to give a rustic appearance.

Rubble; masonry of rough, undressed stones. Scabbled rubble has only the roughest irregularities knocked off by a hammer. Ranged rubble has the stones in each course rudely dressed to nearly a uniform height.

Rundle, or *round;* the step of a ladder.

Rustic; much the same as rockwork.

Saddle; the rollers and fixtures on top of the piers of a suspension bridge, to accommodate expansion and contraction of the cables. The top piece of a stone cornice of a pediment. Has many other applications.

Sag; to bend downward.

Salient; projecting outward. See Re-entering, above.

Sandbag; a bag filled with sand for stopping leaks.

Scabble; to dress off the rougher projections of stones for rubble masonry, with a stone-axe, or scabbling hammer.

Scantling; the depth and breadth of pieces of timber; thus we say, a scantling of 8 by 10 ins, &c.

Scarf; the uniting of two pieces by a long joint, aided by bolts, &c.

Scarp; a steep slope. In fortification the inner slope of a ditch.

Scotia; a receding moulding consisting of a semi-circle or semi-ellipse, or similar figure.

Screeds; long narrow strips of plaster put on horizontally along a wall, and carefully faced out of wind, to serve as guides for afterward plastering the wide intervals between them.

Screw-bolt, a bolt with a screw cut on one end of it.

Screw-jack. See Jack.

Screw-wrench. See wrench.

Scribe; to trim off the edge of a board, &c, so as to make it fit closely at all points, to an irregular surface. The lower edges of an open caisson are scribed to fit the irregularities of a rocky river bottom.

Scroll; an ornamental form consisting of volutes or spirals arranged somewhat in the shape of S.

Scupper nails; nails with broad heads for nailing down canvas, &c.

Scuppers; on shipboard, holes for allowing water to flow off from the deck into the sea.

Scuttle; a small hatchway. To make holes in a vessel to cause sinking.

Sea-wall; a wall built to prevent encroachment of the sea.

Secret nailing; so nailing down a floor by nails along the edges of the boards, that the nail-heads do not show.

Serve; to wrap twine or yarn, &c, closely round a rope to keep it from rubbing.

Set-screw, or *tightening-screw;* a screw for merely pressing one thing tightly against another at will; such as that which confines the movable leg of a pair of dividers in its socket.

Shackle, or *clevis;* a link in a chain shaped like a U, and so arranged that by drawing out a bolt or pin, which fits into two holes at the ends of the U, the chain can be separated at that point.

Shaft; a vertical pit like a well. The body of a column. A large axle.

Shank; the body of a bolt exclusive of its head. The long straight part of many things, as of an anchor, a key, &c.

Shears, or *sheers;* two tall timbers or poles, with their feet some distance apart, and their tops fastened together; and supporting hoisting tackle.

Sheave; a wheel or round block with a groove around its circumference for guiding a rope.

Sheeting, or *sheathing;* covering a surface with boards, sheet iron, felt, &c.

Shingle; the pebbles on a seashore.

Shoes; certain fittings at the ends of pieces; as the pointed iron shoes for piles. The wall shoes into which the lower ends of iron rafters generally fit, &c.

Shore; a prop.

Shot; the edge of a board is said to be shot when it is planed perfectly straight.

Shrink. When an iron hoop or band is first heated, and then at once placed upon the body which it is intended to surround, it shrinks or contracts as it cools, and therefore clasps the body more firmly. This is called *shrinking on* the hoop.

Shuttle; a small gate for admitting water to a water-wheel, or out of a canal lock, &c.

Siding; a short piece of railroad track, parallel to the main one, to serve as a passing-place.

Silt; soft fine mud deposited by rivers, &c.

Siphon culvert; a culvert built in shape of a U, for carrying a stream under an obstacle, and allowing it afterward to rise again to its natural level. The term is improper, inasmuch as the principle of the siphon is not involved.

Skewback; the inclined stone from which an arch springs.

Skids; vertical fenders, on a ship's sides. Two parallel timbers for rolling things upon.

Skirting; narrow boards nailed along a wall, as the washboards in dwellings.

Sledge; a heavy hammer.

Sleeper; any lower or foundation piece in contact with the ground.

Sleeve; a hollow cylinder slid over two pieces to hold them together.

Slide-bars, or *slides;* bars for anything to slide along; as those for the cross-heads of piston-rods, &c. Often called guides.

Slings; pieces of rope or chain to be put around stones, &c, for raising them by.

Slip; the sliding down of the sides of earth-cuts or banks. A long narrow water space or dock between two wharf-piers.

Slope-wall; a wall, generally thin and of rubble stone, used to preserve slopes from the action of water in the banks of canals, rivers, reservoirs, &c; or from the action of rain.

Slot; a long narrow hole cut through anything.

Sluice; a water-channel of wood, masonry, &c; or a mere trench. The flow is usually regulated by a sluice-gate.

Smoke-box; in locomotives, that space in front of the boiler, through which the smoke passes to the chimney.

Snag; a lug with a hole through it, for a bolt.

Socket; a cavity made in one piece for receiving a projection from, or the end of, another piece; as that into which the movable leg of a pair of dividers fits.

Soffit; the lower or underneath surface of an arch, cornice, window, or door-opening, &c.

Solder; a compound of different metals, which when melted is used for uniting pieces of metal also heated. Soft solder is a compound of lead and tin, and is used for uniting lead or tin. There are various hard solders, such as spelter solder, composed of copper and zinc, for uniting iron, copper, or brass.

Sole; that lining around a water-wheel which forms the bottoms of the buckets.

Spandrel; the space, or the masonry, &c, between the back or extrados of an arch and the roadway.

Spanner; a kind of wrench, consisting of a handle or lever with a square eye at one end of it; much used for tightening up the nuts upon screw-bolts, &c. The eye fits over or surrounds the nut.

Spar; a beam; but generally applied to round pieces like masts, &c.

Spelter; zinc.

Spigot; the pin or stopper of a faucet. The smaller end of a common cast-iron water or gas pipe.

Spindle; a thin delicate shaft or axle.

Splay; to widen or flare, like the jambs of a common fireplace, or those of many windows; or like the wing-walls of most culverts.

Splice; to unite two pieces firmly together.

Springer; the lowest stone of an arch.

Sprocket-wheel, or *rag-wheel;* one with teeth or pins which catch in the links of a chain.

Spur-wheel; a common cog-wheel, in which the teeth radiate from a common cen, like those of a spur.

Square; in roofing; 100 square feet.

Square-head; a square termination like that upon which a watch-key fits for winding; or that upon which the eye of the handle of a common grindstone fits for turning it, &c.

Staging; the temporary flooring of a scaffold, platform, &c.

Stanchion; a vertical prop or strut.

Standing-bolt, or *stud-bolt;* a bolt with a screw cut upon each end; one end to be screwed permanently into something, and the other end to hold by means of a nut something else that may be required to be removed at times.

Staple; a kind of double pin in shape of a U; its two sharp points are driven into timber, and curved part is left projecting, to receive a hoop, pin, or hasp, &c.

Starlings; the projecting up and down-stream ends or cutwaters of a bridge pier.

Stay; variously applied to props, struts, and ties, for staying anything or keeping it in place.

Stay-bolts; long bolts placed across the inside of a boiler, &c, to give it greater strength.

Steam-chest; the iron box in locomotive engines and others, through which the steam is admitted to the cylinders.

Steam-pipe; the one which leads steam from a boiler to the steam-chest.

Step; a cavity in a piece for receiving the pivot of an upright shaft; or the end of any upright piece.

Stiles; the flat vertical pieces between and at the sides of the panels in doors, &c.

Stock; the eye with handles for turning it, in which the dies for the cutting of screws are held.

Stove-up, or *stoved*, or *upset;* when a rod of iron is heated at one end, and then hammered endwise so that that part becomes of greater diameter or stouter than the remainder. The heads of bolts are frequently made in one piece with the shank in this way; and the screw ends of long screw-rods are often upset, so that the cutting of the threads of the screw may not reduce the strength of the bar.

Strap; a long thin narrow piece of metal bolted to two bodies to hold them together. A strap-hinge is a strap fastened to a shutter, &c, and having an eye or a pin at one end for fitting it to the other part of the hinge which is attached to the wall.

Stratum; a layer, or bed; as the natural ones in rocks, &c.

Stretcher; a brick, or a block of masonry laid *lengthwise* of a wall. A frame for stretching any thing upon.

Stretcher-course; a course of masonry all of stretchers, without any headers.

Strike; an imaginary horizontal line drawn upon the inclined face of a stratum of rocks. Thus, if the slates or shingles on a roof represent inclined strata of rocks, then either the ridge or the eaves of the roof, or any horizontal line between them, will represent their strike. The inclination is called the *dip* of the strata; and the strike is always at right angles to it by compass.

String; variously applied to longitudinal pieces.

String-board; the boarding (often ornamented) at the outer ends of steps in staircases. It hides the *horses*, as the inclined timbers which carry the steps are called.

String-course; a long horizontal course of brick or masonry projecting a little beyond the others; and often introduced for ornament.

Stringer; any longitudinal timber or beam, &c.

Strut; a prop. A piece that sustains compression, whether vertical or inclined.

Strut-tie, or *tie-strut;* a piece adapted to sustain both tension and compression.

Stub-end; a blunt end.

Stud; a short stout projecting pin. A prop. The vertical pieces in a stud partition.

Stud-bolt. See Standing-bolt.

Stuffing-box; a small boxing on the end of a steam cylinder, and surrounding the piston-rod like a collar; or in other positions where a rod is required to move backward and forward, or to revolve, in an opening through any kind of partition, without allowing the escape of steam, air, or water, &c, as the case may be. The box is filled with greased hemp or other packing, which is kept pressed close around the moving rod by means of a top-piece or kind of cover called the *gland,* which may be screwed down more or less tightly upon it at pleasure. The rod passes through the gland also.

Sumpt, or *sump;* a draining well into which rain or other water may be led by little ditches from different parts of a work to which it would do injury.

Surbase; the inside horizontal mouldings just under a window-sill. Also those around the top of a pedestal, or of wainscoting, &c.

Swage, or *swedge;* a kind of hammer, on the face of which is a semi-cylindrical, or other shaped groove or indentation; and which, being held upon a piece of hot iron and struck by a heavy hammer, leaves the shape of the indentation upon the iron.

Switch; the movable tongue or rail by which a train is directed from one track to another.

Swivels; devices for permitting one piece to turn readily in various directions upon another, without danger of entanglement or separation.

Synclinal axis; in geology, a valley axis, or one toward which the strata of rocks slope downward from opposite directions. The line of the gutter in a valley roof may represent such an axis.

T's; pieces of metal in that shape, whether to serve as straps, or for other purposes. So also with L's, S's, W's, +'s, &c. See figs to Welded Iron Tubes.

Tackle; a combination of ropes and pulleys.

Talus; the same as batter.

Tamp; to fill up with sand or earth, &c, the remainder of the hole in which the powder has been poured for blasting rock. To compact earth generally, as under cross-ties, &c.

Tap; a kind of screw made of hard steel, and having a square head which may be grasped by a wrench for turning it around, and thus forcing it through a hole around the inside of which it cuts an interior screw. To strike with moderate force. To make an opening in the side of any vessel.

Tappet; a pin or short arm projecting from a revolving shaft; or from an alternating bar, and intended to come into contact with, or tap, something at each revolution or stroke.

Teeth; or cogs of wheels.

Temper; to change the hardness of metals by first heating, and then plunging them into water, oil, &c. To mix mortar, or to prepare clay for bricks, &c.

Templet; the outline of a moulding or other article, cut out of sheet metal or thin wood, to serve as a pattern for stonecutters, carpenters, &c.

Tenon; a projecting tongue fitting into a corresponding cavity called a mortise.

Terra cotta; baked clay. Brick is a coarse kind.

Thimble; an iron ring with its outer face curved into a continuous groove. A rope being doubled around this and tied, the thimble acts as an eye for it, and prevents that part of the rope from wearing. Also, a short piece of tube slid over another piece, or over a rod, &c, to strengthen a joint, &c.

Thread; the continuous spiral projection or worm of a screw.

Through-stone; a stone that extends entirely through a wall.

Throw; the radius, or distance to which a crank "throws out" its arm. Applies in the same way to lathes. Some use it to express the diameter instead of the radius. To avoid mistakes, the terms "single" and "double" throw might be used.

Tie; any piece that sustains tension or pull.

Tie-strut; a piece adapted to sustain either tension or compression.

Tightning-screw. See Set-screw.

Tire; the iron ring placed around the outer circumference of the felloe of a wheel.

Tongue; a long slightly projecting strip to be inserted into a corresponding groove, as in tongued and grooved floors.

Tooling; dressing stone by means of a *tool* and mallet; the tool being a chisel with a cutting edge of 1 to 2 inches wide. Tooling is generally done in parallel stripes across the stone.

Torus; a projecting semi-circular, or semi-elliptic moulding; often used in the bases of columns. It is the reverse of a scotia.

Trailing-wheels; in a locomotive, those sometimes placed behind the driving-wheels.

Train; a number of cog-wheels working into each other.

Transom; a beam across the opening for a door, &c. Also, a horizontal piece dividing a high window into two stories, &c, &c. Also, an opening above a door, for ventilation or light.

Tread; the horizontal part of a step.

Treadle; a kind of foot-lever, for turning a lathe, grindstone, &c, by the foot.

Treenail; a long wooden pin.

Trimmer; a short cross-timber framed into two joists so as to sustain the ends of intermediate joists, to prevent the latter from entering a chimney-flue, or interfering with a window, &c.

Trip-hammer, or *tilt-hammer;* a large hammer worked by camb machinery, and used for heavy iron work, especially for hammering irregular masses into the shape of bars, &c.

Truck; a kind of small wagon consisting of a platform on two or more low wheels. Also, those frames and wheels usually placed under railroad cars and engines, and which, by means of a pintle connecting the two, allow them to vibrate or move laterally to some extent independently of each other.

Trundle, lantern-wheel, or *wallower;* used instead of a cog-wheel, and consisting of two parallel circular pieces some distance apart, and united by a central axis, and by cylindrical rods placed around and parallel to the axis, to serve instead of cogs or teeth.

Trunk; a long wooden boxing forming a water channel.

Trunnions; cylindrical projections, as at the sides of a cannon, forming as it were an interrupted axle or shaft for supporting the cannon on its carriage; and allowing it to revolve vertically through some distance.

Tumbler; a kind of spring catch, which at the proper moment falls or tumbles into a notch or hole prepared for it in a piece; thus holding the piece in position until the tumbler is lifted out of the notch.

Tumbling-bay; see "waste-weir."

Tumbling-shaft; in locomotives, a shaft used in the "link motion."

Turntable; the well-known arrangement for turning locomotives at rest.

Undermine; to excavate beneath anything.

Underpin; to add to the height of a wall already constructed, by excavating and building beneath it. Also, to introduce additional support of any kind beneath anything already completed.

Upset. See Stove-up.

Valves; various devices for permitting or stopping at pleasure the flow of water, steam, gas, &c. A SAFETY VALVE is one so balanced as to open of itself when the pressure becomes too great for safety. A SLIDE VALVE is one that slides backward and forward over the opening through which the flow takes place. A BALL VALVE, or spherical valve, is a sphere, which in any position fits the opening. When the pressure below it raises it off from its seat, it is prevented from rolling away by means of a kind of open caging which surrounds it. A CONICAL or PUPPET VALVE is a horizontal slice of a cone, which fits into a corresponding conical seat made in the opening. In rising and falling it is kept in position by a vertical valve-stem or spindle, which passes through its center, and which plays through guide-holes in bridge-pieces placed above and below the valve. A TRAP, CLACK, FLAP, or DOOR VALVE, is a plate with hinges like a door. When two such valves are used, with their hinged edges adjacent to each other, so that in opening and shutting they flap like the wings of a butterfly, they constitute a butterfly valve. A THROTTLE VALVE is one which when closed forms a partition across a pipe; and opens by partially revolving upon an axis placed along its diameter. A ROTARY VALVE works like a common stopcock. A SNIFTING VALVE is one which lets out steam under water; and is so called from the snifting noise thereby produced. The PORT VALVE is the sliding one which admits steam from the steam-chest into the cylinders. A DOUBLE SEAT, or DOUBLE-BEAT VALVE is a peculiar one with two seats, one above the other; and so arranged that the pressure of steam or water against it when shut, does not oppose its being opened. A CUP VALVE is in shape of an inverted cylindrical cup, with a length somewhat greater than its diameter. Its lower or open edge is ground to fit the seat over which it rests. As this cup rises and falls, it is kept in place by a cylindrical caging closed at top, and having for its sides four or more vertical pieces, against the inner sides of which the sides of the cup play. A CHECK VALVE is any kind so placed as to check or prevent the return of the fluid after its passage through the valve into the pipe or vessel beyond it.

Vault; an arch long in comparison with its span. The space covered by such an arch.

Veneer; a very thin sheet of ornamental wood glued over a more common variety.

Wainscot; a wooden facing to walls in rooms, instead of plaster, or over a facing of plaster; usually not more than 3 or 4 feet high above the floor.

Wales; long longitudinal timbers in the sides of a ship, coffer-dam, caisson, &c.

Wallow; a water-wheel, &c, is said to wallow when it does not revolve evenly on its journals.

Wallower. See Trundle.

Wall-plate, or *raising-plate;* a timber laid along the tops of walls for the roof trusses or rafters to rest on, so as to distribute their weight more equally upon the wall.

Warped; twisted, as a board, or the face of a stone, &c, which is not perfectly flat. *To warp;* to haul a vessel ahead by means of an anchor dropped some distance ahead. To flood an extent of ground with water for a short time to increase its fertility.

Washboards; boards nailed around the walls of rooms at the floor, so as to prevent injury to the plaster when washing the floors.

Washers; broad pieces of metal surrounding a bolt and placed between the faces of the timber through which the bolt passes, and the head and nut of the bolt, so as to distribute the pressure over a larger surface, and prevent the timber from being crushed when the bolt is tightly screwed up.

Waste-weir; an overfall provided along a canal, &c, at which the water may discharge itself in case of becoming too high by rain, &c. Sometimes called a tumbling-bay.

Watch-tackle; ropes running in different directions from a boat, and used in bringing it into a desired position.

Water-shed; the sloping ground from which rain-water descends into a stream.

Water-table; a slight projection of the lower masonry or brickwork on the outside of a wall, and reaching to a few feet above the ground surface, as a partial protection against rain, or as ornament.

Ways; the inclined timbers along which a vessel glides when being launched.

Weather-boards; boards used instead of bricks or masonry for the outsides of a building, or bridge, &c. They are nailed to vertical and inclined indoor timbers; and may be either vertical or hor. When hor, they are so placed that the lower edge of one overlaps the upper edge of the one below. When vert, their edges should be tongued and grooved; and narrow slips be nailed over the vert joints, to keep out rain, &c.

Weir, or *wier;* a dam, or an overfall.

Weld; to join two pieces of metal together by first softening them by heat, and then hammering them in contact with each other. In this operation fluxes are used.

Welt; see "butt-joint."

Wharf; a level space upon which vessels lying along its sides can discharge their cargoes; or from which they can receive them.

Wheel-base; the distance from center to center from the extreme front wheels, to the extreme hind ones in a locomotive, car, &c.

Wicket; a small door or gate made in a larger one; as the shuttle or valve in a lock-gate, for letting out the water.

Winch; a handle bent at right angles, and used for turning an axis; that of a common grindstone.

Wind. See Out of wind.

Winders; those steps (often triangular) in a staircase by which we wind, or turn angles.

Windlass; the wheel and axle, or winch and drum, as often used in common wells. Also, a horizontal shaft on shipboard, by which the anchor is raised; the windlass being revolved by means of wooden levers called *handspikes*.

Wing-dam; a projection carried out part way across a shallow stream, so as to force all the water to flow deeper through the channel thus contracted.

Wings; applied in many ways to projections. The flanges which radiate out from a gudgeon; and by which it is fastened to the shaft. Small buildings projecting from a main one. The wings or flaring wing-walls of a culvert or bridge.

Wing-walls; the retaining-walls which flare out from the ends of bridges, culverts, &c.

Wiper. See Camb.

Working-beam, or *walking-beam;* a beam vibrating vertically on a rock-shaft at its center, as seen in some steam-engines; one end of it having a connection with the piston-rod; and the other end with a crank, or with a pump-rod, &c.

Worm; the so-called endless screw, which by revolving without advancing gives motion to a cog-wheel (worm-wheel), the teeth of which catch in the thread of the screw.

Wrench; a long handle having at one end an eye or jaw which may catch hold of anything to be twisted or turned around, as a screw-nut, &c. When it has a jaw which by means of a screw is adaptable to nuts, &c, of different sizes, it is a monkey-wrench, or screw-wrench.

INDEX.

The numbers refer to the **pages.** In the alphabetical arrangement, **minor words,** as "and," "between," "in," "on," "through," etc., **are neglected.** See also Glossary, pp. 1025, etc., and Table of Contents, pp. xxv, etc.

A.

Abacus, defined, 1025.
Abrasion
  of cements, 937.
  by streams, 577, 582.
Abscissa, defined, 1025.
Absorbent bodies,
  specific gravity of—, 211.
Absorbents for nitro-glycerine, 949, 951.
Absorption
  by bricks, 927.
  by earth, etc., 329.
Abutment, Abutments,
  of arch, 617.
  batter, 619.
  courses in —, inclination of—, 620.
  of dams, 645.
  foundations for—, 582.
  masonry, 623.
  piers, 619.
  to proportion—, 617, 618.
Accelerated tests for cements, 941.
Acceleration, 334.
  of gravity, 335, 336, 348, 350, 539.
    equivalents of—, 250.
  on inclined planes, 349.
  units of—, conversion of—, 250.
Acid fumes, effect of— on roofs, 970.
Acre, Acres,
  area of—, 222.
  equivalents of—, 233.
  -foot, equivalents of—, 235.
  required for railroads, 254.
Action,
  line of—, 359.
  and reaction, 333.
Addition of fractions, 36.
Adhesion
  of cement, 934.
  of glue, 922, 1030.
  of locomotives, 413, 860.
  of mortar, 926.
  of nails and spikes, 818.
Adit, defined, 1025.
Adjustable counters, 721.
Adjustment. See the several instruments.
Adjutages, flow through—, 540.
Admiralty knot, 220.
Age, effect of— on cements, 932.
Agonic line, 301.
Air, 320.
  buoyancy, 513.
  chambers, 663.
  compressed—, 320, 597, 681.
    breathing—, 320, 597, etc.
    in diving bells, 321.
    in foundations, 596.
    in rock drills, 681.
  compressors, 681.
    manufacturers, 991.
  density, 320.
  locks, 597.
  pressure, 320, 502.
    barometer, levelling by—, 312.
    of compressed—, 320, 597, etc.
    on water surface, 502.
  in siphons, 521.
  slacking, 925.
  in tunnels, 812.
  valves, 662.
  ventilation, quantity required for—, 320.
  vessel, 663.
  volumes of unit weights of—, conversion of—, 242.
  weight, 320.
  weights of unit volumes of—, conversion of—, 242.
  wind, 321.
Alcohol, weight of—, 212.
Alioth, 285.
Allardyce process, 955.
Alligation, 40.
Alphabet, Greek—, 34.
Alternating stresses, 465, 761.
Alternation of ratios, 38.
Altitude. See Height.
  of the pole, 284.
Alumina, in cement, 930.
Aluminum
  oxide, in cement, 930.
  weight, 212.
Amortization, 43.

Amount, in interest, etc., 41.
Anchorage, Anchorages.
  of suspension bridges, 770.
  wind— in bridges, 759.
Angle, Angles, 92.
  arc, angle subtended by—, 181, 780, etc.
  blocks, 736.
  in built sections, 723.
  chords subtending—, 143, 780, etc.
  circular measure of—, 34.
  complement and supplement, 94.
  co-secants of—, 97.
  cosines of—, 97.
  co-versed sines of—, 97.
  deflection—, 780, 784–789, 840.
  degrees in—, decimals of—, 95.
  of direction, 765.
  of friction, 409.
    in arches, 432.
    in dams, 433.
  frog—, 835, 839.
  hour—, 285.
  iron, 497, 896–899, 912.
  of maximum pressure, 607.
  to measure
    with the hand, etc., 96.
    with the sextant, 152.
    with the tape line, 152.
    with the two-foot rule, etc., 96.
  minutes and seconds in decimals of a degree, 95.
  plates for rail joints, 820–823.
  in polygons, 148.
  rule, 2 ft.—, to measure— by, 96.
  secant of—, 97.
  seconds in decimals of a degree, 95.
  sines of—, 97.
    table, 98.
  of slope, 255–257.
  on sloping ground, 151.
  steel, 896, 898.
    test, 753.
  subtended by arc, 181, 780, etc.
  supplement and complement, 94.
  switch—, 827.
  symbol for—, 33.
  tangent of—, 97.
  tangential—, tables, 784–786.
  in triangles, 148.
  versed sines of—, 97.
Angular velocity, 351.
Animal power, 685.
Anneal, defined, 1025.
Annual
  earnings of railroads, 867, etc.
  expenses of railroads, 867, etc.
  magnetic variations, 301.
Annuity, Annuities, 43.
  equations, 44.
  required to redeem $1000, 46.
Antecedent, 38.
Anthracite,
  heat from—, 317.
  space occupied by—, 215.
  weight, 212, 215.
Anti-bursting device, 665.
Anticlinal axis, 1025.
Anti-component, 362.
Anti-friction rollers, 417, 725, 751, 846.
Antilogarithms, 71.
Antimony,
  strength, 920.
  weight, 212.
Anti-resultant, 362.
Apertures,
  contiguous—, flow through—, 542.
  shape of—, effect on flow, 541.
  in thin partition, 541.
Apex, apex distance, 780.
Apothecaries'
  measure, 223, 224.
  weight, 220.
Apparent solar time, 265.
Application of force, 332, 347.
  point of—, 333, 359.
Applied and imparted forces, 372.
Approach, velocity of—, 556.
Aqueduct, Aqueducts,
  flow in—, 560.
  Kutter's formula, 563.
Arc, Arcs,
  circular—, 179.
    angles subtended by—, 181, 780, etc.
    center of gravity, 391.
    chords, 143, 179, 780, etc.
    co-secant, 97.
    cosine, 97.
    cosine, table, 98.
    co-versed sine, 97.
    graduated—, 292.
    large—, to draw—, 181.
    ordinates, 180, 784, 817, 840.
    radii, 180, 781, etc.
    rise, 180.
    secant, 97.
    sine, 97.
      table, 98.
    tables, 183, 185.
    tangent, 97.
      table, 98.
    time equivalents of—, 265, 290.
    versed sine, 97.
  elliptic—, 189.
  parabolic—, 192.
  semi-elliptic—, 189.
Arch, Arches, 424, 430, etc., 613, 740.
  abutments, 617.
  angle of friction in—, 432.
  brick—, 629, 632.
  bridges, 613.
  centers for—, 631.
  concrete—, 615, 616.
  design of—, 431, 432, 613, etc.
  elliptic—, 616.
    joint in—, to draw—, 190.
  keystone, 613, 615.
  line of pressure, 430.
  line of resistance, 430.
  line of thrust, 430.
  mechanics of—, 430, 432.

Arch, Arches—continued.
  moments in—, 424.
  practical considerations, 432.
  pressure in—, 430, 614.
  pressure, line of—, 430.
  radius, to find—, 614.
  resistance line, 430.
  roofs, 740, 742.
  rubble—, 616.
  settlement of,— 432.
  statics of—, 430, 432.
  stones, 613.
    chamfering, 634.
    pressure in—, 430, etc., 614.
    pressure of—, on centers, 633.
  theory of—, 430, 432.
  thrust line, 430, 432.
Archimedes screw, 687.
Architrave, defined, 1025.
Area, Areas,
  of a circle, to find—, 161.
  of a circle, table, 163–178.
  contraction of— in iron and steel, 752, 754, 873.
  crippling—, of rivet, 775.
  of pipes, 526.
  reduction of—, 752, 754, 873.
  of sections of beams, 468, 892, etc.
  of surfaces. See the surface in question.
  unit—, equivalents of—, 233.
Arithmetic, 35.
Arithmetical complements, 71.
Arithmetical progression, 39.
Arm, lever—, 360.
Arris, defined, 1025.
Arroba, 227.
Artesian wells, 671.
Artificial
  horizon, 298.
  stone, concrete, 943.
Ascent. See Grade, Height, Slope.
  effect of— on power of horses, 683.
  effect of— on power of locomotives, 860.
Ash wood, strength, 459, 476, 957, 958.
  weight, 212.
Ashlar masonry, cost, 601, 602.
Asphaltum, weight, 212.
Atlas powder, 951.
Atmosphere, See Air, 320.
  buoyancy, 513.
  (unit of pressure), equivalents of—, 240.
  weight, 212, 320.
Augers for earth and sand, 670.
Avoirdupois weight, 220.
Axis, See the given surface or solid.
  of buoyancy, 514.
  of equilibrium, 514.
  of flotation, 514.
  neutral—, 466.
  of symmetry, 514.
Axle, friction, 416, 417.
Azimuth, Azimuths, 284–290.

B.

Back, Backs,
  of arch, defined, 613.
  of retaining walls, 603.
Backing of walls, 603.
Backstays for suspension bridges, 766.
Bag scoop or spoon, 581.
Baggage cars, 865.
Bailing by bucket, day's work, 686.
Balk, defined, 1025.
Ballast,
  for railroads, 815.
    cost of—, 855.
Balloon, principle of—, 513.
Balls, weight, 874, 877, 879, 918.
Baltimore truss, 694.
Bar, Bars,
  in built-up sections, 723.
  dredging, 580.
  iron—, weight, 877, 878.
Barbed fence, 854.
Bargeboards, defined, 1025.
Barometer, 312, 320.
Barrel, contents, 223.
Barrow. See Wheelbarrow.
Barschall process, 955.
Bascule or lift bridges, 697.
Base, Bases,
  of logarithmic systems, 72.
  wheel— of locomotives, etc., 856.
Batten plates, 724.
Batter,
  of abutments, 619.
  defined, 1025.
  of retaining walls, 603, etc.
Bazin's formula, 552.
Beam, Beams,
  breaking loads, coefficients, 476.
  channel—, 497, 894, 912.
  coefficients for breaking loads, 476.
  concrete—, strength of—, 945.
  continuous—, 489.
  curved—, 446.
  deflections, 481, 959.
    loads for given—, 480.
    under sudden loading, 460, 959.
  elastic limit, 482.
  elasticity, modulus of—, 457.
  end reactions, 439.
  equilibrium of—, 437.
  floor—, 720, 749.
  forces acting upon—, 437.
  granite—, 924.
  I— and channel—, 892, etc., 912.
    in fire-proof floors, 894.
    as pillars, 497, 902, 912.
    separators for—, 900.
    tables, 892, etc.
  inclined—, 445.

Beam, Beams—continued.
  iron—, safe loads and deflections, 960, 962.
  limit of elasticity, 482.
  loads, 437, etc., 466, etc., 476, 760, 762, 892, etc.
    for given deflection, 480.
    within limit of elasticity, 482.
    suddenly applied—, 460, 959.
  mechanics of—, 437, etc., 466, etc.
  modulus of elasticity, 457.
  moments, 440, 443, 445.
  reactions, 439.
  rolled—. See Beams, I—; Beams, Channel—; etc.
  shear, 446.
  steel and iron—, loads, 892, etc., 960, 962.
  stone—, 476, 924.
  strength. See Beams, Loads.
  suddenly loaded, 460, 959.
  timber—, See Beams, Wooden—.
  and trusses, comparison, 689.
  of uniform strength, 486.
  wooden—, 760, 762, 764.
    deflections, table, 959.
    loads, tables, 959, 962.
Bearing, Bearings,
  of piles, 590.
  power of soils, 583, 593.
  and reverse bearing, 277.
  stresses, permissible—, 762.
  in trusses, 721, 725, 750, 751.
Beats, of clocks and watches, 266.
Beaumé hydrometer, 211.
Bed plates, 721, 750.
Beech-wood, strength, 459, 476, 957, 958.
Beetle, defined, 1025.
Bell,
  diving—, pressure in—, 321, 597.
  joint for pipes, 660.
Belting, cost and mfrs., 987.
Belts, leather—, strength, 922.
Bench mark, 1025.
Bending
  of beams. See Beams, Deflections.
  and compression, combined, 724.
  moments, 466, etc.
  stresses in bridge members, permissible—, 762.
  tests, 871.
    bridge steel, 752.
    iron and steel, 873.
    steel castings, 754.
Bends in water pipes, 537.
Bents in trestles, 814.
Berm, defined, 1025.
Beton, defined, 1025.
  See Concrete.
Beveled joints for rails, 819.
Bibliography, 1008.
Bilge, defined, 1025.
Birch, strength, 459, 476, 957, 958.
Birmingham gauges, 887, 890.
Bismuth,
  strength, 920.
  weight, 212.
Bitumen, weight, 212.
Bituminous coal,
  space occupied by—, 215.
  weight of—, 212, 222.
Black-line prints, 982.
Blasting, 600, 953.
  apparatus, 952.
Bled timber, strength, 957.
Blocks, angle—, 736.
Bloom ton, 216.
Blue-line prints, 982.
Blue prints, 979.
Board measure, table, 269.
Boasting, defined, 1025.
Boat, canal—, 684.
Body, Bodies. See the body in question.
  defined, 330.
  expansion of— by heat, 317.
  falling—, 348, 539.
  floating—, 513.
  rigid—, force in—, 330, 358.
Boiler, Boilers,
  cost and mfrs., 988.
  incrustation, 327.
  iron—, 872.
  thickness, 511.
  tubes, 882
Boiling point, 326.
  leveling by—, 314.
Boiling tests for cement, 941.
Bollman truss, 695.
Bolster, defined, 1026.
Bolts, 883, etc.
  cost and mfrs., 986.
  expansion—, 884.
  iron—, table, 886.
  strength and weight, table, 886.
  stresses, permissible—, 762.
Bonnet, defined, 1026.
Bonzano rail-joint, 823.
Books, 1008.
Boring,
  artesian wells, 671.
  augers for earth—, 670, 671.
  test—, 582, 670.
  wells, 671.
Borrow pit,
  defined, 1026.
  to measure—, 195, 196.
Boss, defined, 1026.
Bottom,
  heading, 812.
  of stream, scour on—, 577.
  velocity, 560.
Bowstring,
  centers, 637, 638.
  truss, 695, 699.
Box
  cars, 865.
  drains, 627.
  sextant, 297.
Boxed timber, strength, 957.

Bracing.
  in bridges, 691, 710, 748, 749.
  counter—, See Counter-bracing.
  cross—, 691.
  for dams, 502.
Brad spikes, 818.
Brake friction, 412.
Branches in pipes, 661.
Brass,
  balls, weight, 918.
  ductility, etc., 459.
  effect of mortar, etc., on—, 936.
  effect of water on—, 327.
  expansion, by heat, 317.
  friction, 411, 415.
  pipes and tubes, seamless—, 919.
  strength, 476, 920, 921.
  tubes, seamless—, 919.
  weight, 212, 875–878, 887.
  wire, 887.
Brasses, defined, 1026.
Braze, defined, 1026.
Breast-wall, defined, 1026.
Breathing,
  air consumed in—, 320.
  in diving-bells, 320.
Brick, Bricks, 926, 927.
  arches, 629, 632.
  cost and mfrs., 985.
  cylinders, sinking of—, 599.
  dust, 925.
  friction, 411.
  incrustations, 929, 936.
  laying, 927.
  strength, 476, 922, 923.
  weight, 212, 213.
  work, 927.
    mortar required for—, 931.
    weight of—, 213.
Bridge, Bridges. See also Arch, Beam, Girder, Trestle, Truss, etc.
  arch—, 613.
  brick—, 629, 632.
  Brooklyn—, foundations, 598.
  camber, 726, 746.
  centers, 631.
  clearance, 746.
  combination—, specifications, 763.
  cross-section, 746.
  design, 720, 745.
  electric railway—, specifications, 745.
  erection of—, 743, 763.
  friction rollers, 725, 751.
  gage on—, 746.
  headway on—, 746.
  highway—, specifications, 745.
  joints (connections), 774.
  manufacturers, 988.
  painting, 763, 764.
  protection, 763, 764.
  railroad—, specifications, 745.
  roadways, drainage, 628.
  specifications, digest of—, 745.
  stone—, 613. See also Arch.
  stone—, centers, 631.
Bridge, Bridges—continued.
  suspension—, 765.
    cables of—, 765, 891, 976, 977.
  test of completed—, 753.
  trusses, 689.
  weight of—, 731, 738.
  Wissahickon—, Phila., 640.
  wooden—, specifications for—, 763.
Briggs logarithms, 70, 78, 80, etc.
Briquettes, cement—, 938, etc.
British
  Imperial measure, 223, 224.
  rod of brickwork, 222, 928.
Broken
  bubble tube, to replace—, 296.
  cross-hairs, to replace—, 296.
  joints, 819.
  stone. See also Rubble, 583.
    for concrete, 943, 945.
    foundations, 583.
    voids in—, 688, 943.
Bronze,
  phosphor— wire, strength, 920.
  weight, 212.
Brooklyn bridge, foundations, 598.
Bubble tube, to replace—, 296.
Buckle plates, 750, 885.
Builder's level, to adjust—, 311.
Building, Buildings,
  specifications for—, 764.
Built beams, 479, 734.
Built sections, truss members, 722.
Bulk, increase of—, broken stone, etc., 688, 943.
Buoyancy,
  of air, 513.
  of liquids, 210, 513, 515.
Burkli-Ziegler formula, 575.
Burned clay, 925.
Burnettizing, 955.
Burr truss, 695.
Bursting,
  anti— device, 665.
  of pipes, 513, 663, 665, 668.
Bush, defined, 1026.
Bushel, equivalents of—, 234.
  volume of—, 223.
Business directory, 983, 996.
Butt-joint, 773.
Buttresses, 612.

C.

Cable, Cables,
  number of wires in—, 891.
  stays, 766.
Caisson, 585.
  Brooklyn bridge, 598.
  work in—. See Diving-bell, 321.
Calcium carbonate, in cement, 930.
Calcium oxide, in cement, 930.
Calking, 660.
Camber, 696, 726, 746.
Camel, defined, 1026.

Canal—Chuck.

Canal, Canals,
  boats, 684.
  flow in—, 560.
    Kutter's formula, 523, 563.
  leakage from—, 329, 561.
  traction on—, 683.
Cantara, 227.
Cantilever, Cantilevers. See also Beams.
  bridges, 696.
  end reactions, 439.
  equilibrium of—, 437.
  forces acting upon—, 437.
  moments, 440, 442, 445.
  reactions, 439.
  of uniform strength, 486.
Cap, for blasting, 952.
Capitalization, 41, 44.
Car, Cars, 865.
  derrick—, 808.
  earthwork, 807.
  friction, 417.
  mfrs., 990.
  pressed steel—, 865.
  resistance, 417.
  wrecking, 808.
Carat, 219, 220.
Carbon,
  dioxide, in cement, 930.
  in steel, 872.
  in steel castings, 754.
Carbonic acid, in cement, 930.
Carnegie beams, channels, etc., 892, etc.
Cart, Carts,
  earthwork, 800, 802, 805.
  excavating (wheeled scrapers), 805.
  road, repairs of—, 801.
  rock, removing, 810.
  traction, 683.
Cartridge,
  dynamite—, 950.
  rack-a-rock, 951.
Cassiopeia, 285.
Castellano, 227.
Castelli's quadrant, 561.
Casting, Castings,
  safety—, 824, 833.
  steel—, requirements, 754, 872.
  weight of—, by size of pattern, 875.
Cast-iron. See Iron, Cast—.
Cattle cars, 865.
Cedar, strength, 476, 957, 958.
Cement, 930–942.
  brick-dust—, 925.
  concrete, 943.
  cost and mfrs., 985.
  and iron pipes, 657.
  for leaks, 971, 973.
  moisture, effect on—, 929.
  slag—, 940.
  strength, 923, 932, etc.
  weight, 212, 934, etc.
Center, Centers,
  for arches, 631.
Center, Centers—continued.
  of buoyancy, 514.
  of circle, to find—, 161.
  for fire-proof floors, 895.
  of force, 399.
  of gravity, 386.
  of gyration, 496.
  of moments, 361.
  of oscillation, 351.
  of percussion, 351.
  of pressure, 399, 501, 506, 514.
Centigrade thermometer, 318.
Centigram, equivalents of—, 236.
Centiliter, 235.
Centimeter,
  centigram, etc., definitions, 231.
  cubic—, weights, 212.
  equivalents of—, 233.
Central forces, 354.
Centre. See Center.
Centrifugal force, 354, 711, 758.
Centripetal force, 354.
Chain, Chains,
  cost and mfrs., 987.
  equivalents of—, 232.
  iron—, 915
  loaded—, 428.
  pump, 687.
  riveting, 774.
  strength of—, 915.
  surveying, 274, 282.
  of suspension bridges, 765.
  weight of—, 915.
Chaining, 282.
Chalk,
  strength, 923.
  weight, 212.
Chamfer, defined, 1027.
Chamfering arch stones, 634.
Channel, Channels,
  flow in—, 523, 560.
    Kutter's formula, 523, 563.
  steel— as pillars, 497, 912.
    table, 894.
Channeling in rock, 681.
Characteristics, 70–79.
Charcoal, weight, 212.
Chart, Charts,
  isogonic—, U. S., 300.
  logarithmic—, 73.
Checking of cement, 938.
Cherry wood, weight, 212.
Chestnut. See Wood.
Chilling, defined, 1027.
Chock, defined, 1027.
Chord, Chords,
  of arcs, to find—, 179.
  in circles, 162, 179.
  in curves, 780, etc.
  increments, 701.
  long—, table, 787.
  members. See Trusses.
  to radius 1, table, 143.
  stresses, 701, 709.
  of trusses, 689.
Chronometers, behavior, 266.
Chuck, defined, 1027.

Churn Drilling—Component.

Churn drilling, 600.
Cippoletti trapezoidal notch, 559.
Circle, Circles, 161, etc. See also Circular.
  angles in—, 94.
  chords, 162, 179.
  great—, 184, 284.
  radius, 161, 180.
  tables, 163–178.
  vertical—, 284.
Circular
  arcs. See Arc, Circular—.
    tables, 183, 185.
  curves for railroads, 780.
  inch, 222, 889.
  lune, 186.
  measure of angles, 34.
  measure of wires, etc., 889.
  motion, 351.
  ordinates, 180.
    tables, 784, 786, 817, 840.
  plates, strength, 493.
  rings, 186.
  sector, 186.
    center of gravity, 393.
  segment, 186.
    center of gravity, 394.
    table, 187.
  spindle, 209.
  zone, 186.
Circulating decimals, 38.
Circumference
  of circle, to find—, 161, 171, 178.
  of ellipse, 189.
Cisterns, 512, 851, 854.
Civil day, month, time, year, 266.
Clamp, Clamps,
  pouring—, for pipe joints, 660.
  rod, switch, 825.
Clay,
  effect on mortar, 925, 926, 935, 936.
  in foundations, 583.
  loosening of—, 800.
  swelling of— by absorption, 583.
Clearance,
  in highway bridges, 726.
  in railroad bridges, 746.
Clearing, cost, 855.
Cleat, defined, 1027.
Climate, effect of— on rainfall, 322.
Clinometer, 256, 311.
Clips, for cement briquettes, 939.
Clock,
  to regulate— by star, 266.
  -wise, 360.
Close piles, 590.
Cloth, tracing—, 978.
Coal,
  cars, 865.
  consumption of— by locomotives, 860, etc.
  corrosive fumes from—, 880, 916.
  for locomotives, 856.
  oil, weight (petroleum), 214.
  space occupied by ton of—, 215.
  ton of—, volume, 215, 222.
  weight, 212, 222.
Cocks, corporation—, 664.
Coefficient, Coefficients,
  of contraction, 542.
  defined, 1027.
  of deflection, 483.
  of friction, 408.
  of kinetic friction, 409.
  for loads within elastic limit, 482.
  of roughness, 523, 564, 565.
    for pipes, 523.
  of safety. See Safety, Factor of—.
  of stability, 423.
Coffer-dam, 585, 586.
  defined, 1027.
Cohesion, 957.
Coin, Coins, 218, 219.
Coke, weight, 212.
Cold,
  effects of—,
    on cement, 932.
    on explosives, 948, 950.
    on iron, 819, 874.
    on mortar, 928.
  -rolled iron, 920.
Colinear forces, 361, 363.
Collision, impact, 347.
  posts, 695.
Cologarithm, 71.
Colors,
  draughtsmen's, 978.
Column, Columns (pillars). See Pillars.
  Gray—, 905.
  Phœnix—, 497, 904, 912, 913.
  water—, 852.
  Z-Bar—, 901–903.
Combination
  bridges, 738.
    specifications, 763.
  and permutation, 40.
Combined stresses, 493, 724.
Commercial
  measures, size of— by weight of water, 224.
  weight, 220.
Common
  denominator, 35.
    to reduce to—, 36.
  divisor, 35.
  factor, 35.
  fraction, 36, 37.
  logarithms, 70–91.
  measure, 35.
  multiple, 35.
Compass,
  to adjust, 298.
  declination, 301.
  variation, 301.
Compensating reservoir, 653.
Compensation, water, 653.
Complement and supplement, 94.
Component, Components.
  equations for—, 365.
  of force, 362.
  normal—, 370.
  rectangular—, 369.

Component—Co-tangent.

Component, Components—cont'd.
  stress—, 371.
  summation of—, 466.
  tangential—, 369.
Composition
  of couples, 405.
  of forces, 362, etc.
  of ratios, 38.
  of steel, 753.
Compound
  interest, 42.
    equations, 44.
    table, 43.
  levers, 420.
  locomotives, 856.
  stresses, 762.
Compressed
  air, 320, 597, 681.
  gun-cotton, 951.
Compressibility
  of air, 320.
  of liquids, 326.
  of sand, 935.
Compression
  and bending, combined, 493, 724.
  members, 721, 722, 732, 747.
  and tension, 359.
Compressive strength
  of cements, 934.
  of concrete, 943.
  of metals, 921.
  of stone, etc., 923.
  of timber, 958.
Compressors, air—, 681, 991.
Concentrated
  excess loads, 705.
  loads, deflections, 484.
  loads, moments due to—, 444.
  loads, shears in beams, 446.
Concrete, 943.
  beams, strength of—, 945.
  defined, 1027.
  -metal construction, cost, 989.
  mixers, 943.
    cost and mfrs., 992.
  strength, compressive—, 923, 943, etc.
  strength, transverse, 945.
Concretions in pipes, 655.
Concurrent forces, 361, 364, 380
Cone, Cones, 200.
  center of gravity of—, 395, 397.
  frustum, 201, 395, 397.
Conical rollers, 846.
Connections, pin and riveted, 721.
Conoid,
  frustum of—, 209.
  parabolic—, 209.
Consequent, 38.
Consolidation locomotives, 857.
Constants. See Coefficients.
Construction,
  bridge—, 720.
  railroad—, cost, 855.
Consumption
  of fuel by locomotives, 860.
  of water, 649.
Contiguous openings, flow thro—, 542.
Continued proportion, 38.
Continuous beams, 489.
Contour lines, 302.
Contracted vein, 541.
Contraction,
  of area, iron and steel, 752, 754, 873.
  coefficient of—, 542.
  by cold, 317, 819.
  incomplete—, 540, 544.
  of outflow, 541.
  of rails, 819.
  of waterway, 623.
Contractor's profit, 801.
Conversion tables of units of weights and measures, etc., 228.
Conveying machinery, cost and mfrs., 991.
Cooper's standard loading, 755.
Co-ordinates in resolution of forces, 372.
Coping,
  defined, 1027.
Coplanar forces, 361.
Copper,
  balls, weight, 918.
  compressibility, etc., 459.
  cost, 919, 987.
  effect of cement, mortar, etc., on—, 926, 936.
  effect of water on—, 327.
  expansion of— by heat, 317.
  pipes, seamless—, 919.
  roofs, 918.
  sheets, 918.
  strength, 499, 500, 920, 921.
  sulphate, for wood, 955.
  tubes, seamless—, 919.
  weight, 212, 875, 878, 886, 887, 918.
Corbel, defined, 1027.
Cord,
  loaded—, 428.
  mechanics of the—, 425.
  polygon, 377, 428.
  of wood, 222, 234.
Cork,
  weight, 212.
Corporation cocks or stops, 664.
Corrections for tapes, 283.
Corrosion,
  by acid fumes, 970.
  by coal fumes, 880, 916.
  by water, 327, 594.
Corrugated
  flooring, 914.
  sheet-iron, 880.
Co-secants, 97.
Cosines, 97, 98.
  logarithmic—, 72.
Cost, Costs. See articles in question.
  price list, 983.
  of operation of railroads, 867.
Co-tangent, 97, etc.

- Cotter-bolt, defined, 1027.
- Counter, Counters,
  - adjustable—, 721.
  - -bracing, 690, 705, 712, 721, 738, 746.
    - of centers, 634.
  - -clockwise, 360.
  - -scarp revetment, 612.
  - sloping revetment, 612.
- Counterforts, 612, 1027.
- Couples, 404.
- Couplings for pipes and tubes, 657, 882.
- Courses
  - of masonry, 603, 620.
- Cover
  - in a butt-joint, 773.
  - plate, 723.
- Co-versed sines, 97.
- Crab, defined, 1028.
- Cradle, defined, 1028.
- Cramp, defined, 1028.
- Crane, defined, 1028.
- Creeping
  - of rails, 819, 820.
- Creo-resinate process, 955.
- Creosote, 815, 954, 984.
- Crescent truss, 695.
- Crib, Cribs,
  - coffer-dam, 645.
  - dams, cost, 645.
  - foundations, 584, 585.
- Criterion,
  - for maximum chord stresses, 709.
  - for maximum web stresses, 706.
- Cross
  - bracing, 691, 710, 748.
  - girts, turntable, 846.
  - -hairs, in level, 306.
  - -hairs, to replace—, 296.
  - section of bridge, 746.
  - section paper, 978.
    - logarithmic, 73.
  - -shaped beam, 492.
  - ties, 815, 855.
- Crowd, weight of—, 726.
- Crown,
  - of arch, defined, 613.
  - (coin), value, 218.
- Crushers, stone—, 943, 992.
- Crushing
  - loads, 923, 934, 943, 958.
  - in timber construction, 732.
- Cube, Cubes, 55, 194, 195.
  - roots, 54, etc. See also Powers.
    - of decimals, to find—, 67.
    - of large numbers, to find—, 66.
    - tables, 54, etc.
  - tables, 55.
- Cubic
  - centimeter, foot, inch, etc. See Conversion Tables.
  - measure, 222.
    - metric, 225.
  - meter, etc. See Meter, etc.
- Culmination of Polaris, 287, 288.
- Culmination
  - of a star, 284.
- Culvert, Culverts,
  - arches for—, 613.
  - box—, 627.
  - foundations of—, 627.
  - lengths of—, 622.
  - quantity of masonry in—, 622.
- Curbs
  - in highway bridges, 750.
- Current meters, 562, cost, etc., 993.
- Curvature of the earth, table, 153.
- Curve, Curves. See Arc, Circle, Ellipse, Parabola, etc.
  - effect of— on distribution of live load on bridge, 712, 756.
  - elastic—, 483.
  - railroad—, 780.
    - gauge on—, 789.
    - tables of—, 784–789.
    - in tunnels, 812.
    - in turnouts, 840.
  - in water pipes, 537.
- Curved
  - beams, 446.
  - chords in trusses, 695.
- Curvilinear motion, 351.
- Cuttings,
  - level—, 790.
- Cutwater, defined, 1028.
- Cycloid, 194.
  - center of gravity, 394.
- Cylinder, Cylinders, 196.
  - contents, table, 197, 223, 525.
  - in foundations, 593, 594, 596, 599. See also Foundations.
  - of locomotives, 856, 861.
  - pneumatic process, 596.
  - pressure in—, 511.
    - locomotive—, 861.
  - strength of—, 511.
- Cylindrical
  - beams, deflections, 485.
  - pillars, 497, 912, 913.
  - ungula, 199.
    - center of gravity of—, 397.
- Cyma, to draw—, 191.

D.

- Dam, Dams, 400, etc., 430, etc., 433, etc., 502, 576, 642.
  - center of pressure against—, 400.
  - coffer—, 585, 586.
  - construction, 585, 642.
  - danger in—, 436.
  - deflection, 436.
  - discharge over—, 547.
  - height of water, 554.
  - leakage through—, 329, 651.
  - masonry—, 400, etc., 430, etc., 433, etc.
  - practical considerations, 436.
  - stability, 433, 508.

Dam, Dams—continued.
  stone, 400, etc., 430, etc., 433, etc.
  timber, 642.
  trembling, 648.
  walls, 508, 611.
  wooden, 642.
Day, Days, 236, 265, 266.
Dead load, Dead loads,
  for bridges, 690, 755.
  in cars, 865.
  defined, 1028.
  in roof trusses, 713.
  stresses, graphic method, 703.
Dead point, defined, 1028.
Decagon, 148.
Decay of timber, 954.
Decigram, equivalents of—, 231, 236.
Deciliter, equivalents of—, 235.
Decimal, Decimals, 37.
  of a degree, mins. and secs. in—, 95.
  of a foot, inches expressed in—, table, 221.
  fractions, 37.
  logarithms, 70.
  roots of—, 67.
Decimeter, equivalents of—, 233.
Deck trusses, 692.
Declination, 284.
  magnetic—, 301.
  of a star, formula, 290.
Deepened beams, 479, 734.
Deflection, Deflections,
  angle, 780, 784–789, 840.
  of arch, 436.
  of beams, 480, 483, 486.
    of uniform strength, 486.
  of cantilevers of uniform strength, 486.
  coefficient, 483.
  of cross-shaped beams, 492.
  of dam, 436.
  distance, 781.
    tables, 784–786.
  and fiber stress, 481.
  of suspension bridges, 765.
  of trusses, 718.
Degree, Degrees,
  of latitude, length of—, 184, 220.
  of longitude, length of—, 221.
  mins. and secs. in decimals of—, 95.
Dekagram, equivalents of—, 231, 236.
Dekaliter or centistere, equivalents of—, 235.
Dekameter, Dekameters, 225.
  chord of 2—, curve, table, 786.
  equivalents of—, 233.
Dekastere or hectoliter, equivalents of—, 235.
Delta Cassiopeia, 285.
Demagnetization of compass needle, 302.
Demi-revetment, 612.
Denominator, 35.
Density
  of air, 320.
  relative—, 210.
Departures and latitudes, 274.
Depreciation, 43, 864.
Depth, Depths,
  for beams, 477.
  conversion table, 239.
  on dams, 554.
  effective—, 759.
  equivalents of—, in volumes per surface, 250.
  flow at different—, 560.
  for given deflection, 483.
  hydraulic mean—, 564.
  of keystone, 613.
  for permissible deflection, 485.
  pressure at different—, 501, 648.
Derrick,
  car, 808.
  defined, 1028.
Detrusion, 499.
Dew-point, 321.
Diagonal, Diagonals,
  counterbraces, 712.
  of parallelogram, 95, 157.
  of trapezoid, etc., 158.
  in trusses, 689.
Diagram, Diagrams,
  for dead load stresses, 703.
  influence— for beams, 449.
    for pressure distribution, 403.
  for Kutter's formula, 570.
  for live load stresses, 706.
  moment and shear, 449.
  for shear in trusses, 702.
  truss—, 707.
  wheel—, 706.
Dial, to make—, 268.
Diameter, Diameters,
  of bolts, 883.
  of circle, to find, 161.
  of pipes, 524, 882, 918.
    actual and nominal—, 526, 882.
  of rivets for safety, 775.
  sq. roots of—, 526.
  of wire, 887–891.
Diamond drill, 675.
Dike, defined, 1029.
Dimensions. See the article in question.
  for beams, 477.
  limiting—, in bridge members, 762.
  in roof trusses, 764.
  in wooden bridges, 764.
Dip, defined, 1028.
Dipper, (constellation), 285.
Direction, angle of—, 765.
Directory, business—, 996.
Discharge, Discharges,
  through adjutages, 540.
  through channels, 560.
  through contiguous openings, 542.
  over dams, 547.
  head for a given—, to find—, 527.

Discharge, Discharges—continued.
  through notches, 559.
  through orifices, 539, 546.
  through pipes, 516, etc.
  through sewers, 574.
  through short tubes, 540.
  tables of—, 261–265.
  through thin partition, 541.
  units of rates of—, conversion of—, 243.
  over weirs, 547, etc.
Disks, centrifugal force in—, 355.
Distance, Distances,
  deflection—, tangential—, 781.
  frog—, 839.
  polar—, 284.
  by sound, 316.
Distributing reservoirs, 653.
Distribution of pressure, 400.
Diurnal magnetic variations, 301.
Diving
  apparatus, cost and mfrs., 992.
  -bell, 321.
  dress, 992.
Division
  of decimals, 37.
  of fractions, 36.
  by logarithms, 71.
  by logarithmic chart, or slide rule, 75.
  of a modified logarithm, 72.
  of ratios, 38.
Divisor, common, 35.
Dodecagon, 148.
Dodecahedron, 194.
Dog-iron, defined, 1028.
Dollar,
  U. S.—, weight, etc., 219.
  value of—, 218.
Dolomitic limestones, 931.
Dome, pneumatic—, 665.
Donkey engine, defined, 1029.
Double
  float, 561.
  intersection trusses, 694.
  riveting, 772.
  rule of three, 39.
  shear, 499, 774.
Dovetail, defined, 1029.
Dowel, defined, 1029.
Draft
  of horses, 683, 685.
  of locomotives, 860.
  of vessels, 515.
Drag
  scrapers, earthwork by—, 805.
  of train on bridge, 711, 758.
Drain, Drains,
  area drained by—, 575.
  box—, 627.
  foundations of—, 627.
  pipe, 575.
Drainage
  of roadways of bridges, 628.
  sewers, 574.
  of tunnels, 812.
Draw-bridges, 696.
Drawing
  instruments, cost and mfrs., 993.
  materials, 978.
Drawn pipes and tubes, 919.
Dredge, Dredges, 580.
  land—, 808.
  mfrs., 992.
Dredging, 580.
  by screw-pan, 596.
Dress, diving—, 992.
Dressing of stone, 601.
Drift, defined, 1029.
Drifting test, 752.
Drill, Drills,
  cost and mfrs., 989.
  rock—, 600, 675.
Drilling,
  artesian well—, 671.
  rock—, 300, 670, 675.
  tunnel—, 812.
Driving
  wheels, 856.
    weights on—, 705, etc., 755, etc., 856, etc.
Drop
  tests, 871.
  timbers, 644.
Drowned or submerged weirs, 554.
Dry
  drains, 627.
  measure, 223.
  rot, defined, 954, 1029.
Dualin, 952.
Dubuat's formula, 555.
Ducat, value of—, 218.
Ductility, 455, 459.
Dump-cars, 865.
Duodecimals, duodenal or duodenary notation, 47.
Duplicate ratio, 38.
Dyke, defined, 1029.
Dynamics, 330.
Dynamite, 949, 984.

E.

E and W line, to run—, 277.
Earnings of railroads, 867.
Earth,
  augers, 670.
  bearing power, 583.
  blasting, 950.
  boring, 670.
  cars (dump-cars), 865.
  curvature, table, 153.
  friction, 612, 683.
  hauling, 801.
  heat of—, 320.
  leakage through—, 329, 651.
  leveling of—, 801.
  loosening of—, 800.
  natural slope, 419, 607, 610.
  pressure, 607.
  resistance of—, 583.
  shoveling of—, 800.
  shrinkage, 799.

- Earth—continued.
  - slope of—, natural—, 607, 610.
  - supporting power, 583.
  - weight, 212.
  - -work, 790–811.
    - cost, 800, 855, 988.
    - in tunnels, 812.
    - volume of—, 790, etc.
- East and west line, to run—, 277.
- Eastern elongation, 284.
- Easting, 274.
- Eccentric,
  - defined, 1029.
  - loads, 712.
    - deflections, 484.
- Efflorescence, 929, 936.
- Effort, total— of force, 371.
- Elastic
  - curve, 483.
  - deflection, trusses, 718.
  - limit, 458, 459, 482.
    - bridge steel, 752.
    - cast iron, 874.
    - iron and steel, 873.
    - steel castings, 754.
  - ratio, 458, 461.
- Elasticity,
  - limit of—, 458, 459, 482.
    - in beams, 482.
  - modulus of—, 456, 459.
    - cast iron, 874.
- Electric
  - blasting machine, 952.
  - railroad bridges, loads for—, 757.
- Electricity in compass box, 302.
- Elevation of outer rail on curves, 787.
- Ellipse, 189, 190.
  - false—, to draw—, 191.
  - ordinate, 189.
  - tangent to—, to draw—, 190.
- Ellipsoid, 208
- Elliptic
  - arc, 189.
    - ordinates, 189.
    - table, 190
  - arch, 616.
    - joints in—, to draw—, 190.
- Elm wood,
  - strength, 459, 476, 957, 958.
  - weight, 212.
- Elongation,
  - bridge steel, 752.
  - by heat, 317.
  - polar distances and azimuths of Polaris at—, table of—, 290.
  - of Polaris, location of meridian by—, 286.
  - of Polaris, times of—, 288.
  - required, iron and steel, 873.
  - of a star, 284.
  - in steel castings, 754.
  - under tension, 455.
  - of truss members, 718.
- Embankment, 790–811.
  - cost, 800.
  - shrinkage, 799.
- End
  - post, design, 723.
  - reactions, 360, 439, 699, 702, 714.
- Energy, 343.
  - kinetic—, 343.
  - potential—, 346.
- Engine, Engines,
  - cost and mfrs., 990.
  - locomotive—, 856.
    - dimensions, 856.
    - performance, 860.
    - weight, 856.
  - pumping—, 852.
  - wheel loads, 705.
- Entry head, 516.
- Equal
  - altitudes, location of meridian by any star at—, 287.
  - shadows from the sun, location of meridian by—, 288.
- Equality of ratios, 38.
- Equation
  - of payments, 42.
  - of time, 265.
- Equilibrium, 358.
  - of beams and trusses, 437, 466, 698.
  - of couples, 405.
  - of floating bodies,
    - axis of—, 514.
  - indifferent—, 387, 514.
  - in levers, 419.
  - of moments, 360.
  - polygon, trusses, 707.
  - stable—, 387, 514.
  - unstable—, 387, 514.
  - vertical of—, 514.
- Equipment, railroad—, cost, 855, 867.
- Equivalence of work,
  - in trusses, 718.
- Equivalents. See Conversion Tables, 230, 231, etc.
- Erection of bridges, 743, 763.
- Erie R. R. locomotive standard, 858.
- Establishment of a port, 328.
- Evaporation, 329, 561.
  - by locomotives, 854.
- Even joints, 819.
- Evolution by logarithms, 71.
- Excavating carts (wheeled scrapers), 805.
- Excavation, 790–811.
  - cost of—, 800, 855, 988.
  - in tunnels, 812.
  - volume, 790.
- Excavators,
  - mfrs., 992.
  - steam— (land dredge), 808.
- Excess loads, concentrated—, 705.
- Expansion
  - bearings, 721, 725, 751.
  - bolts, 884.
  - of cement, 937.
  - by heat, 317. See Heat.
    - of rails, 819.

Expense, Expenses,
  locomotive running—, 864.
  railroad—, 867.
Exploder, Exploders, 952.
Explosive, Explosives, 948.
  cost and mfrs., 984.
Express cars, 865.
Extrados, 613.
Extreme fiber stress, permissible—, 759.
Extremes of ratio and proportion, 38.
Eye-bars, 721, 747.
  design, 722.
  full size—, test of—, 753.

F.

Face of arch, 613.
Face wall, 603.
Facing switch, 824.
Factor, Factors,
  common—, 35.
  and multiples, 35.
  safety—,
    for piles, 593.
    for pillars, 909, 912.
      See also Safety, factor of—.
Fahrenheit thermometer, 318.
Fall, Falls,
  defined, 1029.
  required for a given discharge, 527, 566, 573.
  in sewers, 574.
Falling
  bodies, 348, 539.
  water, horse power, 578.
False
  ellipse, to draw—, 191.
  -works, 743.
    defined, 1029.
Fanega, 227.
Fascines, 599.
  defined, 1029.
Fathom, 232.
Fatigue of materials, 465.
  defined, 1029.
Faucet in pipe joint, 660.
Feather, defined, 1029.
Feet. See Foot.
Felloe or Felly, defined, 1029.
Fence, 854.
Fencing, 987.
Ferris-Pitot meter, 536.
Ferrule,
  defined, 1029.
  for water pipe, 664.
Fiber
  reactions, 466.
  stress, 466, 467, etc.
    and deflection, 481.
    permissible—, 759.
Field tests for cements, 942.
Fifth powers and roots, 67–69.
Figure, Figures, 148.
  areas of—, 160.
Figure, Figures—continued.
  defined, 92.
  to draw—, 159.
  to enlarge—, 160.
  irregular—, to find area of—, 160.
Filler in pin joints, 725.
Filling, spandrel—, 613.
Filters, mfrs., 994.
Fineness,
  of cement, 938, 940.
  of sand and cement, 937.
Finish, hard—, 968.
Fink truss, 695.
Fir, strength, 957, etc.
Fire, Fires,
  heat of—, 317.
  hydrant (fire-plug), 669.
  -proof floors, 894.
  -proofing, cost, 989.
  protection, water for—, 650.
Firing, simultaneous—of blasts, 952.
Fish-plates, 820.
Fittings for pipes, 656, 882.
Flagging,
  strength of—, 476.
Flashings, defined, 1029.
Flasks, casting—, defined, 1029.
Flats, in built-up sections, 723.
Flexible joints for pipes, 661.
Floating
  bodies, 513.
  mills, 578.
Floats, 560, 561.
Floor, Floors,
  beams, 720, 749.
    connections, 730.
  bridge—, 720, 749.
  fire-proof—, 894.
  glass—, 974.
  sections, rolled—, 914.
  systems of bridges, 720, 749.
  trough—, 750, 914.
  wooden— in bridges, 750.
Flooring,
  corrugated—, 914.
  Z-bar—, 914.
Florin, value, 218.
Flotation, 513.
Flow,
  through adjutages, 540.
  in channels, 560.
  through contiguous openings, 542.
  full—, 540.
  Kutter's formula, 523, 563, 564.
  obstructions to—, 537, 576, 578.
  through orifices, 539, 546.
  in pipes, 516.
  in sewers, 574.
  through short tubes, 540.
  in streams, 560.
  in syphon, 520.
  through thin partition, 541.
  in trough, 544.
  over weirs, formula, 549.
Fluid, Fluids,
  friction of—, 415.
Flume, defined, 1030.

- **Flush**, defined, 1030.
- **Fluxes**, defined, 1030.
- **Fly-wheels**, centrifugal force, 355.
- **Follower**, in pile driving, 594.
- **Foot, Feet,**
  - cubic—,
    - equivalents of—, 222, 234.
    - of substances, weight of—, 212.
  - equivalents of—, 232.
  - of mercury (pressure), equivalents of—, 241.
  - per mile, equivalents of—, 237.
  - per second, equivalents of—, 242.
  - -pound, 341.
    - equivalents of—, 237.
- **Forbay**, defined, 1030.
- **Force, Forces**, 330, 332, 358.
  - acting upon beams and trusses, 437.
  - application of—, point of—, 333.
  - applied and imparted—, 372.
  - center of—, 399, 506, 514.
  - centrifugal—, 354.
    - on bridges, 758.
  - centripetal—, 354.
  - classification, 361.
  - colinear—, 363.
  - component, 362.
  - composition of—, 362, 364.
  - defined, 332.
  - diffusion of— through liquids, 506.
  - on inclined planes, 349.
  - internal— in beams, 466.
  - living—, 343.
  - measure of—, 338.
  - parallel—, 382.
    - couples, 404.
    - resultant of—, 399.
  - parallelogram, 364.
  - parallelopiped, 380.
  - point of application of—, 333.
  - polygon, 374, 377.
  - resolution of—, 362, 364.
  - resultant of—, 362.
  - in rigid bodies, 330, 358.
  - total effort, 371.
  - transmission, 358.
  - triangle, 367.
  - units of—, 358.
    - conversion of—, 235.
- **Forcite**, 952.
- **Foreign**
  - coins, 218.
  - explosives, 952.
- **Forgings**, steel—, requirements, 872.
- **Formula.** See also the given problem.
  - Gordon's—, 495.
  - Kutter's—, 523, 563, 564.
  - prismoidal—, 203.
- **Foundations**, 582.
  - of arches, 613.
  - artificial islands, 600.
  - brick cylinders, 599.
  - caissons, 585.
  - for centers, 631.
  - in clay, 583.
- **Foundations**—continued.
  - close piles, 590.
  - coffer-dams, 585, 586.
  - crib—, 585.
  - of culverts, 627.
  - cylinders, 594, 596, 597, 599, 600.
  - of drains, 627.
  - fascines, 599.
  - on gravel, 583.
  - grillage, 590.
  - iron piles, 594.
  - islands, artificial—, 600.
  - loads for—, 583.
  - masonry—, cylinders, 599.
  - Nasmyth pile-drivers, 591.
  - Pierre perdue, 583.
  - pile—. See Pile, Piles.
  - plenum process, 597.
  - pneumatic process, 596.
  - random stone, 583.
  - resistance of—, 583, 592
  - of retaining walls, 612.
  - rip-rap, 583.
  - on sand, 582.
  - sand piles, 599, 670.
  - sand pump, 599.
  - screw piles, 594.
  - sheet piles, 590.
  - sustaining power, 583, 592.
  - for trestles, 814.
  - for turntables, 845.
  - vacuum process, 596.
- **Four-way stop-valve**, 667.
- **Fourth proportional**, 38.
- **Fractions**, 35.
  - logarithms of—, 72.
- **Frames**, blue-print—, 980.
- **Framing**, timber—, 734.
- **Framework**, steel—, specifications, 764.
- **Franc**, value of—, 218.
- **Francis's formula**, 550.
- **Franklin Institute standard dimensions of bolts, etc.**, 883.
- **Free end reaction**, roof trusses, 715.
- **Freezing**
  - of dynamite, 950.
  - effect of— on cement, 932.
  - of explosives, 948.
  - of mercury, 318.
  - of mortar, 928.
  - of nitro-glycerine, 948.
  - in pipes, 656, 665.
  - behind retaining walls, 604.
  - in stand pipes, 663.
  - in track tank, how prevented, 853.
  - of water, 326.
- **Freight,**
  - cars, 865.
  - earnings, 867.
  - locomotives, 856.
  - ton-mile, 867.
- **Friction**, 407.
  - angle of—, 409.
    - in arch, 432.
    - in dams, 433.

Friction—continued.
  axle—, 416.
  of cars, 417.
  coefficient of—, 408.
  of earth, 612.
  head, 516, 527.
  on inclined planes, 350.
  of iron cylinders, 593.
  journal—, 416.
  kinetic—, coefficient, 409.
  launching—, 415.
  longitudinal— of revolving shafts, 419.
  of masonry, 411, 612.
  Morin's laws, 410.
  of piles, 593.
  rollers, 417, 725, 751, 846.
    defined, 1030.
  of walls, 608.
  of water, 415.
Frictional stability, 409.
Frog, Frogs, 834–840.
  angle of—, 835, 839.
  distance, 839.
    graphic method, 842.
  length, 835.
  number, 835, 840.
  point, 835.
Frost
  jacket in fire hydrant, 669.
  -proof tank, 852.
Frustum,
  of cone, 201.
  of parabola, 192.
  of paraboloid, 209.
  of prism, 195.
  of pyramid, 201.
Fteley and Stearns's formula, 552.
Fuel consumption of locomotives, 861.
Fulcrum, 419.
Full
  flow, 540.
  size eye-bars, tests of—, 753.
Fumes, acid—, effect on roofs, 970.
  coal—, effect on iron, 880
Funds, sinking—, 43.
Funicular machine, 427.
Furlong, equivalents of—, 232.
Furrings, defined, 1030.
Fuse, defined, 1030.

G.

G. C. D., 35.
Gage, Gages,
  Birmingham—, 887, 890.
  hook—, 548.
  narrow— cars, 865.
  narrow— locomotives, 857.
  railroad—, 827,
    on bridges, 746.
    on curves, 787, 789.
  rain—, 324.
  stubs—, 890.
  stuff, 968.
  wire—, 887–891.
Gaging of streams, 560.
Gallon, 223, 224, 234.
Galton's experiments, 412.
Galvanic action in water pipes, 656.
Galvanized
  iron, 880.
  pipes, 664.
Gas
  engines, mfrs., 990.
  weight, 211.
Gasket, 660.
  defined, 1030.
  to prevent washing— into pipe, 661.
Gate valves, 666, cost. etc., 995.
Gates for water pipes, 666.
Gauge, Gauges. See Gage, Gages.
Gauging of streams, 560.
Gauthey's pressure plate, 561.
Gearing, ratio of power and weight, 420.
Gelatine, explosive—, 952.
Geographical mile, 220.
Geometrical progression, 39.
Geometrical similarity, 92.
Geometry, 92.
Giant powder, 951.
Gib, defined, 1030.
Gin, 686.
  defined, 1030.
Girders,
  details, 728.
  erection, 743.
  plate—, 731, 747.
    bracing, 749.
  and trusses, comparison, 689.
Glass, 973.
  compressibility, etc., 459.
  cost and mfrs., 974, 985.
  dimensions, etc., 973.
  expansion by heat, 317.
  friction, 411.
  strength, 476, 922, 923, 974.
  weight, 212.
Glazing, 973.
Globe, 204, 205.
Glossary of terms, 1025.
Glue,
  adhesion of—, 922.
  defined, 1030.
Glycerine, nitro—, 948.
Gneiss, weight, 213.
Gold,
  strength, 920.
  value—, 219.
  weight, 213, 219.
Gondola cars, 865.
Gordon's formula, 495.
Grade, Grades, 255–257.
  contour lines, 300.
  etc., conversion of—, 237.
  defined, 255, 256.
  effect on horses, 683.
  effect on locomotives, 860.
  hydraulic—, 519, 521.
  percentage, 255.
  resistance, 683, 860.

Grade, Grades—continued.
  of roads, 255, 683.
  of sewers, 574.
  tables, 255–257.
  traction on—, 683.
  in tunnels, 812.
  on turnpikes, 255.
  of water-pipes, 653.
Gradient, hydraulic—, 519, 521.
Grading, cost, 800, 855.
Grain, equivalents of—, 235
Gram, or Gramme, 217, 226.
  equivalents of—, 236.
Granite,
  beams, 924.
  cost of— blocks, 601.
  expansion by heat, 317.
  rubble, cost, 602.
  strength, 476, 923, 924.
  weight, 212.
Graphic
  method, truss stresses, 703, 706.
  representation of couples, 405.
  statics, 428–431, 435.
Gravel,
  boring in—, 670.
  in concrete, 943.
  dredging in—, 580.
  for foundations, 582.
  natural slope of—, 610.
  weight, 213.
Gravity,
  acceleration of—, 335, 336, 348, 349, 539.
  center of—, 386.
  on inclined planes, 349.
  line of—, 389.
  plane of—, 389.
  specific—, 210.
Gray column, 905.
Great
  bear, constellation, 285.
  circle, 284.
Greatest common divisor, 35.
Grillage, 590.
  defined, 1030.
Groin, defined, 1030.
Gross ton, 216.
Ground lever, 826.
Grout, 926.
  defined, 1030.
Grubbing, cost, 855.
Guard, Guards,
  rails, 750, 828, 833, 835.
  wheel—, 750.
Gudgeon, 416.
  defined, 1030.
Guide-rails, 828, 833, 835.
Guldinus theorem, 194.
Gun
  -cotton, compressed, 951.
  metal, strength, 920.
  -powder, 953.
    pile-drivers, 591.
    weight (under Powder), 214.
Gunter's chain, 220, 232, 282.
Gusset, defined, 1030.
Gutta-percha
  pipe, 657.
  weight, 213.
Gypsum, weight, 213.
Gyration,
  center of—, 496.
  radius of—, 352, 496, 892, etc.

H.

H. C. F., 35.
H. P. See Horse-power.
Hair,
  cross—, to replace—, 296.
  stadia—, 293.
Half-section, equivalents of—, 233
Hand
  level, 310.
  spike, defined, 1030.
Hard finish, 968.
Hardening of cement, 930.
  rate of—, 932.
Hasselmann process, 955.
Haul, mean—, 801.
Hauling, 683, 685, 801, 805.
Haunches, defined, 1030.
Head, Heads,
  block, 825.
  of bolts, 883.
  due to a given velocity, 539.
  entry—, 516.
  friction—, 516, 527.
  for a given velocity, to find—, 527.
  for piles, 593.
  plate, 825.
  pressure—, 258, etc., 518.
  theoretical—, 539.
  tripod—, 292.
  velocity—, 516, 539.
  of water, 516.
  for water supply, 654.
Header, defined, 1030.
Heading, 812.
  defined, 1030.
Headway in bridges, 746.
Heat,
  of the air, 320.
  conduction of—, by air, 320.
  expansion of air by—, 320.
  expansion of rails by—, 819.
  expansion of solids by—, 317.
  expansion of surv. chains by—, 274, 283.
  of fires, 317.
  subterranean—, 320.
  thermometer, 318.
  and work, units of—, conversion of—, 237.
Hectare,
  equivalents of—, 234.
Hectogram, equivalents of—, 236.
Hectoliter, equivalents of—, 235.
Hectometer,
  equivalents of—, 233.

Google

- Heel of frog, 835.
  - of switch, 825, 828, 839.
- Height,
  - effect on temperature, 320.
  - effect on weight, 336, 348.
  - to find— by barometer, 312.
  - to find— by boiling point, 314.
  - to find— by trigonometry, 151.
  - of locomotive smoke-stack, 856.
- Heliography, 979.
- Helve, defined, 1030.
- Hemlock,
  - strength, 476, 499, 958, 965.
  - weight, 213.
- Heptagon, 148.
- Hexagon, 148, 159.
- Hickory,
  - strength, 476, 957, 958.
  - weight, 213.
- High explosives, 948.
- Highest common factor, 35.
- Highway bridges, 745, etc.
- Hip
  - roof, defined, 1031.
  - suspender, stress in—, 709.
- Hoisting
  - engines, cost and mfrs., 990.
  - machinery, cost and mfrs. of—, 991.
- Holes,
  - for blasting, 600.
  - boring— in earth, 670.
  - boring— in rock, 600, 670, 675.
- Homogeneity, tests for—, 871.
- Hook-head spikes, 818.
- Hopkins's pneumatic dome, 665.
- Horizon, artificial—, 298.
- Horizontal,
  - defined, 153.
  - forces, summation of—, 466.
  - loads in trusses, 710.
  - shear in beams, 478.
- Horse, Horses,
  - power of—, 683, 852.
  - -power, 342, 685.
    - equivalents of—, 244.
    - of falling water, 578.
    - -hour, equivalents of—, 237.
    - metric—. See Metric horse-power.
    - of running streams, 578.
  - pumping, day's work, 852.
  - weight, 685.
- Hose,
  - cost, 995.
- Hour, hours,
  - angle, 285.
  - defined, 265.
  - equivalents of—, 236.
- House, engine—, cost, 850.
- Howe truss, 692, 736, 738.
- H. P. See Horse-power.
- Hydrant, Hydrants,
  - cost and mfrs., 995.
  - fire (fire-plug), 669.
- Hydraulic, Hydraulics, 516. See also Water, Flow, Velocity, Discharge, etc.
- Hydraulic, Hydraulics—continued.
  - cement. See Cement.
  - grade line, 519, 521.
  - index, 930.
  - lime, 930.
  - mean depth, 523, 564.
  - radius, 523, 564.
  - ram, 578.
    - cost and mfrs., 991.
- Hydraulicity of cement, 930.
- Hydrogen, specific gravity—, 213.
- Hydrometers, 211.
- Hydrometric pendulum, 561.
- Hydrostatic, Hydrostatics, 501.
  - paradox, 501.
  - press, 506.
- Hyperbolic logarithms, 72.

I.

- I-beams. See Beams, I—.
  - in fire-proof floor, 894.
  - as pillars, 497, 912.
  - separators for—, 900.
  - table, 892.
- Ice,
  - adhesion to piles, 594.
  - blasting of—, 950.
  - in stand pipes, 663.
  - strength, compressive—, 923.
  - weight, 213, 326.
- Icosahedron, 194.
- Illumination
  - of cross-hairs, 286.
  - of stake, surveying, 286.
- Impact, 347.
  - of trains on bridges, 711, 758.
- Imperial
  - gallon. See Gallon.
  - measure, British, 224.
- Impost, defined, 1031.
- Impulse, 337.
- Inch, Inches,
  - equivalents of—, 221, 232.
  - circular—, 222.
  - cubic—, equivalents, 222, 234.
  - in decimals of a foot, 221.
  - per foot, equivalents of—, 237.
  - of mercury (pressure), equivalents of—, 241.
  - miner's—, 546.
  - spherical—, equivalents of—, 222.
  - square—, equivalents of—, 233.
- Inclination. See Grade.
  - of courses in masonry, 603, 620.
  - tables of—, 255–257.
  - in tunnels, 812.
- Inclined
  - beams, 445, 485.
  - plane, 349, 369.
    - descent on—, 349.
    - ropes for—, 976–977.
    - stability on—, 424.
    - tables, 255–257.
    - velocity on—, 349.

Incomplete contraction, 544.
Increments, chord—, 701.
Incrustation,
  of boilers, 327.
  of walls, 929, 936.
Indeterminate stresses, 720.
Index, logarithms, 70.
India rubber, weight, 213.
Indifferent equilibrium, 387, 514.
Inertia, 338.
  moment of—, 351, 468.
Infinity, symbol for—, 33.
Influence diagrams, 403, 449, 702.
Ingot, defined, 1031.
Instability, 514.
Interest, 40.
Internal forces in beams, 466.
Internat'l metric screw thread, 883.
Intersections,
  in railroad curves, 780.
  in trusses, 694.
Intrados, defined, 613.
Inverse proportion, 39.
Inversion of ratios, 38.
Invert, defined, 1031.
Involution, 54–69.
  by logarithms, 71.
Iron,
  balls, weight, 874, 875, 877, 879, 918.
  bars, weight, 877, 878.
  beams. See Beams, iron—.
  bending tests, 873.
  blasting of—, 950.
  bolts, 883, 886.
  in bridges, requirements, 754.
  cast—,
    balls, weight of—, 918.
    cohesive strength, 920.
    compressive strength, 874, 921.
    elastic limit, 459, 874.
    expansion by heat, 317.
    friction, 411.
    malleable—, strength, 874.
    modulus of elasticity, 459, 874.
    pillars, 495.
    pipes, flow in—, 522.
    pipes, weight, 656, 876.
    requirements, 874.
    salt water on—, 327, 594.
    shearing strength, 499.
    strength, 476, 499, 500, 874, 920, 921.
    tensile strength, 874, 920.
    torsional strength, 500.
    transverse strength, 476, 874.
    weight, 213, 875, 918.
  casting, weight, 875.
  and cement, pipes of—, 657.
  chains, 915.
  channels, as pillars, 497, 912.
  cohesive strength of—, 920.
  cold, effect on—, 274, 819, 874.
  cold-rolled—, 920.
  columns. See also Pillars, iron—, 495.
  compressive strength, 921.

Iron—continued.
  contraction of— by cold, 274, 819.
  corrosion of— by coal fumes, 880.
  corrugated sheet—, 880.
  cost, 986.
  crushing strength, 921.
  cylinders, bursting pressure in—, 511, 512.
  cylinders, foundations, etc. See also Foundations, 593–598.
  ductility of—, 459.
  effect of cement on—, 936.
  effect of cold on—, 274, 819, 874.
  effect of heat on—, 274, 317, 819.
  effect of mortar on—, 926, 936.
  effect of water on—, 327, 594.
  elastic limit, 459, 872, 874.
  expansion of— by heat, 274, 317, 819.
  friction of—, 411.
  galvanized—, 880.
  heat, effect on—, 274, 317, 819.
  limit of elasticity, 459, 872, 874.
  malleable cast—, strength, 874.
  manufacture, 870.
  manufacturers, 986.
  modulus of elasticity, 459, 874.
  net, 774.
  paints for preserving—, 763, 972.
  piles, 594. See also Foundations.
  pillars, 495, 497, 901–913. See also Pillars.
  pipes,
    cast—, weight, 656.
    fittings for—, 882.
    flow in—, 522.
    galvanized—, 664.
    joints for—, 656, 660.
    thickness, 512, 656.
    wrought—, 656.
      diams, actual and nominal—, 526, 882.
  plates, buckled—, 885.
  porosity of—, 512.
  prices—, 986.
  re-rolled—, 920.
  rolled—. See Iron, wrought—.
  rolled— for bridges, requirements, 754.
  roofs. See Roofs.
  salt water, effect on—, 327, 594.
  shearing strength, 499.
  sheet—, 880.
  specific gravity, 213.
  specifications, 870.
  spikes, 818.
  strength, 476, 499, 500, 870, 872, 874, 907, 920, 921.
  stretch of—, 459.
  T—, 497, 898, 912.
  tensile strength, 920.
  tests, bending—, 873.
  torsional strength, 500.
  transverse strength, 476, 874.
  tubes, 882.
  water, effect on—, 327, 594.
    salt—, effect on—, 327, 594.

Iron—continued.
  weight, 213, 875–882. See also Iron, cast—; Iron, wrougnt—.
  wire, 891.
    rope, 976, 977.
  -wood (Canadian), strength, 476.
  wrought—,
    bars, weight, 877, 878.
    cohesive strength, 873, 920.
    compressive strength, 921.
    elastic limit, 459, 872.
    expansion by heat, 274, 317, 819.
    friction of—, 411.
    pillars. See also Pillars, 495.
    pipes, 656, 657.
      diams, actual and nominal—, 526, 882.
      fittings for—, 882.
      joints for—, 656, 660.
      weight, 656, 882.
    prices, 986.
    shearing strength, 499.
    strength, 476, 499, 500, 872, 920, 921.
    tensile strength, 872, 920.
    torsional strength, 500.
    transverse strength, 476.
    tubes, weight, 882.
    water pipes, 656.
    weight, 213, 877–882.
Island, artificial—, for foundations, 600.
Isogonic chart and lines, 300, 301.

J.

Jack, defined, 1031.
  rafters, defined, 1031.
Jag-spike, 818, defined, 1031.
Jaw-plate, 724.
Jet
  pile driving, 595.
Jetty, defined, 1031.
Jig-saw, defined, 1031.
Joint, Joints,
  in arches, 629.
  bell— for pipes, 660.
  in bridges, 724.
  butt—, 773.
  in chimneys, etc., cement for—, 971, 973.
  distribution of pressure in—, 400.
  end—, roof trusses, 733.
  flexible— for pipe, 661.
  lap—, 773, 778.
  masonry—,
    distribution of pressure in—, 400.
    inclination of—, 603, 620.
    lead in—, 634, 921.
  net—, 774.
  pin—, 747.
  pin and riveted—, 721.
  for pipes, 656, 660, 882.
Joint, Joints—continued.
  rail—, 819.
    cost and mfrs., 994.
  riveted—, 721, 749, 772.
  in roofs, 733, 916, 971.
  timber—, 734.
  toggle—, 427.
Joule,
  equivalents of—, 237.
  per second, equivalents of—, 245.
Journal friction, 416.
Jumper,
  defined, 1031.
  drill, 600.

K.

Key frog, 836.
Keystone, 613, 615.
  pressure on—, 614.
Kieselguhr, 949.
Kilogram,
  centigrade, equivalents of—, 237.
  equivalents of—, 236.
Kilogrammeter, equivalents of—, 237.
  per second, equivalents of—, 245.
Kiloliter, equivalents of—, 235.
Kilometer,
  equivalents of—, 233.
  per hour, etc., equivalents of—, 243.
Kilowatt, equivalents of—, 245.
Kinetic
  energy, 343.
  friction, 407.
  coefficient of—, 409.
King,
  post, defined, 1031.
  truss, 691.
Knife-edge, strength, 921.
Knot (nautical), 220.
Kutter's formula, 523, 563, 564.
Kyanizing, 955.

L.

L. C. D., 35.
L. C. M., 35.
Lacing, 722.
Lagging,
  for centers, 631, 639.
  defined, 1031.
Laitance, 947.
Land,
  dredge, 808.
  measure, 222, 233.
    metric—, 225.
  required for railroads, 254.
  section of—, area of—, 222, 233.
  surveying, 274.
  ties, 612.
Lap
  joint, 773, 778.
  welded boiler tubes, 882.
  welded pipe, 656, 882.

Lard,
  as a lubricant, 415.
  weight, 213.
Lateral
  bracing, 691, 720.
    timber trusses, 737.
Laths, 968, 969.
Latitude, Latitudes,
  astronomical—, 284.
  degree of—, length, 220.
  and departures, 274.
  effect of— on barometer, 312, 314.
  effect of— on gravity, 336, 348.
Lattice
  bars, 747.
  truss, 694.
Latticing, 722.
Launching, friction of—, 415.
Laying
  bricks, 927.
  out of turnouts, 839.
  pipe, cost, 658.
  track, cost, 855.
Lead,
  balls, weight, 918.
  defined, 1031.
  effect of cement, mortar, etc., on —, 926, 936.
  elasticity, etc., 459.
  expansion by heat, 317.
  in masonry joints, 634, 921.
  paint, 971.
  pencils, 978.
  pipe, 513, 918.
  for pipe-joints, 658–661.
  roofs, 918.
  sheets, 918.
  strength, 920, 921.
  weight, 213, 875–878, 887, 918.
  white— cement for leaks, 971.
  white— paint, 971.
Leaded tin, 916.
Leak in roof, to stop—, 971, 973.
Leakage, 329, 561, 642, 650, 651.
Leap year,
  defined, 266.
  equivalents of—, 236.
Least
  common denominator, 35.
  common multiple, 35.
Leather,
  friction, 415.
  strength, 922.
Legua, 227.
Length,
  per time—, units of—, conversion of—, 242.
  units of—, conversion of—, 232.
Level, Levels, 306.
  builder's—, to adjust—, 311.
  cost and mfrs., 993.
  cuttings, 790.
  engineer's—, 306.
  hand—, Locke—, 310.
  lines, defined, 153.
  note-book, form of—, 309.
  Y—, 306.
Levelling,
  by barometer, 312.
  by boiling point, 314.
  of earth on embankment, 801.
  screws, 292.
Lever, Levers, 419.
  switch—, 826, 830.
  tumbling—, 826.
Leverage, 360, 419.
Libra, 227.
Life,
  average—,
    of cars, 865.
    of shingles, 971.
    of ties, 815.
Lift bridges, 696.
Lignum vitæ,
  strength, 476, 957.
  weight, 213.
Lime, 925.
  in cement, 930.
  hydraulic—, 930.
  paste, 926.
  quick—, 930.
  weight, 213.
Limestone, 213, 923, 930.
Limit,
  elastic—, 458, 459, 482.
    cast iron, 874.
    iron and steel, 752, 873.
  of elasticity. See Limit, elastic—.
Limnoria, 954.
Linch pin, defined, 1031.
Line, Lines, 92.
  of action, 359.
  agonic—, 301.
  center of gravity of—, 391.
  contour—, 302.
  of gravity, 389.
  hydraulic grade—, 519.
  isogonic—, 301.
  of no variation, 301.
  parallel—, to draw—, 94.
  of pressure, 399, 430.
  resistance—, 430, 432, 434–436.
  of resultants, 430, 432, etc.
  thrust—, 430, 432, 434–436.
Lining of tunnels, 812.
Link, equivalents of—, 232.
Liquid, Liquids. See Water.
  buoyancy of—, 513, 514.
  compressibility, 326.
  flow, 516.
  friction, 415.
  measure, 223.
  pressure, 500, 518.
    transmission of—, 506.
  specific gravity, 211.
Liter,
  equivalents of—, 225, 235.
Lithofracteur, 952.
Little bear, constellation, 285, 286.
Live load, Live loads. See Loads live—.
Living force, 343.

- Load, Loads,
  - on bridges, 726, 755.
  - cart—, of earth, 800.
  - chord stresses, 709.
  - on columns, 495, etc., 901, etc., 963, etc.
  - dead—, 690.
  - on driving-wheels, 705, etc., 755, etc., 856–859, 861.
  - on earth, safe—, 583.
  - for given deflection, 480, 481, 483.
  - line, 707.
  - live—, 690, 705, 709, 726, 755, 856–859, 861.
  - locomotive—. See Loads on driving wheels.
  - moving. See Loads on driving wheels.
  - permissible— on beams, 473.
  - for permissible deflections, 485.
  - on piles, 592.
  - on pillars, 495, 901, etc., 963, etc.
  - on roofs, 321, 713.
  - on roof-trusses, 713, 764.
  - on sand, 582.
  - stresses, 705.
    - graphic method, 706.
  - suddenly applied—, 460, 486, 959.
  - on wooden bridges, 764.
- Loaded
  - chain, 428.
  - cord, 428.
- Loading,
  - standard—, 705, 755.
  - of trusses, 690.
- Local time, 287.
- Location of the meridian, 284.
- Lock,
  - air—, 597.
  - gates, spacing of cross-bars, 506.
  - nut, 821, 885.
- Locke level, 310.
- Locomotive, Locomotives, 856.
  - adhesion of—, 413.
  - house, cost, 850.
  - mfrs., 990.
  - statistics, 867.
  - tonnage rating of—, 862.
  - turntables for—, 845.
  - water for—, 327, 851.
  - wheel-load, 705, etc., 755, etc., 856–861.
- Locust, strength, 476, 957, 958.
- Logarithmic
  - chart, 73.
  - plotting, 74.
  - sines, tangents, etc., 72.
- Logarithms, 70–91.
- Long
  - chords, table, 787.
  - measure, 220, 225, 232.
  - ton, 216.
- Longitude, degree of—, length, 220, 221.
- Longitudinal and transverse stresses combined, 493, 724.
- Lower
  - chord, design, timber trusses, 733.
  - chord splice, 736.
  - culmination, 284.
- Lowering of centers, 631, etc.
- Lowest terms, 36.
- Lubricants, 415.
- Lubrication of turntables, 846.
- Lumber. See also Wood, Timber.
  - cost and mfrs., 984.
- Lunation, 266.
- Lune, circular—, 186.

M.

- Machine,
  - drill, 675.
  - funicular—, 427.
  - riveting, 775.
  - for tapping pipes, 657, 664.
- Magnesia in cements, 931.
- Magnetic
  - declination, 301.
  - variation, 301.
- Magneto-electric blasting, 952.
- Mahogany,
  - strength, 459, 476, 957, 958.
  - weight, 213.
- Mail
  - cars, 865.
  - earnings, 867.
- Man power, 686.
- Mandrel, defined, 1032.
- Manganese, in steel, 872.
- Mantissa, logarithms, 70.
- Manufacture of iron and steel, 870.
- Manufacturers, list of, 996.
- Map, to reduce or enlarge—, 160.
- Maple-wood,
  - strength, 476, 957.
  - weight, 213.
- Marble,
  - cost, 602.
  - expansion by heat, 317.
  - strength, 476, 923.
  - weight, 213.
- Mark,
  - German—, 218, 246, etc.
  - Spanish—, 227.
- Masonry,
  - in abutments, quantity, 623.
  - adhesion of mortar to—, 926.
  - in arches, quantity, 622–628.
  - and concrete, 945.
  - cost, 601, 988.
  - courses,
    - inclination of—, 603, 620.
    - lead between, 634, 921.
  - dam, 433.
  - foundations, loads on—, 750.
  - friction of—, 411, 603, 620.
  - incrustation of—, 929, 936.
  - joints, distribution of pressure on—, 400.
  - mortar required for—, 931.

Masonry—continued.
  in piers, quantity, 628.
  quantity
    in arches, 622–628.
    in piers, 628.
    in retaining walls, 610.
    in walls of wells, 198.
    in wing-walls, 624.
  railroad—, cost, 855.
  in retaining walls, 603.
  strength, compressive—, 923.
  weight, 213.
Mass, 334, 336.
Material, Materials,
  fatigue, 465.
  particle, 358.
  point, 358.
  strength, 454.
  weight, 210, etc.
Mathematical symbols, 33.
Mathematics, 33.
Matter, defined, 330.
Mattock, defined, 1032.
Maximum
  intensity of rainfall at points in U. S., table, 323.
  and min. stresses in truss members, 712.
  pressure,
    angle, etc., of—, 607.
  velocity, 560.
Mean,
  depth, hydraulic—, 523, 564.
  haul, 801.
  proportional, 38.
  radius, 523, 564.
  of ratio and proportion, 38.
  solar time, defined, 265.
  sun, 265.
  velocity, 522, 560.
Means, defined, 1032.
Measure, Measures, 216.
  apothecaries', 223, 224.
  circular— of angles, 34.
  circular— of wires, 889.
  commercial—, size of—, by weight of water, 224.
  common—, 35.
  conversion tables, 228, etc.
  cubic—, 222, 234.
  fluid—, 223, 224.
  long—, 220, 232.
  metric—, 217, 225, etc., 228, etc.
  Russian—, 227.
  Spanish—, 227.
  square—, 222, 233.
  weights, etc., conversion tables of units of—, 228.
  wine—, 223, 224.
Measuring weirs, 547, 646.
Mechanics, 330.
  of arch, 430–432.
  of beam, 437, etc., 466, etc.
  of masonry dam, 430, 433–436.
  of trusses, 698, etc.
Melting points, 317.
Mercury,
  barometer, 312, 320.
  foot of—, etc. (pressure), equivalents of—, 241.
  freezing-point, 318.
  thermometer, 318.
  weight, 213.
Meridian,
  location of—, 284.
  of longitude, 220, 221.
  variation of compass, 296, 301.
Metacenter, 514.
Metal, Metals. See also Iron, and Steel.
  blasting of—, 950.
  cohesive strength, 920.
  compressibility, 459.
  compressive strength, 921.
  ductility, 459.
  effect of cement on—, 926, 936.
  effect of heat on—, 317, 819.
  effect of lime on—, 926, 936.
  effect of mortar on—, 926, 936.
  effect of water on—, 327, 594.
  elastic limit, 459.
  expansion by heat, 317, 819.
  limit of elasticity, 459.
  modulus of elasticity, 459.
  preservation of— by cement, 936.
  roof trusses, 740.
  shearing strength, 499.
  sheet—, 880, 881, 887, 916–919.
  strength, 454, 476, 499, 500, 920, 921.
  stretch of—, 459.
  tensile strength, 920.
  torsional strength, 500.
  transverse strength, 476.
  weight, 210, etc., etc.
Meter,
  equivalents of—, 225, 233.
  Ferris-Pitot—, 536.
  length, 217, 225, 233, etc.
  Pitot—, 536, 561, 562.
  radii, etc., of curves in—, 786.
  Venturi—, 532, etc.
  water—, 649, cost and mfrs., 994.
  wheel, 562.
Metric,
  atmosphere, 240, 320.
  horse-power, equivalents of—, 245.
  horse-power hour, equivalents of—, 237.
  measures, 217, 225, etc., 228, etc.
  railroad curves, tables, 786.
  screw thread, international—, 883.
  ton, equivalents of—, 236.
  units, } See Metric measures.
  weights. }
Mica, weight, 213.
Middle
  ordinates, 180, 784, 786, 788, 817, 840.
  third, 402.
Mikron, equivalents of—, 232.

Mil,
  equivalents of—, 232.
Mile, Miles,
  equivalents of—, 232.
  freight ton-mile, 867.
  geographical—, 220.
  per hour, etc., equivalents of—, 242.
  land and sea—, 220, 233.
  nautical—, 220.
  passenger—, 867.
  sea—, 220.
  square— (section), 222, 233
  ton—, 867.
Milligram,
  equivalents of—, 236.
Milliliter, equivalents of—, 235.
Millimeter, equivalents of—, 233.
Mills, floating—, 578.
Miner's friend powder, 951.
Miner's inch, 546.
Minim, 223, 224.
Minimum
  and maximum stresses in truss members, 712.
  sections, 722.
Minute, Minutes,
  in decimals of a degree, 95.
  equivalents of—, 236.
  of time, 265.
Mitred joints for rails, 819.
Mitre-joint, defined, 1032.
Mixing cement for briquettes, 938.
Mizar, 285, 287.
Models,
  of beams, strength and weight, 478.
  in force composition, 380.
Modern explosives, 948.
Modified logarithms, 72.
Modulus. See Coefficient, Strength.
  defined, 1032.
  of elasticity, 456, 459.
  of flow, 540.
  of resilience, 460.
  of rupture, 468.
  section—, 467-8, 473, 892 to 898.
Mogul locomotives, 856.
Moisture,
  effect of—,
    on cement, 934.
    on sound, 316.
    on zinc, 917.
Molded concrete, 945.
Molds for cement briquettes, 939.
Molecular action, 358.
Moment, Moments, 360.
  in arches, 424.
  in beams, 440, 443.
  in cantilevers, 440, 442.
  in continuous beams, 489.
  of couple, 405.
  defined, 360, 1032.
  diagrams, 479.
    trusses, 707.
  of inertia, 351.
    in beams, 468.
  influence diagrams, 449.
  in levers, 419.
Moment, Moments—continued.
  live load—, 709.
  maximum bending—, 474.
  of non-coplanar forces, 381.
  resisting—, 467.
  and shear, relation between—, 452.
  of stability, 422, 508, 514, 608.
  summation of—, 466.
  in trusses, 440, 701.
Momentum, 338, 345.
Money, 218.
Monkey-switch, 826.
Mont Cenis tunnel, 812.
Month, civil—, sidereal—, synodic—, 266.
Morin's laws of friction, 410.
Mortar,
  adhesion of—, 926.
  in arches, 616, 629, 633.
  bricks, etc., 925.
  cement—, 931.
  clay, effect on—, 926, 935, 936.
  effect on iron, 770, 926, 936.
  effect on wood, 926.
  grout, 926.
  in retaining walls, 604.
  rubble—, cost, 602.
    weight. See under Masonry, 213.
  salt, effect on—, 926, 936.
  sand for—, 925, 926, 935, 936.
  strength, tensile—, 933.
  weight, 213, 926.
Mortise, defined, 1032.
Motion, 331.
  circular—, 351.
  quantity of—, 338.
  relative—, 331, 358.
Mould. See Mold.
Movable bridges, 696.
Moving load. See Load, live—.
Muck, defined, 1032.
Mud,
  penetrability, 593.
  in reservoirs, 651.
  weight, 213, 581.
Multiple, common—, 35.
Multiples and factors, 35.
Multiplication
  of decimals, 37.
  of fractions, 36.
  by logarithms, 71.
    chart or slide rule, 75.
Muskrats, 651.
Myriagram, equivalents of—, 236.

N.

Nails,
  cost and mfrs., 986.
  shingling, 971.
  slating, 970.
Napierian logarithms, 72.
Narrow-gauge
  locomotives, 857.
  railroad cars, 865.

- Nasmyth pile-driver, 591.
- Natural
  - cements, 930, 937, 940.
  - logarithms, 72.
  - sines, 97, 98.
  - slope, 419, 604, 606, 610.
- Nautical mile, 220.
- Neat cement tests, 941.
- Needle,
  - compass—, 293, 299.
    - variation of—, 301.
- Negative
  - characteristics, logarithms, 72.
  - exponents, logarithmic chart, 76.
  - numbers, logarithms, 72.
- Net
  - earnings of railroads, 867.
  - iron, net plate, net joint, 774.
  - section of tension members, 759.
  - ton, 216.
- Neutral
  - axis, 466.
  - surface, 466.
- Newel, defined, 1032.
- Niagara cantilever, 696.
- Nicholson hydrometer, 211.
- Nickel steel, 872.
- Nicking test, 752, 871.
- Nitro-glycerine, 948.
- Nonagon, 148.
- Non-concurrent forces, 375.
- Non-coplanar forces, 380.
- Normal component, 370.
- North
  - point, etc., 284.
  - star, 285.
- Northing, 274.
- Number, Numbers,
  - and equivalents in common use, conversion tables, 231.
  - of frog, 835, 840.
  - prime—, 35.
  - by wire gage, 887–891.
- Numerus logarithmi, 71.
- Nut, Nuts, 883.
  - locks, 821, 885.

O.

- Oak,
  - strength, 459, 476, 499, 957, 958.
  - weight, 214.
- Oblique, Obliques,
  - lines, 92.
  - pillars, 498.
  - pressure, 372, 504, 607.
- Obstacles in surveying, to pass—, 281.
- Obstruction, Obstructions,
  - to flow, 575.
  - by piers, 575.
  - in pipes, to prevent—, 655.
- Octagon,
  - area, 148.
  - to draw—, 159.
- Octahedron, 194.
- Ogee, defined, 1032.
- Oil, Oils,
  - coal— (petroleum), weight, 214.
  - olive—, 415.
  - weight, 214.
  - wells, nitro-glycerine, 948.
- Open channels, flow in—, 523, 560.
- Open hearth steel, requirements, 872.
- Openings,
  - flow through—, 540–542.
- Ordinate, Ordinates,
  - defined, 1032.
  - elliptic—, 189.
  - to find—, 180.
  - middle—, 180, 784–789, 817, 840.
  - parabolic—, 192.
  - tables, 784–789, 817, 840.
- Orifices, flow through—, 539, 546.
- Oscillation, center of—, 351.
- Ounce, 220, 235.
  - equivalents of—, 235.
  - fluid—, 223, 224, 235.
- Outer rail, elevation of—, 787.
- Outflow, velocity of—, theoretical —, 539.
- Outlet valves, 653.
- Oval, to draw—, 191.
- Overfall
  - dams, 642.
  - discharge over—, 547.
  - for reservoir, 652.
- Overturning,
  - effect of wind, 710.
  - work of—, 422.

P.

- Packing,
  - defined, 1032.
  - of eye-bars, 722.
  - piece, 775.
- Paint, Paints, 971.
  - cost, 984.
  - for iron, 763, 972.
  - on zinc, 880.
- Painting, 971.
  - of bridges, 763, 764.
- Panel, Panels,
  - diagonal of—, to find length of—, 160.
  - points, 692.
  - reactions, 702.
  - in trusses, 692.
- Paper, 978.
- Parabola, 192, 193. See Parabolic.
  - center of gravity, 394.
  - to draw—, 193.
  - ordinates, 192.
  - semi—, center of gravity of—, 394.
  - tangent to—, to draw—, 193.
- Parabolic
  - arc, 192, 193.
  - conoid, 209.
    - frustum of—, 209.
  - curve, 192, 193.
  - frustum, 192.

Parabolic—continued.
  ordinates, 192.
  zone, 192.
Paraboloid, 209.
  center of gravity of—, 398.
  frustum, 209.
Paradox, hydrostatic—, 501.
Parallel
  forces, 382, 514.
    couples, 404.
    defined, 361.
    resultant of—, 382, 399.
  lines, to draw—, 94.
  plates, 292.
Parallelogram, Parallelograms, 95, 157.
  angles in—, 95.
  force—, 364.
Parallelopiped, 195.
  force—, 380.
Parlor cars, 865.
Partial contraction, 540, 544.
Particle, material—, 358.
Partition, thin—, flow through—, 541.
Passenger
  cars, 865.
  earnings, 865, 867.
  locomotives, 856, etc.
  mile, 867.
Paste, lime—, 926.
Patterns,
  weight of casting, 875.
Paving,
  Belgian—, 602.
  brick—, 927.
  cost, 989.
Payments, equation of—, 42.
P. C., P. I., P. T., 780.
Pedestals, bridge—, 721, 750.
Pencils, lead, 978.
Pendulum, Pendulums, 350.
  hydrometric, 561.
  seconds—, 216, 351.
Pennsylvania R. R.
  locomotives, 857, 859.
  track-tank, 853.
Penstock, defined, 1032. See Forebay.
Pentagon, 148.
Per, Percentage, etc., 40.
  of grade, 255.
  interest, annuities, 40.
Perch, 222.
  equivalents of—, 235.
Percussion, center of—, 351.
  drills, 676.
Perimeter. See also Circumference, wet—, 523, 563.
Permanent way, 815.
Permutation, 40.
Perpendicular, to draw—, 93.
Perpetual snow, limit of—, 324.
Persian wheel, 687.
Petroleum, weight, 214.
Phœnix segment-columns, 497, 904, 912, 913.
Phosphor bronze,
  permissible load, 762.
  requirements, 754.
  wire, strength, 920.
Phosphorus,
  in steel, 753, 754, 872.
Pi, symbol and value, 34.
Picks, wear, 801.
Pier, Piers,
  abutment—, 619.
  foundations, 582.
  masonry, quantity in—, 628.
  obstructions by—, 575, etc.
  of suspension bridges, 768.
Pierre perdue, 583.
Piezometer, 518.
Pig iron ton, 216.
Pile, Piles, 589, etc.
  adhesion of ice, 594.
  bearing—, 590.
  blasting of—, 950.
  cost, 984.
  in cylinders, 600.
  drivers, 590, 591, 687.
    gunpowder—, 590.
    mfrs., 992.
    steam—, 591.
  driving, 590, etc.
    by jets, 595.
  factor of safety, 593.
  foundations, 589, etc.
  friction, 593.
  grillage, 590.
  head for—, 594.
  hollow—, 596.
  ice, adhesion to—, 594.
  iron—, 594.
  jet driving, 595.
  loads for—, 592.
  resistance of—, 592.
  sand—, 599, 670.
  screw—, 594.
  sheet—, 590.
  shoes for—, 593.
  sustaining power, 592.
  water jet for driving—, 595.
  withdrawal, 594.
Pillar, Pillars, 495, etc., 901, etc., 907, etc., 963, etc.
  of angle-iron, 497, 912.
  capitals of—, shapes of—, 498.
  Carnegie Z-bar—, 901-903.
  of channel-iron, 497, 912.
  ends of—, shapes, 495, 498.
  factor of safety, 909, 912.
  Gordon's formula, 495.
  hinged ends, 495.
  of I-beams, 497, 912.
  iron—, 495, etc., 760, 901-913.
    factor of safety, 909, 912.
  masonry—, strength, 923.
  oblique—, 498.
  Phœnix segment, 497, 904, 912.
  pin-ended—, 495.
  radius of gyration, 496.
  with rounded ends, 495.
  safety factor of—, 909, 912.

Pillar, Pillars—continued.
  segment—. See Phœnix—.
  steel—. See Pillar, iron—.
  strength—, 495, 760, 901–913, 963, etc.
  T iron—, 497, 912.
  wooden—, 761, 764, 963, etc.
  Z-bar—, 901–903.
Pillow-block, defined, 1032.
Pin, Pins,
  connections, 721, 724, 725, 747, 762.
  -end columns, 495.
  surveying—, 282.
Pine,
  pillars, 963, etc.
  strength, 459, 476, 499, 957, 958, 963.
  weight, 214.
Pinions and wheels, 420.
Pintle, defined, 1032.
Pipe, Pipes,
  air-valves for—, 662.
  areas and contents—, 197, 526.
  bends in—, 537.
  branches, 661.
  brass and seamless—, 919.
  bursting of—, 513, 663, 665, 668.
    thickness required to prevent—, 511, 513.
  bursting pressure in—, 518.
  cast-iron—, 653, 658, 662, 876.
    cost of— and laying, 658.
    weight, 656, 658, 876.
  cement and iron—, 657.
  concretions in—, to prevent—, 655.
  contents and areas, 197, 526.
  copper seamless—, 919.
  cost of—, 658.
  cost of laying—, 658.
  cost and mfrs.—, 994.
  couplings for—, 656, 660, 882.
  cracks in—, 661.
  curves in—, 537.
  diameter of—, 524, 653, 654, 656.
    actual and nominal—, 526, 882.
    for water-supply, 653.
    square roots of—, 526.
  discharge from—, 516, 522.
  drain—, 575.
  drawn brass—, 919.
  ferrules for—, 664.
  flexible joints for—, 661.
  flow in—, 516, etc.
    Kutter's formula, 523, 563, 564.
  galvanic action in—, 656.
  galvanized—, 664.
  gates for—, 666.
  gutta-percha—, 657.
  iron—,
    cast—, 653–656, 658–662, 876.
      weight, 656, 658, 876.
    and cement, 657.
    fittings for—, 661, 882.
Pipe, Pipes—continued.
  iron—, continued.
    joints for—, 660, 661, 882.
      flexible—, 661.
    laying—, 658, etc., 660.
  lead—, 664, 918.
    thicknesses of—, 513.
  material of—, effect on velocity, 523.
  to mend—, 661.
  obstructions in—, to prevent—, 655.
  pressure of water in—, 511, 518.
  seamless—, 919.
  service—, 657, 664, 918.
  sleeves for—, 661.
  stand—, 663.
  steam—, 882.
  stop-valves for—, 666.
  street—, 653.
  tapping of—, 657, 664.
  terra-cotta—, 575.
  thickness required, 511, 513, 656.
  valves for—, 666.
  of varying diameter, discharge through—, 531.
  velocity in—, 522–524.
  water—, 653, 657, 876.
    cost of— and laying, 658.
    freezing, anti-bursting device, 665.
  weight, 656, 658, 876, 882.
    of water in—, 525.
  wooden—, 657.
  wrought iron—, 656, 657, 882.
    diam, actual and nominal—, 526, 882.
    weight, 656, 882.
Pitch,
  defined, 1033.
  effect of—, on wind pressure, 714.
  of rivets, 776.
  of roofs, 970.
  of screw, 436.
  weight—, 214.
Pitman, defined, 1033.
Pitot's tube, 536, 561.
Plane, Planes, 148.
  of couple, 404.
  of flotation, 514.
  of gravity, 389.
  inclined—.
    See Inclined Plane.
  of moment, 360.
  surfaces, 148.
  trigonometry, 150.
Plank,
  board measure table, 269.
  in foundations, 582.
  sheet piling, 590.
  thickness for a given pressure, 586, 648.
Plaster of Paris, 968.
  effect on metals, 936.
  price—, 985.
  strength, 922, 923.
  weight, 214.

- Plastering, 968.
- Plate, Plates,
  - bed—, 750.
  - buckle—, 750, 885.
  - fish—, 820.
  - frog—, 837.
  - girders, 747.
    - bracing in—, 749.
    - details, 728.
  - glass—, 974.
  - iron—, prices, 986.
  - net—, 774.
  - parallel—, 292.
  - resistance of—, 492.
  - steel—, tinned, 916.
  - strength, 492.
  - terne—, 916.
  - tie—, 816.
  - tin—, 916.
  - tinned steel—, 916.
- Platform,
  - cars, 865.
  - revolving—, 850.
- Platinum, 214, 920.
- Plenum process, 597.
- Plows, cost and mfrs., 992.
- Plug, fire— (fire-hydrant), 669.
- Plumb level, to adjust—, 311.
- Plumbago as a lubricant, 415.
- Plummet, defined, 1033.
- Plunger, defined, 1033.
- Plus, 33, 782.
- Pneumatic
  - dome, Hopkins'—, 665.
  - foundations, 596.
- Pocket sextant, 297.
- Point, Points,
  - of application of force, 333, 359.
  - boiling—, 314, 326.
    - levelling by—, 314.
  - of curve, 780.
  - freezing—, 326.
  - frog—, 835–843.
  - of intersection, 780.
  - material—, 358.
  - melting—, 317.
  - position of—, to find—, 156.
  - switch—, 828, 830.
  - of tangent, 780.
- Pointers—, astronomy, 285.
- Pointing with cement, 936.
- Polar distance, 284.
  - and azimuth of Polaris, table, 290.
- Polaris, 285–290.
- Pole, north—, 284.
- Polygon, Polygons, 148.
  - cord—, 377, 428.
  - force—, 374, 377.
  - irregular—, to find area of—, 160.
  - to reduce to a triangle, 159.
  - regular—, to draw—, 159.
- Polyhedron, Polyhedrons, regular—, 194.
- Pond, discharge of—, time required for—, 545.
- Pony trusses, 692.
- Pood, 227.
- Poplar, strength, 476, 957, 958.
- Porous bodies, specific gravity, 211.
- Port, establishment of—, 328.
- Portal bracing, 691.
- Portland cement, 930, etc.
- Posts, 359. See also Pillars.
  - design of—, 722.
    - timber trusses, 733.
  - fence—, 854.
  - pivot— in turntables, 846.
  - and ties, 689.
- Potential energy, 346.
- Pound
  - equivalents of—, 235.
  - Fahrenheit, equivalents of—, 237.
  - sterling, value, 218.
  - weight, 220.
- Pouring-clamps for pipe joints, 660.
- Powder, 214, 953.
- Power, Powers. See Steam, Water, Wind, Animal, etc.
  - animal—, 685.
  - defined, 342.
  - fifth—, 67–69.
    - sq. rts. of—, 69.
  - finding— by logarithmic chart, 76.
  - finding— by logarithms, 71.
  - finding— by slide rule, 76.
  - of horse, 683, 685, 852. See also Horse-power.
  - in levers, 419.
  - of locomotives, 860, 861.
  - man—, 686.
  - second and third—, tables. 55, etc.
  - tractive—, 683.
  - units of—, conversion of—, 244.
- Pratt truss, 692.
- Precipitation (rainfall), 322.
  - in the U. S., table of details of—, 325.
- Present worth, 41, 42, 44.
- Preservation
  - of metals by cement, 936.
  - of timber, 954.
- Press, Presses,
  - hydrostatic—, 506.
- Pressed
  - brick, 927.
  - steel cars, 865.
- Pressure. See Load.
  - of air, 320, 597.
  - in arches, 614, 616.
  - of atmosphere, 320.
  - barometer—, 320.
    - levelling by—, 312.
  - center of—, 399, 501, 506, 514.
  - on centers of arches, 633.
  - in dams, 648.
  - distribution of—, 400.
  - of earth, 603, 607.
  - on foundations, 583.
  - head, 258, 518.
  - hydrostatic—, 501, etc.
  - on inclined planes, 349.
  - line of—, 430, etc.

- Pressure—continued.
  - maximum—,
    - angle, prism, slope of—, 607.
  - in pipes, 511, 518.
  - plate, Gauthey's, 561.
  - in reservoirs, 651.
  - on retaining walls, 603, 607.
  - of running streams, 578.
  - of running water in pipes, 518.
  - steam cylinder—, 861.
  - transmission of— through liquids, 506.
  - unit—, conversion of—, 240.
  - of water, 501, etc., 516, etc.
    - in cylinders, 511.
    - in pipes, 511, 518.
    - plank to resist—, 586, 648.
    - running, 518, 578.
    - walls to resist—, 508.
  - of wind, 321.
    - on roof trusses, 714.
- Price list, 983.
- Prime,
  - defined, 1033.
  - number, 35.
- Principal, in interest, 41.
- Printing, blue—, 979.
- Prints,
  - black-line—, 982.
  - blue—, 979.
- Prism, Prisms, 195.
  - center of gravity of—, 395.
  - frustums of—, 195.
    - center of gravity of—, 395.
  - of max. pressure, 607.
- Prismoid, 202.
- Prismoidal formula, 202.
- Profile, Profiles, 304.
  - paper, 978.
  - transformation of—, 611.
- Progression, 39.
- Projection, defined, 1033.
- Proportion,
  - by logarithms, 71.
  - and ratio, 38.
- Proportionals, 38.
- Protection of bridges, 763, 764.
- Protracting by chords, 143.
- Puddle,
  - defined, 1033.
  - walls, 651.
- Pug-mill, defined, 1033.
- Pull
  - and push, 359.
  - on tapes, surveying, 282.
- Pulley, 428.
- Pump, Pumps, 852.
  - chain—, 687.
  - cost and mfrs., 991.
  - day's work at—, 686, 852.
  - sand—, 599, 670.
- Purlins, 713.
  - defined, 1083.
- Push and pull, 359.
- Puzzolan cement, 940.
- Pyramid, Pyramids, 200.
  - frustum of—, 201.

R.

- Rabbet, defined, 1033.
- Race, defined, 1033.
- Rack-a-rock, 951.
- Radii, Radius,
  - to find—, 161, 179.
  - of gyration, 352, 353, 495, 496, 892, etc.
    - square of—, 496.
  - mean, 523, 564.
  - of railroad curves, 784–786.
  - of turnouts, 840.
- Rail, Rails, 817.
  - bending—, ordinates for—, 817.
  - cost and mfrs., 994.
  - creeping of—, 819, 820.
  - elevation of outer—, 787.
  - expansion by heat, 317, 819.
  - fence—, 854.
  - frog, 835.
  - guard or guide—, 750, 828, 833, 835.
  - joints, 819.
  - ordinates for bending—, 817.
  - outer—,
    - elevation of—, 787.
  - roads, 780–869.
    - acres required for—, 254.
    - ballast, 815.
    - bridges. See Bridge, Truss, Arch, etc.
    - construction, 855.
    - cost, 855.
    - cross-ties, 815.
    - resistance on—, 417.
    - roadway, 815.
    - shops, cost, 850.
    - slopes, 255–257.
    - spikes, 818.
    - switch, 824.
    - ties, 815.
    - time, standard—, 267.
    - track tank, 853.
    - traction on—, 860.
    - turnout, 824.
    - water stations, 851.
  - safety—, 833.
  - steel—, requirements, 872.
  - stock—, 828.
  - switch-length, 830.
  - way. See Railroad.
- Rain, 322.
  - fall, 322.
    - depths, equiv. volumes, 250.
    - equivalent of snow, 324.
  - gages, 324.
  - reaching sewer, rate, 575.
  - and snow, 322.
  - water, 327.
- Rainy days, table of average number of—, 325.
- Ram, Rams,
  - hydraulic—, 578.
    - cost and mfrs., 991.
  - water—, 513, 663, 668.
- Ramming concrete, 945.

Random—Roller.

- Random stone, 583.
  - defined, 1033.
- Ratio,
  - elastic—, 458, 461.
  - and proportion, 38.
- Reaction, 333.
  - end—, 360, 439.
    - in trusses, 699, 702, 714.
  - of fibers, 466.
  - of soils, elastic—, 593.
- Réaumur thermometer, 318.
- Reciprocal, Reciprocals, 48–53.
  - or inverse proportion, 39.
  - on logarithmic chart, 76.
  - by logarithms, 71.
  - by slide rule, 77.
- Rectangle, Rectangles, 157.
- Rectangular
  - components, 369.
  - plates, strength of—, 492.
- Recurring decimals, 38.
- Reduction
  - of area, 752, 754, 873.
  - of figures, 160.
- Redundant members, 720.
- Reflection, to measure heights by—, 155.
- Refraction and curvature tables, 153.
- Regular figures, 148.
- Regular solids, 194.
- Regulation of time-pieces by the stars, 266.
- Reinforcing plates, 724, 747.
- Relative density, 210.
- Renewal of bridges, 743.
- Repair, Repairs,
  - of bubble-tube, 296.
  - of cars, 865.
  - of cross-hairs, 296.
  - of pipe, 661.
  - in reservoirs, 652.
  - of road, 801.
  - of rolling stock, 865.
- Repeating decimals, 38.
- Reservoir, Reservoirs, 650.
  - evaporation from—, 329.
  - for railroads, 852.
- Resilience, 460.
- Resistance
  - of cars, 417.
  - to flow, 523, 537, 563.
  - of foundations, 583, 592.
  - on grades, 860.
  - line, 430, 432, 434–436.
  - of piles, 592.
  - of plates, 492.
  - on railroads, 417.
- Resolutes, 369.
- Resolution of forces, 362, etc.
- Rest, relative—, 358.
- Resultant,
  - of forces, 362.
  - line of—, 430, 432.
  - of moments, 360.
  - of parallel forces, 382, 399.
  - sense of—, 366.
- Retaining walls, 603.
  - masonry in—, quantity of—, 610, 612.
  - surcharged, 605.
  - theory of, 606.
  - transformation of profile, 611.
- Reverse bearing, 277.
- Revetment, 612.
  - defined, 1034.
- Revolving bodies, 351.
- Rhomb, 157, 195.
- Rhombic prism, 195.
- Rhombohedron, 195.
- Rhomboid, 157.
- Rhombus, 157.
- Rhumb-line, 277.
- Ridge-pole, defined, 1034.
- Right angle, to draw, 93.
- Rigid bodies,
  - force in—, 330, 358.
- Rigidity in bridges, 721.
- Ring, Rings, circular, 186, 209.
- Rip-rap, 583.
  - defined, 1034.
- Rise of arch, 613.
- River, Rivers. See Water, Rain, etc.
  - dams, 642.
  - flow in—, 560.
  - scour of—, 577.
- Rivet, Rivets, 772.
  - stresses in—, permissible—, 762.
- Riveted
  - connections, 721.
  - joints, 749, 772.
- Road, Roads,
  - cart—, repairs, 801.
  - grade, 255, 683.
    - tables, 255–257.
  - maintenance, 801.
  - rail—. See Railroad.
  - rollers, mfrs. of—, 992.
  - traction on—, 683.
  - -way, acres required for—, 254.
    - drainage of—, in arches, 628.
- Rock, Rocks,
  - blasting, 948, etc.
  - broken, voids in—, 688, 810, 943.
  - channeling, 681.
  - crushers, cost and mfrs. of—, 992.
  - drill, 600.
    - hand—, 681.
    - machine—, 675.
  - removal, 810, 811.
  - weight, 212, etc.
  - work in tunnels, 812.
- Rocker bearings, 725, 730.
- Rod, Rods,
  - of brickwork, 928.
  - cost and mfrs., 993.
  - equivalents of—, 232.
  - upset—, 886.
- Rolled iron. See Iron, wrought—, and Steel.
  - cost and mfrs., 986.
- Roller, Rollers,
  - anti-friction—, 417, 751, 846.
  - bearings, 725–728.

Rolling
  friction, 414.
  lift bridges, 697.
  load. See Load, live—.
  stock, 856, 865, 867.
    resistance of—, 417.
Roof, Roofs,
  acid fumes on—, 880, 970.
  copper—, 918.
  iron for—, 880.
  lead—, 918.
  leak in—, to stop—, 971, 973.
  painting, 764, 880, 972.
  pitch of—, 916, 970.
  sheet-iron—, 880.
  shingle—, 971.
  slate—, weight, 970.
  tin—, 916.
  trusses, 698, 713. See Truss.
    loads on—, 764.
    metal—, 740.
    specifications for—, 764.
  wind on—, 321, 713.
  zinc—, 916.
Roofing,
  cost of—, 989.
Root, Roots,
  cube and square—, tables, 54.
  of decimals, to find—, 67.
  fifth—, 67, 68.
  finding— by logarithmic chart, 76.
  finding— by logarithms, 71.
  finding— by slide rule, 76.
  of large numbers, to calculate—, 66.
  square—, tables, 54.
    of diameters, 526.
    of fifth powers, 69.
Rope, Ropes, 975.
  cost and mfrs., 987.
  strength, 922, 975.
  wire—, 976, 977.
Rosendale cement, 931.
Rosin, weight, 214.
Rot, dry—, 954.
Rotary
  drills, 675.
  motion, 351.
Rotating bodies, 351.
Rotundity of the earth, 153.
Rough casting, 973.
Roughness,
  coefficients of—, 523, 564, 565.
Rubble,
  adhesion to mortar, 926.
  arches, 616.
  cost, 602,
  defined, 1034.
  proportion of mortar in—, 213.
  retaining walls, 610.
  strength, 923.
  voids in—, 688, 799.
  weight, 213.
Rule, Rules,
  of three, 39.
  two-foot—, to measure angles by—, 96.
Run-off, equivalents of—, 251, 252.
Rupture, modulus of—, 468.
Russian weights and measures, 227.
Rütger's process, 955.

S.

Sachine, 227.
Safety,
  castings, 824, 833.
  factor of—,
    for beams, 959.
    for piles, 593.
    for pillars, 495, 909, 912, 913.
    for retaining walls, 605.
    for suspension bridges, 767.
  rail, 833.
Sag,
  of tape, correction for—, 282.
  in trusses, 718.
Salt,
  effect of— on mortar, 926, 936.
  water,
    effect of— on iron, 327, 594.
    weight, 326.
  weight, 214.
Sand,
  augers, 670.
  blasting of—, 950.
  cement, 937.
  in cement, effect, 931, etc.
  for cement, quantity, 935.
  for centers, striking, 633.
  in concrete, 943, etc.
  cost, 985.
  dredging, 580.
  effect of— in cement, 932, etc.
  excavating in—, 800.
  for foundations, 582.
  natural slope, 419, 610.
  penetrability, 593.
  piles, 599, 670.
  in plaster, 968.
  pressure, 603.
  price of—, 985.
  pump, 599, 670.
  required in mortars, 931.
  retaining walls for—, 603.
  slope of—, natural—, 419, 610.
  specific gravity, 211, 214.
  stone, expansion by heat, 317.
    strength, 476, 922, 923.
    weight, 214.
  sustaining power, 583, 593.
  voids in—, 214, 935.
  weight, 211, 214.
Sandage, 216.
Sap, 954.
Scabble, defined, 1034.
Scale, Scales, track—, 854.
Scantling, defined, 1034.
Scour of streams, 577.
Scrapers,
  cost and mfrs., 992.
  earthwork by—, 805.

Screeding, 968.
Screw, Screws, 436.
  Archimedes, 687.
  cylinders, 594.
  levelling—, 292, 307.
  piles, 594.
  standard dimensions, 883.
  for striking centers, 633.
  thread, metric—, 883.
Scribe, defined, 1034.
Sea,
  mile, 220.
  tides, 328.
  water, 326, 328, 594.
  worms, 954.
Seamless,
  pipes and tubes, 919.
Seasoning, 954.
Secant, 97.
Second, Seconds,
  in decimal of a degree, 95.
  equivalents of—, 236.
  to estimate—, 266.
  pendulum, 216.
  of time, defined, 265.
Section,
  equivalents of—, 222, 233.
  of land, area, 222, 233.
  of members, minimum—, 722.
    in timber trusses, 733.
  method by—, stresses in trusses, 700.
  modulus, 467, 468, 473, 892, 894, 896, 898.
  net— in tension members, 759.
Sector,
  center of gravity, 393.
  circular, 186.
  spherical—, center of gravity, 396.
Secular magnetic variation, 301.
Sediment in reservoirs, 651.
Segment, Segments,
  circular—, 186, 187, 394.
  columns, Phœnix—, 497, 904, 912, 913.
  spherical—, 208.
    center of gravity, 395.
Sellers' standard dimensions of bolts, etc., 883.
Semi-
  circle, center of gravity, 391.
  parabola, center of gravity, 394.
Sense,
  of force, 359.
  of moment, 360.
  of resultant, 366.
Separators for I-beams, 900.
Series, arithmetical and geometrical—, 39.
Service pipe, 657, 664, 918.
Set-screw, defined, 1034.
Setting
  of cement, 930, etc.
Settling
  of arch, 432.
Settling—continued.
  of backing, 604.
  of centers, 633, 640.
  of embankment, 799.
Sewer, Sewers,
  cost, 989.
  flow in—, 574.
  Kutter's formula, 523, 563, 564.
  rain water, rate of reaching—, 575.
  velocities in—, 574, 575.
Sextant,
  angles measured by—, 152.
  box or pocket—, 297.
  center of gravity of—, 393.
Shackle, defined, 1034.
Shadows, equal— from the sun, location of meridian by—, 288.
Shaft,
  revolving—, longitudinal friction, 419.
  of tunnel, 812.
Shafting,
  friction, 416.
  strength, 500.
Shale, weight—, 214.
Shapes, structural—, tables, 892–898.
  T—, 898.
Sharpening tools, cost, 801.
Shear, Shears,
  in beams, 446.
  in continuous beams, 489.
  defined, 1034.
  diagrams, 479.
    trusses, 702, 706.
  double and single—, 499, 774.
  horizontal— in beams, 478.
  influence diagrams for—, in beams, 450.
  and moments, relation between—, 452.
  in trusses, 702.
    web stresses, 706.
Shearing,
  of rivets, 774.
  strength, 499.
    of cements, 934.
  stresses, permissible—, 762.
  in timber construction, 732.
Sheet, Sheets,
  copper—, 918.
  iron—, 880.
    corrugated—, 880.
    galvanized—, 880.
    roof, 880.
    and steel, cost and mfrs., 986
  lead, 918.
  metals, thickness, 887–890.
  piles, 590.
  zinc, 916.
Sheeting of centers, 631, etc., 639.
Shell,
  -lime, 926.
  spherical—, 208.
    weight, 875, 877.
Shilling, value, 218.

Shingles, 971.
Shoes,
  bridge—, 721.
  for piles, 593.
Shops, railroad—, cost, 850.
Shore, defined, 1034.
Short ton, 216.
Shoveling earth, 800.
Shovels, wear of—, 801.
Shrinkage of embankment, 799.
Sidereal, day, month, time, year, 266.
Sieves, for cement tests, 938.
Signal target, 826, 829, 833.
Silica cement, 937.
Silicate of alumina, 930.
Silver,
  coins, etc., 218.
  strength, 920.
  weight, 214, 219.
Similarity, geometrical—, 92.
Simple interest, 41.
Sine, Sines, 97.
  logarithmic—, 72.
  natural—, defined, 97.
    table, 98.
  by slide rule, 77.
Single
  riveting, 772.
  rule of three, 39.
  shear, 499, 774.
Sinking fund, 43.
Siphon, 520.
Skew
  back, 613.
    defined, 1034.
  bridge, 697.
Skidding of wheels, 413.
Slacking of lime, 925, 926, 930.
Slag cement, 940.
Slaking, 925, 926, 930.
Slate, 969.
  compressibility, etc., 459.
  expansion by heat, 317.
  roofs, weight, 970.
  strength,
    compressive—, 923.
    tensile—, 922.
    transverse—, 476.
  weight, 214.
Sleeping cars, 865.
Sleeves for pipes, 661.
Slide rule, 73.
Slope, Slopes,
  angle of—, 255, 256.
  description of—, 255, 256.
  earthwork, description of—, 256.
  hydraulic—, 523, 564.
  instrument, 256, 311.
  Kutter's formula, 523, 563, 564.
  of maximum pressure, 607.
  natural—, 419, 604, 606, 610.
  railroad—, 255, 256.
  structures built upon—, 424.
  tables, 255–257.
  of tapes and chains, corrections for—, 283.
Slope, Slopes—continued.
  in tunnels, 812.
Sloping weirs, 558.
Sluice, defined, 1035.
Sluices in dams, 645.
Snow, 214, 323.
  load, 713.
  rainfall equivalent, 324.
Soakage, loss by—, 329, 561, 651.
Soap,
  as a lubricant, 415.
  stone, weight, 214.
  wash for walls, 928.
Soffit, defined, 613, 1035.
Soil, Soils,
  boring in—, 670.
  dredging in—, 580.
  excavation of—, 800.
  leakage through—, 329, 561, 651.
  penetrability, 593.
  pressure of—, 603.
  reaction of—, elastic—, 593.
  scour, 577.
  sustaining power of—, 583, 592.
  weight, 212. See under Earth.
Solar time, mean and apparent—, 265.
Solid, Solids, 194.
  center of gravity, 396.
  defined, 92.
  expansion by heat, 317.
  floors, 721, 750, 914.
  measure, 222, 234.
    metric— 225, 234.
  mensuration of—, 194.
  specific gravity, 210.
  surface of—,
    center of gravity, 395.
Sound, 316.
Soundness of cement, 940.
Southing, 274.
Sovereign, 218.
Span, 613, 759.
Spandrel, 613–618.
  defined, 1035.
Spanish weights and measures, 227.
Spanner, defined, 1035.
Specific gravity, 210, etc.
Specifications;
  for bridges and buildings, 745.
  for combination bridges, 763.
  for iron and steel, 870.
  for roof trusses, steel framework and buildings, 764.
  for wooden bridges, 763.
Speed, Speeds,
  of teams, 801, 806.
  of trains, to estimate—, 866.
Spelter. See Zinc.
Sphere, Spheres. See Spherical, 204, 205, 222, 396, 875, 877, 918.
  volume of—, 222.
Spherical
  sector, center of gravity, 396.
  segment, 208.
    center of gravity, 395.

Spherical—continued.
  shell, 208, 875, 877.
  zone, 208.
    center of gravity, 396.
Sphericity of the earth, 153.
Spheroid, 208.
  center of gravity, 395.
Spigot, defined, 1035.
  in pipe joint, 660.
Spikes, 818.
Spindle,
  circular—, 209.
  torsional stress in—, 500.
Splice bars, requirements, 872.
Splice, timber—, 736.
Split switch, 828–830.
Spreading of earth, 801.
Spring, Springs,
  of arch, 613.
  in foundations, 583.
  frog, 838.
Spruce,
  strength, 459, 476, 499, 957, 958.
  weight, 214.
Spudding, 672.
Spur-wheel, defined, 1035.
Square, Squares. See also Powers.
  area, 157.
    equivalents of— in circles, 161.
  center of gravity, 391.
  measure, 222, 233.
    conversion table, 233.
    metric—, 225.
  mensuration of—, 157.
  of radius of gyration, 496.
  of roofing, 970, 1035.
  roots, 54.
    of decimals, to find—, 67.
    of diameters, 526.
    of fifth powers, 69.
    of large numbers, to calculate —, 66.
    tables, 54.
  sides of—, 157, 161.
  tables of—, 55.
Stability, 422, 514.
  of arches, 430, 432, 620.
  of dams, 433, 510.
  frictional—, 409.
  on inclined planes, 424.
  of retaining walls, 603.
Stable equilibrium, 387, 514.
Stadia hairs, 293.
Stand, switch—, 826.
Stand pipes, 663.
  for railroad water-station, 852.
  for water-works, 663.
Standard
  railway time, 267.
  wheel loads, 705, etc., 755, etc.
Starlings, defined, 1035.
Stars, to regulate a watch, etc., by—, 266.
Static friction, 407.
Statics, 330, 358.
  of arch, 430, 432.
Statics—continued.
  of beam, 437, etc., 466, etc.
  graphic—, 428–431, 435.
  of masonry dam, 430, 433–436.
  of trusses, 698, etc.
Station, Stations,
  in surveys, 309.
  water—, 851.
  way—, cost, 854.
Stationary engines, cost and mfrs. of—, 990.
Stays, cable, 766.
Steam,
  dredges, 580.
  engines,
    locomotives, 856.
    pumps—, 852.
  excavator, 808.
  pile drivers, 590, 591.
  pipes, 882.
  rock-drill, 675.
Steel,
  angles, 896, 898.
  beams, 476, 892.
  bending tests, 873.
  in bridges, requirements, 751.
  cars, 865.
  castings in bridges, 754.
    requirements, 872.
  channels, 894.
  cohesive strength of—, 920.
  columns. See Pillars, iron—.
  composition of—, 753, 872.
  compressibility, 459.
  compressive strength, 921.
  cost and mfrs., 986.
  ductility, 459.
  elastic limit, 459.
  expansion by heat, 317.
  forgings, requirements, 872.
  framework, specifications, 764.
  friction, 411.
  I-beams, 892.
  manipulation, 751.
  manufacture, 751, 870.
  modulus of elasticity, 459.
  open hearth—, requirements, 872.
  pillars. See Pillars, iron—.
  plates, tinned, 916.
  price, 986.
  rails, frogs of—, 835.
  requirements, 751, 872.
  roof trusses, 740.
  rope, 976, 977.
  shearing strength, 499.
  shop work on—, 751.
  specifications, 870.
  strength, 476, 499, 500, 870, 920, 921.
  stresses in—, permissible—, 760.
  stretch, 459.
  structural—, requirements, 872.
  tensile strength, 920.
  tests, bending—, 873.
  torsional strength, 500.
  transverse strength, 476.
  weight, 214, 877, 878.

- Steel—continued.
  - wire, 891.
    - rope, 976, 977.
  - yard, 383.
- Stere, equivalents of—, 235.
- Stiffeners, 748.
- Stiffness in bridges, 721.
- Stirrups, timber framing, 734.
- Stock rails, 828.
- Stone, Stones,
  - arch—, 613.
  - in arches, quantity, 622.
  - artificial—, 943.
  - ballast, 815, 855.
  - beams, 476, 924.
  - bridges, 613.
    - centers for—, 631.
  - broken—, voids in—, 688, 943.
  - cohesive strength, 922.
  - compressive strength, 923.
  - cost, 985.
  - crushers, 943.
  - cutter's day's work, 601.
  - dams, 400, etc., 430, etc., 433, etc., 508, 510.
  - dressing, 601.
  - drilling, 600.
  - expansion by heat, 317.
  - friction, 411.
  - key—, 613.
  - quantity of—, in arches, etc., 622.
  - quarrying, 600, 601.
  - random—, 583.
  - strength, 476, 922, 923.
  - tensile strength, 922.
  - transverse strength, 476.
  - weight, 212, etc.
  - work, 600, 809.
    - mortar required for—, 931.
    - strength, 923.
    - weight, 213.
- Stop, Stops,
  - corporation— for pipes, 657, 664.
  - valves for water pipes, 666.
- Storage reservoirs, 652.
- Stove-up, defined, 1035.
- Strain, Strains, 454. 455.
- Straps, timber framing, 735.
- Stratum, defined, 1035.
- Stream, Streams,
  - abrasion by—, 577, 578.
  - flow in—, 560.
  - -flow and precipitation, relation between—, 323.
  - to gauge—, 560.
  - horse-power of—, 578.
  - pressure of running—, 578.
  - scour of—, 577, 578.
  - virtual head, 578.
- Strength, Strengths. See also article in question.
  - of arches, 368, 430, etc., 613.
  - of beams, 466. 473, 476, 478, 892.
  - of cast iron, 874.
  - of cement, 932, etc.
    - tests for—, 939, 940, etc.
- Strength, Strengths—continued.
  - of chains, 915.
  - of channels, 894.
  - cohesive—, 454, 920, 922, 957.
  - compressive—, 454, 921, 923, 958.
  - of concrete, 944.
  - of cylinders, 511.
  - of iron, 476, 499, 500, 870, 907, etc., 920.
  - of materials, 454.
  - of piles, 592.
  - of pillars, 495, etc., 901, etc., 907, etc., 963, etc.
  - of plates, 492.
  - of retaining walls, 603.
  - of riveted joints, 772, etc.
  - of shafting, 500.
  - shearing—, 499.
  - of steel, 476, 499, 500, 870, 920, etc.
  - tensile—, 454, 920, 922, 957.
  - of timber, 476, 499, 500, 764, 957, 958.
  - torsional—, 499.
  - transverse—, 466, 473, 476, 478, 892.
  - uniform—, beams and cantilevers of—, 486.
  - of wood, 476, 499, 500, 764, 957, 958.
- Stress, Stresses, 359, 454, etc.
  - alternating—, 761.
  - bearing—, permissible—, 762.
  - bending— in bridge members, permissible—, 762.
  - in bridges, permissible—, 759.
  - combined longitudinal and transverse—, 493, 724, 762.
  - components, 371.
  - compound—, 493, 724, 762.
  - fiber—, 466, 467, etc.
    - and deflection, 481.
    - permissible—, 762.
      - in bridge trusses, 759, 764.
      - in roof trusses, 764.
  - range of—, 465.
  - repeated—, 465.
  - shearing—, permissible—, 762.
  - in truss members, 698.
  - in trusses, graphic method, 703.
  - unit—, 456. See also Stress, fiber—.
    - in beams, 467.
  - wind— in bridges, 710, 758.
- Stretch,
  - of materials, 459.
  - of tape, correction for—, 282.
  - of truss members, 718.
  - unit—, 456.
- Stretcher, defined, 1035.
- Strike, defined, 1035.
- Striking of centers, 631, 633, 640.
- Stringers,
  - in trusses, 713, 720, 749.
- Structural
  - shapes, tables, 892, etc., 986.
  - steel, 872.

Strut, Struts, 689.
  design, trusses, 722, 733.
  and ties, criterion for—, 359, 699.
Stub switch, 824, 825.
Stubs gauge, 890.
Stucco, 968.
Stuffing box, defined, 1036.
Stumps,
  blasting of—, 950.
Sub-delivery,
  cost, 855.
Submerged weirs, 554.
Subterranean temperature, 320.
Subtraction of fractions, 36.
Sub-verticals, 694.
Suddenly applied loads, 460, 486.
Sulphur,
  in steel, 753, 872.
  weight, 214.
Summation
  of deflections in trusses, 720.
  of forces, 466.
Sump, defined, 1036.
Sun,
  dial, to make—, 268.
  equal shadows from—, location of meridian by—, 288.
  mean—, 265.
Superelevation, 787.
Supplement of angle, 94.
Supported joints, 819.
Surcharged walls, 605, 609.
Surface, Surfaces,
  neutral—, 466.
  per length,
    conversion of—, 238.
  pressure of water against—, 501, etc.
  units of—,
    conversion of—, 233, 238.
  velocity, 560.
Surveying, 274.
Suspended joints, 819.
Suspender,
  hip—, stress in—, 709.
Suspenders of suspension bridges, 770.
Suspension bridges, 765.
Swage, defined, 1036.
Sway bracing, 691, 710, 749.
Swing bridges, 696.
Switch, Switches, 824.
Swivels, defined, 1036.
Sycamore,
  strength, 459, 476, 957, 958.
  weight, 214.
Symbols,
  mathematical—, 33.
Symmetry,
  axis of—, 514.
Synclinal axis, defined, 1036.
Synodic month, 266.
Syphon, 520.
System, metric—, 225.
Système,
  ancien, 226.
  usuel, 226.

T.

T, Ts,
  defined, 1036.
  iron, 896, 898, 912.
  rails, 817.
  shapes, 896, 898, 912.
Table, Tables. See the article in question.
  conversion— of units of measures, weights, etc., 228.
  turn—, 844.
Tackle, defined, 1036.
Tallow, 214, 415.
Talus, 612.
Tamp, defined, 1036.
Tamping, nitro-glycerine, 948.
Tangent, Tangents, 97, 98.
  to circles, to draw—, 162.
  to an ellipse, to draw—, 190.
  logarithmic—, 72.
  natural—, 97, 98.
  to a parabola, to draw—, 193.
  screw, 293, 307.
  by slide rule, 77.
Tangential
  angles, table, 784–786.
  component, 369.
  distance, table, 784–786.
Tank,
  of tender, capacity, 856, 860.
  thickness, 506, 854.
  track—, 853.
  water—, 851, 856, 860.
Tapes,
  surveying—, 282.
    cost and mfrs., 993.
Tapping,
  of pipes, 657, 664.
  of trees, effect on timber, 957.
Tar, weight, 214.
Target, signal—, 826, 829, 833.
Tarpaulin, 946.
Teams, speed of—, 801, 806.
Temperature. See Heat, 317.
  of air, 320.
  altitude, effect on—, 320.
  corrections for tapes, 283.
  effect of— on
    cement, 932.
    evaporation, 329.
    metals, etc., 317.
    rails, 317, 819.
    strength of iron, 874.
    surveying chains, 274, 283.
    velocity of sound, 316.
    weight of water, 326.
  subterranean—, 320.
  thermometers, 318.
Templet, defined, 1036.
Tender, Tenders, 853, 856.
  scoop, 853.
Tensile
  strength, 454, 920, 922, 932, 957.
    of cement, 932, etc., 939.
    of chains, 915.
    of riveted joints, 772, etc.

Tension,
  and compression, 359.
  members, 722, 732, 746.
    flexible and rigid—, 721.
    net section, 759.
  in tapes, 282.
Tents, cost and mfrs. of—, 993.
Teredo, 954.
Terne plates, 916.
Terra-cotta pipes, 575.
Test, Tests. See Requirements.
  bending—, iron and steel, 873.
  borings, 582, 670, etc.
  of cement,
    Am. Soc. C. E., 937.
    U. S. Eng'rs, 940.
  of completed bridges, 753.
  of full-size eye-bars, 753.
  -pieces, iron and steel, 870.
  of surveying instruments, 293, etc.
Testing machine for cements, 941.
Tetrahedron, 194.
Thawing, effect of— on cement, 932.
Theodolite, 296.
Thermometers, 318.
Thimble, defined, 1036.
Thin partition, flow through—, 541.
Third,
  middle—, 402.
  proportional, 38.
Three,
  rule of—, 39.
  -throw switch, 830.
Throat of frog, 835.
Through trusses, 692.
Throw,
  defined, 1036.
  of switch, 827.
Thrust,
  in arch, 430, 432.
  line, 430, 432, 434–436.
Tides, 328.
Tie, Ties,
  cost of—, 994.
  cross—, 815, 855.
  land—, 612.
  plates, 816.
  and struts, criterion for—, 359, 699.
  in trusses, 689.
Timber. See also Wood, Wooden, etc.
  beams, 760, 762, 764, 959, 962.
  bled—, strength, 957.
  board measure, table, 269.
  bridges, 732–740.
  cohesive strength, 957.
  columns, 963, etc.
  compressibility, 459.
  compressive strength, 958.
  cost, 984.
  crushing strength, 958.
  dams, 642.
  decay of—, 954.
  ductility, 459.
Timber—continued.
  elastic limit, 459.
  friction, 411.
  joints, 733, etc.
  limit, elastic—, 459.
  modulus of elasticity, 459.
  pillars, 761, 764, 963, etc.
  preservation, 954.
  requirements, 754, 760, 764.
  roof trusses, 716, 732, 742.
  shearing strength, 499.
  strength, 476, 499, 500, 760, 764, 957, 958.
  stresses in—, 760, 764.
  stretch, 459.
  tensile strength, 957.
  for ties, 815.
  torsional strength, 500.
  transverse strength, 476.
  trestles, 813.
  turntables, 848.
  weight, 212.
Time, 265.
  effect of—, on strength of cements, 932.
  local—, 287.
  -piece, to regulate— by stars, 266.
  standard railway—, 267.
  units of—, conversion of—, 236.
Tin, 916.
  elastic limit, etc., 459.
  expansion by heat, 317.
  leaded—, 916.
  roofing—, 916.
  strength, 920, 921.
  weight, 215, 877.
Tinned steel-plates, 916.
Toe of switch, 825, 828, 839.
Toggle joint, 427.
Ton, 216, 220.
  of coal, volume of—, 215, 222.
  (2240 lbs.), equivalents of—, 236.
  -mile, 867.
  net—, 216.
Tonelada, 227.
Tongue,
  of frog, 834.
  switch, 828.
Tonite, 951.
Tonnage rating of locomotives, 862.
Tonne, or metric ton, equivalents of—, 236.
Tools, wear of—, 801.
Top heading, 812.
Torpedoes, nitro-glycerine—, 948.
Torsion, 499.
Towers,
  of suspension bridges, 768, 770.
  valve—, 652.
Towne lattice truss, 694.
Tracing cloth and paper, 978.
Track. See Rail.
  gauge, 827.
  laying, cost, 855.
  scales, 854.
  tank, 853.
  trough, 853.

- Traction, 683.
  - of cars, 860.
  - on grades, 860.
  - of horses, 683, 685.
  - of locomotives, 860.
- Trailing switch, 824.
- Train,
  - centrifugal force of—, 758.
  - drag of— on bridge, 758.
  - earthwork by—, 807.
  - -shed roof, Broad St., Phila., 740.
- Transformation of profile, 611.
- Transit, Transits,
  - the engineer's, 291.
    - cost and mfrs., 993.
- Transmission,
  - of force, 358.
  - of pressure in liquids, 506.
- Transportation of bridges, 743.
- Transverse
  - and longitudinal stresses combined, 493, 724, 762.
  - strength, 466.
    - of concrete, 945.
- Trap rock, weight, 214.
- Trapezium, 158.
  - center of gravity, 392.
- Trapezoid, 158.
  - center of gravity, 392.
- Trapezoidal notch, 559.
- Tread
  - -wheel, 590, 686.
  - of wheel, 821.
- Trees, blasting of—, 950.
- Trembling of dams, 648.
- Tremie, 946.
- Trenching machine, mfrs. of—, 992.
- Trenton wire gauge, 891.
- Trestles, 813.
- Triangle, Triangles, 148.
  - in or about a circle, 161.
  - element of truss, 690.
  - force—, 367.
  - mensuration of—, 148.
  - right-angled—, 150.
- Triangular truss, 692.
- Trigonometric functions, 97, 98, etc.
  - logarithmic, 72.
- Trigonometry, plane—, 150.
- Trimmer, defined, 1037.
- Trip-hammer, defined, 1037.
- Tripod, 292.
- Trough
  - floors, 721, 750, 914.
  - flow through—, 544.
  - track—, 853.
- Troy weight, 220.
- True or apparent solar time, 265.
- Trundle, defined, 1037.
- Trunnion, friction of—, 416.
- Truss, Trusses, 689.
  - and beams, comparison, 689.
  - bracing in—, 691, 710, 748.
  - for centers, 636, etc.
  - counterbracing, 690, 705, 712, 721, 738, 746.
  - diagonals, to find lengths of—, 160.
- Truss, Trusses—continued.
  - end reactions, 439, 699, 702, 714
  - equilibrium of—, 437.
  - forces acting upon—, 437.
  - loads on—, moving—,
    - See Loads, live—.
  - members, stresses in—, 698.
  - moments in—, 440, 443.
  - moving loads on—,
    - See Loads, live—.
  - rafters of—, 691, 713, etc.
  - reactions, 439.
  - roof—,
    - loads on—, 764.
    - specifications for—, 764.
  - specifications for—, 745.
  - in suspension bridges, 765.
  - weights of—, 731, 738.
- Tube, Tubes. See also Pipes, Flow, etc.
  - boiler—, 882.
  - brass seamless drawn—, 919.
  - bubble—, to replace—, 296.
  - copper seamless drawn—, 919.
  - flow in—, 516.
  - iron—, 882.
  - Pitot's—, 536, 561.
  - pressure of water in—, 511, 518.
  - seamless—, 919.
  - short—, flow through—, 540.
  - welded—, 882.
- Tumbling lever, 826, 830, 833.
- Tun, 216.
- Tunnel, 812.
- Turbines, mfrs., 990.
- Turf, weight, 215.
- Turnbuckle, Turnbuckles, 986.
- Turnouts, 824, 839.
- Turnpike, grades on—, 255.
- Turntables, 844.
- Turpentine, 957, 972.
- Twaddell hydrometer, 211.
- Tympan, 687.
- Typical wheel loads, 705, etc., 755, etc.

U.

- U. S. See United States.
- Undecagon, 148.
- Underpin, defined, 1037.
- Ungula, cylindric—, 199, 397.
- Uniform,
  - live load, 705.
  - loads,
    - deflections, 485.
    - influence diagram, 703.
    - moments due to—, 444.
    - shears in beams due to, 447.
  - strength, beams and cantilevers, 486.
  - velocity, 331.
- Unit, Units,
  - of force, 338, 358.
  - of measures, weights, etc., conversion tables of—, 228.

Unit, Units—continued.
  of moment of inertia, 468.
  pressures, conversion of—, 240.
  of rate of work, 342.
  stress. See Stress, unit—.
  stretch. See Stretch, unit—.
  of work, 341.
United States,
  average precipitation in—, 322.
  coins, 219.
  gallon. See Gallon.
  measures, 223.
  railroad statistics, 867.
  standard dimensions of bolts, etc., 883.
Unstable equilibrium, 387, 514.
Unsymmetrical loading, 690.
Upper chord, 723, 733.
Upper culmination, 284.
Upset rods, 886.
Ursa minor and major, 285.

V.

Vacuum process for sinking cylinders, 596.
Value,
  per length, surface, time, volume, weight, work, etc., conversion of units of—. See Conversion tables, 246, etc.
  present, 42, 44.
Valve, Valves,
  air—, 662.
  cost and mfrs., 995.
  defined, 1037.
  four-way—, 667.
  outlet—, 653.
  stop—, 666.
  tower—, 652.
  for water-pipes, 666.
Vara, 227.
Variation,
  of compass, 301.
  line of no—, 300.
  magnetic, 301.
  vernier, 296.
Vegetation
  in reservoirs, 652.
Vehicles, friction, 414.
Vein,
  contracted—, 541.
Velocity, Velocities,
  of abrasion, 577.
  accelerated—, 331.
  through adjutages, 540.
  affected by material of pipes, 523.
  angular—, 351.
  of approach, 556.
  in channels, 560.
    Kutter's formula, 563, 564.
  defined, 331.
  due to a given head, 539.
  effect of— on friction, 412.
Velocity, Velocities—continued.
  equivalent to discharge per surface, 253.
  of falling bodies, 348, 539.
  head, 516.
    for a given velocity, to find—, 527.
  on inclined planes, 349.
  Kutter's formula, 523, 563, 564.
  material of pipe, effect on—, 523.
  mean—, 522, 560.
  through orifices, 539, 546.
  of outflow, 539.
  in pipes, 516, etc.
  retarded—, 331.
  in rivers, 560.
  in sewers, 574.
  in short tubes, 540.
  of sound, 316.
  theoretical—, of outflow, 539.
  uniform—, 331.
  units of—, conversion of—, 242.
  of wind, 321.
Vena contracta, 541.
Ventilation,
  air, quantity required, 320.
  of tunnels, 812.
Venturi meter, 532, etc.
Vernier, 293.
  variation—, 296.
Versed sines, 97.
Verst, 227.
Vertical, Verticals,
  of buoyancy, 514.
  circle, astronomy, 284.
  defined, 92.
  of equilibrium, 514.
  of flotation, 514.
Vessel, Vessels,
  air—, 663.
  contents of—, 198, 223.
  floating—, 514, 515.
  metallic—, effect of water on—, 327, 917.
Viaduct, Viaducts. See Trestles, and Trusses.
  erection, 743.
Vibrating bodies, 350.
Vibration, 350.
Vinculum, symbol, 33.
Virtual head, 539, 578.
Vis viva, 343.
Voids,
  in broken stone, 688, 943.
  in concrete, 943.
  in rubble, 799, 925.
  in sand, 214, 935.
Volume, Volumes,
  of air, weights of—, conversion of—, 242.
  equivalent depths, 251.
  increase of— of stone, 943.
  occupied by coal, 215.
  unit—, conversion of—, 234.
  of water, weights of—, 241.
Voussoir, 613.

W.

- Wagons, friction, 414.
- Wales, defined, 1037.
- Wall, Walls,
  - backing of—, 603.
  - battered, 605.
  - bricks, number in a sq. ft. of—, 925–927.
  - cost, 601, 602.
  - dams, 508.
  - face—, 603.
  - foundations for—, 582.
  - incrustation of—, 929, 936.
  - to resist water pressure, 508.
  - retaining—, 603.
  - soap-wash for—, 928.
  - spandrel—, 613.
  - stability of—, 508, 606.
  - surcharged—, 605, etc.
  - water,
    - to render impervious to, 928.
    - to resist pressure of—, 508.
  - wharf—, 515, 611.
  - wing—, 624.
- Walnut,
  - strength, 476, 957, 958.
  - weight, 215.
- Warp, defined, 1037.
- Warren truss, 692.
- Washers, 883.
  - defined, 1037.
  - lock-nut or nut lock, 885.
- Washes for walls, 928, 972.
- Waste of water, 649.
- Waste-weir,
  - defined, 1037.
  - for reservoirs, 652.
- Watch, to regulate—, by the stars, 266.
- Water, 326. See also Pipes, Flow, etc.
  - for boilers, 327.
  - boiling—, to measure heights by—, 314.
  - brick work, to render impervious to—, 928.
  - buoyancy, 513.
  - in cement, 932, 936, 938, 941.
  - cisterns, 512, 851–854.
  - column, 852.
  - compensation, 653.
  - composition of—, 326.
  - compressibility, 326.
  - in concrete, 944.
  - concrete under—, 946.
  - consumption of—, 649.
  - corrosion by—, 327, 594.
  - dams for—, 508, 642.
  - discharge. See Discharge.
  - effect on
    - cement, 932, 936, 938, 941.
    - dynamite, 950.
    - iron, 327, 594.
    - lime, 925, 926.
  - effect of zinc on—, 328, 917.
  - evaporation, 329.

- Water—continued.
  - for fire protection, 650.
  - flow of—. See Flow.
  - foot of—, etc. (pressure), equivalents of—, 240.
  - foundations in—, 583.
  - freezing of—, 326, 328.
  - gates, 666.
  - head of—, 258–260, 516.
  - horse-power, 578.
  - jet for pile-driving, 595.
  - leakage, 329, 561, 642, 649, 651.
  - for locomotives, 327, 852.
  - masonry, to render impervious to—, 928.
  - meters, 532, 536, 562, 649.
    - cost and mfrs., 994.
  - motors, mfrs., 990.
  - pipe, prevention of bursting of—, by freezing, 665.
  - in pipes. See Pipes, Velocity, Flow, Discharge, Pressure, etc.
  - pipes, 653, etc., etc.
    - cost of—, 658.
  - power, 578.
  - pressure, 501, etc., 518.
    - in cylinders, 511.
    - in pipes, 511, 518.
    - plank, to resist—, 586, 648.
    - running—, 578.
    - still—, 501, etc.
    - on surfaces, 501, etc.
    - wall to resist—, 508.
  - rain—, 322, 327.
  - ram, 513, 663, 668.
  - resistance to moving bodies, 578.
  - running—, pressure, 578.
  - salt—, effect on metals, 327, 594.
  - scouring action, 577.
  - shed, defined, 1038.
  - size of commercial measures by weight of—, 224.
  - stations, 851.
  - storage of—, 650, etc.
  - supply—, 322, 649.
  - tank, thicknesses, 854.
  - traction on—, 683.
  - in tubes, flow of—, 516.
  - velocity. See Velocity.
  - volumes of unit weight of—, conversion of—, 242.
  - walls, to render impervious to—, 928.
  - walls to resist pressure of—, 508, 515.
  - waste of—, 649.
  - way, contraction of—, 575, 623.
  - weight, 241, 326.
    - in pipes, 525.
    - size of commercial measures by—, 224.
  - wheel, 578.
  - works, 649.
    - depreciation, 45.
- Watt,
  - equivalents of—, 245.
  - -hour, equivalents of—, 237.

Wax—Weed.

Wax, weight, 215.
Way,
  permanent—, 815.
  station, cost, 854.
Wear,
  of cars, 865.
  of locomotives, 864.
  of ties, 815.
  of tools, 801.
Web, Webs,
  members, 689.
  in plate girders, 748.
  plates, permissible shear, 762.
  stresses, 702.
    live load—, 706.
Wedge, Wedges,
  mensuration of—, 203.
  striking—, for centers, 631, 632, 640.
Week, 236, 265.
Weight, Weights. See also article in question, 212.
  of air, 320.
    conversion of unit volumes, 242.
  of beams,
    compared, 478.
    as load, 477.
  of bridges, 731, 738.
  of cement, 939.
  of centers for arches, 639.
  on driving wheels, 705, etc., 755, etc., 856.
  French—, old—, 226.
  in levers, 419.
  and measures, 216.
    conversion tables of units of—, 228.
  metric—, 217, 226, 228, etc.
  of roof covering, 713.
  of roof trusses, 713.
  Russian—, 227.
  of snow, 323.
  Spanish—, 227.
  of steel railroad bridges, 731.
  of substances, table, 212. See also the article in question.
  unit—, conversion of—, 235.
  of water in pipes, 525.
  of wooden bridges, 738.
Weir, 547.
  discharge, formulæ for—, 549.
  measuring—, 547, 646.
  submerged—, 554.
Well, Wells,
  artesian—, 671.
  boring, 670.
  contents, 197.
  machinery, mfrs., 992.
  masonry, quantity in walls, 198.
Wellhouse process, 955.
Western elongation, 284.
Westing, 274.
Wet perimeter, 523, 563.
Wheel, Wheels,
  barrows, earthwork by—, 803, 810.
Wheel, Wheels—continued.
  base, 787, 789, 856, 1038.
  centrifugal force, 355.
  diagram, 706.
  driving—, 856.
    loads on—, 705, etc., 755, etc., 856–859.
  guards, 750.
  loads, 705, etc., 755, etc., 856–859.
  of locomotives, 856.
    loads on—, 705, etc., 755, etc., 856–859.
  meters, 562.
  Persian—, 687.
  and pinions, 420.
  skidding of—, 413.
  tread—, 590, 686.
  tread of—, 821.
  water—, 578.
Wheeled scrapers, 805.
Whipple truss, 694.
White
  effervescence on walls, 929, 936.
  lead paint, 971.
  wash, 973.
Whitworth screw thread, etc., 883.
Winch, 686.
Wind, 321.
  effect on suspension bridges, 766.
  loads, 710.
  mills, 852.
  pressure on roofs, 321.
  on roof trusses, 713.
  stresses, 710, 758.
Wine measure, 223, 224.
Wing,
  dam, defined, 1038.
  defined, 1038.
  of frog, 834.
  walls, 624.
Wire, 887–891.
  circular measurement of—, 889.
  cost and mfrs., 987.
  fence, 854.
  gauges, 887–891.
  iron—, 891.
  rope, 976, 977.
  steel—, 891.
  strength, 920.
Wöhler's law, 465.
Wood.
    See Timber, Wooden, etc.
  board measure, table, 269.
  cohesive strength, 957.
  compressibility, 459.
  compressive strength, 958.
  cost, 984.
  crushing strength, 958.
  ductility, 459.
  effect of lime on—, 926.
  effect of mortar on—, 926.
  elastic limit of—, 459.
  friction, 411.
  limit of elasticity, 459.
  modulus of elasticity, 459.
  pipe, cost, 995.

Wood—continued.
  shearing strength, 499.
  shingles, 971.
  specific gravity, 212.
  strength, 476, 499, 500, 760, 764, 957, 958.
  stretch of—, 459.
  tensile strength, 957.
  torsional strength, 500.
  transverse strength, 476.
  weight, 211.
Wooden. See also Wood.
  beams. See Beams, wooden—, 476.
  bridges, 732, etc.
    specifications for—, 763.
  dams, 642, etc.
  floors in bridges, 750.
  joints, 734.
  pillars, 761, 764, 963.
  pipes, 657.
  roof trusses, 716, 742.
  trestles, 813.
  turntables, 848.
Work, 341.
  equivalence of—, trusses, 718.
  of friction, 418.
  and heat, conversion of units of—, 237.
  of overturning, 422.
  per time [power], conversion of units of—, 244.
  units of—, 341.
  useful—, 342.
Working beam, defined, 1038.
Worm,
  defined, 1038.
  fence, 854.
  sea—, 954.
Worth, present—, 42, 44.
Wrecking car, 808.
Wrought iron. See Iron, wrought—.
  pillars, breaking loads, 910.

Y.

Yard, Yards, 216, 220.
  cubic—,
    of earthwork, 790.
    equivalents of—, 222.
  equivalents of—, 232.
Year,
  civil—, 266.
  equivalents of—, 236.
  sidereal—, 266.
Yield point, 459, 871, 873.

Z.

Z-bars, 901-903.
  flooring, 914.
Zenith, 284.
Zigzag riveting, 774.
Zinc, 916.
  cost, 987.
  effect of cement, mortar, etc., on—, 926, 936.
  effect of— on water, 917.
  effect of water on—, 328.
  expansion of— by heat, 317.
  paint, 971.
  paint on—, 880.
  price, 987.
  roofing, 916.
  sheets, 916.
  strength, 920.
    compressive—, 921.
  weight, 215, 875, 877, 878, 887.
Zone, Zones,
  circular—, 186.
  of circular spindle, 209.
  parabolic—, 192.
  spherical—, 208.
    center of gravity of—, 396.

THE END.

ADVERTISEMENTS.

INDEX TO ADVERTISEMENTS.

PAGE
Association of Engineering Societies, 15
Austin Drainage Excavator, 20
Builders Iron Foundry, 17
Burnham, Williams & Co., 5
Camp, W. M.— 12
Chapman Valve Manufacturing Company, 25
Dexter Portland Cement, 28
East Jersey Pipe Company, 11
Engineering Magazine, 14
Engineering News, 7, 17
Engineering Record, 3
French, Samuel H.—& Co., 28
Heller & Brightly, 8
Hersey Manufacturing Company, 16
Le Clère, Francois, 6, 27
Leschen, A.—& Sons Rope Company, 18
Lippincott, J. B.—Company, 24
Mines and Minerals, 16
Morris, Wheeler & Co., 26
Municipal Engineering and Contracting Company, 4, 20
National Meter Company, 22
National Tube Company, 9
Railroad Gazette, 13
Railway Age, 23
Railway and Engineering Review, 11
Railway Engineering and Maintenance of Way, 10
Raymond Concrete Pile Company, 4
Roebling's Sons Co., John A.— 4
Shelby Steel Tube Company, 9
Suplee, H. H.— 24
Viennot, Jules—Estate of— 7
Wiley, John—& Sons, 19, 20, 21, 26

It is necessary to read

The
Engineering
Record

to become well informed in the latest practice—both standard and special—in all departments of civil engineering.

PUBLISHED WEEKLY

Domestic Subscriptions, $3.00 per Annum
Foreign Subscriptions, $6.00 per Annum

CLUB RATES

Domestic Subscriptions in clubs of two or more, $2.50 each ; at least one of each two subscriptions in the club to be new.

All Subscriptions Payable in Advance.

SAMPLE COPY ON REQUEST

The Engineering Record
114 Liberty Street, New York, U. S. A.

Roebling
Wire Rope,
MADE AT
TRENTON, N. J.
ADDRESS:
John A. Roebling's Sons Co.

The Chicago Sewer Excavator

digs a trench from 14 inches to 60 inches in width and up to 20 feet in depth at a single cut.

It has many records of 500 to 1000 cubic yards excavated in 10 hours

On many kinds of work it will reduce the cost to ONE-TENTH of the EXPENSE of HAND LABOR.

For sale or leased to responsible contractors.

Write for Catalogue No. 131.

MUNICIPAL ENGINEERING
AND CONTRACTING CO.
General Offices
607-611 Railway Exchange, Chicago, U. S. A.
New York Office, 150 Nassau Street

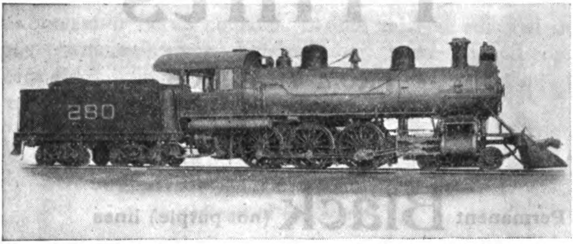
Raymond Concrete Piling
Write us for Estimates
RAYMOND CONCRETE
PILE COMPANY
GENERAL OFFICES
135 ADAMS STREET CHICAGO, ILL.

BALDWIN LOCOMOTIVE WORK

BURNHAM, WILLIAMS & CO.

PHILADELPHIA, PA., U. S. A.

Cable Address, "Baldwin," Philadelphia



MANUFACTURERS OF

LOCOMOTIVES

BOTH SINGLE EXPANSION AND COMPOUND AN
FOR ALL GAUGES OF TRACK

Locomotives particularly adapted for Logging and Industrial Purposes

Locomotives for Mines and Furnaces

Electric Locomotives built in conjunction with the Westinghouse Electric & Manufacturing Co.

Electric Motor and Trailer Trucks for Railway and Suburban Service

Black-Line
Prints

FROM TRACINGS

Permanent **Black** (not purple) lines

on

White (not gray) ground

Enlarged Facilities
Price Reduced

SEND FOR SAMPLE

FRANCIS LE CLÈRE
21 N. Thirteenth St., Philadelphia

1874 **1906**

ENGINEERING NEWS

Has a Larger Paid Circulation than any similar Periodical in the World, without any exception

It circulates in every State of the United States and in nearly 500 different places in 55 foreign countries.

ENGINEERING NEWS contains, besides valuable editorial and descriptive articles, some hundreds of items of proposed Engineering, Building and Contracting Work, averaging in value more than $5,000,000 a week.

Published every Thursday at 220 Broadway, New York.

Subscription Price, $5.00 a Year

to any part of the world

Send 25 cents for copy of the Buyers' Guide (containing 1100 names under 1287 headings) and sample copy of ENGINEERING NEWS.

ESTATE OF JULES VIENNOT

524 Walnut Street, Philadelphia, Pa.

SPECIAL ADVERTISING AGENT FOR

Merchant & Co., Inc., Philadelphia.
Baldwin Locomotive Works,“
J. G. Brill Co., “
Otto Gas Engine Works, “
Latrobe Steel Co., “
Latrobe Steel and Coupler Co.“
Main Belting Co., “
Phosphor Bronze Smelting Co., Ltd., Philadelphia.
C. H. Wheeler Manufacturing Co., Philadelphia.
Newton Machine Tool Works, Philadelphia.
Stow Flexible Shaft Co., Philadelphia.
R. D. Wood & Co., “
Standard Steel Works, “
Samuel H. French & Co., “
Paul S. Reeves & Son, “
Cortright Metal Roofing Co.,“
Thos. H. Dallett Co., “
Abrasive Material Co., “
France Metallic Packing Co.,“
The Green Fuel Economizer Co., Matteawan, N. Y.
Frick Co., Waynesboro, Penna.
Warren Webster & Co., Camden, N.J.
Montross Metal Shingle Co., Camden, N. J.
Keystone Chemical Mfg. Co., Philadelphia.
Wm. Jessop & Sons, Ltd., New York.
U. S. Mineral Wool Co., “
Randolph Brandt, “
Wm. E. Quimby, Inc., “
Peter Pulver & Sons, “
Tight Joint Co., “
Goubert Manufacturing Co., “
Pratt Chuck Co., Frankfort, N. Y.
Harrisburg Pipe & Pipe Bending Co.
Lukens Iron and Steel Co., Coatesville, Pa.
Lobdell Car Wheel Co., Wilmington, Del.
Jas. Leffel & Co., Springfield, Ohio.
Gandy Belting Co., Baltimore.
S. Morgan Smith Co., York, Penna.
Birdsboro Steel Foundry and Machine Co., Birdsboro, Pa.
Shepherd Engineering Co., Franklin, Pa. AND OTHERS,

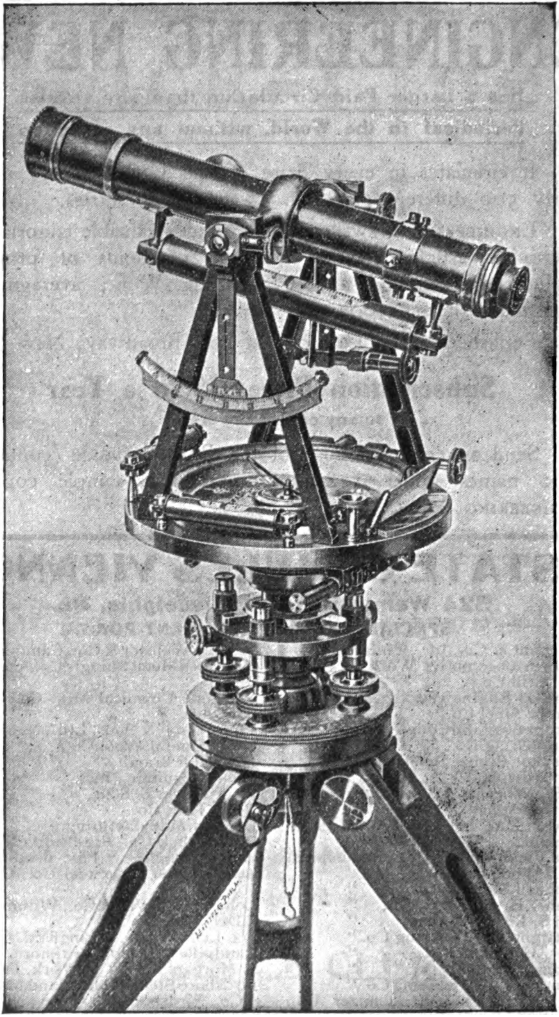
Taking entire charge of all Advertisements, relieving the Advertiser from trouble and annoyance, besides making a great saving in cost.

ADVERTISEMENTS INSERTED IN ALL THE LEADING INDUSTRIAL PAPERS.

(INSTRUMENTS OF PRECISION.)

HELLER & BRIGHTLY,

SURVEYING, OPTICAL, AND ASTRONOMICAL INSTRUMENT MAKERS.



CORNER SPRING GARDEN AND RIDGE AVENUE,
PHILADELPHIA, PENN'A.

Messrs. HELLER & BRIGHTLY publish a book containing tables and maps useful to Civil Engineers and Surveyors; also contains much valuable information respecting the selection, proper care, and use of field instruments. A copy of this book they send by mail, post-paid, to any Civil Engineer or Surveyor in any part of the world, on receipt of a postal card containing the name and post-office address of the applicant.

NATIONAL TUBE COMPANY

MANUFACTURERS OF

Lap-Welded Pipe and Tubes

STEAM, GAS AND WATER

Oil Well Tubing, Casing, Line Pipe
and Drive Pipe

Locomotive Boiler Tubes

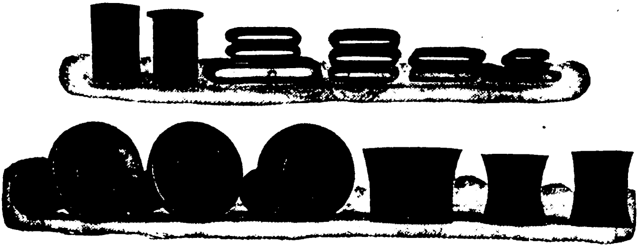
Converse Joint and Matheson Joint Pipe

GENERAL OFFICES
Frick Building, Pittsburg, Pa.

SHELBY STEEL TUBE CO.

MANUFACTURERS OF

Cold Drawn Seamless Boiler Tubes



GENERAL OFFICE
Frick Building, Pittsburg, Pa.

BRANCH OFFICES
New York—Battery Park Bldg. Chicago—The Rookery

RAILWAY ENGINEERING

and Maintenance of Way

Published by the

BRUCE V. CRANDALL COMPANY

OFFICE OF PUBLICATION
Rooms 409-410 Security Building, Corner
Madison Street and Fifth Avenue,
Chicago

EASTERN OFFICE
Room 714, 132 Nassau Street,
New York City

A Monthly Railway Journal

Devoted to the interests
of railway engineering,
maintenance of way,
bridges and buildings.

BRUCE V. CRANDALL
President

CHARLES S. MYERS
Vice-President

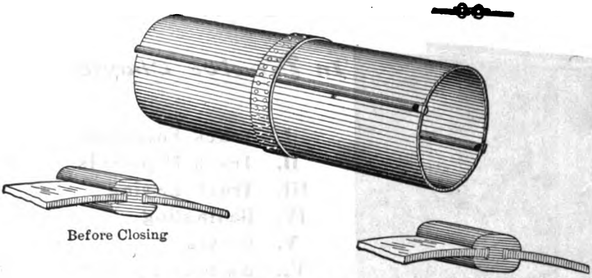
O. W. BODLER, Secretary

O. H. REYNOLDS
Editor

WARREN EDWARDS
Manager

Lock-Bar Pipe, Riveted Pipe, Boilers, Tanks, Caissons, Air Locks, Mud and Steam Drums, High Pressure Cylinders

THE ONE HUNDRED PER CENT. JOINT



LOCK-BAR STEEL PIPE

THE EAST JERSEY PIPE COMPANY

OFFICES
71 Broadway, New York

WORKS
Paterson, N. J.

THE RAILWAY AND ENGINEERING REVIEW

Guaranteed Largest Paid Circulation of any Weekly Railway Paper in the World

Devoted to the Interests of Railway Transportation in all its Phases; Civil, Mechanical, and Electrical Engineering; Political and Social Economy; Traffic and News. A practical paper for practical railway men and all interested, actively or prospectively, in railway or engineering work. Technical, sound, reliable, independent, and readable. Richly illustrated.

SUBSCRIPTION PRICE $4.00 per year in the United States, Canada, and Mexico; **$5.00** per year, postpaid, to other countries.

ADVERTISING RATES will be furnished on application.

THE

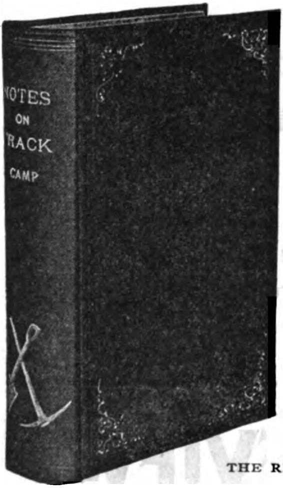
RAILWAY AND ENGINEERING REVIEW

315 Dearborn Street, CHICAGO

NOTES ON TRACK

By W. M. CAMP, M. Am. Soc. C. E.

AN EXHAUSTIVE TREATMENT OF TRACK CONSTRUCTION AND MAINTENANCE, FROM A PRACTICAL STANDPOINT



In Twelve Chapters:

I. **Track Foundation**
II. **Track Materials**
III. **Track Laying**
IV. **Ballasting**
V. **Curves**
VI. **Switching Arrangements and Appliances**
VII. **Track Maintenance**
VIII. **Double-Tracking**
IX. **Track Tools**
X. **Work Trains**
XI. **Miscellaneous**
XII. **Organization**

THE REVISED EDITION CONTAINS

1223 PAGES AND 637 ILLUSTRATIONS

Every phase of the subject is treated. Particular attention has been paid to **Cost of Track Work and to Modern Labor-Saving Machinery in Track Service.** The book also covers in much detail many allied subjects, such as Earthwork and Grading, Culverts, Yard Layouts and Switching Movements, Interlocking Switches and Signals, Automatic Electric Block Signals and Track Circuits, Principles of Rail Design, Fence, Cattle Guards, Section Houses and Tool Houses, Boarding Trains, Bridge Floors, Ash Pits, Handling Ballast and Filling Material, Wrecking, Fighting Snow, Tie Preservation, Metal and Concrete Ties, Track Elevation and Depression, Easement Curves, Crossing Gates, Tree Planting, Capacity of Single Track, Etc.

> "This is the most complete and comprehensive book ever written on the subject of track. The author is a master of his subject."—*American Engineer and R. R. Journal.*

Sent to any Part of the World, Postpaid, for $4.00

WRITE FOR FULL TABLE OF CONTENTS
AND ILLUSTRATED CIRCULAR

W. M. CAMP, Author and Publisher

AUBURN PARK, CHICAGO

THE
RAILROAD
GAZETTE

Contains each week the best and latest information on all branches of RAILROAD ENGINEERING—the best engravings, news, and discussions. Every one who desires to keep informed on this branch of professional work should read it. Sample copy free. Subscription price to United States, Canada, and Mexico, $5.00 a year; elsewhere, $8.00.

We publish RAILROAD BOOKS. Send for catalogue.

83 FULTON STREET
NEW YORK, N. Y.

OLD COLONY BUILDING
CHICAGO, ILL.

QUEEN ANNE'S CHAMBERS
LONDON, ENGLAND

"Every Engineer should make his own pocket-book"—NYSTROM

THE "pocket-book" and treatise gather the principal fundamental materials, but current practice and new commercial methods—the world in which the engineer must work—are found only in the periodical engineering press and the "transactions" of engineering societies.

"The Engineering Magazine" contains monthly a full descriptive index to the current contents of nearly two hundred of the leading engineering periodicals of the world, and provides a means of procuring, at a minimum cost of time and money, the *original text and drawings* of any article on any engineering topic published in any current engineering journal.

It is plain that such an index is of the highest value in collecting and filing engineering data. Ask for a specimen copy of the Magazine and special circular on the use of the card index for filing engineering data. State what branch of engineering you are directly interested in, so that we may point out the special value of the Index to you.

The Engineering Magazine

140 - 142 NASSAU STREET, NEW YORK

THE ASSOCIATION OF ENGINEERING SOCIETIES

ORGANIZED IN 1881, NOW COMPRISES ELEVEN SOCIETIES, AS FOLLOWS:

Boston Society of Civil Engineers,
Civil Engineers' Club of Cleveland,
Engineers' Club of St. Louis,
Civil Engineers' Society of St. Paul,
Engineers' Club of Minneapolis,
Montana Society of Civil Engineers,
Technical Society of the Pacific Coast,
Detroit Engineering Society,
Engineers' Society of Western New York,
Louisiana Engineering Society,
Toledo Society of Engineers.

In the map, each Society is represented by one dot for each one hundred members or fraction thereof over fifty.

THE JOURNAL OF THE ASSOCIATION

issued monthly, contains the proceedings and selected papers of these **Eleven Engineering Societies.**

Engineering News, in its editorial of November 8, 1894, says: "Among the bound volumes of the proceedings of engineering societies upon our shelves, we know of none which contains in a smaller space a greater amount of valuable matter than the dozen volumes thus far issued of *The Journal of the Association of Engineering Societies.*"

THE SUBSCRIPTION PRICE

of THE JOURNAL is $3.00 per annum. Single copies may be had at 30 cts. each.

DESCRIPTIVE INDEX OF
CURRENT ENGINEERING LITERATURE
From 1884 to 1900, inclusive

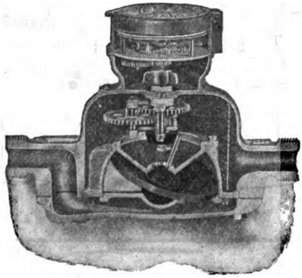
BOUND IN CLOTH

Volume I, 1884-1891, price, $5.00 Volume II, 1892-1895, price, $5.00
Volume III, 1896-1900, price, $7.50

FRED. BROOKS, Secretary
31 MILK STREET, BOSTON, MASS.

HERSEY WATER METERS

DISC
ROTARY
AND
TORRENT



ALSO THE
DETECTOR
METER
FOR FIRE
SERVICES

We are one of the Oldest and Largest Meter Manufacturers in the World, having made and sold over 260,000 in the last twenty years.

HERSEY MFG. CO., So. Boston, Mass.

ESTABLISHED 1881

"MINES AND MINERALS"

S THE MOST PROGRESSIVE AND MOST WIDELY CIRCULATED MINING JOURNAL PUBLISHED.

IT IS CONCEDED TO BE

THE ABLEST AND MOST UP-TO-DATE EXPONENT OF

American Mining and Metallurgical Methods

It is devoted exclusively to Mining and Metallurgy.
It neither booms nor advertises mining stock-jobbing schemes.

MERIT HAS WON FOR IT AN ACTUAL CIRCULATION OF OVER 11,000 COPIES PER ISSUE.

ubscription (U. S., Canada, Mexico, Hawaii, Philippines, and Porto Rico), ... $2.00 per Year
oreign Countries (except Canada and Mexico), ... 3.00 " "

SEND FOR FREE SAMPLE COPY

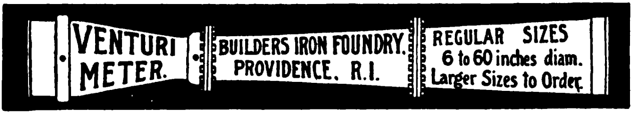
"MINES AND MINERALS," Scranton, Pa., U. S. A.

THE VENTURI METER

is the only practical meter for measuring large quantities of water

ITS ACCURACY is unquestioned, and it is frequently employed to test the accuracy of pumps.

NO BY-PASS is necessary, and the cost of tees, elbows and gate-valve is avoided.


VENTURI METER.
BUILDERS IRON FOUNDRY. PROVIDENCE. R.I.
REGULAR SIZES
6 to 60 inches diam.
Larger Sizes to Order.

One city has two Venturis on 60-inch mains and fifty-three Venturis on mains from 8 to 48 inches in diameter.

Pages 532 to 535 of this book describe the scientific principles upon which these meters are based. Pamphlets fully describing the construction of the meters mailed upon application.

Employment of Experienced Technical Help

Securing the services of only the **best men** is recognized throughout the world as the only means of keeping in the front rank in the business world. No profession or business requires a more strict adherence to this rule than that of engineering. Men of tried experience want to work for firms who offer better opportunities and responsibilities. Those engaging, or ever intending to engage, the very best technical help should not fail to write our "Situation Wanted" advertisers, and, if time allows, to advertise for further names in our "Situation Open" columns.

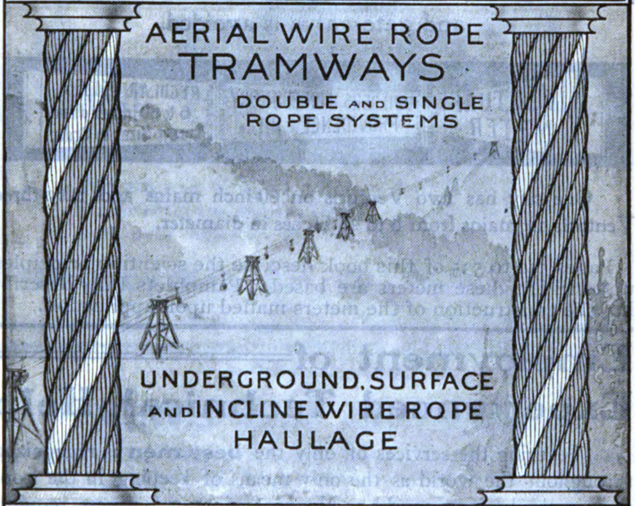
Rate for "Situation Open" advertisements, 3 cents a word, set solid.

Subscription, $5.00 a year; $2.50 six months.

ENGINEERING NEWS

220 Broadway, New York

A. LESCHEN & SONS ROPE Co.
ST. LOUIS, MO.
NEW YORK, DENVER, SEATTLE, CHICAGO.



AERIAL WIRE ROPE
TRAMWAYS
DOUBLE AND SINGLE
ROPE SYSTEMS

UNDERGROUND, SURFACE
AND INCLINE WIRE ROPE
HAULAGE

SOLE MANUFACTURERS
PATENT FLATTENED STRAND
AND
"HERCULES"
(TRADE MARK REGISTERED)
WIRE ROPE

ALSO WIRE ROPE OF EVERY DESCRIPTION
ALL GENUINE HERCULES WIRE ROPE HAS ONE COLORED STRAND

NOW READY

Wiley's 1906 Catalogue of

TEXT-BOOKS AND INDUSTRIAL WORKS

For Schools, Colleges, Polytechnic Institutes, Engineers, Architects, Chemists, Military and Naval Officers, etc.

Agriculture
Architecture
Army and Navy
Assaying
Astronomy
Botany
Chemistry
Civil Engineering
  Bridges and Roofs
Civil Engineering
  Hydraulics
  Materials of Engineering
  Railway Engineering
Drawing
Electricity and Physics
Law
Manufactures
Mathematics
Mechanical Engineering
  Materials of Engineering
  Steam Engines, Boilers
Mechanics and Machinery
Metallurgy
Mineralogy
Mining
Sanitary Science
Miscellaneous

This Catalogue will be sent free by mail on request

The following sections are to be had separately:

Assaying, Metallurgy, Mineralogy and Mining, Chemistry, Civil Engineering including Bridges and Roofs, Hydraulics, Materials of Engineering, Railway Engineering, Drawing including Kinematics, Machine Designs.

JOHN WILEY & SONS
43-45 East 19th St., New York
London: Chapman & Hall, Limited
1906

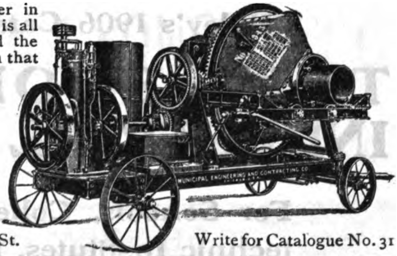
THE CHICAGO IMPROVED CUBE CONCRETE MIXER

holds the entire batch together in mixing in a single mass; it is all thoroughly mixed together and the form of the mixing drum is such that there is no angle of repose for the contents.

Sizes and mountings for every requirement.

Municipal Engineering and Contracting Co.

General Offices
607-611 Railway Exchange
Chicago, U. S. A.
New York Office, 150 Nassau St.


MUNICIPAL ENGINEERING AND CONTRACTING CO.

Write for Catalogue No. 31

RECLAIM THE SWAMPS AND ARID LANDS

Worthless lands are now made the garden spots of the country at small expense with the

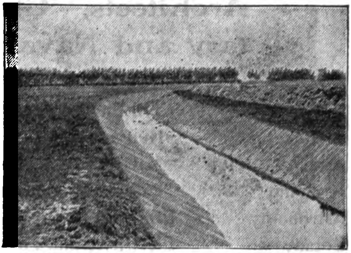
AUSTIN DRAINAGE EXCAVATOR

It works on an entirely new and economical principle.

Sizes for both ditching and levee building.

Write for Catalogue No. K.

General Offices
607-611 Railway Ex., Chicago. Ill.
New York Office, 155 Nassau St.



KUTTER'S FORMULA.

THE ONLY TRANSLATION OF

GANGUILLET AND KUTTER'S

"Versuch zur Aufstellung einer neuen allgemeinen Formel für die gleichförmige Bewegung des Wassers in Canälen und Flüssen,"

which is the only work treating fully of **KUTTER'S FORMULA**, and explaining its derivation.

By RUDOLPH HERING,
M. Am. Soc. C. E., M. Inst. C. E.,
AND
JOHN C. TRAUTWINE, Jr.,
Assoc. Am. Soc. C. E., Assoc. Inst. C. E.,

with a Preface by the authors, many original Appendices and Illustrations, and an entirely new and very comprehensive Table of Experiments upon flow in channels and in pipes

SECOND EDITION.

240 Pages, with Eight Folding Plates, and many other Illustrations, $4.00.

JOHN WILEY & SONS, Scientific Publishers,
43 AND 45 E. NINETEENTH STREET, NEW YORK.

ENGINEERING WORKS

OF

JOHN C. TRAUTWINE

CIVIL ENGINEER

The Civil Engineer's Pocket Book

12mo, 832 pages. Illustrated. Morocco, Gilt Edge, Eighteenth Edition, Eighty-fourth Thousand, 1906, . . $5.00

The Field Practice of Laying Out Circular Curves for Railroads

Thirteenth Edition, 1903

12mo, 192 pages. Illustrated. Morocco, Gilt Edge, . $2.50

A Method of Calculating the Contents of Excavations and Embankments

8vo, 71 pages. Illustrated. Cloth, Eleventh Edition, 1903, $2.00

JOHN WILEY & SONS, Scientific Publishers

43 and 45 E. Nineteenth Street

NEW YORK

THE LARGEST WATER METER
MANUFACTURERS IN THE WORLD

CROWN, EMPIRE,
NASH, GEM, AND
PREMIER

WATER METERS

National Meter Company

JOHN C. KELLEY, President

NEW YORK BOSTON
CHICAGO LONDON

THE Railway Age

CHICAGO

A Journal of

Railway Transportation

for

Railway Managers
Railway Engineers
Manufacturers and
Dealers in Railway
Equipment and Supplies

Published Every Friday
Subscription Price
$4.00 Domestic
$6.00 Foreign

THE Railway Age

CHICAGO

SUPLEE'S
MECHANICAL ENGINEER'S REFERENCE BOOK

Second Edition

A HAND-BOOK OF

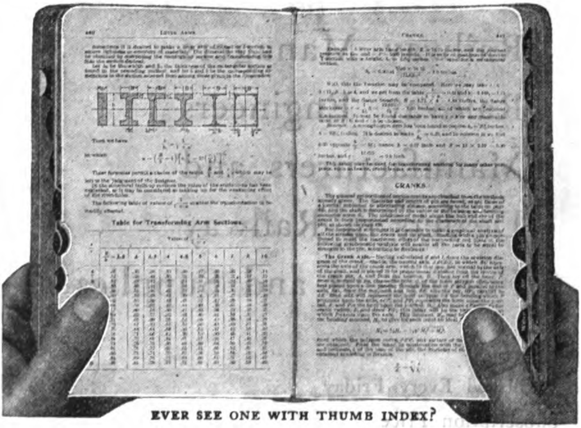
TABLES, FORMULAS, AND METHODS

FOR ENGINEERS, STUDENTS, AND DRAFTSMEN

BY

HENRY HARRISON SUPLEE, B.Sc., M.E.

MEMBER AMERICAN SOCIETY OF MECHANICAL ENGINEERS
MEMBRE DE LA SOCIÉTÉ DES INGÉNIEURS CIVILS DE FRANCE
MITGLIED DES VEREINES DEUTSCHER INGENIEURE
MEMBER OF THE FRANKLIN INSTITUTE



EVER SEE ONE WITH THUMB INDEX?

$5.00 NET. 50c ADDITIONAL, PATENT THUMB INDEX.

3 POINTS TO REMEMBER

I. The latest on the market.
The arrangement of the matter unexcelled and amply illustrated.

II. That Mathematics has more space than in any other similar reference book in English.

III. The subject of Machine Design is amply treated.

J. B. LIPPINCOTT COMPANY
PHILADELPHIA

CHAPMAN VALVE MFG. COMPANY

MANUFACTURERS OF

VALVES AND SLUICE GATES

Are prepared to furnish reliable information and estimates regarding Valves for Steam Power Plants, Traction and Lighting Power Stations, Steam Ships, Plumbing and Steam Heating, Water Works and Fire Protection Systems, Refrigerating Plants, Compressed Air and Hydraulic Systems, Blast Furnaces and Steel Mills, Gas Works.

ELECTRICALLY OPERATED VALVES.

FIRE HYDRANTS.

Designed upon engineering principles and accurately constructed of the highest grade materials, especially adapted to fluids to be controlled.

GENERAL OFFICE AND WORKS

INDIAN ORCHARD, MASS., U. S. A.

BRANCHES

BOSTON,	94 Pearl Street
NEW YORK,	28 Platt Street
PHILADELPHIA,	18 N. Seventh Street
CHICAGO,	1628 Monadnock Block
CLEVELAND,	97 Superior Street
ST. LOUIS,	1610 Chemical Building
SAN FRANCISCO,	63 First Street
PITTSBURGH,	914 Farmers Bank Building

CATALOGUE ON APPLICATION

MORRIS, WHEELER & CO.

1608 Market Street, PHILADELPHIA
39 Cortlandt Street, NEW YORK

BARS, BEAMS, CHANNELS AND SHAPES

Sheared and Universal Mill Plates
of Iron and Steel
For Ships, Boilers and Bridges

IRON AND STEEL OF ALL DESCRIPTIONS
IRON AND STEEL BOILER TUBES

Large and Well-assorted Stock always in Warehouse

BAZIN'S WEIR EXPERIMENTS.

Recent Experiments on the Flow of Water over Weirs, etc.

By M. BAZIN,
Inspector-General des Ponts et Chaussées.

TRANSLATED FROM ANNALES DES PONTS ET CHAUSSÉES,

By ARTHUR MARICHAL and JOHN C. TRAUTWINE, Jr.

PUBLISHED IN THE
Proceedings of the Engineers' Club of Philadelphia.

EXPERIMENTS
UPON
CONTRACTION OF THE LIQUID VEIN ISSUING FROM AN ORIFICE,
AND UPON THE
Distribution of the Velocities Within It.

By H. BAZIN,
Inspecteur Général des Ponts et Chaussées.

Translated by JOHN C. TRAUTWINE, Jr., C. E.

JOHN WILEY & SONS, Scientific Publishers,
43 AND 45 E. NINETEENTH ST., NEW YORK.

Black-Line Prints

FROM TRACINGS

Permanent **Black** (not purple) lines

on

White (not gray) ground

Enlarged Facilities
Price Reduced

SEND FOR SAMPLE

FRANCIS LE CLÈRE
21 N. Thirteenth St., Philadelphia

DEXTER

PORTLAND
CEMENT

The Highest Standard Attainable

SOLE AGENTS

SAMUEL H. FRENCH & CO.

York Ave., Fourth and Callowhill Sts.

Established 1844
PHILADELPHIA

R

D

T

able

).

I A

$ m. alti
ESw S

THIS BOOK IS DUE ON THE LAST DATE STAMPED BELOW

AN INITIAL FINE OF 25 CENTS

WILL BE ASSESSED FOR FAILURE TO RETURN THIS BOOK ON THE DATE DUE. THE PENALTY WILL INCREASE TO 50 CENTS ON THE FOURTH DAY AND TO $1.00 ON THE SEVENTH DAY OVERDUE.

SEP 21 1934
OCT 11 1935
SEP 30 1937
SEP 28 1938
SEP 30 1940
NOV 18 1942
AUG 12 1943
20Dec'48RF

LD 21-100*m*-7,'33

YA 02262

TA151
T7
1906

501816

UNIVERSITY OF CALIFORNIA LIBRARY